Mathematical Modelling of Mandibular Metamorphosis from 3D CT

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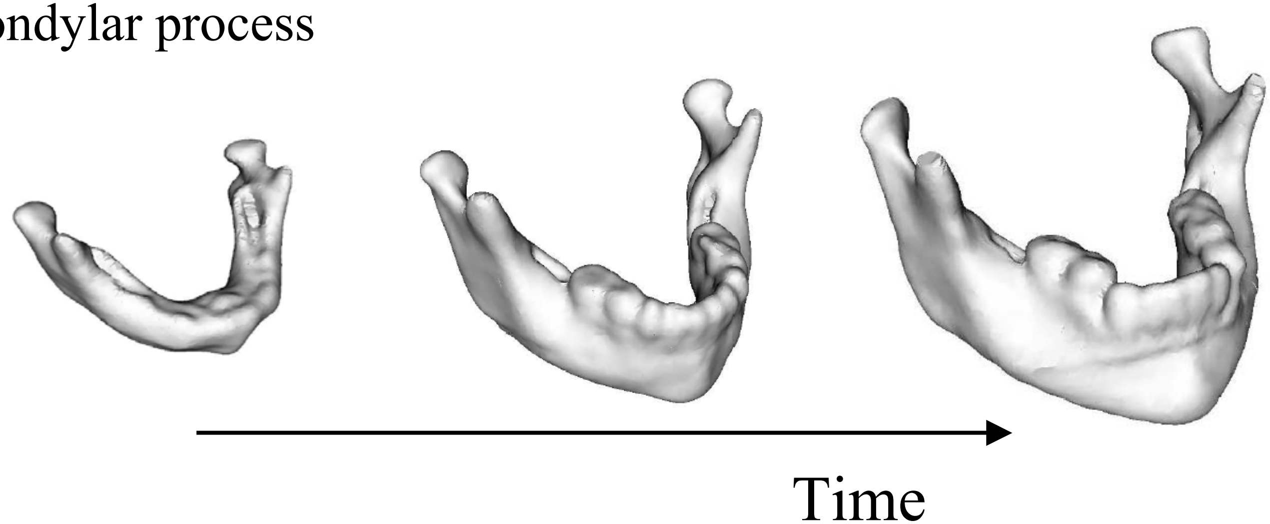
Metamorphosis!?

- Etymology: Latin, from Greek *metamorphosis,* from *metamorphoun* to transform, from *meta-* + *morphe* form
“*change of physical form, structure, or substance*”

- The goal is to obtain better insight and improve the understanding of mandibular growth
- The growth of the mandible is particularly complex; due to asynchronous teeth eruption and changes in the angular direction of the condylar process



The Data

- The data are 31 mandibular surfaces acquired from CT volume scans of a total of six subjects with the Apert syndrome
- All scans where acquired for treatment and diagnostics purposes
- In the Apert syndrom the mandible is not affected by the primary anomaly

Age in month vs. CT scan number for each patient

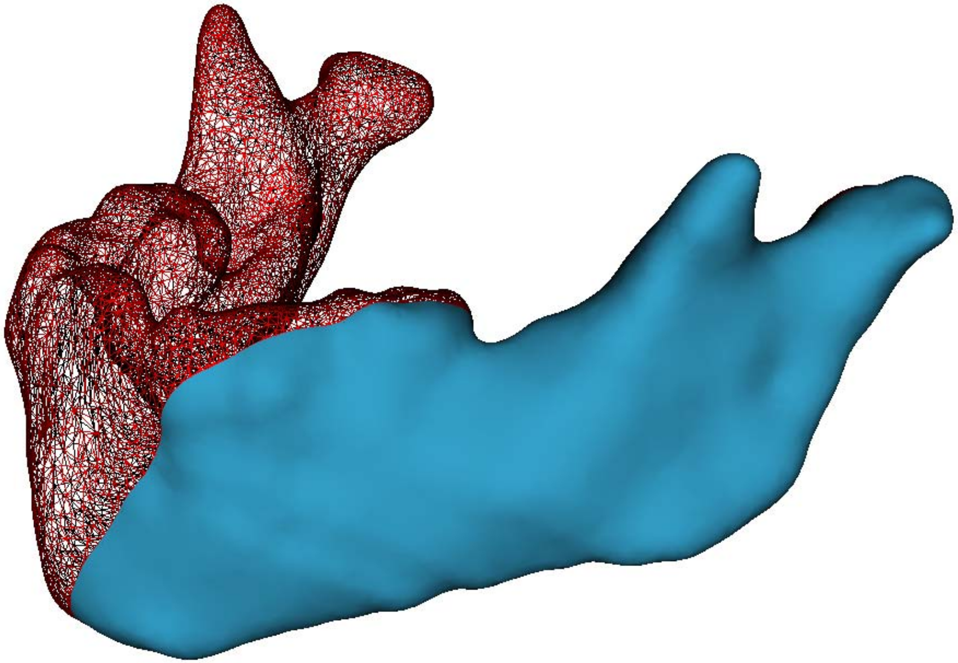
		Scanning no.						
Patent no.	Sex	1	2	3	4	5	6	7
1	M	3	16	21	23	34	-	-
2	M	1	7	23	54	56	60	72
3	M	1	5	17	32	36	-	-
4	F	3	27	46	62	131	132	144
5	M	3	4	21	72	-	-	-
6	F	9	21	84	-	-	-	-

