Beneath the Wheel - Greenwood Engineering

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Publication date:
2005

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Dias, K., Gravesen, J., Hjorth, P. G., Larsen, P., Please, C., Radulovic, N., ... Aagaard Pedersen, L. (2005). Beneath the Wheel - Greenwood Engineering. University of Southern Denmark.

Beneath the Wheel: Greenwood Engineering

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Figure 1: The Greenwood Working Group

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1 Introduction – The Bearing Capacity of Roads

In the maintainance of roads it is of great importance to know the surface properties, such as, roughness, rutting, friction and surface damage, and in particular the so called *bearing capacity* [1] of the road. The quantity can be estimated from data describing the deflection created by a heavy load. See the basic geometry in figure 2.

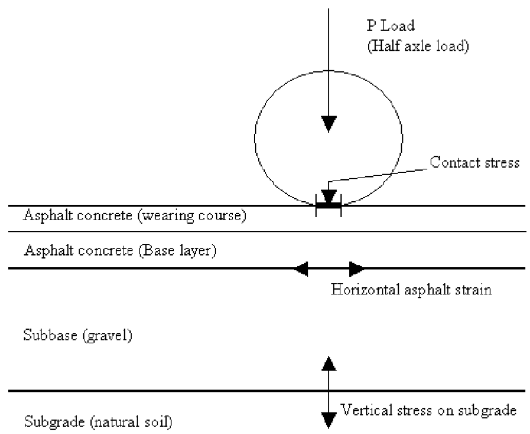

Figure 2: The basic geometry of the problem.

Until now, most bearing capacity measurements have been performed by stationary or very slow moving measuring equipment. This can disturb the traffic flow and result in dangerous situations. An example of such equipment

Figure 3: Traditional 'falling weight' deflectometer equipment.

is a so called Falling Weight Deflectometer (figure 3), which applies a force to the road corresponding to the force of a heavy loaded truck and then measures the pavement surface deflection. Based on these measurements and back calculation (see below), the deflection basin, the maximal deflection, and the Structural Curvature Index can be found with this equipment.

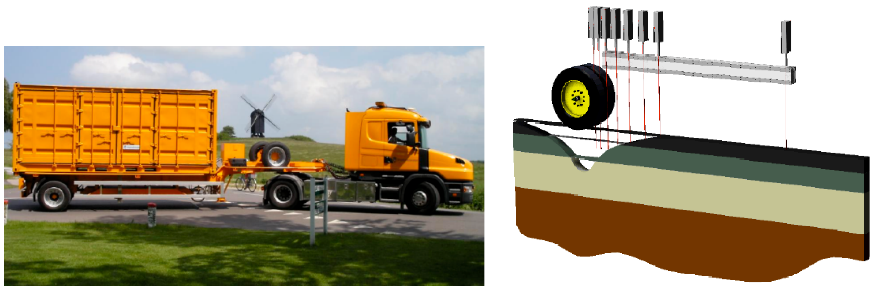

Figure 4: The High Speed Deflectograph (HSD) is able to perform measurements of road deflection from a vehicle travelling at normal traffic speeds.

Greenwood Engineering has developed and patented a new device which measures deflection of roads at high speed - the High Speed Deflectograph (HSD) (figure 4). The HSD operating continuously and moving at normal traffic speeds measures the deflection *velocities*. From these measurements the Structural Curvature Index can be calculated.

1.1 Definitions

A well known road quantity among road engineers is the *Structural Curvature Index* (SCI_{300}). It is defined as the difference

$$\mathrm{SCI}_{300} = d_0 - d_{300},$$

where d_0 is the deflection below the center of the wheel and d_{300} is the deflection 300 mm in front of the wheel, see figure 4 and 5. From SCI_{300} one can obtain the *Asphalt Strain* (AS) of the road, which can be converted into a measure for the remaining life of the road.

Greenwood Engineering is currently able to obtain SCI_{300} and AS, allthough they take $\mathrm{SCI}_{300} = d_{300}$, assuming that the integration constant is zero.

1.2 Problem description

ESGI54 is asked to model the deflection caused by a wheel. The shape and size of the deflection depends on the elasticity of the pavement construction layers. A typical deflection bowl can be seen on the figure 5 (below the horizontal axis).

