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Load flow analysis considering wind turbine generator power uncertainties

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Abstract—In this paper the effect of wind turbine generator (WG) output uncertainties on the distribution system node voltages has been investigated. For this purpose a new approach viz., interval method of analysis has been proposed. In this method, the wind speed variations are represented as intervals and the WG output uncertainties are computed. Further, an interval load flow method has been proposed to compute the effect of this WG output uncertainties on the distribution system node voltages. The performance of the proposed interval method has been compared with the Monte Carlo method.

Index Terms—Distributed generation, Load flow analysis, Uncertainty, Wind turbine

NOMENCLATURE

$[\underline{a}, \bar{a}]$: Indicates that a is an interval and \underline{a} is the minimum value/lower bound, \bar{a} is the maximum value/upper bound.

P_{WG} : Active power output of WG

Q_{WG} : Reactive power of WG

u_w : Wind speed

V : Voltage magnitude

$u_{w_{nominal}}$: Nominal wind speed (Minimum wind speed at which the power output of the wind turbine is equal to its rated value)

P_p : Active power at bus p (subscripts p, q have been used to indicate the bus number)

Q_p : Reactive power at bus p

V_p : Voltage magnitude at bus p

θ_p : Voltage angle at bus p

$Y_{pq} = G_{pq} + jB_{pq}$, G_{pq} and B_{pq} are the real and imaginary parts of the pq^{th} element of the admittance matrix.

$V_{p0}, \theta_{p0}, P_{p0}, Q_{p0}$: Mid point of voltage magnitude, voltage angle, active and reactive power intervals at bus p . These are deterministic scalar values and calculated as the average of their respective interval upper and lower bound values.

R_1 : Induction generator stator resistance

X_{l1} : Induction generator stator leakage reactance

R_2 : Induction generator rotor resistance

X_{l2} : Induction generator rotor leakage reactance

X_m : Induction generator magnetizing reactance

X_c : Reactance of shunt capacitor compensation

N : Number of poles (Induction generator)

$u_{w_{CI}}$: Cut-in wind speed (Lowest wind speed at which the WG starts generating power)

$u_{w_{CO}}$: Cut-out wind speed (Wind speed at beyond the WG is shutdown)

I. INTRODUCTION

In the last decade generation of electricity from wind energy has gained world wide attention. At present large number of wind turbine generating units (WG) are being connected to the distribution systems. Due to intermittency in the WG output, the connection of these sources could effect the voltages, power flows and losses of the distribution systems. Quantification of these effects is important in determining the maximum allowable wind power penetration in a distribution network.

In an attempt to quantify the effects of WG output uncertainties on the distribution system node voltages the use of probabilistic load flow methods has been suggested in [1], [2] while in [3] a sequence of deterministic load flow simulations have been carried out.

In the context of load flow analysis considering uncertainties, the probabilistic approach has been suggested by many investigators. The probabilistic methods used in [1], [2], [4]–[10], requires probability distribution function(PDF) of powers at each node, which is difficult to obtain [2]. Further, in most of these investigations [1], [5], [9], [10] the linearized load flow equations are used. In some of the investigations [2], [7], [8], where in the linearity assumption is not made, sequence of deterministic load flow solutions are used to compute the node voltage statistics (mean/standard deviation/PDF). However, the node voltage statistics obtained using these methods depends either on the quantization step of the nodal power PDF [2], [7] or on the manner in which the node voltage ranges are chosen [2], [8]. Further, the methods suggested in [2], [7], [8] require considerable computational effort.

In this paper, a new method of analyzing the effect of WG power output uncertainties on the power system network steady state voltage has been proposed. This method is based on interval method of analysis, wherein the uncertainties are represented by intervals. Since this method requires the uncertainty in the inputs to be specified as interval range (WG active and reactive powers), first a method has been presented here to calculate interval range for the power output of the WG. In order to determine the upper and lower bound (interval range) for the voltage at each node of the network an interval load flow algorithm has been suggested here. Simulation studies have been presented to demonstrate the effect of WG output uncertainties on the distribution system node voltages. Further, the results obtained using the proposed interval method have been compared with the one obtained using the Monte Carlo method suggested in [8].

