GENDER DIFFERENCES IN MATHEMATICS PERFORMANCE

Analysis of attainment and attitudes in mathematics of girls and boys; detailed appraisal of theories and pressures that influence girls' underachievement and underparticipation in the subject

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Thesis Abstract

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Statistics show that boys perform better in mathematics tests than girls. In order to make a refined assessment of the magnitude of gender differences in mathematics performance, a study was made of one thousand 16+ mathematics scripts to find the precise topics on which girls and boys differ significantly in performance. These concepts were found to be concerned with scale or ratio, spatial problems, space-time relationships and probability questions.

Differences were found in performance between girls and boys at each ten-percentile level through the ability range. A longitudinal study also revealed differences in mathematics performance through the years of secondary education. There is no convincing evidence that the discrepancy can be accounted for by innate or genetic reasons. Intervention programmes have been found to improve the performance of girls in the weak areas of spatial awareness, proportionality and problem solving.

In addition, a study was made of gender attitudes towards mathematics. Ten secondary schools were surveyed and the results revealed a marked decrease in the attitudes of third and fourth form girls. During these difficult adolescent years girls and boys are susceptible to strong internal and external pressures. Corresponding differences were also found across the ability range. These social pressures are concerned with teacher influence, social interaction, type of grouping, sex stereotyping, choices, teaching materials and careers advice.

Gender Differences in Mathematics Performance

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Gender Differences in Mathematics Performance

1. Introduction

Since Lucy Sells (1976) identified mathematics as the 'critical filter' that prevented many women from having access to higher paying, prestigious occupations, there has been much rhetoric and many investigations focused on gender differences in mathematics performance. The question of girls' achievement in mathematics provided the focus for an appendix in the Cockroft Report (Shuard, 1982) and is now a well recognised issue of public concern.

Few subjects in the school curriculum are as important to the future of the nation. This value and usefulness is perceived in different ways. For many it is seen purely in terms of the basic arithmetic skills which are needed for use at home or in the office or on the shop-floor. Some see mathematics as the basis of scientific development and modern technology while others emphasise the increasing use of mathematical techniques, as a management tool in industry and commerce. All these perceptions arise from the fact that mathematics provides a means of communication which is powerful, concise and unambiguous. It must be seen as a discipline which is important both in one's place of employment and in everyday life. There can be no room for discrimination on the grounds of class, race or gender.

Accordingly, mathematics is compulsory in the United Kingdom for virtually all pupils up to the age of sixteen. It is seen by educationalists as an important basic subject. Indeed, it forms one of the core subjects in the National Curriculum. It occupies a comparatively large share of the timetabled curriculum. In secondary schools, it is common for five periods of mathematics to be taught in a forty period week or four periods in a thirty five period timetable. The National Curriculum gives guidance on the time to be made available for mathematics. It suggests an average of 20% in primary schools and 10% for secondary pupils in years 1-5. However, the Council recognises that this is not a matter for Orders and that the guidance allows for some flexibility. In the consultative documents, suggestions were made to increase the 10% to 12.5%, in line with current practice (National Curriculum Council, 1988).

In addition, many mathematical concepts are extended or reinforced in cross-curriculum studies such as the sciences, technology or applied humanities. Its relevance to school and post-school life is equally great for girls and boys.

It is therefore with good reason, that educators at all levels and employers have become increasingly concerned in recent years over the relative underachievement of girls in mathematics, notably from age 11, compared with their male contemporaries. The disparity between success of the sexes in mathematics examinations is acute. In 1984, there were only ten girls achieving good grades (A, B and C) to every fifteen boys at the age of 16. In 1987, the figures were ten girls to every twelve boys (D.E.S. statistics).

This situation must be regarded as unsatisfactory and worthy of further investigation. The problem is exacerbated by the knowledge that over <u>all</u> subjects at 16, girls are performing better than boys in external examinations.

It is the aim of this study to investigate the relative achievement in mathematics of girls and boys and to identify and examine specific concepts which give rise to the greatest discrepancy of scores. The study surveys the theories and observations and outlines the many and varied reasons suggested for the under-representation of girls in the subject. At any one stage of a pupil's mathematical education many causative factors interact to influence relative mathematical achievement. However intricate the reasons, it is disturbing that girls are underachieving to a significant degree within an educational system with 'apparently' equal opportunities.

Teaching styles, methods of classroom interaction, the examination syllabuses and modes of assessment are all influential in establishing patterns of learning. These, together with the attitudes and affective beliefs held by girls, their attitude to themselves and to the subject, their interaction with and expectations of their teachers, parents and peers form other important variables. The cognitive and affective components are so intertwined that it is sometimes difficult to separate them. They are also developed over a period of years in a complex social matrix which involves home, community and school.

Many important questions need to be addressed. For example, what differences exist in performance between the sexes? What specific concepts give rise to the greatest differences in raw scores? Also, in the light of equal opportunity drives, what relative improvements have been made in recent years? Are the gaps getting smaller?

The first part of this study aims to answer these basic questions. Consideration is then given to differences in attitudes and affective variables held by the sexes and the influence these may have on overall performance. First, however, it is important in order to establish the correct setting, to review the mathematical policies in education from a historical perspective.

2. Historical Review of Educational Policies

Mathematics established itself in the curriculum of boys' public and secondary schools, in the first half of the 19th century. Although pioneers of girls' education wished to introduce mathematical studies into their curriculum, the forum of public opinion was against them. Professor Maurice suggested in a lecture in 1848 that 'women students were unlikely to advance far in mathematics' (Special Reports on Educational Subjects, Vol. 26). He did acknowledge however, that there were positive benefits which girls would gain from its study.

The impetus to the teaching of mathematics and arithmetic in particular, in girls' secondary schools came in 1863 with the opening to girls of the Cambridge Local Examinations. Of the first 25 candidates from the North London Collegiate School, 10 failed in arithmetic. This alarmed the then headmistress Miss Buss, and arithmetic at once became a matter of extreme importance in her school.

Only three years later, girls were doing as well in arithmetic as in other subjects in the Cambridge Local Examinations and when substantial numbers of girls' secondary schools were founded after 1873, mathematics became a regular subject in the curriculum.

In the public elementary schools, both girls and boys studied arithmetic but the Royal Commission on the Elementary Education Acts reported in 1888 that since girls' time was largely devoted to needlework, the time they could give to arithmetic was less than that given by the boys. They therefore recommended that the arithmetical requirements of the curriculum should be modified in the case of girls.

An important contribution to the debate was made in 1912 by the British Board of Education. The Board issued a report on the teaching of mathematics which included four papers discussing mathematics education with reference to girls. The views expressed in these papers differed on the desirability for girls to study mathematics beyond basic arithmetic. Many influential educators both male and female were of the opinion that mathematics had no interest to most girls, that its utilitarian value to them was negligible, and that its difficulty put a strain on pupils out of all proportion to the benefit received. Presumably, the lack of interest was in part due to its apparent disregard of relevant value. There were others who did not agree with these assertions and their objections are recorded in the 1912 Report.

It was argued that mathematics offered unique opportunities to the teacher for recognising and encouraging independent thought. Girls needed a 'greater stimulus' to independent thought than boys. The weakness of girls was that they submitted to 'so much dullness without resentment'. 'Many girls who are apparently good workers are really mentally lazy, they reproduce, but they do not produce. A teacher needs to be alive to this danger, and to realise that it is her business to stimulate intellectual curiosity, the desire to know, and not only to know, but to find out at first hand. She has every opportunity in mathematics.' (Special Reports on Educational Subjects, Vol. 26).

In fact, as Shuard (1982) points out, the reasons given for girls' need for mathematics as part of their education in 1912, are now advanced as reasons for their failure to perform better at the subject.

In 1923, the Consultative Committee of the Board of Education reported on differentiation of the curriculum for girls and boys. The regulations for secondary schools supported the girls' study of science and mathematics over 15 years of age. Yet, out of 230 advanced courses in mathematics and science provided in secondary schools, less than one quarter were in girls' schools.

The Committee did note that the degree of girls' inferiority in mathematics should not be regarded as permanent. They cited 'teaching of an old-fashioned kind' and 'impressions among parents' as reasons for the view that mathematics was unsuitable for girls.

Suggestions for the Guidance of Teachers were issued by the Board of Education at intervals between 1905 and 1937. These suggestions were addressed to teachers in elementary schools, including in the 1937 edition, the senior schools which were the forerunners of the secondary-modern schools. In the early issues there was little suggestion that the arithmetic curriculum should differ for boys and girls but as standards rose and children stayed longer at school, comment on curriculum differences for the sexes began to appear.

From 1918 onwards, it was suggested that the sex of the pupil affected to some extent the treatment of subjects in the curriculum. These differences in the education of girls and boys seem closely linked to the division of labour in both the home and the labour market. It was suggested that girls should deal with accounts accompanying shopping and housekeeping while boys establish by experimental methods some of the more important geometrical theorems.

In 1927, it was noted that in the senior classes girls spent less time on mathematics and were less likely to be using it in other subjects. However, it was acknowledged that the work missed in other subjects such as scale drawing and practical measurement was a reason for doing more of such work in mathematics and threw more responsibility on the teacher for providing a basis of reality for girls' mathematical work.

By the next edition of the handbook in 1937, the education of girls and boys had come closer together and for the first time was noted the important observation that the range of difference in either sex, was greater than the difference between the sexes. It also acknowledged a greater equality in intellectual interests and in mental capacity, though the report was still reserved about future roles of men and women in society.

Since this time, there has been a gradual change in attitudes. The 1944 Education Act gave pupils a greater equality of opportunity, though it is debatable if it actually intended to extend the notion of equality to gender. No longer are girls banned from subjects such as woodwork, metalwork or technical drawing, or boys from subjects such as domestic science or child care. Craft, design and technology (C.D.T.) is all embracing. Public examinations are equally available to girls and boys. Mixed comprehensive schools are now a feature of our education system with the apparent non-discriminatory advantages which they enhance.

The Sex Discrimination Act of 1975 also brought education within its scope. There is a much greater awareness within the United

Kingdom and indeed within the European Economic Community to equate the numbers of men and women in different spheres of mathematical study. Special funds have been made available, for example, to train women in areas of engineering and science.

At Ruskin College, Oxford, in 1976, the then Prime Minister, James Callaghan, launched the 'Great Debate' in which he made reference to the needs of industry and the production of a curriculum which led 'talented young people into science and engineering'. As part of this concern, he posed the question as to why such a high proportion of girls abandon science before leaving school. The concern about girls' performance in mathematics can hardly be said to result solely from that moment, but it provides a way of understanding the background to the debate.

Britain has a tradition of research on inequality groupings. Early post-war work concentrated almost entirely on social class, defined as the wastage of talent and the problems of upward mobility for working class boys. The climate of the economic crisis of the 1970s together with the more recent birth of the women's liberation movement, created conditions which searched for industry and talent. Girls could no longer be ignored as previous work had done.

Following the Education Reform Act 1988, school governors have increased powers over the appointment of staff and the allocation of resources. In the future, governors as well as Local Education Authorities (L.E.A.) will have a vital role to play in promoting equal opportunities in education. With the introduction of the National Curriculum, mathematics now has a clearly defined content for all pupils up to 16 year olds. Also, pupils will not be able to opt out of science or foreign languages in the early years of secondary school. These subjects will be compulsory until 16.

In the job market, it seems that with the fall in the birthrate, women will be increasingly sought after to return to full-time employment in the 1990s. This may particularly target women who have had their families and are available for employment. For many of these jobs, a good, basic grounding in mathematics will be important. Women who give up careers in favour of having families and looking after the home, may well find they are persuaded to return to either part-time or full-time employment. Well-educated women who return to employment will clearly be a great asset to the economy of the country.

From the historical perspective, there are clear chronological landmarks of a development towards greater equality in the teaching of mathematics. Given this greater awareness of equal opportunity and the 'apparent' implementation of equal opportunity policies in mixed schools, what are the differences that exist now? What exactly is the gap between the sexes in examination results?

3. Contemporary Picture

a. Examinations

Information about the General Certificate of Education (GCE) showed that more boys than girls entered for Ordinary level (O-level) mathematics and that they achieved better results. In 1979, in England, boys formed 56% of the entry to O-level. In 1985, boys formed 52% of the entry, and 60% of these gained a higher grade (A to C) as opposed to 52% of the girls. This meant that girls formed only 44% of those gaining higher grades. However, these figures referred to all candidates including those from colleges of further education and overseas, and may not have reflected the position in schools accurately.

The discrepancy between boys' and girls' results is nevertheless, getting smaller. In 1974, in England, the ratio of boys to girls obtaining a higher grade (A to C) at O-level, Certificate of Secondary Education (CSE) and 16+ examinations was 1.76. In 1984, the ratio was 1.51 (see Tables 1-4). By 1987, this ratio had reduced to 1.20. However, it is still cause for concern.

A leavers' survey by the Department of Education and Science (DES) sampled 10% of school leavers. This showed that, subject to sampling error, 27.9% of all girls leaving school in 1985 obtained grades A-C in mathematics at O-level or grade 1 in CSE, not necessarily at their first attempt. The comparable figure for boys was 33.3%, and these percentages remained unchanged in 1987. Detailed figures are given in Table 5.

Discrepancies between the proportion of boys and of girls gaining a particular grade were greatest at the extremes, with more boys achieving a grade A or grade B, and more girls a grade 4 or grade 5. In 1985, 19% of the boys leaving school had not attempted mathematics at O-level or CSE, compared with 17% of the girls; the comparable figures in 1987 were 20% and 17% respectively.

Table 1. Passes at O-level, GCE, CSE and 16+ in England. Numbers in thousands.

Year	English Language		Mathematics	
	Boys	Girls	Boys	Girls
1974	102.6	130.3	103.9	59.2
1975	106.96	145.6	107.8	61.2
1976	109.6	146.4	115.3	66.7
1977	113.1	150.2	123.2	71.3
1978	114.2	154.4	129.3	76.9
1979	116.7	157.6	134.5	83.4
1980	114.5	154	138.6	89.1
1981	115.4	158.2	142.9	95.5
1982	121.2	159.7	149.3	98.8
1983	118.2	157	154.0	102.1
1984	118.8	159.4	157.4	104.2

(Source: DES Statistics)

An O-level pass is taken to mean grade A, B or C. Also included are CSE grade 1. The percentage increases from 1979-1984 are Boys' English: 1.8%, Girls' English : 1.1%, Boys' Maths : 17.0% and Girls' Maths : 25.0%.

Table 2. Ratio of boys to girls obtaining an O-level grade A, B, C or CSE 1 in mathematics in England.

Year	Ratio b:g	Year	Ratio b:g
1974	1.76	1980	1.56
1975	1.76	1981	1.50
1976	1.73	1982	1.51
1977	1.73	1983	1.51
1978	1.68	1984	1.51
1979	1.61	1985	1.20

(adapted from DES Statistics)

Table 3. Passes at A-level, GCE in England. Numbers in thousands.

Year	Mathematics		Physics		Chemistry		Biology	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
1974	33.6	9.5	22.6	5.1	16.1	6.6	11.01	10.6
1975	34.7	10.0	22.98	5.1	16.1	6.6	10.98	10.89
1976	38.3	10.2	23.0	4.9	16.5	7.0	11.7	10.9
1977	37.4	10.9	25.2	5.5	18.9	7.9	12.7	12.8
1978	39.8	11.8	26.5	5.8	19.7	8.3	13.1	14.15
1979	42.5	13.4	27.8	6.5	20.8	9.3	12.5	14.5
1980	42.6	14.8	28.77	6.9	20.7	10.1	12.5	15.7
1981	45.9	16.5	29.6	7.4	21.8	10.8	12.6	16.4
1982	46.6	18.2	30.6	7.9	22.6	12.0	12.4	17.7
1983	48.5	19.5	30.8	8.1	22.96	12.5	13.3	19.1
1984	49.2	20.1	31.1	8.2	22.7	12.98	12.95	18.95

(Source: DES Statistics)

The percentage increases from 1979-84 are Boys' Maths 15.9%, Girls' Maths 49.3%, Boys' Physics 11.9%, Girls' Physics 27.1%, Boys' Chemistry 9.3%, Girls' Chemistry 39.7%, Boys' Biology 3.7%, Girls' Biology 30.8%.

Table 4. Ratio of boys' to girls' passes in A-level, GCE in England.

Year	Maths	Physics	Chemistry	Biology
1974	3.54	4.43	2.44	1.04
1975	3.47	4.51	2.44	1.01
1976	3.75	4.69	2.36	1.07
1977	3.43	4.58	2.39	0.99
1978	3.37	4.57	2.37	0.93
1979	3.17	4.28	2.24	0.86
1980	2.88	4.17	2.05	0.80
1981	2.78	4.00	2.02	0.77
1982	2.56	3.87	1.88	0.70
1983	2.49	3.80	1.84	0.70
1984	2.45	3.79	1.75	0.68

(adapted from DES Statistics)

Table 5. Best grades in O-level or CSE mathematics as a percentage of all leavers, England 1985 and 1987.

O-level or CSE	1985		1987	
	Boys	Girls	Boys	Girls
A	7.6	4.0	7.3	4.3
B	10.1	7.9	9.5	7.6
C	10.8	10.8	11.4	11.0
1	4.8	5.2	4.3	4.8
All 'higher grades'	33.3	27.9	32.5	27.7
D or 2	8.8	9.2	9.1	9.6
E or 3	11.9	12.7	12.6	13.4
4	12.8	15.4	13.2	15.3
5	7.7	9.8	7.2	9.3
All 'lower grades'	41.2	47.1	42.1	47.6
U or failed	6.2	7.9	5.6	7.3
Total entry	80.7	82.9	80.2	82.6

(Source: DES 10% School Leavers' Survey)

Table 6 shows the percentage distribution in the General Certificate of Education examination in mathematics in 1988. Here 40.2% of the boys are gaining high grades (A-C) compared to 33.1% of the girls.

Table 6. Grades in GCSE mathematics (mode 1), 1988.

	Percentage of candidates gaining grade						
	A	B	C	D	E	F	G
Girls	4.8	7.7	20.6	16.4	16.3	16.4	7.8
Boys	7.4	9.6	23.2	15.4	16.4	14.2	6.2
Total	6.0	8.7	21.9	15.9	16.3	15.3	7.1

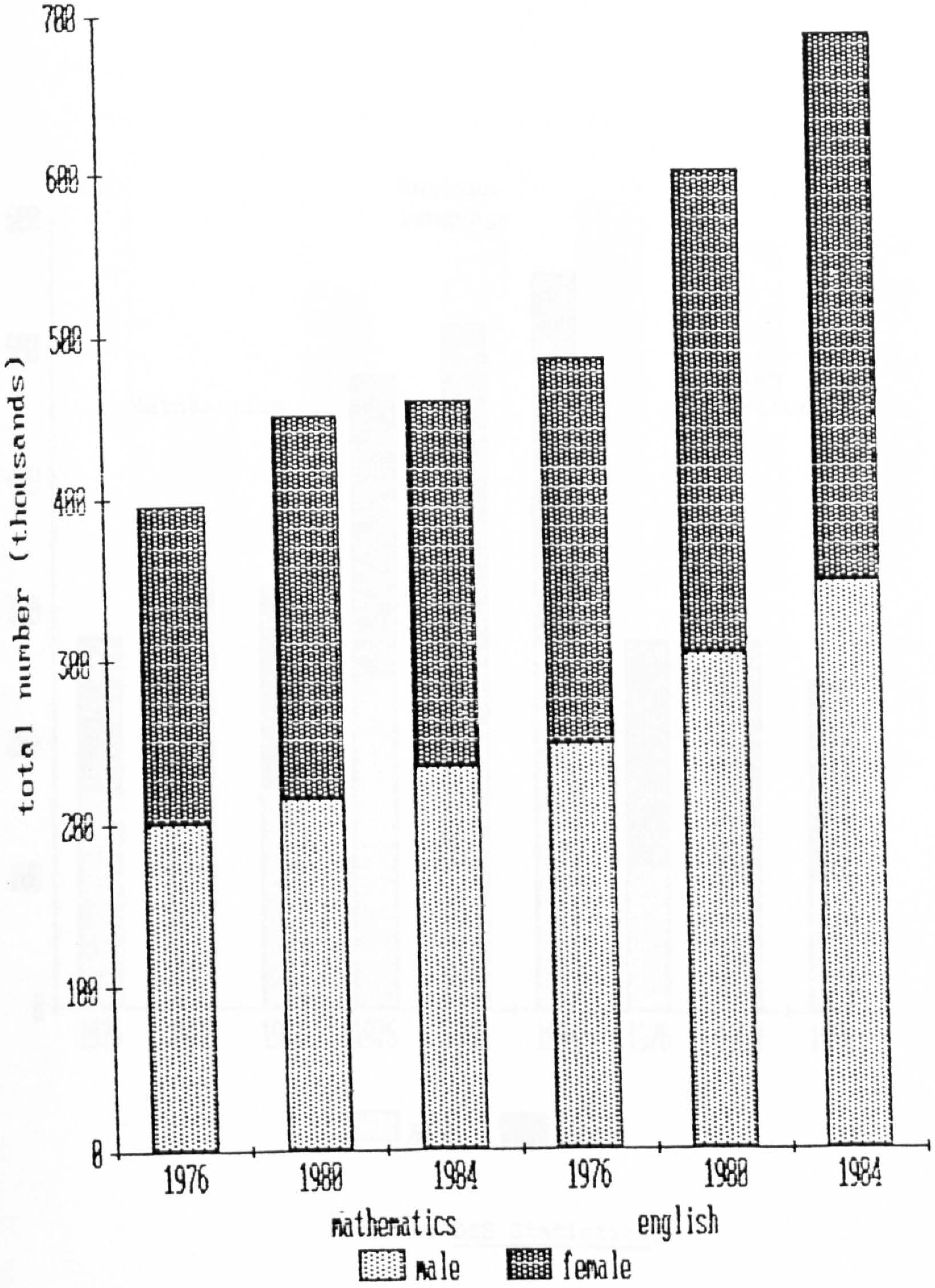
(Source: DES Statistics)

These are the first set of statistics relating to the new GCSE examination in mathematics at 16. They show a boy to girl ratio of those pupils gaining high grades (A-C) of 1.21. This compares to the 1987 figure of 1.20. So, there has been no improvement in the discrepancy of boys' to girls' results with the introduction of the new examination at 16. There are still twelve boys gaining high mathematics grades to every ten girls.

Figures 1-3 show the total numbers of girls and boys entering CSE, GCE O-level and A-level examinations in England and Wales in 1976, 1980 and 1984 (English and Welsh examination boards).

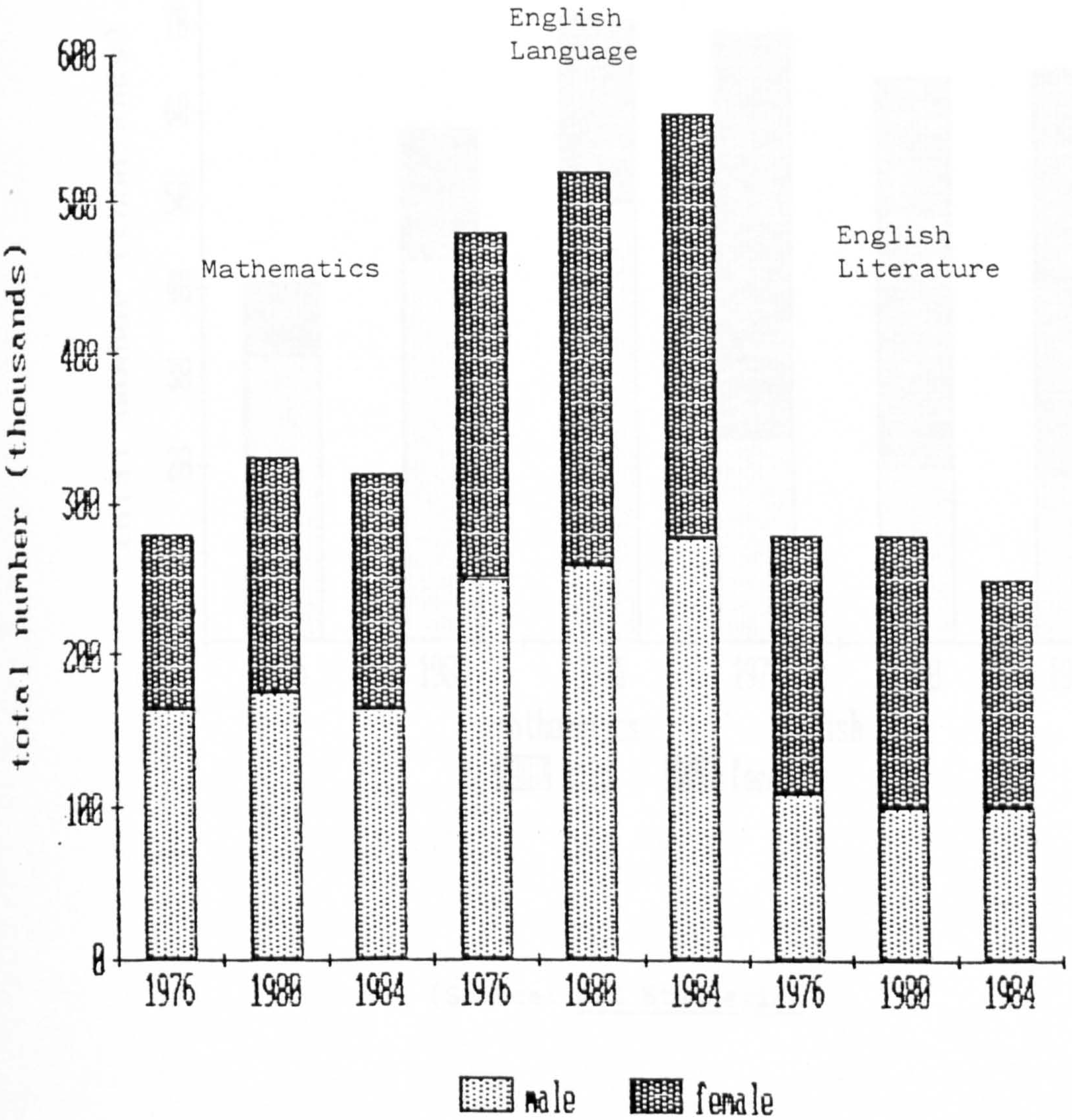
The numbers of entries of girls and of boys at CSE for mathematics and English are approximately even. However, more boys offered O-level mathematics than girls and the disparity is large at A-level when only 30.3% of the entry in 1984 were girls. The reverse trend is evident for English. The overall numbers of pupils entering for mathematics O-level have dropped and again, the reverse trend is evident for English. Harding (1979) raises the question, that if mathematics is a core subject (one that is taught to all pupils in the same way as English), why is it that substantially fewer pupils offer it in public examinations?

Figure 1. Total numbers entering CSE examinations (English and Welsh examination boards) in summer 1976, 1980 and 1984, by gender. The school population in England and Wales in 1985 at 15+ was boys 392980, girls 377539.



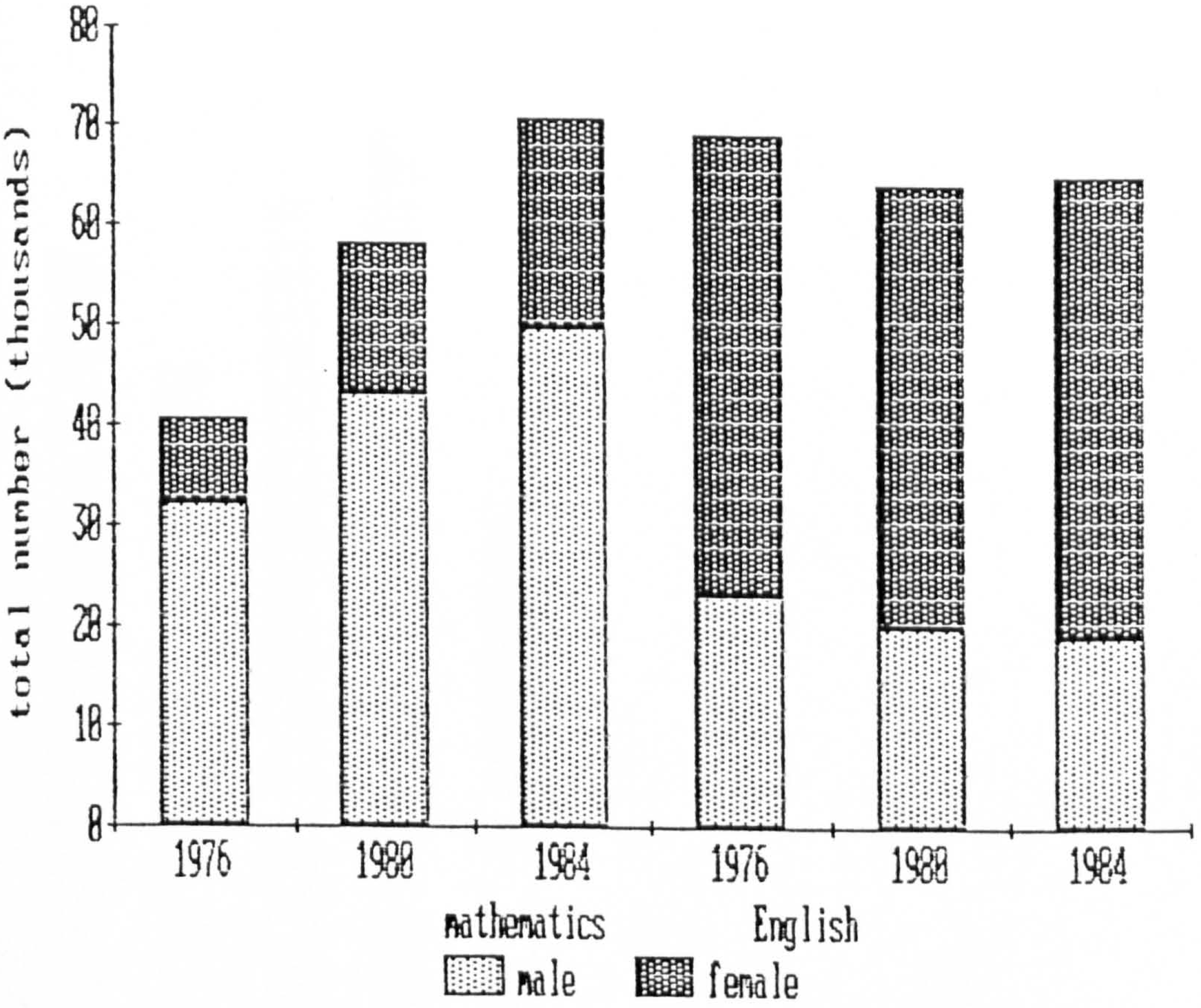
(Source: DES Statistics)

Figure 2. Total numbers entering GCE O-level examinations (English and Welsh examination boards) in summer 1976, 1980 and 1984, by gender.



(Source: DES Statistics)

Figure 3. Total numbers entering GCE A-level examinations (English and Welsh examination boards) in summer 1976, 1980 and 1984 by gender. The school population in England and Wales in 1985 at 17+ was boys 79476, girls 78566.



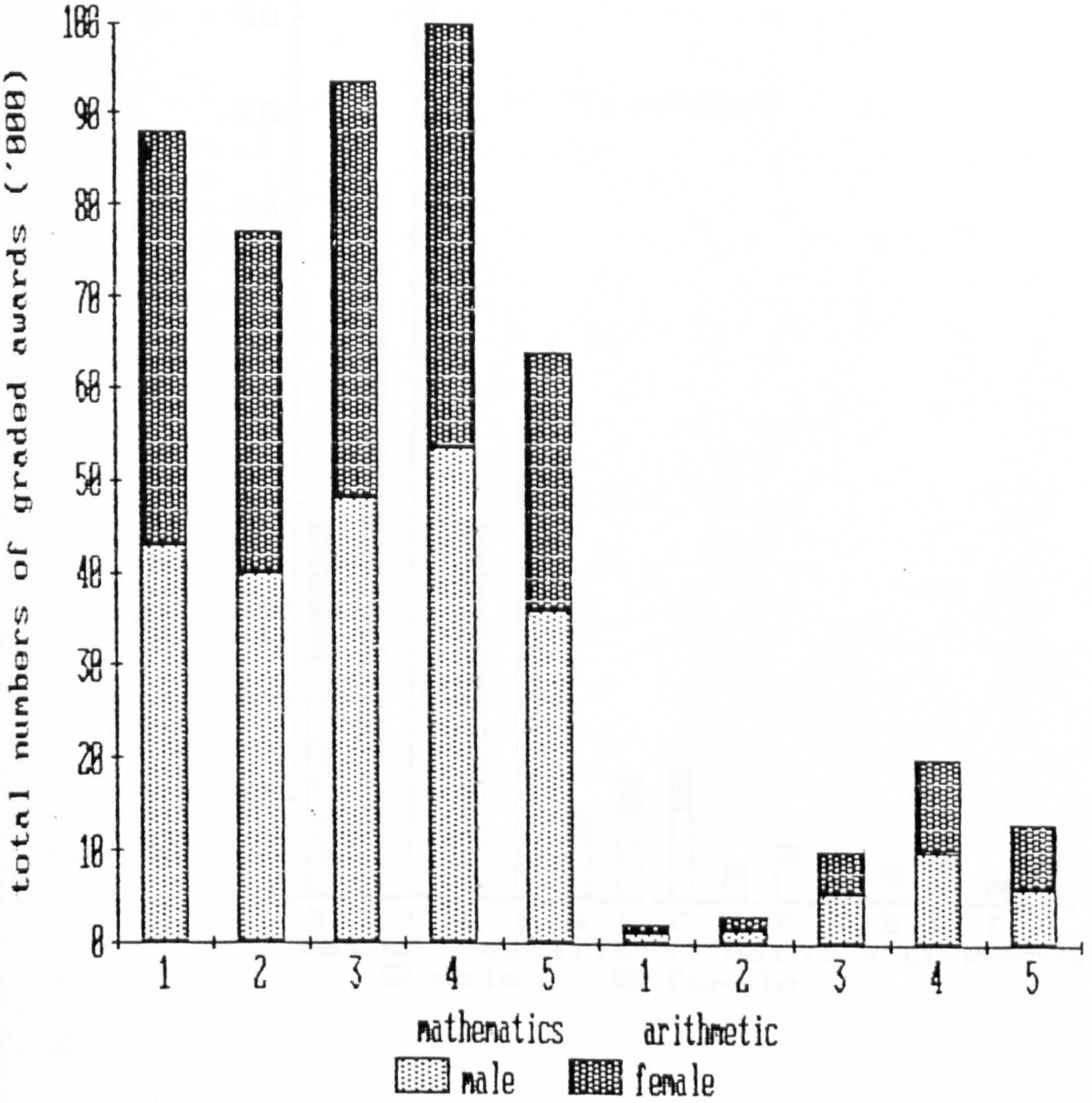
(Source: DES Statistics)

The examination grades achieved by boys and girls at CSE, GCE O-level and A-level in summer 1984 are shown in Figures 4-6.

Figure 4. CSE examinations achieved by grade and gender in summer 1984 (English and Welsh examination boards). The school population in England and Wales in 1985 at 15+ was boys 392980, girls 377539.

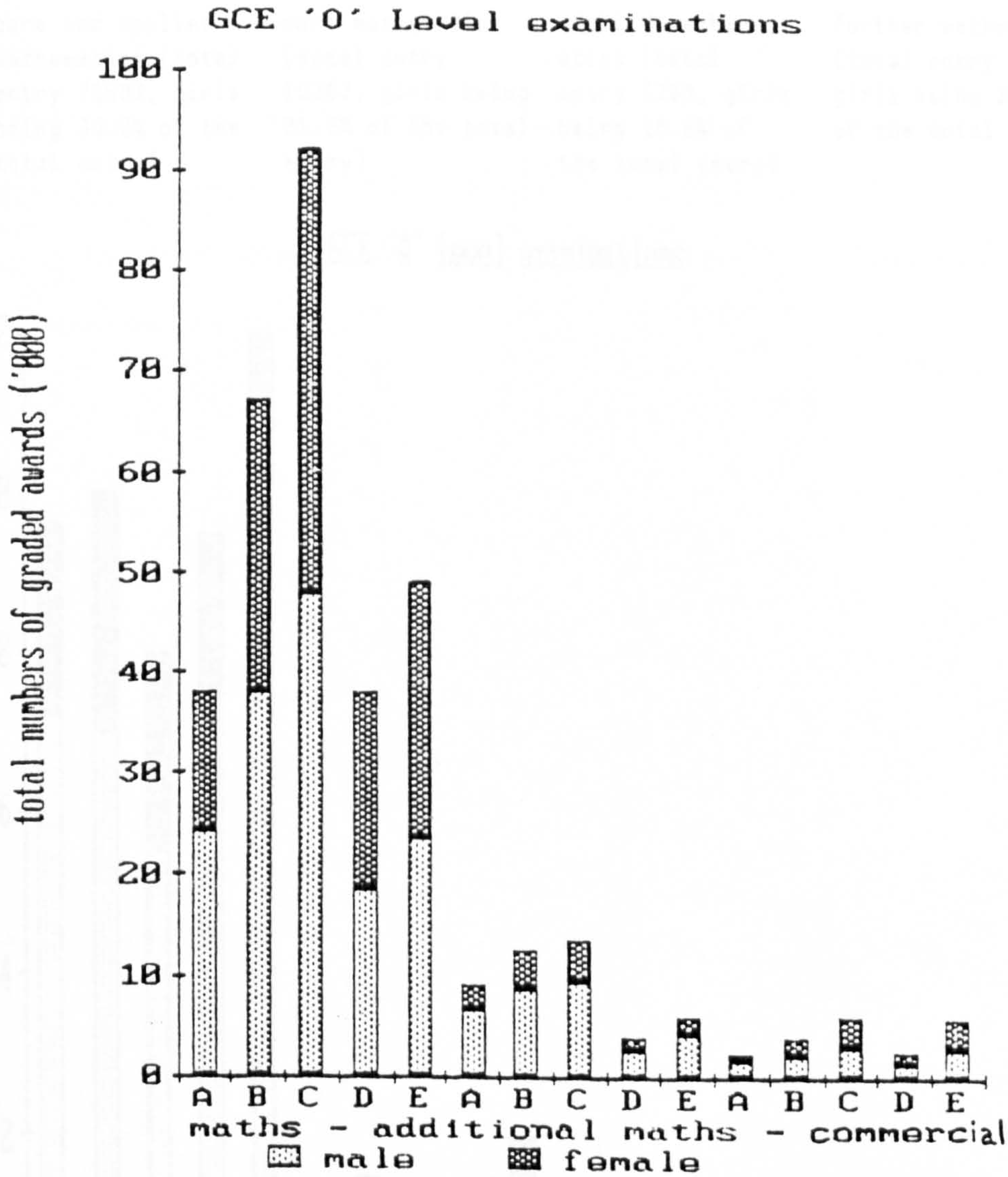
mathematics (total entry 228753, girls being 52.7% of the total entry)

arithmetic (total entry 60773, girls being 49.9% of the total entry)



(Source: DES Statistics)

Figure 5. GCE O-level examinations achieved by grade and gender in summer 1984 (English and Welsh examination boards). The school population in England and Wales in 1985 at 15+ was boys 392980, girls 377539.



(Source: DES Statistics)

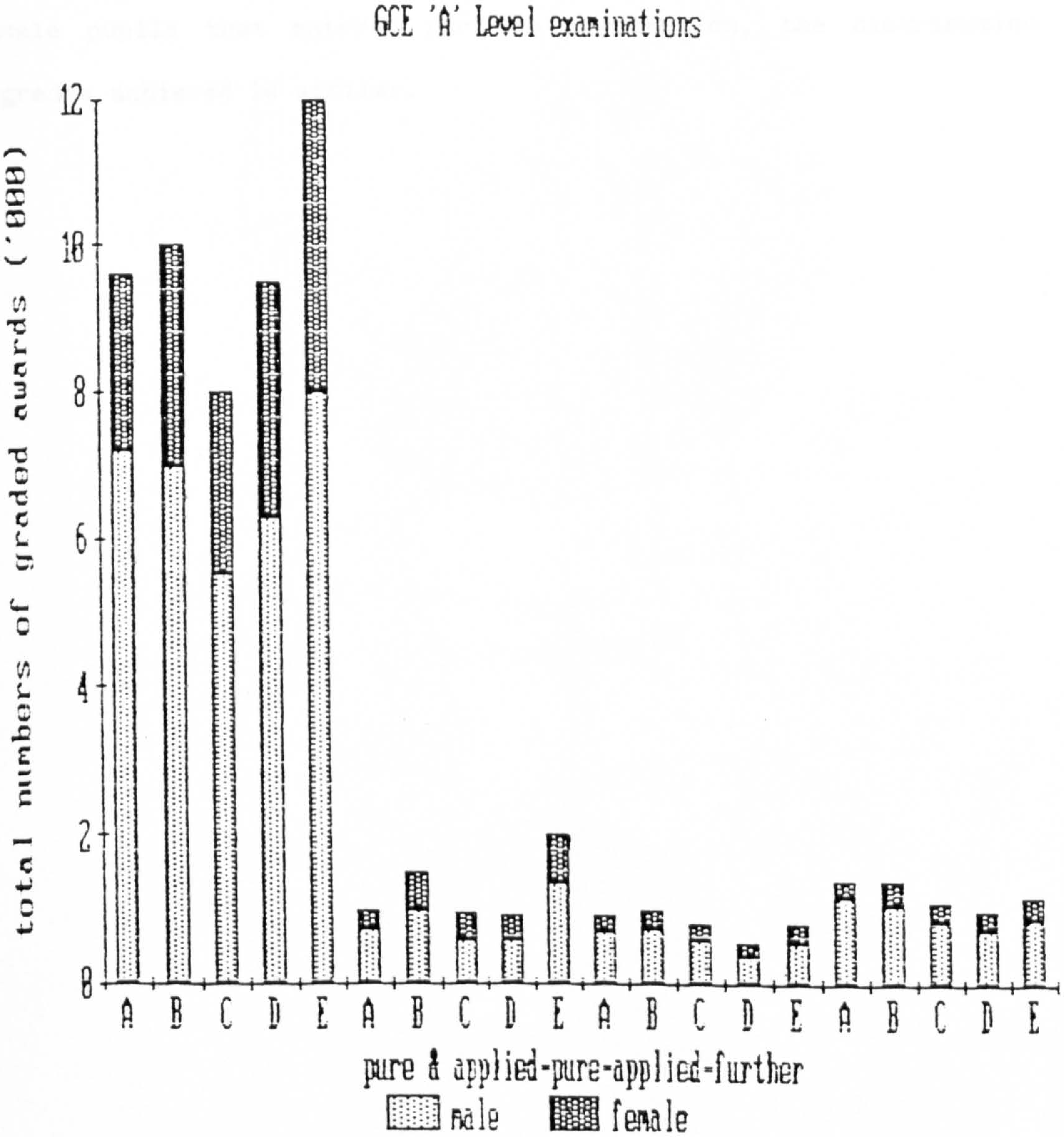
Figure 6. GCE A-level examinations achieved by grade and gender in summer 1984 (English and Welsh examination boards). The school population in England and Wales in 1985 at 17+ was boys 79476, girls 78566.

pure and applied mathematics (total entry 70587, girls being 30.3% of the total entry)

pure mathematics (total entry 10367, girls being 31.7% of the total entry)

applied mathematics (total entry 5763, girls being 26.6% of the total entry)

further mathematics (total entry 7187, girls being 23.2% of the total entry)



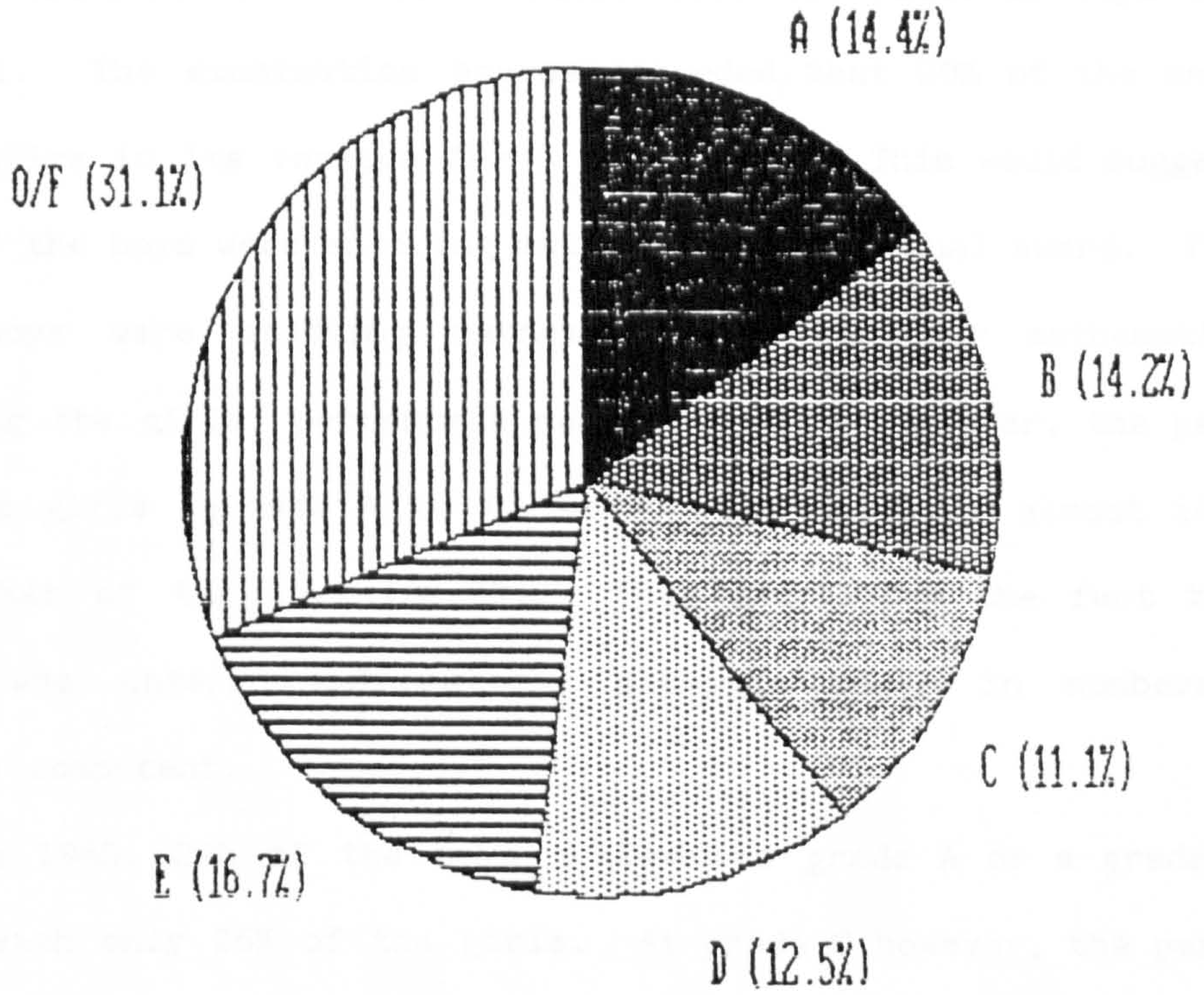
(Source: DES Statistics)

Examination results for boys and girls taking CSE mathematics and arithmetic differ little. At O-level, in mathematics, additional mathematics and commercial and statistical mathematics a higher proportion of boys than girls achieve good grades (A-C).

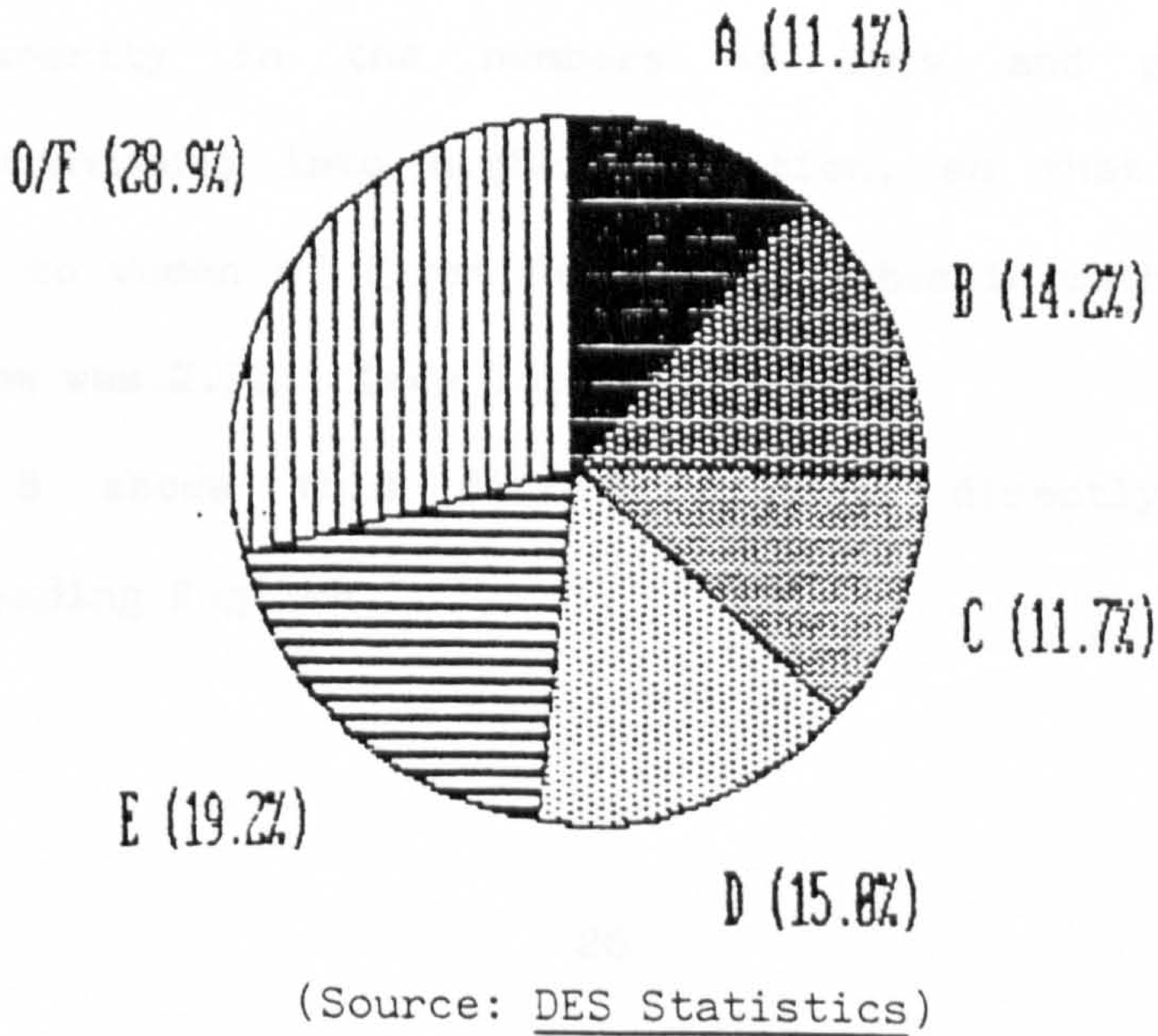
Figure 7 shows two pie charts of the relative achievements of boys and girls by grade in pure and applied mathematics, A-level, summer 1984. Despite a large disparity in the number of female and male pupils that entered for the examination, the distribution of grades achieved is similar.

Figure 7. Entry and achievement of males and females in A-level pure and applied mathematics in summer 1984 (English and Welsh boards) graded A-E and O/F (O-level/fail).

Males
(total entry 49196)



Female
(total entry 21391)



(Source: DES Statistics)

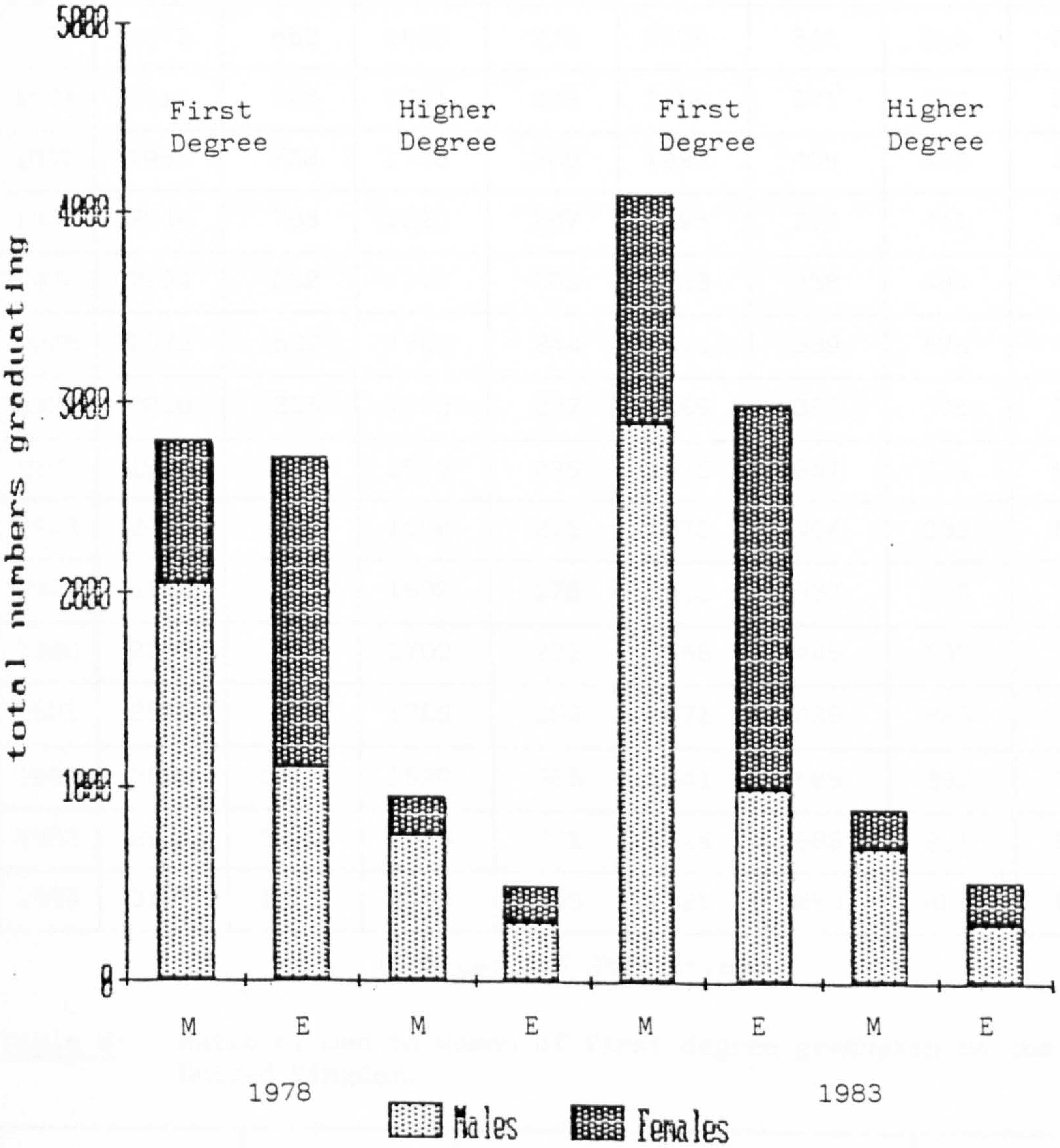
In the academic year ending July 1985 there were just over 58,000 boys taking one or more GCE A-level subjects, and a similar number of girls. However, 51% of the boys were studying mathematics compared to only 25% of the girls. This meant that girls formed 33% of the mathematics classes overall, i.e. the ratio of boys to girls was 2:1. The examination boards recorded that 29% of the entry for mathematics in its various forms were girls. This would suggest that more of the boys were entering mathematics for a dual award. That is, more boys were studying mathematics and further mathematics, so reducing the girls' percentage subject entry. However, the pass rate for the girls (grades A to E) was 70.1%, which is almost identical with that of the boys (69.9%). This underlines the fact that the girls who entered mathematics, although fewer in numbers, were equally competent.

In 1985, 29% of the boys achieved a grade A or a grade B compared with only 26% of the girls. At grade C however, the percentage was the same for both (11.5%). This pattern was also evident in the 1987 results, although in that year the proportion of girls entering had risen to 32%.

The disparity in the numbers of boys and girls studying mathematics continues into higher education, so that in 1984, the ratio of men to women of first degree graduates in mathematics in the United Kingdom was 2.32:1 (see Tables 7 and 8).

Figure 8 shows this difference when directly compared to candidates reading English.

Figure 8. Total numbers of degrees obtained in Great Britain in 1978 and 1983.



Figures for mathematics are marked on the left bar of each pair, and English on the right. Higher degree results follow the results for first degrees in each year, with 1978 results on the left and 1983 on the right.

(Source: DES Statistics)

Table 7: First degree graduates in the United Kingdom. Actual numbers.

Year	Mathematics		Physics		Chemistry		Biology	
	Men	Women	Men	Women	Men	Women	Men	Women
1970	1871	652	1698	278	2206	341	316	262
1971	2017	723	1701	243	2156	371	373	300
1972	1955	738	1746	269	1993	409	433	333
1973	2044	788	1825	287	1695	356	441	401
1974	2294	862	1777	286	1783	358	484	435
1975	2072	803	1769	244	1579	339	575	455
1976	2010	813	1603	227	1566	353	575	505
1977	1965	850	1519	235	1476	347	739	582
1978	2050	790	1606	215	1573	404	839	654
1979	2134	866	1602	178	1653	437	846	727
1980	2203	977	1702	232	1655	445	797	739
1981	2519	972	1789	254	1671	488	866	705
1982	2632	1155	1920	355	1641	569	893	791
1983	2932	1200	2076	331	1716	683	901	923
1984	3140	1351	2152	369	1725	690	857	936

(Source: DES Statistics)

Table 8: Ratio of men to women of first degree graduates in the United Kingdom.

Year	Maths	Physics	Chemistry	Biology
1974	2.66	6.21	4.98	1.11
1975	2.58	7.25	4.66	1.26
1976	2.47	7.06	4.44	1.14
1977	2.31	6.46	4.25	1.27
1978	2.59	7.47	3.89	1.28
1979	2.46	9.00	3.78	1.16
1980	2.25	7.34	3.72	1.08
1981	2.59	7.04	3.42	1.23
1982	2.28	5.41	2.88	1.13
1983	2.44	6.27	2.51	0.98
1984	2.32	5.83	2.50	0.92

(Source: DES Statistics)

Clearly, girls are underachieving in public examinations in mathematics, compared with boys. However, it must always be clearly understood that there is considerable overlap in the distribution of scores. Indeed, the variability of boys' scores is usually greater (see Table 5). This means that conclusions drawn from information about mean scores provides no clear information about individuals of either sex. Many girls can equal or surpass their male peers in some or all of the qualities under discussion. There are boys who do not reach even the basic levels of mathematical competency compared with girls who achieve first class degrees in mathematics and continue into research work.

b. Surveys

Although it is generally accepted that there is little difference between girls' and boys' mathematical scores until early adolescence (Walden, R. and Walkerdine, V., 1985), differences in favour of boys do begin to appear in the secondary school so that, by the age of sixteen these differences become much more apparent.

At the age of eleven, the Assessment of Performance Unit (APU) Primary Survey found certain specific differences between girls and boys in the results of its written tests. Girls' average scores were higher in computation of whole numbers and decimals whereas boys' average scores were higher in questions relating to length, area, capacity, the application of number, rate and ratio (APU, 1978).

In the 1979 APU Primary Survey there were additional categories in which boys scored significantly higher. These were the measurement of money, time, weight, temperature and fractions. Both the 1978 and 1979 surveys also found differences between the sexes in practical investigational testing. This is significant in view of the twenty five per cent compulsory assessment of project and investigational work in 1991 in GCSE mathematics.

Other results have borne this out (Bradberry, 1986). Boys were found to be significantly better at building a model from a diagram which required hidden blocks to make it stand.

Part of a five year investigation carried out in Sheffield involved a large sample of twelve-year-old pupils who were examined at the beginning and end of a school year in arithmetic and problem-solving (Eddowes and Sturgeon, 1981).

On the first tests, there was no overall significant difference between the sexes. Indeed, individual school differences were greater. On the arithmetic test, girls excelled in questions which involved integer and decimal operations and boys excelled in questions on fractions. Most pupils performed badly on problem-solving, though boys did better on average.

Problem-solving is essentially the process of performing established mathematical calculations in the context of novel situations. It appeared that the context in which a problem was set influenced the pupil's ability to solve it. For example, in the Cockcroft Foundation List: <u>APU Results</u> (1979),it was found that 81% of fifteen-year-olds could answer the question 'What is 1/3 of £3-90?' However, only 60% of fifteen-year-olds could answer the question: 'In a sale, everything is reduced by 1/3. How much will you save on something which used to cost £3-90?' In the second set of tests, both sexes had improved performances but marked sex differences were still not apparent in mean scores.

In the project reported in Mathematics and the 10 Year Old, (1979) Ward tested 2,296 children in England and Wales. Girls performed significantly better than boys on 11 items out of 91. These were items on computation of whole numbers and money. Others involved verbal responses in naming geometrical shapes, and making a deduction from given verbal (non-numerical) information.

Boys performed significantly better on 14 items out of 91. These items were on place value, measurement, word problems, reversing an operation and visuo-spatial concepts.

It seems that many of the items on which boys did better at the age of ten form the basis of concepts which become more important as children get older and proceed towards examinations at sixteen.

To make a refined assessment of the magnitude of gender differences in mathematics performance, Hyde, Fennema and Lamon (1990) performed a meta-analysis of 100 studies. They yielded 254 independent effect sizes, representing a sample size of over three million. They found no gender differences in problem solving in elementary or middle school in the USA. Differences favouring men emerged in high school, with greater differences in college mathematics. They also found that the magnitude of the gender difference has declined over the years. They compared studies published before 1973 with those published after, and found a narrowing in difference. However, they make it very clear in conclusion, that the lower performance of women in problem-solving that is evident in high school, requires attention.

Over the years, educators have become more aware of sex differences in mathematical education. This has been made more apparent in the 1988 Education Reform Act, and the requirement of Local Education

Authorities to draw up equal opportunity policies. Many schools are now mixed which in theory offers equal opportunity across the curriculum. In practice this may not be so (see NUT document, Towards Equality for Girls and Boys, 1988). Legislation and functioning may not always lead to good practice in the classroom.

