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## **Two wind power prognosis criteria and regulating power costs**

### **Abstract**

The objective of the present work is to investigate the consequences of the choice of criterion in short-term wind power prognosis. This is done by investigating the consequences of choice of objective function in relation to the estimation of the power curve that is applied in the prognoses. Basically, the choice is between focusing on predicting the energy content of the wind and focusing on the cost of buying regulating power to compensate for the prognosis errors. It will be shown that it can be expected that the two power curves thus estimated will differ, and that therefore also the hourly wind power production predicted will differ. In turn this will influence the operation and economics of the system.

The consequences of this are illustrated by application to the integration of wind power in the Danish parts of the Nordpool area, using recent data. Using a regression analysis the prices of regulating power will be estimated. Then the two mentioned power curves may be estimated using wind speed production from the numerical weather prediction model from the Danish Meteorological Institute and the corresponding short term prognoses of wind power will be elaborated. From wind power production measurement the errors may be calculated. Combining this information it is possible to find the consequences of inconsequent use of prognosis criterion, i.e., using one criterion in estimating the power curve and another in assessing the quality of the prognosis.

### **1. Introduction**

Associated with the introduction of wind power into the electricity system are two activities, making short-term wind power prognoses and neutralising the prognosis errors. The key point in the present paper is that the way the cost (in a general sense) of neutralising the prognosis errors is determined will have consequences for how short-term wind power prognoses should be made. Or, in other words, it will have consequences for the choice of criterion for evaluation of short-term wind power prognoses.

In the present work the objective is to investigate the consequences of the choice of criterion in short-term wind power prognoses. More specifically, the focus is on the consequences in the choice of objective function in relation to the estimation of the power curve applied in the prognosis. The criteria considered are presented in Section 2 together with a brief description of

the background for the problem. Section 3 presents an analysis of the regulating prices, in both Eastern and Western Denmark throughout the year 2002. This analysis is then used as the base of one of the two types of criteria investigated in the case study given in Section 4, using wind speed measurements and wind speed production data from Taastrup in Eastern Denmark.

## 2. Wind power prognosis and criteria

In Denmark, the system operators (Eltra [1] in Western Denmark and Elkraft System [2] in Eastern Denmark) make a wind power prognosis each morning and makes bids based on this prognosis to the Nordic electricity spot market, NordPool [3], for each of the 24 hours of the following. The bidding procedure closes at noon and hourly market crosses are formed, determining the quantities of electricity traded each hour of the following day, as well as the associated price.

Prognosis errors on wind power production are a significant source of deviation from the spot market plan, and when taking into account the potential amount of wind power available, particularly in Western Denmark, this can make quite an impact on the regulating costs and prices in the system, as they are precisely the costs that the system operator must meet to smooth out imbalances in the system. See e.g. Holttinen et al. [4] for a description and analysis in relation to this.

It is therefore in the interest of the system operator, and others, to have as good a prognosis as possible available when bidding to the spot market, in order to avoid having to pay for either up or down regulation in the system. However, 'good' is a term often hard to define precisely, as several qualities may be desirable, but in practise hard to obtain simultaneously.

When evaluating the quality of wind power prognoses, typical criteria include minimising the root mean square error, the mean absolute error, and the mean error. All these criteria are well based in statistical theory for prognoses.

The choice of criterion for a wind power prognosis naturally depends on the type of decision making the prognosis is to be the base for. When a system operator needs to evaluate the quality of a wind power prognosis, three criteria, as stated in [5], arise naturally:

- The prognosis value of the wind power production should be close to the average of the realised values.
- The sum of deviations between prognosis values and realised values should be small.
- The prognosis should result in low cost of the consequences of prognosis errors.

The first criterion corresponds to minimising the sum of squared deviations (i.e., the mean or average value), and the second to minimising the sum of absolute deviations (i.e., the median). As for the final criteria, it concerns the cases when the cost of wind power prognosis errors is asymmetric, i.e., specified with a cost  $c^+$  when the realised value of wind power production lies above the prognosis and a cost  $c^-$  when the realised value lies below the prognosis, where  $c^+ \neq c^-$ . This would naturally relate to the regulating costs where the up and down regulating costs are  $c^-$  and  $c^+$ , respectively.