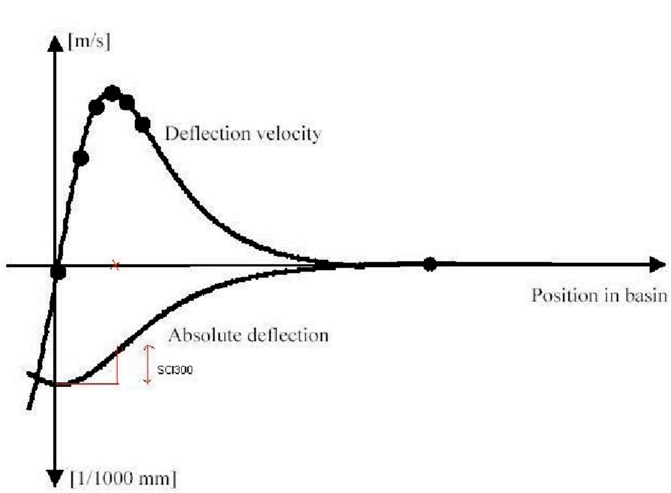

Figure 5: Curves of the absolute deflection and the deflection velocity as a function of the position in front of the wheel to the left. A curve of the slope as a function of the position in front of the wheel is just a scaling of the deflection velocity curve.

Currently, the Greenwood Engineering HSD employs 3 sensors (not 6 as suggested in figure 5) positioned at 100 mm, 200 mm and 300 mm in front of the wheel. The HSD measures the deflection velocities (v_d), by using doppler sensors. Using the relation

$$v_d = v_{dr}\beta \tag{1}$$

we compensate for the driving speed (v_{dr}) and obtain the slope β (mm/m).

The current treatment of the measurements is the following. Based on the slope, and the assumption that the deflection velocity is zero at (0,0), a curve is fitted. This curve is then integrated to obtain the relative deflection bowl. As sensors are positioned relativly close to the wheel, our obtained curve fits cover the range from 0 mm to 300 mm. It is expected that the curve should look approximately as the curve above the horizontal axis in figure 5.

1.3 BISAR

In the late 1960s, Shell Research developed the BISTRO (BItumen STructure in ROads) program, which calculates stresses, strains, and displacements at any position in a multi-layer system, where the number of layers is arbitrary (though typically no more than three), and the system is subjected to one or more circular normal loads. After further extensions the BISTRO program was renamed BISAR (BItumen Stress Analysis in Roads). Originally only suitable for mainframe computers, the BISAR program was in 1987 adapted for use on personal computers.

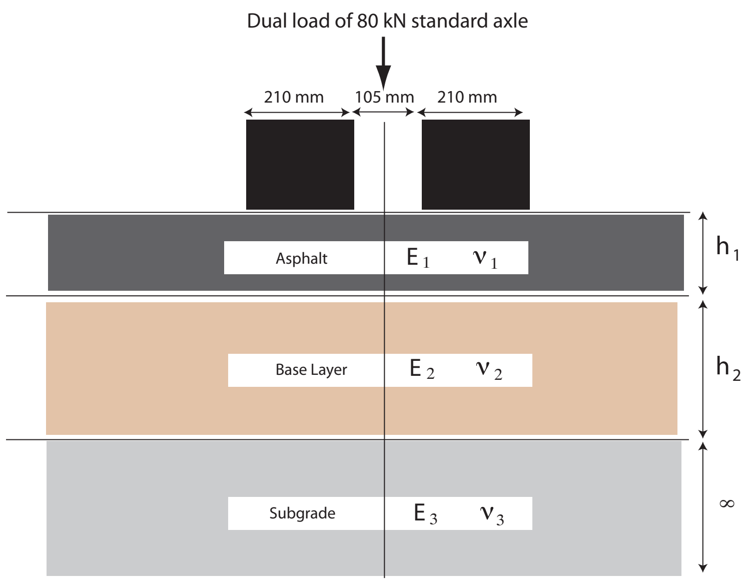

Figure 6: The model underlying the BISAR program is a multi-layer pavement model. Each layer is assigned a Youngs Modulus E and a Poission ration ν.

With the BISAR program, stresses, strains and displacements can be calculated in an elastic multi-layer system which is defined by the following configurations and material behaviour:

1. The pavement consists of horizontal layers of uniform composition, resting on a final semi-infinite half-space.
2. The layers are unbounded in the horizontal direction.

3. The materials are homogeneous and isotropic.
4. The materials are elastic with a linear stress-strain relationship.

Assumption 3 and 4 reduce the stress-strain relation to Hooke's Law. Assumptions 1 and 2 simplify the boundary value problem.

The BISAR program provides a good numerical model for most types of pavements, and can be used as a check of the analytical model.