II. INTERVAL MODEL FOR WG POWER OUTPUT

For a given/obtained wind speed range the WG power output range is computed by modifying the steady state WG models developed in [11], [12]. In [11] a steady state model has been developed for each type of WG i.e., stall regulated fixed speed, pitch regulated fixed speed, semi-variable and variable speed WG. However, these steady state models can not be directly used here since it requires a deterministic value for wind speed, voltage and computes a (deterministic) value for the WG power output. In order to quantify the WG power output uncertainty by intervals, some modifications have been proposed here to the steady state WG models suggested in [11]. In particular, the method suggested here is based on the sensitivity of the power output of different types of WG (fixed-stall and pitch regulated, semi variable and variable speed) to the wind speed and voltage. In the following sections, the procedure for calculating the active and reactive power intervals of each of the four types of WG has been discussed.

A. Determination of interval range for a given wind speed variation

The wind speed range is generally obtained from the wind speed measurement data at a given WG site. The wind speed measurements or recording (at a wind site) could correspond to either instantaneous or the average values of wind speed. If average wind speed data is available then it would not contain the high frequency components (few milliseconds) of wind speed variation. Hence, the wind speed range can be obtained directly i.e., maximum and minimum values of the average wind speed data. In cases where the instantaneous values of wind speed data is available it is necessary to eliminate the high frequency components. This is because the high frequency wind speed variations may not affect the WG power output [13]. Hence, the high frequency components present in the wind speed data is first filtered out using a low pass filter (the method is illustrated later in simulation studies) and from this the wind speed range is obtained.

B. Stall regulated fixed speed WG

From the study carried out in [11] it is evident that the active power output increases with wind speed up to the nominal wind speed. However, beyond nominal wind speed, increase in wind speed decreases the power output of the WG. Further, it has been shown that voltage variation has little impact on the active power output of the stall regulated fixed speed WG and hence can be neglected.

The reactive power demand varies with voltage as well as wind speed and it increases with increase in wind speed (up to nominal) and beyond nominal it decreases. However, with increase in voltage the reactive power demand decreases and vice-versa.

For the given wind speed interval, the maximum as well as minimum values of the power output are computed using the WG models developed in [11] and the following procedure:

Method of computing stall regulated fixed speed WG power output range ($[P_{WG}, \overline{P_{WG}}]$ and $[Q_{WG}, \overline{Q_{WG}}]$)

Given intervals for wind speed $[u_w, \overline{u_w}]$ and voltage $[V, \overline{V}]$
If $u_{w,nominal} \in [u_w, \overline{u_w}]$ then

- a) Minimum value of the active and reactive power ($\underline{P_{WG}}, \underline{Q_{WG}}$):

Compute the active and reactive power using the method suggested in [11] for the following cases:

- i) Minimum wind speed and maximum voltage
- ii) Maximum wind speed and minimum voltage

Minimum value of active and reactive powers are

$$\underline{P_{WG}} = \min(P_{WG}(u_w, \overline{V}), P_{WG}(\overline{u_w}, V)) \quad (1)$$

$$\underline{Q_{WG}} = \min(Q_{WG}(u_w, \overline{V}), Q_{WG}(\overline{u_w}, V)) \quad (2)$$

- b) Maximum value of active and reactive power ($\overline{P_{WG}}, \overline{Q_{WG}}$):

Compute the active and reactive power, for nominal wind speed and minimum voltage, i.e.,

$$\overline{P_{WG}} = P_{WG}(u_{w,nominal}, V) \quad (3)$$

$$\overline{Q_{WG}} = Q_{WG}(u_{w,nominal}, V) \quad (4)$$

Else

- a) Minimum value of active and reactive power ($\underline{P_{WG}}, \underline{Q_{WG}}$):