It is also true that career guidance has improved and that there is less male-stereotyping in literature and examination papers. It might therefore, be expected that most of these perceived differences have been removed through careful schooling.

What then are the differences in performance between girls and boys? Is there a narrowing of the gap in mathematical achievement?

Much of the recent debate has been concerned with the importance of teacher interactions (Economic and Social Research Council, 1988), the nature of biological influences, and the function of mathematics, as a filter into various career options (Girls and Mathematics, The Royal Society and the Institute of Mathematics and its Applications, 1986). However, these studies do not always address the basic questions: Where are girls underachieving? What are the precise concepts that give the widest discrepancy between the sexes? What sort of problem seems to cause the greatest anxiety and lower marks? Are there links across different mathematical concepts? What are the underlying cognitive mechanisms that generate the observed sex differences?

It is the aim of this study to investigate and identify those mathematical areas where girls and boys differ significantly in performance. It attempts to pinpoint the nature and extent of those difficulties in the light of current educational philosophy.

It is important to survey and discuss all the different aspects of mathematical education with reference to current literature. These have been observed as relevant factors in possible differences between the sexes in mathematical attainment. First, however, it is necessary to consider exactly what is meant by mathematical ability.

4. Mathematical Ability

What is mathematical ability? It is important to establish the nature of the subject to then consider the relative performance of girls and boys. Mathematical ability may be different to, say, verbal ability where girls achieve better results than boys at 16+ (Table 1, Page 12). So what is the nature of mathematical ability which the girls may find more difficult? In what areas do girls equal the boys or even excel? For example in a meta-analysis of 100 studies Hyde, Fennema and Lamon (1990) concluded that females are superior in computation. Since much of the mathematics in junior schools is concerned with basic arithmetic, this may account for girls' better average performance up to 11 years. They go on to say that where gender differences do exist, they are in critical areas. For example, they highlight problem-solving as such a factor. From the study, they found that girls in secondary schools perform less well than males, on mathematical problem-solving tasks.

An analysis of mathematical ability has been the subject of many studies which have taken a variety of different forms. At one extreme has been the method based on the statistical procedure of factor analysis applied to test scores. At the other extreme has been the anecdotal approach, often based on reflections of mathematicians about their own ability. The outcome has been the indication that overall intellectual capacity is the most dominant influence on mathematical ability, and it is a matter of what other, more specific abilities can be shown to exist.

A major study of mathematical ability in pupils was carried out by Krutetskii (1976). The study was, in essence, based on observation of, and conversation with, pupils. The origins of mathematical ability were seen by Krutetskii to lie in the existence of 'inborn inclinations'. He suggested that people have inborn characteristics in the structure and functional features of their brains which are favourable to the development of mathematical abilities. If this is true, it may account for boys' superior performance in certain mathematical areas. On the other hand, it might be expected that there would be a greater difference in performance than has been observed. Why, for example, are some girls extremely good at mathematics?

In more detail, the components of mathematical ability were seen by Krutetskii to be:

* An ability to extract the formal structure from the content of a mathematical problem, and to operate with that formal structure.
* An ability to generalise from mathematical results.
* An ability to operate with symbols, including numbers.
* An ability for spatial concepts.
* A logical reasoning ability.
* An ability to shorten the process of reasoning.
* An ability to be flexible in switching from one approach to another, including both the avoidance of fixations and the ability to reverse trains of thought.
* An ability to achieve clarity, simplicity, economy and rationality, in mathematical argument and proof.
* A good memory for mathematical knowledge and ideas.

It is useful to compare this analysis with that of Suydam and Weaver (1977) who characterise good problem-solvers in mathematics as those who have the:

* Ability to estimate and analyse.
* Ability to visualise and interpret quantitative facts and relationships.
* Ability to understand mathematical terms and concepts.
* Ability to note likenesses, differences and analogies.
* Ability to select correct procedures and data.
* Ability to note irrelevant detail.
* Ability to generalise on the basis of few examples.
* Ability to switch methods readily.
* Higher scores for self-esteem and lower scores for test anxiety.

Suydam and Weaver also noted that 'more impulsive students are often poor problem-solvers, while more reflective students are likely to be good problem-solvers' (The Arithmetic Teacher, 25(2), 40-42).

In comparing these two theories, it is interesting to note that both agree on the ability to generalise from mathematical findings. Both agree on the ability to understand mathematical terms and symbols, and the ability to be flexible in switching from one approach to another. The ability in spatial concepts in the first, also corresponds with the ability to visualise in the second.

When looking at the differences, Krutetskii mentions the importance of a good memory for mathematical knowledge and ideas, whereas Suydam and Weaver do not, although it may be implicit in their list. Also, Suydam and Weaver place importance on high self-esteem and low anxiety, whereas Krutetskii does not mention motivational aspects at all. This will be discussed later.

Krutetskii also suggests that there are different kinds of mathematical thinking. Some pupils have an analytical mind and prefer to think in verbal, logical ways. Others have a geometrical mind and prefer a visual or pictorial approach. Yet others have a harmonic mind and are able to combine characteristics of both the analytical and the geometrical. He suggests that pupils with a harmonic type of mind are the most likely to show real mathematical aptitude.

Poor pupils often show 'blind', unmotivated manipulations. They are chaotic and unsystematic attempts to find a solution. Able pupils on the other hand, often adopt procedures which involve trying out ideas systematically. They appear to be able to see which ideas are worth pursuing and which are not.

Factor-analytic studies have been used to justify the existence of group factors such as verbal ability, spatial ability and numerical ability which are required over a whole range of school subjects. Mathematical ability may be a particular hybrid drawn from a number of group factors. These factors may include numerical, spatial, verbal and non verbal reasoning, and convergent and divergent thinking.

The existence of different forms of mathematical ability together with the elusiveness of a single mathematical ability as revealed by factor analysis, suggests that mathematical ability can take many forms, each form derived from a different mix of other abilities. It is important therefore, that in considering the gender differences in mathematical exercises or tests, the content is closely examined. If that content is purely numeric and computational, it might be expected that girls excel. If the content is purely problem-solving or spatial, boys might excel.

This is a difficulty when exploring the research and comparing different examinations and syllabuses. Some are very general in content; others are much more specific acting as filters for different career options. The National Curriculum has done much to stabilise the content of mathematics teaching in England and Wales. However, a comparison with foreign syllabuses cannot be exact, with the weighting of different mathematical skills being varied. This study will look at the different cognitive skills and compare the performance of girls and boys on the content of a varied mathematical syllabus.

Also to be considered are the varied approaches of the teaching of mathematics which might favour a particular mode of thinking or a particular sex. Again, when test results are given, it is not always clear what teaching approach has been used. Has it been purely didactic or has the process of investigation been used? Has the assessment been verbal, practical or by examination? Has it been the combination of different approaches? There is for example, a danger of attempting to assess the potential of pupils on the basis of verbal tests alone. Orton (1987) concludes that verbal ability tends to correlate negatively with mathematical ability.

Do girls respond better to the traditional rote learning approach or do they welcome an open investigational approach?

In the Post-War period, new ideas from psychology about child development and learning came to be incorporated into policy documents and reports, culminating in the policies of the 1960s. Perhaps the best example of this was the Plowden Report of 1967. The new policies discouraged old methods of teaching which had stressed rote learning, remembering, regurgitating facts, in favour of the 'active learner'. By this was meant a process of discovering concepts by means of experimentation and investigation. This was a new view of a child's learning and a new pedagogy. Certain ideas about learning were now obsolete. How could a child who was rote learning really understand a particular concept?

In England and Wales there has been a shift of emphasis over the last twenty years towards a policy of learning by discovery. There has been an equivalent shift in the form of wording in examination questions. Questions are much more practically based. Does this favour the girls or the boys? The narrowing difference in the performance between girls' and boys' mathematics results would suggest that this has been a positive factor for the girls. It may be that the relevance of a particular practical problem has been a stimulus to further interest and success.

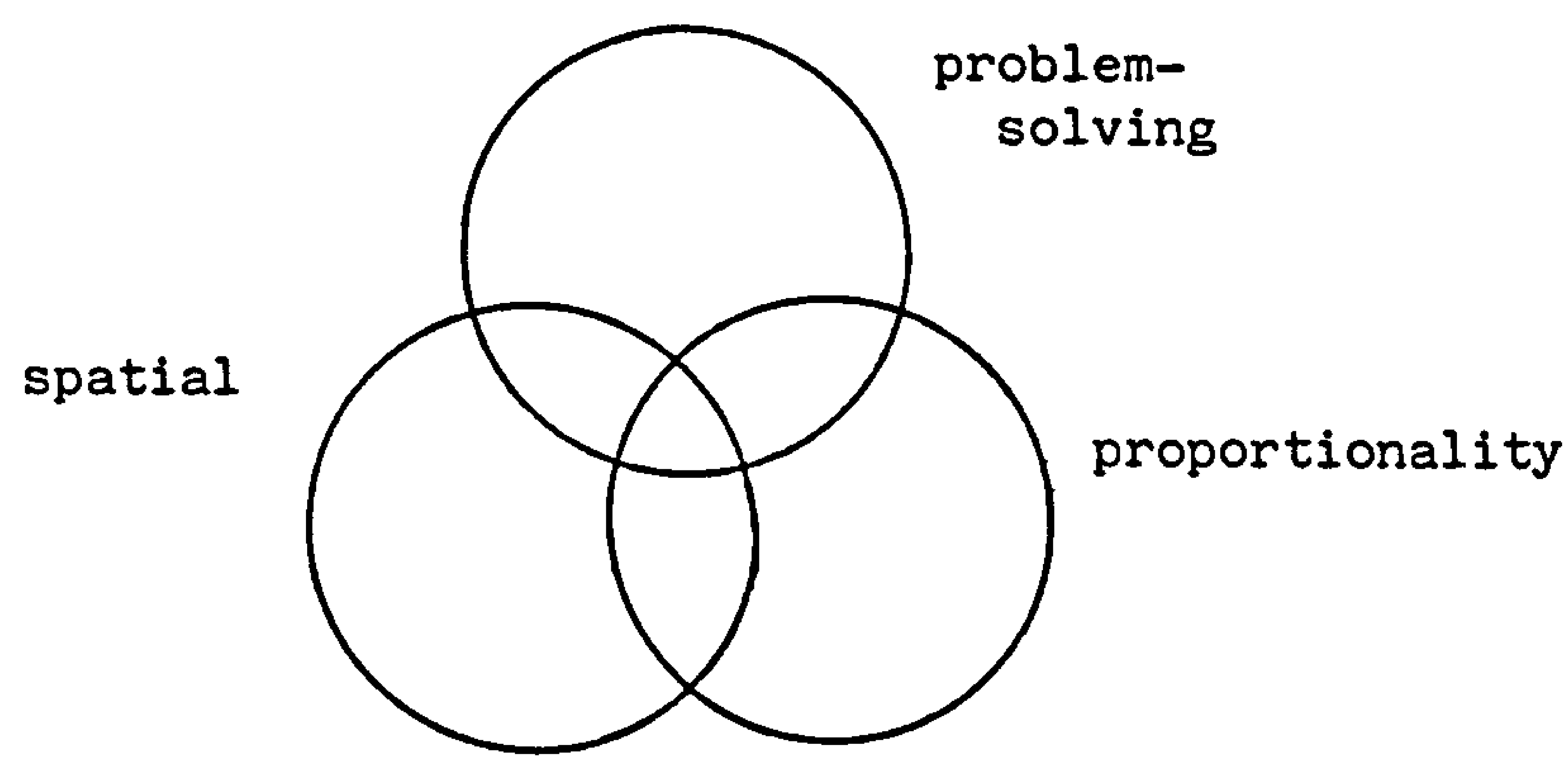
It is significant that it was the boys who were often singled out as exemplars of the 'active learner' while girls represented the passive approach. 'Boys will be boys - boys need a sense of adventure' (Plowden Report, P.742, 269, 1967). Unfortunately, there appears to be a history of equating 'natural' masculinity with 'natural' reason.

So, as can be seen, mathematical ability can take many forms and be weighted in a number of directions. Most studies however, would to a larger or lesser degree place importance on basic numeric computational ability, spatial ability, reasoning and problem-solving. This is consistent with the meta-analysis cited earlier (Hyde, Fennema and Lamon, 1990).

The Fennema study also found a slight superiority in computation in favour of the girls. In terms of mathematical content, there was no gender difference in arithmetic or algebra. This might have been expected since much of the arithmetic and algebra is based on standard computational exercises. This deserves further investigation later.

The study above also found no gender differences in the understanding of concepts. This is significant because it assumes no concept deficiency on the part of the girls compared with the boys.

The areas of greatest difference in favour of the boys which result in the differences in overall mathematical performance, can be found in spatial awareness skills, reasoning or proportionality skills and problem-solving skills. These skills are related as shown in the diagram.



Problem-solving skills may be needed in both proportionality and spatial questions. Indeed, there may be questions which involve all three skills or just one or two. For example, a question involving the use of similar figures may have all three characteristics. It will be interesting to investigate the relative performance of girls and boys within the intersection areas of this diagram where it might be expected that girls perform less well compared to the boys.

First, it is important to discuss what is meant by spatial, proportionality and problem-solving skills.

5. Spatial Ability

Spatial awareness skills have been highlighted as skills related to mathematics performance. Indeed, many studies have found spatial skills to be positively correlated with measures of mathematics performance (Connor and Serbin, 1985: Fennema and Sherman, 1977). What then are spatial skills, and how and why are they related to mathematics? What specific aspects of spatial awareness do girls find more difficult?

In general, spatial skills are considered to be those mental skills concerned with understanding, manipulating, reorganising or interpreting relationships visually (see Spatial Orientation Skill and Mathematical Problem Solving, Tartre, 1990). Learning mathematics involves the pupils with pictures, diagrams, graphs and visual presentations of a wide variety of forms. One particular problem is the two-dimensional representation of three-dimensional objects.

Spatial visualisation involves visual imagery of objects, movement by the objects themselves or change in their properties. In other words, objects or their properties must be manipulated mentally. Spatial representations are increasingly being included in the teaching of mathematics. For example, the Piagetian conservation tasks which are a focus of many pre-school programmes involve focusing on correct spatial attributes. These come before concepts of quantity, length and volume are conserved.

Smith (1964) carried out extensive studies into spatial ability which ultimately led him to conclude that spatial ability was a key component of mathematical ability. This is consistent with the work of Krutetskii (1976) and Suydam and Weaver (1977) in their attempts to define mathematical ability.

A meta-analysis of gender differences in spatial ability, indicated that the magnitude of the gender difference depended considerably on the type of spatial ability tested (Linn and Petersen, 1985). They identified three spatial ability categories: spatial perception, mental rotation, and spatial visualisation. They defined spatial visualisation as 'those spatial ability tasks which involve complicated multi-step manipulations of spatially presented information'. They distinguished it from the other two categories 'by the possibility of multiple solution strategies' (Child Development, 56, Page 1484).

The smallest gender difference was on questions relating to spatial visualisation. This was followed by questions on spatial perception. However, the greatest difference was for measures of mental rotation. In all cases the differences favoured males. This is consistent with the difficulty that many girls seem to have compared with boys on questions relating to bearings and transformation geometry. These questions require the skills of mental rotation. They will be examined further.

The issue of whether spatial skills are general indicators of a particular way of mentally organising information that might be helpful in certain areas of mathematics, has been discussed by a number of authors. One hypothesis is that mathematical reasoning and problem-solving are facilitated by a 'mental blackboard' on which the activity may be organised and the components visualised (Anglin, Meyer, and Wheeler, 1975). Bishop (1980) also theorised that spatial training might help students to organise the situation with mental pictures during problem-solving in mathematics. He proposed that the structure of the problem might be understood through a spatial format. The frequent use of tree diagrams, Venn diagrams, charts and other figures to organise information and show relationships among components of a problem demonstrates the plausibility of this hypothesis. Polya (1962) also argues that the drawing of a diagram or a mental image, assists in mathematical thinking. He insists that in certain stages of a problem-solving exercise, it is even essential.

In mathematical terms, spatial visualisation requires essential skills of rotation, reflection and translation of two and three dimensional figures. These are important skills in geometry. Fennema (1983) sees a direct relationship between mathematics and spatial visualisation. Indeed, if spatial visualisation terms are geometrical ideas, then spatial visualisation and mathematics are inseparably intertwined.

School pupils in science and mathematics lessons are frequently presented with two-dimensional drawings as representations of three-dimensional structures. Such drawings, known as stereograms, attempt to give an impression of three-dimensions. On other occasions, the drawings are of plane sections through a three-dimensional structure and the pupil is expected mentally to construct the structure in three-dimensions. However, the extent to which pupils are able to construct such visualisations and the difference between the sexes is open to doubt.

Classical work on the subject was carried out by Piaget and Inhelder and published in The Child's Conception of Space (1956) in which a section is devoted to geometric sections and their place in the overall development of representational space. They suggest that children entering secondary schools are able to internalise spatial images. Also, they can represent such images by drawing, even after a time delay. Furthermore, they suggest that such children are able to imagine plane sections through solid objects. At this level, there were no recorded differences between the sexes.

Similar studies by Boe (1968) in the USA and Langton (1969) in this country have tested pupils over an age range 11-18. Their results indicate that it is around the first year of the secondary school (11-12) that pupils are beginning to achieve the attainment of geometric sectioning. This appears to confirm the views of Piaget and Inhelder. However, Langton (1969), in investigating pupils' ability to predict plane sections through a sphere, found that one third of pupils had not reached this stage by 11 years. Although there was some improvement during the next year of development, there was no further improvement beyond the age of 13. This is confirmed by Boe's work which suggests that once the age of 14 or so is reached, age makes little difference in the pupils' ability to make geometric sections. Twenty per cent of pupils never reach this level of geometric sectioning. This lack of attainment has considerable implications for the teaching, not only of mathematics, but also of science, geography and other related subjects.

In a previous study, (Bradberry, 1986) an investigation was made into the skills of 11-year-old girls and boys in constructing models from a two-dimensional picture. The pupils were individually shown a series of diagrams of models and were asked to construct each in turn with $2cm^3$ wooden bricks. It was found that even at this early age fewer girls than boys were able to complete the task. Out of a sample of fifty pupils, of which twenty five were girls and twenty five were boys, only 60% of the girls completed the task compared to 80% of the boys. The greatest single difficulty was experienced by the girls in deducing where the hidden bricks were located.

Boe (1968) conducted a series of tasks which involved pupils drawing the shape of the flat surface formed by a cut in the solids. An analysis of the drawing response scores yielded a statistically significant source of variation in favour of the boys ($p < 0.05$) (The Mathematics Teacher, 61, 417).

Given that spatial ability forms an integral part of mathematical ability and problem-solving, why are some girls not performing as well as the boys? Is this an innate difference, or is it one of social conditioning? Can the relative performance of the girls be improved by intervention programmes of space-related activities?

The relative difference in performance can be attributed to environmental and social influences although there have been theories of a biological nature.

A child rearing practice which may have an effect on mathematical attainment is the type of toys given to girls and boys. Brierley (1975) found that boys of three to five years are actively exploring and manipulating the environment. Girls become increasingly skilled in verbal and social functions, and occupied in sedentary activities such as crayoning, cutting out and plasticine work. Delamont (1980) suggests that sex divisions in early learning are accentuated by the expectations of parents, and by the toys given to pre-school children. She suggests that the world of toys and games offers girls a far more restricted range of roles than it does boys. The roles offered girls are essentially passive, home-centred, non-scientific and non-technical. Throughout childhood, boys play more with constructional toys and take part in more physical games, both of which may promote greater spatial awareness and problem-solving activity.

Boys are encouraged to be more independent, a valuable characteristic for problem-solving, while girls are expected to be more passive and conformist. Delamont (1980) suggests that girls spend time helping mother around the house, rather than helping father with 'do-it-yourself' and with the car, both of which are more directly related to measurement, shape and calculation.

Straker (1986) suggests that children who come to school having already played at home with wheeled vehicles, clockwork or battery-powered toys, a calculator or a computer, are more likely to enjoy and benefit from the technological, scientific and mathematical experiences they will have in school.

Suggestions of comparatively poorer spatial ability in girls and comparatively better verbal ability, have led to biological considerations. In particular, study has concentrated on brain lateralisation. The left hemisphere of the brain controls verbal abilities, whilst the right hemisphere controls spatial abilities. The question arises as to whether the two hemispheres are differently developed in girls and boys? Research does not confirm that such a difference exists (Springer and Deutsch, Left Brain, Right Brain, 1981).

If there were such marked differences, then it might be expected that test results would reveal a far greater discrepancy between the sexes. Why for example, are some girls excellent mathematicians and excel in the areas of spatial awareness, proportionality and problem-solving? These are concepts which often give a wide discrepancy in test results between boys and girls. This theory does not provide an alternative to the hypothesis that the differences in attainment are a product of environmental and social influences. However, it may be that these external influences do to some degree, determine the development of the various mental faculties within the brain.

Bruner (1973) raised the question of the extent to which spatial ability may be enhanced through teaching. 'I don't think we have begun to scratch the surface of training in visualisation,' he said (Beyond the Information Given, P.60). The potential value of manipulative materials was described by Bishop (1980), who found that children who have used such materials extensively, tend to perform better on spatial visualisation tests. This was also suggested by Mitchelmore (1980), who concludes that the best approach is to base all learning both in arithmetic and geometry, on manipulative materials.

Evidence referred to by Badger (1981) indicates that girls do indeed show an improvement in scores on spatial tests after they have been involved in space-related activities, whether these are in the form of explicit training for test results or take a less directed form. Again, an innate difference would imply a much larger difference between boys' and girls' scores on spatial tests. There are many girls who score highly on spatial tests and many boys who do not. This study will investigate the type of question which gives the greatest and smallest relative difference in scores.

To summarise, it is evident that spatial awareness skills are inseparably intertwined with mathematics. In particular, the skill of mental rotation is found to give the greatest difference between the sexes on test questions. This skill is basic to mathematics, and forms the basis of reflections, rotations and bearing questions.

The differences in attainment in spatial skills seem to be the product of environmental and social influences. Boys generally have a greater opportunity to develop early spatial skills. Girls do show an improvement in scores on spatial tests, after they have been involved in space-related activities. An intervention programme of this sort suggests that this difference is not innate but part of a social conditioning exercise.

Another particular ability which has attracted research attention is the reasoning skill of proportionality. This is now considered separately.

6. Proportionality

An understanding of ratio and proportion may be considered to be fundamental to the learning of mathematics. Proportionality skills form the structure of hierarchical concepts upon which much of mathematics is based. It forms the basis of reasoning and provides a useful problem-solving technique.

Geometrical theorems and results based on similarity and on parallel lines and intercepts require an appreciation of proportionality. The idea of gradient, which is important in the algebra of graphs and in calculus, also depends on ratio and proportion. Simple trigonometry, likewise, has its beginnings in a study of equal ratios. Rational numbers are studied throughout most years of a child's school life, progressing through operations on fractions, decimals and percentages, and culminating in a more formal study of the number system itself. Ratio also underlies pie charts, scale factor and probability. It is clear that it is important to discuss proportionality in the context of this study because it pervades mathematics (see Orton, Learning Mathematics, 1987).

The development of scientific understanding also relies on the ability to handle ratios, for example, in the definitions of density, velocity and acceleration; in calculating chemical equivalents; in applications of the ideal gas laws and in using many laws of physics. Likewise, other school subjects make use of proportionality through simple calculations such as percentages, through scale and through graphical representation.

Researchers agree (Orton, 1987), (Hart, 1981), that a true understanding of proportionality develops late, if it develops at all.

For intellectually weaker children it is beyond their capabilities at 14 and for some it may never be within their capabilities. What then, if any, are the gender differences in performance? Can the skills of proportionality be taught?

The Assessment of Performance Unit (APU, 1988) reports on a proportionality question involving scale and it illustrates the improvement by boys relative to girls at age 15 compared with age 11.

> '1 cm on a map represents 1 km on the ground. What is the actual distance between two towns whose distance apart on the map is 5.5 cm?' (Attitudes and Gender Differences, 1988, P.24).

The results were found as follows:

Correct	Boys	Girls	Boys-Girls
Age 11	49%	38%	11%
Age 15	87%	73%	14%

What is evident from these questions is that although ratio and proportion are important in mathematics, pupils often struggle and find difficulty in grasping the basic concept. Also evident is the better performance by boys than girls, particularly at 15.

Wood (1977) suggested that the topic of fractions was a major source of the difference between the sexes. Fractions are basically a comparison between two quantities. He identified 'a comparison factor' which is the scaling up and down, which is so important in coming to an understanding of proportionality. He suggested that girls in general are not as competent in the use of this skill. He recommended a concentration on the teaching of fractions and proportion and the use of a comparison factor to compensate for differences in the test results between girls and boys.

So, have the girls a slower conceptual development, or are there practical and experiential issues here? The answer would again seem to lie in the practical and experiential areas. Cognitive, conceptual difficulties by the girls would expect to produce far greater divergence of scores. Many girls outperform boys on proportionality questions. Performance differences suggest experience differences and this again has implications for programmes of intervention.

Renner and Paske (1977), and Hart (1981), would see a practical approach as being a way forward to help both the girls and boys who find difficulty with proportionality concepts. It is possible that ratio and proportion be taught in such a way that learning is optimised, though possibly not greatly accelerated. They suggest a practical approach on the basis that 'learn how to do it' approaches are soon forgotten or even misunderstood. They suggest the need of discovery approaches which can then be extended to other problem situations.

There is evidence (Kuntz and Karplus, 1977; Renner and Paske, 1977) to show that children who are presented with practical problems which need a ratio for solution (eg. gears), do improve and abandon false strategies.

Another possible eradication strategy described by Hart (1981) is that of showing gross distortions that occur when false methods are used. Orton (1987) suggests that 'a carefully devised programme of work geared towards parts of the numerical work might, after all, accelerate the growth of understanding of proportionality' (Learning Mathematics, P.18).

The problems for girls generally, may again lie in early play experiences where they may not have the same exposure as the boys to activities relating to ratio and proportionality concepts. However, this argument is not as convincing as that for spatial skills. There are few activities in early childhood that could be considered as prerequisite for the understanding of proportionality concepts. General comparisons of size and distance and speed, may be some of the early important experiences. Indeed, in Wood's study (1977) girls' performance on questions relating to speed, distance and time, which are essentially proportionality concepts, gave some of the biggest differences in performance in favour of the boys.

It may be also that the approach taken by teachers is not conducive to girls' understanding of proportionality concepts or it may be that girls are just not interested in the topics. Indeed, it is easy to see how girls (and boys) may lose interest in topics such as map scales, speed, acceleration and probability when they have opted out of science courses and have no scientific career aspirations. It is therefore, important to know what girls see as relevant and not to allow them to 'switch off'. Attitudinal and motivational issues are important here, and these will be considered in detail. 'Girl friendly' teaching styles need to be assumed as the norm. Proportionality concepts can be approached from many different angles and from many different practical situations, eg. proportion and ratio can be developed from relative ingredients in food recipes or in scale models of aeroplanes. What is important is that girls and boys see the relevance of such practical concerns and that the impetus of their motivational attitude is developed to the full.

So an understanding of ratio and proportion is fundamental to the successful study of mathematics and the development of proportionality skills form a basic tool in problem-solving questions. What is important is to discover the extent to which the test scores of girls and boys differ on proportionality questions. This will give the information on which future programmes of intervention can be based.

7. Problem-Solving

Problem-solving is characterised by some form of novel situation in which there is a synthesis of either conceptual or procedural knowledge, or both, resulting in meaningful learning.

In a meta analysis involving 100 studies Hyde, Fennema and Lamon (1990) found that girls in secondary schools perform less well than boys on mathematical problem-solving tasks. This then is clearly an important cause for concern as problem-solving lies at the heart of mathematics. As well as investigating gender variations, it is appropriate to analyse what is involved in the problem-solving process, so that appropriate learning environments and instructional techniques can be developed.

Lester (1977) argues that due to the complex nature of problem-solving there is little universally accepted knowledge about the best way to enhance children's problem-solving abilities. Even the most successful problem-solvers have difficulty in describing why they are successful, and even the best mathematics teachers are hard pressed to pinpoint what causes their students to become good problem-solvers. There are at least two reasons for this condition.

First, a variety of tasks have been used in problem-solving research. The tasks found in research literature include puzzle problems, anagram problems, concept-identification problems, arithmetic-computation problems and standard textbook problems. Secondly, problem-solving research has been conducted by experimenters with very different positions on the nature of problem-solving. Behaviourists, developmental psychologists, gestaltists and proponents of information processing approaches, have all contributed to the research literature.

Orton (1988) says that 'problem-solving implies a process by which the learner combines previously learned elements of knowledge, rules, techniques, skills and concepts to provide a solution to a novel situation' (Learning Mathematics, P.35). It is now generally accepted that mathematics is both product and process. That is, it is both an organised body of knowledge and a creative activity in which the learner participates. In fact, it could be claimed that the real purpose of learning rules, techniques and content generally is to enable the learner to do mathematics, indeed, to solve problems.

Gagné (1970, 1977) has expressed the view that problem-solving in mathematics is the highest form of learning. Having solved a problem, one has learned. It might be that the person has learned to solve that one problem, but it is more likely that the learning can be extended to solve a variety of similar problems, and perhaps a variety of problems possessing some similar characteristics. Descartes said, 'Each problem that I solved became a rule which served afterwards to solve other problems.'

An early, and famous, study of problem-solving in mathematics was by Polya (1945) in which he suggested ways of improving the teaching and the learning of problem-solving, an aim subsequently taken up by Wickelgren (1974). More recent research into human problem-solving abilities, has drawn attention to comparisons with the use of a computer to solve problems. Problem-solving involves the processing of information, an activity for which computers are well-suited, particularly when the testing out of many possibilities is involved.

The essence of Polya's How to Solve It (1945), was the justification of a self-questioning technique to be carried out by the solver. This technique involved four stages: understanding the problem, devising a plan, carrying out the plan and looking back.

It is interesting that Hadamard (1945) suggested four stages in the solution of a problem. His stages were preparation, incubation, illumination and verification. The first and last of these stages are clearly similar to those of Polya. The difference lies in the middle two stages where Polya's implied belief, that by practising a routine, pupils would become better problem-solvers, might seem at variance with Hadamard's implication that the pupil must sit back and wait for illumination to occur.

Maier (1970) sees efficient problem-solving as both a matter of perceiving obstacles that can be readily surmounted and of ingenuity in dealing with a particular obstacle. He sees finding a solution to a problem as one of idea-getting and idea-evaluating. Idea-getting is concerned with generating alternatives whereas idea-evaluating is concerned with selecting the best alternatives. A common difficulty for a problem-solver is the tendency to evaluate and select an alternative before the best one has been generated. This often results in a 'snatching' at the solution and giving an answer appropriate to an intermediate stage of a solution.

Pask (1976) has established a strong case for the existence of two distinct learning strategies - serialist and holist. He suggests that learning performance is regulated by the level of uncertainty at which the learner is prepared to operate. Serialists proceed from certainty to certainty, learning, remembering and recapitulating a body of information in small, well-defined and sequentially ordered 'parcels'. They may appreciate topics ahead of those they understand, but they tend not to look far ahead. They are cautious, 'one step at a time' learners who are confident that the necessary knowledge will be gained steadily.

Holists, on the other hand, prefer to start in an exploratory way, working first towards an understanding of an overall framework, and then filling in the details. Birch and Rabinowitz (1968) consider that a combination of both serialistic and holistic skills is desirable.

Some researchers, for example Wood 1974, suggest that girls work more to the serialist model and boys to the holist. If this is true, then a careful error analysis would help to show whether or not girls are giving solutions appropriate to an intermediary stage. This will be investigated further in a later part of the study.

Dweck and Licht (1980) also point to the degree to which the problem-solver will respond to a challenge and the length of time the individual will stick with a problem. They raise the problem of a person's tolerance of ambiguity. An individual's performance may vary depending on the person's 'frustration threshold'.

Again, these are important considerations when looking at the performance of girls and boys in mathematics. The length of time a candidate perseveres with a problem or spends time generating alternatives or even a person's 'frustration threshold', may be easily affected by outside influences. For example, if a girl does not see mathematics to be relevant to her, or if she is imbibing cues that mathematics is for the boys and not for her, then all these important qualities will be reduced, even though she is as capable and intelligent as any boy.

By definition, problems are not routine, each one being to a greater or lesser degree new to the learner. Successful solution of a problem is as Polya (1945) said, dependent on the learner not only having the knowledge and skills required, but also being able to use them and establish a network or structure. This again, requires not only skill, but it requires a degree of confidence and assurance. These affective variables may work more against the girls than the boys and will be developed in a later chapter. The confidence of many girls, particularly in the 3rd and 4th years of secondary school, may be lower than that of the boys. This in turn, may affect their performance in any novel, problem-solving situation.

So, problem-solving depends on having both an organised body of knowledge and a creative activity in which the learner participates. It assumes a degree of mathematical ability as defined earlier in the study. It depends on a level of competence in the use of spatial, logical, numeric and symbolic skills. It may also depend on proportionality skills, as this is essentially a development of logical thought.

Most problems are capable of being represented in a variety of ways and the difficulty of the problem may be greatly affected by the representation chosen. As Frazer (1982) says, 'No problem exists in isolation - a problem is perceived by the individual' (Chemical Society Reviews, P.171). So, how a problem is construed is clearly important. This may depend on the experience of the problem-solver and the extent to which the individual relates and interprets the problem. The problem may well be viewed differently by a girl or boy depending on the background experience, even if they have the same mathematical ability. This has implications in the type of problems that pupils are asked to do and the context in which they are set.

There is interest too, in the aim to improve problem-solving skills of pupils in school. Polya (1945, 1962) has led the way in the consideration of how to establish a routine for problem-solving and how to train people to become better problem-solvers. Wickelgren (1974) too, suggests that it is possible to produce more competent problem-solvers. If this is true, then it might be possible for the teacher in the classroom to redress any imbalance in the performance of both girls and boys.

Gagné (1977) has however, stated that it is probably not possible to teach people to become better problem-solvers. This is because of his belief that thinking skills cannot be taught in a vacuum - each problem involves its own content and context. Ausubel (1964) too, whilst accepting that training in problem-solving within a fairly narrow and well-defined subject discipline might achieve some success, is careful to point out the transfer problem.

If a girl or boy finds particular difficulty with problem-solving exercises, and if they have a sound basic mathematical ability, then it may be that supervised practice and guidance in the classroom will lead to greater success. An illustration of this guidance is contained in the Joint Matriculation Board/Shell Centre pack Problems with Patterns and Numbers (1984). If it is possible to teach girls and boys to become better problem-solvers, then the advice contained within the pack and the emphasis on a problem-solving routine, would be very helpful. The particular list of steps in the routine are as follows: Try some simple cases; Find a helpful diagram; Organise systematically; Make a table; Spot patterns; Use the patterns; Find a rule; Check the rule; Explain why it works. Advice and practice of this kind must be of assistance to the struggling pupil, either girl or boy.

Having analysed what is involved in the problem-solving process, it is appropriate to look at examples of performance variation. This will then lead to the present study where particular types of problem-solving questions can be analysed for variation in performance.

Armstrong's (1981) review of the two American national surveys of achievement in mathematics gave evidence of a significant difference between the scores of male and female 12th grade students in the Women in Mathematics survey on problem-solving in favour of the boys. This result was paralleled in the 1978 National Assessment of Educational Progress survey. Boys scored higher in the problem-solving section.

This evidence supports the findings that girls perform less well on one or two stage problems, where techniques cannot be applied in a routine way. It was shown however, that in the Women in Mathematics survey, where students had participated in higher level courses, differences between the scores of girls and boys were not mathematically significant. This is consistent with the fact that there is always a large distribution in the scores of girls and boys in examinations in mathematics and the better girls can equal or surpass their male peers.

The first APU Secondary Survey (1980) also found differences in performance between girls and boys in practical problem-solving activities. For example, in one question relating to mass, candidates were asked to find the mass of one peg from a bag of equal small pegs, when given only a 20g mass and a balance. It was found that in the 15/16 year age range, 19% more boys succeeded in finding the correct answer.

A study by Fennema (1974) of children aged between ten and fourteen found that girls performed better than boys in skills of mathematical computation, eg. money, time and decimals. This is consistent with other studies of a similar nature. However, in tests of a problem-solving nature involving the more complex skills of analysis, comprehension and application, the boys did better. She concluded that girls tend to perform better in tests of basic, mathematical, rote computation, and boys tend to perform better in tests of mathematical problem-solving, requiring comprehension and reasoning.

Tests carried out in 1964 as part of the International Study of Achievement in Mathematics (Husen, 1967) showed a similar pattern. In all twelve developed countries which took part in the study, the performance of boys was higher than that of girls at the age of 13. The performance of boys was further ahead on mathematical problem-solving than on basic computational questions. However, there was difference between countries, the sex differences in performance being greatest in Belgium and Japan, and least in the USA and Sweden. This may add weight to the nurture debate and the many socialisation patterns which may affect performance.

Another example of performance variation is given by Wood (1976). He studied the relative performance of girls and boys in the London

Board's O-level, mathematics examination. He studied the responses to questions set in June 1973 and June 1974 papers and analysed these for girls and boys separately. He found that even when school effects had been allowed for, differences in favour of boys on certain kinds of problems persisted. These problems were concerned with scale or measurement, probability and space-time relationships. He found evidence that girls from girls-only schools as well as from mixed schools experienced particular difficulty with certain problems.

Wood found that none of the items on which girls did better than the boys required what could be termed problem-solving behaviour. Instead, they called for 'the supply of definitions, recognition or classification, application of techniques and theorems and substitution of numbers into algebraic expressions, just the type of operations which are most susceptible to drilling' (Educational Studies, 2, 2, 1976, P.156).

The skill of problem-solving is not specific to one area of mathematics. The mathematical content may relate to any of the areas of number, algebra, measures, shape and space or data handling. What is important to discover in terms of this study, is the difference in performance between girls and boys in each of these content areas. What are the precise concepts that give the widest discrepancy in performance between the sexes? Studies show apparent differences in mathematical ability, notably in skills relating to visuo-spatial awareness, proportionality and problem-solving processes; but an in depth study is needed to highlight more specific content variations.

To summarise, it is evident that problem-solving involves both knowledge and process. Skills are needed to use the knowledge in establishing networks and structures. Girls may find more difficulty with the spatial and proportionality concepts but there is no evidence to suggest that they are less able to establish appropriate networks and structures. It may be that more girls approach a problem from a serialist model, and more boys from a holistic approach.

The experience of the problem-solver and the extent to which the individual relates and interprets the problem is important. A problem-solving exercise may be construed differently by a girl or boy depending on their background experience. If a boy has had greater play experience with constructional toys and a greater practical background he may do better in certain problem-solving situations. One aspect of problem-solving in mathematics is that often problems are divorced both from the mainstream subject matter and also from the real world (see Wickelgren, 1974). Such problems may contain great interest for some children, but others may not see the point and become demotivated. Such problems are unlikely to produce knowledge or rules which are useful or applicable elsewhere. This raises the question as to whether the problems which are set are of equal interest and relevance to both girls and boys. It may be that certain pupils are just not interested in a particular problem. This has implications for the teacher in the classroom to be aware of the interests and backgrounds of all pupils in their charge.

The extent to which a candidate perseveres with a problem may vary, depending on a person's 'frustration threshold'. Mathematics students, both girls and boys, often report feelings of frustration or satisfaction when they work on non routine problems. These affective responses are an important factor in problem-solving and deserve increased attention in research. If a student does not see mathematics to be relevant to his/her future or if there is a lack of confidence and an increase in anxiety and panic, then they may perform less well. McLeod (1988) specifies several dimensions of the emotional state of problem-solvers including the magnitude and direction of the emotions, their duration, and the student's level of awareness and level of control of the emotions.

Are these affective factors important in problem-solving performances of girls? Does a greater anxiety affect mathematical performance? Are girls more prone to this debilitating anxiety than boys?

8. Affective Variables

Recent research has made substantial progress in characterising the cognitive processes that are important to success in mathematical problem-solving. However, the relationship of affective factors to these processes and the variable responses of girls and boys needs closer attention.

One of the difficulties in discussing research on affect is the confusion over terminology. For this study, 'affect' is used as a general term to represent all of the feelings related to mathematics learning. 'Emotion' is used to signify a more visceral kind of affect, a response that is quite intense but relatively short in duration. Following Simon (1982), emotion will refer to affect that is sufficiently powerful to redirect attention. 'Attitude' is a term that is used for less intense affective responses, especially responses that are relatively consistent.

When students are engaged in solving non routine mathematical problems, emotional feelings may be expressed. If the work on the problem extends over a period of time these emotional responses may become quite intense. Students who fail to reach a solution frequently report feelings of frustration (Confrey, 1984) or even panic (Buxton, 1981). The intensity of their feelings is often reflected in muscle tension or rapid heartbeat (Ginsburg and Allardice, 1984). If the students obtain a solution to the problem, they typically express feelings of satisfaction, even joy. As Mason, Burton and Stacey (1982) note, these emotions, both positive and negative, are important factors in mathematical performance.

From Mandler's (1984) point of view, a major source of emotion is the interruption of a person's plans or planned behaviour. When an interruption occurs, the normal pattern of completion of these sequences of thought or action cannot take place. The result of the interruption is physiological arousal. These interruptions of planned, organised sequences of thought or action, are also referred to as blockages, or discrepancies between what is expected and perceived. In mathematical problem-solving, the solution or goal is not immediately attainable and there may be no obvious algorithm for the student to use. In other words, the student's initial reaction to the problem is that no solution is evident. The problem-solver is blocked. The initial plan to solve a non routine problem is often inadequate and the plans are interrupted. New strategies or heuristics may be applied. In other words, the definition of a mathematical problem is exactly the situation that Mandler uses to describe how interruption and arousal lead to emotion.

He emphasises the point that the cognitive evaluation that is combined with the arousal, can result in either a positive or a negative emotion. Both of these reactions can be seen in students in problem-solving. What is stimulating for one student can be depressing for another. Either kind of emotion can result from the same type of interruption. Therefore, the way that students interpret the effect of the interruption is very important.

Mandler points out that the intensity of the reaction to the interruption is related to the degree of organisation of the student's mental activity. In mathematics, where students spend so much of their time doing routine exercises, students' actions are very highly organised. So, the blocks that inevitably interrupt problem-solving activities may lead to intense emotions.

The confidence-anxiety dimension as it relates to mathematics may be one of the more important affective variables that helps to explain sex-related differences in mathematics learning. In discussing differences between the sexes however, it is important not to assume that these differences are absolute. Although there may be differences on average, there is always considerable overlap and this is true about anxiety levels, as it is true about other observed differences.

Literature seems to support the fact that there are sex related differences in the confidence-anxiety dimension. The relationship of anxiety and mathematics learning has been explored by a variety of methodologies and with instruments purported to measure debilitative or facilitative anxiety specific to the subject. Callahan and Glennon (1975) concluded that anxiety and mathematics are related and that in general, high anxiety is associated with lower achievement in mathematics. An American study (Crosswhite, 1975), found that between grades four and ten, facilitating anxiety decreased. Girls' scores decreased more than boys'. However, debilitating anxiety increased for girls between these grade levels. Sutherland (1983) supports this same theme when she says, 'Women are more anxious than men, girls are more anxious than boys' (Sex Differentiation and Schooling, P.60).

These conclusions are supported by Hembree (1990). The results of 151 studies were integrated by meta-analysis to scrutinise the construct of mathematics anxiety. He found that mathematics anxiety is related to poor performance on mathematics achievement tests. It relates inversely to positive attitudes toward mathematics and is bound directly to avoidance of the subject. He also found that across the age range girls report higher mathematics anxiety levels than boys. However, higher ability levels do not seem to translate into more depressed performance or to greater mathematics avoidance on the part of female students. Indeed, male students in secondary schools exhibit stronger negative behaviours in both these regards. In other words, the girls with high ability are performing very well even though they express greater anxiety.

This paradox may be explained along two lines. The first is that females may be more willing than males to admit their anxiety, in which case their higher levels are no more than a reflection of societal norms. Sarason (1960) suggested that the greater apparent anxiety in girls than in boys, might be due to the fact that it is socially more acceptable for girls than for boys to admit to such reactions. The second explanation is that females may indeed cope with anxiety better than males.

In Sarason's study, the levels of mathematical anxiety increased through junior school, peaked near Grades 9-10 and levelled off in upper high school and college. High levels appeared in remedial mathematics and declined with more advanced study. Mathematics anxiety seemed somewhat higher in slow and average students, but no difference was found between these groupings. Females consistently displayed higher levels than males especially at the college level.

Higher achievement consistently accompanies reduction in mathematics anxiety but there is no compelling evidence that poor performance causes mathematics anxiety.

Positive attitudes toward mathematics consistently related to lower mathematics anxiety, with strong inverse relations observed for an enjoyment of mathematics and self confidence in the subject. Small correlations were found between mathematics anxiety and desire for success, and a view of mathematics as male-oriented. Highly anxious students viewed parents and teachers as somewhat negative towards mathematics. Also, highlyanxious students took fewer high school mathematics courses and showed less intention in high school and college to take more mathematics.

So, it seems reasonable to believe that lesser confidence or greater anxiety, on the part of girls, is an important variable which may help to explain sex-related differences in mathematics.

Leviton (1975) and Primavera (1974) reviewed the literature dealing with self confidence, and both concluded that a positive relationship exists between academic achievement and self esteem. Gallaher and Glennon (1975) concluded that there is a positive relationship between self-esteem and achievement in mathematics. Others have also recognised the importance of academic self-confidence in learning mathematics (Bachman, 1970, Fink, 1969).

Crandall (1962) concluded that girls underestimate their own ability to solve mathematical problems. Others have concluded that females may feel inadequate when faced with a variety of intellectual, problem-solving activities (Kegan, 1964). Maccoby and Jacklin (1973) reported that girls tend to underestimate their own intellectual abilities more than do the boys.

In the Fennema-Sherman study (1978), at each grade level from six to twelve, boys were significantly more confident in their ability to deal with mathematics than were girls. In most instances this was true when there were no significant sex-related differences in mathematics achievement. In addition, confidence in learning mathematics was more highly correlated with mathematics achievement than was any other affective variable. Confidence was almost as highly related to achievement as was spatial visualisation. The APU Primary Surveys (1980, 1981) confirm that there is a comparative lack of mathematical self-confidence among girls at an age as early as 11 and as schooling progresses, girls have been found to display greater anxiety of a debilitating type.

So, what causes anxiety and a low degree of self-esteem? It may well be that a multiplicity of causes is present, some in the individual, some in the environment and experiences. The degree of uncertainty as to the outcome of a situation or problem seems to be important. This depends partly on the individual's estimate of ability to cope with the situation.

An individual capacity to endure uncertainty may also depend on characteristics of the central nervous system or biochemical reactions within the individual (Sarason, 1960). Or, in other cases, the physical developments of adolescence may make the individual unsure of ability to control events (Barker-Lunn, 1972). Certainly in girls, anxiety develops well before adolescence (Bennett, 1976).

Sunderland (1983) speculates as to whether the observed tendency of girls in childhood to attend to adult approval, while boys tend to concentrate on peer-group reactions, could mean a greater uncertainty about approval, leading to greater anxiety about performance on the girls' part.

Is it always a bad thing to be anxious? In common experience it is true that some degree of anxiety is helpful. From the educational point of view, it is often felt that it is indeed time some pupils were anxious about the results of their lack of application. A certain amount of anxiety seems to correlate with the best performances. Too much or too little anxiety means a performance which is less good. This finding has been replicated in a number of researches dealing with a variety of learning or skill situations (Cronbach and Snow, 1977).

What complicates research results and causes problems in the teaching situation is the variety of other factors which affect the relationship between anxiety and performance. One is the difficulty level of the mathematics problem. For simple questions, anxiety is best low and for difficult tasks again, it should be low. Yet, the feedback which is received or not received during the performance can be important. Uncertainty about whether the individual can cope may be reduced, progressively, by the assurance of doing well (eg. getting the answers right). Negative feedback on the other hand, can increase anxiety, and absence of feedback leaves the anxiety and uncertainty for the most part, unaltered. Sometimes however, if there is a time delay, anxiety may again be increased (eg. waiting for examination results).

Feedback may also depend on how well the individual is actually performing, which in turn depends on the ability of the individual. Thus, in a learning or performance situation, there may be four types of interaction. First, there are individuals who are high in ability and high in anxiety. Then there are those who are low in ability and high in anxiety, or those who are high in ability and low in anxiety, or low in both ability and anxiety.

Educationalists frequently recognise such types. There are those pupils who have the ability to perform well but do not bother to try. There are the able pupils who on occasion do not do themselves justice because of 'nerves' and there are those who try without success or who go completely 'to pieces'. Then there are those who seem to have little ability but also little concern about the situation.

Certainly then, the indications are that anxiety in just the right amount may be of value. It may be concluded that the task of the teacher is to decrease the anxiety of those who have too much and to induce anxiety in those who have too little to ensure optimal performance all round. Yet, what is important is how the individual decides to cope with an anxiety-causing situation. Clearly, the individual has to face the anxiety-causing problem and to cope with it, effectively or ineffectively.

One popular strategy for coping with an anxiety-causing situation is avoidance. If there is a danger of loss of self-esteem in a certain situation, and if that situation can be avoided, then the obvious action to take is to evade the situation. So, if girls have anxieties about their ability to do mathematics or science, then they take advantage of the school system and opt out.

Certainly the statistics of entries for external examinations show the extent to which girls avoid the subjects in which girls are thought to be less likely to succeed than boys, ie. science and mathematics. Yet, in avoiding these subjects, girls are renouncing a very great range of occupations. They are deliberately excluding important work possibilities, even if, at the moment of decision, not all girls are clear as to the future consequences of avoiding certain subjects. Unfortunately, society readily accepts the modest aspirations of girls. When girls do aspire to uncharacteristic careers or subjects, there are often social reactions which seem calculated to reinforce anxiety.

In his book, Do You Panic about Mathematics? (1981) Buxton provides a variety of reactions of individuals to mathematical problems. A range of emotions, including embarrassment, irritation, frustration and fear have all been mentioned in connection with mathematics performance. He argues that, whatever a person's reasoning capacity, its effectiveness is strongly dependent on the extent to which the emotions aid or impede the particular task on which it is engaged.

Fox (1979) suggests that there may be as many boys as girls who are anxious about mathematics but that this does not inhibit them to such an extent from taking mathematics courses. More boys perceive the courses as useful or even unavoidable.

Burton and Townsend (1985), commenting on the varying attitudes of boys and girls towards success and failure, show that a boy more often attributes his success to 'ability', while a girl claims it is as a result of 'effort'. In the case of failure, boys more often 'externalise' while girls 'internalise'.

In a study by Hoyles (1982), pupils evaluated grades and assessment as information as to their mathematical ability and therefore, judged themselves highly if they did well, but found it difficult to rationalise any failure. This in turn, led them to associate such failure with feelings of inadequacy and anxiety. Pupils wanted to be given mathematics of an 'appropriate' standard but quickly lost confidence if teachers left them behind or put pressure on them. Pupils did not talk about what their mathematics was about, or how it may be used. They did not appear to see that the subject could be of any interest in itself but only as something to be done, something to be mastered.

A Sheffield study involving 12 year old secondary pupils (Eddowes and Sturgeon, 1981), looked at the attitudes of pupils at the beginning and end of a school year. In the first survey, sex differences were small overall compared with school differences, though girls showed less confidence in their ability to cope with mathematics and thought the subject was difficult. Both girls and boys saw the subject as suitable for both sexes and there was no difference in its perceived usefulness. At the end of the year, the overall mean score was significantly lower for girls. Measures of anxiety and lack of confidence were increased, though this was also true of the boys, but to a lesser extent.

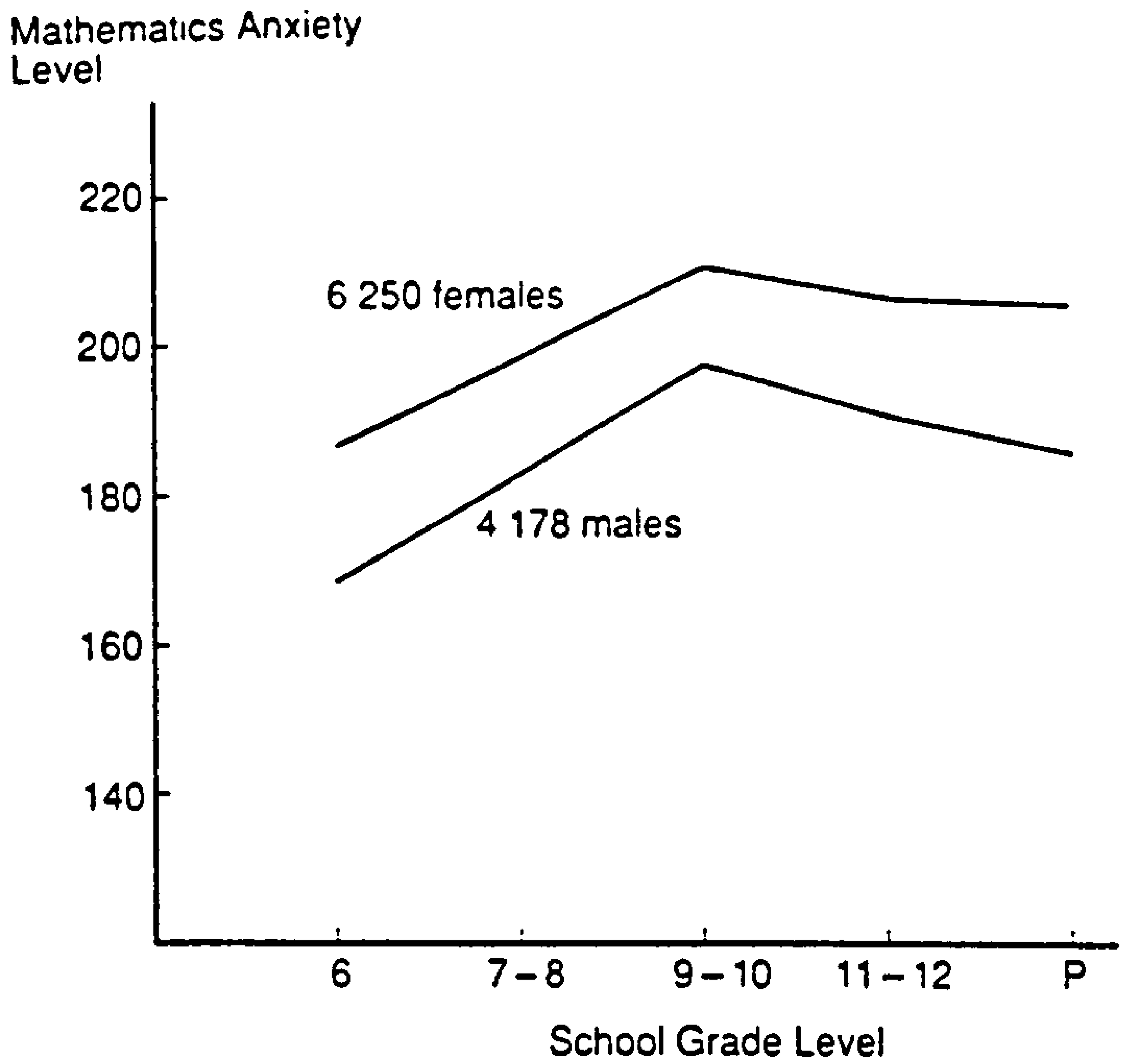
Attitude scores followed attainment scores fairly closely for boys, but not so well for girls. It seems that success in mathematics does not necessarily generate positive attitudes towards it in some girls. In fact, a further survey showed that most able girls under-estimated their success in mathematics compared with the teacher's estimate, whereas boys did not (Marland, 1983).

In the final stage of the Sheffield study a questionnaire survey of 10-15 year old pupils found that girls' overall attitude to mathematics tended to be more positive than boys' at 10+ but much more negative by 14+; the cross-over occurring at about 12.

A survey of Scottish school leavers by Bibby (1980) found that most pupils who had avoided mathematics after the second year of secondary school did so simply because they did not like it. Girls seemed more likely to relate 'liking' and 'choosing' and to admit 'difficulty' as reasons for not taking mathematics. Even well-qualified girls seemed less convinced about the value of the subject.

Clearly, there are still many questions left unanswered and which require further investigation. For example, are feelings of confidence stable within girls and boys across time and across a variety of mathematical activities? Do levels of confidence and anxiety affect girls differently than they do boys? The meta-analysis study (Hembree, 1990) clearly showed that levels of anxiety were greater for girls across the age range. Figure 9 shows average mathematics anxiety levels for girls and boys in Grades 6 to post secondary, based on 10428 measurements of the construct. The levels increased through junior high school, peaked near Grades 9-10, and levelled off in upper high school and college.

Figure 9. Average mathematics anxiety levels for Grades K-12 and undergraduates (USA).



(Hembree, R., 1990, The Nature, Effects and Relief of Mathematics Anxiety in Journal for Research in Mathematics Education, P.41. The anxiety levels are based on the Mathematics Anxiety Rating Scale of Richardson and Suinn, 1972.)

Mathematics is a subject which is highly structured and hierarchical in nature and systematic progress is essential. This is why it is of particular importance to study the particular emotional responses and attitudes of girls and boys towards mathematics across the important adolescent years of secondary education. This study will attempt to do this.

Problems which arise early in a person's mathematical development are difficult to resolve if they are not realised early. It may be that a student's attitude and motivation improves as he/she gets older but by then the damage may be irreparable. This may be true for girls who lose interest in the subject during the 3rd and 4th years (National Curriculum, Years 9 and 10), only to find that when they make the extra effort in the 5th year (Year 11), they are too far behind. Clearly, it is important to know what students are feeling when they are learning. What are their emotions as they are being taught? Certainly in some subjects, expression and the development of feeling are part of the material to explore, and such explorations on the part of the students are encouraged. Mathematics however, is regarded by many as an area of study where this is not the aim. It is characterised as stimulating the cognitive area without necessarily any regard to emotional responses. Yet, it is clear that many people have very strong feelings about it - mostly negative.

Anxiety is a more general feeling and may often relate to the presence of authority relationships. Implicit in anxiety is the feeling of being judged either by the teacher, parents or peers. There is the feeling of not being able to live up to the expectation of others or of one's own standards. The reaction of pupils to allay anxiety may be to lower the expectation of others and to lower personal standards.

Perhaps the most widespread response to mathematics, occurring in large numbers of classrooms and throughout the population at large, is almost a non-response - boredom. It is difficult to categorise this as an emotion, and like bewilderment and panic, it is a 'state of mind'. Unlike the others it is unresponsive, but it is a central issue in teaching mathematics. Boredom stems from a lack of interest or of the relevance of the studies to the real world. The material within mathematics is so varied that this situation need never occur.

However, the repetition of 'method' can become so familiar as to occasion boredom, whether the idea is fully understood or not. The result can be an unwillingness, an inertia, a feeling of 'Why should I bother?' or 'How does it matter to me?' Once it reaches this level, further attempts to teach will result in sharp reactions. These suggest feelings which are far more reactive than the person would accept. This may well imply that boredom is a defence mechanism built to guard against further unwelcome experiences of a sort previously undergone. It may also be true that in comparison with other interests of teenagers at this stage mathematics has a very low rating. The question then becomes:- Is mathematics of more interest to girls or boys?, and How does this interest affect the relative performance of girls and boys? What are the implications for examination results at 16? A clear survey of the attitudes of pupils from 11-16 is needed to establish some of these affective and motivational issues.

In summary, mathematical anxiety is related to poor performance on mathematical achievement tests. Girls display higher levels of anxiety than boys, though they may be more ready to admit it or cope with it. A high anxiety level is bound directly to avoidance of the subject. Variables that exhibit differential mathematics anxiety levels include ability and year groupings.

Models formulated to describe girls' and boys' learning behaviour in mathematics typically include cognitive and affective components but they also include socialisation factors. We are all social creatures and are affected by pressures of various kinds which may directly or indirectly influence mathematical performance. These need to be considered further.

9. Social Variables

The cultural and societal environment in which children grow up has a significant effect on the expectations they develop. As Burton (1978) puts it, the gender of a child determines the expectations of parents, teachers and peer group. From infancy, children's ideas about appropriate roles and behaviour are influenced by the actions and attitudes of their parents and other adults in their environment. Weiner (1980) suggests that many facets of experience outside school may combine to generate in children concepts of mothering, of fathering, of behaviour appropriate to their gender, of manliness or of womanliness.

Bishop and Nickson (1983) also acknowledged the relationship between socialisation processes and the learning of mathematics when they argued that research in mathematics education should be directed away from the individual child as a learner and towards an increased understanding of the effects of the social context of schools on the learning of mathematics. Some schools may try hard not to reflect these ideas. However, it is probable that despite schools trying hard to counter some of these pressures, they are still evident in a variety of ways. These socialisation processes can be conceptualised in terms of cultural expectations, parental expectations and beliefs, school and teacher practices, as well as through peer group pressures. Their collective influence has been highlighted in the debate on sex differences in mathematics learning. The important question however, is whether social pressures affect mathematical performance and whether these pressures are greater for girls or for boys?

So, what are these pressures? The cultural and social environment in which children grow up can exert a variety of different pressures from peer groups, teachers and parents. School organisation, subject choice, career advice and books and teaching materials can influence pupils' ideas and values. These pressures in turn can increase the anxiety pupils experience affecting mathematical performance.

With adolescence, the physical attributes of the sexes diverge. Girls mature earlier, going through puberty on average two years earlier than boys. They also go through the change more quickly. This creates social complications when girls find boys of their own age immature and uninteresting as companions. There are also problems for individuals who are earlier or later than average in reaching puberty.

From the educational point of view, there are two interesting facets of the earlier maturation of girls. The first is the temporary superiority they may enjoy in height and weight when compared to boys of their class group. This may well be accompanied by feelings of greater confidence. It may also lead to teachers interpreting the girls' appearance as indicating a greater social and emotional maturity so that girls at this stage are regarded as being more responsible than boys and more suited to being given duties and responsibilities. The second is the apparent spurt in intellectual development, particularly on the verbal level. Here the differences in the rate of development have been recognised in some tests of verbal reasoning for these age groups, by the provision of separate norms for girls and boys. The 11+ selection which still operates in some LEAs, occurs at a point when the two sexes are at markedly different stages of development.