Computed Tomography (CT) Imaging

Active Shape Modelling

- Surface segmentation and shape representation
- Alignment of the set of shapes
- Tangent space projection into a metric space containing of shape variability
- Shape variability decompositioning
- Growth modelling and future shape prediction

Shape Representation



- Each shape is represented by 14851 homologous semilandmarks

Alignment

- Align the set of shapes by removing: scale, translation and orientation. (Generalized Procrustes Alignment using a similarity transform, GPA)
- Formally identical to Multiset Canonical Correlations Analysis (MCCA) under similar constraints

- Set of shapes:

$$\boldsymbol{S} = \{\boldsymbol{s}_i\}_{i=1}^{S} \ , \ \boldsymbol{s}_i \in \mathbb{R}^{\mathrm{Nk}}$$

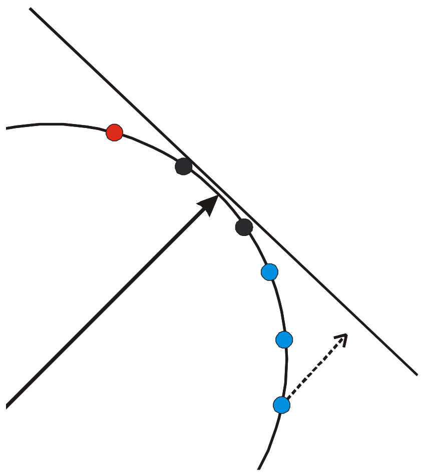
- Objective to maximize:

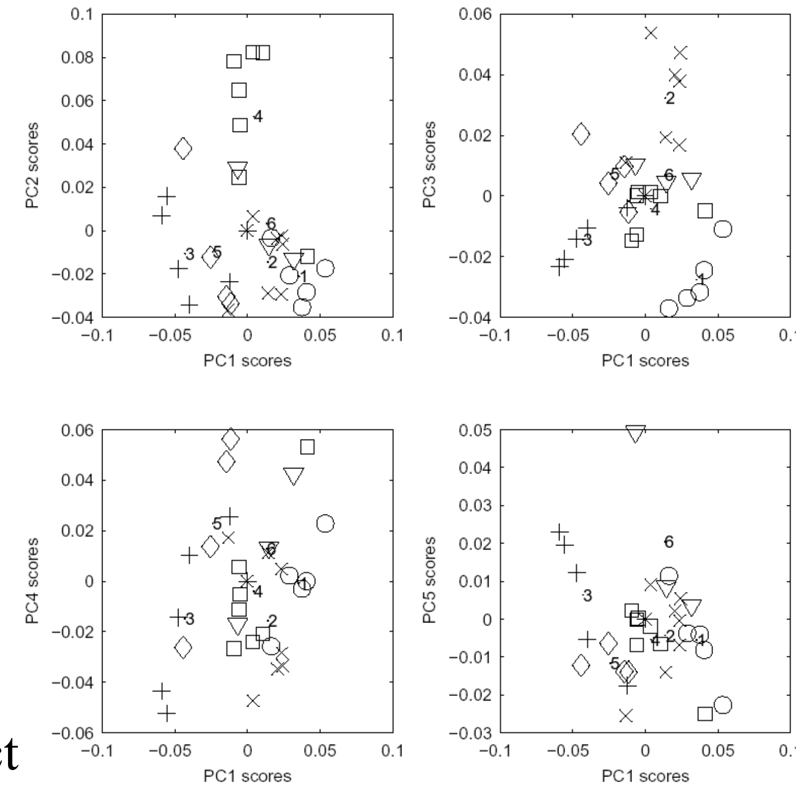
$$\{\hat{T}_i(\cdot)\}_{i=1}^{S} : \mathrm{argmax}\{\mathrm{R} = \sum_{\mathrm{ij}} \mathrm{Corr}\{\mathrm{T_i}(\boldsymbol{s}_\mathrm{i}), \mathrm{T_j}(\boldsymbol{s}_\mathrm{j})\}\}$$