Compute the active and reactive power, for minimum wind speed and maximum voltage, viz.,

$$\underline{P_{WG}} = P_{WG}(u_w, \overline{V}) \quad (5)$$

$$\underline{Q_{WG}} = Q_{WG}(u_w, \overline{V}) \quad (6)$$

- b) Maximum value of active and reactive power ($\overline{P_{WG}}, \overline{Q_{WG}}$):

Compute the active and reactive power, for maximum wind speed and minimum voltage, viz.,

$$\overline{P_{WG}} = P_{WG}(\overline{u_w}, V) \quad (7)$$

$$\overline{Q_{WG}} = Q_{WG}(\overline{u_w}, V) \quad (8)$$

C. Pitch regulated fixed speed and Semi-variable speed WG

The active and reactive power of these two types of WG increases with increase in wind speed up to nominal wind speed. For wind speeds greater than nominal, the active power output of this WG remains constant. Further, the active power output does not vary appreciably with voltage. However, the variation of reactive power with voltage needs to be considered. The reactive power demand increases with decrease in voltage and vice-versa.

The procedure used to compute the active and reactive power output range for this WG has been given below:

Method of computing pitch regulated fixed speed WG power output interval ($[P_{WG}, \overline{P_{WG}}]$ and $[Q_{WG}, \overline{Q_{WG}}]$)

Given wind speed $[u_w, \overline{u_w}]$ and voltage $[V, \overline{V}]$ intervals

If $u_{w,nominal} \in [u_w, \overline{u_w}]$

- a) Minimum value of the active and reactive power ($\underline{P_{WG}}, \underline{Q_{WG}}$):

Compute the active and reactive power, for minimum

wind speed and maximum voltage, using the method suggested in [11] i.e.,

$$\underline{P}_{WG} = P_{WG}(\underline{u}_w, \overline{V}) \quad (9)$$

$$\underline{Q}_{WG} = Q_{WG}(\underline{u}_w, \overline{V}) \quad (10)$$

- b) **Maximum** value of active and reactive power ($\overline{P}_{WG}, \overline{Q}_{WG}$):

Compute the active and reactive power for nominal wind speed and minimum voltage i.e.,

$$\overline{P}_{WG} = P_{WG}(u_{w_{nominal}}, \underline{V}) \quad (11)$$

$$\overline{Q}_{WG} = Q_{WG}(u_{w_{nominal}}, \underline{V}) \quad (12)$$

Else

- a) **Minimum** value of the active and reactive power ($\underline{P}_{WG}, \underline{Q}_{WG}$):

Compute the active and reactive power, for minimum wind speed and maximum voltage i.e.,

$$\underline{P}_{WG} = P_{WG}(\underline{u}_w, \overline{V}) \quad (13)$$

$$\underline{Q}_{WG} = Q_{WG}(\underline{u}_w, \overline{V}) \quad (14)$$

- b) **Maximum** value of active and reactive power ($\overline{P}_{WG}, \overline{Q}_{WG}$):

Compute the active and reactive power for maximum wind speed and minimum voltage i.e.,

$$\overline{P}_{WG} = P_{WG}(\overline{u}_w, \underline{V}) \quad (15)$$

$$\overline{Q}_{WG} = Q_{WG}(\overline{u}_w, \underline{V}) \quad (16)$$

D. Variable speed WG

In the case of variable speed WG it has been shown in [11] that for normal range of voltage variations, the active as well as reactive power do not vary. Further, the reactive power either remains constant at the specified value (irrespective of the wind speed variations) or corresponds to the one required to maintain the specified power factor. Hence, in the normal operating range, the active power output depends on the wind speed alone and reactive power is constant or depends on the active power output of the WG. Hence, the maximum and minimum value of the active power output corresponds to the one at maximum and minimum wind speeds respectively.