1.4 The Questions

Based on our measurements ($(p_1, v_{d1}), (p_2, v_{d2}), (p_3, v_{d3})$, acceleration (x, y, z), data from the gyro, driving speed, temperature, etc.), known mechanical pavement models, and descriptive mathematical modelling, we would like to:

A. obtain a continuous function describing the deflection slope $V_{slope}(x)$ as a function of the positive position from the center of the wheel.
   - Which type of curve fitting?, Polynomial, B-Spline ?
   - Perhaps Boussinesq for distances $>$ 300mm

B. obtain a continuous function describing the absolute deflection $D_{bowl}(x)$ as a function of the positive position from the center of the wheel.
   - The deflection $D_{bowl}(0) =$ the maximal deflection under the wheel.
   - Perhaps based on back calculations through knowledge of pavement stess, strain and layer thickness.

C. obtain a continuous function Curvature(x) describing the curvature as a function of the positive position from the center of the wheel.

Other things to consider

- How the dynamic spatial load variation influences the measurements when driving on bumpy roads or in curves
- Estimate the number of sensors required to obtain a certain precision.

2 Study Group: A two-parameter Model

The objective for the Study Group is (see section 1.4) to propose a family of functions of sufficient complexity to provide a model of the physical process (i.e., the resilient deformation of the road surface) yet simple enough to permit analysis of the shape of the deflection velocity and also of the curvature of the deformation.

The measurements available for the Study Group have been taken at three points along the x axis: 100 mm, 200 mm and 300 mm. A mathematical model with many free parameters can generically be made to fit perfectly to a few measurement points. There should in general be strictly more data points than free parameters in the model. In view of this, the Study Group decided to consider a two-parameter model for the road.

2.1 Winkler foundation Model

This model consists of an elastic beam with Youngs modulus E supported on a base of a material of uniform elasticity characterised by a 'spring constant' k. This is a variant of a two-layer model where only vertical displacements and no compressions of the base material is modelled. The beam extends infinitely in the horizontal direction. See figure 7.

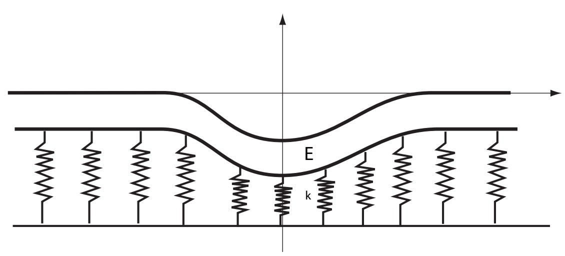

Figure 7: The Winkler foundation model. An elastic beam of thickness H and with Young modulus E rests on a perfectly elastic substrate characterised by a 'spring constant' k.

Suppose such a system is subjected to a point force of magnitude F in the

$-z$ direction. The resulting beam equation (see e.g., [2]) is:

$$E\frac{\mathrm{d}^4}{\mathrm{d}x^4}w(x) = -F\delta(x) + kw(x) \tag{2}$$

has boundary conditions that the displacement $w(x)$ vanish at $x \to \pm\infty$ and that $(\mathrm{d}/\mathrm{d}x)w(0+) = 0$.

This results in a two-parameter $(A > 0, B > 0)$ family of solutions for the displacement $d(x)$ which, without loss of generality, has the form:

$$d(x) = -\frac{A}{2B}e^{-Bx}\left[\cos(Bx) + \sin(Bx)\right] \tag{3}$$

Here A and B are connected with the model parameters in the following way:

$$E = \frac{F}{4AB^2I}, \qquad k = \frac{FB^2}{A}$$

or

$$A = \frac{F}{2\sqrt{kEI}}, \qquad B = \sqrt[4]{\frac{k}{4EI}}$$

The family (3) of d-functions displays the indentation which damps exponentially away and oscillates about the 0-reference level. See figure 8.

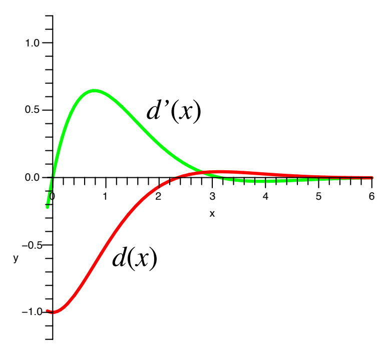

Figure 8: Representatives of the family of functions $d(x)$ and their derivatives $d'(x)$ (for positive values of x).