- Average shape:

$$\boldsymbol{s}_i^P = \hat{T}_i(\boldsymbol{s}_i) \ \rightarrow \ \bar{\boldsymbol{s}} = \sum_i \boldsymbol{s}_i^P / S$$

Tangent space projection





- Projection into a subspace of the tangent space containing the *patient specific* mean shapes, reveals dominating inter-patient variability
- Compensate by centering each subject group to the common average shape
- No significant structure in the scattering indicating no gender related differences in shape

Decompositioning

- Assuming a generative linear model

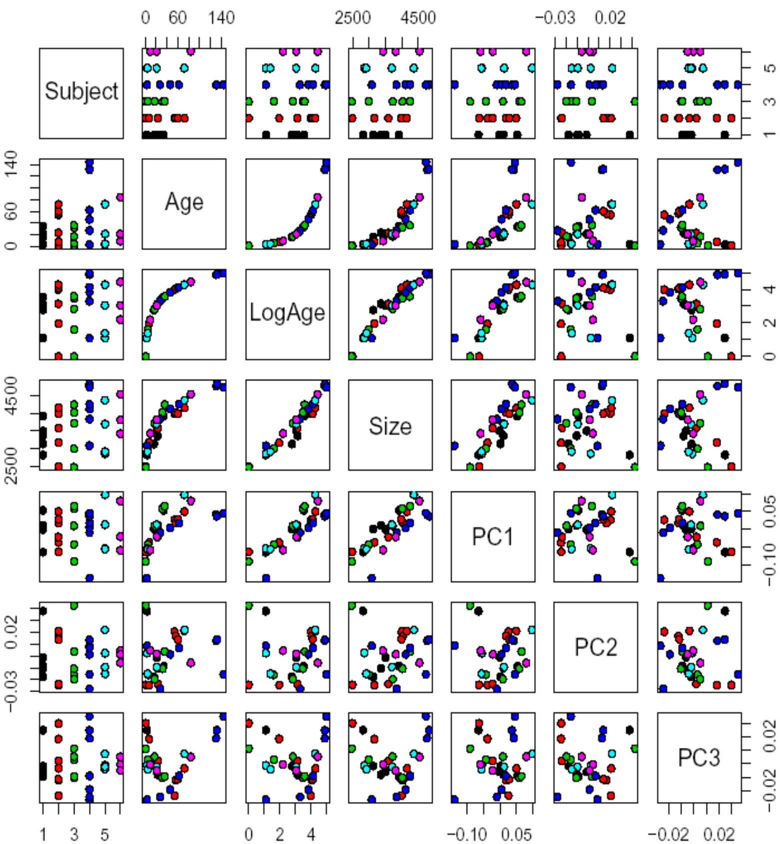
  $$\boldsymbol{s}_{new} = \bar{\boldsymbol{s}} + \boldsymbol{P}\boldsymbol{b}$$