III. INTERVAL LOAD FLOW METHOD

The interval load flow method essentially means a procedure to find a solution for the following interval load flow equations i.e., solving (17),(18) for $[V_p, \overline{V}_p]$ ($p \in \text{PQ buses}$) and $[\theta_p, \overline{\theta}_p]$ ($p \neq \text{slack}$) for given $[P_p, \overline{P}_p]$ ($p \neq \text{slack}$), $[Q_p, \overline{Q}_p]$ ($p \in \text{PQ buses}$) and $[V_p, \overline{V}_p]$ ($p \in \text{PV buses}$).

$$\begin{aligned} [P_p, \overline{P}_p] = [V_p, \overline{V}_p] \sum_{q=1}^n \{ [V_q, \overline{V}_q] (G_{pq} \cos[\theta_{pq}, \overline{\theta}_{pq}] \\ + B_{pq} \sin[\theta_{pq}, \overline{\theta}_{pq}]) \}, p = 1, \dots, n; p \neq \text{Slack} \end{aligned} \quad (17)$$

$$\begin{aligned} [Q_p, \overline{Q}_p] = [V_p, \overline{V}_p] \sum_{q=1}^n \{ [V_q, \overline{V}_q] (G_{pq} \sin[\theta_{pq}, \overline{\theta}_{pq}] - B_{pq} \\ \cos[\theta_{pq}, \overline{\theta}_{pq}]) \}, p = 1, \dots, n; p \neq \text{PV}, p \neq \text{Slack} \end{aligned} \quad (18)$$

where,

n is the number of buses

$$[\theta_{pq}, \overline{\theta}_{pq}] = [\theta_p, \overline{\theta}_p] - [\theta_q, \overline{\theta}_q]$$

It must be noted here that (17),(18) differs from the standard load flow equations in polar coordinates [14] since the active and reactive power at all the PQ buses and active power and voltage magnitude at all the PV buses are intervals.

The procedure for solving the non-linear interval load flow equations (17),(18) involves two main steps [12]. The first step is to find the load flow solution (voltage magnitude/angle, say V_{p_0}/θ_{p_0}) at the mid points of the intervals for $[P_p, \overline{P}_p]$ and $[Q_p, \overline{Q}_p]$ i.e., at $P_{p_0} = (P_p + \overline{P}_p)/2$ and $Q_{p_0} = (Q_p + \overline{Q}_p)/2$. The next step is to calculate the elements of the standard load flow Jacobian in polar coordinates are computed at V_{p_0} and θ_{p_0} . From this, the interval for voltage magnitude and angle are obtained. The procedure for computing the intervals for voltage magnitude and angle is given below:

Algorithm for solving interval load flow equations

Given intervals for $[P_p, \overline{P}_p]$, $[Q_p, \overline{Q}_p]$ at all PQ buses and $[P_p, \overline{P}_p]$, $[V_p, \overline{V}_p]$ at all PV buses, the interval for $[V_p, \overline{V}_p]$ at all PQ buses and $[\theta_p, \overline{\theta}_p]$ at all PQ and PV buses are computed in the following way:

- i) Obtain the load flow solution at $(P_{p_0}, Q_{p_0}$ and $V_{p_0})$ the mid points of each of the intervals.

Let the solution be V_{p_0} and θ_{p_0} ($p = 1, 2, \dots, n+m+1$).

- ii) Let $[V_p, \overline{V}_p] = [V_{p_0}, V_{p_0}]$ and $[\theta_p, \overline{\theta}_p] = [0, 0]$. At the slack bus the voltage magnitude and angle are point intervals and are deterministic values equal to V_{p_0} and zero respectively.

- iii) Compute the elements of the load flow Jacobian matrix (J_{Polar}) at V_{p_0} and θ_{p_0} .

