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BESSEL FILTER AND CHAOS: THREE-IN-ONE ACTION

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Abstract—*Low-pass active Bessel filter is proposed to using in a chaotic oscillator. The Bessel unit plays the role of three-in-one: the delay line, the amplifier, and the filter. Results of Spice simulations and hardware experiments are presented.*

I. INTRODUCTION

A number of chaotic oscillators based on time-delayed feedback have been described in literature, e.g. [1-7]. The main units of a delay oscillator are the following: (1) a nonlinear element, (2) an amplifier, (3) a delay line, and (4) a low-pass filter. Various types of the delay line have been used to build an oscillator: (i) a coaxial transmission cable [3,4,7] most suitable for microwave range, (ii) an ultrasonic delay device [1] for kHz frequencies, (iii) so-called ‘bucket-brigade’ time-discrete tuneable device [2,6] for low frequencies from several tens of Hz to several tens of kHz. Another example is a high-order low-pass filter, composed as a network of T-type LCL passive circuits (an artificial delay line), operating up to 3 kHz and providing 3 ms delay [5].

In the present paper we suggest to exploit high-order low-pass active Bessel filter for the delay line. The main motivation behind such a choice is that the Bessel filter is characterized by linear phase shift versus frequency, i.e. by constant delay time. In addition, the Bessel filter when implemented as an active unit has inherent gain necessary for the oscillator. Finally, the Bessel device plays its natural role of a filter.

II. CIRCUITS

Circuit diagram of the Bessel filter is presented in Fig. 1. Other components of a delay oscillator are depicted in Fig. 2. While Fig. 3 and Fig. 4 show the characteristics of these units, respectively.

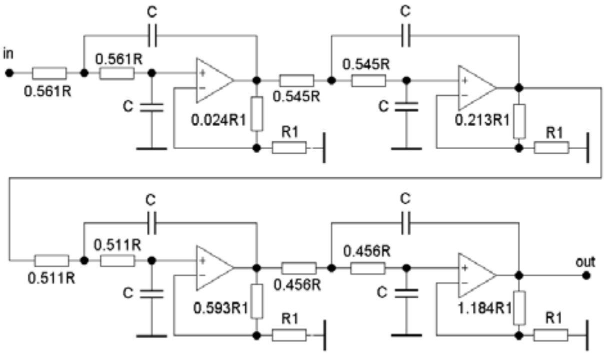
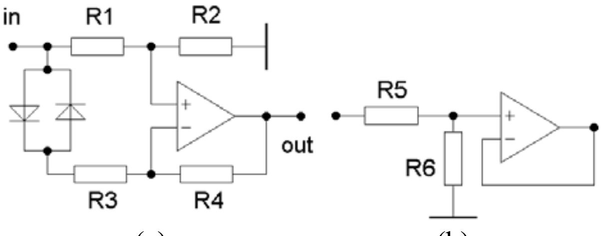


Fig. 1. Circuit diagram of the 8th-order active Bessel filter.



(a) (b)

Fig. 2. Circuit diagrams of the nonlinear unit (a) and the divider (b). Positive and negative slope of the nonlinear function are: $a = R_2/(R_1+R_2)<1$, $b = a(R_4/R_3+1)-R_4/R_3$. Transfer coefficient of the divider $\gamma = R_6/(R_5+R_6)<1$.

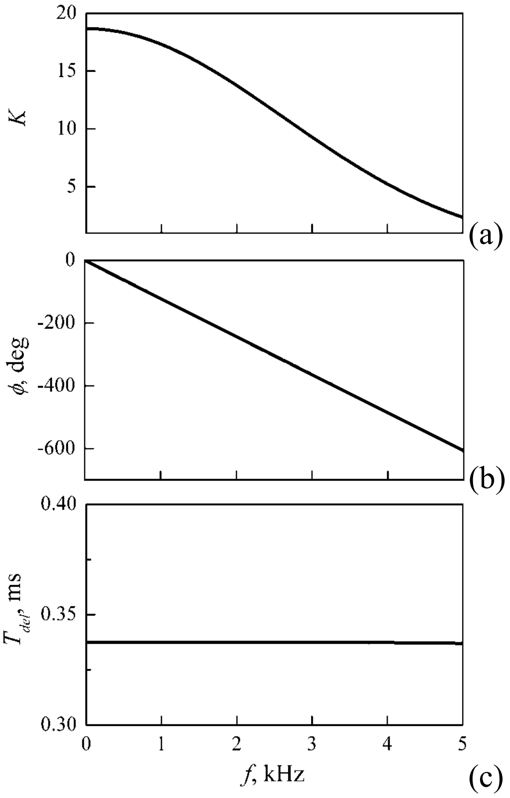


Fig. 3. Transfer characteristics of the Bessel filters (two 8th-order filters coupled in series): (a) gain $K(f)$, (b) phase $\phi(f)$, (c) delay time $T_{del}(f)$ *vs.* frequency. Circuit values of the Bessel filter: R =5.31 kΩ, C =10 nF, R_1 =10 kΩ.