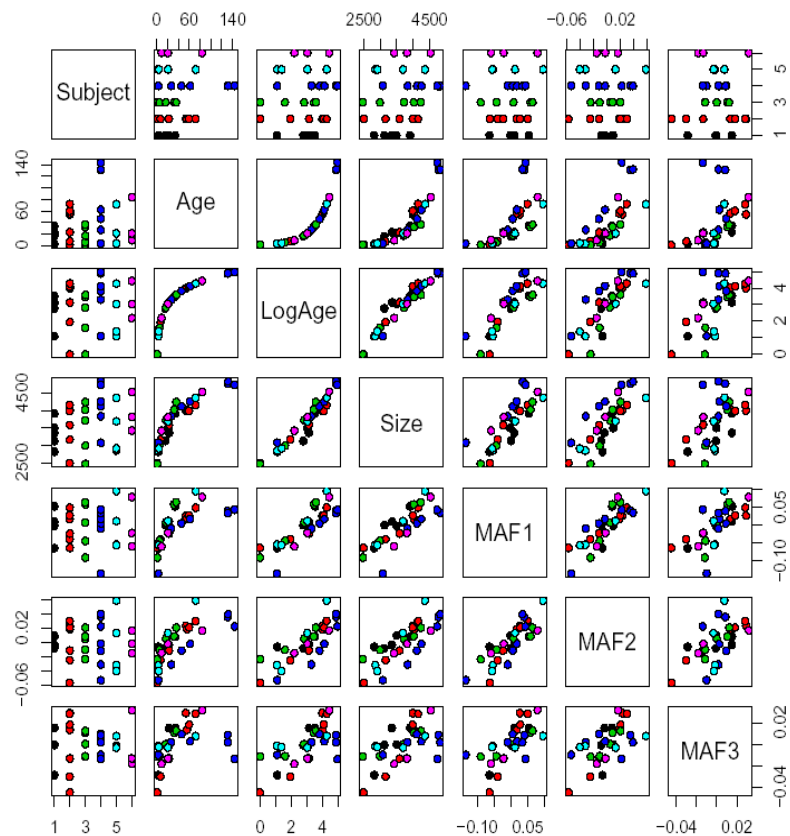
  constrained so that

  $$\boldsymbol{P} : \operatorname{argmax}\{\mathrm{J}(\boldsymbol{P}, \boldsymbol{S})\}$$

  where $\mathrm{J}(\boldsymbol{P}, \boldsymbol{S})$ can be e.g. variance, auto-correlation or the signal-to-noise ratio in the new components
- The traditional approach to maximize variance (PCA) operates in an Euclidean metric
- Decompositioning under different constraints e.g. auto-correlation (MAF) results in uncorrelated components in a non-Euclidean metric