- iv) Solve the following equations for $[\underline{\Delta V}, \overline{\Delta V}]$, $[\underline{\Delta \theta}, \overline{\Delta \theta}]$

$$[\underline{\Delta S}, \overline{\Delta S}] = (J_{Polar}) \begin{pmatrix} [\underline{\Delta \theta}, \overline{\Delta \theta}] \\ [\underline{\Delta V}, \overline{\Delta V}] \end{pmatrix} \quad (19)$$

where

$$[\underline{\Delta S}, \overline{\Delta S}] = \begin{pmatrix} [P_p - P_{p_0}, \overline{P}_p - P_{p_0}] \\ [Q_p - Q_{p_0}, \overline{Q}_p - Q_{p_0}] \end{pmatrix} \quad (20)$$

and J_{Polar} is the standard [14] load flow Jacobian matrix i.e.,

$$J_{Polar} = \begin{pmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{pmatrix} \quad (21)$$

where, $\frac{\partial P}{\partial \theta}$, $\frac{\partial P}{\partial V}$, $\frac{\partial Q}{\partial \theta}$, $\frac{\partial Q}{\partial V}$ are the elements of the standard load flow Jacobian matrix in polar coordinates [14] and these elements are calculated at V_{p_0} and θ_{p_0} .

In order to solve the above equation, first the inverse of the Jacobian matrix (J_{Polar}^{-1}) is computed. Later interval arithmetic [15] is used to compute the product $J_{Polar}^{-1}[\underline{\Delta S}, \overline{\Delta S}]$. It must be noted here that interval addition and multiplication is quite different from the usual addition and multiplication [15].

- v) $[V_p, \overline{V}_p] = [V_0 + \underline{\Delta V}, V_0 + \overline{\Delta V}]$ and $[\theta_p, \overline{\theta}_p] = [\theta_0 + \underline{\Delta \theta}, \theta_0 + \overline{\Delta \theta}]$

IV. INTERVAL LOAD FLOW ALGORITHM WITH WG OUTPUT UNCERTAINTY

For the interval load flow analysis suggested here, the WG are modeled as PQ buses. However, the interval load flow algorithm described in the previous section can not be directly used because the interval range for P,Q of the WG can be calculated only if the node voltage interval is known at the WG bus (Section II). However, the node voltage interval is not known till the load flow solution is obtained. In order to account for this interdependency, a two step procedure (interval load flow equations are solved twice) has been suggested here.

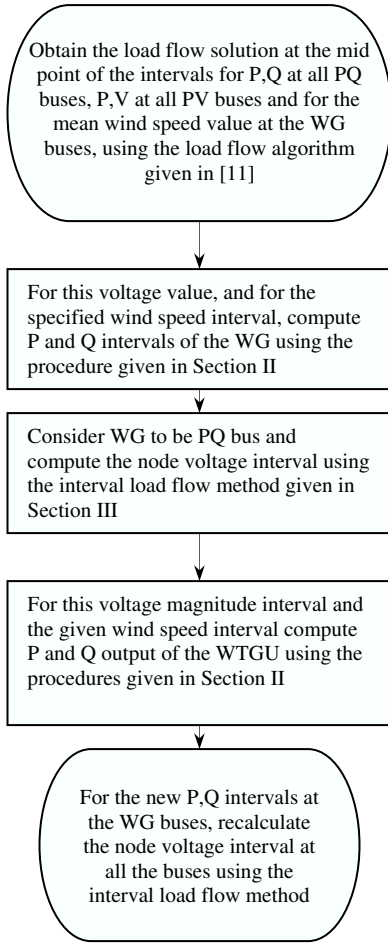


Fig. 1. Flow chart : Load flow analysis with WG (considering wind speed variation)

A flowchart giving all the steps involved in computing the node voltage intervals at all the buses in a power system network considering the WG output uncertainties is shown in Fig. 1. Here, the WG bus is considered as a PQ bus and at the beginning of each iteration, for a given voltage magnitude and wind speed interval, the interval range for active as well as the reactive power output of the WG ($[P_{WG}, \overline{P_{WG}}]$ and $[Q_{WG}, \overline{Q_{WG}}]$) is obtained using one of the methods (depending on the type of WG) given in the Section II. For these

interval ranges of power at the WG bus, the interval range at all the buses is calculated using the interval load flow algorithm given in the previous section. Further, if there is uncertainty in power/voltage magnitude at the other buses (load/generator bus) then these can also be considered by representing them as intervals. However, if there is no uncertainty then the power/voltage magnitude are deterministic values and in the interval load flow flowchart shown in Fig. 1 i.e., they are considered to be point intervals. For example if the active power at bus q is a deterministic value (say 0.1pu) then the interval for the active power at bus is $[\underline{P}_q, \overline{P}_q] = [0.1, 0.1]$, i.e., $\underline{P}_q = \overline{P}_q$ (this type of interval is called as point interval).