10. Peer Group and Stereotyping

In the study of peer group pressures and stereotyping, the age of the pupils could be an important factor. A study by Leder (1980) found that at adolescence many boys value success most highly while many girls consider popularity with peers most important. She found that girls are far more likely to have high aspirations if they have friends who have similarly high expectations. Weiner (1980) also suggested that peer-group pressure increases in adolescence, and is enhanced by the influence of pop culture, the media and teenage magazines, some of which put forward stereotypes which confirm the restricted image of the girl.

As part of Leder's study (1980) pupils were asked to describe a pupil of their own age who was clearly good at mathematics. It was worded as follows: "Anne (John) came top of her (his) mathematics class. Describe Anne (John)." The Anne cue was given to the girls and the John cue to the boys. It was found that generally children are very unflattering about anyone who is good at mathematics. Two typical responses were:

'Anne is a hard-working girl who finds maths rather easy. She has ginger hair with glasses and buck-teeth. Her hair is greasy and she has a spotty forehead. She has lots of freckles. She always works quietly and does her homework. She wears pebble glasses.'

'John is fairly brainy, works hard, also concentrates on his work. I think John is a boffin. I think John would be a puny, skinny kid with National Health glasses. Also, he does his top button on his shirt up with his tie done right up to his neck. He would wear stupid horrible shoes as well.' (Girls into Mathematics, Page 40).

The roles were then reversed, with the boys describing Anne and the girls describing John. Comments on physical appearance all tended to be derogatory but particularly of Anne who was described as 'flat chested' with 'straight hair' with 'spots' and 'goofy teeth'. The results suggest few differences in the opinions of the girls and boys other than that both disliked anyone good at mathematics, although the girls did think more favourably of a boy good at mathematics, than the boys.

Whilst there is no evidence of a direct relationship between pupils' perceptions of each other, and girls' under-achievement in mathematics, this type of stereotyping and attitude may have an effect on confidence and motivation. The popular view seems to be that being good at mathematics and being attractive and feminine are incompatible and so mathematics is perceived as somehow inappropriate and masculine. Pupils' perceptions of a subject may affect what they learn from it. If a subject is seen as relevant and if appropriate encouragement is given, then it is more likely that a positive effort will be made.

Stein (1971) has also provided evidence of mathematical stereotyping. However, he found that this did not occur until adolescent years and even during these years mathematics is not ranked as highly masculine as are spatial and mechanical tasks.

The Fennema-Sherman study (1977) indicated that females in grades 6-12 deny that mathematics is a male domain. While the males in the study did not strongly stereotype mathematics as a male domain, at each grade they stereotyped it at a significantly higher level than did females. This is a significant finding because the cross-sex influence on all aspects of behaviour is strong during adolescent years. Since males stereotype mathematics in this way, they undoubtedly communicate this belief in many subtle and not so subtle ways. This may have an effect on girls' willingness to study mathematics. It also has strong implications for the development of intervention programmes designed to increase female participation.

The 1985 Report, Mathematics from 5 to 16 states clearly that 'specific efforts need to be made to ensure that mathematics is a less male oriented subject than at present ... There must be higher expectations of what girls can do, with more encouragement given them to participate in all activities.'

A simple illustration of how pupils' performance is affected if they believe the skills involved are not appropriate to their sex was demonstrated by psychologist D. Davies (An Equal Start, 1987). She asked a group of 11, 13 and 15-year-old pupils in a London comprehensive to undertake a practical test involving the 'loop wire game'. The girls and boys were told separately about the purpose of the exercise. One group of girls was told that it was to measure needlework skills and one group of boys that it was to measure their skills in electronics. Then two other groups of girls and boys were given the opposite information. Those pupils told that the test was measuring skills they saw as inappropriate to their sex had significantly lower scores than those who saw the skills as appropriate to their sex. Clearly, it is important that mathematics is seen as both useful and relevant to girls and boys equally.

There is evidence that in a variety of ways, girls have been discriminated against in terms of mathematical education. The NUT document, Towards Equality for Girls and Boys (1988), is concerned about the effects of this action. They contend that women who are seen by some to be passive and who then attempt to resist that position are seen as a threat, because they are stepping over the boundary of gender differences into 'masculine' behaviour. This 'deviance' may result in abuse from their peers, both male and female.

Sex-stereotyping is experienced by children even before they reach school. By the age of five, sex-stereotyping has already had a significant effect on the child's perceptions of the world and his/her role in it. A study of 'free choice' among four and five year-olds in primary school showed that when the girls were given a chance to choose an activity, they generally opted to play with dolls, play in the playhouse, tidy up the classroom, read or draw, while the boys tended to use constructional equipment and move around the room with cars. Their choices were already gender related (Janet Hough, Deprivation of Necessary Skills, 1984).

11. Teachers' Influence

Teachers are amongst the most important educational influences on a student's learning of mathematics. From primary to secondary school pupils spend hundreds of hours in direct contact with teachers. While other educational agents may have influence on educational decisions, it is the daily contact with teachers that is the main influence of the formal educational institution. Part of the teachers' influence is in the learners' development of sex-role standards. It may be that this influence is exerted by teachers through their differential treatment of the sexes as well as through their expectation of sex related differences in performance. Clearly, a key factor affecting what pupils learn within the formal curriculum is the teacher's attitude and presentation of the mathematics.

The NUT document, Towards Equality for Girls and Boys (1988) contends that, even as early as nursery and primary level, teachers' expectations affect pupils' learning processes. It suggests that, despite boys' relatively poorer achievement in the primary years, teachers appear to have higher expectations of them and that boys are expected to be more creative and more 'truly' intelligent than girls.

Whyte (1983) suggests that where boys do less well, for example, in acquiring reading and language skills, special efforts are made to improve their performance by offering remedial help. There is much less concern with girls' need to develop visuo-spatial skills and greater confidence in premathematical or scientific learning.

A considerable amount of attention has been directed toward the effect of the teacher's perception of pupils on pupil achievement. One such study, involving a sample of 16 schools, found that while assessments of achievement were closely related to objective measures, these assessments were to some extent affected by the teacher's perception of other characteristics of their pupils (Morrison, McIntyre and Sutherland, 1965).

It was also found that teachers tended to make a more general evaluation of girls than boys. They were less analytic in their approach to rating girls and tended to associate attainment with traits such as sociability and leadership. In particular, they associated girls' attainment in arithmetic more with 'good behaviour' than they did in the case of boys. They appeared to make a more complete assessment in terms of one or two major dimensions of girls than they did of boys and they varied much less in the qualities which they looked for in girls. The 'ideal girl' seemed to be the same whatever her social class or background while there was greater variation with respect to the 'ideal boy' depending on possibilities or limitations of the individual's environment.

Whyte (1983) suggests that because teachers assume girls are more highly motivated, their mistakes and failures are often put down to lack of intelligence, while boys in the same situation are thought to be just lazy or 'playing up'. Consequently, boys tend to become less discouraged by failure. They are told and believe what they need to do is try harder or pay attention next time. This excuse is not usually available to girls, and even small mistakes can make them feel inadequate. In the long term they are more likely to avoid new and difficult work, for fear of being shown up. Whyte says that 'Boys learn persistence and become more confident while girls are initiated into 'learned helplessness'.' (How Girls Learn to be Losers, Primary Education Review, 1983).

Although this scenario will not be true in all cases, it demonstrates the extensive effect of teachers' attitudes, particularly at nursery and primary level, on how pupils learn and their attitudes to certain skills. Clearly, this can have long term consequences in relation to girls' and boys' approaches to learning.

At secondary level teachers' attitudes and expectations continue to shape pupils' learning experiences. Good (1973) found that in all subjects, high achieving boys received the most favourable treatment and encouragement, and that boys were more active and interacted frequently with the teacher.

Becker (1981) reported on research in the USA which was specifically aimed at determining whether differential treatment of the sexes occurred in high school mathematics classes, and more generally, to identify what, if anything, is occurring in such classes, that may negatively affect the decision of girls to continue to study mathematics. The classes were studying geometrical problems and the students were aged 14 and 15. The results indicated distinct differential treatment. Girls were asked fewer questions and received less encouraging feedback. Throughout the period of investigation, female students were observed to withdraw and become more passive in lessons. Becker concluded that teacher, community and school beliefs and values compounded the impression that mathematics is not a subject in which women have an active role. He also concluded that generally teachers were not aware of their differential treatment of girls and boys.

Casserly (1980) presented a more encouraging picture. Where girls come into contact with teachers who positively encourage them to participate in mathematics and science studies and who hold an equal balance of treatment in the classroom, they often respond favourably. Attitudes about mathematics are related to the remembered impression of a teacher more clearly for girls than for boys. Fox (1977) reported that successful women mathematicians often mentioned inspirational teachers as a major factor in their choice of career - male or female.

A survey by the National Foundation for Educational Research (1984) focused on teacher attitudes as an influential factor in sex differentiation at secondary level. Although the survey revealed that most teachers were sympathetic to schools encouraging equal opportunities between the sexes, there were significant differences according to the subjects taught. In subjects such as mathematics and science there was less positive commitment to the concept and practice of equal opportunities.

The same survey noted that the minority opposed to equal opportunities argued that schools should <u>not</u> positively intervene in the process of choice by pupils - they should supply 'neutral' information and let pupils decide. Many however, would claim that an attempt to maintain neutrality, simply means allowing the many and powerful pressures on pupils to follow traditional education and career patterns to operate unchallenged. In 1983, 53% of teachers favoured anti-sexist policies in schools (TES-MORI). By 1987 this had risen to 68% (TES 12.6.87). This significant increase indicates that policies to promote gender equality in education are gaining support and credibility.

It is important therefore, for teachers to help girls recognise their success in mathematics is a result of their mathematical ability, and not attribute this only to hard work and good luck. This can be done alongside other techniques of ensuring girls are confident about each technique and process before progressing to more difficult concepts. Positive steps need to be taken to assure girls that they can continue to do well in mathematics after primary school. This can involve not just mathematics teachers, but other teachers whose subjects include the use of mathematics and who can ensure that girls get further support and encouragement in their classes.

12. Parents' Attitudes and Expectations

Parents have a very important part to play at every point in their children's schooling. Support or lack of support in mathematical studies can make enormous differences to the feelings of pupils and to their consequent performance in examinations.

Parents can shape and reinforce stereotyped views and can have considerable influence when it comes to career and option choices at secondary school. Kelly (1986), in Girls into Maths Can Go, found that when parents were asked to rate the suitability of various jobs for their children on a 1-5 scale, they had quite different occupational aspirations for girls and boys. Many were happy to see their children in traditionally stereotyped jobs, ie. their daughters as nurses, secretaries, social workers and hairdressers, and their sons as engineers, electricians and draughtsmen.

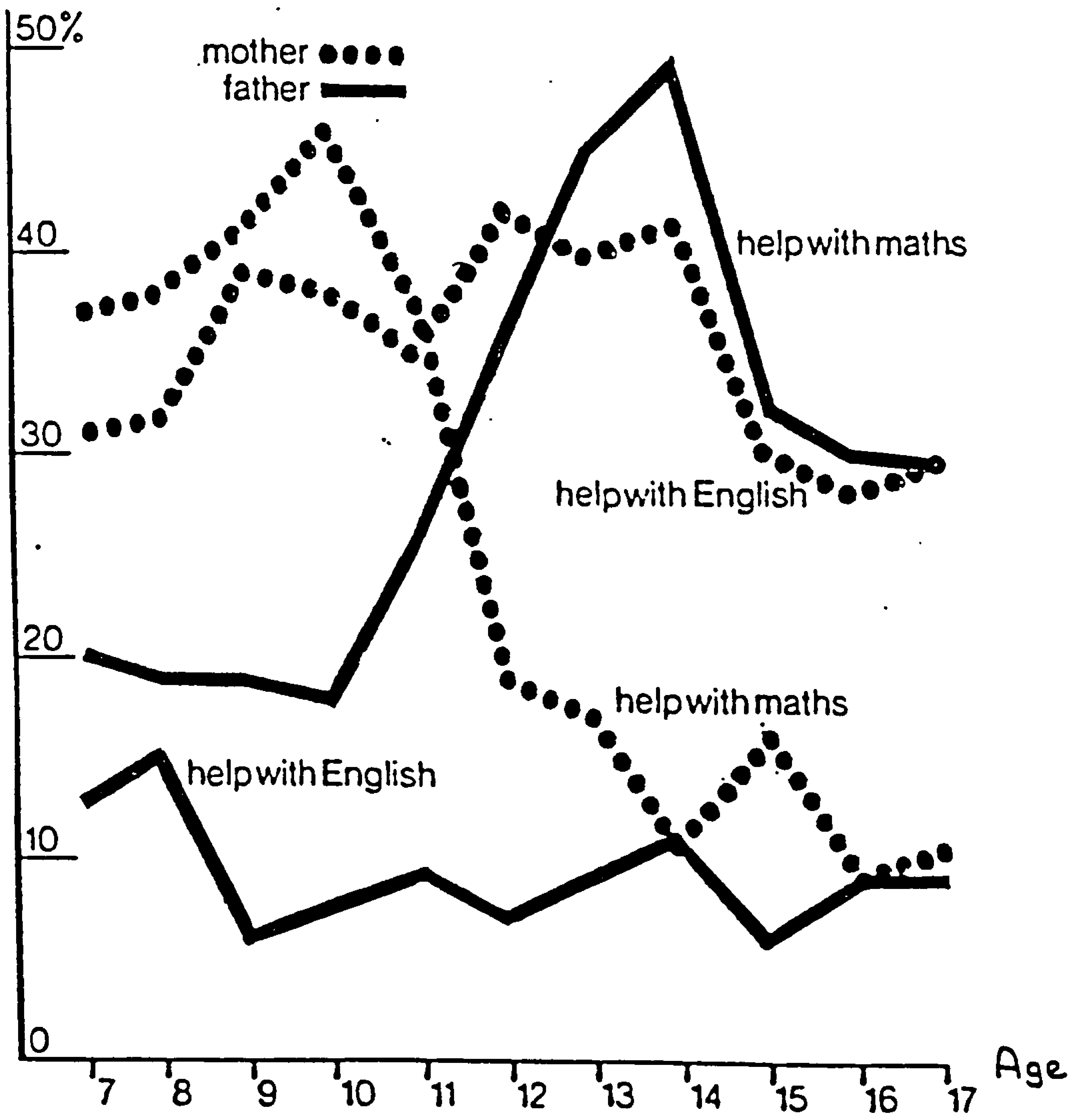
There is also some evidence (Millman and Weiner, 1985) that parents are more likely to coerce boys into useful career subjects while many girls are left to choose the subjects they like. So, many boys might be encouraged to take physics or mathematics in addition to other science or technical subjects but not the girls.

Other studies, eg. Armstrong and Price (1982), have shown that students' attitudes towards mathematics and a decision to continue with the subject are linked with their parents' conception of the educational goals of the school mathematics course, and with the extent of the mathematics education desired for their children by the parents.

There is another important parental factor - the parent as role-model. Luchins and Luchins (1980) have highlighted that parents are perceived as encouraging their sons' mathematical studies more strongly than those of their daughters.

Furthermore, Ernest (1981) found that mothers tend to help their children (both girls and boys) more than fathers in the early stages of formal education in both mathematics and English. However, from the age of ten, fathers increasingly are the ones who give help in mathematics. This peaks at the age of fourteen with fathers giving 50% of help with mathematical studies. The help mothers give during this time reduces in the same proportion to just 10% of help in mathematics. Yet, the help mothers give in English remains relatively stable between 28% and 42%. This trend is shown in Figure 10.

Figure 10. Percentage help given by mothers and fathers in mathematics and English to pupils between the ages of 7 and 17.



(Ernest, J., 1981, Are Girls Really Good at Maths? In New Society, 5.3.81)

These results are significant because if parents are perceived as encouraging their sons' mathematical studies more strongly than those of their daughters, and if that role model is mostly father, then this could have a significant effect on the child's perception of mathematics. It may well add to the male stereotyping of mathematics in the eyes of some girls during the difficult adolescent years.

However, Russel (1983),has shown that girls are more likely to study mathematics to a higher level if both their parents like it and are good at it. Moreover, fathers' high level of mathematical education significantly relates to their daughters' choice in taking mathematics at A-level.

Husen (1967), in an international study, found that in the countries participating in his cross-cultural study, student achievement in mathematics was related to parents' educational and socio-economic status.

Clearly, parents have an important part in shaping and perhaps unwittingly reinforcing stereotyped views. This in turn may begin to shape a pupil's attitude and motivation and 'liking' of mathematics. The result is the pupil's mathematical performance - for better or worse.

13. Mathematical Study in the Classroom

It is important to consider the study of mathematics in the setting where children learn. This may highlight some of the factors which influence attitudes and indeed performance in mathematics. What, for example, is the best way to group girls and boys in teaching mathematics? What are the effects of classroom interaction? What advice is given concerning subject choices and career prospects? What effects do books and teaching materials have in terms of sex stereotyping?

Type of School/Grouping

Some research has suggested that girls achieve more in mathematics in single-sex schools than they do in mixed schools (Smith, 1986). Boys on the other hand, tend to perform better in a mixed school. Wood (1976) in his analysis of O-level mathematics papers also found that girls in single-sex schools did rather better than girls in mixed schools. However, the performance of girls on those items in which boys generally excel, still remained lower than the boys. The comparison between single-sex and mixed schools needs to be treated with great caution because many of the single sex schools are selective and the mixed schools are comprehensive.

Keeves (1973), after a careful and thorough review of mathematics and science education in ten countries, concluded that the extent to which a community provides for education in single-sex schools would appear to indicate the extent to which it sees its girls and boys requiring different preparation for different societal roles. He argues that in so far as a community has different expectations for different groups, and proceeds to mould its future members through different schools, then it fails to provide equal opportunities for individual development. Before single-sex classrooms are embraced as a panacea for educational equity for females, there must be careful examination of long term effectiveness of such programmes. Because of what has happened to females over the last century, single-sex classrooms must be approached with caution.

There is some evidence that schools do influence sex stereotyping. Minuchin (1971) concluded that children who attended schools categorised as 'traditional' differed in their sex-type reactions from those who attended schools categorised as 'modern'. In the most traditional school, boys became leaders in problem-solving, while the girls followed. This was not so in the less traditional schools. The sex role behaviour of children attending traditional schools was more rigid than that of children attending liberal schools.

Some schools are more effective in persuading females to attempt high achievement in mathematics. Casserly (1980) identified 13 high schools that had an unusually high percentage of females in advanced courses in mathematics and science. The schools had identified these girls as early as the fourth grade and the girls' teachers and peers were supportive of their progress and achievement.

There are those who advocate that girl-only classes result in equity in mathematics. The argument is that because peer group pressure against female competitiveness is too strong, females will not compete against males in mixed sex classrooms. It is argued that female leadership in problem-solving for example, is only able to emerge when competition with males is eliminated. Teachers will not have different sex-related expectations of behaviours if only one sex is present. Single-sex classrooms appear to provide a simple solution to a complex problem.

Segregation as a response to sex-differentiated behaviour in the classroom was first reported by Smith (1983). He outlined the experiences at Stamford High School where an amalgamation of an academic boys' school with a less academic girls' school resulted in the observation that the girls were passive in class and under-achieving across the ability range. First-year girls were equal to boys when tested on entry but by the end of their first year had already fallen behind in test results as well as showing evidence of negative attitudes. By the fourth year, boys outnumbered girls by four or five to one in the two top mathematics sets. The few girls in these sets reported feeling uncomfortable in the masculine environment. They feared ridicule and were observed to be adopting a passive role in class.

A two-year experiment was undertaken in which half the top band students were assigned to single-sex and half to mixed-sex mathematics sets in their first year. When tested in November of the second year, the single-sex girls' performance was about equal to the mixed-sex and single-sex boys' and clearly superior to the mixed-sex girls'. This gap continued to increase. The girls' results are shown in Table 9.

Table 9. Stamford results in mathematics of single-sex and mixed groupings.

Mean scores on tests

	Oct.1978 (initial test)	Nov.1979	Feb.1980
Single-sex girls' set	58.9%	55.1%	54.7%
Girls in equivalent mixed-sex set	58.1%	50.0%	43.9%

(Smith, S., 1983, Single Sex Setting : A Possible Solution.)

The October 1978 scores indicate that at the time of the initial set selection there was little to choose between the girls in either set. By February 1980, the average score of the girls in the mixed-set had fallen well behind that of the boys in the same set. In other words, these girls were conforming to the typical pattern for the school. The girls in the single-sex set, however, achieved a far better average score than the girls in the mixed-sex set and were only slightly below the average score achieved by the boys.

Whereas nine of the sixteen girls in the mixed-sex set failed to achieve 40% in the February test, only four out of thirty-one girls in the single-sex set failed to obtain this score. Because of the success of this experiment, the school changed to single-sex mathematics sets throughout the school, and as a result the number of girls taking and passing O-level mathematics increased, as did the number of girls who opted to take A-level mathematics.

Alan Eales (1986) in Girls into Maths Can Go, reports on a similar experiment conducted with fourth year pupils. An interesting feature of this experiment was that it was done in the context of a whole-school approach to equal opportunities and the particular outcomes of the single-sex division could in part relate to the policy of anti-sexism throughout the school. Using single-sex teaching is one strategy which can promote girls' confidence and performance in mathematics. Perhaps what is more important however, is this strong commitment by staff in schools to the concept and practice of equal opportunities for girls.

14. Classroom Interaction

As has already been noted, the reasons for girls' under-achievement in mathematics and their withdrawal from the subject are many and varied. Part of the problem is concerned with the different feelings that pupils often associate with mathematics. Debilitating, negative attitudes towards the subject or high anxiety may depress performance of either girls or boys. It is important therefore to look at the setting and the environment under which mathematics is taught to try to establish the conditions which might foster these negative feelings. The learning of mathematics takes place in a social environment which itself influences that learning. Pupils not only learn the subject content but they also learn about learning and pick up expectations relating to themselves as individuals and as members of a group. It may be that what pupils learn at school is different from what is intended they learn.

> eg. "David, aged five, came home during his first week in school and announced, 'Boys are best'. When asked to explain, he said: 'Well, they must be because they are first on the register.'"
>
> (Girls into Mathematics, Page 63)

Researchers have suggested that active assertiveness and confidence when adopted by children, are the characteristics necessary for full participation in the learning process (Eynard and Walkerdine, 1981). Unfortunately, assertiveness and confidence can be interpreted in the classroom as challenges to the teacher's authority and can then directly affect the way in which that teacher perceives the task of motivating and controlling the class.

Another area of concern, given the importance of pupil oral participation in the learning process, is the way in which linguistic space is dominated by boys in the classroom. Research (Zimmerman and West, 1975) showed that in male/female conversations nearly all the interruptions (94%) were by men and that females were more silent than males. Spender (1978) found that girls' and boys' sex stereotyping is reinforced through the processes of linguistic interaction. It was found that the teacher ignored the girls for longer periods of time. It was more normal for boys to call out, move from their seats and push each other. It was more normal for girls to be addressed collectively and boys by their individual names. It was more normal for boys to dominate classroom talk.

Horrocks (1984) found that men dominated mixed sex talk not by quantity alone. She analysed the discussions of seven student teacher groups engaged in problem-solving exercises. She found not only that men talk more than women, but also that women pause more often and do not exclude others by occupying time. If a person wished to contribute, he/she would have more room to do so where there was a greater amount of pause. Men seemed to occupy 'centre stage' much more readily than women. The longest time of any of the recorded conversations that a woman spoke uninterrupted by a man, was 49 seconds. The longest time a man spoke uninterrupted was 2 minutes 28 seconds. Lengths of between 20 and 40 seconds were consistently recorded for male speech whereas female sequences lasted very often for only one or two seconds. It seemed that men established their 'right' to talk longer by leading the conversation from the beginning and in the process establishing its framework.

Michelle Stanworth (1983) supports the findings outlined above. She presented data which showed that for every four boys who participated in classroom discussion, there was one girl. For every two boys who asked questions there was one girl. Three boys to one girl received praise and encouragement. Both the girls and the boys stated that teachers are generally more concerned about boys. They said that the teachers considered the boys more conscientious and capable, that the teachers get on better with the boys and that they are twice as likely to consider boys as the model pupils.

This of course, may not be true generally but even if it occurs at all, is a cause for concern. It seems that, as has been noted, girls are the model pupils at 11 but this reverses through secondary education. Can this then be corrected?

Spender (1980) suggests that boys in a mixed-sex class receive roughly two-thirds of the teacher attention. This differential treatment is usually unwittingly given and stems from a lack of awareness of the problem. However, this is not always the case. Vivienne Griffiths (1977) writes: 'I am fully aware that during my own lessons, I frequently treated girls and boys differently. It is remarkably difficult to break through behaviour seen as the norm. For example, when a group of loudly disruptive boys threatened to reduce a whole class to chaos, it often seemed simpler and less wearing to focus attention and content on them and try to prevent further disturbance than to stick to principles about 'not paying more attention to boys'.' (Sex Roles in the Secondary School, P.10).

Since the professional competence of teachers is partly judged in terms of skills in class management and control, the dilemma about whether to opt for equal attention for girls and boys or whether to risk greater noise in the classroom, is a very real one.

Frequently, boys initiate interactions with the teacher by calling out and guessing. Becker (1981) found that sometimes girls do appear to try to redress the imbalance in interactions by initiating contacts with the teacher. After a while however, they may stop trying if the teacher is unreceptive. Is this because they identify as normal, assertive behaviour in boys and passive behaviour in girls and that their patterns of interaction reflect this?

Mathematics is often seen as a class-taught subject in which information is given by the teacher and the success of its transfer is judged by a sequence of questions from the teacher and answers from pupils. This expository style of teaching tends to encourage a competitive atmosphere in the mathematics classroom. There is seldom any deliberate intention to reveal inadequacies of particular pupils to peers but unfortunately this is precisely what can happen. Holt (1969) provides evidence that pupils can find this approach threatening. It appears to be highly conducive to anxiety, especially among those whose confidence in the subject is low. As we have seen, in general, low levels of confidence are associated with more girls than boys, so a shift away from expository teaching might therefore be expected to support girls and indeed some boys, in learning mathematics.

What about the use of individualised schemes of work which is a popular method of teaching today? There is no obvious competitive component here. However, many teachers have found that, although individual workcards 'appear' to be non-competitive, they can induce a highly competitive atmosphere. Pupils may compete to be seen to be further ahead through the scheme. Individualised work in itself does not necessarily provide pupils with a supportive environment for their learning (Girls into Mathematics, 1986).

Eggleston (1976), reporting on a study of O-level science classes, identified three specific teaching styles.

Style 1: 'the problem solvers' - where the initiative was held by the teachers who, by questioning, challenged the pupils to observe, speculate and solve problems.

Style 2: 'the informers' - who presented a non-practical, fact acquiring image.

Style 3: 'the enquirers' - who used pupil-centred enquiry methods.

He discovered that style 1 was popular with boys but not so with girls; style 3 was most effective in maintaining girls' liking for science; more women teachers used style 3; nearly half the men teachers used style 1.

Harding (1983), in reporting this study, suggests that style 3 may enable girls to participate more fully in the class activity, sorting things out for themselves. Public interaction with the teacher has been removed.

Not only are different styles of teaching a factor in the experience pupils have of mathematics but also in the different ways they learn. This too, may help to explain girls' under-achievement in the subject. Two important approaches to learning have already been mentioned - serialist and holist.

Scott-Hodgetts (1986) suggests that the learning performance is regulated by the level of uncertainty at which the learner is prepared to operate. Serialists proceed from certainty to certainty, learning, remembering and recapitulating a body of information in small, well-defined and sequential packets. They tend not to look far ahead and are cautious. Holists prefer to start in an exploratory way, working first towards an understanding of an overall framework, and then filling in the details. They will tend to speculate about relationships and remember and recall bodies of knowledge in terms of 'higher order relations'.

Further, she conjectures that those pupils who are predisposed to learn by a serialist strategy are unlikely to develop into versatile learners of mathematics unless offered a role-model who uses a holistic strategy as well. Her hypothesis is that in the primary mathematics classroom, the teacher (often female), tends to be serialist and encourages that approach. More girls than boys follow the teacher's lead and tend to adopt an exclusively serialist approach which leads to success - at least in the primary school. Boys, on the other hand, tend to be holists but adopt the serialist strategies offered by the teacher as well. By using a serialist strategy exclusively, it is argued that pupils dampen their long-term mathematical development since, although they are better at assessment of familiar content in familiar contexts, they are more thrown by unfamiliar situations than holists, eg. problem-solving.

Hilary Shuard (1986), in Girls into Maths Can Go, holds similar views about the role of the teacher. She comments on a Schools Council Project in Primary School Mathematics undertaken by Ward (1979) in which 2300 children were given tests in mathematics. An interesting thing about these tests results was the teachers' opinion of the importance of the different questions. They were asked how important they thought it was that a ten-year-old child of average ability would be able to answer questions of each type.

The questions where significantly more girls were successful than boys tended to be the purely computational ones involving number and money, and these were ranked by the teachers as more important than those involving for example, understanding of place value, measurement, reasoning and spatial visualisation - where boys were significantly better. Shuard suggests that this demonstrates that the type of questions which primary teachers emphasise in class as being important are therefore the straightforward, computational ones and that girls seem to respond to this lead, but that the strategies of boys in answering the 'problem' type of question may be advantageous in their later study of mathematics. These ideas are certainly consistent with the statistics of the performance of girls and boys in mathematics where girls' results up to eleven are certainly as good, if not better, than those of the boys. It also highlights the areas of spatial visualisation, reasoning and problem-solving, which have been highlighted earlier, where boys seem to gain an advantage.

A school is a social organisation and its structures will reflect the conditions and norms that are current in the society at large. So, it is not surprising to find that the sex and race distribution amongst senior management in schools does not reflect the distribution across all pay grades. For example, in ILEA secondary schools, 51% of full-time teachers are women of whom 60% are on scales 1 and 2 - whereas 47% of the male teachers are on the lower scales. In primary schools, 80% of the teachers are women of whom 77% are on scales 1 and 2, compared with 49% of men. Inevitably, the sex and race distribution amongst teachers can influence pupils' attitudes as part of the hidden curriculum, ie. the learning that takes place in school, but which is not part of the formal curriculum. Certainly there are more male mathematics teachers.

There appears to be conflicting evidence as to the effect of the sex of the teacher on mathematical achievement of girls and boys. While O'Brien (1975) reports no sex-of-teacher effect, Good, Sikes and Brophy (1973) report that male students do best in quantitive scores when taught by male teachers.

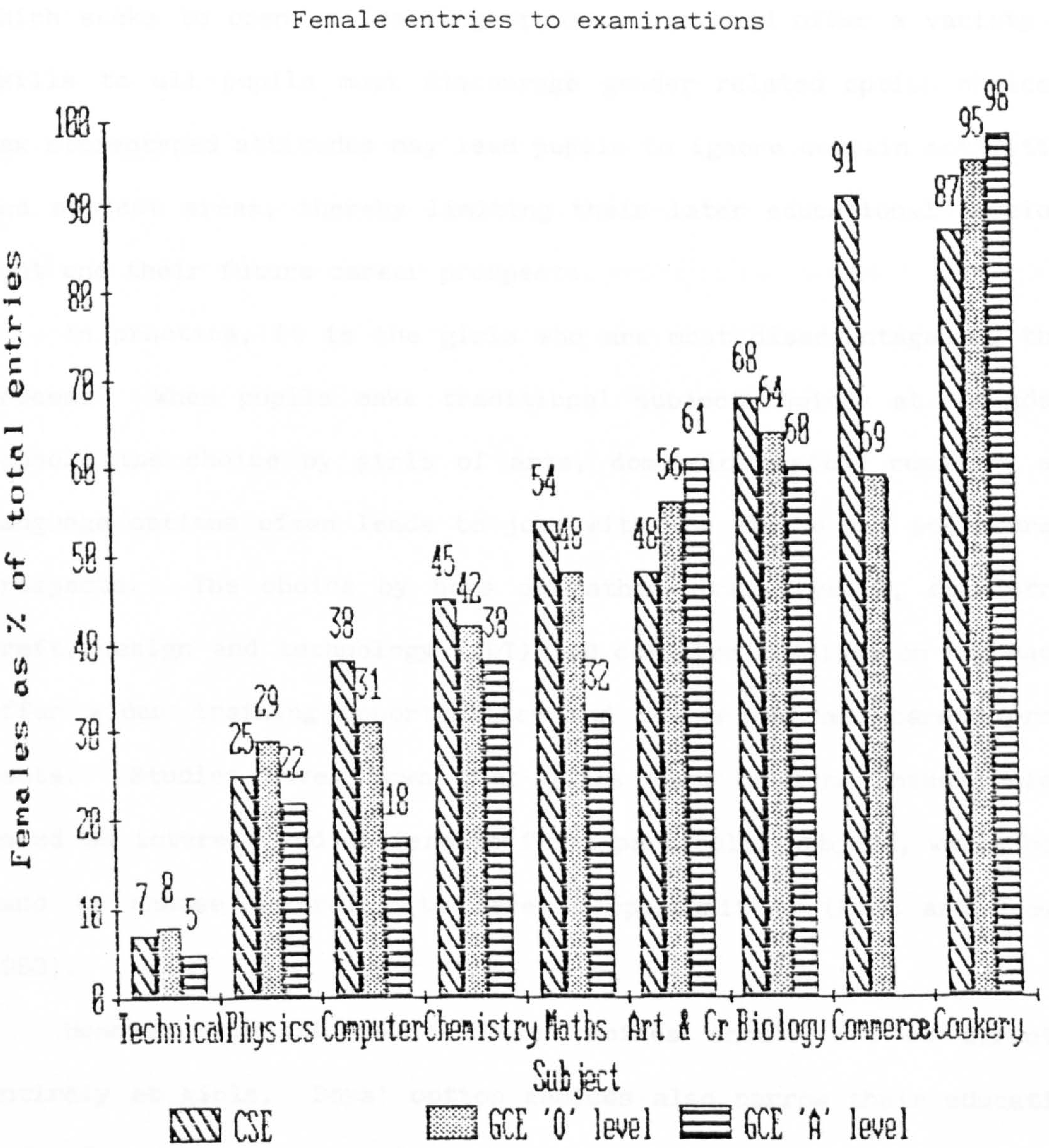
15. Subject Choices

Many girls and boys take different subjects as soon as they have the chance to choose. In England and Wales this choice comes at the end of the third year of secondary education; at the age of 13/14. Making such choices at this early age may be subject to a variety of stereotyping pressures. It may be a question of liking or disliking the subject; it may be peer group pressures or even liking or disliking the teacher. The choice of subjects is clearly shown in the gender differences in entries, and results of 16-plus examinations.

In 1985, four times as many boys as girls took O-level and CSE physics. In computing, boys outnumbered girls by two to one, while forty times as many boys as girls took CSE woodwork, and a hundred times as many took metalwork.

These imbalances were reversed in English literature, modern languages, home economics and needlework. Figure 11 demonstrates how considerable the weighting is towards stereotyped subject choices.

Figure 11. Females as a percentage of entrants in selected subjects for CSE, GCE 'O' and 'A' level, Summer examinations, England and Wales, 1985:



(Source: DES and Welsh Education Committee)

The options chosen by pupils at 13 plus are often regarded as 'free choices' but this is undermined by evidence of recurring sex differentiation patterns in subjects taken. An education system which seeks to open up learning opportunities and offer a variety of skills to all pupils must discourage gender related option choices. Sex stereotyped attitudes may lead pupils to ignore certain activities and subject areas, thereby limiting their later educational development and their future career prospects.

In practice, it is the girls who are most disadvantaged by this process. When pupils make traditional subject choices at secondary school, the choice by girls of arts, domestic crafts, commerce and language options often leads to jobs with low status and poor career prospects. The choice by boys of mathematics, physics, chemistry, craft, design and technology (CDT) and computer studies, on the whole offer wider training opportunities and better pay and career prospects. Studies have shown that girls tend to make these choices based on interest and preference for a particular subject, while boys tend to choose according to career opportunities (Kant and Brown, 1983).

However, concern about these choices should not be directed entirely at girls. Boys' option choices also narrow their education and often fail to provide them with essential understanding of personal and social relationships and the basic skills needed for parenting, child care and basic self maintenance.

Conditioning on the pupils' part does not, on its own, lead to these narrow subject choices. Assumptions and practices adopted by some schools may reinforce this trend. Many schools, for example, limit options and encourage certain subject choices by one sex.

A NFER survey (1984) provided encouraging news that most pupils in the first and second years of secondary school took a wide range of courses that included home economics, needlework, metalwork, woodwork and technical drawing (Option Choice: A Question of Equal Opportunity by J. Pratt, J. Bloomfield and C. Seale).

However, the same survey also found that 40% of schools allocated pupils to streams or bands, taking gender into account to get a balance of numbers. In addition, a study of 127 booklets about schools' option choices revealed only 11% contained explicit commitment to equal opportunities. 26% of schools assumed that one sex only would study certain optional subjects. Legally, however, no pupil can be excluded from an option on the basis of sex. This would contravene the Sex Discrimination Act.

Schools need to be actively encouraging pupils to make wider option choices. It should be clear to pupils that all subjects are open to both sexes. Perhaps there should be a move away from a subject based curriculum. In this way a home economics course which involves scientific principles and practices, or art work which involves mathematical principles and formulae, can be a first step to ensuring that pupils keep their options open. The important aim must be for girls and boys to acquire a common range of skills.

So, in terms of mathematics performance, those pupils who choose subjects which are mathematics related, ie. physics, chemistry, computer studies, and craft, design and technology (CDT), will at least have a greater experience of problem-solving activities. This may lead to a greater confidence in the mathematics classroom with work of a similar nature.

16. Careers Advice

Throughout pupils' school lives, it is important that they not only reach academic standards but also that they develop perceptions of their own skills, knowledge and abilities. If this learning process is predominantly sex differentiated, then pupils' self-images will be shaped more on stereotypes rather than individual characteristics and attributes. For example, if pupils are perpetually told that boys are clever with technical apparatus and girls are very caring, then they begin to adopt these characteristics as their own. As Kant and Brown point out, these sex differentiated self-concepts will inevitably lead girls and boys into different job choices (Jobs for Girls, 1986). They not only lead girls and boys into developing a different range of skills, knowledge, interests and eventually qualifications, but also different attitudes, values and self-images.

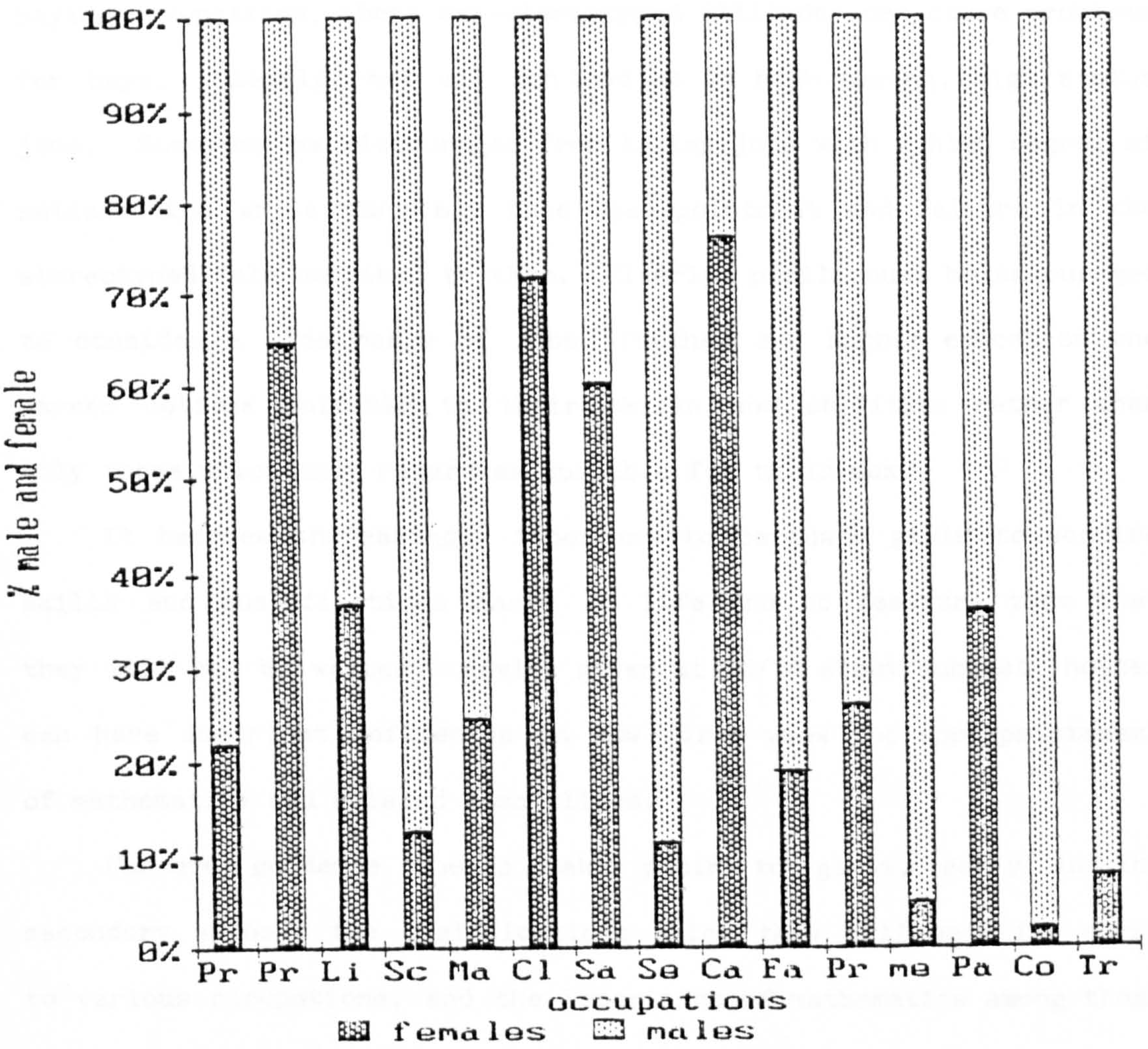
This is highly significant in the teaching of mathematics because if girls do not see the subject as being relevant and important for the future, they may be less motivated. It is clear that the attitudes, values and self-images of 13 and 14 year-old pupils need to be carefully nurtured.

This sex stereotyping may mean girls are likely to picture their future job in relation to their likely role of wife and mother. Boys may be more likely to have the idea of a job involving future responsibility for providing not only for themselves, but a family as well. This may mean that they view well-paid secure employment linked to promotion as important, even if it involves more training.

However, there is no justification for believing that girls need only consider acquiring those skills necessary for marriage and raising children. For instance, the United Kingdom has one of the highest percentages of women working in paid employment in Europe. 63% of all women aged between 16 and 59 are economically active, including just over half of those with dependent children (The Fact About Women Is, EOC, 1987). Women made up 41% of the workforce in 1984 (Women and Men In Britain: A Statistical Profile, EOC). In addition, changes in family size, the high rate of unemployment and of marital breakdown, no longer justify these ideas.

Yet, although most women are either in the paid workforce, or seeking paid employment most of their lives, their career choices are not as wide as men's. Most women work in a narrow range of occupations which pay on average 66% of men's average wage. Figure 12 shows the distribution by sex into different occupations.

Figure 12. Percentage of male and female workers in different occupations:



(Source: Labour Research, 1988)

Pr Professional and related supporting m/m & admin.
Pr Professional & related in educ. welfare & health
Li Literary, music & sports
Sc Prof & related in science, engineering, tech & similar
Ma Managerial
Cl Clerical & related
Sa Security & protective service
Se Selling
Ca Catering, cleaning, hairdressing & other personal service
Fa Farming, fishing & related
Pr Processing, making, repairing & related (excl. metal & elec.)
Me Processing, making, repairing & related (metal & elec.)
Pa Painting, repetitive assembly, product insp. packaging & related
Co Construction, mining & related, not identifiable
Tr Transport operating, material, moving & stores rel.

As well as limiting girls to a narrow range of low status, low paying occupations, these sex-stereotyped attitudes can cause problems for boys. Clearly, not all men end up in high paying, high status jobs. Some may be discouraged from taking jobs with a high degree of satisfaction while many may face disappointment and failure in the stereotyped role ascribed to them. Clearly, pupils must be encouraged to consider a wide range of jobs, further and higher education and career options suitable to their skills and abilities rather than only those which they regard as suitable for their sex.

It becomes increasingly important to persuade girls to acquire skills and qualifications early in life and to reassure them that they will not be wasted. Advice given at 13/14 about subject choices can have important influences on how girls view the appropriateness of mathematics and related disciplines.

Careers guidance should make plain to girls, early in the secondary school, the qualifications which they will need for entry to various occupations, and the importance of mathematics among those qualifications. Mathematics often acts as a 'filter', whose absence as a qualification, can exclude girls from many fields of employment, training and further education.

Careers education needs to go further than informing pupils that jobs and training courses are open to both sexes. Positive depictions of women and men in non-traditional roles are an essential part of encouraging pupils to think more widely about their future lives. Meeting women working in the engineering industry and girls undertaking craft and technician apprenticeships can lessen prejudice. It is important at an early age to make it quite clear that any apparent division is neither permanent nor fixed and that women, past and present, have been mathematicians, scientists and explorers and that men now work as secretaries, cleaners and nurses.

When interviewing pupils, it is essential that advisers consciously try to expand and extend girls' and boys' perceptions of the careers open to them. One way of doing this is to arrange work experience in non-traditional areas. However, care must be taken to ensure that the experience is a positive one and to provide proper support to pupils who undertake this work. Byrne (1979) has recommended to the Commission of European Communities that girls should be encouraged to enter non-traditional areas of further education, vocational training and employment.

It is important however, that careers education should be realistic. As Cockburn points out in the EOC's booklet Training for Her Job and For His (1986), 'occupational choice' is in some ways a misnomer, because class, race and sex strictly circumscribe the occupational possibilities for young people. Often the harder the economic circumstances, the more these factors will tell. Many pupils will face unemployment and many girls will have little option but low-paid, low-status work. What is important is that each pupil is actively encouraged to consider all the options available to them with the skills, knowledge and abilities they have as individuals and not those they see as appropriate for their sex.

There is some evidence that boys see more clearly than do girls that mathematics will be useful in their future lives and work. Fox (1977), in a survey of different studies, found that girls are less orientated towards careers outside the home than are boys and that the usefulness of mathematics in the traditional women's careers of business, nursing, teaching and the social services is less plain than is its usefulness in traditional men's careers.

Several studies have provided evidence that girls tend to believe that mathematics is personally less useful to them than to boys. For example, Russel (1983) showed that although girls, particularly those who were more academic, often rated mathematics as their 'most liked' subject, they tended to rate biology as more useful. Moreover, Russel found that girls tended not to specialise in mathematics because it was not seen as useful or directly relevant to any of the careers presented as suitable for them. They may well have believed biology to be more relevant to their lives as mothers and in predominantly female occupations, eg. nursing/caring occupations. The APU (1981) similarly found that girls perceived mathematics as less useful to them than did boys. They also suggest that pupils are more likely to find 'useful' subjects more interesting to study. If this is true, then it has serious implications with respect to gender differences in mathematical performance. However, there are possibilities of change if girls realise that mathematics can be useful to them. A survey of attitudes of boys and girls across the secondary years of education may help to shed more light on this problem.

Millman and Weiner (1985) found evidence to suggest that employers too, had strong stereotyped views of girl/boy occupations. He also found that, given equal qualifications, boys had a higher chance of achieving their job aspirations than girls and able girls frequently found their career course diverted by their employer's attitude.

Stereotyped views about careers must have implications for mathematical performance. If pupils take up subjects which they see as useful in adulthood and if they see employers appointing only boys to jobs using mathematics and science, then girls are unlikely to look at mathematics and science as appropriate subjects for them to take. In their view, what is the point in studying mathematics to a high level if it will be of no real use? The question is, how soon do these ideas become entrenched and what steps can be taken to allay these feelings?

17. Teaching Materials

Pupils' perceptions of the appropriateness of mathematics may be reinforced by the type of materials used. Many textbooks are highly stereotyped in the type of task assigned to males and females. Authors, publishers, examiners and teachers often attempt to show how the 'everyday' can be viewed mathematically. However, the 'everyday', according to many texts, is a world of football, cricket, men driving cars and traditional boys' hobbies, whether the topic is statistics, arithmetic or algebra. When girls are mentioned it is often in a strictly feminine role.

'A girl makes a cake using ... '

'Stephen helps his father make ... '

'Helen goes shopping with her mother ... '

'A boy rides a bicycle at $3ms^{-1}$ ... '

If the teacher's own examples and exercises follow this pattern, and restrict mathematical applications to science and engineering, then most girls, and the non-technically minded boys, may conclude that mathematics has nothing to offer them.

The Schools Mathematics Project books are more abstract than many, depending less on pictures; but again boys feature in the way in which questions are phrased. This attitude is conveyed in many subtle ways. In one SMP volume (Book F, 1970, 4-5) a flow diagram discriminates between girls and boys, directing the girls to consider a flow chart dealing with a knitting process, while the boys are to examine a flow chart of a cement mixing operation. The two sexes are told what they should be interested in. Even when examples could be non-sex specific, some authors are not aware of the fact.

Many modern textbooks have appreciated the bias and try to avoid sex-stereotyping by the removal of all mention of human beings.

'A dice is thrown ... '

'A particle moves at 10 ms^{-1} ... '

This can also be prejudicial to pupils' attitudes as it implies that mathematics has no place in activities involving social interaction, which may be of importance to both girls and boys.

Northam (1982) draws attention to a number of forms of possible gender bias which were highlighted by her examination of some mathematics textbooks covering the 5-13 age range. She found that:

: there are more references to males than females.

: the number of references to girls decreases as the target age of the book increases.

: illustrations show girls as lacking individuality; boys and men have distinguishing features.

: famous mathematicians referred to are all men.

: the roles that girls play are different from those of boys. Boys are depicted as assertive problem-solvers, whereas girls are generally portrayed co-operating with, or helping, other people.

There may well be a case to compensate the gender bias by positive discrimination in favour of the girls. It is easy to see how this may balance the equation in the short term. However, there are always inherent dangers of tipping the balance too far. Positive discrimination is a dangerous path to follow, however innocent the motives. It may well be seen as threatening to the boys and perhaps condescending to the girls. Surely, it is far better to be seen to be always unbiased and absolutely fair.

Male and female should have equal representation in pictures, written examples and use of personal pronouns. They should participate equally both in physical and intellectual activities and they both should be seen to be competent and decisive. Examples and illustrations should be drawn equally from girls' and boys' lives and activities - this may include non-traditional areas. Language should be non-sexist using for example, 'police officer' instead of 'policeman' and 'refuse collector' instead of 'dustman'.

However, it is important not to lose sight of reality when removing stereotyping from teaching materials. A book in which all the nurses are male and all train drivers are female would be so much at variance with the children's experiences as to be totally unreal, so again divorcing the association of mathematics and practical reality.

The Cockcroft report (1982) supports the view that in many ways mathematics literature is male dominated. 'The applications of mathematics, which are found in many textbooks and examination questions, reflect activities associated with men more often than with women.'

Graham (1985) raises three issues with regard to the context in which mathematics is set. First, the contexts used often imply unnecessarily that girls and boys have different interests. Secondly, the so-called boys' interests are catered for more than those of girls. Thirdly, the context often conveys the impression that boys' interests are superior or more important.

Other studies confirm that the context of a question does seem to be important in relation to gender differences in mathematical performance. Not only do girls seem to be slower than boys in tackling questions framed in a male context, but they may also not perform so well even if time is not a factor. For example, in a study of 1750 second-year pupils in a comprehensive school, Eddowes and Sturgeon (1980) found that girls performed better on a question referring to the area of dress material than to an equivalent one referring to the area of metal needed for a template. They also performed better on a question asking them to calculate quantities required for a recipe than on a similar question relating to a blast furnace. Boys were found to do equally well on both.

A closer survey of examination questions and the context in which they are set is part of this study and may help to illuminate this apparent problem for girls.

Confidence and assurance in handling mathematical apparatus was also found, by the Assessment of Performance Unit (APU, 1987), to be the province of boys rather more than girls. It seems that the use of equipment which is intended to make mathematics more enjoyable and more relevant for everyone could again favour the boys. Clearly, the teacher has an important responsibility here to see that girls are encouraged to use relevant equipment and that boys are not in a position to take it off them.

Straker (1986) found that boys are almost twice as likely as girls to have their own calculator, to have a micro-computer at home and to possess a digital watch. Furthermore, girls from 'girls-only families' are the least likely to have access to this technology out of school. The results of this study must give rise to cause for concern with regard to girls' mathematical performance. Familiarity with this technology at home must increase the confidence with which it is used at school and the range of activities to which it can be applied.

Time and again, we have seen examples of areas in the teaching of mathematics where girls' confidence may have been damaged or at least, not reinforced. This in turn links directly to the affective variables discussed earlier and the apparent increase in anxiety and even panic that can be experienced by pupils in mathematics exercises or tests with the consequences of poor performances.

Summary

The debate concerning the place of mathematics in the education of girls and boys is rooted in history. For a long time it was commonplace to discuss the education of girls and boys separately, even to the extent as to which mathematics courses they should follow. It may be that many of today's problems and perceptions are rooted in these early practices. Attitudes and feelings are not easily or quickly changed. One important step forward is that people actually become aware that a problem exists and acknowledge it as such.

Much of the debate hinges on the nature versus nurture issue. Are girls for example, cognitively less capable of mathematical thinking, or does their relative failure in certain areas have something to do with the way girls are taught to perceive themselves? Or is this perception external rather than internal? Could it be that girls are expected to perform less well in certain mathematical concepts by those who teach them or by society at large? There may well be a dual social-psychological problem here which relates to both internal and external perceptions of performance.

Another difficulty is the diverse nature of mathematical ability. Factor-analytic studies have justified the existence of group factors. These may include numeric, spatial, verbal and non-verbal reasoning, and convergent and divergent thinking. The elusiveness of a single mathematical ability, suggests that care is needed when looking at the conceptual skills needed in varying group factors. Where do boys perform better than the girls? Where do girls perform better than the boys, and what concepts give rise to equal performances?

It is clearly critical to highlight the areas of the subject in which the girls' performance varies, and does not vary, compared to the boys. Is it true for example, as Fennema (1990) suggests, that girls are superior to boys in computation? If mathematics is just about computation then the girls might be expected to excel. On the other hand, if problem-solving, spatial awareness and proportionality are important features of mathematics, how do girls perform then?

It is important to examine closely the type of questions which are presented to girls and boys to establish the nature of those questions and the relative performance on them. An analysis of mathematical ability can only be judged in the context and character of the problems to be solved. So, what are these problems and how can they be classified? What is the mathematics that is being tested? What are the skills needed to give the correct solution?

Many mathematics specialists have been conscious of the need to develop all six elements of the recommendations of paragraph 243 of the Cockcroft Report (1982), as a basis of good practice. These encompass exposition by the teacher; discussion between teacher and pupils and between pupils themselves; appropriate practical work; consolidation and practice of fundamental skills and routines; problem-solving, including the application of mathematics to everyday situations; and investigational work. There is a need to examine closely the results of examination papers since the publication of this popular report in 1982, to look for a relative narrowing of the gender gap.

The statistical evidence (page 12, 13, 14, 15) showed that in 1981 the ratio of boys to girls gaining good grades (A, B or C) was 1.50:1. This increased to 1.51:1 in the years 1982-1984, but fell to

1.20:1 in 1985. In 1987, 32.7% of boys obtained higher grades in mathematics compared to 27.7% of girls (a ratio of 1.18:1). With the advent of the new General Certificate in Secondary Education (GCSE) examination, this might have been expected to narrow in the light of current educational philosophy. However, in 1988, 40.2% of boys gained higher grades compared with 33.1% of girls (a ratio of 1.21:1). Clearly, there is a continuing problem here, but exactly where are the differences greatest?

At A-level, there is a large disparity in the number of female and male candidates that enter for the mathematics examination (page 25). The ratio of entry of boys and girls is approximately 2:1. Yet, a close study of the results shows that the distribution of grades is very similar. This underlines the fact that the girls entering mathematics at A-level are equally competent to the boys. The problem is how to attract more girls to study mathematics to a higher level. Or to put the question another way, why do not more girls choose to study the subject at a higher level? What are the pressures exerted on them?

The answer to these questions raises the key issues of affective variables, peer group pressures, stereotyping, the influence and expectations of teachers and parents, the type of school, classroom interaction, teaching materials and careers advice. We are all social creatures and are affected by pressures of various kinds which may directly or indirectly influence mathematics performance. These need to be considered further. However, it is still important to try to highlight the particular conceptual areas of greatest divergence given that these sociological problems exist and may affect performance. Where are the girls under-achieving compared to the boys?

Over the years, educators have become more aware of gender differences in mathematics performance. This has stemmed from the Equal Opportunities Act of 1975 and the Education Reform Act of 1988 where local education authorities have been required to draw up equal opportunity policies. Many schools are now mixed which in theory offers equal opportunities across the curriculum. However, legislation may not always lead to equality. Many prejudices may be hidden and even unnoticed by those who hold them.

Much of the recent debate following the Cockcroft Report (1982) has been concerned with raising the awareness of educators to gender differences in mathematics performance. In 1986, the Royal Society, in conjunction with the Institute of Mathematics and its Applications, published Girls and Mathematics. This contained valuable statistical information and commented on research. In 1986, a teaching pack called Girls into Mathematics was published as a co-operative venture between the Open University (OU) and the Inner London Education Authority (ILEA). It incorporated many activities which involved observation in the classroom and was followed by reflection and discussion. A companion volume, Girls into Maths Can Go (1986) is a collection of relevant articles by mathematics educators. Separate Tables (1987) and Separate Beginnings (1988) gave accounts of single-sex groupings. Although the results were inconclusive, the teaching staff involved in the initiative recognised that their own levels of awareness had been significantly raised. However, this literature does not always address the basic question: What are the concepts that give the widest discrepancy in gender scores? However, it does highlight certain broad areas of concern.

These include spatial ability, proportionality and problem-solving. The meta-analysis of gender differences conducted by Linn and Petersen (1985) indicated that the magnitude of the difference in spatial ability depended considerably on the type of spatial ability tested. This may be spatial perception, mental rotation, or spatial visualisation. It is clearly important then, to establish the type of question and wording which gives the greatest discrepancy in scores.

Also, as has been discussed (Orton, 1987 and Hart, 1981), the understanding of proportionality develops at a late age, if at all. The early years of secondary school are the formative years of proportionality learning. The question arises whether fewer girls are developing this skill and if so, why?

It was found also that girls in secondary schools perform less well than boys on mathematical problem-solving tasks (page 55). Since problem-solving lies at the heart of mathematics, it is appropriate to examine the different problem-solving processes adopted by the girls and boys. Are there different methods which are favoured by the girls/boys? It may be that a particular approach gives consistently better results and should be encouraged in all pupils. For example, if an holistic approach gives better results than a serialist approach, how can this be encouraged? At what stage in a problem-solving exercise do the mistakes occur? Are girls more prone to give one or two step solutions, as some literature might suggest?

It is important also, to compare the level of difficulty of a question across the sexes. This is not clearly discussed in the literature. For example, are there questions which are found to be very easy by the boys and very hard for the girls or vice-versa? Is a hard question found to be relatively hard for both sexes? There is a need to look at the consistency of performance between the sexes across varying difficulty levels.

Another key issue is the extent to which the difference in gender performance extends across the years of secondary education. There are many published figures of gender differences in mathematics performance at 16 and 18 but what about the formative years from 11-16? When do the differences become apparent? The literature is sparse in coming to terms with this question and this thesis will help to shed light on this question by the analysis of a longitudinal survey across the secondary years 1-5.

Another important issue is the extent to which the performance of girls and boys varies across the ability range. The literature provides statistics which examine the good grades (A, B and C) against the total percentage examination entry. They give a general picture of differences in performance between girls and boys at the higher level. Yet, what are the gender distributions at <u>all</u> levels? Are the poorer girls better than the poorer boys, for example? This thesis will attempt to look at gender differences in performance across the ten percentile levels.

Not only that, but what are the concepts which give the greatest divergence in scores at the higher level compared to the lower levels? If, for example, proportionality is too difficult for the weaker pupils, then what concepts separate the girls and boys in the lower percentiles, if any? This requires further investigation.

Many girls do have unfavourable attitudes towards mathematics and these are many and varied. Cognitive, psychological and sociological factors are all involved. There is a need to examine girls' own fears and feelings as well as their mathematical performance. The detrimental social influences on girls may take a long time to combat. What can be done in the short term is to support intervention programmes which attempt to work with the contradictory position in which some girls are placed. The use of intervention programmes has shown that the performance of girls can be increased by careful schooling. This indicates that the girls are not innately less capable of mathematical thinking. Indeed, as has been noted, many girls can equal or excel their male peers in any or all of the conceptual areas highlighted. The success of intervention programmes of work amongst the girls clearly weakens the nature (cognitive deficiency) argument. If girls were innately less capable of mathematical understanding, the differences in performance might be expected to be much greater than they are.

If, as we have seen, the mathematical performance of girls can be increased by the use of intervention programmes, then the precise areas of concern in terms of poor performance need to be clearly identified so that remedial action can be put into effect. This has not yet been done systematically across the ability range and is clearly a priority area. Where are girls under-achieving compared to the boys? What are the concepts which give rise to the greatest concern in performance? Once these important details have been established, appropriate action can be taken to redress the balance.

This study will attempt to identify these specific areas of mathematical performance across the examined ability range at 16+.

Other studies, notably Wood (1973) have examined mathematical performances of girls and boys with particular reference to the top 20% of candidates (GCE, O-level). This study will examine differences in performance across 80% of the ability range. The remaining 20% were below the examination level. The differences will also be classified according to National Curriculum categories. It will highlight which topics are found relatively difficult across the whole sample and which are easy in terms of raw scores. It will examine the pattern of differences between girls and boys across the ability range and across three different examination papers. It will also give some evidence in terms of performance of the manner in which different questions have been tackled.

19. 16+ Examination Papers

Much is made today of 'equal opportunities' between the sexes in education. In the light of recent equality drives it might be expected that differences in external mathematics examinations be negligible. As the literature review shows, concern has been expressed in specific areas of mathematical study. These have been notably in the areas of visuo spatial studies, proportionality and problem-solving.