PCA versus MAF

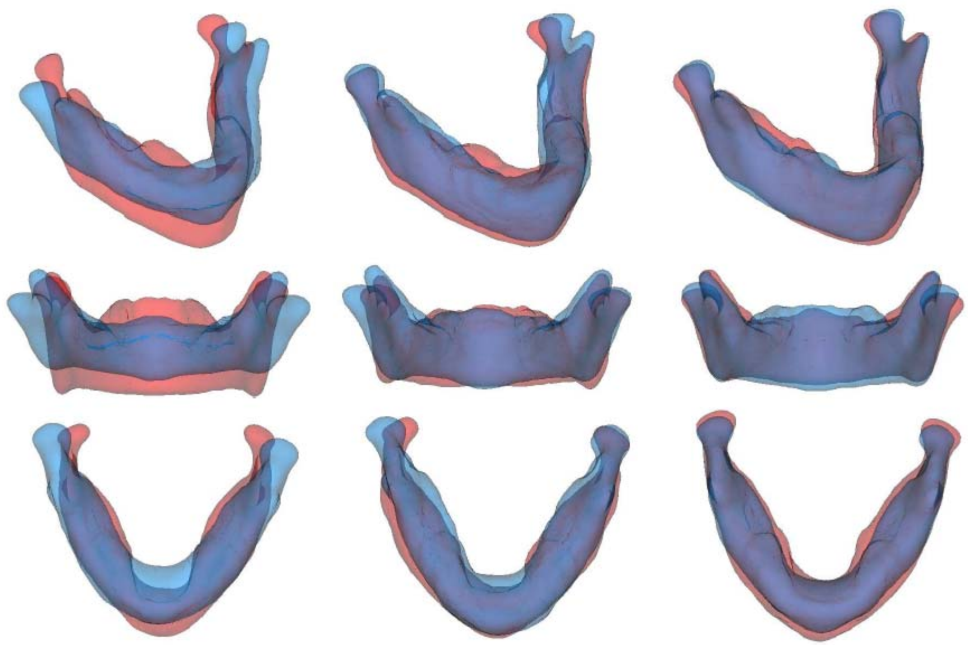


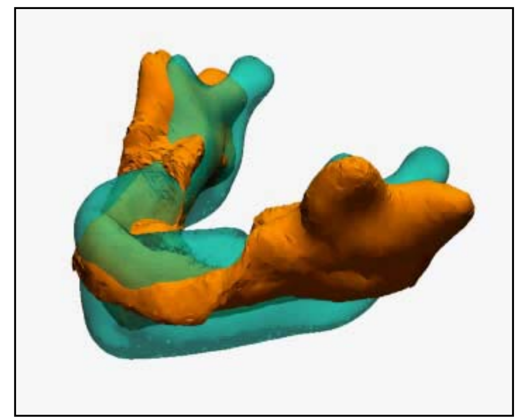


Uncorrelated Modes of Variation

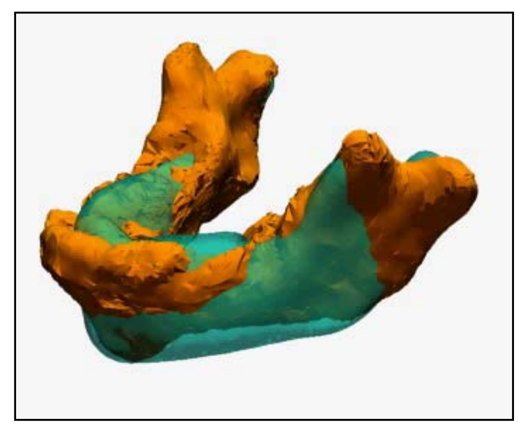
- The principal modes of variation using the MAF decompositioning:

