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TOPOLOGY OPTIMIZATION OF OPTICAL BAND GAP EFFECTS IN SLAB STRUCTURES MODULATED BY PERIODIC RAYLEIGH WAVES

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Summary This paper is concerned with topology optimization of a coupled optical and mechanical wave propagation problem in photonic crystals. It is motivated by the potential gain in functionality of optical devices where mechanical Rayleigh waves (travelling in the surface of the material) play a leading role. The practical applications cover novel optical modulators, switches and frequency shifters. The work uses COMSOL Multiphysics which is a modern and finite element based modelling language.

INTRODUCTION

Topology optimization is a computational method that can be used to optimize the response of continuum structures subject to given loads and constraints [1]. It has proven a very successful design tool in mechanical problems in general (see, e.g., [1]) and also in photonic crystal problems (see, e.g., [2]). This work contributes with knowledge on how to apply topology optimization to the coupled problem of a photonic crystal subjected to mechanical surface waves.

The unique feature of a crystal is that a band gap can form – meaning that eigenstates within a limited band of frequencies do not exist, see, e.g., [3] for examples from optics. A requirement for the existence is that a material contrast is present in the structure, e.g., for optical as well as mechanical waves, one can create a band gap in 1D by a Bragg grating and in 2D via an arrangement of circular inclusions. Computationally a unit cell analysis is often employed, motivated by the periodicity of the crystal. Typically the relevant material parameter (e.g. the dielectric permittivity in optics and the Youngs modulus in solid mechanics) is taken to be cell periodic whereas the state variables are represented using a time-harmonic Floquet-Bloch solution, see, e.g., [4].

MODELLING

Optical model

In this contribution we focus exclusively on the special case of band gaps between *guided optical modes* in a photonic crystal slab being a structure which is in-plane periodic with a finite out-of-plane height, see figure 1(a). A guided optical mode is simply a localized eigenstate in the slab analogous to total internal reflection of light in ray optics. The modelling of the guided optical modes in the slab relies on two subtle arguments as explained in [5]. When computing the band diagram, the guided modes appear under the light line l being defined as $l = |k_w|/2\pi$ where $|k_w|$ is the length of the wave vector, see figure 1(d). Secondly, to avoid the computation with a large volume of surrounding air and non-reflecting boundary conditions, we use the trick of periodicity in the z-direction. This is acceptable since our focus is the guided modes which have a rapidly decaying amplitude outside the slab where the z-periodicity changes the response significantly. Furthermore we assume linear constitutive relations and non-magnetic materials which allow us to compute the optical state by a 3D eigenvalue problem formulation of Maxwell's equations implemented using Nédélec elements. By evaluation of the energy in the $x-$, $y-$ and $z-$direction a procedure is developed which enables a classification of a 3D mode as being either transversely polarized (TM-like or TE-like) or as a mixed mode without a simple polarization. This is useful for comparing topology designs with results from the literature.

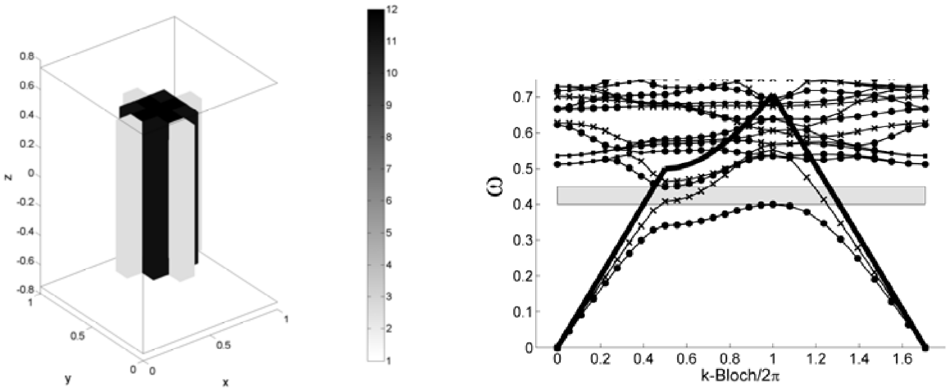
Mechanical model

The Rayleigh wave is represented as a time-harmonic Floquet-Bloch solution and we consider the (typical) case where the Rayleigh wave is unaffected by the optical wave such that there is a one-way coupling in the problem. The wave length of the Rayleigh wave is approximately one order of magnitude larger than the size of the periodic cell which justifies the assumption that the geometric features of the perforated layer do not change the Rayleigh wave. Thus this model does not cover the full geometric coupling.