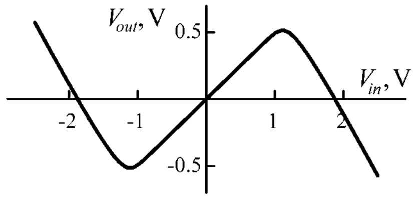


Fig. 4. Simulated transfer function of the nonlinear unit in Fig. 2(a). R_1=R_2=R_3=1 kΩ, R_4=3 kΩ (a = 0.5, b = –1.0).

Full diagram of the delay oscillator including the nonlinear unit (N), the divider (D) and two Bessel filters (BF) is depicted in Fig. 5.

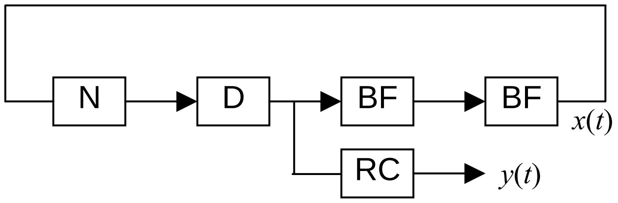


Fig. 5. Block diagram of the delay oscillator. Divider D is used to set a desired open-loop gain K_0=$\gamma K(0)$. RC unit is not in the oscillator's loop; it is an auxiliary first-order low-pass RC filter used to construct the phase portraits.

III. OSCILLATIONS

The oscillator in Fig. 5 has been simulated using the PSpice software based Electronic Workbench simulator. Various types of attractors controlled by the open-loop gain K_0=$\gamma K(0)$ are illustrated in Fig. 6. They range from periodic 1T and 2T attractors to chaotic mono-scroll and two-scroll attractors.

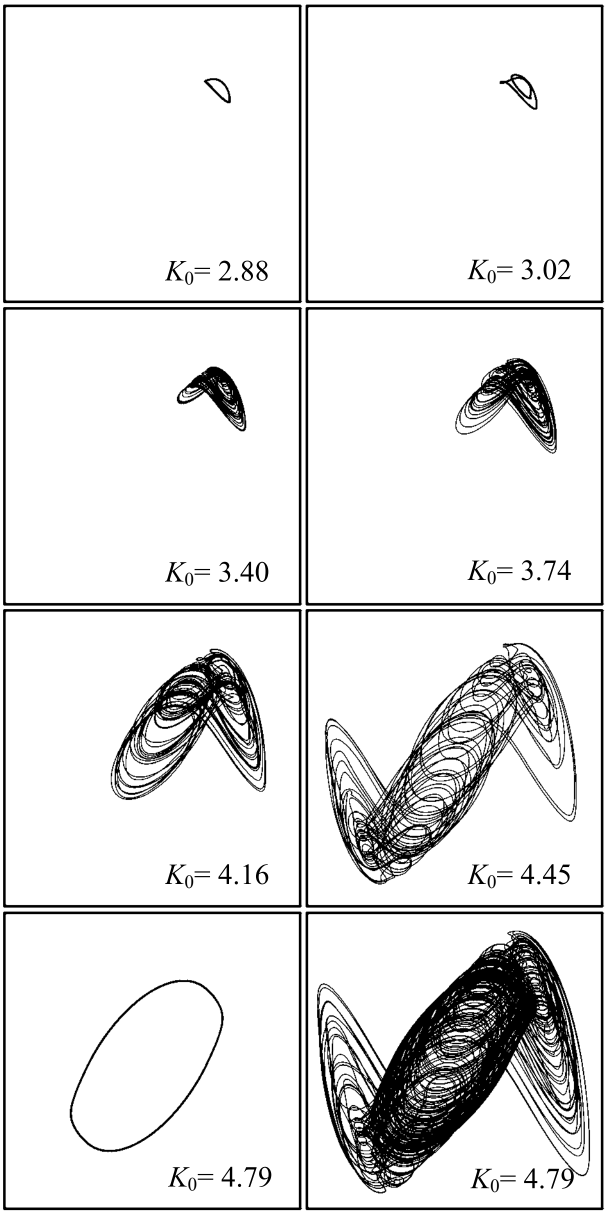


Fig. 6. Simulated phase portraits for the circuit in Fig. 5 at different gain K_0. The two bottom plots have the same gain, but in the right one a small DC bias of 4 mV is added to the input of the first Bessel filter. Horizontal – main output signal $x(t)$, vertical – auxiliary output signal $y(t)$. Circuit values are given in captions of Fig. 3 and Fig. 4. Time constant of the auxiliary RC filter is 75 μs.