This may be stated mathematically in the following manner. Given the observations  $y_j, j \in J$ , and letting  $\beta$  denote the prognosis value, the objective would be to minimise

- $\sum_{j \in J} (y_j - \beta)^2$  for the first criterion,
- $\sum_{j \in J} |y_j - \beta|$  for the second criterion, and
- $\sum_{j \in J^-} c^- (\beta - y_j) + \sum_{j \in J^+} c^+ (\beta - y_j)$  in the third criterion,

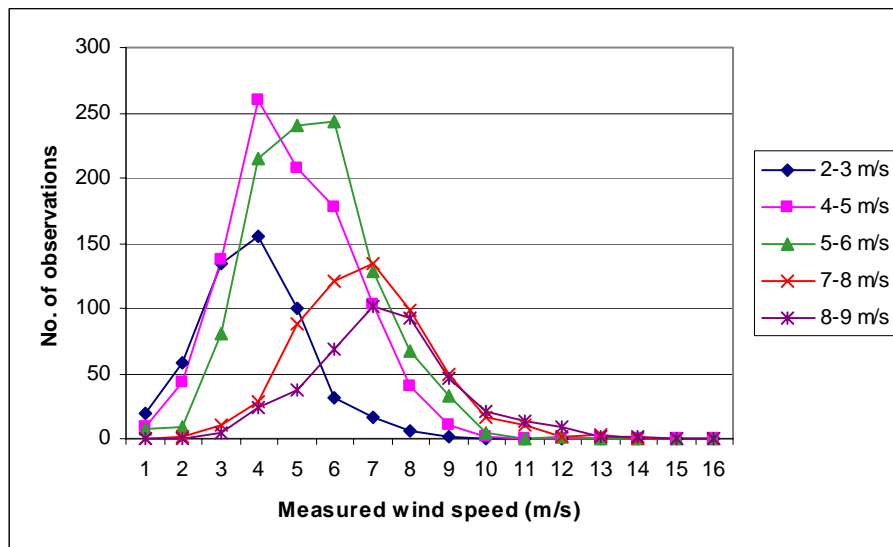
where the subset  $J$  of observations divides into three subsets such that  $y_j < \beta$  for  $j \in J^-(\beta)$ ,  $y_j = \beta$  for  $j \in J^0(\beta)$  and  $y_j > \beta$  for  $j \in J^+(\beta)$ . Note, that the second criterion is in fact a special case of the third criterion, with  $c^+ = c^- = 1$ .

In this paper, the focus lies on the first and third criteria, henceforth referred to as the quadratic and the cost ( $c^+ : c^-$ ) criterion, respectively.

If the distribution of the prognosis errors were symmetric, and  $c^+ = c^-$  then there would be no difference between the two criteria. However, as illustrated in Figure 1 and Table 1, the distribution for the observation site (Taastrup in Eastern Denmark) seems to be right skewed, i.e., the mean value is larger than the median. A contributing factor to this skewness is that there is a natural lower limit on errors in the meteorological forecast for wind speed, as wind speeds cannot be negative. The significance of this will become clear when considering the power curves based on the different criteria in Section 4.

Interval	2-3 m/s	4-5 m/s	5-6 m/s	7-8 m/s	8-9 m/s
Mean	2.65	3.88	4.67	6.21	7.28
Median	2.50	3.80	4.60	6.20	7.2