MAF1 MAF2 MAF3





MAF1



MAF2

Evaluation and Prediction

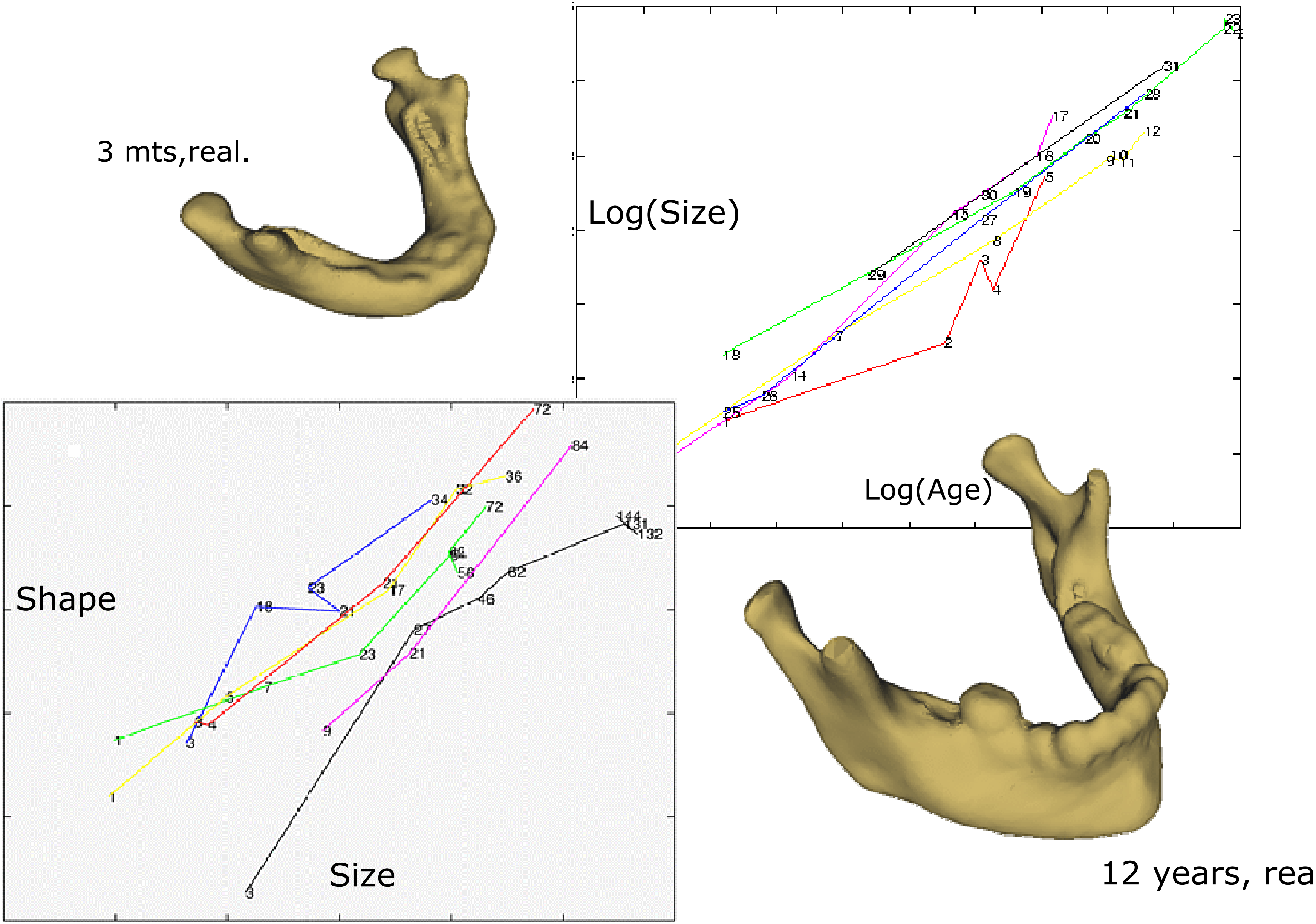
- Robustness of the decompositioning is examined by cross-validation (CV) excluding one patient at a time