The object of this study is to pinpoint precise differences in mathematical achievement between girls and boys at a time when educators are more 'equality conscious'. It might be expected for example, that since Wood's study in 1973/74 equal opportunities within single sex and comprehensive schools would eradicate any observed differences in performance. Option choices at 14 are generally open to all, literature is much less stereotyped, career guidance is well established, and LEAs have implemented equal opportunity programmes. It might be expected then, that any differences be minimal and certainly not as great as in 1973/74.

In order to study specific differences in attainment between girls and boys, it was decided to find an examination, the questions and results of which could be examined in detail. It was decided to find a paper which covered the greatest possible ability range. Unlike other studies, such as that of Wood who studied the London Board's O-level mathematics examination, a Joint GCE O-level and CSE examination was chosen. At best, O-level papers only tested the top 20% of the ability range which was a rather restricted range. Those who were entered for O-level mathematics clearly had a good basic competence in the subject. In choosing the Joint examination, the ability range of pupils tested was greatly increased since the examination covered the top 80% of pupils. This gives a better picture of relative achievement of girls and boys in the subject. It also offers the opportunity to compare the best achieving girls and boys and the less able girls and boys. Is for example, the pattern of differences the same across the ability range between girls and boys? It also allows a more comprehensive study of those questions which are found to be more difficult in a cross-section of 16-year-old pupils.

20. Data

The Northern Examining Association (NEA) Joint GCE O-level and CSE examination for 1986 was used as the basis for study. The examination consisted of three components.

a) Paper 1 was based on the core syllabus and composed entirely of multiple choice questions. Candidates were required to answer all sixty questions and there was a time allowance of one and a half hours.

b) Paper 2 was also based on the core syllabus and composed a variety of types of questions which were designed to test abilities and techniques not easily assessed by the use of objective questions. Candidates were required to answer all questions in a time allowance of two hours.

c) Paper 3 was based on a topic syllabus.

   (A) Algebra, Trigonometry and Calculus

   (B) Choice, Chance and Statistics.

   Candidates were allowed to answer questions from either topic. The questions in each topic were divided into two sections. The first consisted of shorter, straightforward questions and the second, ones which were more difficult. Section 1 questions counted as one unit and section 2 questions as two units. Candidates were asked to answer any combination of questions which gave a total of not more than twelve units.

Each of the three papers carried an equal weighting. Reference leaflets were provided for the use of candidates in all three papers and calculators were permitted in Papers 2 and 3.

For the purpose of this study, a sample of one thousand scripts in each of the three papers was selected. This was made up of five hundred girls and five hundred boys. The scrips were selected at random, from a random selection of centres throughout the north of England. The majority of the centres were mixed comprehensive schools.

The assessment objectives of the examination were designed to test, in conjunction with the subject matter contained in the syllabus, the following abilities:

a) Knowledge and Information: recall of definitions, notations and concepts.

b) Techniques and Skills: computation and manipulation of symbols and the use of mathematical instruments.

c) Comprehension: the capacity to understand problems, to translate symbolic forms and to follow and extend reasoning.

d) Application: of appropriate concepts in both familiar and unfamiliar mathematical situations.

The total entry for this examination was 39,968. Papers 1, 2 and 3 may be found in Appendix B (in the portfolio).

21. Question Papers

Tables 10-12 show the overall results for the NEA Joint O-level/ CSE examination in mathematics for the year 1984-1986. They give the distribution of marks, grades and standard deviations for each paper across the three years together with the respective cumulative percentage of candidates within each grade. It is unfortunate however, that the examining authority do not give a breakdown on their results by gender.

The tables show grades A-U and grades 1-U. In fact, because the examination was a Joint examination, each candidate was graded twice. One grade A-U was that given for O-level and the other 1-U was that given for CSE. The tables show the consistency of grades awarded by the Board over the three years.

Table 10: Joint Mathematics (NEA/Joint O-level/CSE) 1984

	Max. Mark	Mean	Standard Deviation
Paper I	60	34.6 (57.6%)	10.8 (17.9%)
Paper II	60	34.5 (57.5%)	10.7 (17.9%)
Paper III (Written)	60	34.5 (57.6%)	10.7 (17.9%)
Total	180	103.2 (57.4%)	30.7 (17.0%)

Grade Distribution (Ordinary level grades)

Grade	A	B	C	D	E	U
Max. Mark	180	151	128	105	95	85
Cum. % of cands.	7.1	21.1	47.5	59.2	70.8	100.0
Top mark	180					
Bottom mark	1					

Grade Distribution (CSE grades)

Grade	1	2	3	4	5	U
Max. Mark	180	105	91	77	63	55
Cum. % of cands.	47.5	63.9	78.5	89.6	94.1	100.0

Total entry 31 073 (includes 323 candidates offering internal assessment alternative to Paper III)

Table 11: 1985 Joint Mathematics

	Max. Mark	Mean	Standard Deviation
Paper I	60	34.9 (58.1%)	10.4 (17.4%)
Paper II	60	34.8 (58.0%)	10.3 (17.2%)
Paper III (Written)	60	34.9 (58.1%)	10.4 (17.4%)
Total	180	104.1 (57.8%)	29.8 (16.5%)

Grade Distribution (Ordinary level grades)

Grade	A	B	C	D	E	U
Max. Mark	180	154	129	105	95	84
Cum. % of cands.	6.1	20.6	47.1	59.7	73.0	100.0

Top mark 180
Bottom mark 0

Grade Distribution (CSE grades)

Grade	1	2	3	4	5	U
Max. Mark	180	105	91	77	64	56
Cum. % of cands.	47.1	64.0	80.0	91.2	95.9	100.0

Total entry 35 734 (includes 375 candidates offering internal assessment alternative to Paper III)

Table 12: 1986 Joint Mathematics

	Max. Mark	Mean	Standard Deviation
Paper I	60	32.7 (54.6%)	11.3 (18.9%)
Paper II	60	31.4 (52.4%)	12.1 (20.1%)
Paper III (Written)	60	32.7 (54.6%)	11.3 (18.9%)
Total	180	97.8 (54.3%)	32.5 (18.0%)

Grade Distribution (Ordinary level grades)

Grade	A	B	C	D	E	U
Max. Mark	180	151	126	102	89	77
Cum. % of cands.	6.8	20.3	43.0	56.9	70.9	100.0

Top mark 180
Bottom mark 0

Grade Distribution (CSE grades)

Grade	1	2	3	4	5	U
Max. Mark	180	102	88	74	60	52
Cum. % of cands.	41.9	57.4	73.7	87.2	93.1	100.0

Total entry 39 968 (includes 572 candidates offering internal assessment alternative to Paper III)

a) <u>Paper 1</u>

Overall in Paper 1, there was a statistically significant difference in favour of the boys ($F=30.31, d.f.=1, p < 0.0001$). The mean mark for the girls was 30.772 out of 60 compared to 34.696 for the boys. These means compare well with the overall mean for the whole NEA examination in Paper 1 which was 32.7 (see Table 12). Similarly, the standard deviation for the girls was 11.1733 and for the boys 11.3655. This compares with the NEA standard deviation overall in Paper 1 of 11.3. The range of marks in the sample was 5-60, out of 60 in total.

The boys scored more highly on the preponderance of items. In fact, on the multiple choice questions, there were only six questions out of the sixty where the girls did better than the boys. There was a general trend across the whole sample so that an easy question was done well by both sexes and a hard question less well by both. Nevertheless, there are specific items where the boys peak much higher than the girls.

The percentages of girls and boys giving the correct responses to each question in Paper 1 are given in Figure 13. The successful percentages of girls and boys on each question mirror one another closely. The peaks and troughs generally coincide although there is a range difference of 20.8.

Appendix A, Table (i), page 328, gives the percentage differences between girls and boys on correct scores for each question.

Figure 14 again shows the percentage differences between boys and girls on correct scores for each question. These percentage differences between the sexes have been transformed to linearise the percentages and coded according to the character of the question. A

Figure 13 Percentage of Girls and Boys giving the correct responses to each question in Paper 1

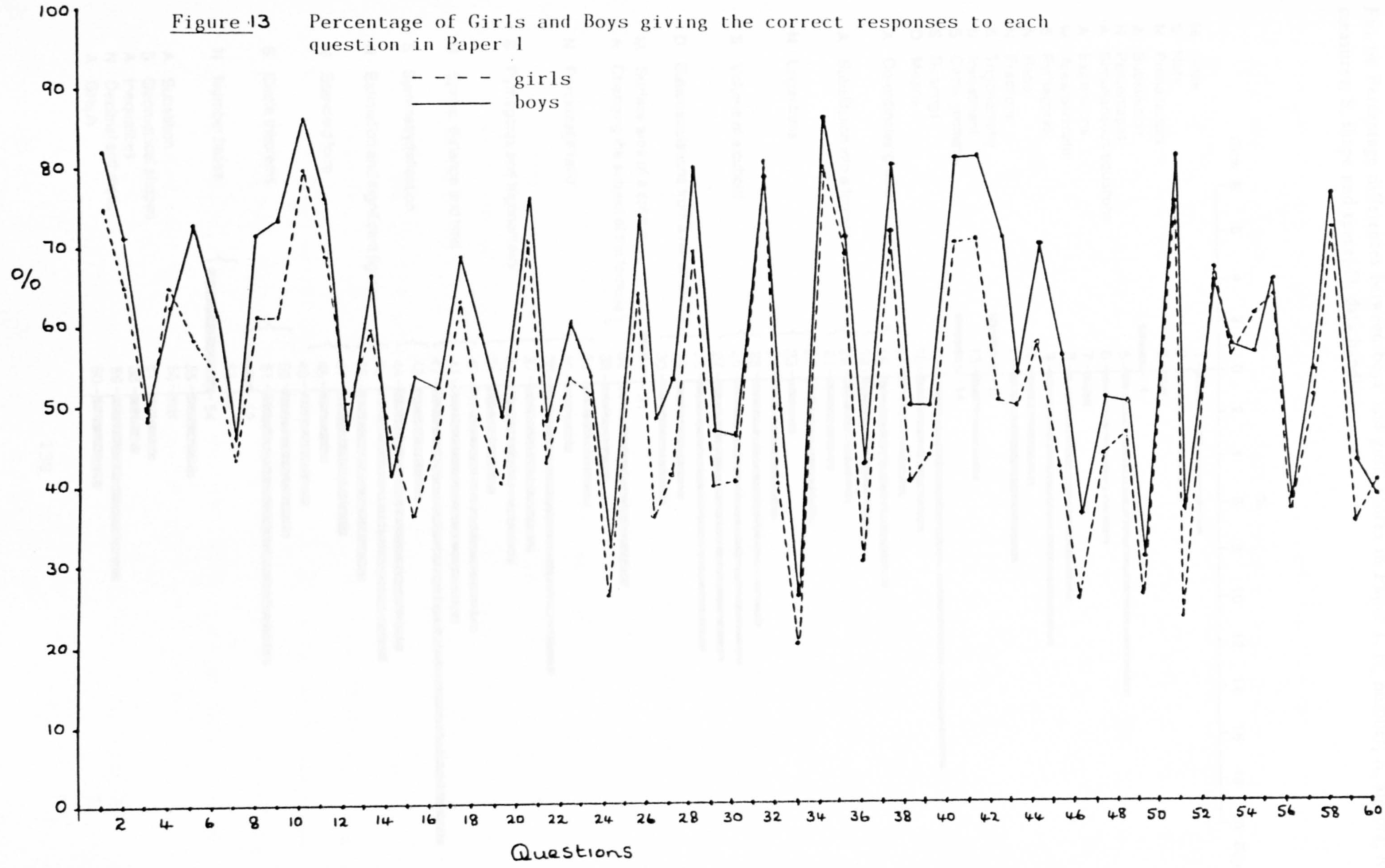
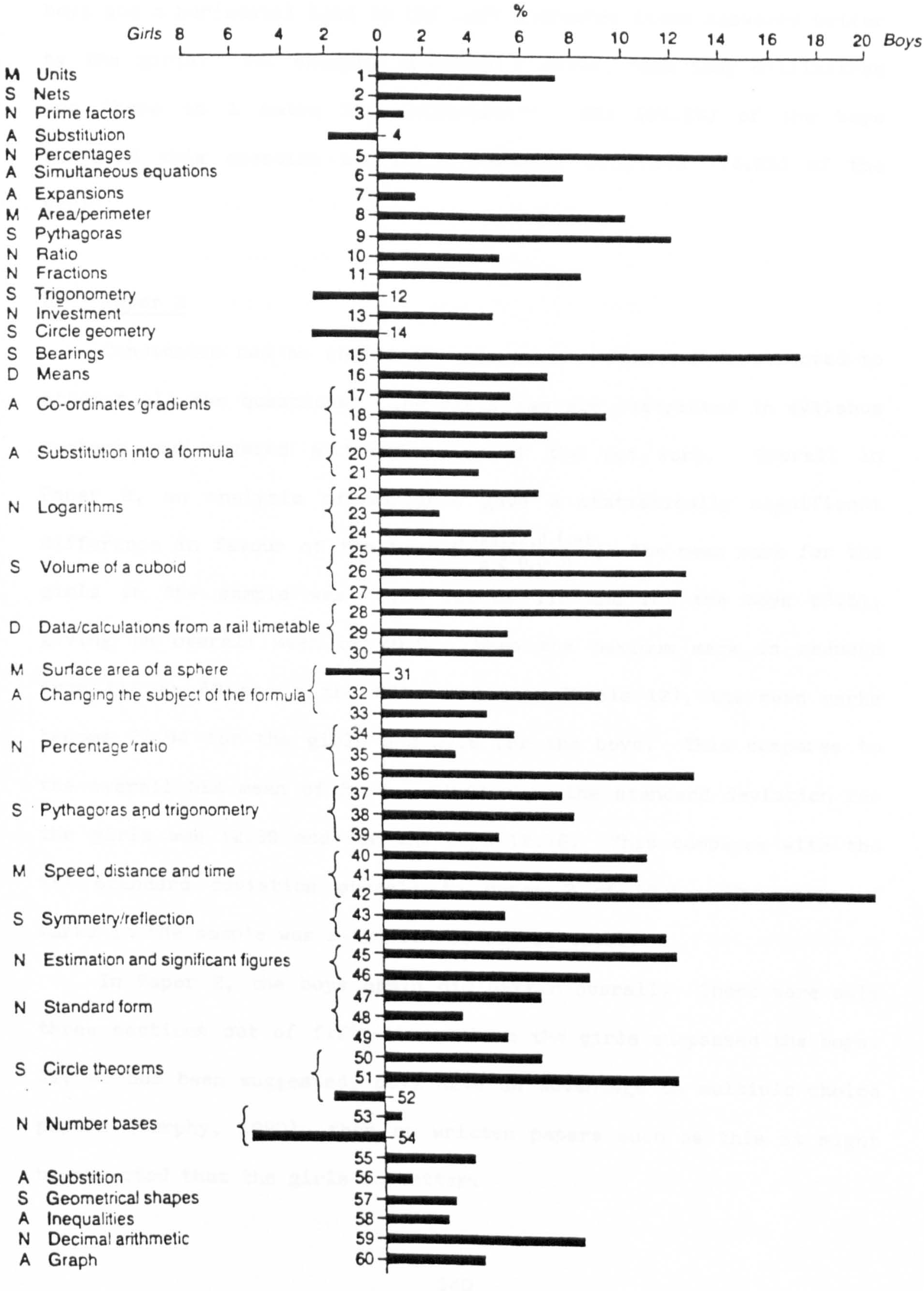


FIG. 14 Percentage differences between boys' and girls' scores in Paper 1. N, number; A, algebra; M, measures; S, shape and space; D, data handling.



horizontal line to the right indicates an item answered better by the boys and a horizontal line to the left indicates items answered better by the girls. For example, question 1 asked, 'how many millimetres are there in 1 metre 10 centimetres?' 411 (82.2%) of the boys answered this question correctly compared with 374 (74.8%) of the girls.

b) Paper 2

Candidates had no choice in this paper. They were instructed to attempt all the questions. The paper was not restricted in syllabus content and covered a broad range of the set work. Overall in Paper 2, an analysis of variance gave a statistically significant difference in favour of the boys ($F=22.12$, d.f.$=1$, $p < 0.0001$). The mean mark for the girls in the sample was 46.29 out of 116 and for the boys 53.51, giving an overall mean of 49.9. When the maximum mark is reduced from 116 to 60 as in the NEA tabulation (Table 12), the mean marks become 23.94 for the girls and 27.6 for the boys. This compares to the overall NEA mean of 31.4. Similarly, the standard deviation for the girls was 12.39 and for the boys 12.76. This compares with the NEA standard deviation overall in Paper 2 of 12.1. The range of marks in the sample was 2-113 out of 116 in total.

In Paper 2, the boys again did better overall. There were only three sections out of fifty-three where the girls surpassed the boys. If, as has been suggested, boys have an advantage on multiple choice papers (Murphy, 1980), then on written papers such as this it might be expected that the girls do better.

The percentages of girls and boys giving correct responses to each question in Paper 2 are given in Figure 15. Again, the two graphs mirror one another closely. An easy question has been done well by both sexes and a hard question less well by both.

Appendix A, table (ii), page 329, gives the percentage differences between girls and boys on correct scores for each question.

Figure 16 again shows the percentage differences between girls and boys on correct scores for each section transformed to linearised percentages and coded according to the character of the question. Again, a horizontal line to the right indicates an item answered better by the boys and a line to the left by the girls.

c) <u>Paper 3</u>

Overall in Paper 3, there was a statistically significant difference in the marks in favour of the boys $\binom{F=8.57, d.f.=1}{p=0.0035}$. This variability analysis in Paper 3 shows a closer margin in mark differences than in the other two papers. The mean mark for the girls in the sample was 40.22 out of 96 and for the boys 44.24, giving an overall mean of 42.33. When the maximum mark is reduced from 96 to 60 as in the NEA tabulation (Table 12), the mean marks become 25.13 for the girls and 27.65 for the boys. This compares to the overall NEA mean of 32.7. Similarly, the standard deviation for the girls was 13.31 and for the boys 13.77. This compares with the NEA standard deviation overall in Paper 3 of 11.3. The range of the marks in the sample was 0-96, out of 96 in total.

Again, the boys did better overall in Paper 3. There were only four questions out of twenty-four where the girls surpassed the boys

Figure 15 Percentage of girls and boys giving correct responses to each section in Paper 2

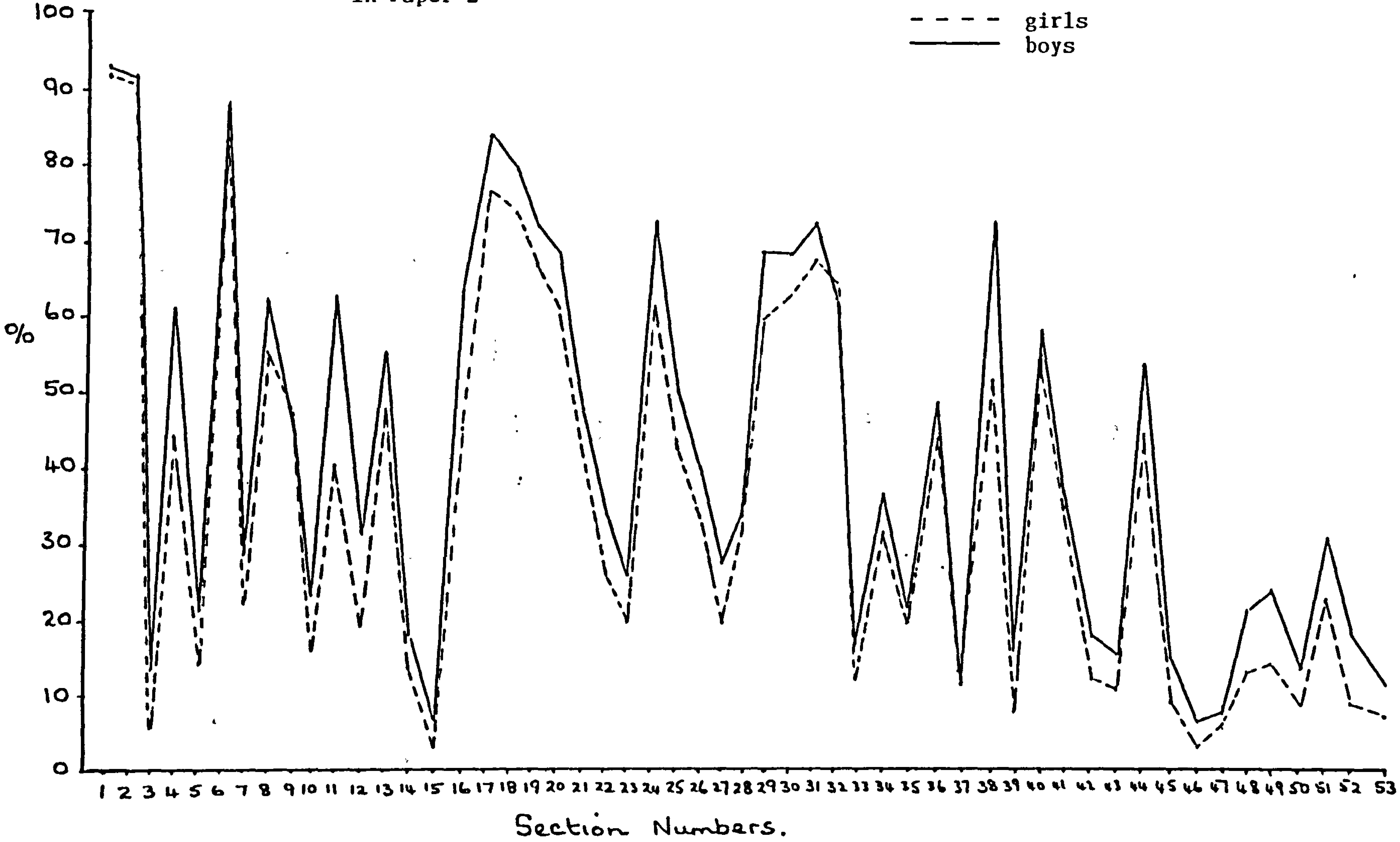
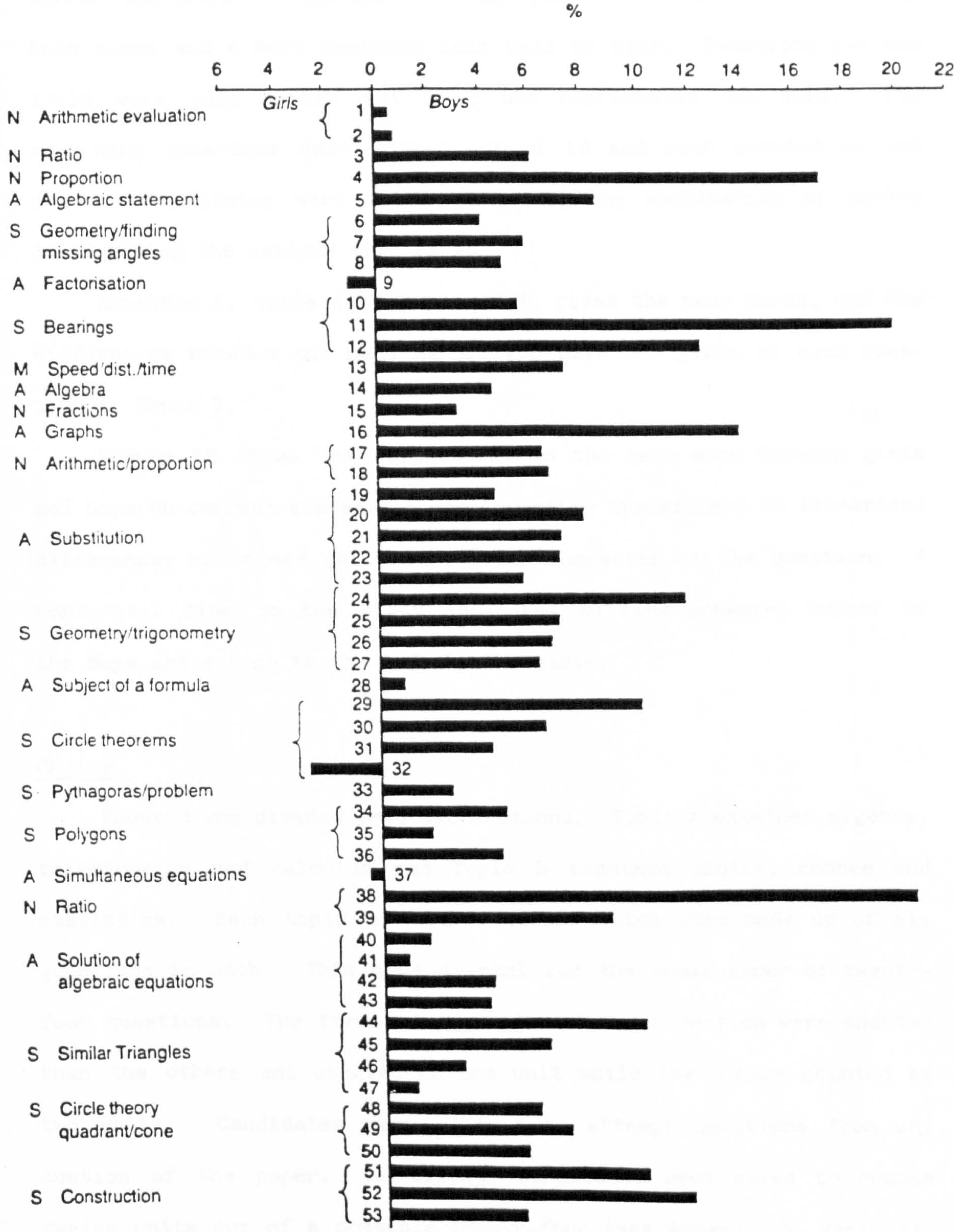


FIG.16 Percentage differences between boys and girls giving correct solutions to each section of Paper 2. N, number; A, algebra; M, measures; S, shape and space; D, data handling.



in test scores. Figure 17 shows the differences in the average mark between boys and girls for each question. Again, the two graphs mirror one another closely. An easy question has been done well by both sexes and a hard question less well by both. Questions 1-6 and 13-18 were each marked out of 8 and represented one unit. The remaining questions were marked out of 16 and each counted as two units. Candidates were asked to answer any combination of twelve units giving the maximum mark of 96.

Appendix A, table (iii), page 330, gives the mean marks, and the differences between the mean marks, for boys and girls on each question in Paper 3.

Figure 18 shows the differences in the mean mark between girls and boys on correct scores for each question transformed to linearised differences and coded according to the character of the question. A horizontal line to the right indicates an item answered better by the boys and a line to the left by the girls.

Choice

Paper 3 was divided into two sections. Topic A examined algebra, trigonometry and calculus and Topic B examined choice, chance and statistics. Each topic had two sections which were made up of six questions in each. This gave a total for the whole paper of twenty-four questions. The first six questions in each section were shorter than the others and counted as one unit while the others counted as two units. Candidates were allowed to attempt questions from any section of the paper. In effect, candidates were asked to choose twelve units out of a possible twenty-four (see Appendix B, Paper 3).

Figure 17 Mean Scores of Girls and Boys in each question in Paper 3.

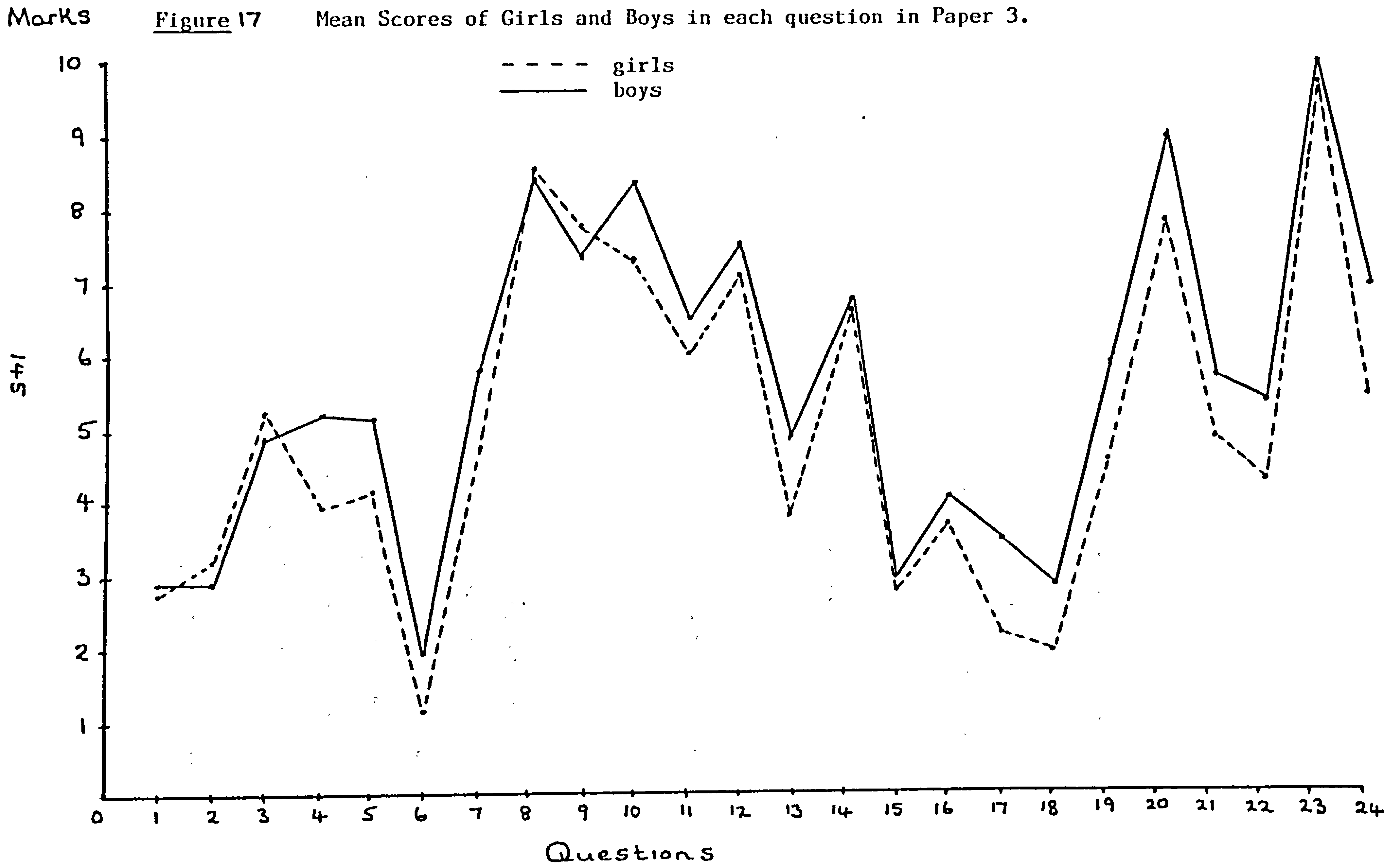
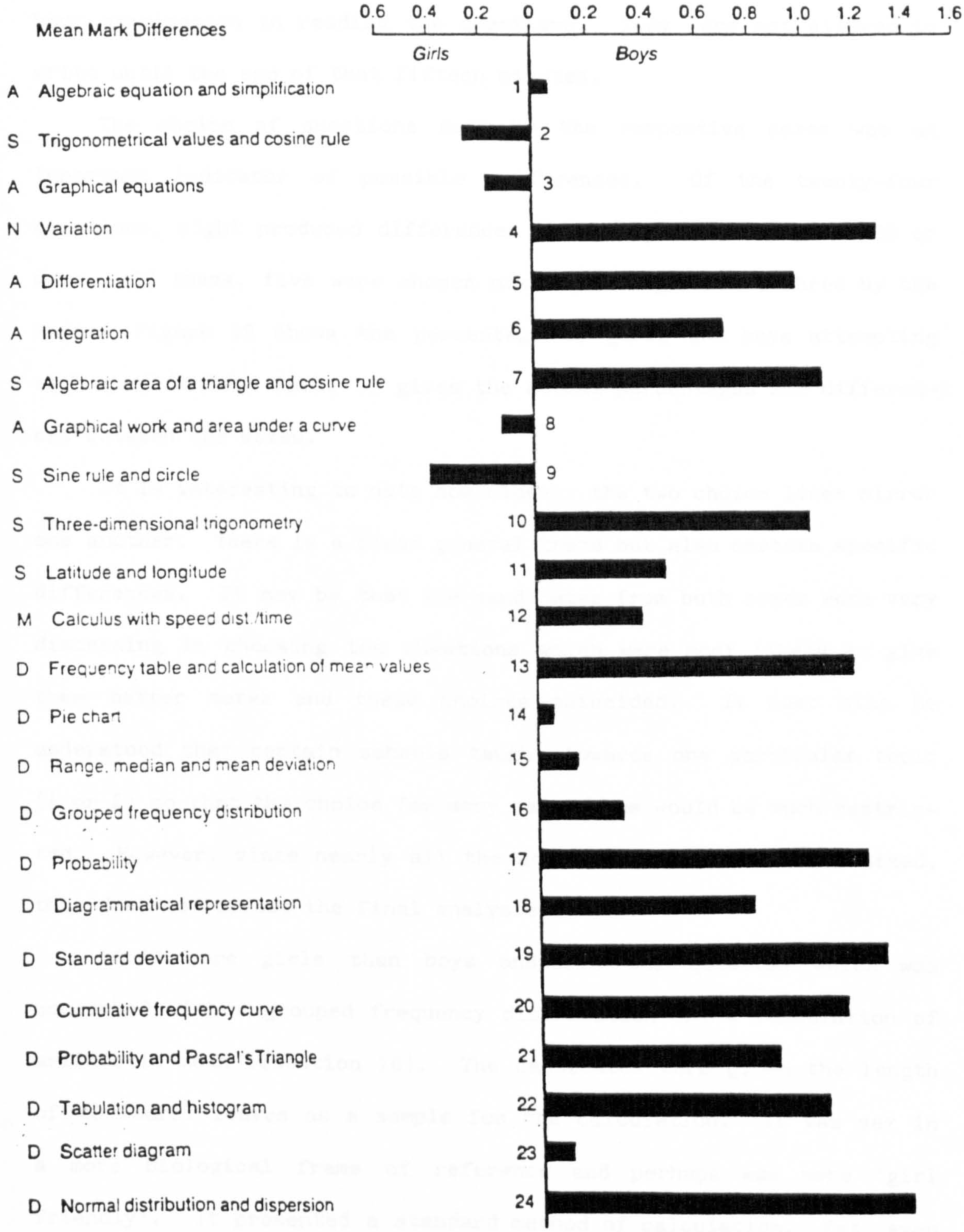


FIG.18 Differences in the average mark between boys and girls in Paper 3. N, number; A, algebra; M, measures; S, shape and space; D, data handling.



In view of the length of the paper and the wide choice available, candidates were told to spend the first fifteen minutes of the two hour examination in reading the questions. They were not allowed to write until the end of that fifteen minutes.

The choice of questions made by the respective sexes was an important indicator of possible preferences. Of the twenty-four questions, eight produced differences between girls and boys of 5% or more. Of these, five were chosen most by the girls and three by the boys. Figure 19 shows the percentage of girls and boys attempting each question and Table 13 gives the actual percentages and differences between the sexes.

It is interesting to note how closely the two choice lines mirror one another. There is a clear general trend but also certain specific differences. It may be that the candidates from both sexes were very discerning in choosing the questions which were most likely to give them better marks and these choices coincided. It must also be understood that certain schools taught towards one particular topic (A or B) so that the choice for many candidates would be much restricted. However, since nearly all the schools in the study were mixed, this does not affect the final analysis.

16.4% more girls than boys answered the question which was concerned with a grouped frequency distribution and a calculation of arithmetic mean (Question 16). The candidates were given the length of 40 laurel leaves as a sample for the calculation. It was set in a more biological frame of reference and perhaps was more 'girl friendly'. It presented a standard method of calculation. Yet, even

Figure 19 Percentage of girls and boys attempting each question in Paper 3

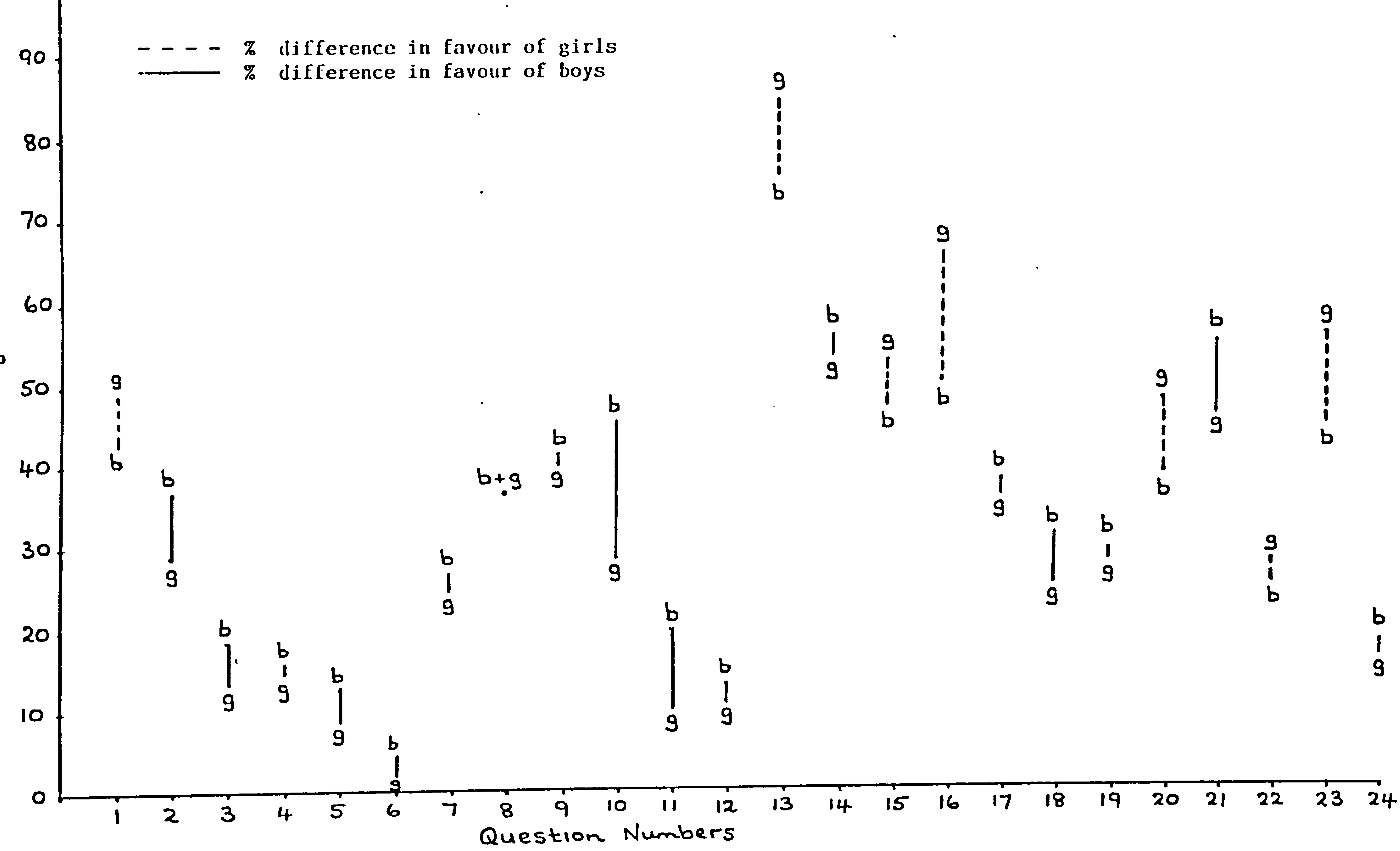


Table 13: Percentage differences between Girls and Boys attempting questions on Paper 3 (M-F).

Ques.	F		M	% M-F	
1	49	-	43	- 6.0	Algebraic equation and simplification
2	29	-	36.2	7.2	Trigonometrical values and cosine rule
3	12.4	-	17.2	4.8	Graphical equations
4	16.2	-	16.8	0.6	Variation
5	9.8	-	11.0	1.2	Differentiation
6	3.2	-	3.8	0.6	Integration
7	24.6	-	27.4	2.8	Algebraic area of a triangle and cosine rule
8	37.2	-	37.2	0	Graph work and area under a curve
9	40.2	-	40.4	0.2	Sine rule and circle work
10	29	-	44.2	15.2	Three dimensional trigonometry
11	10.2	-	19.8	9.6	Latitude and longitude
12	10.6	-	11.4	0.8	Calculus with speed/distance/time
13	82.2	-	76.2	- 6.0	Frequency table and calculation of mean values
14	53.6	-	54.2	0.6	Pie chart
15	51.8	-	47.2	- 4.6	Range, median and mean deviation
16	65.6	-	49.2	-16.4	Grouped frequency distribution
17	36.2	-	37.4	1.2	Probability
18	25.8	-	30.2	4.4	Diagramatic representation
19	29.2	-	29.6	0.4	Standard deviation
20	47.6	-	38.8	- 8.8	Cumulative frequency curve
21	21.2	-	24.4	3.2	Probability and Pascal's triangle
22	27.8	-	24.4	- 3.4	Tabulation and histogram
23	56.2	-	43.2	-13.0	Scatter diagram
24	15.8	-	16.8	1.0	Normal distribution and dispersion

though more girls chose the question, the mean mark was still in favour of the boys.

	Female	Male	Total
Number attempted	328	246	574
% of total	57.1	42.9	
% of each	65.6	49.2	
Mean mark (out of 8)	3.69	3.99	

Again, 13% more girls than boys answered the question which was concerned with a scatter diagram (Question 23). It concerned ten women who had joined 'weight watchers' and gave details of their weights and average daily food consumption. Relationships were asked about the weight of the women and their food consumption in calories per day.

	Female	Male	Total
Number attempted	281	216	497
% of total	56.5	43.5	
% of each	56.2	43.2	
Mean mark (out of 16)	9.76	9.89	

The choice of a question may well relate to the context in which it is set. As Frazer (1982) pointed out, the solving of a problem depends on the experience of the problem-solver and the extent to which the individual relates and interprets the problem. No problem exists in isolation - a problem is perceived by the individual.

The question which most boys chose compared to the girls (15.2% difference) was concerned with visuo-spatial skills (Question 10). It was a question concerned with three dimensional trigonometry.

	Female	Male	Total
Number attempted	145	221	366
% of total	39.6	60.4	
% of each	29.0	44.2	
Mean mark (out of 16)	7.32	8.35	

It is interesting to observe that of those girls who did choose this question (29%), many of the marks were comparable to those of the best boys. If girls have a difficulty with spatial questions as the literature shows, this does not apply to <u>all</u> girls. Had this been a compulsory question, a greater mean mark difference in favour of the boys might have been expected, in the light of research work.

22. Analysis

In order to characterise more clearly the individual questions on the three papers, it was helpful to classify the mathematics under five headings. These are Number, Algebra, Measures, Shape and Space, and Data Handling. This is consistent with other reports and documents, notably the National Foundation for Educational Research (1987) and Mathematics in the National Curriculum (DES, 1989). Indeed, the mathematics taught throughout England and Wales (apart from independent schools) is now based under these five headings. They form a convenient method of classification and cover the whole spectrum of mathematical education.

It would be wrong however, to assume that each question is purely based on one respective category. It may be that a given question relates to two or more. In such cases, the question has been classified according to that part of the question which forms the basis of the work involved. For example, the question from Paper 3, number 23, requires not just knowledge of scatter diagrams (data handling), but skill in plotting points and selecting the correct scales for the axes (algebra). It also involves a skill in the computation of the mean, in estimation and in logical deduction (number). It is therefore relevant to consider each question and the extent to which the girls' and boys' scores differ, and to analyse the respective components.

The literature highlights three areas of interest. These are spatial awareness, proportionality and problem-solving. These skills are not restricted to one area alone but may cross each of the headings. For example, problem-solving is a skill which may be found when concerned with Number, just as it may be found in Shape and Space. Similarly, the concept of proportionality may equally apply to work on ratio (Number) and to similar triangles (Shape and Space) and to probability (Data Handling). The five headings form a clear base upon which to classify, from which the elements of spatial awareness, proportionality and problem-solving can then be considered.

In Paper 1, twenty of the sixty questions can be classified as Number, thirteen as Algebra, six as Measurement, seventeen as Shape and Space and four as Data Handling. These results are shown in Table 14. It illustrates the numbers of questions in each of the five categories across each of the three papers. It also shows the relative weighting of Shape and Space questions in the examination as a

Table 14. Numbers of questions in each of the five mathematical categories.

	Number	Algebra	Measurement	Shape and Space	Data Handling	Total
Paper 1	20	13	6	17	4	60
Paper 2	9	15	1	28	0	53
Paper 3	1	5	1	5	12	24
Total	30	33	8	50	16	137

whole. If girls find this an area of difficulty, as the literature suggests, then it is not surprising that their overall marks are statistically lower than those of the boys. Figures 20-22 show the distribution of scores for girls and boys in each of the five categories. It can be seen that the two graphs for girls and boys are convergent in the Algebra questions but much more divergent in the Shape and Space section and in certain of the Number, Measurement, and Data Handling questions. It may be that these are questions which require further skills of problem-solving and proportionality concepts. This requires further investigation.

Figure 20 Percentage of Girls and Boys giving the correct responses to each of the Five Categories in Paper 1

Figure 21 Percentage of Girls and Boys giving the correct responses to each of the Five Categories in Paper 2

Figure 22 Mean Scores for Girls and Boys in each of the Five Categories in Paper 3.

a) <u>Number</u>

The study of number forms the central element of mathematics. It grows from the counting of separate or discrete objects in a set. In measurement, number is applied, not to separate objects, but to continuous quantities such as length, time and weight. In algebra, number properties are extended and generalised, involving patterns and relationships.

Number questions require candidates to understand the four basic rules (+, -, x, ÷). They require that candidates can acquire competence in number skills - to be able to use them sensibly, obtaining correct answers consistently using appropriate methods. They also require that candidates know how to check that the answers make sense. It is important for pupils at all levels to develop a sound and confident mental facility with number. Often a complete calculation can be done mentally, but when this is not possible, it is important for pupils to be able to do a mental check on the reasonableness of answers obtained. This again requires a certain confidence. If girls are more prone to feelings of anxiety and panic, as the literature seems to suggest, then this may affect their performance.

Questions categorised under the number heading, were ones which require appropriate methods of calculation, estimation and approximation, the recognition and use of patterns, relationships and sequences and the ability to make generalisations. Typical of number questions are ones requiring the understanding of terms such as prime, square root, cube, multiples and factors. Fractions, decimals, common factors, standard form, percentages, directed numbers and ratio, also form a basis in this category.

In the light of the literature, it may be expected that girls find the greatest difficulty compared to the boys in number questions which in some way involve proportionality skills. This may be further compounded when the questions are enveloped in a problem-solving situation. For example, a candidate may understand the concepts of speed, distance and time; but when these are asked in a novel situation which has not previously been seen, they are unable to answer the question. The items on speed, distance and time, and indeed questions on percentages, are in essence about proportionality.

Again, in the light of the literature, it might be expected that some candidates struggle because they are unfamiliar with the context of the question. This is often revealed when answers are given which are totally inappropriate for that question. It may be that some girls are less aware of the 'real life' setting of some of the questions.

Table 15 shows the differences on correct scores between the boys and girls in the Number category across each of the three papers. The greatest difference is that given first.

The question which gave the greatest difference between the sexes in Paper 1 was one involving the use of percentages (Question 5). 'The price of a car changed from £800 to £840. What is the increase in price expressed as a percentage of the original price?'

The number of correct solutions were:-

	Female	Male	Total
Number	290	361	651
% of total	44.5	55.5	
% of each	58.1	72.5	

This question involves the use of a two stage solution. First, the subtraction and then the proportionality.

Table 15. Differences on Correct Scores between Boys and Girls in the Number category - the greatest difference first.

Question Number	F%	M%	% diff M-F
Paper 1			
5	58.1	72.5	14.4
36	29.6	42.6	13.0
45	41.8	54.0	12.2
46	26.3	34.9	8.6
11	67.4	75.9	8.5
59	33.8	42.1	8.3
22	53.1	59.7	6.6
47	42.7	49.3	6.6
24	25.6	31.9	6.3
13	59.4	65.3	5.9
34	78.9	84.6	5.7
10	79.1	84.3	5.2
49	24.7	29.9	5.2
48	45.8	49.1	3.3
35	67.1	70.2	3.1
23	50.8	53.2	2.4
55	62	63.9	1.9
3	48.2	49.4	1.2
53	56.3	57	0.7
54	59.6	54.1	-5.5
Paper 2			
Section			
38	52.8	73.6	20.8
4	43.4	60.4	17.0
39	8	17	9.0
18	72.4	79.2	6.8
17	77	83.2	6.2
3	6	12	6.0
15	3.4	6.4	3.0
2	91.6	92.2	0.6
1	92.2	92.6	0.4
Paper 3 Question NO.	Mean F	Mean M	Mean Difference
4*	3.9	5.2	1.3

* 1 unit question

This is also illustrated in a question from Paper 2, section 38. 'A model of a racing yacht is made to scale of 1:40. The length of the yacht is 8m. Calculate, in its simplest form, the length of the model.' This question gave the results:-

	Female	Male	Total
Number	264	368	632
% of total	41.8	58.2	
% of each	52.8	73.6	

The percentage and ratio questions revealed an inability of many candidates, of whom the majority were girls, to deal with different orders of units. The weakness was compounded by the blindness to the 'real world' significance of problems, sometimes resulting in ridiculous answers.

One question which caused particular difficulty with regard to unit notation was Section 4, Paper 2.

'The mass of a new 2p coin is 7g. Calculate in Kg the mass of £35 worth of new 2p coins.'

The number of correct solutions were:-

	Female	Male	Total
Number	217	302	519
% of total	41.8	58.2	
% of each sex	43.4	60.4	

It is interesting to note that 131 girls (26.2%) and 82 boys (16.4%) managed to gain one mark out of three in this question. They established that in £1 there are fifty 2p coins, but were then confused by the change in units.

A question on 'average' caused some difficulty, especially when the question was worded in a problem format. This again, created more difficulties for the girls.

Paper 1. Number 36: There are 1500 employees in a factory, of whom 600 are female. The average weekly wage of the male employees is £60 and of the female employees is £45. What is the average weekly wage of all the employees?

The number of correct solutions were:-

	Female	Male	Total
Number	148	213	361
% of total	41	59.0	
% of each sex	29.6	42.6	

It is interesting to note that 458 candidates gave an answer of £52.50 which is found by averaging the £60 and £45. Of these, 239 were girls and 219 were boys. Again, it shows a lack of understanding of the real life situation.

Another number question which highlighted difficulties was this:

Paper 1. Number 45: 'The value of $\frac{1}{2.068}$ correct to 4 significant figures is 0.4836. What is the value of $\frac{10}{2.068}$ to 3 significant figures?'

The number of correct solutions were:-

	Female	Male	Total
Number	207	268	475
% of total	43.6	56.4	
% of each sex	41.8	54.0	

The questions highlighted above gave the greatest difference in favour of the boys. The question which gave the best relative response from the girls in this concept area was one concerned with number bases. Paper 1, Number 54.

'How would the number 40 (base TEN) be written in base THREE?'

The number of correct solutions were:-

	Female	Male	Total
Number	294	269	563
% of total	52.2	47.8	
% of each sex	59.6	54.1	

It is unfortunate that although this topic provided the greatest difference in favour of the girls in the whole of Paper 1, it has been omitted from the present GCSE, Northern Examining Association (NEA) Syllabuses (A, B and C).

Table 16 shows the percentage of boys and girls giving correct scores on each of the questions in the Number category when placed in rank order - the best first. This gives an indication of the intra-gender differences - with boys and girls ranked separately. The two lists mirror one another closely with both boys and girls finding difficulty with Paper 1, question 49, which was concerned with standard form.

A closer examination of the order seems to indicate that the boys and girls are performing better in those questions which are straightforward replicas of basic mathematical questions. They are more comfortable (scoring higher) on questions where clear basic methods can be reproduced and applied. Greater difficulty is experienced by both sexes when questions require a greater depth of application or when questions are written in a novel context. For example, Paper 2, Section 3, 'Given that 3p = 7q, calculate the value of the ratio p:q.' Although most candidates were familiar with the

Table 16. Percentage of Boys and Girls giving correct scores on each of the questions in the Number category: Placed in rank order - the best first.

Question Number	Boys %	Rank Order	Girls %	Question Number
Paper 1				
34	84.6	1	79.1	10
10	84.3	2	78.9	34
11	75.9	3	67.4	11
5	72.5	4	67.1	35
35	70.2	5	62.0	55
13	65.3	6	59.6	54
55	63.9	7	59.4	13
22	59.7	8	58.1	5
53	57.0	9	56.3	53
54	54.1	10	53.1	22
45	54.0	11	50.8	23
23	53.2	12	48.2	3
3	49.4	13	45.8	48
47	49.3	14	42.7	47
48	49.1	15	41.8	45
36	42.6	16	33.8	59
59	42.1	17	29.6	36
46	34.9	18	26.3	46
24	31.9	19	25.6	24
49	29.9	20	24.7	49
Paper 2 Section				Section
1	92.6	1	92.2	1
2	92.2	2	91.6	2
17	83.2	3	77	17
18	79.2	4	72.4	18
38	73.6	5	52.8	38
4	60.4	6	43.4	4
39	17.0	7	8.0	39
3	12.0	8	6.0	3
15	6.4	9	3.4	15
Paper 3 Question Number	Boys Mean	Order	Girls Mean	Question Number
4	5.2	1	3.9	4

ratio concept as other questions showed, they had particular difficulty with this question in the context in which it was set. Only 12% of boys (60) and 6% of girls (30) answered this question correctly.

In summary, the boys were more successful than the girls in the number section in all but one question. The girls found greatest difficulty with questions of a problem-solving nature, and seemed less ready to try different options if one failed. This may relate to the confidence/anxiety dimension referred to in the literature or it may refer to Pask's (1976) discussion of the serialist and holistic approaches to problem-solving. In other words, more boys are more prepared to view the problem as a whole in attempting the questions whereas some girls are approaching the question on a step by step basis. Also, girls and boys are finding questions harder depending on the context and familiarity in which they are set.

b) Algebra

Algebra develops out of a search for pattern, relationships and generalisations. Work on number patterns and relationships between them lay the foundation for the subsequent development of algebra. Krutetskii (1976) sees mathematical ability in part, as being able to generalise from mathematical results. However, in moving away from the more concrete foundations of basic number work as discussed in the last section, it might be expected that marks in this section be generally lower.

Algebra includes those questions which test the recognition and use of functions, formulae, equations and inequalities. It also extends to the use of graphical representations of algebraic functions. That is, the use of graphical methods to solve simultaneous linear equations, plotting regions, simple function mappings, tangents to graphs to determine the gradient and finding the area under a graph. In view of the sometimes spatial nature of some of the graphical questions, it might be expected to cause girls the greatest difficulty. Yet, because of the standard procedures of some of the algebra questions, it might be expected that the careful approach exhibited by most girls would be advantageous.

Table 17 shows the differences on correct scores between the boys and girls in the Algebra category across each of the three papers. The greatest difference is that given first.

In general, the girls did well on questions which required the use of standard procedures. For example, in Paper 1, number 4, candidates were asked, 'What is the value of the expression (x - 1) (x + 3) when x = -2?' In the survey, 314 girls (62.8% of all

Table 17. Differences on Correct Scores between Boys and Girls in the Algebra Category - the greatest difference first.

Question Number	F%	M%	% diff M-F
Paper 1			
18	47.0	56.5	9.5
32	38.9	48.0	9.1
6	53.1	60.8	7.7
19	40.4	47.4	7.0
20	70.1	75.8	5.7
17	61.6	67.0	5.4
33	20.0	24.4	4.4
21	41.7	46.0	4.3
60	39.2	37.1	4.2
58	71.1	73.7	2.6
7	43.1	44.7	1.6
56	36.3	37.6	1.3
4	63.2	61.4	-1.8
Paper 2 Section			
16	47.2	61.2	14.0
5	13.0	21.6	8.6
20	60	68	8.0
21	39.8	47	7.2
22	26.6	33.8	7.2
23	20.4	26	5.6
14	13.6	18	4.4
19	66	70.4	4.4
42	15.8	20	4.2
43	12	16	4.0
40	56.8	58.6	1.8
28	37.4	38.2	0.8
41	37.6	38.4	0.8
37	13	12.8	-0.2
9	46.8	46	-0.8
Paper 3 Question No.	Mean	Mean	Mean Diff
5	4.18	5.16	0.98
6	1.19	1.89	0.7
1	2.89	2.94	0.05
8*	8.53	8.42	-0.11
3	5.15	4.99	-0.16

* 2 unit question

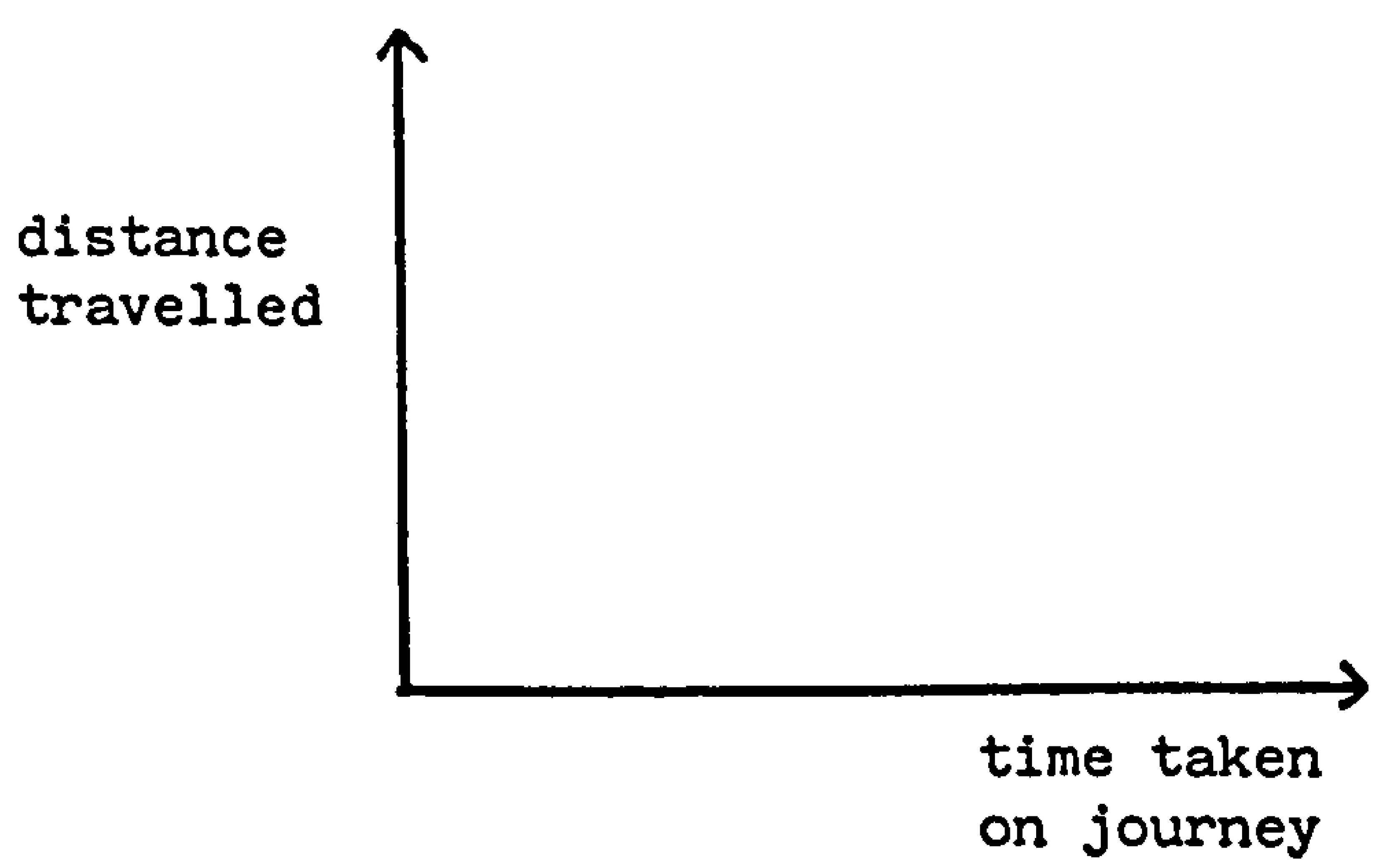
girls) answered this correctly compared with 306 boys (61.2% of all boys). Similar results were found in a question on factorisation, ie. Paper 2, section 9. Here candidates were asked to factorise the expression $9y^2 - 25$. 234 girls (46.8% of all girls) answered this correctly compared with 230 boys (46% of all boys).

When a question was extended to include some 'problem' element, the girls did less well. For example, in Paper 1, number 32, candidates were given the formula for the surface area of a sphere $s = 4\pi r^2$. They were then told, 'Air is added to a spherical balloon, initially of surface area X, so that its radius is doubled. What is the new surface area of the balloon?' The number of correct solutions were:

	Female	Male	Total
Number	194	238	432
% of total	44.9	55.1	
% of each	38.9	48.0	

Similar problems were experienced with a question of a graphical nature (Paper 2, section 16). This also involved the concepts of distance and time.

'A girl leaves home to travel to school. She walks at a constant rate to her friend's home, where she waits until her friend is ready to leave. The two girls are then taken to school by car which travels at a constant speed.



Draw three straight lines on the given diagram to illustrate the journey of the first girl.'

The number of correct solutions were:

	Female	Male	Total
Number	236	306	542
% of total	43.5	56.5	
% of each	47.2	61.2	

One question which favoured the girls was the direct solution of a pair of simultaneous equations, Paper 2, section 37. These equations were $3p - 2q = 200$

$2p + q = 180$

The number of correct solutions were:

	Female	Male	Total
Number	65	64	129
% of total	50.4	49.6	
% of each	13	12.8	

On none of the algebraic questions on which the girls did better than the boys was the difference statistically significant.

Table 18 shows the percentage of boys and girls giving correct scores on each of the questions in the Algebra category when placed in rank order - the best first. This again gives an indication of the inter-gender differences, with boys and girls ranked separately. Again, the two lists are compatible, but it is evident that there are some low percentage success rates, particularly in some of the written sections of Paper 2. This seems to confirm the expectations from the literature review that the development of generalisations is not a process which is easily extended from practical numerical ability.