**Table 1.** Means and medians of the illustrated intervals.

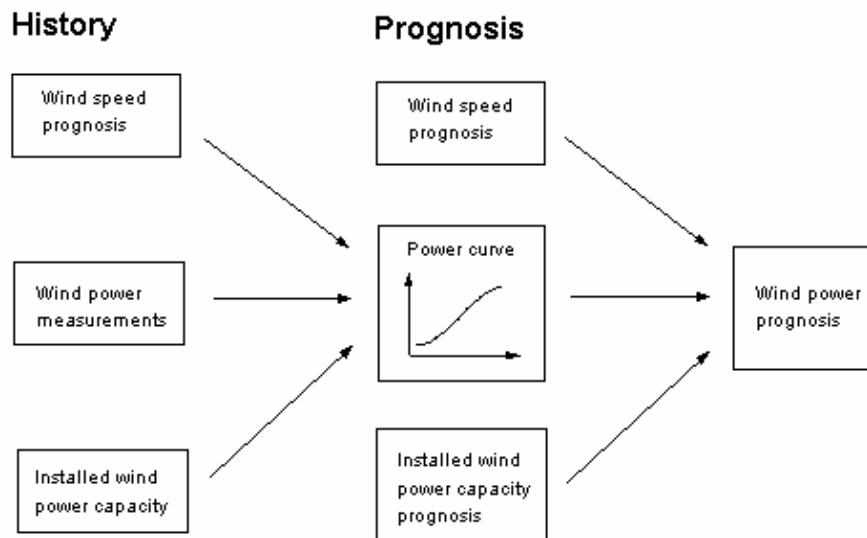


**Figure 1.** Measured wind speed for various prognosis intervals, Taastrup.

The data used here consists of wind measurements from Taastrup in Eastern Denmark, from April 9 to December 31, 2002, provided by Elkraft System and the hourly wind speed prognosis for

Abed (the location closest to Taastrup) from the Danish Metereological Institute (DMI) [6] during the same period.

The wind power prognosis system considered here is built primarily upon two sets of time series: wind speed forecasts, supplied by DMI, and local measurements of wind power production. The combination of these two time series for a specific location is the base of the wind power prognosis in the manner illustrated in Figure 2.



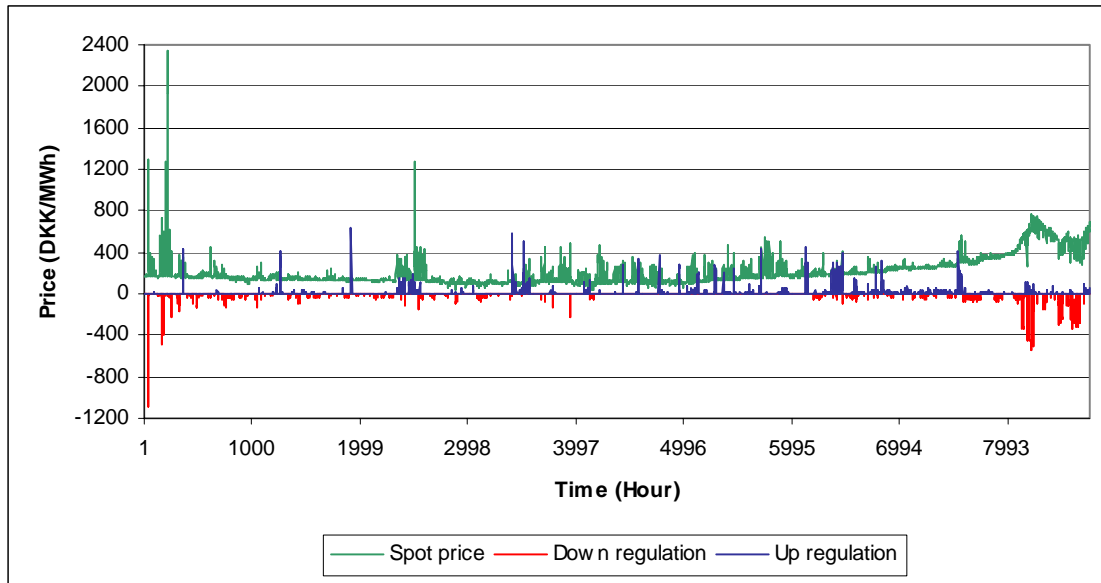
**Figure 2.** The structure of a wind power prognosis for a given location.

The wind speed prognosis supplied by DMI is generated by the model HIRLAM (High Resolution Limited Area Model), which every 6 hours gives a forecast for the following 48 hours.

For a more detailed description of how the prognosis is made, see [5]. A brief introduction to short-term wind power prediction may be found in [7] and a more thorough overview may be found in [8].