Further, it may be seen from the interval load flow algorithm (Fig. 1) that the node voltage at all the buses will be intervals if there is uncertainty in power/voltage magnitude at even one bus. For example if the active power only at bus-p is an interval (not point interval) then the voltage magnitude (at all PQ buses) and angle (at all PV, PQ buses) are intervals.

V. SIMULATION STUDY

For the simulation study a 33 bus radial distribution system [16] with all the four types of WG and a set of actual measured wind speed data have been considered. Each WG is of 900kW capacity and the data for these types of WG have been given in Table I.

TABLE I
WG DATA

Parameter [12]	Types of WG			
	Stall	Pitch	Semi-variable	Variable
MW	0.9	0.9	0.9	0.9
kV	0.69	0.69	0.69	0.69
R_1 (pu)	0.00643	0.0051	0.0051	0.0051
X_{l1} (pu)	0.10397	0.04726	0.04726	0.04726
R_2 (pu)	0.00567	0.00416	0.00416	0.00416
X_{l2} (pu)	0.0794	0.08696	0.08696	0.08696
X_m (pu)	3.0246	2.6087	2.6087	2.6087
X_c (pu)	3.0246	2.6087	2.6087	2.6087
N	4	4	4	4
u_{wCI}	4	4	4	4
u_{wCO}	25	25	25	25

System base = 0.9MVA,0.69kV

In order to assess the impact of wind speed variation on WG output and the distribution system voltage profile, the wind speed variation is needed in addition to the WG and system data. The wind speed interval is obtained from the measured wind speed data at a typical wind site. Three different measured wind speed variation have been considered and these measurements have been made over a period of 10 min. Since the sampling frequency used in all these measurements is 35Hz, the measured wind speed data captures some fast wind variations too. In [13] it has been stated that the high frequency wind speed variations are very local and will not cause power output variations. In order to eliminate the high frequency components present in the wind speed measurements, it has been suggested that the measured wind speed data should be passed through a low pass filter.



Fig. 2. Wind speed filter: Block diagram

1) *Computing wind speed interval/range*: Fig. 2 shows the block diagram of the low pass filter.

The raw data (measured wind speed) is first passed through the low pass filter (Fig. 2). Later the filtered wind speed data is used to find the minimum and maximum values. This has been illustrated below for one of the wind speed measurements considered here. For the study carried out here the time constant of the low pass filter is taken to be $\tau = 4$ sec. The filtered output as well as the raw (measured) wind speed data have been shown in Fig. 3.