Boys and girls both found that questions requiring basic use of algebraic skills were straightforward. Paper 1, number 20 required

Table 18. Percentage of Boys and Girls giving correct scores on each of the questions in the Algebra category; placed in rank order - the best first.

Question Number	Boys %	Rank Order	Girls %	Question Number
Paper 1				
20	75.8	1	71.1	58
58	73.7	2	70.1	20
17	67.0	3	63.2	4
4	61.4	4	61.6	17
6	60.8	5	53.1	6
18	56.5	6	47.0	18
32	48.0	7	43.1	7
19	47.4	8	41.7	21
21	46.0	9	40.4	19
7	44.7	10	39.2	60
56	37.6	11	38.9	32
60	37.1	12	36.3	56
33	24.4	13	20.0	33
Paper 2				
19	70.4	1	66.0	19
20	68.0	2	60.0	20
16	61.2	3	56.8	40
40	58.6	4	47.2	16
21	47.0	5	46.8	9
9	46.0	6	39.8	21
41	38.4	7	37.6	41
28	38.2	8	37.4	28
22	33.8	9	26.6	22
23	26.0	10	20.4	23
5	21.6	11	15.8	42
42	20.0	12	13.6	14
14	18.0	13	13.0	5
43	16.0	14	13.0	37
37	12.8	15	12.0	43
Paper 3	Mean		Mean	
8 *	8.42	1	8.53	8 *
5	5.16	2	5.15	3
3	4.99	3	4.18	5
1	2.94	4	2.89	1
6	1.89	5	1.19	6

* 2 unit question

the skill of substitution and 75.8% (378) boys and 70.1% (349) girls answered this question successfully. Similarly, Paper 1, number 58 concerned a straightforward algebraic inequality question. Again, 73.7% (365) boys and 71.1% (347) girls answered this correctly. This same principle was again demonstrated in Paper 2, sections 19 and 20 where both boys and girls performed well in questions of algebraic substitution.

The girls did less well when the question was expressed in a problem format. For example, Paper 2, section 5.

'A woman hires a car for her holidays. The charge is £x for each day the car is hired and y pence for each kilometre driven. Find an expression in x and y for the total cost, in £, if she hires the car for 10 days and drives a total of 1200m.'

21.6% (108) boys answered this correctly compared to 13.0% (65) girls.

In summary, the boys were more successful than the girls in the algebraic section in all but five parts. The marks were generally lower than those in the Number category. Straightforward algebraic skills were evident from both sexes and the girls showed success in areas of standard computation. However, when a question was set in an unfamiliar setting or was of a problem-solving nature, the girls did less well. It is also worthy of note, that difficulties may be more compounded when questions impinge in other problem areas. For example, in Paper 1, number 32 discussed earlier, the question related to a spherical balloon which is a visuo spatial concept. Also Paper 2, section 16 was set with reference to speed-distance and time which is a proportionality concept.

c) Measures

Measurement is essentially concerned with comparison and it is important for pupils to appreciate that all measurement is approximate. Many pupils have used 'relatively' precise measuring instruments to obtain improved accuracy but an element of inaccuracy remains even then. This is reflected in practical contexts in industry where certain 'tolerance' is accepted.

Basic measures lead to work in compound measures such as speed and density. The main development is through the range of the applications of all the measures. Most of the questions under this heading are those concerned with the understanding of relationships between units, degrees of accuracy and the use of knowledge and skills in length, area, and volume, to carry out required calculations in plane and solid shapes.

It may be expected that if girls have had less experience within these areas, then, as the literature suggests, they will get poorer marks. Also within this section, are questions which relate to speed, distance and time. This was one concept area which Wood (1974) found to produce the greatest difference between boys and girls in the O-level 1973 papers.

Table 19 shows the differences on correct scores between the boys and girls in the Measures category across each of the three papers. The greatest difference is that given first.

In general, girls found greatest difficulty compared with the boys on questions relating to speed, distance and time, and to questions involving mixed units. The difficulties are again compounded when the question is in a problem format. For example,

Table 19. Differences on Correct Scores between Boys and Girls in the Measurement category - the greatest first.

Question Number	F%	M%	% diff M-F
Paper 1			
42	49.0	69.8	20.8
40	69.0	80.0	11.0
41	69.8	80.4	10.6
8	60.7	70.8	10.1
1	74.8	82.2	7.4
31	79.9	77.6	-2.3
Paper 2 Section			
13	48.6	56.0	7.4
Paper 3	Mean	Mean	Mean Diff
12	7.08	7.47	0.39

Table 20. Percentage of Boys and Girls giving correct scores on each of the questions in the Measurement category; placed in rank order - the best first.

Question Number	Boys %	Rank Order	Girls %	Question Number
Paper 1				
1	82.2	1	79.9	31
41	80.4	2	74.8	1
40	80.0	3	69.8	41
31	77.6	4	69.0	40
8	70.8	5	60.7	8
42	69.8	6	49.0	42
Paper 2 Section				
13	56	1	48.6	13
Paper 3	Mean		Mean	
12	7.47	1	7.08	12

Paper 1, numbers 40-42.

40. 'A moped is travelling at a constant speed of 30 km per hour. How far will the moped travel in 10 minutes?'

The correct solutions were as follows:

	Female	Male	Total
Number	342	400	742
% of total	46.1	53.9	
% of each sex	69.0	80.0	

41. How long will it take to travel 45 km?

	Female	Male	Total
Number	349	402	751
% of total	46.5	53.5	
% of each sex	69.8	80.4	

42. How long will it take to travel 500 metres?

	Female	Male	Total
Number	244	349	593
% of total	41.1	58.9	
% of each sex	49.0	69.8	

This last question was one of the three giving the greatest percentage difference between the performance of boys and girls. Between questions 41 and 42 there is a drop in correct scores of 10.6% (53) for the boys and 20.8% (105) for the girls. This gives an indication of the hierarchical nature of mathematics and the extent to which further steps render the question more difficult. It may be that some candidates just give up. Again, the context within which a question is set may be of importance.

Another question relating to speed-distance and time was from Paper 2, section 13.

'An athlete runs 800m in 2 minutes. Calculate her average speed in km per hour.'

It is interesting here that the examiners have set the question in the female gender, but it is not really possible to assume it has increased performance.

The correct solutions were:

	Female	Male	Total
Number	243	280	523
% of total	46.5	53.5	
% of each sex	48.6	56	

When the questions were presented in a different form as in Paper 3, Number 12, the results were closer.

a) 'A ball is thrown vertically downwards from a height of 20m above the ground. The distance, s metres, through which it travels in t seconds after being thrown is given by

$$s = 8t + 5t^2$$

Find:-

(i) how far the ball is above the ground half a second after being thrown,

(ii) an expression, in terms of t, for its speed t seconds after being thrown,

(iii) the speed with which it was thrown.'

b) Evaluate $\int_0^2 (2x - 1)^2 \, dx$.

The results from this question gave a difference of the mean marks of 0.39 in favour of the boys. There was no statistically significant difference between the sexes. It must be remembered however, that since this question comes from Paper 3, the element of choice may have had an effect. Most of those attempting the question would have some level of confidence in tackling the calculus.

Another question in which the boys scored more highly than the girls, was one which was related to the difference in measure between area and distance. Paper 1, number 8.

'The area of a square is 121 cm^2. What is the perimeter of the square?'

The numbers of correct solutions were:

	Female	Male	Total
Number	303	352	655
% of total	46.3	53.7	
% of each sex	60.7	70.8	

The girls performed better than the boys on a question relating to substitution into a measure formula. Paper 1, number 31:

'The surface area of a sphere is given by the equation $s = 4\pi r^2$ where r is the radius. What is the surface area of a sphere of radius 7 cm?'

The numbers of correct solutions were:

	Female	Male	Total
Number	397	387	784
% of total	50.6	49.4	
% of each sex	79.9	77.6	

This straightforward, clear question requiring careful handling and substitution skills, has favoured the girls. Others, which have required skills of transposition of units clearly have not.

Table 20 shows the percentage of boys and girls giving correct scores on each of the questions in the Measures category when placed in rank order - the best first. This gives the inter-gender differences of boys and girls.

The question above, Paper 1, number 31 was the most successful as far as the girls were concerned whereas more boys did better on the straightforward question (Paper 1, number 1), 'How many mm are there in 1m 10 cm?' 82.2% (411) boys answered this correctly.

In summary, it is evident that the girls have particular difficulty compared with the boys on questions relating to speed, distance and time, and to questions involving mixed units. Particular difficulties were again experienced when the question was in a problem format. Again, it must be pointed out the importance of the setting of a question if it is not to be biased to one sex or the other. Clearly, if a candidate is motivated to attempt an interesting, relevant question, it may make all the difference.

d) Shape and Space

These are questions which test the recognition and the use of the properties of two and three dimensional shapes. They include the recognition of location and the use of transformation geometry. They include the understanding of congruence of simple shapes, Pythagoras' theorem, trigonometry, the angle properties of circles, and the calculation of distances and angles in solids using plane sections.

Candidates were expected to be familiar with the properties of the isosceles triangle, parallelogram, rectangle, rhombus, square and trapezium. They were required to have a knowledge of angle properties relating to parallel lines, bearings and polygons. Skills in scale drawings, loci and constructions were required. This included knowledge of the bisection of angles and straight lines, perpendiculars to a given line, angles equal to a given angle, an angle of 60°, triangles, quadrilaterals and circles from simple data. It also included the inscribed and circumscribed circles of a triangle, division of a straight line into a given number of parts or in a given ratio, and tangents to a circle.

It might be expected from earlier discussion in the literature review, that shape and space questions show a greater difference in performance between the sexes than other concepts. This may be more pronounced when dealing with three dimensional problems which required an element of mental visualisation.

Table 21 shows the differences on correct scores between the boys and girls in the Shape and Space category across each of the three papers. The greatest difference is that given first.

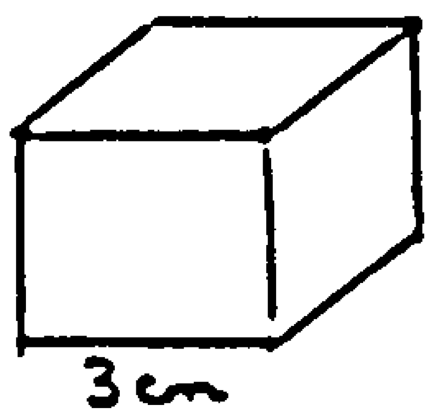
Table 21. Differences on Correct Scores between Boys and Girls in the Shape and Space category - the greatest first.

Question Number	F%	M%	% diff M-F	Question Number	F%	M%	% diff M-F
Paper 1				25	43.0	50.0	7.0
15	36.1	53.5	17.4	26	33.2	40.0	6.8
26	35.1	47.6	12.5	30	63.6	70.2	6.6
27	40.8	53.2	12.4	45	9.8	16.4	6.6
51	22.6	34.8	12.2	27	19.6	25.8	6.2
9	60.6	72.7	12.1	48	15.6	21.8	6.2
44	56.4	68.2	11.8	7	22.2	28.0	5.8
25	63.3	74.3	11.0	50	11.0	16.8	5.8
38	39.6	47.6	8.0	10	17.2	22.8	5.6
37	71.1	78.6	7.5	53	8.8	14.4	5.6
50	73.7	80.1	6.4	8	55.2	60.2	5.0
2	65.7	71.7	6.0	34	31.2	36.0	4.8
43	47.4	52.6	5.2	36	45.4	50.0	4.6
39	42.8	47.6	4.8	31	68.2	72.6	4.4
57	49.5	52.6	3.1	6	83.2	87.2	4.0
52	66.0	64.1	-1.9	46	2.8	6.0	3.2
12	50.0	47.4	-2.6	33	16.4	19.2	2.8
14	45.1	42.5	-2.6	35	18.4	20.4	2.0
Paper 2 Section				47	5.4	6.6	1.2
				32	65.8	63.0	-2.8
11	39.4	61	21.6	Paper 3	Mean	Mean	Mean Diff
12	19	31.6	12.6	7	4.75	5.84	1.09
52	10.6	22.8	12.2	10	7.32	8.35	1.03
24	59.8	71.8	12.0	11	5.96	6.43	0.47
44	45.2	55.6	10.4	2	3.12	2.86	-0.26*
51	28.6	39.0	10.4	9	7.68	7.27	-0.41
29	60.6	70.8	10.2				
49	17.4	24.8	7.4				

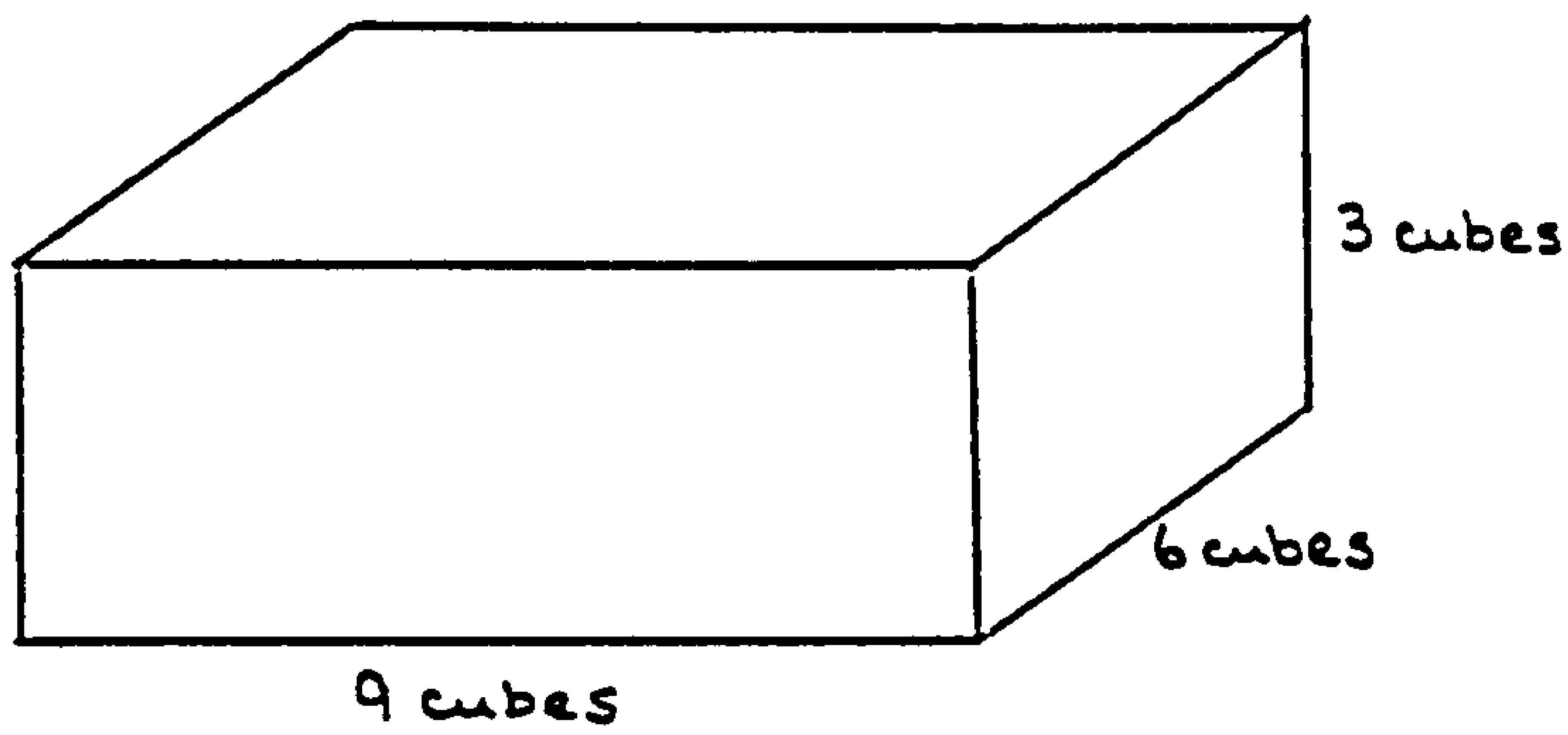
* 1 unit question

These questions revealed some of the greatest discrepancy between the girls' and boys' results. There were marked differences in performance on those questions which were related to solid blocks and bearings. For example, Paper 1, Numbers 25-27:

A solid rectangular block is built up of cubes of edge 3 cm.



The block is 9 cubes long, 6 cubes wide and 3 cubes high.



25. What is the number of cubes needed to build the block?

The correct solutions given were:

	Female	Male	Total
Number	315	371	686
% of total	45.9	54.1	
% of each sex	63.3	74.3	

26. What is the area of the base of the block, in cm^2?

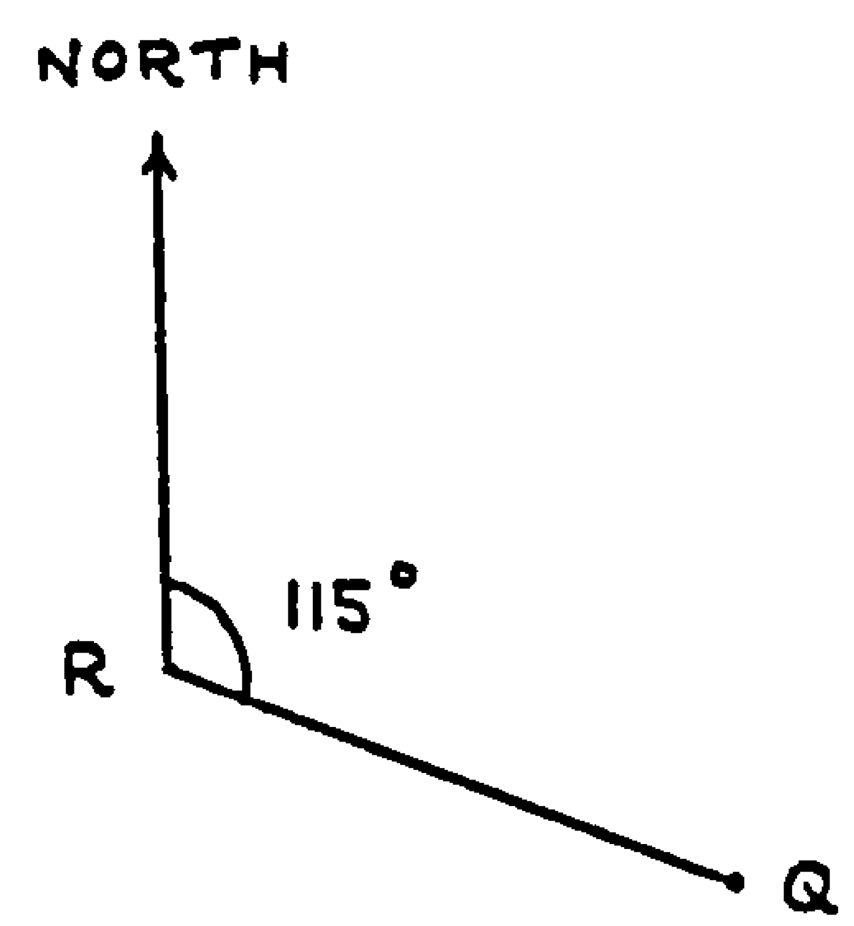
	Female	Male	Total
Number	175	237	412
% of total	42.5	57.5	
% of each sex	35.1	47.6	

27. What is the number of cubes of edge 9 cm, needed to make a block of the same dimensions?

	Female	Male	Total
Number	203	264	467
% of total	43.5	56.5	
% of each sex	40.8	53.2	

Many candidates found difficulty with questions relating to position and bearings. The majority of girls in the sample were unable to answer correctly the question from Paper 1, number 15.

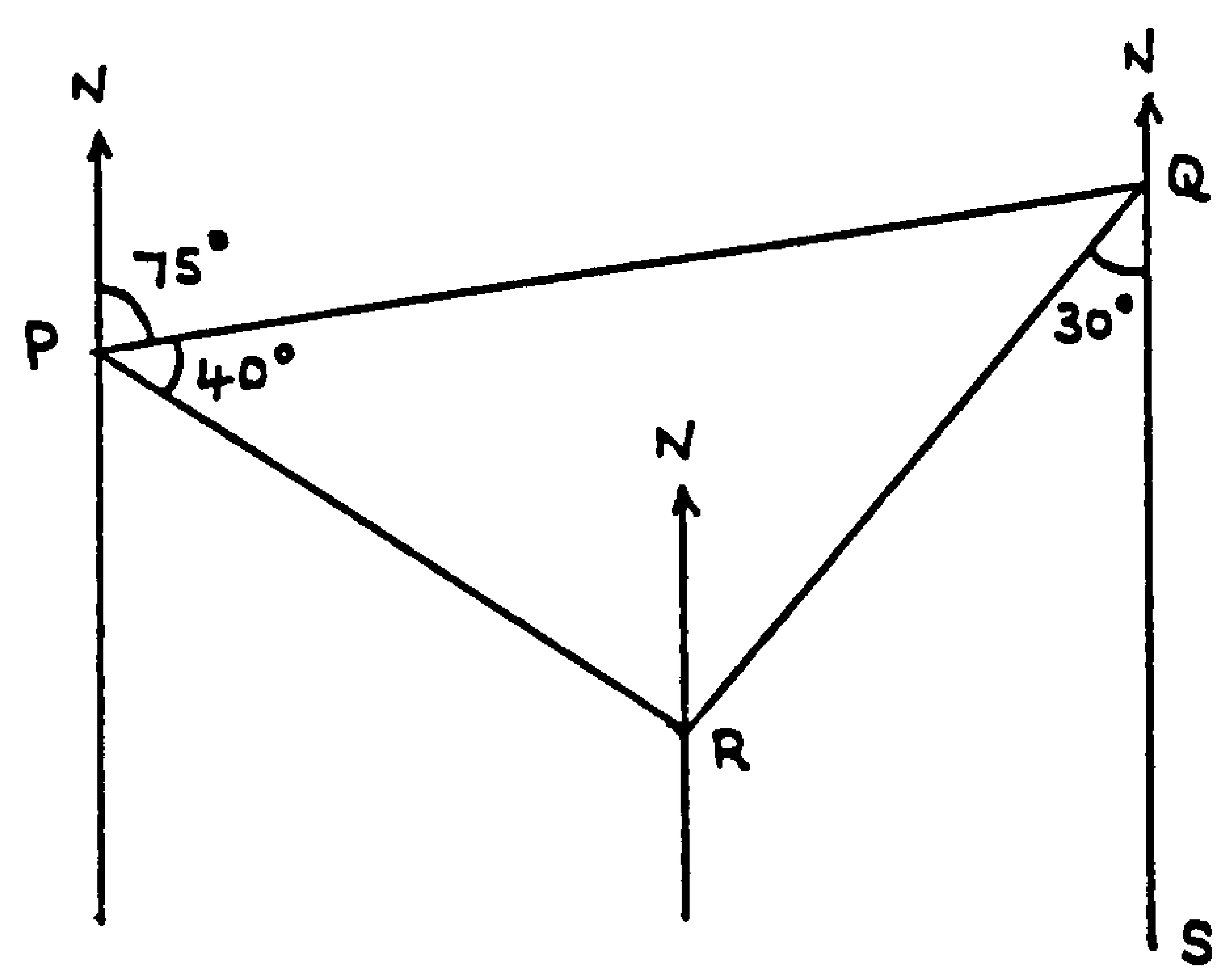
'The bearing of Q from R is 115°.
What is the bearing of R from Q?'



The number of correct solutions were:

	Female	Male	Total
Number	179	267	446
% of total	40.1	59.9	
% of each sex	36.1	53.5	

Similar differences were found in a bearing question in Paper 2, section 11-12.



In the diagram (not drawn to scale) the bearing of Q from P is 075°.

Angle QPR = 40° and angle RQS = 30°.

11. Calculate the bearing of R from Q.

The number of correct solutions were:

	Female	Male	Total
Number	197	305	502
% of total	39.2	60.8	
% of each sex	39.4	61	

12. Calculate the bearing of P from R.

	Female	Male	Total
Number	95	158	253
% of total	37.5	62.5	
% of each sex	19	31.6	

The volume and bearing questions demand spatial visualisation. Spatial visualisation involves visual imagery of objects, movement by the objects themselves or change in their properties. In mathematical terms, spatial visualisation requires that objects be mentally rotated, reflected or translated. In fact, most concrete and pictorial representations of arithmetic, geometric and algebraic ideas appear to be heavily reliant on spatial attributes. Even the number line itself, which is used extensively to represent whole numbers and operations on them, is a spatial representation. Indeed, the number line, algorithms for the four basic arithmetic operations, fractions, ratio, scale, bearings, co-ordinates, mappings, algebraic identities, limits and graphs may all be represented or illustrated in ways involving spatial qualities. Even illustrating the commutativity of two numbers, involves a direct spatial visualisation skill of transposition. In algebraic terms, the changing of the subject of a formula involves the same skill. The degree to which a question involves spatial qualities may well influence the correct scores on that question, particularly for girls. This needs further investigation.

Table 22 gives the differences on correct scores on each of the questions in shape and space placed in rank order with the best first. For Paper 1 and 2, the percentage of boys and girls answering each section correctly is given. For Paper 3, the mean mark is given.

Table 23 gives the rank order of questions for girls in the Shape and Space category showing the character of the spatial element. The best done questions are given first. From the table, it is possible to see a little more clearly the different types of geometrical questions in a comprehensive structure.

It is evident that the questions which involve the use of circle geometry are done well by the majority of girls. This may be because circle theorems are taught as a set standard procedure. Once the theorems have been mastered they can be comprehensively used in other similar questions. There is no element of problem-solving or of proportionality involved. The exception to this was Paper 1, number 51, but this was a hard question and, as Table 22 shows, appeared also at the bottom of the boys' list. Greater difficulty was found with the three-dimensional block questions Paper 1, numbers 26 and 27; but number 25 which required the counting of blocks to make a solid rectangular block, was done better. There is a hierarchy of concepts here from basic volume, to finding the area of the base block.

Table 22. Percentage of Boys and Girls giving correct scores on each of the questions in the Shape and Space category: placed in rank order - the best first.

Question Number	Boys %	Rank Order	Girls %	Question Number	Question Number	Boys %	Rank Order	Girls %	Question Number
Paper 1					44	55.6	9	45.2	44
50	80.1	1	73.7	50	25	50.0	10	43.0	25
37	78.6	2	71.1	37	36	50.0	11	39.4	11
25	74.3	3	66.0	52	26	40.0	12	33.2	26
9	72.7	4	65.7	2	51	39.0	13	31.2	34
2	71.7	5	63.3	25	34	36.0	14	28.6	51
44	68.2	6	60.6	9	12	31.6	15	22.2	7
52	64.1	7	56.4	44	7	28.0	16	19.6	27
15	53.5	8	50.0	12	27	25.8	17	19.0	12
27	53.2	9	49.5	57	49	24.8	18	18.4	35
43	52.6	10	47.4	43	10	22.8	19	17.4	49
57	52.6	11	45.1	14	52	22.8	20	17.2	10
26	47.6	12	42.8	39	48	21.8	21	16.4	33
38	47.6	13	40.8	27	35	20.4	22	15.6	48
39	47.6	14	39.6	38	33	19.2	23	11.0	50
12	47.4	15	36.1	15	50	16.8	24	10.6	52
14	42.5	16	35.1	26	45	16.4	25	9.8	45
51	34.8	17	22.6	51	53	14.4	26	8.8	53
Paper 2					47	6.6	27	5.4	47
Section					46	6.0	28	2.8	46
6	87.2	1	83.2	6	Paper 3	Mean		Mean	
31	72.6	2	68.2	31	10	8.35	1	7.68	9
24	71.8	3	65.8	32	9	7.27	2	7.32	10
29	70.8	4	63.4	30	11	6.43	3	5.96	11
30	70.2	5	60.6	29	7	5.84	4	4.75	7
32	63.0	6	59.8	24	2	2.86	5	3.12	2*
11	61.0	7	55.2	8					
8	60.2	8	45.4	36					

* 1 unit question

Table 23. The rank order of questions for girls in the Shape and Space category showing the level of spatial element. The best done questions are first.

Rank Order	Girls %	Qn No	Character
Paper 1			
1	73.7	50	2-Dim, circle geometry Finding missing angles
2	71.1	37	2-Dim, right angle triangle, Trigonometry
3	66.0	52	2-Dim, circle geometry Finding missing angles
4	65.7	2	Nets folding up to make a cubical box
5	63.3	25	3-Dim. Finding number of cubes to build a block
6	60.6	9	Right angle triangle. Use of Pythagoras
7	56.4	44	2-Dim, reflection
8	50.0	12	2-Dim, trigonometry
9	49.5	57	Understanding 2-Dim mathematical shapes
10	47.4	43	2-Dim, number of lines of symmetry
11	45.1	14	2-Dim, circle geometry Finding missing angles
12	42.8	39	2-Dim, right angle triangle, Trigonometry
13	40.8	27	3-Dim. Finding number of cubes in a block
14	39.6	38	2-Dim, right angle triangle, Trigonometry
15	36.1	15	2-Dim bearings
16	35.1	26	3-Dim. Finding area of base of a cubical block
17	22.6	51	2-Dim, circle geometry Finding missing angles
Paper 2		Sec	
1	83.2	6	2-Dim diagram, finding missing angles
2	68.2	31	2-Dim, circle geometry Finding missing angle
3	65.8	32	2-Dim, circle geometry Finding missing angle
4	63.4	30	2-Dim, circle geometry Finding missing angle
5	60.6	29	2-Dim, circle geometry Finding missing angles
6	59.8	24	2-Dim, calculating the area of a trapezium
7	55.2	8	2-Dim diagram, finding missing angles
8	45.4	36	2-Dim, angles of an isosceles triangle

Rank Order	Girls %	Qn No	Character
9	45.2	44	2-Dim, Similar Triangles
10	43.0	25	2-Dim, calculating a line - trigonometry
11	39.4	11	2-Dim, finding the bearing
12	33.2	26	2-Dim, trigonometry
13	31.2	34	2-Dim, interior angles of a pentagon
14	28.6	51	2-Dim, constructing a convex quadrilateral
15	22.2	7	2-Dim diagram, finding missing angles
16	19.6	27	2-Dim, trigonometry
17	19.0	12	2-Dim, finding the bearing
18	18.4	35	2-Dim, Sum of interior angles of a pentagon
19	17.4	49	3-Dim, the radius of the base of a cone
20	17.2	10	2-Dim, find the bearing
21	16.4	33	2-Dim, circle problem and Pythagoras
22	15.6	48	3-Dim, Circumference of the base of a cone
23	11.0	50	3-Dim, Area of the base of a cone
24	10.6	52	2-Dim, constructional problem
25	9.8	45	2-Dim, Similar triangles involving ratio
26	8.8	53	2-Dim Constructional problem
27	5.4	47	2-Dim Similar triangles. Ratio of area
28	2.8	46	2-Dim Similar triangles. Ratio of area.
Paper 3	Mean		
1	7.68	9	2-Dim circle geometry Sine Rule
2	7.32	10	3-Dim pyramid. Find sides and angles
3	5.96	11	3-Dim Latitude and Longitude
4	4.75	7	2-Dim Area of triangle Cosine Rule
5	3.12	2*	2-Dim, trigonometry Sine/Cosine Rule

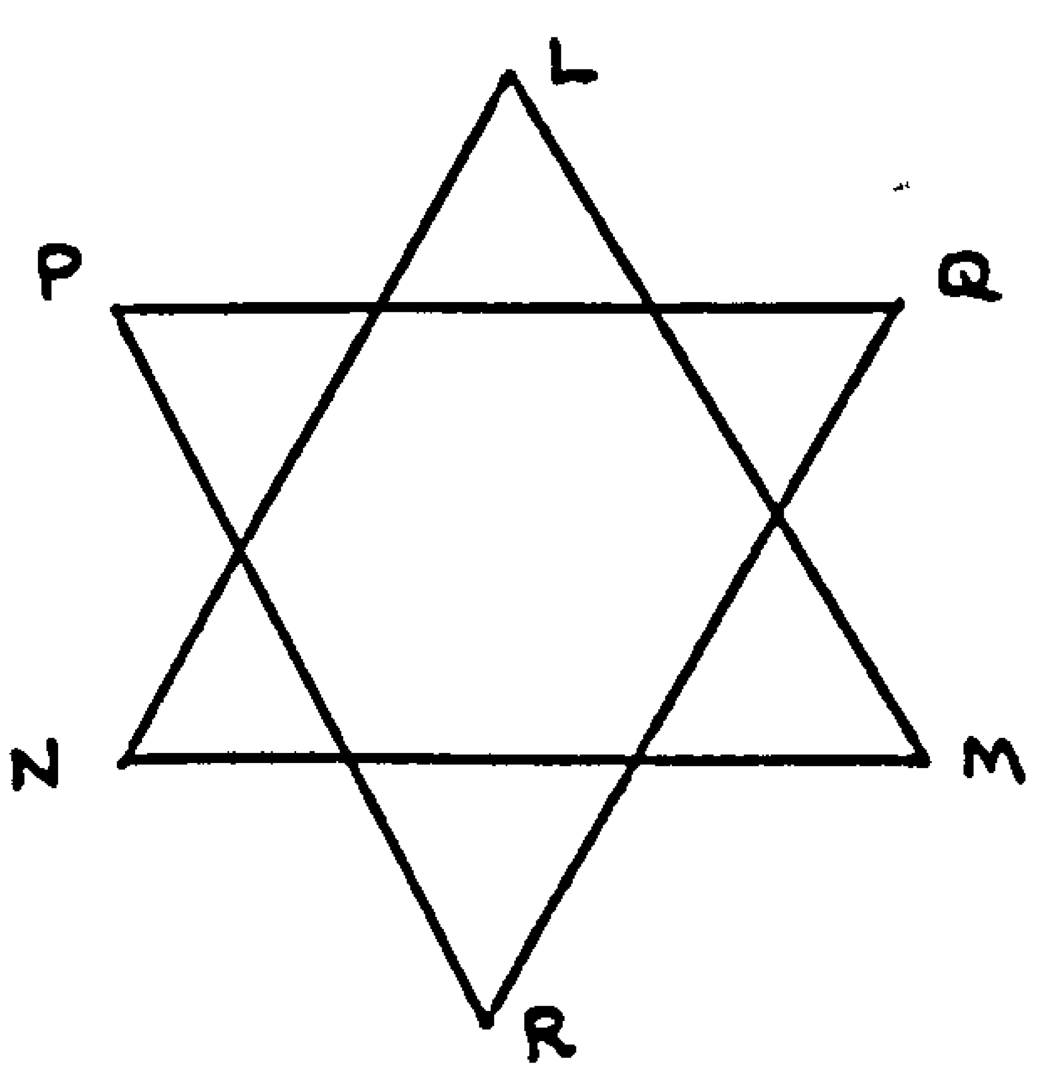
* 1 unit question

Another section of Shape and Space work which the girls found difficult was the construction work. This was ranked 14, 24 and 26 in Paper 2. It may be that girls have had less experience of the practical use of constructional equipment or do not have the same confidence in their use.

Again, bearings come well down the ranking. This involves the ability to measure using a protractor and to have a general spatial orientation. Similar triangles also come down the rankings. In Paper 2 they are ranked 25, 27 and 28 out of 28. This may not be surprising in view of the fact that these questions involve not only spatial awareness, but also proportionality considerations. However, the boys also found these concepts difficult and they appeared at the bottom of their rankings (Table 22). The differences are present but are not large (1.2% on Paper 2, number 47). It is reasonable to suppose that these ideas and their applications can be learned as discrete items by experience and practice. That is, they may not be necessarily generalised or related to further learning and understanding.

This difficulty of spatial visualisation and orientation, is well illustrated in a question from Paper 1 (number 44).

'The triangles PQR and LMN are equilateral. The points P, L, Q, M, R, N form a regular hexagon.



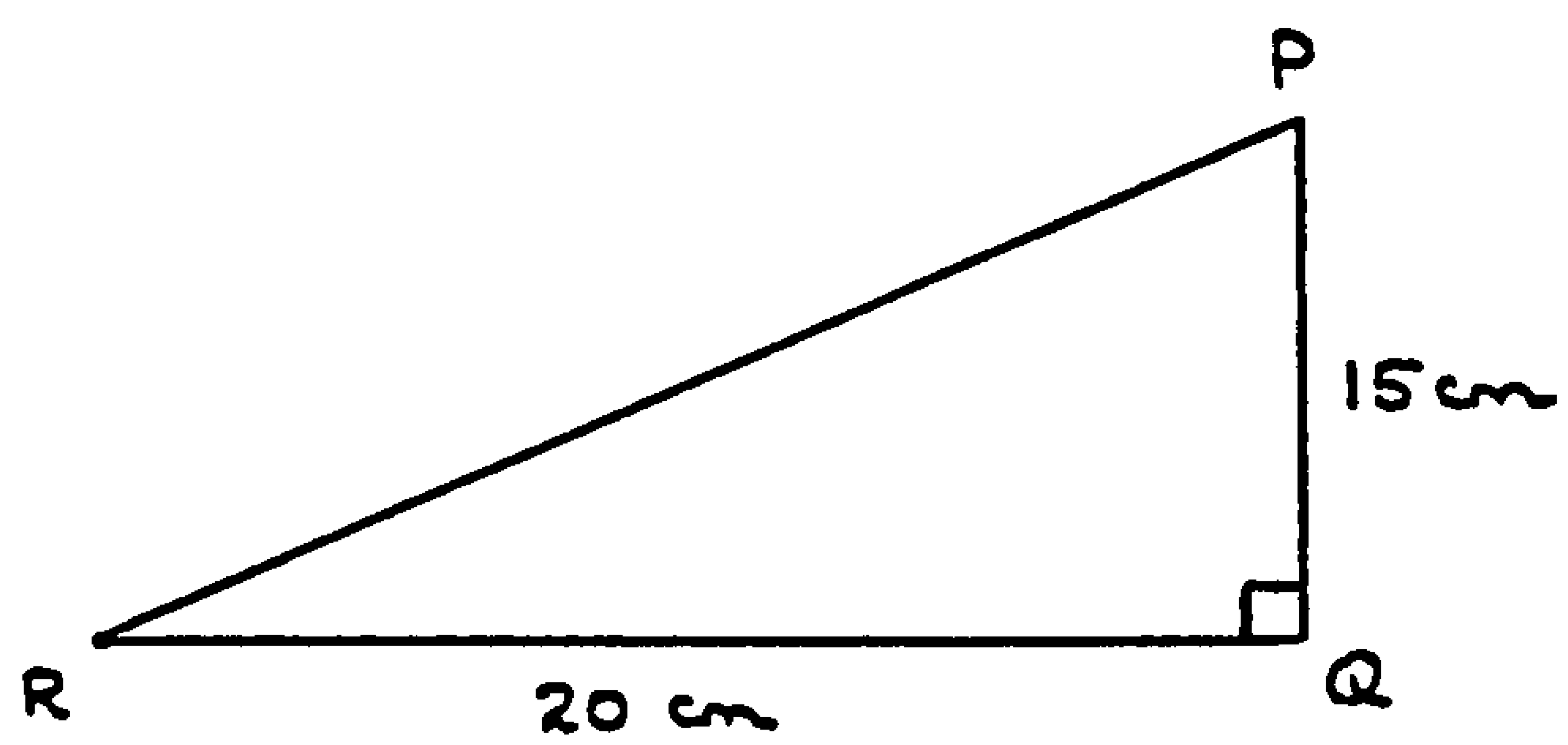
What is the reflection of point N in a straight line drawn through L and R?'

The number of correct solutions were:

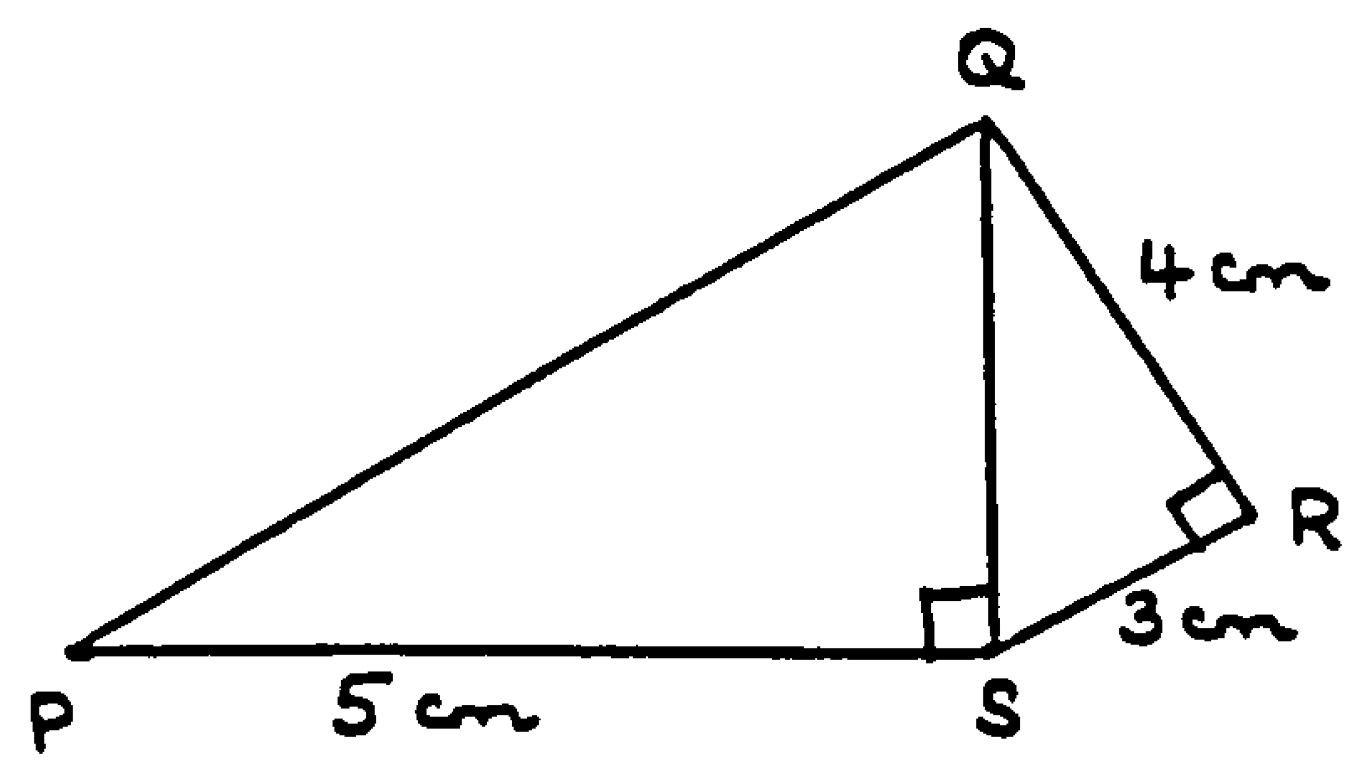
	Female	Male	Total
Number	281	341	622
% of total	45.2	54.8	
% of each sex	56.4	68.2	

It is interesting to note that 29.9% of all girls (149) and 20.8% of all boys (104), gave an answer of Q. It would seem that the visual imagery of reflection is not fully understood particularly by the girls.

The work on Pythagoras' theorem was mixed. In Paper 1, number 12, candidates were given the diagram as shown:



They were then asked to find the value of sin P. 249 girls (50.0%) and 236 boys (47.4%) gave the correct answer of 4/5. Yet in question 9, in a two stage pythagorean problem, the boys did better.



QRS and QSP are right angled triangles. QR = 4 cm, RS = 3 cm, PS = 5 cm. Find the length of PQ.

The number of correct solutions were:

	Female	Male	Total
Number	302	363	665
% of total	45.4	54.6	
% of each sex	60.6	72.7	

In the shape and space work associated with construction, the boys' results were again better overall. Candidates were asked to use only ruler and compass to attempt the question - Paper 2, Sections 51, 52 and 53.

(i) Use the given line AB to construct the convex quadrilateral ABCD such that angle BAD = 60°, AD = 7.3 cm, CD = 5.4 cm and BC = 9.3 cm.

(ii) without actual measurement, construct the midpoint M of the line AD. Hence mark clearly the positions of the points P and Q which are the centres of the two circles of radius 4 cm which touch the line AD at its mid-point.

(iii) construct the line CX where X is the point on AB such that CX is as short as possible.

The numbers of candidates gaining full marks in each section were:

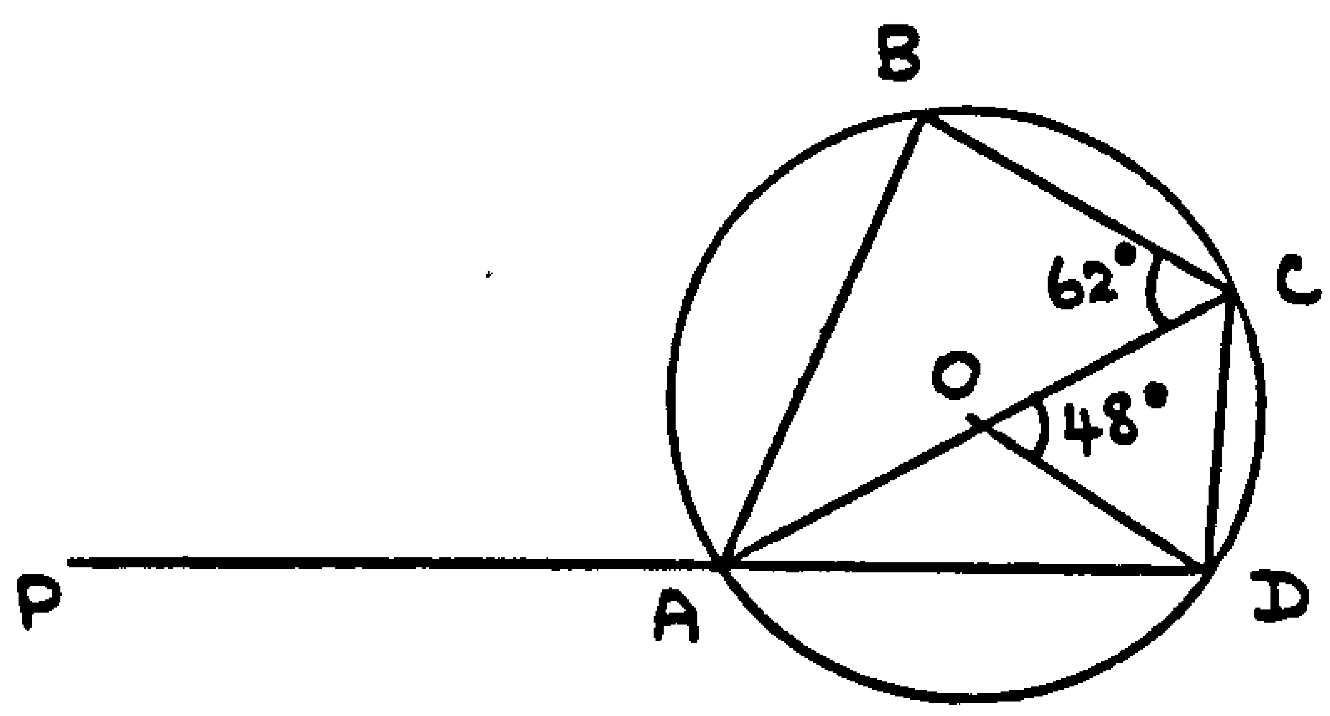
(i)	Female	Male	Total
Number	143	195	338
% of total	42.3	57.7	
% of each sex	28.6	39	

(ii)	Female	Male	Total
Number	53	114	167
% of total	31.7	68.3	
% of each sex	10.6	22.8	

(iii)	Female	Male	Total
Number	44	72	116
% of total	37.9	62.1	
% of each sex	8.8	14.4	

It may be that the experience of some boys in technical and scale drawing has had an influence. There may be a greater cross-curricula transfer of skills from the traditionally 'boys'' subjects than those of the girls.

In the work on the circle, the results between the girls and boys were more mixed. The theorem rules were generally applied by both sexes with equal skill. Candidates were given a circle as shown:



'O is the centre of the circle and DAP and AOC are straight lines. Angle COD = 48° and angle ACB = 62°.'

Candidates were asked to calculate:

(i) the size of angle ABC

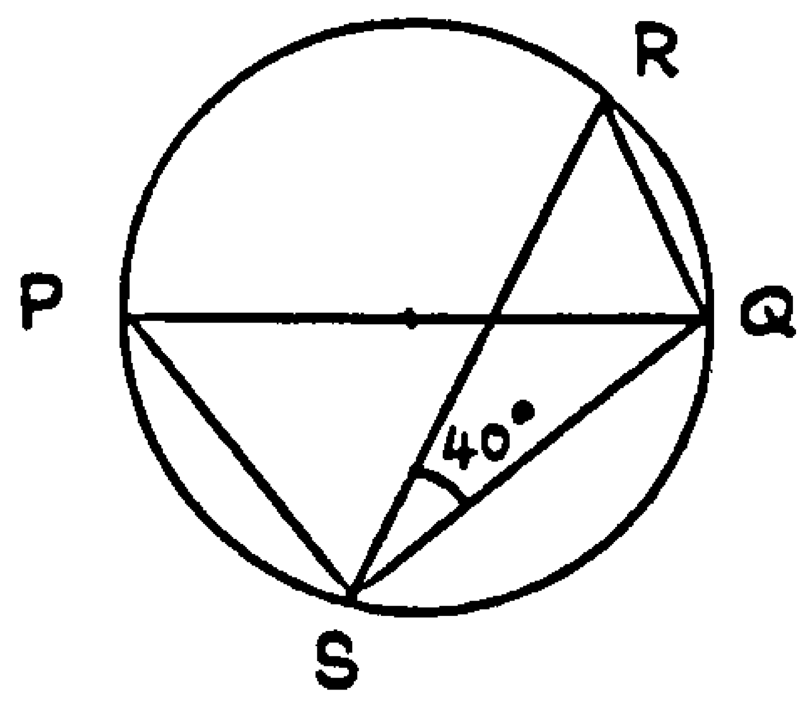
(ii) the size of angle DAO

(iii) the size of angle ACD

(iv) the size of angle BAP.

In part (i) 303 girls (60.6%) and 354 boys (70.8%) answered correctly. A similar result was found for part (ii) where 318 girls (63.6%) and 351 boys (70.2%) answered correctly. The results in part (iii) were more even with 341 girls (68.2%) and 363 boys (72.6%) answering correctly. However, in part (iv) the girls did better, with 329 (65.8%) answering correctly compared with 315 (63%) boys.

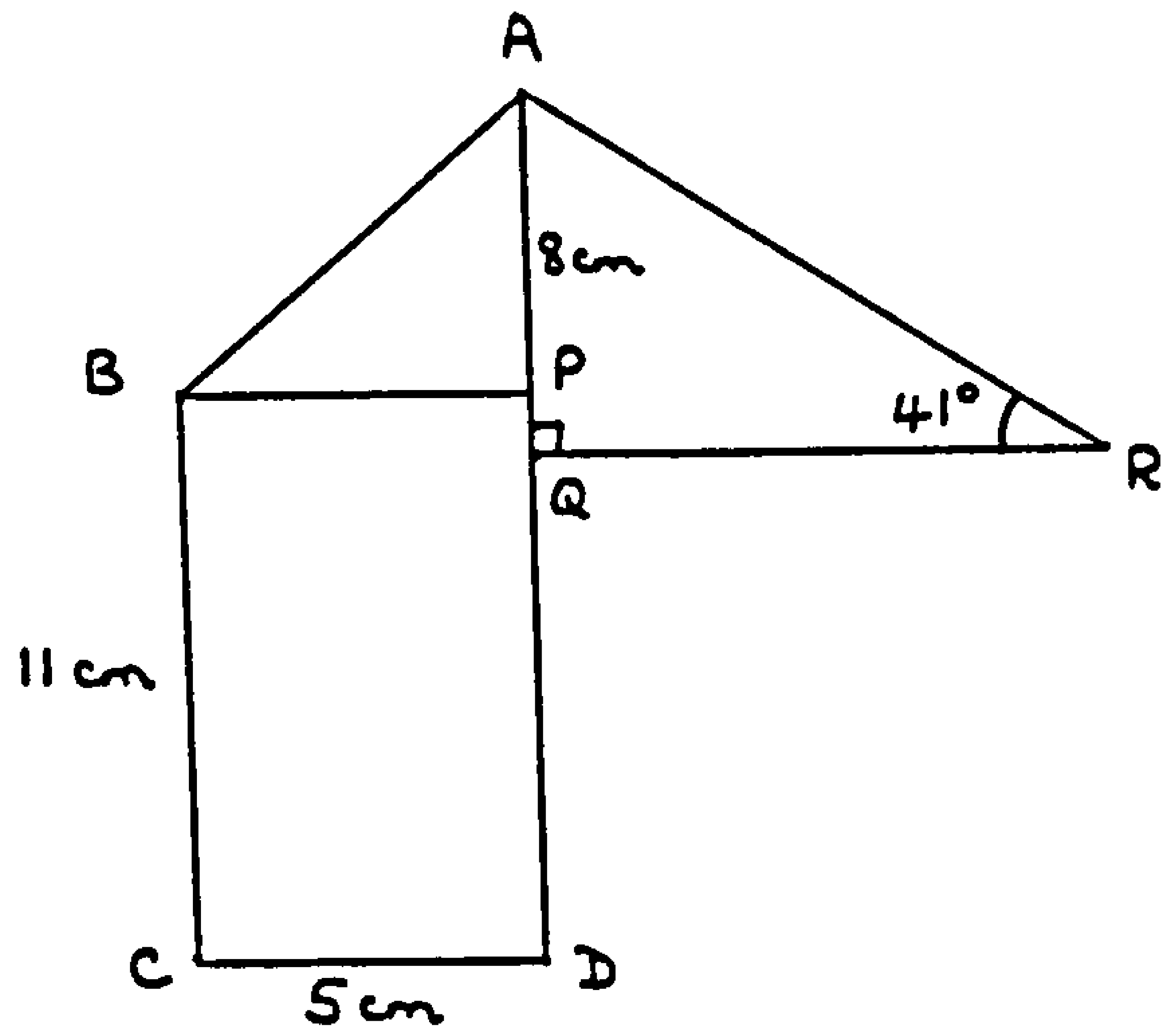
The girls did well compared to the boys in a similar question from Paper 1, number 14.



PQ is a diameter of the circle and RS is a chord. The size of angle QSR is 40°. What is the size of the angle PQR?

223 (45.1%) girls answered this correctly compared to 212 (42.5%) of boys. It may be that girls in general have mastered the rules associated with this type of question, compared to the more general spatial questions such as bearings or construction.

The girls did less well than the boys in a question which related to lengths and area from Paper 2, section 24.



'In the diagram (not drawn to scale), BCDP is a rectangle, APQD is a straight line and angle PQR is a right angle.

AP = 8 cm, BC = 11 cm, CD = 5 cm, QR = 10 cm and angle ARQ = 41°. Calculate the area of the trapezium ABCD.'

The number of correct solutions were:

	Female	Male	Total
Number	299	359	658
% of total	45.4	54.6	
% of each sex	59.8	71.8	

In summary, girls are performing well in questions which require the use of previously learned theorems or formulae. They can equal and surpass the boys in the use of circle theorems and established rules, eg. Pythagoras' theorem, basic geometry and trigonometry. They do less well in three-dimensional and conceptual questions. A question is found to be harder for both sexes but more so by the girls when the question is set in problem form with proportionality skills. Orientation problems such as bearings are found to be harder for some girls as are questions involving constructional work.

e) <u>Data Handling</u>

Data handling is concerned with the collection, recording and processing of data, as well as the representation and interpretation of that data. This includes scatter graphs for both discrete and continuous variables, frequency polygons, histograms, cumulative frequency curves, standard deviations and the normal distribution curve. It includes also the understanding, estimation and calculation of probabilities. It involves calculating the probability of a combined event given the probability of two independent events, and the illustration of combined probabilities of several events using tabulation, tree-diagrams or Venn diagrams. In essence, probability is a comparison or a ratio of two sets of numbers and is a measure of the chance of a particular event happening (or not). As such it is a concept of proportionality. It has been included under the umbrella of data handling because of its specific link to statistical analysis. As the literature shows (Wood, 1976), the ability to deal with proportionality is a critical prerequisite for the successful quantification of probabilities. This is a good example of the way in which the skills of proportionality, problem-solving and spatial awareness can cross the categories of number, algebra, measures, shape and space and data handling.

It may be expected that the girls do well on the careful analysis of data and the verbal statistical analysis but not so well on the reasoning probability questions.

Table 24 shows the differences on correct scores between boys and girls in the data handling category. The greatest differences are given first. Paper 1 are percentage differences and Paper 3 are mean

Table 24. Differences on Correct Scores between Boys and Girls in the Data Handling category - the greatest first.

Question Number	F%	M%	% diff M-F
Paper 1			
28	67.1	79.2	12.1
29	38.8	46.1	7.3
16	44.6	51.6	7.0
30	40.1	45.8	5.7
Paper 3	Mean	Mean	Mean diff
24	5.41	6.82	1.41
19	4.54	5.86	1.32
17 *	2.19	3.43	1.24
13 *	3.69	4.89	1.20
20	7.76	8.93	1.17
22	4.19	5.28	1.09
21	4.75	5.64	0.89
18 *	1.92	2.72	0.80
16 *	3.69	3.99	0.30
15 *	2.80	2.94	0.14
23	9.76	9.89	0.13
14 *	6.62	6.68	0.06

* 1 unit questions

score differences. There were no data handling questions in Paper 2. The Paper 3 scores must be treated with caution, since the 1 unit questions were marked out of 8 and the two unit questions out of 16.

Again, we see that the differences were in favour of the boys. As was expected, the gap was greatest between the boys and girls in questions which related to probability concepts. For example, in Paper 3, number 17.

(a) 'What is the probability that

(i) on one throw of a fair die it will show an even number

(ii) two fair dice thrown together will each show a 4?

(b) In a certain town it has been calculated that the probability of a child catching measles is 0.13

(i) Out of 1000 children in that town how many can be expected to catch measles?

What is the probability that

(ii) a child chosen at random in that town will not catch measles

(iii) two children chosen at random from that town will both catch measles?'

The results from question 17 gave a girls' mean mark of 2.19 and a boys' mean mark of 3.43 out of a possible 8. This gave an overall mean of 2.82, and was again statistically significant in favour of the boys ($F=22.64, d.f.=1$, $p < 0.0001$). Some girls found difficulty with this probability question.

The boys again did better than the girls on a question which involved finding and using given data from a table. This question, Paper 1, number 28, produced the greatest difference between the boys and girls in the data handling category:

'An extract from the time-table shows the times for two trains.

Manchester Victoria Depart	13.00	13.45
Bolton	-	14.02
Chorley	-	14.17
Preston	-	14.40
Blackpool North Arrive	14.10	15.02

How much longer does the slower train take for the journey than the express?'

The number of correct solutions were:

	Female	Male	Total
Number	335	396	731
% of total	45.8	54.2	
% of each sex	67.1	79.2	

The tendency of many candidates with 'time' questions, of whom the majority are girls, is to subtract the figures in a decimal fashion rather than on a 60 minutes = 1 hour basis.

In another question from Paper 3, number 13, candidates were given a frequency table:

'The results of a class test marked out of 10 are shown in the table below.

Mark	Frequency
0	0
1	1
2	3
3	4
4	3
5	5
6	5
7	0
8	2
9	2
10	0

(i) 5 members of the class were absent when the test was taken.

How many pupils are there in the class?

(ii) What was the mean mark achieved by those who took the test?

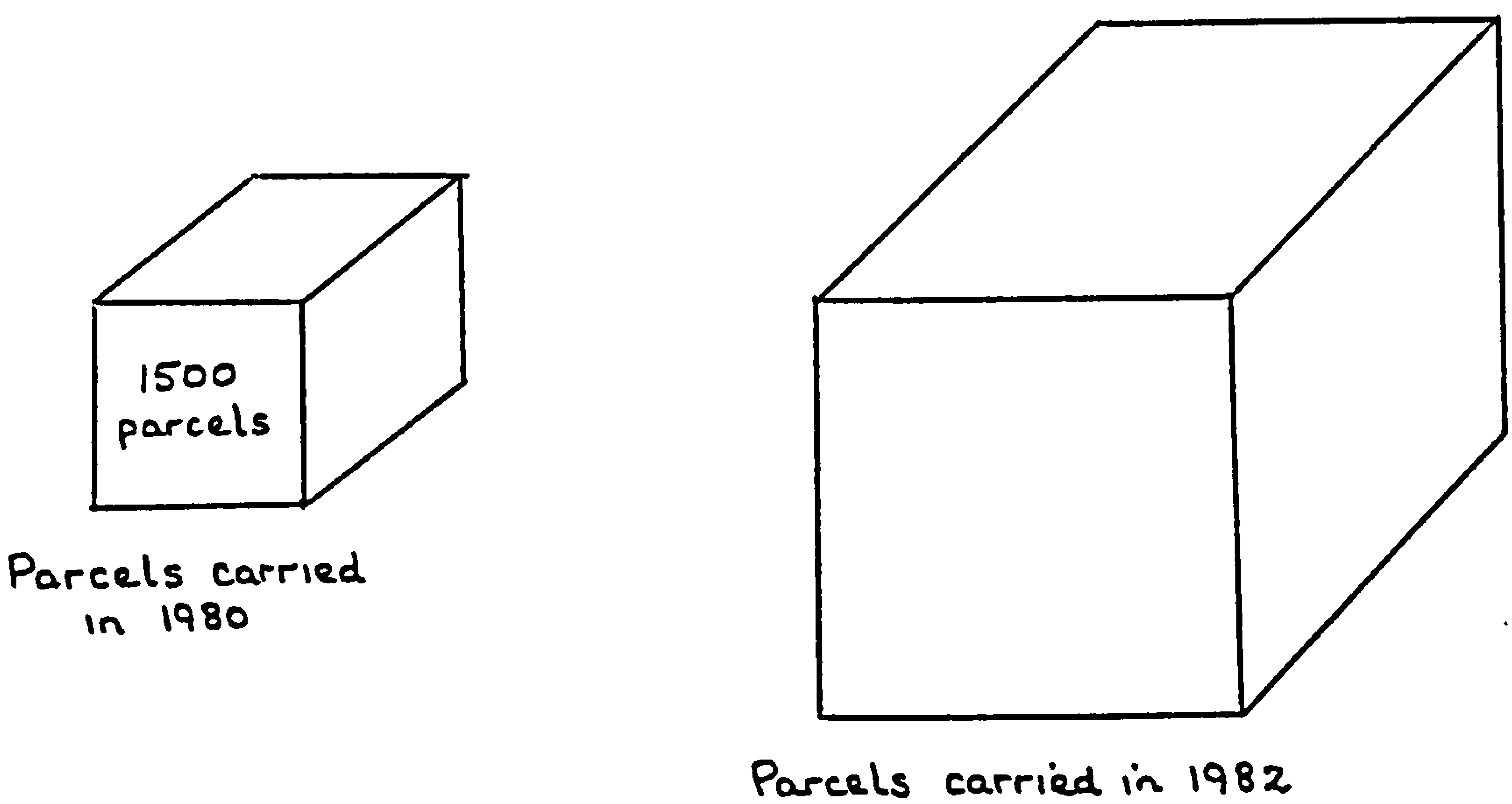
The 5 absentees took the test at a later date, and their mean was 5.4.