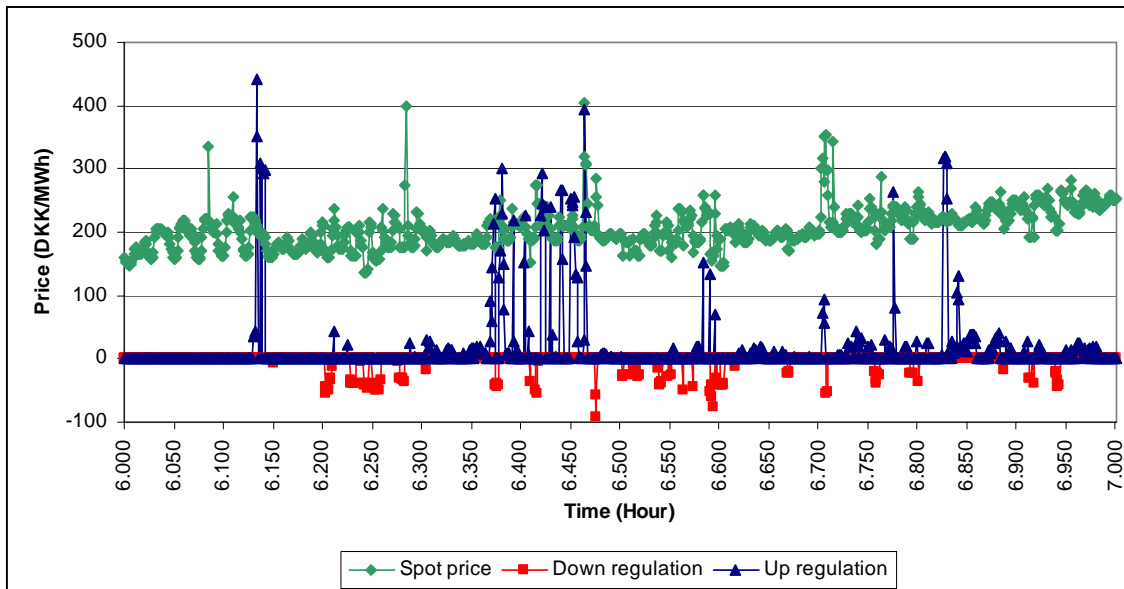
### 3. Analysis of regulating power costs

When determining the values for  $c^+$  and  $c^-$  in the cost criterion, it is necessary to look at the regulating prices for the relevant price area. In this case, as Taastrup is the location considered, the regulating prices for Eastern Denmark during 2002 are used. Here the average down and up regulating prices are 47.50 DKK/MWh and 45.69 DKK/MWh, respectively. The regulating prices as well as the spot prices for the entire year are illustrated in Figure 3. In this figure, the spot prices are given as the upper curve and regulating prices are given by the lower curve where the down regulating prices are negative and the up regulating prices are positive. The regulating prices indicate the price difference relative to the spot price.



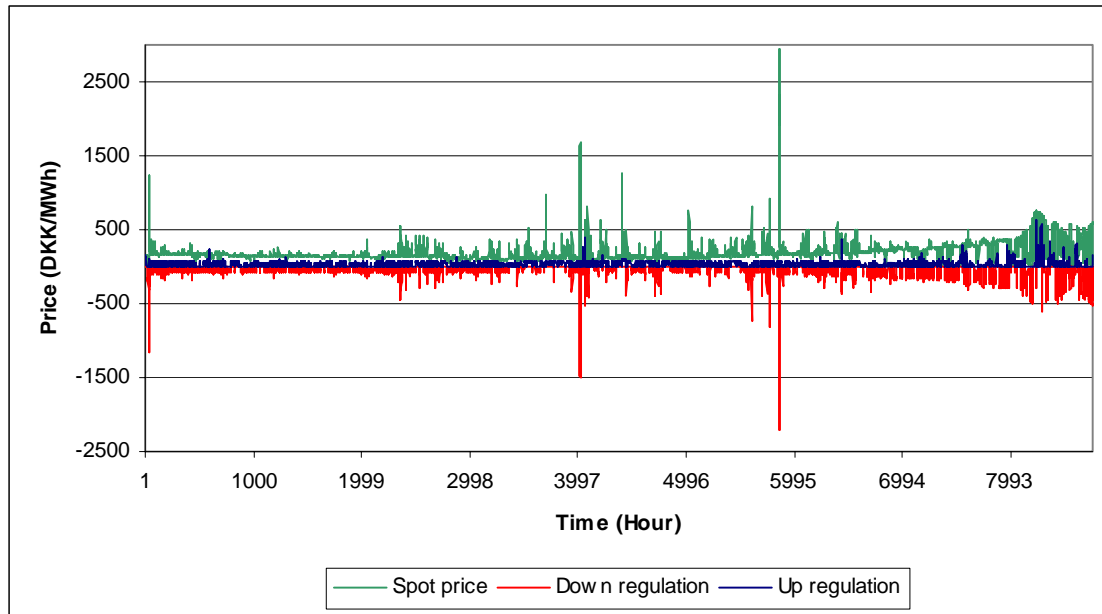
**Figure 3.** Spot and regulating prices for Eastern Denmark, 2002