CROSS-VALIDATION OF THE ORIENTATION OF THE EIGENVECTORS FROM THE REDUCED DATA SETS AGAINST EIGENVECTORS DETERMINED ON THE BASIS OF ALL PATIENTS

patient excluded	PC1	MAF1	MAF2
1	4.1°	3.4°	9.7°
2	5.8°	6.2°	38.4°
3	6.5°	9.3°	38.7°
4	8.3°	6.1°	32.9°
5	5.9°	6.5°	16.9°
6	3.9°	5.7°	32.5°

- The results shows the models ability to generalize. Note that no patient controls the variablility of the pooled analysis

Linear Dependence in Procrustes Tangent Space



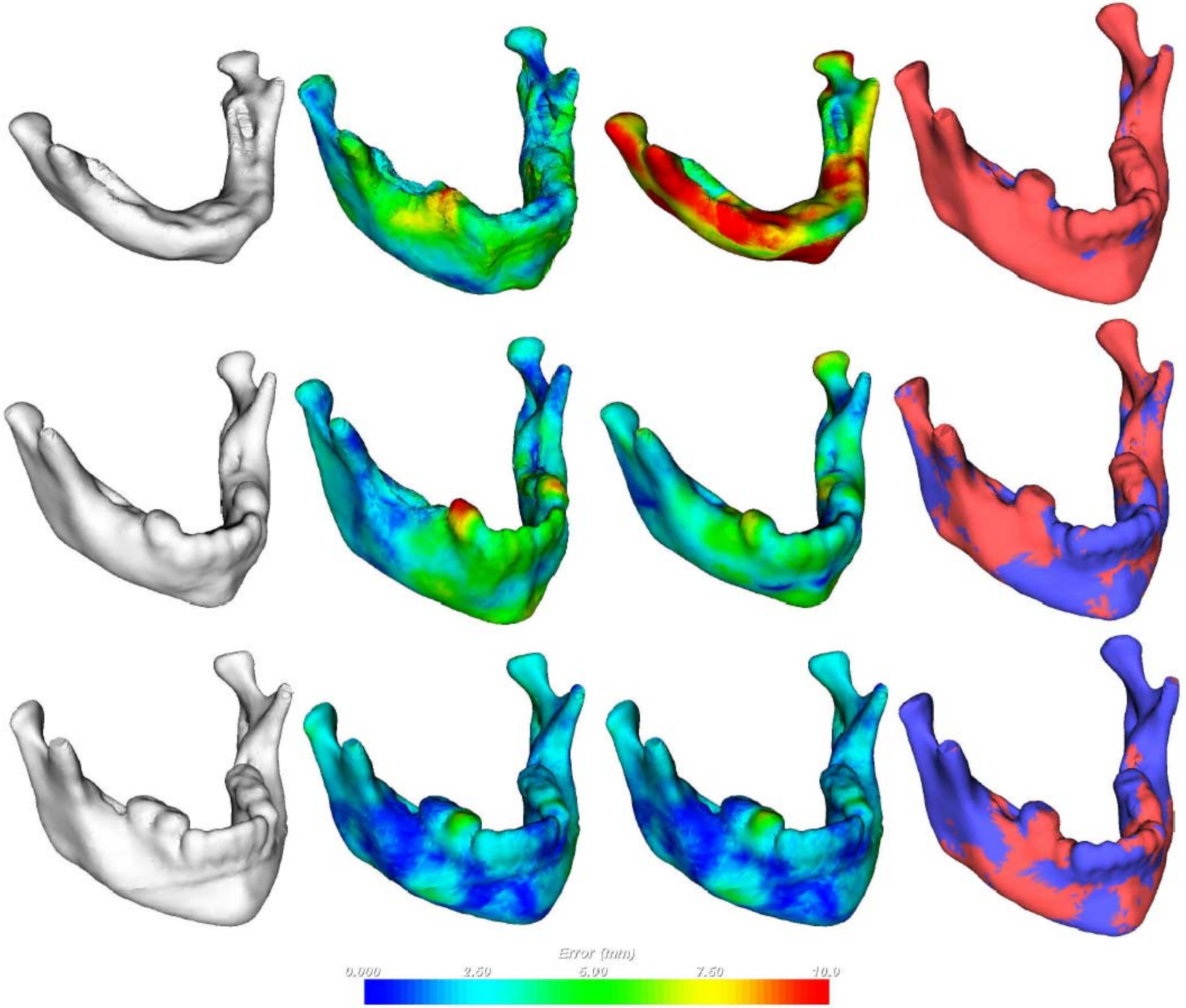
Growth Simulation

- Leave-one-out CV study predicting the most recent scan applying the earlier scannings as sources
- Notice the ability to predict the evolution of patient 4 - no other subject has been scanned at such a high age
- This strengths the hypothesis of linear growth in Procrustes space

Prediction errors of the cross-validation study

Patient		PC1 model		MAF1 model		MAF2 model	
no.	scan	mean	std	mean	std	mean	std
1	1	*2.4*	*1.3*	*2.4*	*1.3*	4.3	1.4
	2	1.8	0.8	**1.7**	**0.7**	2.2	0.9
	3	1.8	0.7	**1.7**	**0.7**	2.3	0.9
	4	1.4	0.6	**1.3**	**0.6**	1.8	0.7
2	1	3.5	1.4	**3.4**	**1.4**	4.3	1.9
	2	2.4	1.1	**2.3**	**1.1**	3.2	1.9
	3	2.3	1.3	**2.2**	**1.3**	3.0	1.8
	4	*1.5*	*0.7*	*1.5*	*0.7*	1.7	0.8
	5	*1.6*	*0.8*	*1.6*	*0.8*	1.8	0.8
	6	*1.4*	*0.6*	*1.4*	*0.6*	1.5	0.7
3	1	*2.8*	*1.3*	*2.8*	*1.3*	5.4	2.2
	2	2.5	1.2	**2.4**	**1.2**	4.1	1.6
	3	*2.1*	*0.9*	*2.1*	*0.9*	2.7	1.2
	4	1.0	0.4	1.0	0.4	**0.9**	**0.5**
4	1	3.7	1.6	**3.6**	**1.6**	7.2	2.6
	2	3.1	1.6	**3.1**	**1.5**	3.3	1.6
	3	*2.8*	*1.4*	*2.8*	*1.4*	3.0	1.4
	4	*2.8*	*1.4*	*2.8*	*1.4*	2.9	1.3
	5	*2.0*	*1.0*	*2.0*	*1.0*	*2.0*	*1.0*
	6	2.0	0.9	2.0	0.9	**1.9**	**0.9**
5	1	**2.6**	**1.0**	2.7	1.1	5.5	2.1
	2	*3.1*	*1.2*	*3.1*	*1.2*	5.7	2.3
	3	*2.1*	*1.0*	*2.1*	*1.0*	3.4	1.5
6	1	**2.8**	**1.0**	2.9	1.0	5.3	1.8
	2	*2.8*	*1.2*	*2.8*	*1.2*	4.2	1.7

Selected predictions of subject 4



Summary

- Mandibular growth is approximately linear in Procrustes tangent space
- Technical high-lights:
  - removal of inter-patient variability
  - decompositioning in non-Euclidean metric

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