IV. HARDWARE EXPERIMENTS

Hardware prototype of the oscillator has been built using the LM741 opamps and general-purpose diodes. Phase portraits are presented in Fig. 7 for different values of gain $K_0=\gamma K(0)$. The nonlinear unit is characterized by transfer function in Fig. 8.

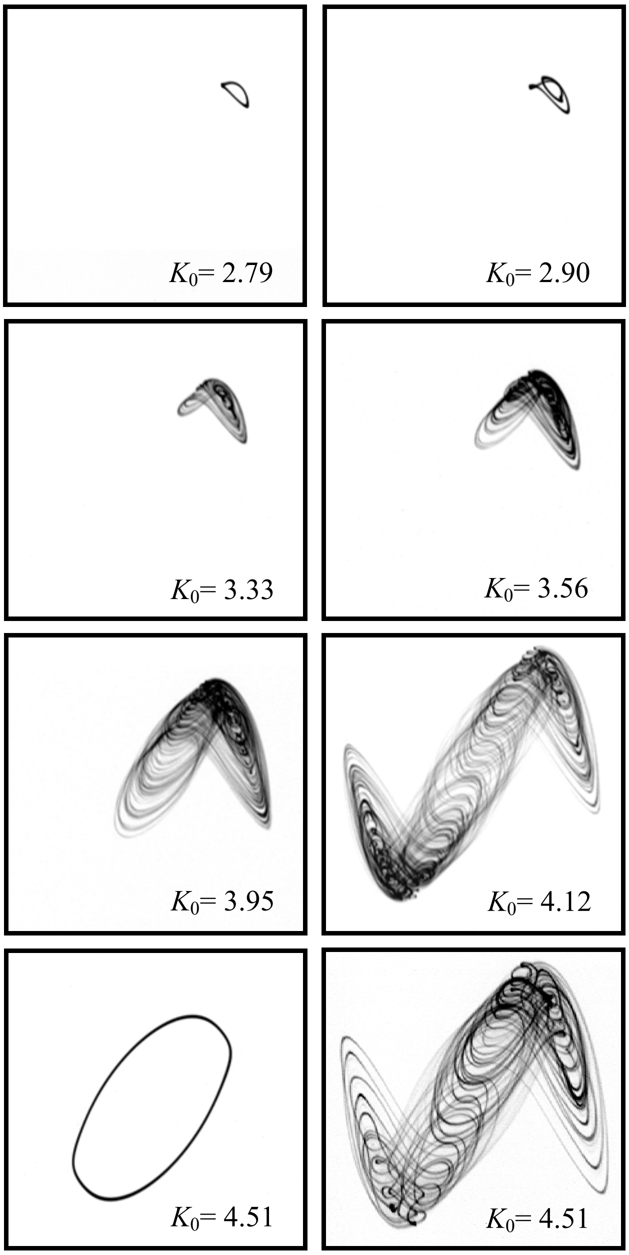


Fig. 7. Experimental phase portraits taken from the screen of an oscilloscope at different open-loop gain. The two bottom plots have the same gain, but in the right one a small DC bias of 4 mV is added to the input of the first Bessel filter. Horizontal – main output signal $x(t)$, vertical – auxiliary output signal $y(t)$. Other circuit parameters are the same as in Fig. 6.

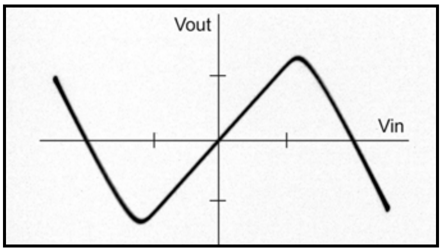


Fig. 8. Experimental transfer function of the nonlinear unit. $R_1=R_2=R_3=1$ kΩ, $R_4=3$ kΩ ($a = 0.5$, $b = -1.0$). Vertical scale 0.5 V/div., horizontal scale 1.0 V/div.

Spectral analysis has been carried out by means of a real time spectrum analyser and the results are presented in Fig. 9.