The relationship between the average up and down regulating prices in Eastern Denmark is 1:1.04, which corresponds to  $c^+ = 1.04$  and  $c^- = 1$ , and which, in turn, makes the cost criterion in this case practically identical to the absolute value criterion. Recall, that  $c^+$  is the cost of regulating down, as the realised value in this case lies *above* the prognosis value and the system operator must pay providers of regulating power to produce. Conversely,  $c^-$  is the cost of regulating up, as the realised value then lies *below* the prognosis value. The behaviour of the spot and regulating prices are illustrated in larger detail in Figure 4.



**Figure 4.** Spot and regulating prices from early September to mid-October, Eastern Denmark, 2002.

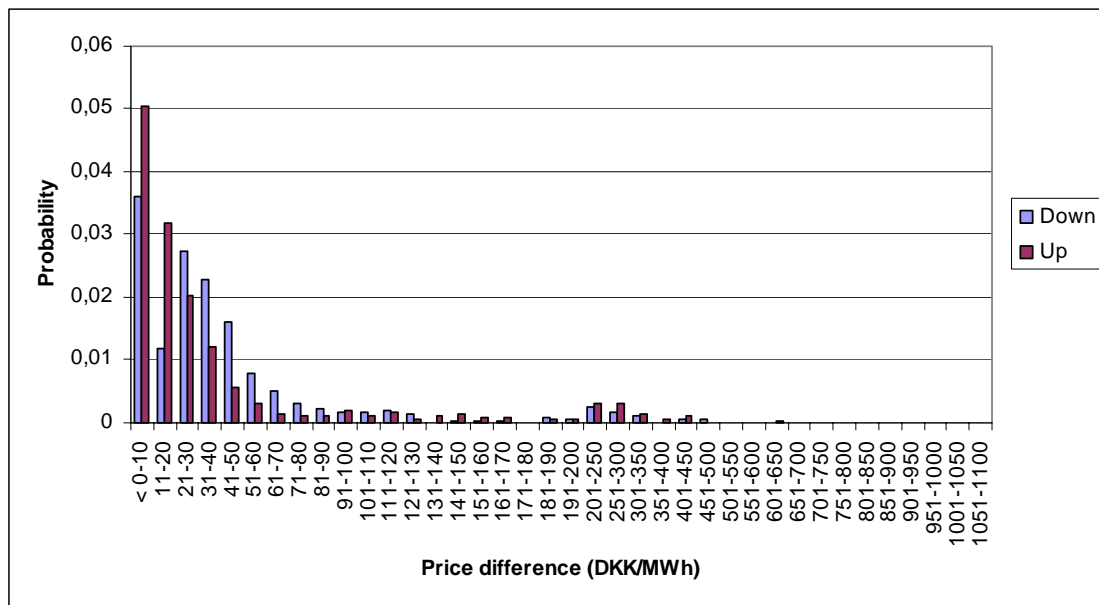
The average up and down regulating prices in Western Denmark, illustrated in Figure 5, during 2002 were 59.27 DKK/MWh and 101.94 DKK/MWh, respectively, corresponding to  $c^+ = 1.81$  and  $c^- = 1$ .