This family $d(x; A, B)$ of indentation function has the following properties:

$$d'(x) = A\sin(Bx)e^{-Bx}$$

$$d''(x) = AB(\cos(Bx) - \sin(Bx))e^{-Bx}$$

$$\text{Curvature} \quad \kappa(x) = \frac{d''}{\sqrt{1 + (d')^2}} = \frac{AB\left[\cos(Bx) - \sin(Bx)\right]e^{-Bx}}{\sqrt{1 + A^2\sin^2(Bx)e^{-2Bx}}}$$

Using this family of curves (which are smooth for $x > 0$) we can provide an answer to the set of questions posed by GREENWOOD.

In terms of $d(x; A, B)$,

- The maximal indentation, at $x = 0$: $d(0) = -\frac{A}{2B}$
- The maximal slope, occurring at $x = \frac{\pi}{4B}$ is $d(\frac{\pi}{4B}) = \frac{e^{-\pi/4}}{\sqrt{2}}A$
- The curvature at $x = 0$ is $\kappa(0) = AB$

Remark. Using a δ-function as the force term corresponds to constructing the Green's function for the problem. A more realistic model should integrate $d(x; A, B)$ over a line segment (the 'footprint' of the tire, assuming, say, an even force distribution throughout the footprint). This should provide a more accurate model of the indentation resulting from an extended load.

2.2 Comparison with Measurements and BISAR data

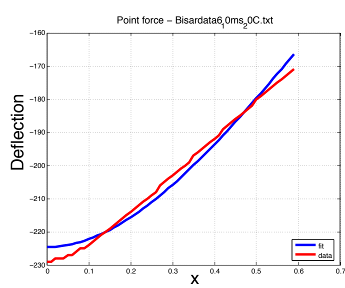

Figure 9: Comparison between the point force elastic beam model function $d(x; A, B)$ and numerical computation data from the BISAR program, assuming a 3-layer model. Blue = model, Red = data

Figure 9 shows a comparison between the best (parameters A and B chosen by a least squared method) elastic beam method function $d(x; A, B)$ and the curve constructed from the output of a BISAR computation assuming a 3-layer model.

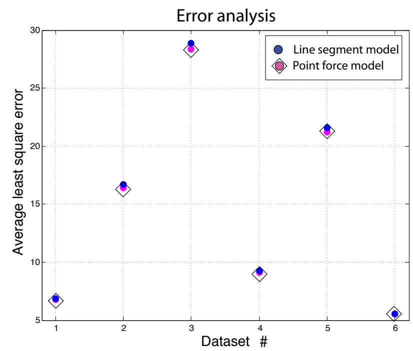

Figure 10: Comparison between the least-squared errors of the point force model and the line-integration model, relative to BISAR data. Note the two methods give very nearly the same error.

As remarked at the end of the previous section, one would expect an integral over an interval (in 2D an integral over an area) better to model the shape resulting from a tire pressing down on the surface. Initial numerical experiments however indicate that this is not the case. Figure 10 shows the difference in the average error (relative to a data from BISAR output) comparing a best point force fit with a best line integral fit. The two calculations have very nearly the same errors.

A more systematic study of the $d(x; A, B)$ function fit to BISAR data was carried out. Figure 11 shows a typical comparison between numerical output from the BISAR program and the best fit of $d(x; A, B)$ function (labelled APPROX) to the data. The 'best fit' is based on a numerical minimization. The minimization problem is one-dimensional, based on an error term E of form

$$E = \sum_{n=1}^{N} \frac{\sum_{n=1}^{N} B_n \chi_n}{\sum_{n=1}^{N} \chi_n^2} (\chi_n - B_n)^2$$

where B_n denotes the BISAR data points at positions x_n and χ_n are defined as $\chi_n = e^{-Bx_n}(\cos(Bx_n) + \sin(Bx_n))$, while A has been separately determined from

$$A = \frac{\sum_{n=1}^{N} B_n \chi_n}{\sum_{n=1}^{N} \chi_n^2}$$

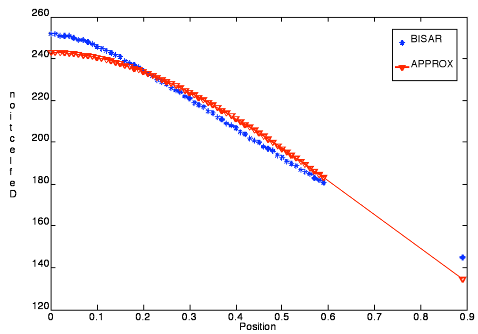

Figure 11: Near $x = 0$ best fit between the BISAR predicted indentation (shown positive) and the $d(x; A, B)$ indentation function.