(iii) What was the mean mark achieved by the whole class?'

82.2% of girls attempted this question compared to 76.2% of boys. Out of 8, the girls' mean mark was 3.69 and that of the boys was 4.89. This was statistically significant in favour of the boys ($F=30.35, d.f.=1, p < 0.0001$).

In another question, which combined concepts of data handling with volume, girls got a lower mean mark. Paper 3, number 18:

'A firm of carriers set up business in 1980 and in their first year of trading carried 1500 parcels. By 1982 they had increased their trade and in a report illustrated their progress by diagrams in which the numbers of parcels carried were represented by the volumes of cubes. The number of parcels for 1980 was represented by a cube of length of edge 2 cm. The two diagrams used are reproduced below.



(i) Calculate the number of parcels carried in 1982.

In 1984 the number of parcels carried increased to 40,500.

(ii) Illustrate similarly the 1984 trade of the firm.

(iii) Suggest an alternative diagramatic method of showing all this information, stating briefly one disadvantage of your method.'

In this question the mean mark for the girls was 1.92 out of 8 and for the boys was 2.72. The difference was significant in favour of the boys ($F=8.34, d.f.=1$, $p=0.0042$). Many candidates failed to appreciate the significance of the volumes of the cubes and used linear or squared factors. In part (iii), pie charts and histograms were common unacceptable suggestions.

The question which produced the largest difference in mean scores on Paper 3, required familiarity with basic statistical concepts. Number 24:

'(a) Name three measures of average used in statistics. Consider the set of seven numbers. 5 5 5 9 9 11 12.

(i) Which one of the three measures of average for this set of numbers is the smallest?

(ii) The numbers 3 and x are added to the given set of numbers. The three measures of average are unchanged. What is the value of x?

(b) The marks of 400 candidates in an examination are normally distributed. The 10th percentile mark is 24 and the 90th percentile mark is 80.

(i) How many candidates scored more than 24 marks?

(ii) What is the median mark of the distribution?

(iii) At which percentile points would the marks need to be known in order to calculate the semi-interquartile range of the distribution?

(c) State one advantage and one disadvantage of using the range as a measure of dispersion in statistics.

This question gave a difference in mean scores of 1.41 (girls 5.41, boys 6.82), and was statistically significant in favour of the boys ($F=6.60, d.f.=1$, $p=0.0111$). Many candidates showed that they were unfamiliar with the basic properties of the normal distribution curve.

The question in this section which produced the highest mean mark for girls and boys was the one mentioned earlier, concerned with a scatter diagram. Paper 3, Number 23:

'Ten women joined a weight-watchers' class and details of their weights and average daily food consumption were measured. The results are shown in the table given below.

Weight (kg)	84	93	65	95	72	86	78	70	90	75
Food consumption (100 calories/day)	32	37	26	39	27	35	31	28	35	30

(i) Use these figures to plot a scatter diagram. Take 2 cm to represent 5 units on both axes, starting the weight axis at 65 and the 'food consumption' axis at 20.

(ii) Calculate the mean weight and mean daily food consumption for these 10 women. Plot clearly the point on the scatter diagram representing these mean values and identify it by the letter M.

(iii) On the scatter diagram draw in a line of best fit.

Use the diagram to estimate

(iv) the weight of a person whose food consumption is 3300 calories per day.

(v) the average loss of weight that a reduction in food consumption of 500 calories per day could produce.

The mean mark for the girls in this question was 9.76 out of 16 and for the boys was 9.89. There were some errors in the interpretation of the scales when plotting points. The calculations leading to the values of 'M' were often omitted and a clearly plotted point representing these values was frequently missing on the graph. Part (v) proved to be difficult for many candidates. However, candidates were generally better prepared for this question than the others in Paper 3.

Table 25 shows the correct scores for boys and girls on each of the questions in the data handling category. These questions are written in rank order with the best first. The table shows little variation between the sexes. Generally, the hard questions are found to be hard for both sexes, including the questions on probability.

In summary, the boys performed better overall, although there was a close similarity in the ranking order. Probability questions were found to be difficult particularly by the girls. There was a general competence in dealing with the statistical data by both sexes - particularly in comparison with the algebraic category (Table 17). Data questions by their very nature, are verbose. The wording is much greater than the other general mathematics questions. Skills of comprehension are needed in addition to mathematical ability. There is no evidence to suggest that this had any effect on the scores.

Table 25. Percentage of Boys and Girls giving correct scores on each of the questions in the Data Handling category: placed in rank order - the best first.

Question Number	Boys %	Rank Order	Girls %	Question Number
Paper 1				
28	79.2	1	67.1	28
16	51.6	2	44.6	16
29	46.1	3	40.1	30
30	45.8	4	38.8	29
Paper 3	Mean		Mean	
23	9.89	1	9.76	23
20	8.93	2	7.76	20
24	6.82	3	6.62	14 *
14 *	6.68	4	5.41	24
19	5.86	5	4.75	21
21	5.64	6	4.54	19
22	5.28	7	4.19	22
13 *	4.89	8	3.69	13 *
16 *	3.99	9	3.69	16 *
17 *	3.43	10	2.80	15 *
15 *	2.94	11	2.19	17 *
18 *	2.72	12	1.92	18 *

* 1 unit questions

1 unit questions are marked out of 8

2 unit questions are marked out of 16

23. Review of Findings

The girls scored more highly than the boys on several sections (see Figures 14, 16 and 18), although none was statistically significant. Girls' marks were comparable to those of the boys on the basic algebra and also on shape and space work which involved the use of standard methods and techniques in their solution. The Sine Rule and Cosine Rule questions as well as the established rules of trigonometry and Pythagoras' Theorem, were well used. Girls did well in spatial questions where diagrams were already drawn and candidates were required to use previously learned methods. This could be why girls did relatively well in questions requiring the use of circle theorems.

In the number category, girls did relatively well in the work requiring the use of basic mathematical techniques, ie. decimals and number bases. So, too, in the data handling category there was general, relative competence in dealing with the statistical data.

It must always be remembered that there is always a considerable overlap in the distribution of scores between the girls and the boys. Many girls surpass the boys in some or all of the questions and qualities under discussion. It is interesting to note also, that generally, the scores of girls and boys mirror one another closely (see Figures 13, 15 and 17). The frequency polygons of the percentage successes were similar. Questions in which there were low percentage success rates for boys corresponded to low percentage success rates for girls.

In the number category, girls found the greatest difficulty with questions of a problem nature. A question requiring clear use of mathematical knowledge was comparatively well done but those questions requiring the use of that knowledge in an unfamiliar context were comparatively less well done.

In the algebra category, the marks for girls and boys were generally lower than in the number category. Straightforward algebraic skills were evident from both sexes with high percentage success in those questions. However, when the setting was unfamiliar or involved a problem approach, the marks were lower but more so for the girls.

In the space and shape category again, girls did relatively less well than the boys. A question is found to be harder for both sexes but more so by the girls when the question is set in problem form which requires proportionality skills, eg. questions on similar triangles. A hard question is one in which percentage success rates are low ($<$ 35%). An easy question may be defined as one in which success rates are high ($>$ 65%). Orientation problems such as bearings produced a greater divergence in scores between the girls and boys as did questions involving constructional work.

In the measures category, it is evident that the girls have particular difficulty compared with the boys on questions relating to speed, distance and time, and to questions involving mixed units. Particular difficulties were again experienced when the questions were in problem format.

In the data handling category, the boys performed better overall. The girls found relatively greater difficulty with reading from a train time-table - practical experience may be important here. Also girls found relative difficulty with questions on the normal distribution curve, percentiles, standard deviation, probability and frequency distributions.

Table 26 shows the rank order of performance of girls and boys across all of the five category headings in the three papers. The greatest differences are given first. The questions have been selected under the criteria of a 10% or greater difference in success rate between girls and boys in each question in Papers 1 and 2. In Paper 3, the questions have been chosen which give a mean mark difference of one or greater. This gives in total the thirty-four questions which gave the greatest differences in success. The questions have also been classified according to category and according to character.

Out of these 34 questions, 6 are in the number category, 1 in the algebra, 4 in the measures, 16 in the shape and space and 7 in the data handling.

A closer examination of these questions shows that in the number category, the greatest differences were in questions involving ratio, units, percentages, averages, significant figures and variation. The Algebra question involved a graph of speed, distance and time. The measures questions concerned three questions on speed, distance and time and one on perimeter and area. The space and shape questions concerned bearings (3 questions), three-dimensional block work (3 questions), geometry/algebra (2 questions), the right angled triangle, the area of a trapezium, reflection, similar triangles, construction work and geometry. The data handling questions concerned the reading of a time-table, the normal distribution curve, percentiles, standard deviation, probability and frequency distributions.

Table 26. The rank order of performance of Boys and Girls across all of the five categories in the three papers - with difference ≯ 10% or mean ≯ 1.

Paper	Question Number	Rank	Mean diff % M-F	Classif^{n}	Character
2	11	1	21.6	S	2 Dim bearing/orientation
2	38	2=	20.8	N	Ratio - scaled model
1	42	2=	20.8	M	Question involving speed/dist/time
1	15	4	17.4	S	2 Dim bearing/orientation
2	4	5	17.0	N	Units kg/£/p. Finding mass
1	5	6	14.4	N	Increase in price as a % of the original
2	16	7	14.0	A	Graph involving speed/dist/time
1	36	8	13.0	N	Calculation of average weekly wage
2	12	9	12.6	S	2 Dim bearing/orientation
1	26	10	12.5	S	3 Dim finding the area of the base of a solid block
1	27	11	12.4	S	3 Dim equivalent dimension question with a solid block
1	51	12=	12.2	S	2 Dim Circle geometry/algebraic
2	52	12=	12.2	S	2 Dim Circle geometry/algebraic
1	45	12=	12.2	N	Equalities and significant figures
1	9	15=	12.1	S	2 Dim Right angled triangle. Use of Pythagoras' Theorem
1	28	15=	12.1	D	Reading from a train time-table
2	24	17	12.0	S	2 Dim. Finding the area of a trapezium
1	44	18	11.8	S	2 Dim - reflection
1	25	19=	11.0	S	3 Dim No. of blocks needed to build a solid block
1	40	19=	11.0	M	Question involving speed/dist/time
1	41	21	10.6	M	Question involving speed/dist/time
2	44	22=	10.4	S	Ratio - Naming similar triangles
2	51	22=	10.4	S	2 Dim - Construction
2	29	24	10.2	S	2 Dim - Finding a missing angle/circle
1	8	25	10.1	M	Finding perimeter of a square given its area
			Mean Scores		
3	24	1	1.41	D	Normal distribution curve/percentiles mean/mode/median
3	19	2	1.32	D	Standard Deviation
3	4	3	1.30	N	Variation (joint)/Problem
3	17	4	1.24	D	Probability/Problem
3	13	5	1.20	D	Frequency distribution/mean
3	20	6	1.17	D	Frequency graph/interquartile range
3	22	7=	1.09	D	Tabulation/histogram
3	7	7=	1.09	S	2 Dim. Use of Cosine Rule. Area of a triangle
3	10	9	1.03	S	3D Pyramid. Finding sides and angles

Although nearly half the questions outlined are concerned with shape and space, it must be remembered that more shape and space questions were set in the examination, ie. 50 out of 137 (see Table 14). If girls find shape and space a difficult conceptual area, relative to the boys, then the weighting of the examination itself is by nature, biased towards the boys.

These findings give a picture of the differences in attainment of girls and boys in precise mathematical areas and the findings are substantially similar to those found by Wood in 1973/74. He found that of all the items he analysed, the one that showed the biggest difference in favour of the boys was a question relating to the ability to visualise in three-dimensions. Closely following this, were items concerned with the scale of maps, the distance-time graph and probability. In this study, as discussed above, the biggest difference in favour of the boys was a question on bearings, followed by scaling, speed, distance and time and units. Also, the list above (see Table 26) includes the elements of three dimensional visualisation in the block questions (ranked 10, 11 and 19=) and also probability, as Wood found. The close resemblance in the two lists highlights a continuing problem. Clearly, the situation has not substantially improved since 1973/74.

Topics such as bearings, speed, distance and time, percentages, ratio, proportion, probability and spatial problems demonstrate the greatest differences in scores. It is however, on these topics that much of mathematics up to 16 is based. It is also relevant that in the above list, all but the spatial problems demand some skill in proportionality.

Boys are still performing better overall in problems which require the use of spatial and proportionality skills. It may be that girls regard topics such as bearings, scale and speed, distance and time as male orientated and therefore outside their experience or perhaps they just do not find them interesting or relevant.

As this study has shown, girls' scores are relatively high in questions which require the use of well established methods, the use of standard formulae, and repetitive techniques. It may be that they have a greater tendency to show caution, to avoid being wrong, and to use processes with which they feel confident and secure. Wood also found that none of the items on which girls did relatively better than the boys, required what could be termed problem-solving behaviour. Instead they called for 'the supply of definitions, recognition or classification, application of techniques and theorems, and substitution of numbers into algebraic expressions, just the type of operations which are most susceptible to drilling' (Educational Studies, 2, 2, P.156).

There is also a greater tendency for girls to attempt to solve a problem in a sequential manner, having broken the question down into several stages. This can cause difficulties and is not always the most efficient method. Often the more stages that are created, the greater is the chance of making a mistake or of missing the point. An error analysis illustrates this one or two stage solution in problems. This can be easily done from the Paper 1 analysis, as in the multiple choice questions there are answers which relate to middle stages. For example, number 40: 'A moped is travelling at a constant speed of 30 km per hour. How far will the moped travel in

10 minutes?' 99 (20%) of all the girls gave an answer of 3 km. Presumably, they have divided the 30 km by the 10 minutes. 68 (13.6%) of boys gave the same answer.

Again, when calculating the base area of a solid block measuring 9 cubes by 6 cubes by 3 cubes in which each cube is 3 cm by 3 cm by 3 cm (number 26), 241 (48.3%) of girls gave an answer of 54 cm^2, compared to 207 (41.6%) of boys. It seems that the candidates have just multiplied the 9 cubes by the 6 cubes. In fact, only 175 (35.1%) of girls and 237 (47.6%) of boys gave a correct answer to this question.

There can often be a blind realisation of the mathematical processes involved. In any given question, there may be a number of steps or stages in reaching the correct answer. To these stages may be awarded method marks and accuracy marks. Many candidates are often aware of the correct methods but do not necessarily see the question in context. There is not always a clear regard to the meaning of the units and the 'feeling' for the question as a whole. As a result answers are given which, with a little thought, could be seen to be impossible. This criticism is particularly true of the girls who may not be able to see the problem in a practical setting.

This difficulty covers each of the five category areas. Again, in the number question 5, 'The price of a car changed from £800 to £840. What is the increase in price expressed as a percentage of the original price?', 67 (13.7%) of girls gave an answer of 40% compared to 21 (4.2%) of the boys. Here, the candidates have subtracted £840 and £800, one step in the solution, but have not continued with the problem to give the correct solution.

There does seem to be a greater propensity for girls to choose answers which are one or two step solutions. It may be that more girls approach the questions in a sequential manner as opposed to the holistic method as described by Pask (1976). This may help to explain, as Wood found, that girls often 'snatch' at solutions. It may be that girls in general are less likely to see mathematics in the 'real-life' context and so are less aware of implausible solutions.

24. Discussion

a) Concept Understanding

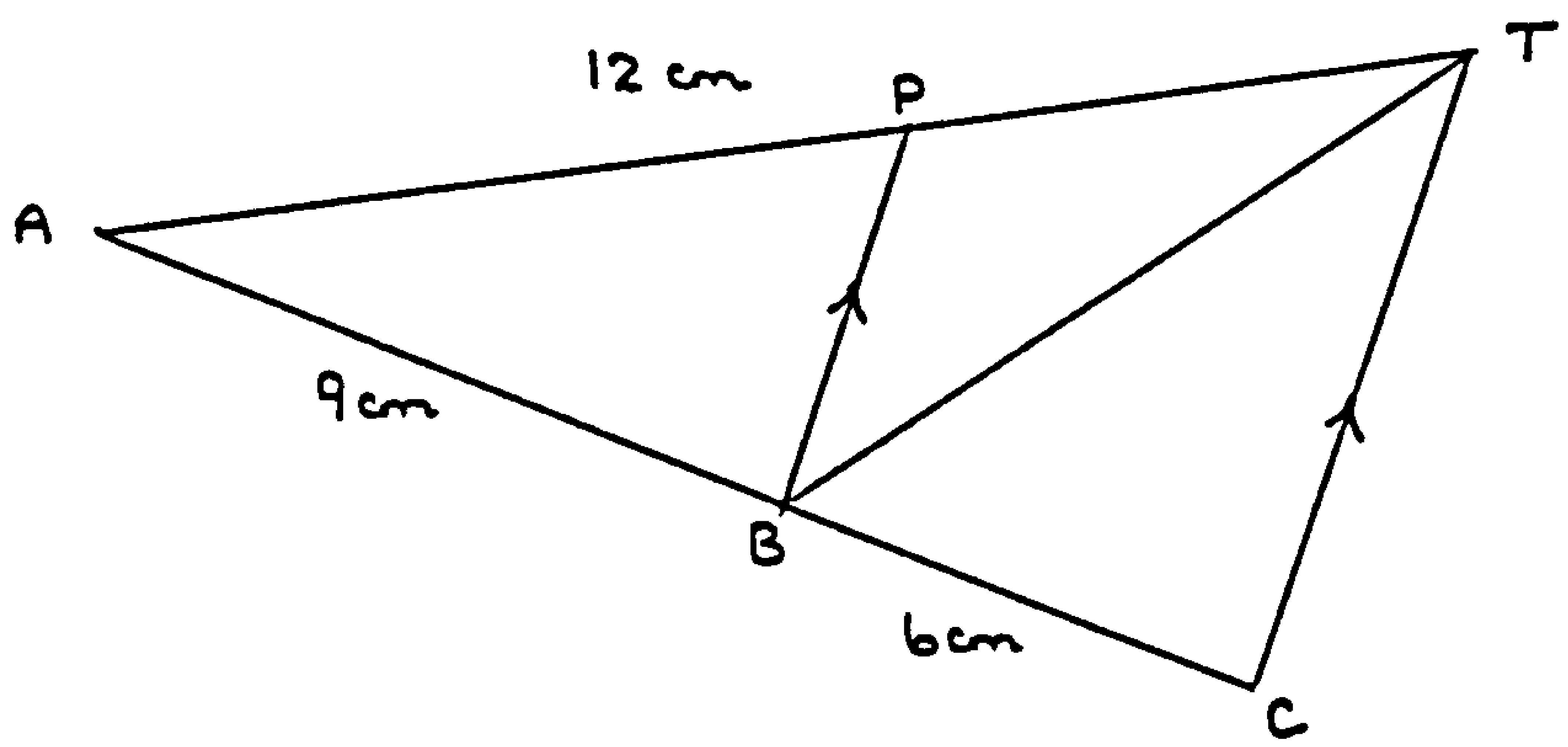
Mathematics is hierarchical in nature and often builds in a sequence of steps. There are problems in remembering facts and there are difficulties in learning algorithms, but it is the conceptual structure of mathematics which is perhaps the hardest aspect of all.

Mathematics consists very largely of building understanding of new concepts onto previously learned concepts. Strangely, it is not easy to explain what a concept is. Perhaps the definition by Novak (1977) explains it best. 'Concepts describe some regularity or relationship within a group of facts and are designated by some sign or symbol' (A Theory of Education in Learning Mathematics, P.31). It is not hard to appreciate why, in the National Curriculum, there are 14 Attainment Targets, each of which has 10 levels of understanding. However, they are still categorised under the same 5 headings described as part of this study.

In some subjects there might be considerable freedom as regards the order in which topics are taught. In mathematics it is much more important to establish the right sequence for the learner. Yet, learners are not identical in their needs nor do they all achieve identical levels of understanding of particular topics in a hierarchy. Ausubel (1968) summed up the problem neatly when he said:

> 'If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.' (Educational Psychology: A Cognitive View in Learning Mathematics, P.34).

Yet, it is evident from the survey, that there are specific concepts which have not been adequately mastered. This is particularly clear in some of the proportionality questions. Consider, for example, the question in Paper 2, section 44-47, where there is a sequence or hierarchy of concepts. Here there is the development of first, naming a triangle, then calculating a length (using similar triangles), then the ratio of two triangles with the same height, and then the ratio of two similar triangles.



'In the diagram (not drawn to scale), APT and ABC are straight lines and BP is parallel to CT. AB = 9 cm, BC = 6 cm and AT = 12 cm.

44. (i) Name a triangle which is similar to the triangle ABP.

45. (ii) Calculate the length of PT.

46. (iii) Calculate in its simplest form, the ratio $\frac{\text{area of } \triangle \text{ ABP}}{\text{area of } \triangle \text{ BPT}}$

47. (iv) Calculate in its simplest form, the ratio $\frac{\text{area of } \triangle \text{ ACT}}{\text{area of } \triangle \text{ ABP}}$'

In section 44, 226 (45.2%) of girls gave the correct result, compared to 278 (55.6%) of boys.

In section 45, 49 (9.8%) of girls gave the correct solution compared to 82 (16.4%) of boys.

Section 46 gave the lowest combined number of correct solutions across the three papers - 14 (2.8%) of girls compared with 30 (6%) of boys.

In section 47, 27 (5.4%) of girls gave the correct solution compared with 33 (6.6%) of boys.

A similar result was found in another question relating to proportionality from Paper 2, section 15.

'Given that $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, calculate the exact value of c when $a = 2\frac{1}{6}$ and $b = \frac{13}{15}$.'

Only 17 (3.4%) of girls and 32 (6.4%) of boys gave the correct result.

Clearly, these questions caused considerable difficulty and were only successfully completed by a handful of candidates, the majority of whom were boys. True understanding of proportionality, as has been discussed earlier, Orton (1987), Hart (1981), develops late, if at all, in a pupil's life. As can be seen, the majority of pupils have not fully understood the concept at age 16.

Are there then, more girls than boys who have not yet developed full understanding in the conceptual areas of proportionality and spatial awareness? On the basis of this study, the answer may be yes, but as Kuntz and Karplus (1977) demonstrated, the skills of proportionality can be improved by careful schooling. Also, as Badger (1981) found, girls do indeed show an improvement in scores on spatial tests after they have been involved in space-related activities.

b) Level Differences

Assuming that there are mathematical concepts which are not fully understood at 16, this begs the question as to the distribution of conceptual development across the full range of ability. What is the gender distribution at all levels? For example, the study gives a picture of general differences in attainment between girls and boys, but are the best girls as good as the best boys? Are the poorer girls better than the poorer boys? This requires further investigation.

In order to examine more closely the distribution of the sexes across the ability range, it is expedient to look at each of the papers in terms of 'cut off' marks. For example, Table 27 shows the range of marks for given percentiles of all 1000 candidates in Paper 1 of the study. The maximum mark on Paper 1 is 60.

Table 27. The range of marks for given percentiles of boys and girls in Paper 1.

	Percentile	Boys' Mark	Girls' Mark
Bottom	10	< 21	< 16
	20	< 24	< 20
	25	< 26	< 22
	Median	< 34	< 30
Top	25	> 42	> 37
	20	> 44	> 40
	10	> 49	> 45

It is significant that it is not just the best boys performing better than the best girls, as might be expected from DES statistics; the same trend continues. The pattern is found to be remarkably even through the ability range. Tables 28 and 29 show a similar pattern in Papers 2 and 3.

Table 28. The range of marks for given percentiles of boys and girls in Paper 2.

	Percentile	Boys' Mark	Girls' Mark
Bottom	10	< 24	< 17
Bottom	20	< 31	< 24
Bottom	25	< 35	< 28
	Median	< 50	< 43
Top	25	> 69	> 62
Top	20	> 74	> 65
Top	10	> 88	> 80

The maximum mark for Paper 2 is 116.

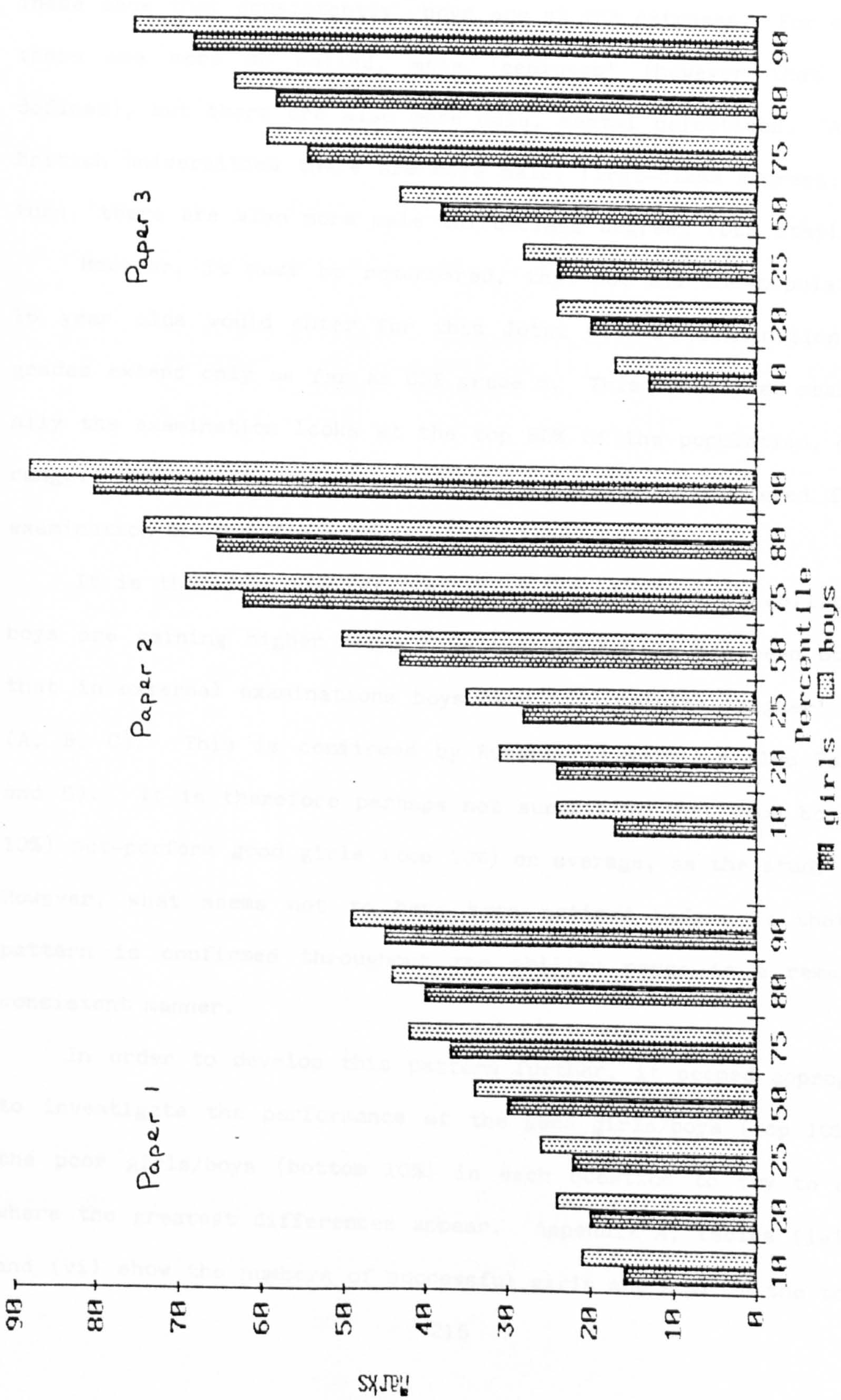
Table 29. The range of marks for given percentiles of boys and girls in Paper 3.

	Percentile	Boys' Mark	Girls' Mark
Bottom	10	< 17	< 13
Bottom	20	< 24	< 20
Bottom	25	< 28	< 24
	Median	< 43	< 38
Top	25	> 59	> 54
Top	20	> 63	> 58
Top	10	> 75	> 68

The maximum mark for Paper 3 is 96.

These tables show a remarkable consistency across the ability range. These figures are more clearly illustrated in Figure 23 and clearly show that at each tenth percentile across the ability range, the performance of the boys on test scores exceeds that of the girls.

Figure 23: Distribution of marks by given percentiles of candidates



It might have been assumed that the poorer girls would produce better results than the poorer boys, in the light of DES statistics. These show that consistently, boys are at the extremes. For example, there are more so called, male 'geniuses' (however that may be defined), but there are also more male, mental defectives. Also, in British universities there are more male, first-class degrees; but in turn, there are also more male third-class degrees (DES statistics).

However, it must be remembered, that not all the population of 16 year olds would enter for this Joint GCE/CSE examination. The grades extend only as far as CSE grade 5. This means that realistically the examination looks at the top 80% of the population, ability range. 20% of the candidates would either not be entered for any examination or else for a limited grade CSE paper.

It is therefore significant, that at each level of ability, the boys are gaining higher marks than the girls. It has been observed that in external examinations boys have been gaining 'higher' grades (A, B, C). This is confirmed by Russell (1984) (see also Tables 5 and 6). It is therefore perhaps not surprising that good boys (top 10%) out-perform good girls (top 10%) on average, as the study shows. However, what seems not to have been noticed before is that this pattern is confirmed throughout the ability range in a remarkably consistent manner.

In order to develop this pattern further, it seemed appropriate to investigate the performance of the good girls/boys (top 10%) and the poor girls/boys (bottom 10%) in each question to try to detect where the greatest differences appear. Appendix A, tables (iv), (v) and (vi) show the numbers of successful girls and boys in the top and bottom ten percentiles. Appendix A, figures (i), (ii) and (iii) illustrate this information by linearising the numbers in horizontal lines - girls against boys for direct comparison.

In Paper 1, the questions which gave the largest difference in numbers of successful girls/boys for the top 10% and the bottom 10% are given in Table 30. These are listed in rank order with the greatest difference first. Only the ten questions which produced the greatest difference are considered.

Table 30. Questions giving the largest difference in numbers of successful girls and boys for the top and bottom 10% in Paper 1, placed in rank order - the greatest difference first.

	Question	Category	Rank	Category	Question	
Top 10%	27	S	1	N	34	Bottom 10%
	24	N	2	N	5	
	29	D	3	M	1	
	49	N	4	N	13	
	51	S	5	N	10	
	36	N	6	M	42	
	25	S	7	A	58	
	46	N	8	S	37	
	2	S	9	N	59	
	30	D	10	N	11	

It is interesting to note that there are no common questions between the top 10% and the bottom 10% of candidates. Six questions producing the greatest difference in numbers of correct responses in the bottom 10% of girls/boys were in the number category. The number question which produced the largest difference (number 34) was concerned with percentages. The second largest difference was also concerned with percentages (number 5). Clearly, the poorer girls found greater difficulty relative to the boys, with this concept. Indeed, number 13 also concerned percentages though in a different form. Percentage work requires skills of proportionality and it seems that a greater number of boys in the bottom 10% have a better grasp of these than the girls. The other number questions were concerned with ratio (number 10), decimals (number 59) and fractions (number 11).

The measures question (number 42) was concerned with speed, distance and time. The algebra question (number 58) related to inequalities, and the shape and space question (number 37) to trigonometry. It is worthy of note, that some of the greatest differences (numbers 59 and 11) are on topics which may be regarded as foundation concepts in the mathematical hierarchy, ie. decimals and fractions. If these concepts are not clearly understood, then the compounded work which follows may falter.

In the top 10% there were four number and four shape and space questions giving the largest differences in success rate. The number questions were concerned with logarithms (24), standard form (49), averages (36) and significant figures (46). The shape and space questions were concerned with a three-dimensional solid block (numbers 25 and 27), circle geometry/algebra (number 51), and a series of nets folding into an open cubical box (number 2). The two data handling questions (numbers 29 and 30) were both concerned with reading data from a railway time-table. This is not a conceptual development in a mathematical hierarchy but is more a skill which is developed from practice and experience. Clearly, the girls were at a disadvantage with these questions.

Table 31 shows a similar analysis in Paper 2 giving the questions in rank order which gave the largest differences in numbers of successful girls and boys for the top and bottom 10%.

Table 31. Questions giving the largest difference in numbers of successful girls and boys for the top and bottom 10% in Paper 2, placed in rank order - the greatest difference first.

	Section Number	Category	Rank	Category	Section Number	
	39	N	1	N	17	
Top 10%	52	S	2	S	29	Bottom 10%
	53	S	3	M	13	
	15	N	4	N	18	
	5	A	5	A	20	
	7	S	6	N	4	
	12	S	7	S	24	
	3	N	8	N	38	
	46	S	9	A	40	
	47	S	10	S	44	

In the two section number columns, there are again, no questions which overlap. The number questions which produced the greatest differences in scores for the bottom 10% were concerned with percentages (section 17), proportion (section 18), units (section 4) and ratio (section 38). The shape and space questions were concerned with circle geometry (section 29), the area of a trapezium (section 24) and similar triangles (section 44). The measures question concerned speed, distance and time (section 13) and the algebra questions concerned substitution (section 20) and solving equations (section 40). Again, the percentage question is significant in this list for the bottom 10% as it was for Paper 1. Also, topics such as work on basic units, substitution and solving simple equations appear lower down the mathematical hierarchy. (See the levels in the National Curriculum.)

In the top 10%, there were six questions which produced the greatest differences in scores between girls and boys in the shape and space category. These were concerned with construction work (sections 52 and 53), angle geometry (section 7), bearings (section 12) and similar triangles (sections 46 and 47). The number questions were concerned with ratio (sections 39 and 3) and a complex fraction problem (section 15). The algebra question was concerned with a problem involving algebraic statements (section 5). These results are consistent with the overall pattern. They again highlight the difficulty girls experience in relation to boys in questions of construction and ratio.

Table 32 shows a similar analysis in Paper 3 giving the questions in rank order which gave the largest difference in number of successful girls and boys for the top and bottom 10%. The numbers have been analysed on the basis of a candidate gaining half marks or greater, on a particular question. This is because so few candidates gained full marks to questions in Paper 3 in the botom 10%. It is apparent that the weaker candidates found this paper difficult. In fact, there were only four candidates to gain full marks in any one question in Paper 3 (see Appendix A, table (vi)). These were all boys. It is encouraging however, to see that many of the good girls (top 10%) were performing well in this paper (see table (vi)). So, in the context of the analysis of Paper 3, 'successful' means gaining half marks or greater.

Table 32. Questions giving the largest difference in numbers of successful girls and boys for the top and bottom 10% in Paper 3, placed in rank order - the greatest difference first (gaining $\geqslant$ half marks).

	Question Number	Category	Rank	Category	Question Number	
	9	S	1	D	13	
Top 10%	1	A	2	A	8	Bottom 10%
	17	D	3	A	1	
	21	D	4	S	2	
	5	A	5	A	3	
	13	D	6	N	4	
	14	D	7	D	14	
	8	A	8	D	15	
	19	D	9	D	16	
	23	D	10	D	23	

There is a greater overlap in the relative difficulties experienced by the girls in this paper compared with Paper 1 and 2 (5 questions). This is partly because there are fewer questions in this paper (24) compared with 60 questions in Paper 1 and 53 sections in Paper 2.

The number question which gave the greatest difference in scores for the bottom 10% was concerned with joint variation (number 4). The algebra questions were concerned with fractional expressions (number 1) and graph work and calculus (numbers 3 and 8). The data handling questions were concerned with frequency distributions (number 13), pie chart (number 14), mean deviation (number 15), a grouped frequency distribution (number 16) and a scatter diagram with line of best fit (number 23).

In addition to question numbers 13, 8, 1, 14 and 23, the greatest differences in scores between the girls and boys in the top 10% were one algebra question, one shape and space question, and three data handling questions. The algebra question was concerned with the gradient of a curve (number 5), the shape and space question was concerned with the use of the Sine Rule in the context of circle geometry (number 9), and the data handling questions were concerned with diagramatic representation (number 17), probability (number 21), and standard deviation (number 19).

Over all the three papers, it is important to note the divergence of the two lists outlining the greatest differences in performance between girls and boys. It appears, for example, that the best girls (top 10%) have mastered the concept of speed, distance and time, although they still have relative difficulty with orientation in the form of bearings and with construction work. Also, they may have less experience in the practical skills of reading a time-table. The three-dimensional concepts also, are giving greater differences. It may be that particular intervention programmes of teaching aimed specifically at these concepts, would be of great help to the girls.

The differences for the bottom 10% are unique in that they involve greater differences in lower order concepts which are not found in the top 10%. This does not mean there were no differences in higher order concepts. Candidates had limited success in these areas to give adequate statistical differences. Concepts such as units, fractions, substitution and simple linear equations are foundation principles upon which the more complex work is built. This underlines the hierarchical nature of mathematics. Also, the girls found greater relative difficulty with percentages. This too, involves a basic understanding of terminology and process. It involves skills of proportionality.

In order to find out if the top 10% of girls were performing as well as the top 10% of boys, a cross tabulation analysis was conducted across each of the three papers. The aim was to find how many girls/boys were in the top ten percentiles in one paper, in two papers, and in all three papers. In other words, how consistent are the best girls in relation to the best boys?

Table 33 shows the number of girls and boys who were in the top ten percentile in each paper.

Table 33. Numbers of girls and boys in the top ten percentile - Papers 1, 2 and 3.

Sex	Paper 1	Paper 2	Paper 3	Total
Girls	59	52	55	166
Boys	59	52	51	162

Table 34 shows the number of pupils in one or more paper while Table 35 shows the split between boys and girls of those who appeared in one paper only.

Table 34. Numbers of girls and boys in the top ten percentile in one paper, in two papers, and in all three papers.

Sex	In one paper	In two papers	In all three papers
Girls	43	24	25
Boys	37	22	27
Total	80	46	52

Table 35. Numbers of girls and boys in the top ten percentile in one paper only, either Paper 1, 2 or 3.

Sex	Only in Paper 1	Only in Paper 2	Only in Paper 3	Total
Girls	20	11	12	43
Boys	16	9	12	37

These figures show a relatively even match between the girls and boys. This supports the view of Russell (1984), that the good girls are as competent as the boys in many of the topics under discussion. In fact, there were more girls in the top ten percentile in Paper 3 than there were boys. Also, there were equal numbers of girls and boys in the top ten percentile in Paper 1. If, as has been suggested, boys have an advantage on multiple choice papers (Murphy, 1980), then it does not seem to have an effect on the relative distribution in the sexes. Indeed, the girls appeared not to be adversely affected by the multiple choice paper at all. (Compare Tables 27, 28 and 29.)

There were marginal differences in the figures of this analysis. There were six more girls in the top ten percentile in one paper only and there were two more boys in the top ten percentile in all three papers. Generally, the figures are comparable. It might appear that the top 10% of girls are performing as well as the top 10% of boys, but it must be remembered that the 'cut off' marks for girls and boys were different. These were, greater than 49 out of 60 for boys, and greater than 45 for girls in Paper 1. In Paper 2, where the maximum mark was 116, the top 10% of boys was greater than 88 and that for girls was greater than 80. In Paper 3, where the maximum mark was 96, the top 10% of boys' 'cut off' mark was 75 and that for girls 68. What can be said, is that the best girls are as consistent as the best boys across the three papers.

In this study of the ranking of questions for the top and bottom 10% of girls/boys, it has become apparent that the hierarchical nature of mathematics is important. This was demonstrated in the lower order concepts which appeared in the tables of differences in the bottom 10% of candidates. Consideration is now given to programmes of study because the ordering of topics may be an important factor in performance variation.

c) Programmes of Study

The ordering of topics in the teaching of mathematics is clearly important, particularly in view of the problems many girls may have with particular concepts. This is a factor with which the National Curriculum has wrestled, though not with particular reference to girls. Tables 36/37 show the depth of mathematical learning required at levels 6 and 10. Level 6 is the standard which the top girls and boys might be expected to reach at thirteen and which the poorer girls/boys may only just reach (or never reach). Level 10 is the standard which only the top girls/boys might be expected to reach at sixteen.

In terms of this study, what is of concern as far as many girls of all ability is concerned, is the content of level 6. The Number section has a predominantly proportionality content; for example, fractions, ratios and percentages. The measures section considers speed and the shape and space section is concerned with representations and transformations of two and three-dimensional shapes as well

Table 36. National Curriculum Programme of Study
(Source: Mathematics in the National Curriculum, 1989)

Level 6

Using and applying mathematics	Designing a task and selecting mathematics and resources; checking information and obtaining any that is missing; using trial and improvement methods. Presenting findings using oral, written, visual or concrete forms. Making and testing generalisations and simple hypotheses; defining and reasoning with some precision.
Number	Understanding and using equivalence of fractions and ratios. Working out fractional and percentage changes. Calculating using ratios in a variety of situations. Converting fractions to decimals and percentages. Using estimation and approximation to check answers to multiplication and division problems are of the right order.
Algebra	Determining rules for generating sequences and using different methods to explore pattern. Exploring number patterns using spreadsheets or other computer facilities. Solving linear and simple polynomial equations by trial and improvement methods. Using and plotting Cartesian coordinates to represent simple function mappings.
Measures	Understanding and using compound measures, eg. speed, density. Recognising that measurement is approximate and choosing degree of accuracy required for measurement.
Shape and space	Classifying and defining types of quadrilaterals. Using angle and symmetry properties of quadrilaterals and polygons. Using 2-D representation of 3-D objects. Using computers to generate and transform 2-D shapes. Understanding and using bearings to define direction. Determining the traversability of networks. Reflecting a figure in mirror lines in different positions. Enlarging a shape by a whole number shape factor. Determining, with the aid of a computer, a rule that will give rise to a desired path or shape.

Continued

Table 36 (continued)

Handling data	Designing and using observation sheets; collating and analysing results. Surveying opinions taking account of bias, using a questionnaire. Creating scatter graphs for continuous variables. Constructing and interpreting information through two-way tables and network diagrams. Identifying outcomes of two combined events which are independent. Knowing that the total sum of the probabilities of mutually exclusive events is 1 and that the probability of something happening is 1 minus the probability of it not happening.

Table 37. National Curriculum Programme of Study
(Source: Mathematics in the National Curriculum, 1989)

Level 10

Using and applying mathematics	Designing, planning and carrying through a mathematical task to a successful conclusion; presenting alternative solutions and justifying selected route. Giving definitions which are necessary, sufficient or minimal. Using symbolisation with confidence; constructing a proof including proof by contradictions.
Number	Calculating the upper and lower bounds in calculations involving a variety of numbers expressed to a given degree of accuracy.
Algebra	Using a calculator or computer, investigate whether a sequence given iteratively converges or diverges. Manipulating a range of algebraic expressions in a variety of contexts. Constructing tangents to graphs to determine the gradient. Finding area under a graph and interpreting the result. Sketching the graph of functions derived from other functions.
Measures	Determining the possible effects of error on calculations involving measurements.
Shape and space	Knowing and using angle properties of circles. Sketching the graphs of sine and cosine functions for all angles. Using sine and cosine rules to solve problems in 2-D and 3-D contexts. Understanding how transformations are related by combinations and inverses. Using matrices to transform vectors, representing points in 2-D and 3-D space. Using matrix algebra to define transformations.
Handling data	Describing a range of variables through different measures of dispersion; calculating standard deviation of a set of data. Interpreting various types of diagram including critical path diagrams and linear programming. Consideration of different shapes of histograms representing distributions with special reference to mean and dispersion, including normal distribution. Understanding and applying conditional probability to an event. Understanding and applying the probability rule for any two events, ie. probability of an event (A or B).

as the understanding and use of bearings and networks. The Data handling section is concerned with probabilities.

These are all concepts in which some girls are experiencing difficulty relative to the boys, and as this study will go on to show, are presented at a time when girls are most susceptible to outside influences. Nor is the National Curriculum unique as a programme of study. Many text books introduce these topics in the third year (Year 9) of secondary education. As this study will show, this may be the point at which many girls lose interest in the subject.

d) A Longitudinal Study

Consideration has been given to the levels of performance of girls/boys at 16 and to concept attainment at the higher and lower levels, but this does not address developmental questions. Are there distinguishable and consistent sex differences in the learning of mathematics at all levels? As children mature, do the types of observed differences change? Is there any one year when the differences are more pronounced? The study of these questions requires a longitudinal approach. Only by examining the same students over time can we begin to determine whether the magnitude of differences shift as pupils progress through school.

In order to study these differences, a longitudinal study was made of girls'/boys' mathematics results over a period of five years from 1982-1987, in a comprehensive school of 900 pupils. The entrance and end of summer term examination marks, plus fifth form mock examination marks were taken as the basis of study.

The school in question was a four-form-entry mixed comprehensive whose catchment area encompassed twenty feeder primary schools. The school was an 'aided' school which meant that parents had chosen to send their children to the school. This resulted in a good caring relationship and good liaison between staff, pupils and parents. The school was mixed ability and followed a traditional mathematics scheme leading to the NEA external examinations at 16+.

The year studied had a population entry of 128 pupils, 74 of whom were girls and 54 boys. On entry to the school, pupils were required to take a basic mathematics test and on the basis of these results were split into four sets. Set 1 was the highest, down to the lowest which was set 4. The groups were further subdivided so that in the third year they were split into five sets. The aim was to accelerate the more able pupils and at the same time help those pupils with greater learning difficulties at the lower end. Often it is the pupils at the lower ability end who demand the greater attention and for this reason the lower sets were kept as small as possible compared to the higher sets.

There was a process of movement between the sets at the end of the autumn term and again after the summer term. This depended on the results and progress of individual pupils. This amounted to two or three pupils being moved up and down across each set on each occasion.

The tests which were set at the end of each year were teacher based. Each teacher followed a set programme of work and then at the end of the year set a test appropriate to the set and to the work covered. These tests were not the same across all the sets in any one year although the teacher had covered the same work. Set 1 would cover the work at a quicker pace and at a greater depth than the other sets. The marks of tests at the end of the year and the number of pupils in each set by gender are given in Table 38.

Table 38. Average percentage scores and number of girls and boys in each school examination from 1982-1987.

Year		Set 1 b	Set 1 g	Set 2 b	Set 2 g	Set 3 b	Set 3 g	Set 4 b	Set 4 g	Set 5 b	Set 5 g
Entrance 1982	No	13	22	15	20	13	22	13	10		
	Av	50.9	50.5	39.7	41.6	30.2	30.2	19.3	16.0		
First 1983	No	13	22	14	21	13	22	13	10		
	Av	80.6	75.5	52.2	55.9	38.9	41.5	41.5	33.2		
Second 1984	No	15	20	10	25	17	18	12	11		
	Av	52.6	42.4	57.4	50.8	37.4	41.6	44.3	35.5		
Third 1985	No	13	14	9	16	6	20	14	11	13	10
	Av	71.9	73.7	62.1	56.2	40	36.9	32.7	33	27.5	24.6
Fourth 1986	No	14	13	11	16	9	15	9	15	13	9
	Av	64	62	62	47.8	38.9	46.3	27.7	23.5	39.5	30
Fifth 1987	No	14	13	11	15	9	17	9	15	11	9
	Av	57.9	54.9	40.9	29.3	45.3	47.1	55.9	51.7	50.1	34.2

The figures cover the whole of the ability range over the five years leading to the external examination. The fluctuation in set numbers is due not only to movement between the sets at Christmas and summer but also to late entrants to the school and mid-school leavers.

The results of the girls' and boys' examinations across each of the sets in years 1982-1987 are illustrated in Figures 24 and 25. Overall, the boys' results are higher than those of the girls. In sets 1 and 4 the results mirror each other closely. In sets 2, 4 and 5 however, there is a greater divergence. This supports the findings

Figure 24. Mathematics examination marks for Sets 1-3, 1982-1987.

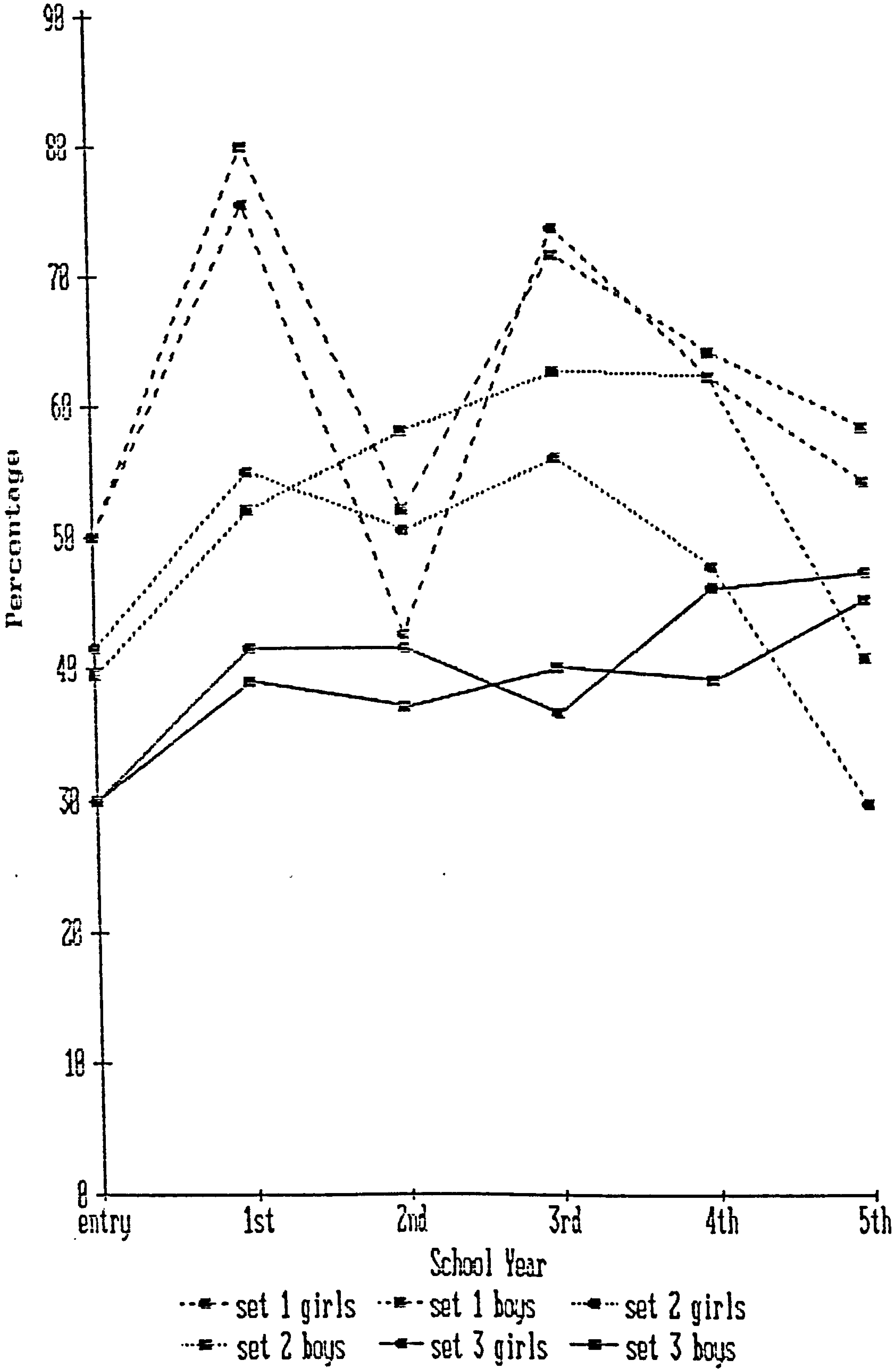
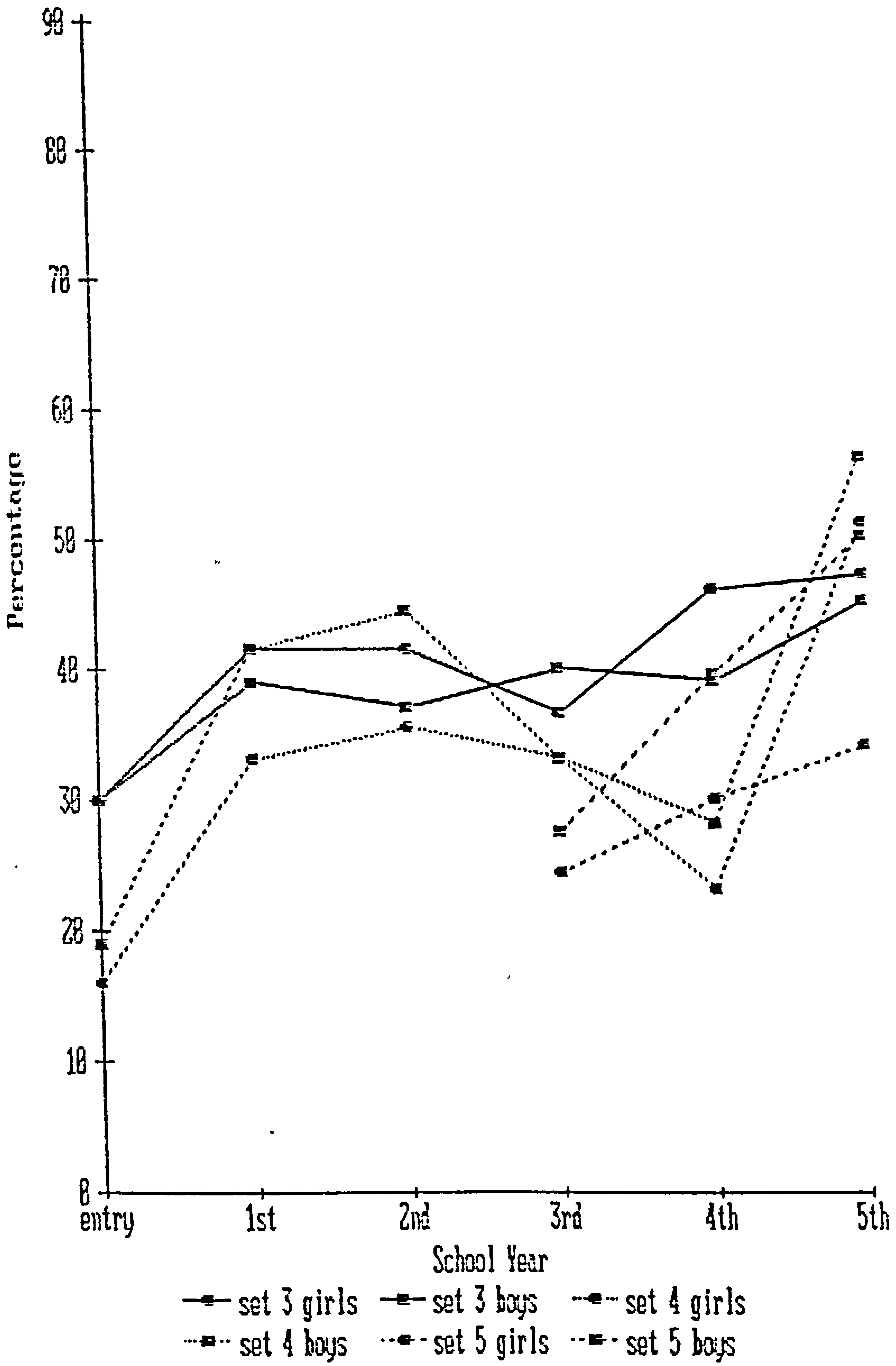


Figure 25. Mathematics examination marks for Sets 3-5, 1982-1987.

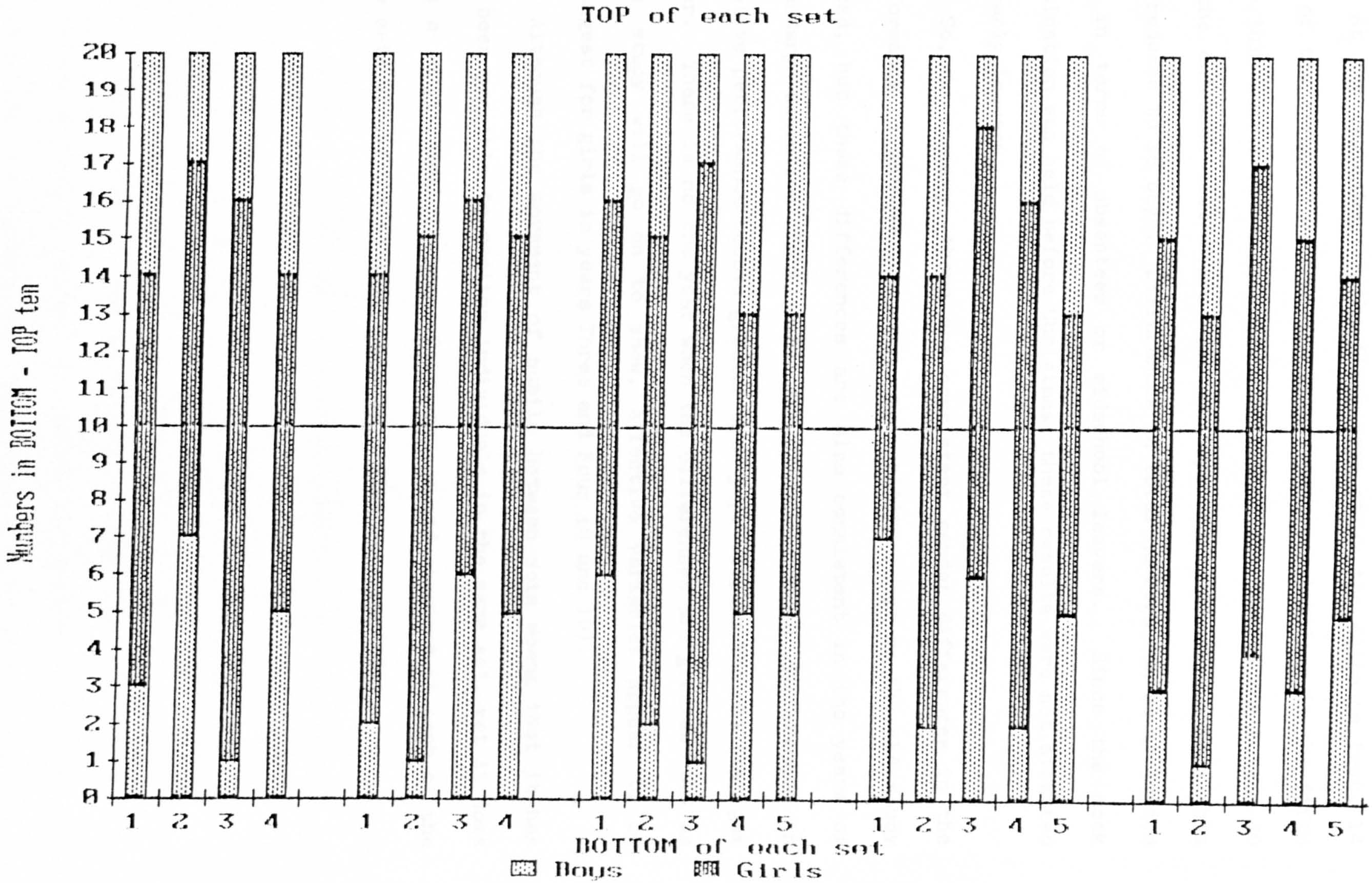


in the previous section that the percentile results for girls and boys are consistent across the ability range. Any inconsistencies in this pattern from the data may well be due to other external influences at work. For example, since the tests were different across the sets, one may have been more 'girl friendly' than another. The teacher of the particular set may have had an influence in the gender differences in terms of particular gender interaction. It may be too, that other influences, eg. class discipline or peer group pressures, had an effect. However, no clear deduction can be made.

The sets of figures are almost identical at the beginning of the first year (entrance). This is because it was on the basis of these figures that the pupils were put into the four sets. The results of set 3 show that the girls at this level were working and achieving very well compared to the boys. At the end of the 1st, 2nd and 4th year the girls were ahead, but even here, by the time the mock examinations were held in the 5th form, the gap had narrowed.

In order to study the high and low achievers in each set, a count was made of the number of boys and the number of girls who came in the top ten and bottom ten in each examination. These figures are illustrated in diagramatic form (see Figure 26). Above the line are the bars representing the numbers of boys and girls in the top ten and below the line are those boys and girls in the bottom ten in each set. There are more girls in the bottom ten and more boys in the top ten on average. The average number of boys in the top ten in each group across the five years is 5.04 and girls 4.96. In the bottom ten, the average is 3.78 boys and 6.21 girls. It must be remembered that there were 74 girls and 54 boys in the study, although there is no reason to suppose this would affect the means.

Figure 26. Numbers of girls and boys in the top ten and bottom ten of each set 1-5 in each of the years 1-5.



At the beginning of the first year after the entrance test, 14 out of the first 40 were boys. By the mock examinations in the fifth form this number had increased to 23 out of 40. Out of the bottom 40 in the entrance test there were 19 boys and by the fifth form this had reduced to 18 boys. Differences in total numbers can be accounted for in terms of absentees or midschool leavers. Since the mock examination was held before Christmas, these results were not affected by early leavers.

So, in summary, there are consistent gender differences in the performance of mathematics across the ability range as the NEA study showed, but these differences are also consistent in the years of secondary education from years 1-5. Also, these differences in relative performance become greater from year One, as the children get older. There is no one year when the differences are greatest but as this study will go on to show, affective variables appear to be strongest for girls in years Three and Four (9 and 10).

Although the movement of pupils between sets means that it has not been possible to monitor individuals in the same set, yet it does give a longitudinal picture of the change and movement in each of the five sets across the ability range in a five year period.

25. Conclusions

To investigate sex differences in any sphere of activity, is sometimes to be accused of being biased, especially if females seem to come out badly in the comparison, and the writer is male. The purpose of this study has been to examine systematically in a logical manner the evidence available. It is in the interests of everyone to pay attention to these differences in performance in mathematics, and to allow an awareness of them to influence decisions about the content of curricula and examinations.