**Figure 5.** Spot and regulating prices in Western Denmark, 2002.

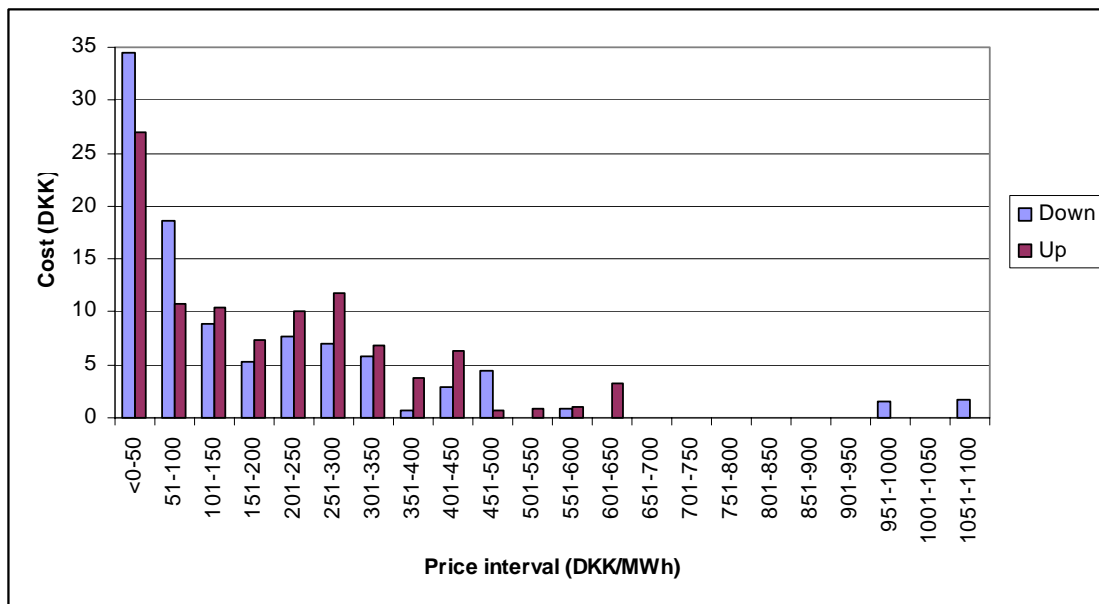
It should be noted that in this paper it is assumed that variations in wind power are the main cause of regulation in the system. In reality this need not be the case, as regulation is incurred by general imbalance in the system of which wind power variations are not necessarily the only contributing factor. However, in the Western Danish system the share of wind power capacity is large (more than 2000 MW out of a total of 6600 MW installed capacity) and so, consequently, may the variations in wind power production also be; therefore wind power is in fact the major contributing factor with regards to imbalance in the system. This makes an analysis of the Western Danish system particularly relevant in relation to wind power prognoses and certainly an avenue to be investigated further using the tools presented in [5] and here.

The probabilities of regulating in Eastern Denmark, 2002, are illustrated in Figure 6.



**Figure 6.** Probabilities of regulating up and down for different regulating price intervals, Eastern Denmark, 2002. Observe that the price scale has a finer division for prices less than 200 DKK/MWh.