Figure 12 shows four comparisons between HVD deflection velocity data (blue) and the deflection velocity (red) predicted by the $d(x; A, B)$ functions.

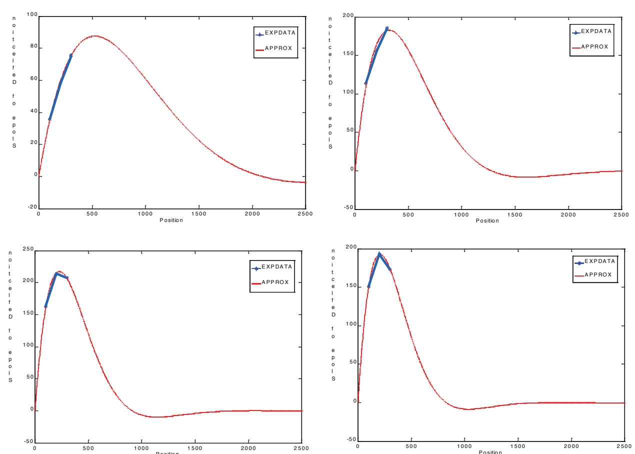

Figure 12: Four comparison between measured deflection velocity data and best fit to the data of the deflection velocity predicted by the $d(x; A, B)$ function.

In figure 12, the continuous deflection velocity curve has been obtained from a paramter fit. Again, given the deflection data points D_n, the parameter A has first been taken as

$$A = \frac{\sum_{n=1}^{N} D_n \zeta_n}{\sum_{n=1}^{N} \zeta_n^2}$$

where $\zeta_n = e^{-Bx_n} \sin(Bx_n)$ are the model predictions, and then B is determined by minimizing the error function

$$E = \sum_{n=1}^{N} \frac{\sum_{n=1}^{N} D_n \zeta_n}{\sum_{n=1}^{N} \zeta_n^2} \left(\zeta_n - D_n\right)^2 \ .$$

Note from figure 12 that the deflection data are only on one side of the maximum of the deflection velocity curve. A better fit, and also a more critical test of the model, would be obtained with more sensor points at at greater distance from $x = 0$.

3 Suggestions for future work

An increase in the number of sensors on the HSD will open up for a more accurate modelling. By replacing the 'beam' in the Winkler Model by a layer characterised by both a Young's Modulus E *and* a Poisson ration ν, one may be able to develop an analytic, or perhaps semi-analytic more accurate 3-parameter model.

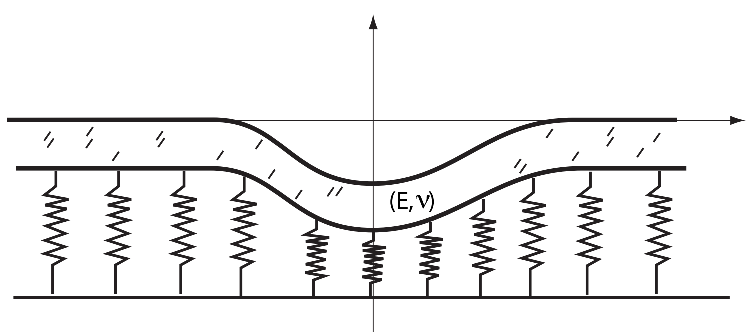

Figure 13: A three-parameter 'Elastic Layer' model.

In a model of this type one cannot expect the deflection d to have the simple paramter dependence of the Winkler foundation model. On the other hand, the perhaps unrealistic oscillations around the unperturbed surface level inherent in the Winkler model might not be present in this Elastic Layer model.

4 Conclusion

In this report we have discussed a simple analytical model, the 'Winkler foundation' model, resulting in a 2-parameter explicit family $d(x; A, B)$ of functions modelling the deflection of a point force as a function of distance x from the impact point.

We have observed very little change in the indentation shape when the point force was integrated over a rectangular 'footprint'.

The family $d(x; A, B)$ are continuous (even twice continuously differentiable), and thereby permit explicit formulas predicting the maximal deflection, and also the curvature as a function of distance x from the wheel impact point.

We have compared the model prediction to simulated (BISAR) data and conclude that we can obtain very good agreement.

We have compared the model to HSD experimental data and found that we can fit all sets of three data point quite well with the 2-parameter family of functions.

We propose (but do not solve) a three-parameter model which allows for compression of the top layer.

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