The opto-mechanical interaction is modelled geometrically based on the amplitude field of the Rayleigh wave. In this way the optical state is computed using the eigenvalue formulation but now with a geometrically distorted unit cell.

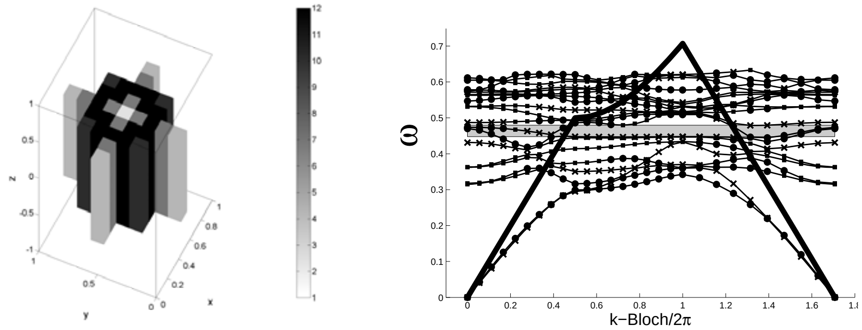
Optimization problem

We apply the gradient driven topology optimization procedure described in [6] and use the dielectric permittivity as the design variable. Rather than the double bound formulation known from the literature [1] this work uses an active set strategy to select a small set of critical eigenvalues on which the mathematical optimization problem is based.



(a) View of the unit cell with a topology optimized design. The design is restricted to have rectangular symmetry and the design domain is $-0.5 \leq z \leq 0.5$. Outside the design domain the slab is suspended in air. The Colorbar: the dielectric permittivity.

(b) Band diagram for figure 1(a). Grey box relative bandgap beteween TM-like modes of 11.5%. The fat line is the light line. Mode symbols: ’•’ TM-like, ’×’ TE-like and ’■’ mixed.



(c) View of the unit cell with a topology optimized design (which is hollow). The design is restricted to have rectangular symmetry and the design domain is $-0.5 \leq z \leq 0.5$. Outside the design domain the slab is suspended in air. The Colorbar: the dielectric permittivity.

(d) Band diagram for figure 1(c). Grey box relative bandgap between guided modes below the light line of 6.9%. The fat line is the light line. Mode symbols: ’•’ TM-like, ’×’ TE-like and ’■’ mixed.

Figure 1. Topology design and bandgaps for the optical problem. The relative bandgap measure used is $2(\omega^{i+1} - \omega^i)/(\omega^{i+1} + \omega^i)$.

PRELIMINARY RESULTS

Topology optimization of photonic crystal slabs is introduced by comparing the band gaps of topology optimized slabs to the literature [5]. As seen from figure 1(a) this allows for an increase of known band gaps between TM-like modes. Furthermore, figure 1(c) shows a slab structure which displays a complete bandgap. The results of using topology optimization to design crystal slabs with a 3D model are novel, unpublished and form a necessary prerequisite for solving the coupled problem.

It is natural to extend this study with the slight variation where the photonic slab is not suspended in air, but mechanically supported on an optical insulator. Having established a well-posed mechanical problem, the final goal of this work is to apply topology optimization to the coupled problem where a periodic model of a Rayleigh wave is introduced in the optical slab model as described above.

A small part of this talk has its focus on the implementation in COMSOL Multiphysics which is a finite element based modelling software which is sufficiently flexible to provide analytical sensitivities. Moreover, computational performance results such as efficiency of the 3D eigenvalue solver and parallel scalability are discussed. These issues are critically important since they limit the resolution in the topology design.

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