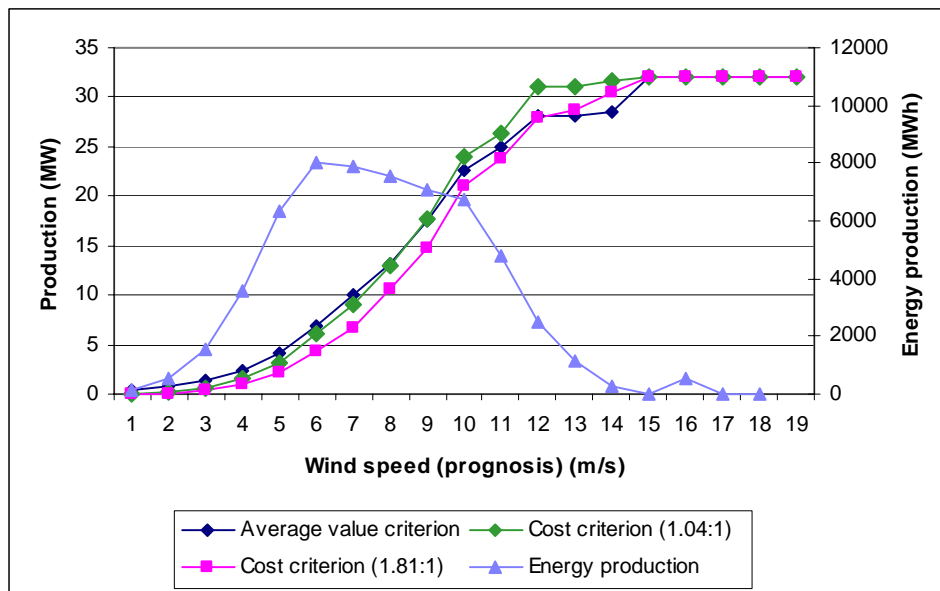
Figure 7 illustrates that although the probabilities in Figure 6 indicate that the amount of small regulating prices (between 0 and 100 DKK/MWh) are predominant, the impact to the system of larger, although more seldom, regulating prices is significant.



**Figure 7.** Distribution of cost of regulating one MWh every hour throughout the year (Eastern Denmark, 2002).

#### 4. Numerical results

Using the data presented in the previous sections, i.e., wind power production data and measured wind speeds from Taastrup, and regulating prices from Eastern Denmark, power curves for the quadratic criteria and the cost (1.04:1) criteria are estimated and shown in Figure 8. For comparison, the cost (1.81:1) criteria relating to Western Danish regulating prices is also estimated and shown in Figure 8. One ought however be careful when concluding anything based on this latter estimation, as the location from which production data and measured wind speeds are obtained after all is situated in Eastern Denmark. The estimation was performed by discretising the wind speed prognosis values into intervals of 1 m/s length. For each interval an optimal value relative to the observations according to both the quadratic and the cost criteria functions specified in Section 2. Also shown in the figure is the distribution of energy (the pale blue or bell-shaped curve). Note that the major part of the energy is produced at wind speeds (between 0 and approximately 10 m/s) where the power curve is upwards curving.



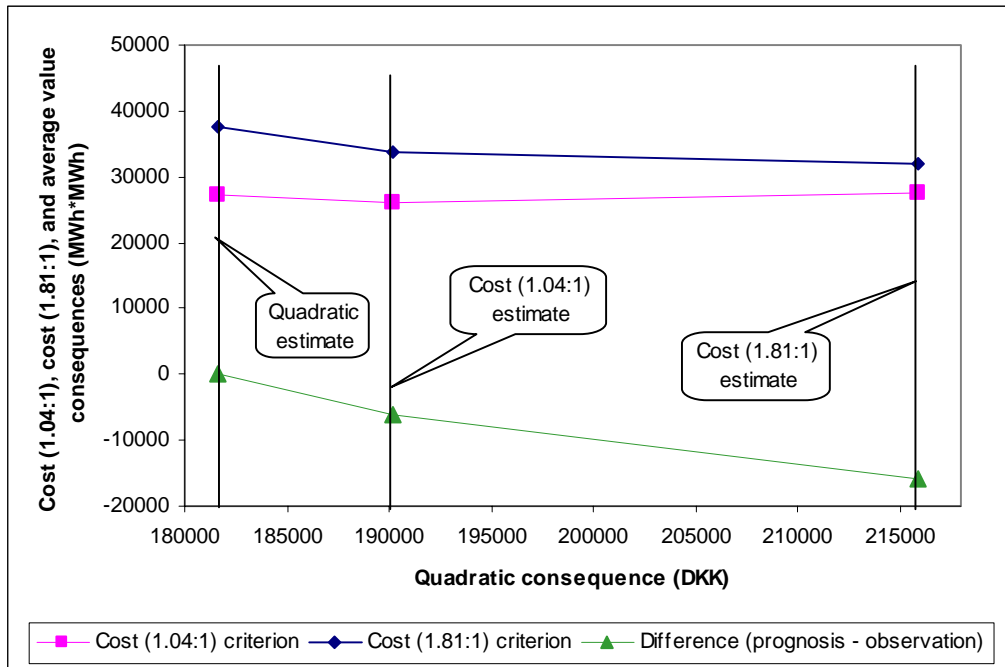
**Figure 8.** Estimated power curves and distribution of wind power, Taastrup.