It has been the purpose of this study to pinpoint precise differences in mathematical performance and the results are more startling than might have been expected. It might be expected for example, that since Wood's study in 1973/74 which was one of the first to look at specific differences, equal opportunity policies would have reduced differences in performance between girls and boys. With a well established career guidance system, an open option choice at fourteen, and apparently less stereotyped mathematics texts, it could be expected that differences in scores would be reduced. Most LEAs have been keen to introduce equal opportunity plans in school. However, a careful analysis of the results shows that girls are under-achieving compared to boys to an extent which is similar to that in 1973/74.

Wood (1973) found that of all the items he analysed, the biggest differences in favour of the boys were concerned with spatial visualisation, scaling, the distance-time graph and probability. In this study a similar pattern was found, the greatest differences in favour of the boys being items concerned with bearings (spatial orientation), scaling, speed, distance and time, the use of units and probability. The close resemblance in the two lists highlights a continuing problem.

The implications of these results are important because they show that policies of equal opportunity alone cannot solve the problem. A change of ideas cannot immediately affect the situation. There has to be a change in attitude on the part of pupils, parents and teachers before positive progress is made. There must be changes both to the content of the syllabuses and to the teaching approach. For example, concepts must be presented in a way which are 'girl-friendly'. They must be seen to be relevant in a practical situation by both girls and boys.

On the surface, the results of the study seem to give weight to the argument that there are innate differences between girls and boys in mathematical attainment. After all, differences in spatial questions have remained the same over thirteen years. However, this theory does not provide an alternative to the hypothesis that the differences in performance are a product of environmental and social influences. Many girls do excel in the areas of spatial awareness, proportionality and problem-solving. Added to this is the evidence by Badger (1981) and Bruner (1973) referred to earlier (page 48) which demonstrate that girls do indeed show an improvement in scores on spatial tests after they have been involved in space-related activities, whether these are in the form of explicit training for test results or take a less directed form. The argument for innate differences is consequently very much weakened.

What this study does highlight however, are the particular concept areas where intervention programmes for girls may be of value.

Specific intervention programmes in shape and space questions, scaling and ratio, speed, distance and time and probability, would help to reduce the greatest differences in performance between girls and boys.

Wood's findings related to girls and boys who were in the top 20% of the ability range. This study has produced similar average results in an 80% section of the ability range. However, an examination of the top and bottom 10% of the sample revealed certain differences in performance in mathematical topics. In fact, in the ten questions which produced the greatest differences there were no common questions between the top 10% and the bottom 10%. The top 10% had differences similar to those already described. However, the bottom 10% had differences in topics which may be regarded as foundation concepts in the mathematical hierarchy, eg. percentages, decimals and fractions. Again, this is significant for intervention programmes with the lower ability range and has not been highlighted in previous studies. In other words, intervention programmes are of value but must be related to the ability of the individual and to their level of conceptual development.

This does not mean that there are no differences in higher order concepts for these children. What it does suggest is that they may not have been sufficiently successful in these topics to give adequate statistical differences. Concepts such as units, percentages, fractions and substitution form the foundation upon which more complex work is built and underline the hierarchical nature of mathematics. This suggests that there are girls and boys who may not attain high levels of expertise in the features of mathematical ability outlined by Krutetskii (1976) and by Suydam and Weaver (1977) (pages 34 and 36). It may be that there are limits of concept development, however sophisticated the intervention programme. It also suggests that it is possible for a person to be more competent in one skill compared to another. For example, it may be possible for a candidate to have a high level of logical reasoning but be poor in the ability to generalise from mathematical results. It would seem that intervention programmes have to be carefully tailored to a person's ability if they are to be successful.

One feature of particular interest which has not been clearly documented before, is the remarkable consistency in performance difference between girls and boys across the ability range. At each tenth percentile across the ability range, the performance of the boys on test scores exceeds that of the girls. This was found to be true across each of the three papers. It is significant that it is not just the better boys performing better than the best girls. The pattern is found to be remarkably even through the ability range. DES statistics as shown earlier (pages 14 and 15) confirm that a greater number of boys get good grades compared to the girls (see Tables 5, 6), but do not give clear information regarding the middle and lower bands. The implication of this finding is that schemes designed to increase the performance of girls compared to boys must be targetted across the whole ability range and not confined to the top 20%.

Having established the top 10% of girls and the top 10% of boys, it is interesting to note that there was a relatively even match of the numbers in the top ten percentile in each paper. This supports the view of Russell (1984) that the good girls are as good as the boys in many of the topics under discussion. It must however, be remembered that the 'cut off' mark to obtain the top 10% of girls was lower than that for the boys. The important fact is that the best girls are as consistent as the best boys across the three papers. This supports the findings that the best girls are performing better in mathematics tests compared with the majority of boys through the ability range. What is the argument for innate ability here?

It seems that the differences in attainment are a product of environmental and social influences. This supports the views of Bishop (1973) and Mitchelmore (1980) who concluded that the best approach is to base all learning both in arithmetic and geometry on manipulative materials. They found that children who have used such materials extensively tend to perform better in mathematical tests. However, it seems illogical to provide experiential learning situations for girls and boys if the material used is not interesting and relevant. This raises the important issue of motivation and the need for material and content to be 'gender-friendly'.

The problem then seems to be what girls regard as being interesting and friendly and when these perceptions are established. The APU (1984) survey looked at some of the interests and activities engaged in by children. It was found that boys were more likely to be interested in making models, playing snooker, fishing and watching birds; where the girls were more interested in sowing seeds, looking after small animals, cooking, knitting and sewing. It was found that in general, boys' interests rest more in the physical sciences whereas girls' interests lie in the biological sciences. Boys are more interested in space exploration, satellite communication, robotics and nuclear power. Girls however, are more interested in test-tube babies, heart transplants, and cancer research. Such general differences seem to persist and polarise as children get older. Many of these ideas are carried through to career aspirations and follow traditional patterns.

It may be that the most important influences in terms of experience for girls and boys take place early in life. Girls are more likely to receive dolls, a pram, a dolls' house and miniature kitchen equipment, while boys get toy cars, lego, working models, guns and computer games. The outcome is that each sex learns appropriate roles, including acceptable emotions: girls - caring, boys - aggression. Girls' toys can rarely be taken apart nor are they designed to interact with other materials. The early experience of boys therefore, gives a much more appropriate foundation for the physical sciences. Children are reared in an ever changing environment and the internal processes of development within growing children are varied.

Obviously mathematics syllabuses cannot be tailor-made for the girls alone. They must look towards the concepts which form the foundation of future studies and items which are the basis of being able to deal with future life. However, the context in which a problem is set does affect the relative performance. As Eddowes and Sturgeon (1980) found, girls performed better on a question referring to the area of dress material than to an equivalent one referring to the area of metal needed for a template. Clearly, care is needed in setting appropriate questions.

However, the new Schools Examinations and Assessment Council (SEAC, 1990) suggests that mathematics problems set in context are usually harder for girls than for boys, even at age 11. This effect does not seem to be always due to the differential familiarity of the context of the two genders. Simply making sure that the contexts used do not reinforce stereotyping, may not be sufficient to equalise success rates. As the profiles of performance of the genders across mathematical topics are established by 11, the reasons for boys doing better than girls in measures and contextualised questions must generally lie in their experiences before and/or during the time they are of primary school age.

It is important to be aware of this for two reasons. One is that differences between girls and boys in mathematics in primary school are often not spotted from the results of tests they take which only give overall scores. This supports the findings of Eddowes and Sturgeon (1981). The second reason is that the National Curriculum following Cockcroft, places considerable emphasis on mathematics in context and this is likely to favour boys rather than girls on current performance.

There is a Number/Algebra Target in the National Curriculum which is concerned with patterns and generalisations. Since girls are slightly (though not significantly) ahead in 'problems and patterns' tests (Schools Examinations and Assessment Council, 1990), additional emphasis on this work should be an advantage to the girls. (See Attainment Targets 1 and 9.)

One of the difficulties for the girls found from the study of NEA scripts is when questions are enmeshed in a problem-solving exercise. This supports the findings of Hyde, Fennema and Lamon (1990), who found that girls in secondary schools perform less well than boys on mathematical problem-solving tasks. It seems that girls are more comfortable with well established methods. They appear to have a greater tendency to show caution, to avoid being wrong and to use processes with which they feel confident and secure. Boys on the other hand, seem to be far more perspicacious. They appear to demonstrate greater flair in restructuring problems and to use the relevant cues in a novel situation.

There does seem to be a greater tendency for girls to attempt to solve a problem in a sequential manner, having broken the question down into several stages. This may not be the most efficient approach. Wood also found that none of the items on which girls performed relatively better than the boys required what could be termed problem-solving behaviour. Boys on the other hand, generally looked at the problem as a whole, as an entity, and then proceeded to solve it accordingly.

This supports the view held by Pask (1976) that girls approach questions sequentially as opposed to the holistic approach by many boys. It also helps to explain Wood's finding that girls often 'snatch' at solutions or give implausible results. This may again relate back to the relevance issue and the extent to which girls can identify with the 'real-life' context. As a result, they may be more prone to present answers which, with a little thought, are obviously wrong.

It is evident from the NEA scrips that girls are competent in basic computational skills. This is supported by other studies, notably those conducted by the Assessment of Performance Unit (1978) and the meta-analysis of Hyde, Fennema and Lamon (1990). Indeed, these studies found that girls are superior in computation. Yet, computation is a prime example of a topic whose relative importance declines as pupils get older and especially as their mathematics becomes more advanced. The availability of the calculator and the computer mean that much computational work is no longer as important through the years of secondary education.

As pupils get older, problem-solving and the understanding of mathematical concepts such as spatial-visuo and proportionality items, become increasingly important. As Orton (1987) suggests, proportionality skills form the structure of hierarchical concepts upon which much of mathematics is based. Burton (1986) says that 'somehow', by the age of ten, more boys than girls have got themselves into a position where they are able to cope with these aspects of mathematics' (Girls into Maths Can Go, P.34).

The implications of these findings are important because they mean that girls need to be encouraged not just to produce mountains of good, neat exercises, but rather to understand and to use mathematical concepts and principles. This should include practical work, problem-solving, discussion and investigation. This clearly places a heavy responsibility on the teacher to provide these appropriate learning environments and to encourage and support. Teachers need to be made aware of these findings and to take the necessary action.

The development of girls' and boys' mathematical attainment from age eleven can be seen from the longitudinal study described earlier. This shows that generally, there is a consistent difference in mathematics performance in the years of secondary education from years 1 to 5. Also, these differences in relative achievement become greater from year one, as the children get older. Again, this supports the view held by Burton (1986) and Hart (1981), that boys are better placed from year one to build the new skills which are needed as the mathematics becomes more advanced. Hart for example, suggests that a true understanding of proportionality develops late and for intellectually weak pupils, it is beyond their capabilities at fourteen.

There seems to be no one year when the differences in performance between girls and boys are greatest, but as this study will attempt to show, girls are more prone to strong affective variables in years three and four (9 and 10). This is why, as Walden and Walkerdine (1985) suggest, approaches need to be made which examine girls' own fears and feelings.

Attempts to introduce better role models for girls, to change stereotyped images in books and presentation of subject matter and to allow greater curriculum choice and provision, are good in themselves. Nevertheless, a clear understanding of both positive and negative feelings concerning the learning of mathematics is important. The motivation and attitudes of girls and boys towards the subject may well affect performance and also affect the possibility of future study in mathematics.

26. Affective Variables

There is no convincing or conclusive evidence that differences in performance can be adequately accounted for by innate or genetic factors. Yet, there are certain specific areas that need to be highlighted where girls seem to be in difficulty. These concern the attitudes and affective beliefs held by girls compared with the boys, their attitude to themselves and to the subject, their interaction with, and expectations of, their teachers, parents and peers.

There are internal as well as external pressures. In particular, consideration needs to be given to the image of mathematics in schools, its perceived usefulness, the view of mathematics as a masculine subject, subject choices and career advice. These together with teaching styles, methods of classroom interaction, the examination syllabuses and modes of assessment form important variables. They can all contribute to feelings of anxiety, instability, confusion, lack of confidence, boredom and submission.

The cognitive and affective components are enmeshed, and it is not always possible to separate them. They are developed over a number of years in a complex social structure, involving home, community and school.

In seeking to understand why inequality exists it is necessary to study cognitive and affective components affecting the acquisition of mathematical skills and knowledge in the social environment where they are developed. It is not possible to study the totality of causative behaviour but it has been possible and indeed profitable to select variables which exert a major influence and to look at the developments, interrelationships and effects of these variables on the learning of mathematics.

It is the aim of this study to look at the attitudes of girls and boys to their studies in mathematics. A comparison of the attitudes of girls and boys is used to highlight particular areas of concern. For example, if girls have a low esteem of their mathematical performance, then this may be an inhibiting factor in choosing to study mathematics to a higher level. Also, if more girls are expressing a disinterest in the subject, then this may be a factor which is affecting their performance at sixteen.

This study also examines some of the attitudes of the mathematics teachers. How aware are the teaching staff of possible difficulties experienced by the girls? Since the teacher spends much of his/her time in the learning process with the pupils, then the teacher's attitudes and beliefs are important. The teacher can exert an effective influence on the pupil and feelings and attitudes can be very quickly imbibed by the pupil.

So, are there differences in attitudes between girls and boys towards mathematics? Do boys express more confidence than girls in their own ability? Do they have a greater expectation of success? How do these attitudes vary from year to year, set to set, and from school to school? The thoughts and feelings of pupils towards the activities they engage in at school are important features of their learning. The hypothesis is that there are indeed differences in attitudes of girls and boys towards mathematics which in turn affect their performance. Data concerning factors which may influence a positive approach to the subject have important implications for remedial action.

27. Attitudinal Evidence

a) Data

Ten schools co-operated in the study. One was a grammar school and the other nine were 11-18 comprehensive schools. Each school was asked to supervise the completion of twenty questionnaires from each year group (Year One to Year Five). This was made up of ten girls and ten boys giving a total of one hundred from each of the ten schools.

The pupils were selected at random (on an alphabetical basis), across the full ability range and were asked to indicate sex, year group, age, set and school. Otherwise, the questionnaires were anonymous and pupils were invited to express their feelings freely. A copy of the questionnaire is given in Figure 27.

To complete the questionnaire, pupils were asked to tick the appropriate box of their choice for each question in one of the four columns. These columns represented those who strongly disagreed, disagreed, agreed or strongly agreed. A middle fifth column was purposely omitted to ensure that pupils did make a choice. Those who found that none of the categories applied, in the main, either ticked across two boxes or left the question out.

In the event, a sample size of 938 was obtained with 479 girls and 459 boys. The numbers were not equal largely because one of the participating centres had only girls in their upper school. This school was in the process of changing from a single-sex to a mixed school. All the other participating schools were mixed.

In addition, each member of the respective mathematics departments took part in a staff survey answering questions specifically

Figure 27. MATHEMATICS QUESTIONNAIRE   Boy ... ☐   Girl ... ☐

Your Age ....
Year Group in School ....
Maths Set ....

Put a tick in one of the columns for each question

	Strongly Disagree →			→ Strongly Agree
1. I am always keen to start doing maths	......	......	......	......
2. Maths is interesting	......	......	......	......
3. I used to like maths, but not now	......	......	......	......
4. Maths won't be important to me in future	......	......	......	......
5. Maths makes me feel confused	......	......	......	......
6. I have always liked maths	......	......	......	......
7. Maths is more important for boys than girls	......	......	......	......
8. Boys are better at maths than girls	......	......	......	......
9. Boys ask more questions and get on faster	......	......	......	......
10. I get lost if I miss work in Maths	......	......	......	......
11. I like maths because we are always doing something interesting	......	......	......	......
12. I never expect to do well in Maths	......	......	......	......
13. Girls usually choose a job which needs Maths	......	......	......	......
14. I like working out problems in Maths	......	......	......	......
15. Maths is my best subject	......	......	......	......
16. My mother liked Maths at school	......	......	......	......
17. My father liked Maths at school	......	......	......	......
18. I panic in Maths tests	......	......	......	......
19. Jobs needing Maths are usually for boys	......	......	......	......
20. I am no good at Maths	......	......	......	......
21. I like Maths because I can do it	......	......	......	......
22. A lot of the Maths I'm taught I don't need	......	......	......	......
23. Without Maths our lives would be harder	......	......	......	......
24. I am disappointed when I miss Maths lessons	......	......	......	......
25. I like Maths because I like working with numbers	......	......	......	......
26. Whether or not I like Maths depends on the teacher	......	......	......	......

related to the gender aspect of mathematics teaching. A copy of the questionnaire is given in Figure 28.

There was a sense of reluctance on the part of some members of staff, male and female, in completing the questionnaire. The feeling was that in their capacity as professional teachers there could be no possible reason to investigate equal gender opportunities. 'I treat girls and boys absolutely equally' was one response. They were seemingly above reproach and could not identify a problem. This, in itself, speaks volumes about raising awareness at the very root of the situation.

Also, there was something of a back-lash against the feminist movement in supposedly trying to push girls forward unnecessarily. 'Not another feminist ploy' was another response. There is a need here for clear and careful presentation of the facts.

Forty staff completed the questionnaire, of whom thirty were male and ten female. This was out of a possible fifty full-time mathematics teachers, giving a response rate of 80%.

In addition, each school was asked to supply details of their mathematics results in external examinations at the end of the fifth form for the years 1984, 1985, 1986 and 1987. The purpose of this was to see if the pattern of results for girls and boys was the same as the national pattern. Had they been different, this would have shed a different light on the results of the attitude survey. In addition, it was interesting to see if there was a consistency in the results over the four years. That is, are the differences narrowing or not?

Figure 28. STAFF SURVEY QUESTIONNAIRE

	Strongly Disagree →			Strongly Agree
1. There is a difference in mathematical ability between boys and girls				
2. Boys are better at maths than girls				
3. Girls are less enthusiastic in maths lessons				
4. Girls see less relevance in maths compared with boys				
5. Boys are more dominant in maths lessons				
6. I give more of my attention to the boys				
7. I make a conscious effort to motivate girls in the maths lessons, ie. positive discrimination				
8. The maths department is trying to redress the imbalance of performance between boys and girls				
9. Girls are more willing to admit failure				
10. Boys have a greater expectation of success				
11. Boys find maths more interesting				
12. Girls often under-achieve in maths tests/lessons				
13. Boys have more flair in problem-solving				
14. Girls present their work more neatly than boys				
15. Girls are more methodical than boys				
16. Boys are more likely to experiment/ take risks				
17. Girls do not see maths as necessary for most of their future jobs				
18. Girls reach their limit of mathematical understanding sooner than boys				
19. A deterioration in girls' mathematical understanding is more pronounced in the upper school than that of boys				
20. Boys tend to have a more logical and clear view of maths				
21. Girls dislike maths because their mothers also disliked it				
22. Boys are better at solving spatial, geometrical questions				
23. Maths text-books and work sheets are male orientated				
24. Maths is a male stereotyped subject				
25. Boys have more ambition in life therefore they work harder				

The results are shown in Table 39. They give the relative proportion for every one hundred successful candidates. That is, they show the percentage of girls and boys gaining grades A, B and C (GCE) or grade 1 (CSE).

Table 39. The Number of successful girls and boys in every 100 in the surveyed schools

	1984	1985	1986	1987
Girls	42	43	42	44
Boys	58	57	58	56

Again, it was found that the boys were more successful overall. This is consistent with the results of the NEA 16+ mathematics papers and also with the DES statistics. Again, it must be remembered that there is always a considerable overlap in the distribution of scores and the variability of boys' scores is usually greater.

b) Pupil Questionnaire

Table 40 shows the percentage of girls against the percentage of boys giving responses for each written statement. The literature supports the conclusion that there are sex related differences in the confidence-anxiety dimension. This is borne out in this study. Response to the statement, 'maths makes me feel confused' (5) revealed that 36.9% (176) of girls agreed or strongly agreed compared with 24.3% (111) of boys. 40.1% (183) of boys strongly disagreed with this same statement compared to 30.5% (146) of girls. 'I get lost if I miss work in maths' (10) also received positive agreement from the girls (51.6%) (245), compared with the boys (46.5%) (211).

Table 40. The percentage of girls and boys giving a response to each of the statements in the mathematics questionnaire

	Strongly disagree		Disagree		Agree		Strongly agree	
	g	b	g	b	g	b	g	b
1. I am always keen to start doing Maths	9.0	9.7	26.6	25.8	45.6	39.7	18.8	24.7
2. Maths is interesting	7.7	9.4	19.4	20.7	44.7	38.6	28.2	31.4
3. I used to like Maths, but not now	43.6	48.8	27.9	26.6	18.0	13.0	10.5	11.6
4. Maths won't be important to me in the future	74.2	77.5	8.8	6.3	5.4	4.4	11.7	11.8
5. Maths makes me feel confused	30.5	40.1	32.6	35.5	24.3	13.8	12.6	10.5
6. I have always liked Maths	18.4	18.1	26.2	21.6	27.9	32.6	27.5	27.8
7. Maths is more important for boys than girls	87.1	64.0	6.1	17.0	2.1	8.1	4.7	10.9
8. Boys are better at Maths than girls	88.1	58.6	7.8	19.2	1.5	9.2	2.7	13.1
9. Boys ask more questions and get on faster	74.3	44.8	13.4	27.8	6.1	12.0	6.3	15.5
10. I get lost if I miss work in Maths	17.5	25.1	30.9	28.4	25.7	24.7	25.9	21.8
11. I like Maths because we are always doing something interesting	14.3	17.7	32.8	30.2	27.0	27.8	25.8	24.3
12. I never expect to do well in Maths	24.6	35.1	35.9	32.0	24.8	19.3	14.7	13.6
13. Girls usually choose a job which needs Maths	35.8	33.8	32.4	36.7	18.4	18.0	13.4	11.4
14. I like working out problems in Maths	20.2	13.7	25.1	17.2	28.8	32.9	25.9	36.2
15. Maths is my best subject	37.5	36.5	27.9	26.9	20.8	22.8	13.8	13.8
16. My mother liked Maths at school	32.1	30.1	23.3	24.0	22.6	25.6	22.0	20.3
17. My father liked Maths at school	21.4	23.8	22.1	19.9	23.6	24.7	32.8	31.6
18. I panic in Maths tests	19.9	33.8	17.4	22.0	24.1	22.4	38.6	16.8
19. Jobs needing Maths are usually for boys	66.3	42.0	21.4	29.8	4.2	12.0	8.1	16.2
20. I am no good at Maths	31.0	49.3	37.1	27.5	20.3	11.8	11.5	11.4
21. I like Maths because I can do it	21.2	15.8	28.8	24.8	32.1	31.1	17.9	28.3
22. A lot of the Maths I'm taught I don't need	41.4	40.0	27.0	27.2	17.2	17.2	14.4	15.7
23. Without Maths our lives would be harder	7.5	10.1	12.1	7.0	22.1	16.4	58.3	66.5
24. I am disappointed when I miss Maths lessons	29.2	31.9	33.4	31.1	20.6	22.5	16.8	14.4
25. I like Maths because I like working with numbers	18.3	19.7	30.2	29.9	31.5	31.4	20.0	19.0
26. Whether or not I like Maths depends on the teacher	29.6	26.5	15.2	14.3	24.4	22.4	30.8	36.8

The relationship of anxiety and mathematics learning has been explored by a variety of methodologies measuring debilitative and facilitative anxiety. It is reasonable to believe that lesser confidence or greater anxiety on the part of girls is an important variable which may help to explain sex related differences in mathematical attainment. This is supported by Fennema and Koehler (1982). There is a similar, positive, relationship between self-esteem and achievement.

38.6% (184) of girls compared to 16.8% (77) of boys strongly agreed that they panicked in mathematics tests. In response to the statement 'I am not good at maths' (20), 31% (148 of girls strongly disagreed compared with 49.3% (226) of boys. Both of these statements (18 and 20) are mathematically significant ($\chi^2=66.73$, d.f.=3, $p < 0.0001$), ($\chi^2=36.81$, d.f.=3, $p < 0.0001$). Again, in response to the statement 'I never expect to do well in maths' (12), there were more boys (35.1%) (160), compared to girls (24.6%) (117), strongly disagreeing. Either the girls were showing more anxiety towards the subject or the boys were less willing to admit their anxiety.

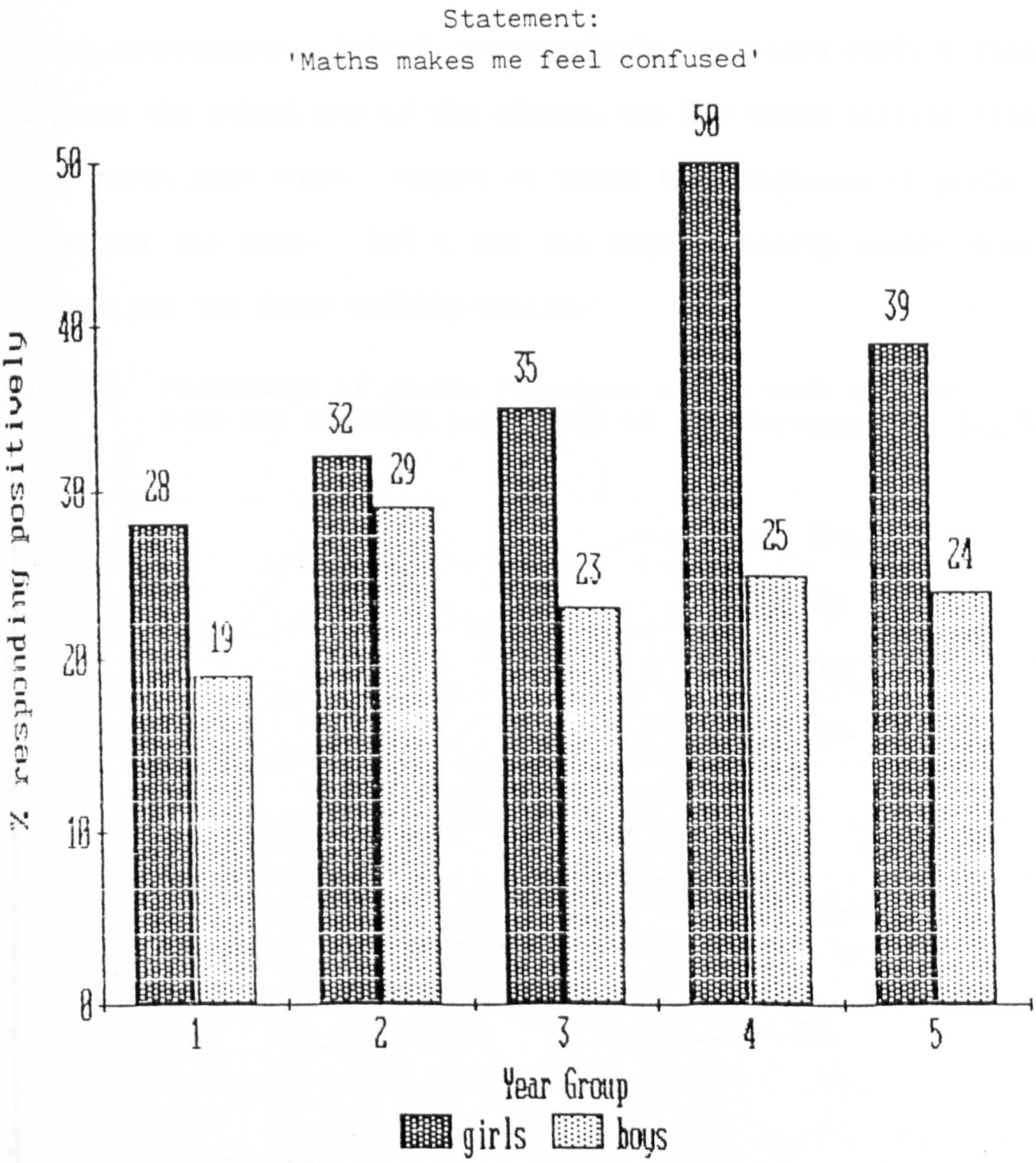
Buxton (1981) suggests a variety of reactions of individuals to mathematical problems. He argues that, whatever a person's reasoning capacity, its effectiveness is strongly dependent on the extent to which the emotions aid or impede the particular task on which it is engaged. This can also have long term effects on the person's willingness to continue studying the subject. For example, in response to the statement 'I used to like Maths, but not now' (3), 28.5% (136) of the girls compared to 24.6% (112) of the boys either agreed or strongly agreed.

The boys are clearly expressing a greater confidence in their own ability. For example, in response to the statement 'I like Maths because I can do it' (21), 50% (238) of girls compared to 59.4% (271) of boys either agreed or strongly agreed. Also, the boys are more prepared to attribute their liking of maths to the teacher (26). 30.8% (91) of girls compared to 36.8% (168) of boys strongly agreed with this statement.

What is important is to try to establish the attitudes of girls and boys across the five years. At what stage for example, do girls display greater anxiety of a debilitating type? Figure 29 shows this pattern in histogram form for girls and boys in years 1 to 5 when responding to the statement 'Maths makes me feel confused' (5). Here, there can be seen a marked difference in the responses of the 3rd, 4th and 5th form girls compared with the boys.

Some girls may be more prepared to admit their inadequacies concerning intellectual, problem-solving activities and consequently under-estimate their ability to solve mathematical problems. These results are consistent with other studies showing sex differences in self confidence - for example, that of Schildkamp-Kündiger (1980). However, the degree to which this happens is unclear. Research suggests that even high achieving girls still have not got the same confidence in their own ability compared to the boys. On the other hand, it may also be argued that over-confidence on behalf of boys could be equally detrimental to performance.

Figure 29. Percentage of gender responses in each year group who answered positively to the statement 'Maths makes me feel confused' (5)

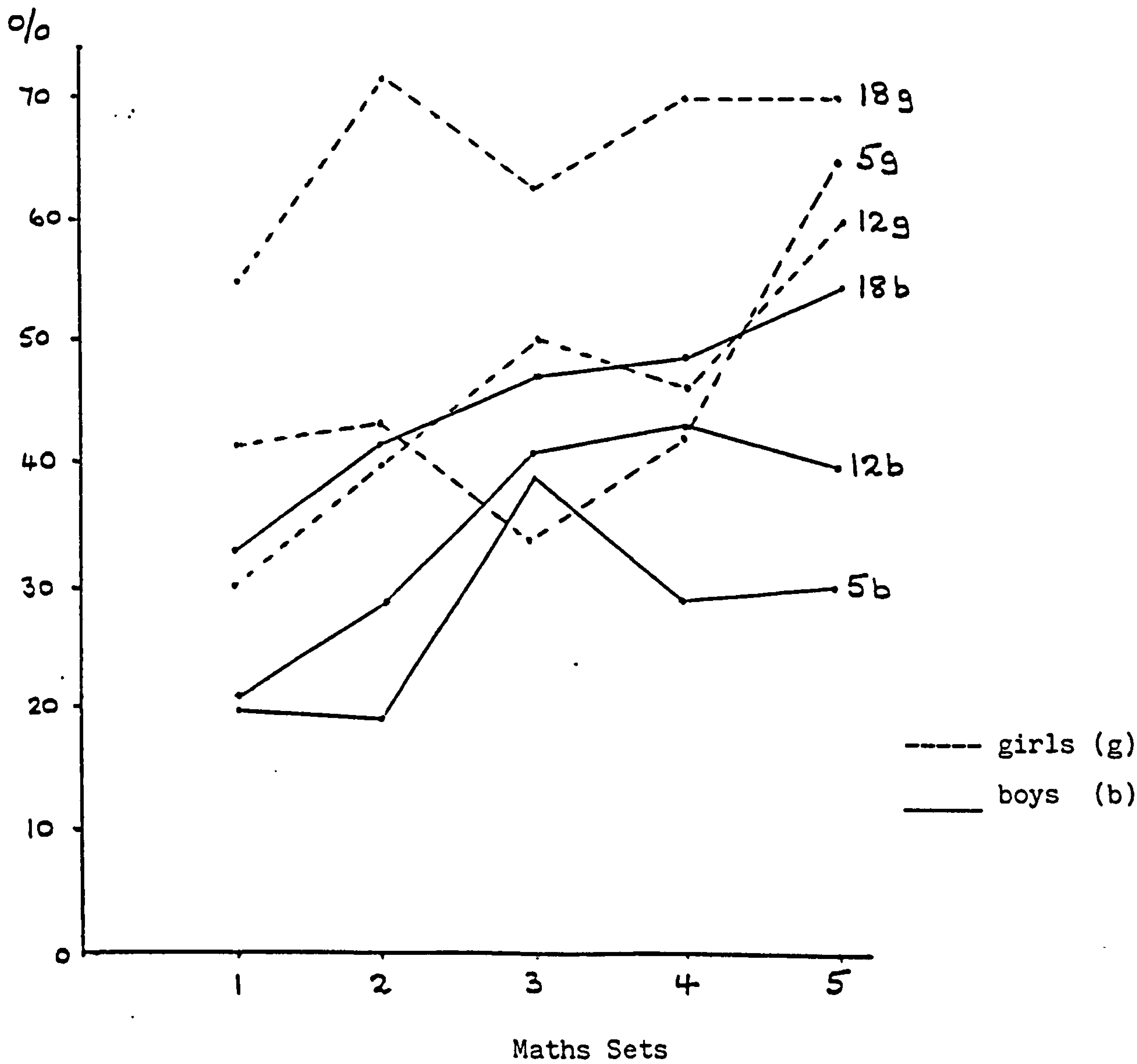


Year	1		2		3		4		5	
Sex	g	b	g	b	g	b	g	b	g	b
Strongly Disagree	42	48	46	44	19	35	23	31	25	42
Disagree	30	33	23	27	47	42	27	44	36	34
Agree	20	9	17	14	26	14	30	13	28	18
Strongly Agree	8	10	15	15	9	9	20	12	11	6

GIRLS: $\chi^2 = 39.83$, d.f. = 12, p = 0.0001
BOYS: $\chi^2 = 16.27$, d.f. = 12, p = 0.1794

Not all the schools in the survey had a setting system of teaching mathematics. Indeed, three schools had mixed ability classes throughout the school and of the others, two had mixed ability classes in the first year only. Figure 30 shows the responses of girls and boys across the sets. Set 1 are the higher ability pupils down to Set 5 who are the lower ability pupils.

Figure 30. Percentage of gender responses across each of five sets who answered positively to the statements 5, 12, 18.



Statement 5 : 'Maths makes me feel confused'

12 : 'I never expect to do well in Maths'

18 : 'I panic in Maths tests'

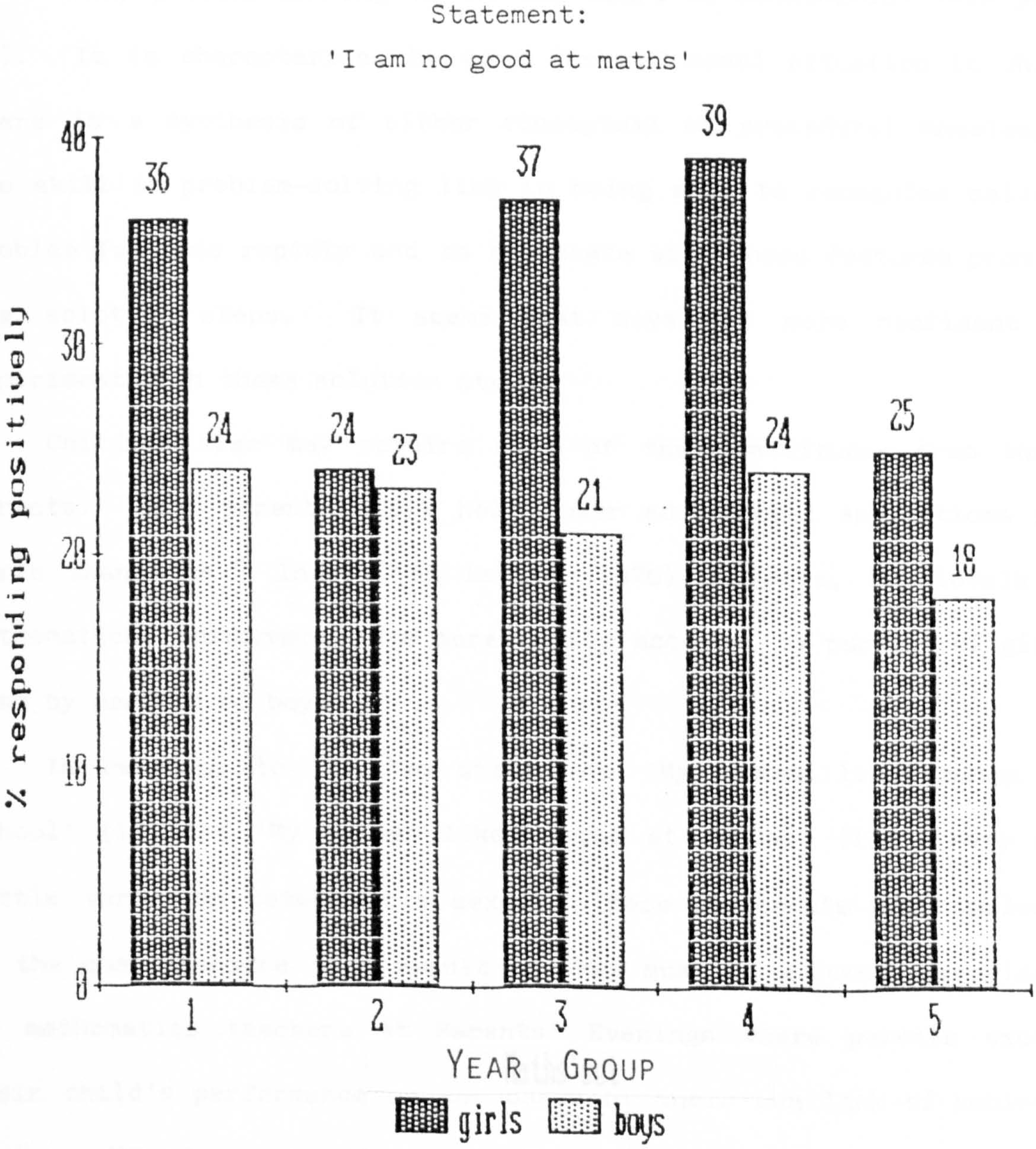
Girls seem to be consistent across the sets in their lack of confidence in their approach to mathematics. There is a clear difference between Set 1 pupils, girls and boys but generally this is maintained across the weaker sets. This is a remarkably similar picture to that obtained in the survey of performance in the NEA survey. There is a good correlation between confidence and performance within each ability level.

It is apparent, that girls in the lower sets are more ready to admit to being 'no good at maths' (20). The boys on the other hand, seem to maintain their confidence. This represents a significant difference in the perceptions of their respective abilities and may well support the sex stereotype syndrome.

Although there is a correlation between confidence and performance within each ability level, this does not seem to extend across the whole ability range. In other words, the best girls and boys in the whole sample are not necessarily the most confident. Perhaps the brighter pupils are more aware of the difficulties involved in the work. Even when females succeed in mathematics, they may attribute their success to factors other than their own ability, such as luck, much more than do males (Wolleat, 1980). So, it seems that success in mathematics does not necessarily generate positive attitudes towards it in some girls.

Overall, looking at the figures generally, there was little difference expressed in interest in mathematics between the sexes (2 and 11), but in response to the statement 'I like working out problems in maths' (14), the differences become greater. 69.1% (313) of boys answered positively compared to 54.7% (262) of girls and the

Figure 31. Percentage of gender responses in each year group who answered positively to the statement 'I am no good at maths' (20)



Sets	1		2		3		4		5	
Sex	g	b	g	b	g	b	g	b	g	b
Strongly Disagree	41	55	42	50	30	52	10	30	33	50
Disagree	23	21	35	26	33	27	50	26	42	33
Agree	22	9	12	10	24	9	28	15	17	13
Strongly Agree	14	15	12	13	13	12	11	9	8	5

GIRLS: $\chi^2 = 39.45$, d.f. = 12, p = 0.0001
BOYS: $\chi^2 = 16.24$, d.f. = 12, p = 0.1803

results are again mathematically significant (χ^2=21.88, d.f.=3, p=0.0001). As we have seen, problem-solving is at the heart of mathematics (see page 55). It is characterised by some form of novel situation in which there is a synthesis of either conceptual or procedural knowledge. The skill in problem-solving lies in being able to recognise salient problem features rapidly and to associate with those features promising solution steps. It seems that boys are more confident to experiment with these solution steps.

Children also may acquire some of their attitudes from their parents. Some parents still hold lower educational aspirations for girls than boys. Indeed, as Levine (1976) suggests, low levels of mathematical performance are more easily accepted by parents of girls than by parents of boys.

In response to the two statements 'My mother liked maths at school' (16) and 'My father liked maths at school' (17), there was little variation between the sexes. These statements were included in the questionnaire as a result of what must be a common experience of mathematics teachers at Parents' Evenings where parents excuse their child's performance on the basis of their own lack of achievement. Clearly, not all pupils knew what their parents liked or disliked at school but of those who responded, 45.3% (362) said that their mothers liked mathematics at school compared to 54.7% (438) who said that their fathers liked mathematics at school. This shows a greater affinity for a male-parent predominance in mathematics.

Teachers too, may unconsciously play a part in the sex role stereotyping which reinforces children's attitudes to mathematics. In response to the statement 'Whether or not I like maths depends on the teacher' (26), 52% (488) of the total sample answered positively.

There appear to be differing views as to the effect of the sex of the teacher on the mathematical performance of girls and boys. Most mathematics teachers in secondary schools are men; of the schools in the survey for this study, 70% (30) of the mathematics teachers were male. While some researches report no sex-of-teacher effect, others, eg. Good, Sykes and Brophy (1973) report that male students do best in quantitative scores, when taught by male teachers.

There was little difference between the sexes in response to the statement 'Maths won't be important to me in the future' (4) and it was encouraging to find that three-quarters of all girls and boys strongly disagreed with this statement. There was a similar unanimity in response to the statement 'Girls usually choose a job which needs maths' (13). Unfortunately this time the response was negative. Only 31.8% (152) of girls and 29.8% (134) of boys agreed or strongly agreed with this statement. Care is clearly needed to educate both girls and boys in the importance of mathematics as a basic qualification. The absence of a mathematics qualification could exclude girls and boys from fields of employment for which they might well be capable. In this sense it may act as a 'critical filter' as discussed earlier (page 1).

Most girls and boys acknowledged that without mathematics their lives would be harder (23), although there was a stronger agreement from the boys, 66.5% (304) compared to 58.3% (280).

There was a larger variation in the response to the statement 'Jobs needing maths are usually for boys' (19). 28.2% (129) boys answered positively compared with 12.3% (59) of girls. The responses to this statement were mathematically significant ($\chi^2 = 62.72$, d.f. $= 3$, $p < 0.0001$).

66.3% (319) of girls, against 42% (192) of boys, strongly disagreed with the statement.

This may suggest that girls are becoming increasingly aware of the importance of mathematics in their future careers and are contradicting the notion that jobs and mathematics are just for boys. If so, this is an encouraging step forward.

On the other hand, the positive response by the boys may either mean they are perpetuating the sex role stereotyping or that they do see more clearly than the girls that mathematics will be useful in their future lives. Some studies suggest that girls are less oriented towards careers outside the home than are boys, and that the usefulness of mathematics in the traditional women's careers in business, nursing, teaching and the social services is less plain than is its usefulness in traditional men's careers (Fox, 1980).

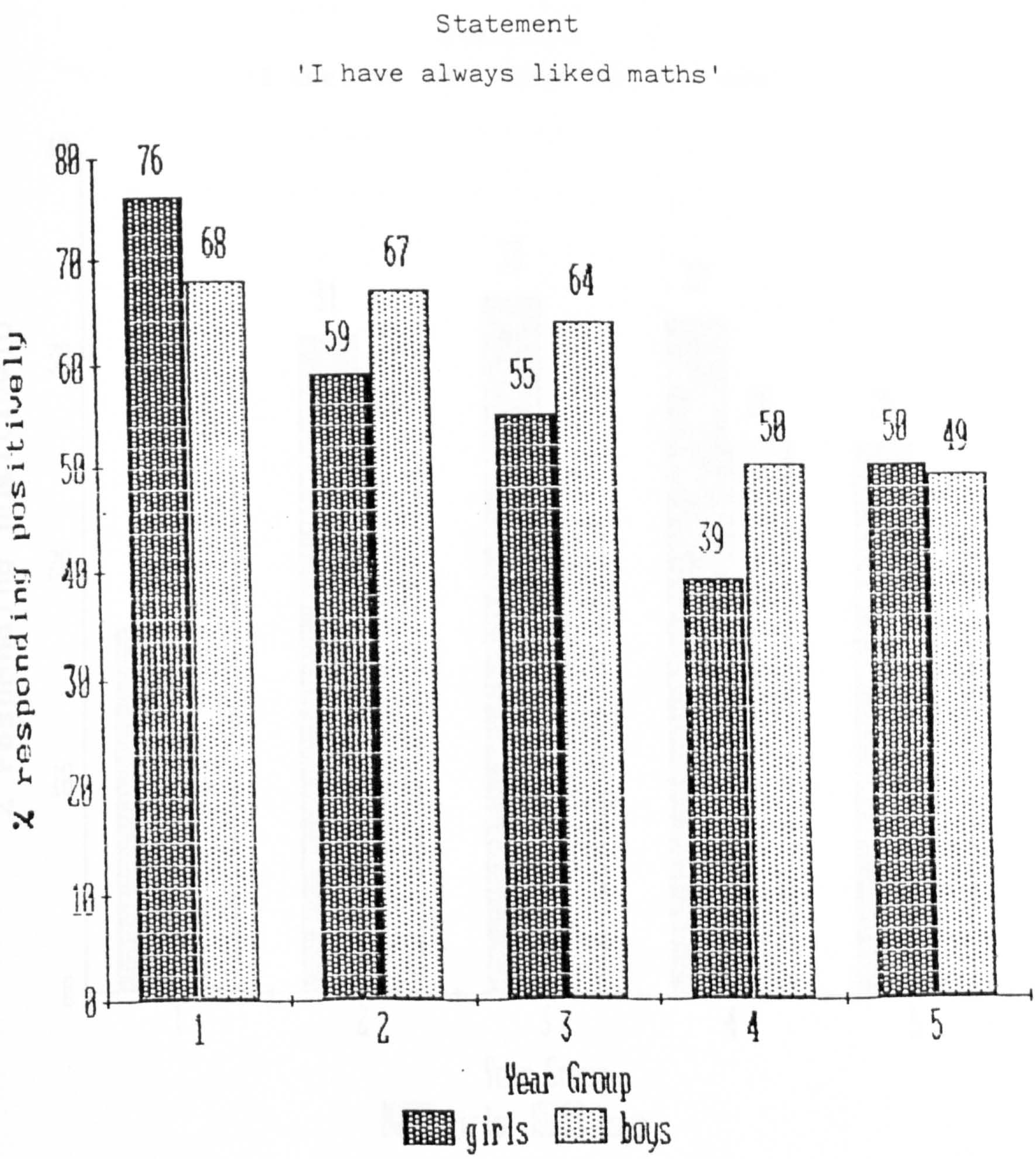
In response to the statement 'Boys ask more questions and get on faster' (9) there was a strong denial particularly by the girls (74.3%) (356), compared to the boys (44.8%) (205). Yet, data presented by Stanworth (1983) showed that for every four boys who participated in classroom discussion, there was one girl. Three boys to one girl received praise and encouragement. In another study by Spender (1980), it was found that the boys in a mixed-sex class received close to two-thirds of the teacher attention (see page 101). All too often this differential treatment is unwittingly given and stems from a lack of awareness of the problem. The majority of good, professional teachers are fair and apparently unbiased. They may not even realise or appreciate the relative attention they give. So too with the pupils, who may not actually appreciate the respective attention given to girls and boys.

It is evident from the study that the attitudes of girls towards mathematics deteriorates across the years (1-5) at a greater rate than that of the boys. This trend is shown in Figure 32 which shows the percentage of gender responses in each year group who answered positively to the statement 'I have always liked maths' (6). It is significant that in the first year (Year 7), girls' overall attitudes to mathematics are more positive than boys' but much more negative in the third (Year 9) and fourth form (Year 10). As can be seen, they pick up again in the fifth form (Year 11), but by then it may be too late.

A similar pattern is found across the year groups in response to the statement 'I used to like maths, but not now' (3) (Figure 33). It is evident here that the confidence of girls in their own ability decreases during the same crucial third and fourth years. These early adolescent years are the foundation years when the girls in particular are coming to terms with their adulthood and their role in society. They are especially susceptible to outside influences during these years and perhaps more particularly to male peer pressures (see page 82).

While the boys in the study did not strongly stereotype mathematics as a male domain, at each year they stereotyped it at a higher level than did the girls. This is well illustrated in Figure 34. This is a significant finding because the cross-sex influence on all aspects of behaviour is strong during these adolescent years. Since boys tend to stereotype mathematics in this way, they undoubtedly communicate this belief in many subtle and not so subtle ways. This may influence girls' willingness to study mathematics to the same

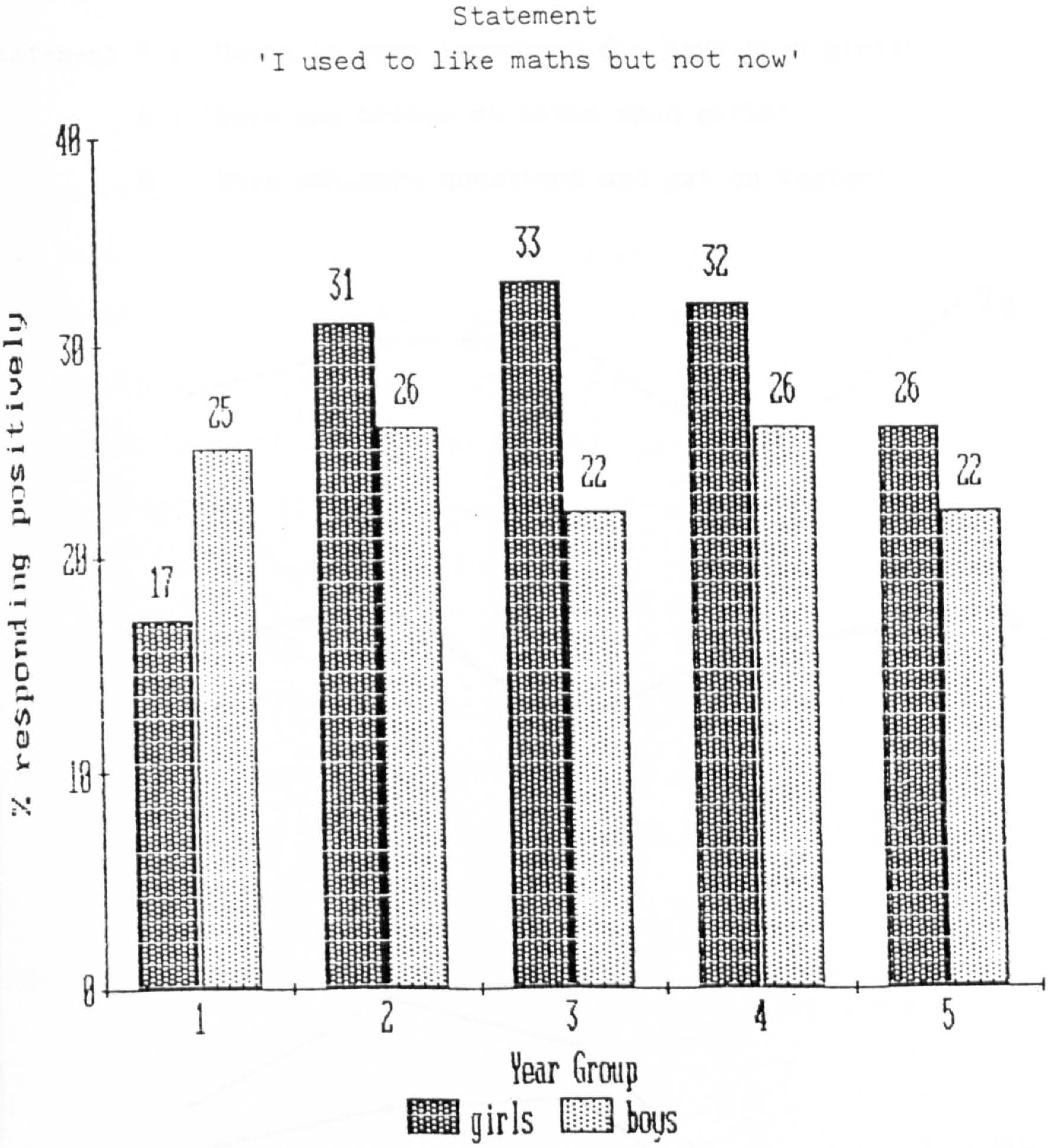
Figure 32. Percentage of gender responses in each year group who answered positively to the statement 'I have always liked maths'



Year	1		2		3		4		5	
Sex	g	b	g	b	g	b	g	b	g	b
Strongly Disagree	15	15	16	15	20	16	22	22	19	23
Disagree	9	16	25	18	24	20	39	28	31	28
Agree	26	29	27	37	34	34	24	39	28	23
Strongly Agree	50	39	32	30	21	30	15	11	22	26

GIRLS: $\chi^2 = 45.07$, d.f. = 12, $p < 0.0001$
BOYS: $\chi^2 = 24.41$, d.f. = 12, $p = 0.0179$

Figure 33. Percentage of gender responses in each year group who answered positively to the statement 'I used to like maths but not now'



Year	1		2		3		4		5	
Sex	g	b	g	b	g	b	g	b	g	b
Strongly Disagree	62	55	47	53	35	51	33	33	43	49
Disagree	21	20	22	20	30	27	35	41	31	28
Agree	9	10	20	16	21	13	23	13	16	11
Strongly Agree	8	15	11	10	12	9	9	13	10	11

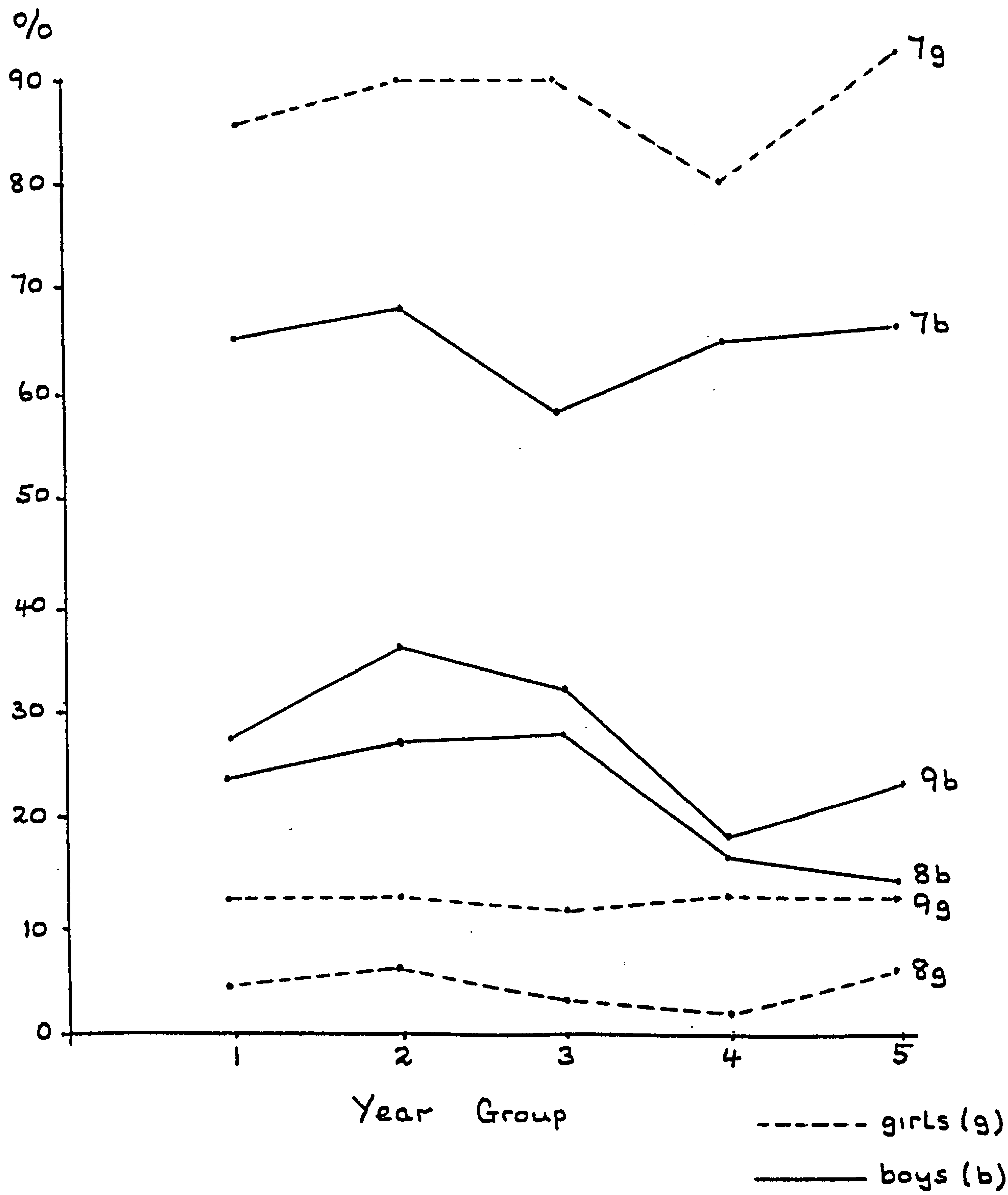
GIRLS: χ^2 = 23.16, d.f. = 12, p = 0.0264
BOYS: χ^2 = 17.39, d.f. = 12, p = 0.1357

Figure 34. Percentage of gender responses in each year group who answered positively to statements 8 and 9 and strongly disagreed with statement 7.

Statement 7 : 'Maths is more important for boys than girls'

8 : 'Boys are better at maths than girls'

9 : 'Boys ask more questions and get on faster'



degree. It has strong implications for the development of intervention programmes designed to increase female participation and for single-sex classes.

An important common feature in this study is the way in which the attitudes and beliefs held by the girls deteriorate in comparison to the boys in the middle years of secondary education. This is an important discovery and highlights the particular years of concern in the teaching of mathematics in secondary schools. These are the important years of the National Curriculum's Key Stage 3 leading on to Key Stage 4. If the girls are experiencing confidence and anxiety problems, and if they are failing to see the relevance of mathematics for future careers, then this may have an effect on their performance at sixteen. It is in these formative years that much of the basic mathematics curriculum is taught (see page 224).

The following figures highlight this mid-school dip with reference to responses to the given statements.