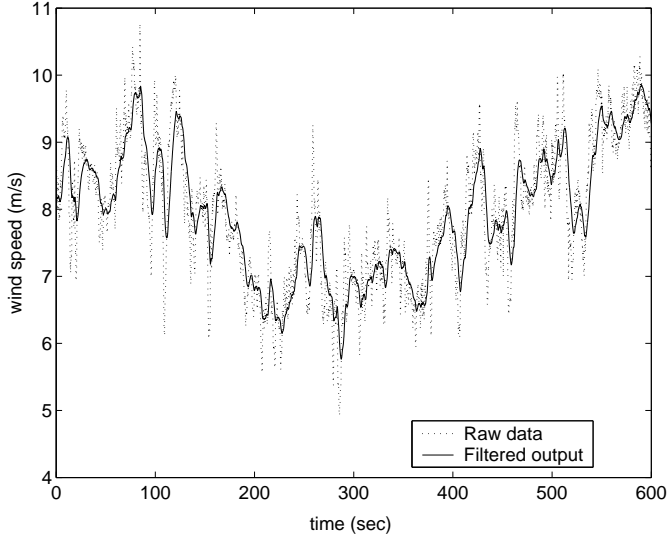


Fig. 3. Wind speed variation with time

From Fig 3 it may be seen that the maximum and minimum values of the filtered wind speed output are 5.7m/s and 9.8m/s respectively while that of the measured wind speed data are 4.9m/s and 10.7m/s respectively. The same procedure has been followed to obtain the wind speed interval for all the wind speed data at the other three WG and these are given in Table II.

TABLE II
WIND SPEED INTERVALS CONSIDERED AT EACH WG

Wind speed details		WG details	
$[u_w, \bar{u}_w]$ (m/s)	Mean wind speed	WG type	Location Bus No.
[5.7, 9.8]	7.8	Fixed speed (stall)	33
[5.3, 7.5]	6.4	Semi-variable speed	6
[5.5, 9.2]	7.3	Fixed speed (pitch)	18
[5.3, 7.5]	6.4	Variable speed	25

For this case, the WG power output and the node voltage intervals have been obtained using the interval method suggested here and the Monte Carlo simulation. A summary of the load flow results obtained using the proposed interval method and using the Monte Carlo method are given in Table III. From this table it may be seen that, even for the same WG output intervals, the node voltage intervals obtained using the interval method and the Monte Carlo method [8] are different. It may be observed that the intervals obtained using the Monte Carlo method is a subset of that obtained using the proposed interval method. Further, it must be noted here that the Monte Carlo method used in practice does not always guarantee that the bounds obtained will correspond to the worst case. This is because in reality, it is generally difficult to ensure that the entire sample space is covered. However, the interval method does not have this limitation and the manner in which the interval arithmetic is carried out ensures that the upper and lower bounds obtained correspond to the worst case. Hence, for a given uncertainty in nodal power/wind speed, this method always guarantees that the node voltage magnitude can never be outside the computed interval. However, one of the limitations of the interval method is that it is very difficult to use it for large nodal power uncertainties (interval range is broad). This is because, the interval method could compute a voltage interval range which is so broad that it may have little practical significance. Hence, the proposed interval method could be effectively used to analyze the impact of small nodal power uncertainties on the power system nodal voltages.

TABLE III
SUMMARY OF LOAD FLOW RESULTS-DISTRIBUTION SYSTEM

Bus No.	WG power output		Voltage Magnitude
	$[P_{WG}]$	$[Q_{WG}]$	[V]
Interval Method			
33	[0.1190 0.5577]	[-0.0097 0.1345]	[0.8819 0.9208]
6	[0.0847 0.2770]	[-0.0150 0.0077]	[0.9352 0.9559]
18	[0.0331 0.4920]	[-0.0163 0.1118]	[0.8957 0.9499]
25	[0.0960 0.3106]	[0.0000 0.0000]	[0.9668 0.9763]
Monte Carlo Method			
33	[0.1190 0.5577]	[-0.0081 0.1219]	[0.8877 0.9142]
6	[0.0847 0.2770]	[-0.015 0.0076]	[0.9379 0.9533]
18	[0.0331 0.4920]	[-0.0163 0.1117]	[0.9037 0.9413]
25	[0.0960 0.3106]	[0.0000 0.0000]	[0.9678 0.9759]

VI. CONCLUSION

In this paper a new approach has been used to quantify the effect of WG output uncertainties on the distribution system node voltages. This approach is based on the interval method of analysis. In this method for a given wind speed interval the WG output uncertainties are computed. To compute the effect of these uncertainties on the distribution system node voltages an interval load method has been proposed. A comparison of the results obtained from the proposed method with Monte Carlo method has shown that the Monte Carlo method could under-estimate the effect of WG output uncertainties on the distribution system nodal voltages.

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