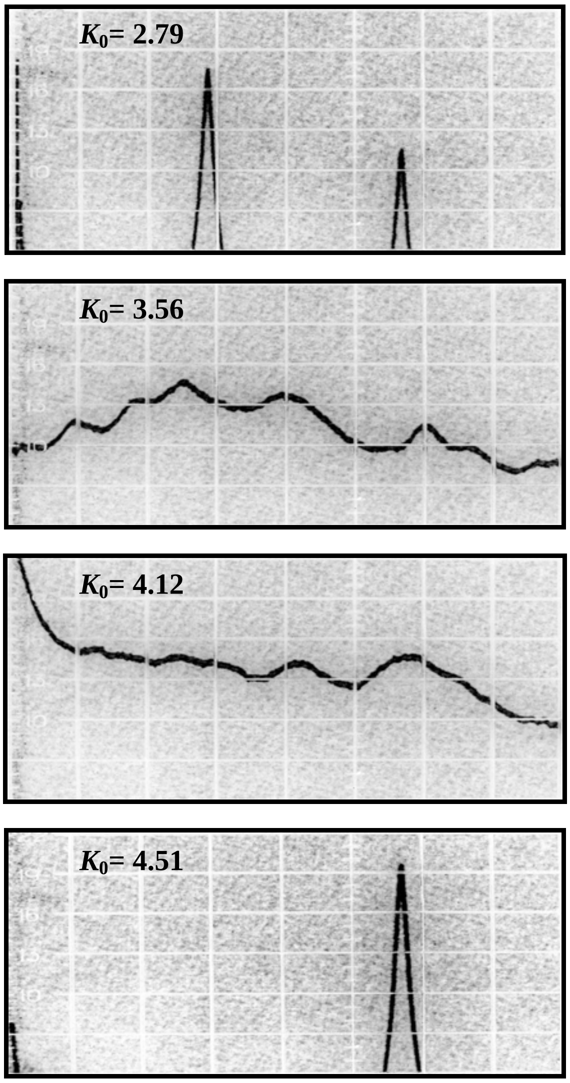


Fig. 9. Power spectra of variable $x(t)$ taken from the screen of a spectrum analyser at different values of open-loop gain K_0. Spectral range: 0…4 kHz. Horizontal scale: 500 Hz/div. Resolution: 30 Hz. Vertical scale: 10 dB/div.

V. DISCUSSION

The employed Bessel filter (two units in series) is characterized by low frequency gain $K(0)=18.7$, constant delay time $T_{del}= 0.34$ ms, and the cut-off frequency (at minus 3 dB level) $f_{th}=2.1$ kHz. The latter value corresponds to the cut-off frequency of a first-order RC filter with $R_0C_0=75$ μs. The effective dimensionless delay parameter $\tau = T_{del}/R_0C_0 \approx 4.5$, i.e. a moderate value for the common delayed feedback oscillators, like the Mackey-Glass system [5,8]. By keeping this parameter fixed and tuning the open-loop gain $K_0=\gamma K(0)$ various dynamical states can be observed. The system remains stable for $K_0<2.6$. For higher gains it exhibits 1T, 2T periodic oscillations evolving to mono-scroll and two-scroll chaotic attractors (Fig. 6, Fig. 7).

A distinctive feature of the Bessel filter based oscillator is the fact that the fundamental frequency $f^* \approx 1.5$ kHz (Fig. 9a), i.e. $f^*=1/2T_{del}$, in contrast to the conventional delay oscillators with the first-order filter where $f^*=1/3T_{del}$. Indeed, if we insert in the feedback loop an external first-order RC filter with time constant $RC > R_0C_0$ the f^* moves to 1 kHz, i.e. to $f^*=1/3T_{del}$.

Another interesting feature is the possibility to destroy 1T periodic attractor ($f^*=1/T_{del}$) emerging at certain gains (e.g. at $K_0=4.79$ in Fig. 6 or $K_0=4.51$ in Fig. 7) by means of adding only small DC bias.

In addition, we have carried out the experiments with a single Bessel unit. In this case low frequency gain $K(0)=4.3$, constant delay time $T_{del}= 0.17$ ms, and the cut-off frequency is $f_{th}=3.0$ kHz. The time constant of an equivalent first-order RC filter is $R_0C_0=53$ μs and the dimensionless delay parameter $\tau=T_{del}/R_0C_0 \approx 3.2$, that is somewhat lower than for two units in series. Nevertheless, various periodic and chaotic states can be observed by tuning $K_0=\gamma K(0)$, except the two-scroll attractors, because the maximal open-loop gain at $\gamma=1$ is $K_0=K(0)=4.3$ appears to be insufficient. An additional low-gain amplifier is necessary to achieve two-scroll chaotic oscillations with a single 8th-order Bessel filter.

VI. CONCLUSION

High-order (preferably the 8th- or higher-order) low-pass active Bessel filter is a good option for building delayed feedback chaotic oscillator. The Bessel unit acts like "three-in-one" device: delay line + amplifier + filter.

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