Note, that the power curve for the quadratic criterion intersects both with the cost (1.01:1) and the cost (1.81:1) criteria. The intersection with the cost (1.04:1) criterion occurs around a forecasted wind speed value of 9 m/s, whereas the intersection with the cost (1.81:1) criterion does not occur until at a forecasted wind speed value of 12 m/s. In other words, at low forecasted wind speeds prognoses based on either cost criterion have a tendency to underestimate the production in comparison to a prognosis based on the quadratic criterion. At high forecasted wind speeds, the situation is the reverse.

The right skewness of the observations could be the reason that the power curves for the cost criteria lie below the power curve for the quadratic criterion. These are the wind speeds for which the power curves are upwards curving on Figure 8. A similar observation was made in [5] based on data from Nybølle in Eastern Denmark.



The consequences of using a specific criterion when estimating the power curve for prognosis purposes are illustrated in Figure 9.



**Figure 9.** Consequences of using a specific criterion when estimating the power curve for prognosis purposes.

Firstly, estimating the power curve using the quadratic (or average value) criterion incurs the consequences for the criteria indicated on the left vertical line. The value where this line intersects the x-axis is the sum of squares (181662) and the difference between the sum of the prognosis values and the observed values is naught. Next point on the vertical line is the value of the cost (1.04:1) criterion (corresponding to the Eastern Danish regulating prices), which is 27307. Lastly, the cost (1.81) criterion, which corresponds to the Western Danish regulating prices, attains a value of 37614.

When the power curve is estimated using the cost (1.04:1) criterion (i.e., Eastern Danish regulating prices – practically corresponding to the absolute value criterion), the consequences are indicated on the second vertical line. The quadratic consequence is 190146 where the second line intersects the x-axis, a higher value than when the power curve is estimated using the power curve is estimated using the quadratic criterion. The difference between the sum of the prognosis values and the observations has declined into the negative (-6245), which indicates that the prognosis on the average underestimates the observed values of energy. The cost (1.81:1) criterion practically maintains its value at 26141.

Finally, and this was done mostly as an exercise, the last vertical line indicates the consequences of estimating the power curve using the cost (1.81:1) criterion. The sum of squares is 215895 where the line intersects the x-axis, and the difference between the prognosis values and the observed values has decreased further (to -15749), indicating that the prognosis now underestimates the observed energy values to an even larger degree than the cost (1.04:1) criterion does. The very same criterion continues rises a little in this last case, attaining a value of 27429, and lastly the cost (1.81:1) criterion continues its decline, attaining the value 31837.

Once again, note that this last criterion has mostly been added for the sake of curiosity and, being based on the regulating prices of Western Denmark, perhaps does not pertain sensibly to this particular location (Taastrup, Eastern Denmark).

Be that as it may, it is still clear from the values mentioned in the previous paragraphs and from Figure 9, that the criteria are mutually conflicting, regardless of whether one wishes to take the cost (1.81:1) criterion into account or not. But, as mentioned in Section 2, both the quadratic and the cost criterion (as well as the absolute criterion, which is in fact implicitly considered in the cost (1.04:1) criterion) are relevant to the system operator. Thus, it might be prudent to consider finding the 'best' prognosis estimate a multi-criteria problem.

## 5. Concluding remarks

The paper has dealt with criteria in wind power prognosis. In particular two criteria have been considered, viz., the average value criterion and the cost criterion, related to forecasting with the objective of minimising the error in terms of energy and minimising the cost of acquisition of regulating power to compensate forecast errors, respectively.

The regulating prices for Eastern and Western Denmark were presented and used as a basis for the two criteria considered in this paper. It was shown that the power curves estimated differ according to the criterion that is selected, and the consequences in terms of prognosis error consequences were presented.

The analysis confirms that making short term wind power prognosis should be considered a multi-criteria problem.

## 6. References

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