Figure 35. Percentage of gender responses in each year group who strongly agreed with the statement 'I am always keen to start doing Maths' (1)

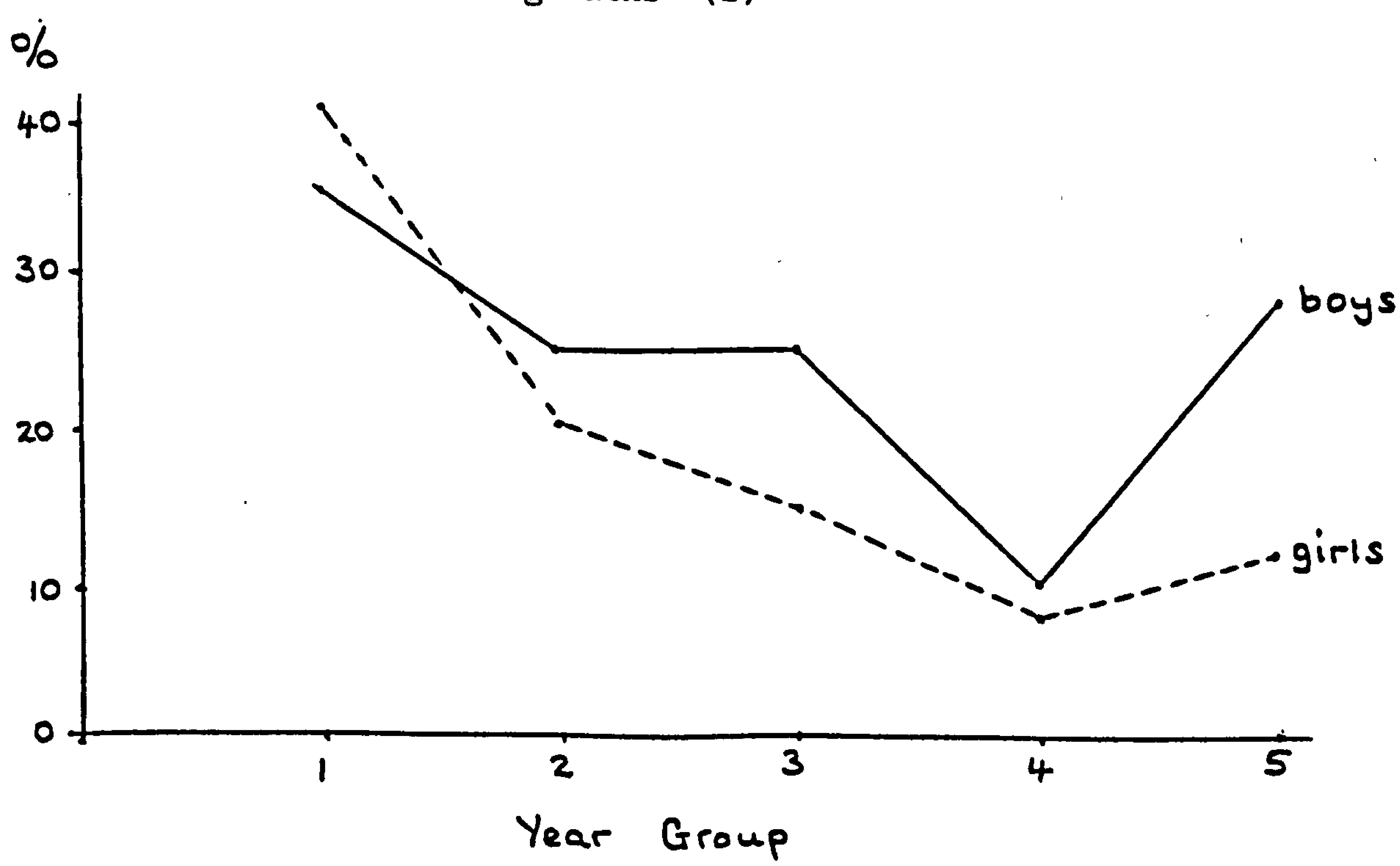


Figure 36. Percentage of gender responses in each year group who answered positively to the statement 'I never expect to do well in Maths' (12)

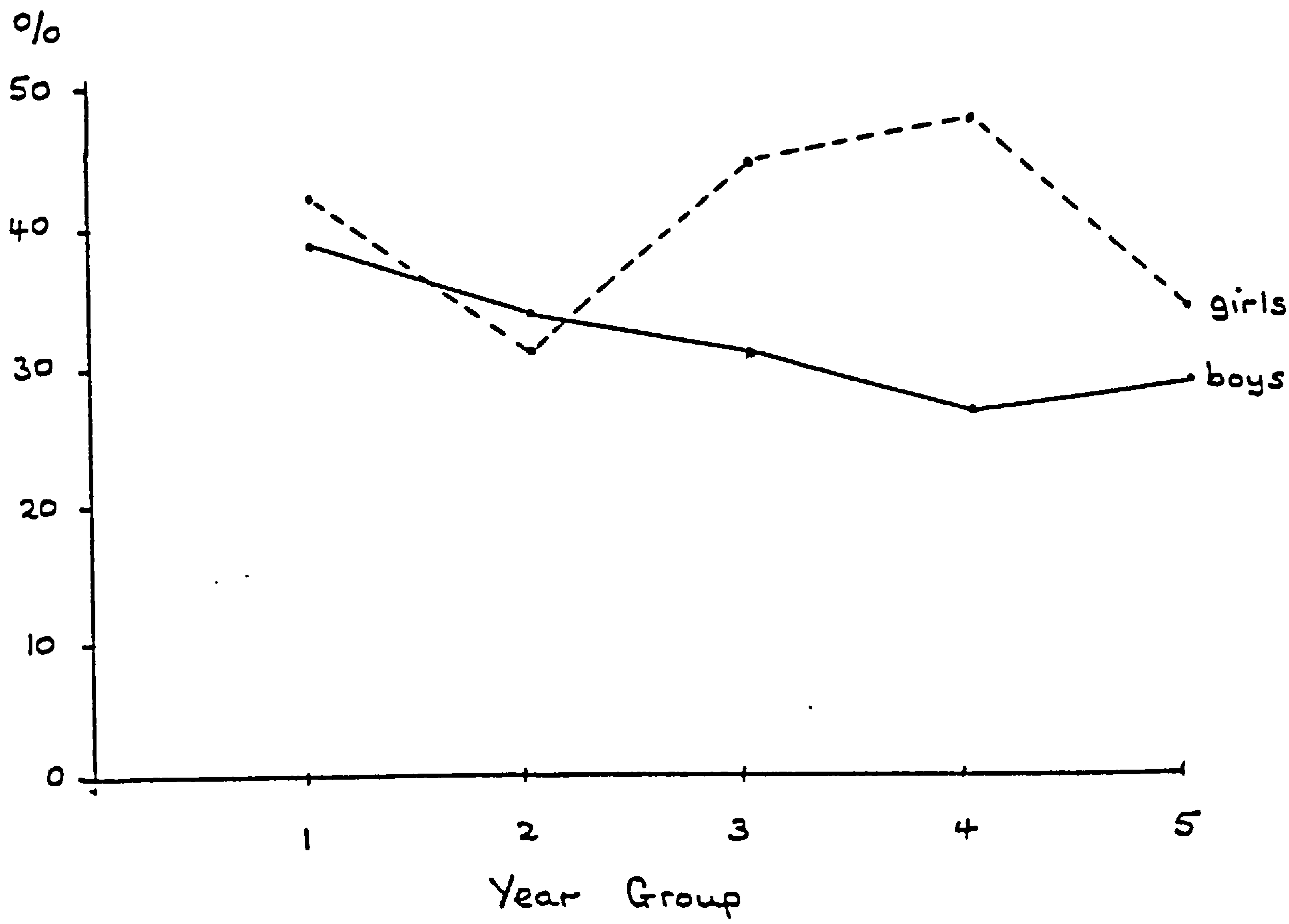


Figure 37. Percentage of gender responses in each year group who answered positively to the statement 'I like working out problems in Maths' (14)

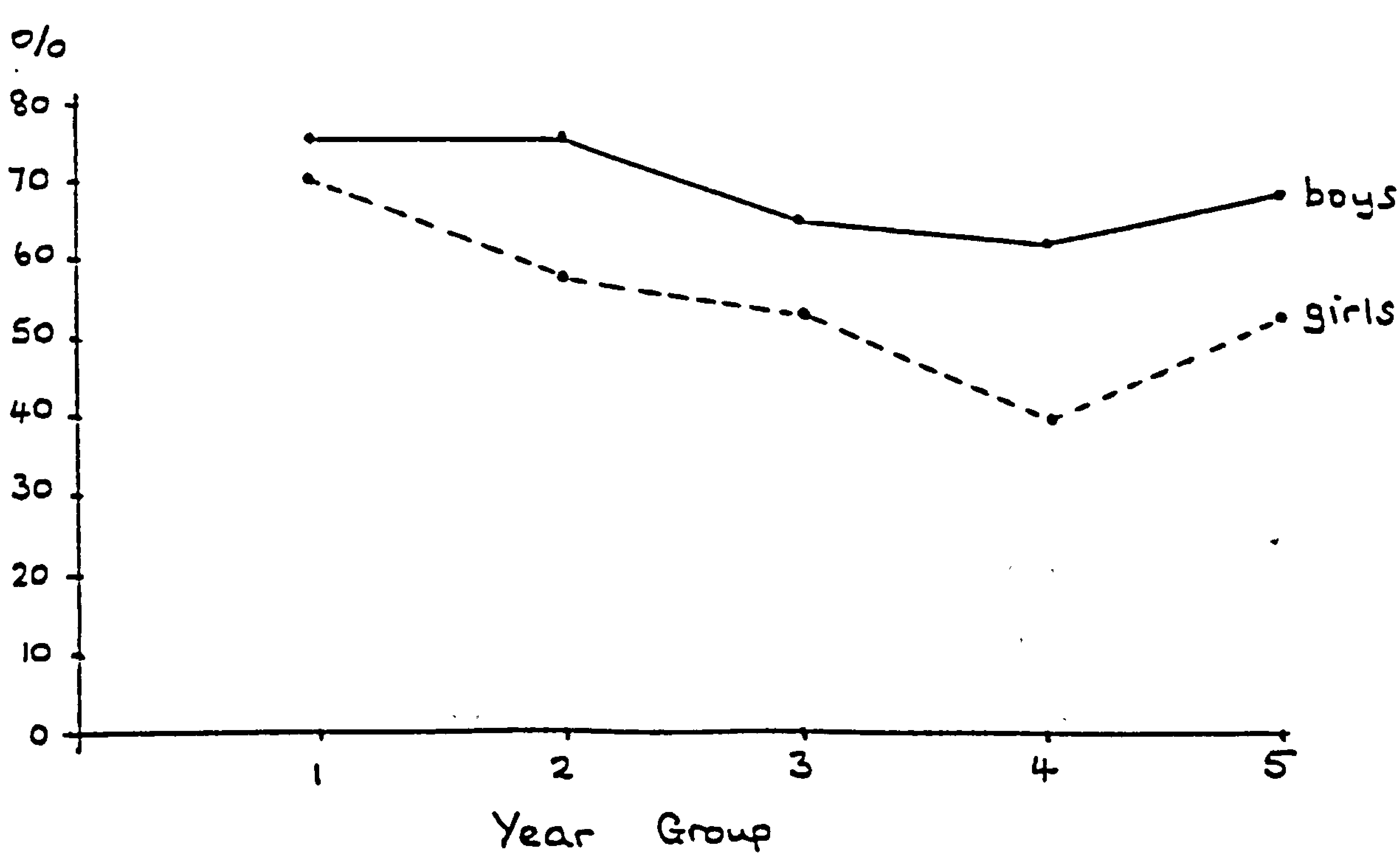


Figure 38. Percentage of gender responses in each year group who answered positively to the statement 'I panic in Maths tests' (18)

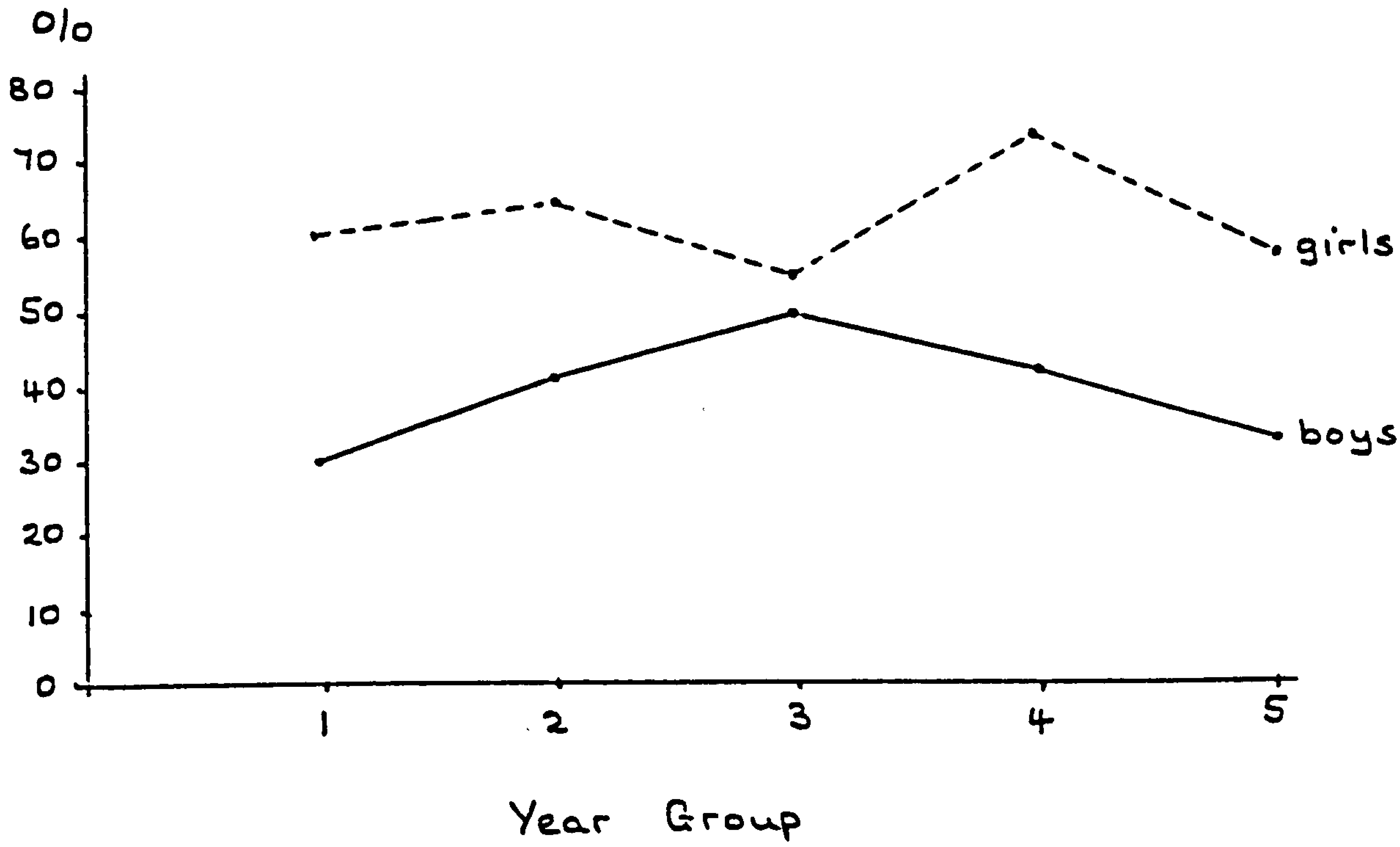
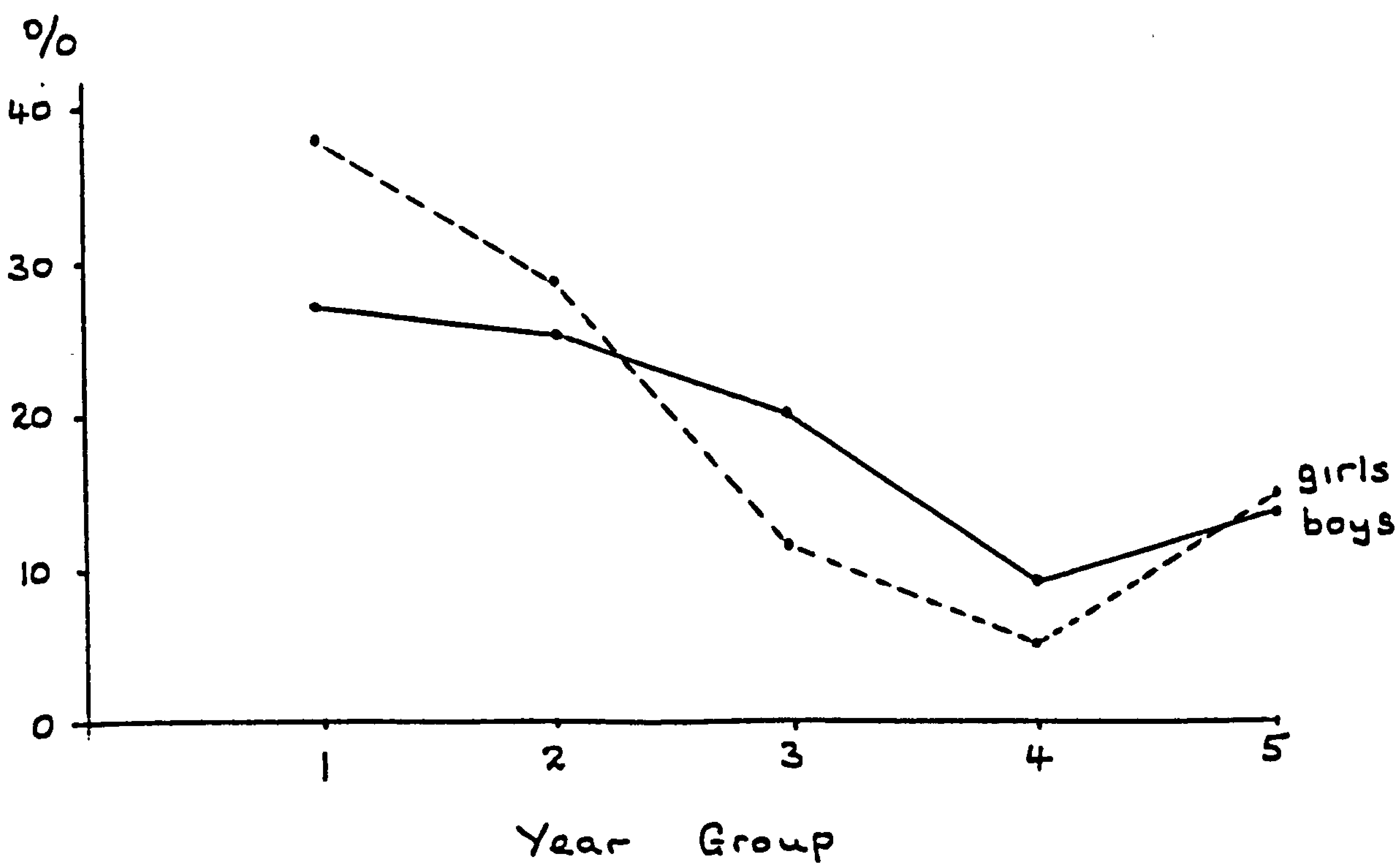


Figure 39. Percentage of gender responses in each year group who strongly agreed with the statement 'I like Maths because I like working with numbers' (25)



This evidence of the falling away of attitudes of the girls in the mid-school years, represents a significant step forward in an understanding of girls' approach to mathematics. The scale of the differences in attitude scores between the sexes do not exactly mirror differences in performance but they may be a factor which helps to explain those differences.

Boys are more likely to attribute their successes to stable causes such as ability and their failures to unstable causes such as lack of effort. Girls attribute their successes to unstable causes such as the effort they put into their work, and their failures to stable causes such as lack of ability (Whyte, 1983). It seems that girls often feel inadequate about 'intellectual', problem-solving activities and underestimate their ability to solve mathematical problems.

Comparisons across the schools showed no significant differences. Figure 40 gives the responses to statement 11. These figures are encouraging. Nearly 50% of pupils in each school were prepared to answer positively to the statement 'I like maths because we are always doing something interesting' (11).

Figure 41 shows a comparison across the ten participating schools of the gender responses to statements 9 and 14. These patterns follow what might be expected from the overall percentages given earlier.

Figure 40. Percentage of gender responses in each of the ten participating schools who answered positively to the statement 'I like maths because we are always doing something interesting'

Statement

'I like maths because we are always doing something interesting'

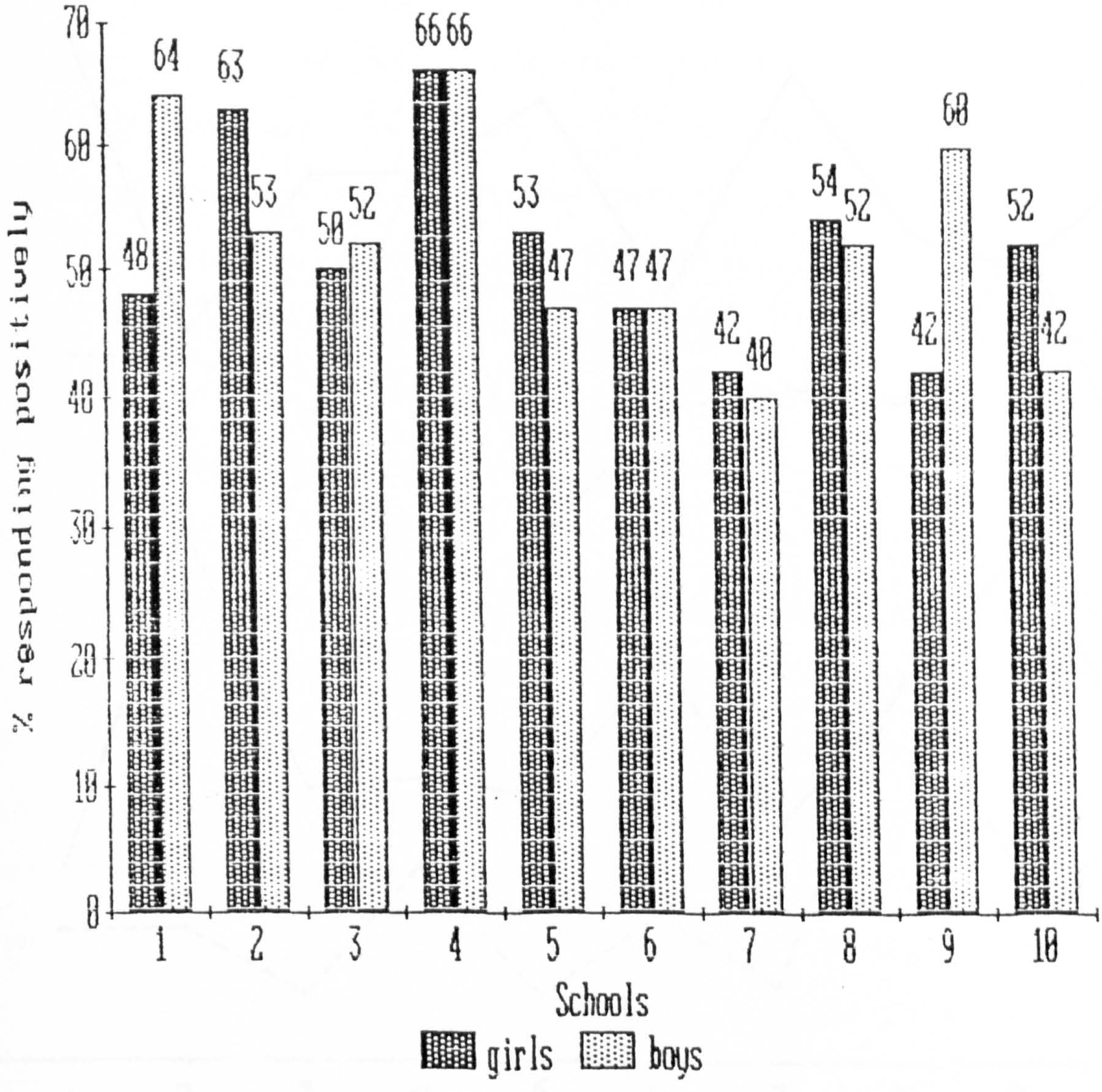
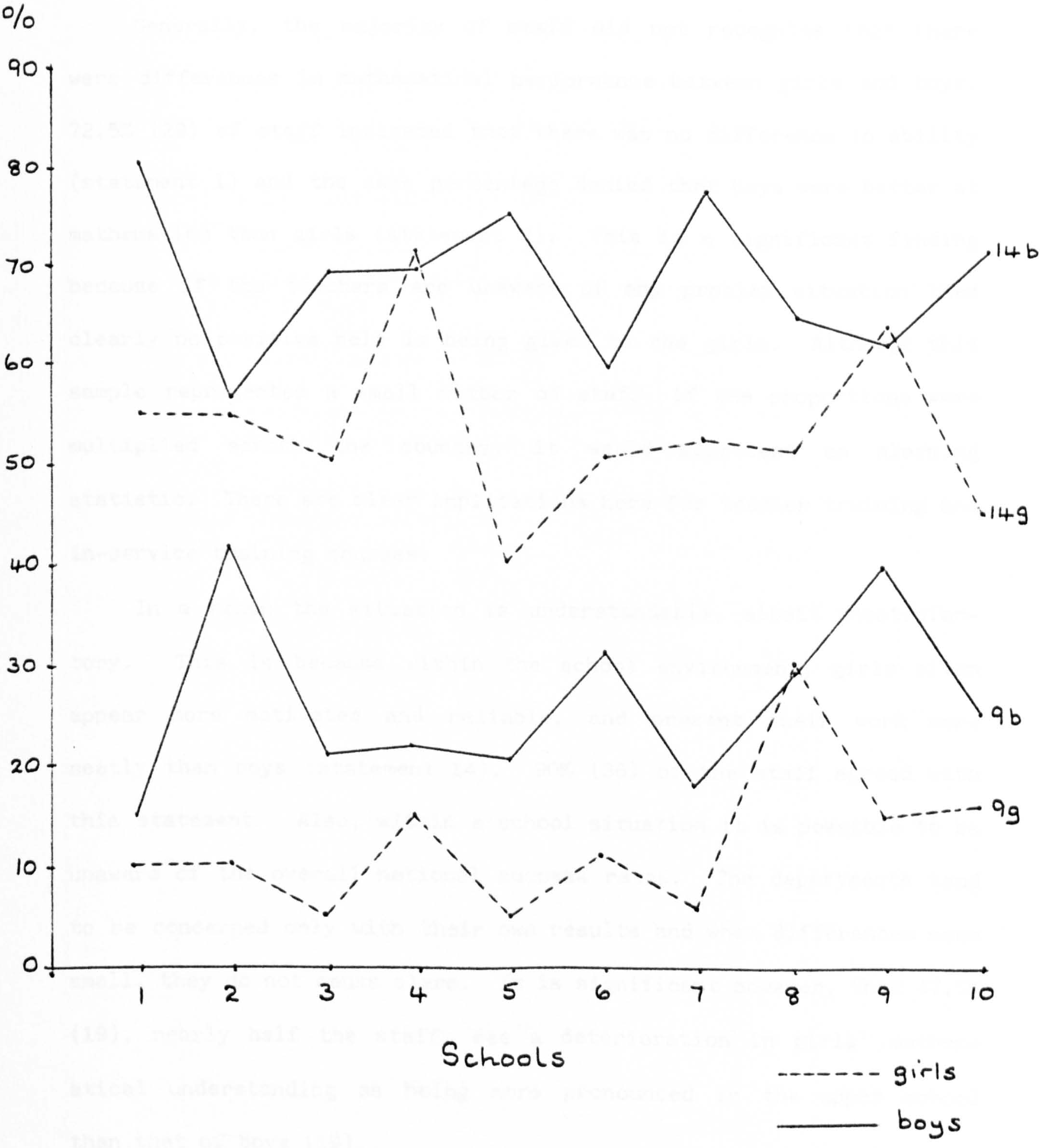


Figure 41. Percentage of gender responses in each of the ten participating schools, who answered positively to the statements 9 and 14.

Statement 9 : 'Boys ask more questions and get on faster'

14 : 'I like working out problems in Maths'



c) Staff Questionnaire

Table 41 shows the percentage of staff giving responses to each of twenty-five statements. They were all teachers in the mathematics departments of the participating schools.

Generally, the majority of staff did not recognise that there were differences in mathematical performance between girls and boys. 72.5% (29) of staff indicated that there was no difference in ability (statement 1) and the same percentage denied that boys were better at mathematics than girls (statement 2). This is a significant finding because if the teachers are unaware of any problem situation then clearly no positive help is being given to the girls. Although this sample represented a small number of staff, if the proportions were multiplied across the country, it would represent an alarming statistic. There are clear implications here for teacher training and in-service training courses.

In a sense the situation is understandable, albeit unsatisfactory. This is because within the school environment, girls often appear more motivated and reliable, and present their work more neatly than boys (statement 14). 90% (36) of the staff agreed with this statement. Also, within a school situation it is possible to be unaware of the overall national success rates. The departments tend to be concerned only with their own results and when differences seem small, they do not cause alarm. It is significant however, that 47.5% (19), nearly half the staff, see a deterioration in girls' mathematical understanding as being more pronounced in the upper school than that of boys (19).

Table 41. The percentage of staff giving a response to each of the statements in the questionnaire.

	strongly disagree	disagree	agree	strongly agree
1. There is a difference in mathematical ability between boys and girls	50.0	12.5	27.5	10.0
2. Boys are better at maths than girls	50.0	12.5	30.0	7.5
3. Girls are less enthsiastic in maths lessons	40.0	37.5	17.5	5.0
4. Girls see less relevance in maths compared with boys	27.5	40.0	20.0	10.0
5. Boys are more dominant in maths lessons	25.0	30.0	30.0	15.0
6. I give more of my attention to the boys	62.5	22.5	7.5	5.0
7. I make a conscious effort to motivate girls in maths lessons, ie. positive discrimination	32.5	35.0	17.5	12.5
8. The maths department is trying to redress the imbalance of performance between boys and girls	30.0	32.5	27.5	7.5
9. Girls are more willing to admit failure	22.5	32.5	37.5	5.0
10. Boys have a greater expectation of success	17.5	40.0	30.0	10.0
11. Boys find maths more interesting	27.5	40.0	30.0	2.5
12. Girls often under-achieve in maths tests/lessons	30.0	27.5	35.0	7.5
13. Boys have more flair in problem-solving	32.5	22.5	35.0	7.5
14. Girls present their work more neatly than boys	0.0	10.0	37.5	47.5
15. Girls are more methodical than boys	7.5	27.5	42.5	22.5
16. Boys are more likely to experiment/take risks	17.5	25.0	37.5	20.0

continued

Table 41 (continued)

	strongly disagree	disagree	agree	strongly agree
17. Girls do not see maths as necessary for most of their future jobs	30.0	35.0	32.5	2.5
18. Girls reach their limit of mathematical understanding sooner than boys	45.0	30.0	20.0	5.0
19. A deterioration in girls' mathematical understanding is more pronounced in the upper school than that of boys	22.5	27.5	35.0	12.5
20. Boys tend to have a more logical and clear view of maths	32.5	37.5	25.0	5.0
21. Girls dislike maths because their mothers also disliked it	35.0	25.0	22.5	7.5
22. Boys are better at solving spatial, geometrical questions	17.5	32.5	40.0	5.0
23. Maths text-books and work sheets are male orientated	30.0	27.5	35.0	5.0
24. Maths is a male stereotyped subject	42.5	25.0	22.5	5.0
25. Boys have more ambition in life therefore they work harder	60.0	22.5	5.0	5.0

Some staff were incredulous that it could be even possible that they, as 'good professional teachers', may give more attention to the boys or that there needs to be any redress in balance (6 and 8). A good teacher may not discriminate consciously in any way and teaches all to the best of his/her ability. Yet, teachers need to become aware of the fact that they may unconsciously give cues to both girls and boys, and that this may affect not only attitudes, but also the learning of mathematics (see page 86).

If a teacher responds to a pupil in a way which conveys the message to a boy that mathematics is important for him, that he is expected to succeed and that lack of success is due to his lack of effort; while a girl receives the message that her lack of success is due to lack of ability and that lack of mathematical ability is common and unimportant in girls, then it is not surprising that the girl gives up trying while the boy tries harder. So, teachers need to be consciously aware of the importance of helping girls and to see their successes as the result of their good mathematical ability and not solely due to their hard work.

Only 12.5% (5) of the teachers said they gave more attention to the boys with 62.5% (25) strongly disagreeing. Yet, 45% (18) agreed or strongly agreed that boys were more dominant in mathematics lessons.

In response to the statement 'Boys have a greater expectation of success' (10), 40% (16) of teachers gave a positive reply. Also, 42.5% (17) of teachers gave a positive response to the statement that 'girls are more willing to admit failure' (9). Burton and Townsend (1985), commenting on the varying attitudes of girls and boys towards failure, suggest that boys more often 'externalise' while girls 'internalise'. This then, may provoke the debilitating affective variables as described earlier.

Dweck (1976) investigated a phenomenon she calls 'learned helplessness' which exists when a pupil believes that failure at a task is insurmountable, and it is then accompanied by a deterioration in performance. She found that some children become less competent following a failure, while others rise to the challenge, persist, and improve their performance. It may be that girls are more likely than boys to fall into this state. This may be true, if as Burton and Townend (1985) suggest, girls 'internalise' failure.

42.5% (17) of teachers acknowledged that girls often under-achieve in maths tests/lessons (12). This is an interesting result in view of the dissent to the statement 'Boys are better at maths than girls' (2) 62.5% (25). In response to the statement 'Boys have more flair in problem-solving' (13), 42.5% (17) replied positively, although 32.5% (13) strongly disagreed. Similarly, in the statement 'Boys are better at solving spatial, geometrical questions' (22), 50% (20) of staff replied negatively. This suggests that although the literature and NEA study highlight problem-solving and spatial visualisation as difficult areas for girls, this is not an obvious and evident fact in the classroom.

Most teachers do not see mathematics as a male stereotyped subject (24) (67.5%, 27). They do not regard girls as being less enthusiastic in mathematics lessons (3) (77.5%, 31), or that boys find mathematics more interesting (11) (67.5%, 27). There seems to be a general view of equality of opportunity for all. Yet, 40% (16)

of teachers agreed or strongly agreed, that mathematics text-books and work sheets are male orientated (23).

As discussed earlier (page 118), pupils' perceptions of the appropriateness of mathematics may be reinforced by the type of materials used. Authors, publishers, examiners and teachers are often keen to show how the 'everyday' can be viewed mathematically. However, the 'everyday' for some texts is a man's world. If exercises follow this pattern, and restrict mathematical applications to boys' interests, then some girls will assume that mathematics has nothing to offer them.

d) Summary

The combined results in external examinations at age sixteen in mathematics across the ten participating schools were generally consistent with national figures. The overall percentage figures from the ten schools were higher in terms of pass rate (page 252). This might have been expected because one of the participating schools was a grammar school. However, the ratio of boys' passes to girls' are remarkably similar. Table 42 shows the results of ratio passes of boys to girls using national figures and the survey figures.

Table 42. A comparison of the ratio of boys' to girls' mathematics results between national figures and sample figures.

Year	1984	1985	1986	1987
National Figures	1.51	1.20	1.25	1.18
Sample Figures	1.38	1.33	1.38	1.27

In order to draw any conclusions from the study, it was important that the sample conformed to the national pattern. As can be seen, it clearly is representative of the national trend.

If sex-related differences in mathematics cannot be explained wholly by cognitive variables, then the affective variables may provide important insights into some of the difficulties which girls experience. The affective domain is a complicated one, and has received less attention than the cognitive domain because of its characteristics. It has to do with feelings, beliefs and attitudes.

All too often, the affective variables have been classified into one large conglomerate and labelled as attitudes. Yet, this type of labelling can often mask important variations.

The literature strongly supports the conclusion that there are sex-related differences in the confidence-anxiety dimension (eg. Fennema and Koehler, 1982). It is reasonable to believe that lesser confidence, or greater anxiety on the part of girls, is an important variable which helps to explain gender-related differences in mathematics performance. The difficulty is knowing what effects feelings of confidence have on cognitive processes and whether these feelings of confidence are stable within individuals across time and across a variety of mathematical activities.

What is clear from this study is that attitudes of girls towards mathematics deteriorate across the years of secondary education at a greater rate than that of the boys. In the first year (Year 7), girls' attitudes to mathematics are more positive than the boys'. However, this attitude becomes much more negative in the third and fourth forms (Years 9 and 10). Boys' attitudes also decline during these middle years but to a lesser extent. The attitudes of the girls become more positive in the fifth form (Year 11), but by this time they may have lost-out on much of the foundation work. This cannot be attributed to differential dropout as in American schools where students are given the choice of studying mathematics at an earlier age. In England and Wales, mathematics is compulsory to the end of the secondary school (usually up to the age of sixteen).

It seems that whatever a person's intellectual ability, its effectiveness is dependent on the extent to which the emotions support, or hinder, the question on which it is engaged. For girls generally, this debilitating anxiety appears to be more pronounced during these formative years.

Self confidence in mathematics performance is also lower at this time. More girls admitted to feeling 'no good at maths' (20) and 'not expecting to do well in maths' (12). Fennema and Koehler (1982) found that just as there is a relationship between anxiety and performance, so there is a similar relationship between self-esteem and achievement.

This has long term repercussions for the girls in continuing to study mathematics beyond the age of sixteen. If some girls are experiencing acute feelings of anxiety and low self esteem, then they may decide that mathematics is not for them.

However, care must be taken when interpreting statistics relating to attitudes in mathematics. Some girls may be more prepared to admit feelings of tension and anxiety than the boys. Conversely, over-confidence on the part of the boys could be equally damaging in terms of performance.

What is interesting to note, is that girls seem to be consistent across the sets in their lack of confidence in their approach to mathematics. The picture mirrors very closely the performance of girls and boys across the percentile study of the NEA survey (page 212).

Within each ability level there is a good correlation between confidence and performance. Yet, this correlation does not necessarily mean that most able pupils are the most confident across the whole range. Some high achieving girls readily admit to feelilngs of anxiety and lack of confidence.

In response to the statements 'My mother liked maths at school' (16) and 'My father liked maths at school' (17), there was little variation in response between the girls and boys. Nor was there a great positive response to the statement 'Whether or not I like maths depends on the teacher' (26). Only 52% (488) of the total sample answered positively to this statement. Yet, it is from the home and from the teacher that much of the influence on the pupils is presumed to lie. This is exemplified in the studies of Spender (1980) and Whyte (1985). If there are pressures from these sources, then either the pupils do not see them, or alternatively choose to ignore them.

Again, there was little difference between the sexes in response to the statement 'Maths won't be important to me in the future' (4). Three-quarters of all girls and boys strongly disagreed with this statement. It is encouraging to see a general recognition of the importance of mathematics. However, there is also a perception of the reality of the practical situation. In response to the statement 'Girls usually choose a job which needs maths' (13), only 31.8% (152) girls and 29.4% (134) boys agreed or strongly agreed (Table 40). Clearly, it is important for both girls and boys to recognise the importance of mathematics as that 'critical filter' which may prevent many girls from having access to higher paid, prestigious jobs.

While the boys in the study did not strongly stereotype mathematics as a male domain, at each year they stereotyped it at a higher level than did the girls (page 266). This is an important finding because the peer group pressures are strong during these adolescent years. Teenagers are very sensitive and susceptible to group opinions and reactions both of their own and the opposite sex. If boys do stereotype mathematics in this way, or if girls collectively decide mathematics is not for them, then this can have a marked effect on commitment.

This has clear implications for the timing and development of intervention programmes. The third and fourth years (Years 9 and 10) are the important formative years of mathematics learning in Key Stages 3 and 4 of the National Curriculum. The combined effect of increased anxiety levels and peer group pressures, may be to reduce the learning potential for mathematics amongst some girls.

The question then must be asked whether the same situation is true in other subjects. Why, for example, are girls' results better overall than the boys' at sixteen across all subjects?

The answer to this question may lie in the nature of the structure of mathematics. It is a subject which is highly structured and hierarchical in nature. Systematic progress is essential for good performance. One concept builds upon another and if the foundations are weak, the whole structure falters. For example, until a child can master the basics of place value in numbers, little progress can be made with the further work of addition and subtraction. Mathematics is unique in this way and the formative years are very important.

In other school subjects, eg. English and the humanities, there is not the same degree of structure. Pupils are able to 'catch up' much more easily on work missed. Future work does not depend as rigidly as in mathematics on what has gone before. The programmes of study (page 224) illustrate the highly structured nature of mathematics. For example, how can a student master Pythagoras' Theorem until he/she understands the concepts of right angled triangles, area, squares and square-roots?

The study shows a better attitude of girls towards mathematics in the fifth form. With most other subjects, there would be little of a problem, in that girls could increase their work input and manage satisfactorily. For example, many subjects can be taken as one year courses in the sixth form without the need of much background knowledge. However, with the structured and hierarchical nature of mathematics, this is not so. For girls with an improved attitude and commitment in the fifth form, it may be too late. Some girls may need to go back to the basics of work covered earlier in the third and fourth forms but is there time or even the motivation to do it?

In mathematics, new concepts require new modes of thinking and may require the pupil to think in novel ways, for example, conceptualising an imaginary number or solving simultaneous equations in algebra. A careful application of old skills and thinking may not be sufficient. To enjoy mathematics and to perform well it may be necessary to maintain both confidence and concentration in the face of novelty and in the face of failure. It is unlikely that an individual can grasp new concepts without experiencing a fair degree of confusion. This is precisely the kind of situation that is poorly matched to the achievement orientations which some girls are likely to hold.

In contrast, given boys' tendency to view a novel task with a moderate risk of failure as challenging, this characteristic of mathematics may serve to make it attractive and to facilitate performance.

The differential responses of males and females may be exacerbated further by the fact that each new unit in mathematics begins with a new name, for example 'algebra', 'geometry', 'calculus', 'trigonometry' and 'statistics'. This may serve as a reminder that a whole new set of skills must be mastered. Whilst this suggestion may be acceptable to most boys, it may cause concern to some girls who already have a tendency to discount previous successes as predictive of future success.

Comparisons across the ten participating schools showed no significant differences in response. Individual school responses were consistent with the overall pattern (Figures 40, 41).

The staff survey revealed some interesting results. One of the most significant of these was the fact that 72.5% (29) staff indicated that there was no difference in mathematical performance between girls and boys. This is alarming because if the teachers are unaware of performance differences, then they are not in a position to offer any positive encouragement to the girls.

Furthermore, some teachers were indignant that as professionals there could be any differential treatment of girls and boys. Yet, as studies have shown (eg. Reyes and Fennema, 1982), males appear to be more salient in the teachers' frame of reference. In general, teachers interact with boys more than girls in both blame and praise contacts. More questions are asked of boys. They are given the opportunity to respond to more high cognitive questions than the girls.

The problem of staff awareness is also supported by the Girls into Science and Technology (GIST) project. This reports that teachers in general, did not believe that they ever treated girls and boys differently. Gender differences in pupil performance were seen as linked to extra-school factors such as social expectation or the 'natural' differences between the sexes or youth culture (Payne, Hustler and Cuff, 1984). Yet, as the research has shown, teachers unwittingly confirm stereotyped views and are often firm proponents of traditional values.

Paradoxically, nearly half the staff said they saw a deterioration in girls' mathematical understanding as being more pronounced in the upper school than that of boys (19). Teachers need to be aware of the importance of helping girls to realise their full potential and that their success is the result of ability rather than just hard work. Girls' confidence in their own ability needs to be built up and reinforced.

So, the study highlights the particular problems of confidence and anxiety as it relates to mathematics across the ten schools. It clearly demonstrates the deteriorating attitudes of girls in the third and fourth years of secondary education with regard to the subject. It also highlights the particular problems of raising staff awareness to the difficulties experienced by the girls in the classroom. The implications of these results are significant and these need to be discussed further.

Conclusions and Implications

Conclusions

a) The Problem

The debate concerning the place of mathematics in the education of girls and boys, has been raging for more than a century. For a long time it was commonplace to discuss separately the education of girls and boys, including the mathematics courses they should follow.

The concept of equality of opportunity, was first given official recognition in the 1944 Education Act. No longer were girls banned from woodwork, metalwork or technical drawing. Public examinations were equally available to girls and boys. The Sex Discrimination Act of 1975 also brought education within its scope, and there is now a much greater commitment within the European Community, to equate the numbers of men and women in different spheres of mathematical study. For example, special funds have been made available to train women in the areas of engineering and science.

Although it is generally accepted that there is little difference between girls' and boys' mathematical attainment until early adolescence, differences in favour of boys do begin to appear in the secondary school. This may be due to the fact that much of the mathematics in junior education is predominantly based on computation. This is a part of the subject where girls seem to do well. Concepts such as geometry, trigonometry, proportionality and abstract development (algebra), where boys seem to do well, are introduced much later. It may therefore, be misleading to suppose, as some of the literature suggests, that the girls are better than the boys in mathematics up to the secondary stage. It may be true in terms of performance but then that depends on the concepts being tested.

However, it does seem that up to the secondary stage the motivation to study mathematics is high for both girls and boys. Thereafter, fewer girls do mathematics voluntarily and the performance of those who do study is worse (on average) than the boys.

This raises two important issues. One concerns the motivation and commitment of pupils to study mathematics. This in turn affects the representation of pupils in the further study of mathematics beyond the age of sixteen. The other is concerned with the performance and ability of pupils and their possible under-achievement in the subject.

A pupil's performance in mathematics may be influenced by a variety of different agents. These may come from the teacher, but also from the size and type of the class grouping, by the literature and materials, by the role models, and by the teaching styles.

Studies show that girls are more susceptible to debilitating anxiety towards mathematics. They are exposed to pressures from parents, from peers and from the demands of the subject. For many girls there is a dual social-psychological effect, which relates to both internal and external perceptions of performance. The question is, when are these pressures at their greatest, and what can be done to relieve them?

These debilitating pressures give cause for concern, in the light of the importance of mathematics as a critical filter into many fields of employment. Many see mathematics as the basis of scientific development and modern technology. It is seen as a discipline which is important not only as a tool, but also as a means of communication.

There can be no room for discrimination on the grounds of class, race or gender.

It is worrying also that in the light of so-called 'equal opportunities' there are still significant discrepancies in the performance of girls and boys in mathematics. The performance in public examinations sheds light on the problem that exists. At sixteen, the girls are not as successful as the boys in mathematics examinations. This is the immediate, practical concern. In 1987, 32.7% of boys obtained higher grades (A, B or C), compared to 27.7% of girls (a ratio of 1.18:1). In 1988, with the introduction of the new General Certificate in Education (GCSE), 40.2% of boys compared to 33.1% of girls obtained higher grades (a ratio of 1.21:1). Clearly, there is a continuing problem.

At A-level, there are twice as many boys entering for mathematics subjects. Yet, the distribution of grades of those girls and boys who enter, are remarkably similar. The girls who enter mathematics at A-level are equally competent. This again underlines the considerable overlap in the abilities of girls and boys. Girls can equal, or surpass, many of their male contemporaries.

The pattern of fewer girls choosing to study mathematics continues into higher education. In 1984, the ratio of men to women of first degree graduates in the United Kingdom, was 2.32:1. The situation in physics was even more alarming at 5.83:1.

b) The Aim of the Thesis

The problem then, is, what are the gender differences in mathematical performance? Why are fewer girls choosing to study mathematics at a higher level? Is it that there are innate differences between girls and boys, or is the problem one of nurture in the socialisation process? In other words, are there basic biological factors which give rise to these differences such as brain lateralisation or hormone development? Or, is the problem one of attitudes and experience based on the manner in which girls and boys are brought up under the pressures of a complex social structure?

The aim of this thesis is two-fold. The first is to examine the nature of the differences in examination performance. The purpose of this is to establish any common pattern of variation in scores across the different mathematical concepts. Where exactly are the girls under-achieving compared to the boys? Rather than investigate general, global issues, it is important to get to the root of the differences. There is a need to establish which concepts give rise to large differences as well as those where the scores are close together. Having established where precise differences exist, programmes of intervention can be more readily put into effect to help both girls and boys.

The second aim of the thesis is to offer possible explanations for these detailed differences and to identify the implications for teaching. On the surface, research seems to support the argument that there are innate differences between girls and boys in mathematics performance. However, this argument is very much weakened by the evidence, which shows that girls do indeed, show an improvement in scores on mathematical activities, after appropriate intervention programmes of study.

These two aims have been satisfied by a review of relevant literature and by original, empirical research.

c) <u>The Research</u>

The thesis attempts to answer some of these questions, in two major studies. The first was concerned with finding the precise concepts, where the differences in mathematics performance were greatest. The second study was a survey designed to examine the attitudes of pupils and staff towards mathematics, and to compare the responses of girls and boys. Here the object was to examine the whole ability range, across different ages and across different schools.

In the first study, 1000 mathematics scripts (500 girls and 500 boys) were analysed. These were the 16+ mathematics papers of the Northern Examining Association. The examination consisted of three papers resulting in an analysis of 3000 separate papers. Each question was classified according to National Curriculum criteria and the statistics were analysed under these headings.

The second study was a questionnaire for staff and pupils across ten schools. A sample size of 40 staff and 938 pupils (479 girls and 459 boys) was used. The questionnaire was designed to examine the attitudes and motivation of pupils towards mathematics.

d) <u>Differences in Performance</u>

The literature shows that mathematical ability can take many forms and be weighted in a number of directions. Although differing in emphasis, most studies would, to a larger or lesser degree, place importance on basic numeric, computational ability, spatial awareness, reasoning and problem-solving. This is consistent with the meta-analysis mentioned earlier (Hyde, Fennema and Lamon, 1990, page 31). This particular study highlights the difficulties experienced by girls, compared to boys, in problem-solving tasks.

Girls are found to be generally better than the boys on the basic computational work. This may account for girls' better average performance in mathematics, up to eleven years. Here the content is concerned very much with basic arithmetic. However, where gender differences do exist, they are in critical areas. These can be classified as spatial ability, proportionality and problem-solving.

In Wood's analysis of London O-level mathematics papers (1973), the biggest differences in favour of the boys, were concerned with spatial visualisation, scaling, the distance-time graph and probability. Concepts such as scaling, ratio and probability, are all encompassed under the umbrella of proportionality. This is a skill which research has found to develop at a relatively late age. The early years of secondary school are the important years of proportionality learning. Yet, these are the years of greatest social awareness of pupils when they are most susceptible to social-psychological pressures.

Girls' lack of spatial ability is often explained by suggesting that they play less often with spatial toys as young children. Toys such as bricks, Lego or Meccano could be related to developing such abilities. The early experience of boys does seem to give a much more appropriate foundation for physical understanding.

Studies have highlighted particular categories of spatial ability. These are spatial perception, mental rotation and spatial visualisation. The greatest difference in performance in favour of the boys was for measures of mental rotation. This is consistent with the difficulty that many girls seem to have with questions relating to three dimensional work and bearings.

The existence of different forms of mathematical ability, together with the elusiveness of a single component as revealed by factor analysis, suggests that mathematical ability can take a variety of forms. This means that when comparing the performance of girls and boys in mathematical examinations, care needs to be taken to look closely at the precise questions being asked. What is the examination testing? How can the concepts best be classified? Clearly care needs to be taken in view of the variety and number of different mathematical syllabuses and examinations.

One of the major weaknesses of previous studies is the bias of the sample population. Many studies have centred around high ability pupils, particularly in relation to success in O-level papers or college progress. This has been perpetuated by the apparent assumption that high ability pupils are the only successful performers in mathematics. It is important however, to look at the overall performance of pupils in mathematics. What are the differences in performance through the ability range? Are the concepts which give the greatest divergence in performance between girls and boys the same for high ability children, as for children with learning difficulties?

Another problem with some of the research findings, particularly those from America, is that many of the studies have been conducted amongst pupils in High School who have chosen to study mathematics. This does not always give a clear picture of the performance on mathematics across the whole year population. Those who choose to study mathematics have in some way already been motivated to continue and are aware of what the process of study involves. They may already have come to terms with the affective variables that may hinder progress in others.

The research literature is also essentially pupil centred and does not clearly address the attitude of the teachers. Mathematics teachers have an influence on the learning environment in which the pupils are placed. It is important therefore, to examine the processes by which teachers may be helped to see where gender differences in performance arise.

Research suggests that mathematical performance of girls and boys can be increased by the use of intervention programmes. However, this remedial action cannot be put into effect until it is clear where the difficulties lie. These need to be identified systematically across the ability range. It is not good enough to introduce an intervention programme of proportionality, for example, to <u>all</u> girls. Indeed, some girls may be so competent that they do not need it while for others, it may be beyond their comprehension.

Intervention programmes must be tailor-made for the individual. Nor should such programmes be restricted to the girls. There may well be many boys who have the same difficulties with spatial ability, proportionality, and problem-solving concepts. Remedial action should <u>not</u> just be for girls compared to the boys, but rather for the weak compared to the strong.

In the first study of this thesis which was aimed at the top 80% of the ability range, specific differences in performance were found. Of all the items analysed, the greatest differences in favour of the boys were concerned with bearings (mental rotation), scaling, speed, distance and time, the use of units and probability. These results confirm the evidence of the literature that many girls experience difficulty in the areas of spatial ability, proportionality and problem-solving.

The study found that in an analysis of the ability range, topic differences varied considerably. The ten questions which produced the greatest differences for the top 10% of the ability range, were different for the lower 10%. The bottom 10% revealed differences in topics which form foundation concepts. This shows that the use of intervention programmes, valuable as they are, must relate to the ability of the pupils and to their mathematical understanding. A pupil may, for example, have a clear grasp of proportionality concepts but be poor in problem-solving techniques.

The study also found a remarkable consistency in performance difference between girls and boys across the ability range. At each tenth percentile across the ability range the performance of boys exceeds that of the girls. This important finding implies that schemes designed to increase the performance of girls must be targetted at each level across the whole ability range.

Some girls found particular difficulty with the Northern Examining Association questions when they required the skill of problem-solving. It seems that girls are more comfortable with well established methods. They appear to have a greater tendency to show caution, to avoid failure, and to use methods with which they feel confident and secure. None of the items on which girls performed better than the boys required what could be classified as problem-solving processes.

e) <u>Explanations</u>

Pupils who followed a sequential approach to a problem as opposed to a holistic approach did not perform as well. This may help to explain how pupils often 'snatch' at solutions, or give implausible results. One reason for this is that pupils, mostly girls, do not see mathematical problems in context. They need to have more experience in the 'hands-on' aspects of the subject.

This raises the important issue of motivation, and the need for material and content to be 'gender-friendly'. Intellectual abilities are not distinct from the social context in which children live. There is a need to think about the relationship between the physical and the mental, the biological and the social. Pupils are not static beings acted on by their environment; rather they are active in the process of making sense out of their world.

The context in which a question is set does have an effect on performance (Eddowes and Sturgeon, 1980). Care is needed in setting the context of questions which are relevant to both girls and boys. Mathematics syllabuses cannot be biased towards the girls. They must be concentrated towards concepts which form the foundation of future studies, and give pupils the skills they need in a technological age. Further research is needed to examine the context in which questions are set for the benefit, rather than the detriment, of girls.

There is a danger that greater accessibility to mathematics may be interpreted as devaluing or softening the subject. No suggestion is made here that standards should be lessened, but rather that pupils should be set challenging work, without being made to feel that success is unattainable.

Literature suggests that pupils do show an improvement in scores on tests after relevant intervention programmes (Badger, 1981). If this is true, then it should be possible to assist pupils by carefully prepared programmes of work. For many girls this would target the concepts of shape and space, scaling and ratio, speed, distance and time, and probability. However, it must be remembered that there are differences in performance in mathematical topics across the ability range. Also, it is possible for a pupil to be more competent in one skill, compared to another.

Literature also supports the view that learning is enhanced by the use of practical exercises and on manipulative materials (Mitchelmore, 1980). The practical, handling aspects of mathematics are clearly important and have important implications in teaching methods. A familiarity with the practical situation and 'real-life' context would help to eliminate the sequential approach to problem-solving (Pask, 1976).

The first study of this thesis also showed a consistent difference in mathematics performance between girls and boys longitudinally across the years of secondary education. This supports the view (Burton, 1986) that boys are better placed from year one. Differences are apparent from age eleven and not just at sixteen, although the deterioration in performance is progressive.

In the nature versus nurture debate, the evidence for natural, innate differences is very weak. However, if sex-related differences in mathematics cannot be explained wholly by cognitive variables, then the affective variables may provide greater insight. It is reasonable to believe that lesser confidence, or greater anxiety on the part of girls, is an important variable which helps to explain differences in performance.

There are internal as well as external pressures. These include the image of mathematics, its perceived usefulness, sex stereotyping, subject choices and career advice. Then there are the pressures of the pupils' interaction with, and expectations of, teachers, peers and parents. These together with motivation, self-confidence, teaching methods and modes of assessment can all contribute to feelings of anxiety, instability, confusion and submission.

So, what are the differences in attitudes between girls and boys towards mathematics? Do boys have a greater expectation of success and more confidence in their own ability? How do these attitudes vary from year to year? Answers to these questions have important implications for remedial action.

The second study of this thesis found that the attitudes of girls towards mathematics do indeed deteriorate across the years of secondary education, at a rate greater than that of the boys. In the first year (Year 7), girls' attitudes to mathematics are more positive than the boys'. This to some extent is a reflection of primary school education. Good teaching methods have resulted in high motivation for both girls and boys. It may also be due, however, to the type of mathematics being taught. The syllabus content becomes much more diverse in secondary education, embracing concepts which may favour the interest and experience of boys.

The attitude towards mathematics was found to deteriorate and become much more negative in the third and fourth years (Years 9 and 10), particularly amongst the girls. Boys' attitudes also declined, but to a lesser extent. Attitudes of girls become more positive in the fifth form (Year 11), but because of the hierarchical nature of mathematics it may be too late. For some girls and boys the important foundation work of the third and fourth years (Years 9 and 10) may not have been fully understood.

This is an important new step forward in the understanding of gender differences in mathematics performance. It has significant implications in terms of the timing of intervention and remedial programmes of work.

This second study also revealed some important findings through the staff survey. Nearly three quarters of the teachers in the sample indicated that there was no difference in mathematical performance between girls and boys. Some teachers were indignant that as professionals, there could be any differential treatment of girls and boys. Any gender differences were linked to extra-school factors such as social expectation or 'natural' differences between the sexes.

f) Conclusion

The studies showed that the attitudes of girls and boys towards mathematics deteriorate across the third and fourth years of secondary education (Years 9 and 10). These are the years when girls and boys are coming to terms with their role in society. These are the years of adolescence, when they are most susceptible and vulnerable to internal and external pressures. These are the years when many of the more fundamental mathematical concepts are taught.

The peer group pressures are very strong during these adolescent years. Teenagers are very sensitive to authority and to reactions, both of their own and the opposite sex. The disciplined structure, and hierarchical nature of mathematics may give rise to feelings of frustration and failure. The result is increased anxiety levels amongst the girls. Self confidence in performance decreases, and more girls admit to being no good at mathematics. Again, this follows the same pattern in boys, but the problem is not as acute.

Whilst it may be that more girls are prepared to admit their feelings of anxiety, nevertheless the pattern of the statistics, in comparison with the boys in the third and fourth year (Years 9 and 10), gives cause for concern.

The studies also showed that girls are consistent across the ability range in their lack of confidence in mathematics. This correlates with the performance of girls and boys across the ten percentile study of the NEA survey. In other words, within each ability level confidence seems to relate to performance. Yet surprisingly perhaps, the most able girls and boys do not necessarily display the greatest confidence. In fact, as some studies have shown, eg. Wolleat (1980), the more able girls show a proportionately lower level of confidence than other ability levels.

What is not clear, and requires further investigation, is knowing the precise effects feelings of confidence have, on cognitive processes. How do these feelings vary across proportionality and spatial problems? These are questions as yet unanswered, and require further study.

What is clear is that programmes of intervention need to be specifically targetted to individual pupils, girls and boys alike. For the higher ability pupils, this may include concepts of bearings, scaling, speed, distance and time, the use of units and probability. For the lower ability pupils, it may include foundation concepts such as percentages, decimals and fractions. However, great care is needed with the teaching of such topics. Many pupils may abhor the sight of fractions having met them several times before without success. Practical teaching methods may be needed to gain the confidence of pupils in the work they are doing.

It is also evident that pupils may vary in their performance across the skills of spatial ability, proportionality and problem-solving. It may be that certain pupils have had particular experience with one element of mathematical expertise. For example, one girl has a particular aptitude for weight and measure because she works in a butcher's shop at the weekends. The practical experience of mathematics cannot be over emphasised.

What is not clear is the relationship between the skills of spatial ability, proportionality and problem-solving. What common cognitive mechanisms are involved? Is there a link, or are there other more suitable methods of classification? It has been convenient for the purposes of this study, to use the classifications of the National Curriculum for the individual questions on the examination papers, and then to examine the results in the light of the skills of spatial ability, proportionality and problem-solving. It is within these areas where the greatest differences in performance have been found. However, it may be that other means of classification are also appropriate. For example, the headings of 'arithmetic', 'algebra', 'geometry', 'trigonometry', 'graphs' and 'statistics' could also have been used. However, whatever classification is given to the mathematical content, it will still relate to the basic mathematical skills as outlined.

This thesis has concentrated on the particular difficulties experienced by girls and boys in mathematics. Reference has been made to the social pressures experienced by girls and boys from a general perspective. However, there are particular groups of people with added pressure placed upon them. Ethnic minorities, for example, may have increased difficulties placed upon them, as a result of cultural pressures. Indeed, the problem for black pupils is further exacerbated by possible racial tension. Some black girls may suffer the simultaneous oppression of sex and race. Also, there is the problem of social mix and social class. Are fee paying parents getting a better mathematical education for their daughters? How does the mathematical performance of girls vary from families of one daughter, two sisters, or a brother and sister?

In seeking to understand why differences exist, it is necessary to study cognitive and affective components which affect the acquisition of mathematical skills and knowledge, in the social environment where they are developed. The attitudes or affective beliefs held by girls, male peers, parents and educators, are all important influences on the learning of mathematics. The cognitive and affective components are intertwined, and develop over a period of years in a complex social matrix, which involves home, community and school.

29. Implications

Given the validity of these conclusions based on the literature review and the two studies, where do teachers go from here to improve performance? What are the factors which must be tackled?

The first, most important objective is to increase the self-confidence and motivation of all pupils but particularly the girls. This is often caused by feelings of failure, lack of practical experience, lack of relevance and lack of interest.

Teachers need to be aware of these possible feelings, of hidden prejudices and social pressures. They need to be aware of the attitudes, confidence and anxiety experienced by their pupils. Positive expectations of girls will help to foster positive attitudes to the subject. All pupils should be allowed to experience success rather than failure.

Interest and involvement need to be fostered and developed. If girls have had little experience with constructional skills, then they should be encouraged to use apparatus and equipment that will develop visuo-spatial awareness. Pupils should be encouraged to talk about mathematics, and to listen to each other, so as to bring a social, 'girl-friendly' element to the teaching. Girls need the confidence to develop their ideas and a helpful, constructive atmosphere needs to be created where this can best happen.

It is all too easy for boys to stereotype mathematics as a male domain. This can result in girls assuming that mathematics is not for them. Care must be taken to ensure that equal access is given to girls and boys in the use of all mathematical equipment, and that particular encouragement is given to the girls in their use. Positive expectations of girls by teachers will help to foster positive attitudes to the subject from girls.

Many of the most exciting developments in mathematics in developed countries in recent years, have been made available through the power of modern computers and their software. Logo, spreadsheets and programmed learning packages are becoming increasingly used in schools. It is important, given the new technology, that both girls and boys get a fair share of their use.

Unfortunately, even when equal opportunities are provided, there is a noticeable tendency for the girls to be 'elbowed out' by the boys. Although no differences have been found in the programming ability of girls and boys, girls generally show less interest in computers. The implications of these findings are significant, in that boys are gaining more experience with mathematical equipment. Familiarisation and 'hands on' experience is important in all aspects of mathematics in building confidence and performance.

Mathematics skills can be taught by a carefully devised programme of work for each individual. These individualised schemes of work such as the <u>Schools Mathematics Project (SMP)</u> or the <u>Kent Mathematics Project (KMP)</u> allow pupils to work at their own pace. Pupils are not forced-on at a pace which would lose them. In a similar way specific intervention programmes need to be targetted at pupils with concept difficulties. In particular, girls need to be helped with their mathematics studies in the third and fourth years of secondary education.

The HMI Report, <u>Education Observed, Girls Learning Mathematics</u> (1989), suggested that girls succeed in mathematics when teaching is sensitive and perceptive. This successful practice can be identified in schools where neither girls nor boys are disadvantaged in their allocation of teaching groups. There is the use of examination and assessment procedures which enable all pupils to perform well. These may include oral tests, practical and assignment work and discussion. They may take place in class, in groups or as individuals.

It becomes clear that girls need to be encouraged not just to reproduce standard exercises, but rather to understand, appreciate and use mathematical principles in the widest possible way. Mathematics departments should ensure that teaching methods are effective, by using a wide range of materials. It is important, for example, that practical work is attempted, so that the subject is seen in context.

If girls are affected by the gender of the teacher in the learning of mathematics, and evidence suggests there may be some connection, then it is important that more women are encouraged to teach mathematics. A good role model in the form of a teacher or visiting speaker may be important for the girls. In 1983, only one third of mathematics teachers were women. In 1985, in England and Wales there were 2062 (15.7%) women secondary school headteachers, and 2062 (14.7%) women secondary school heads of mathematics departments. Clearly, more women need to be appointed to positions of responsibility. Also, there is evidence that fewer women apply for senior posts.

Teachers must try to encourage their pupils and to give girls in particular a feeling of success, rather than failure - a sense of involvement, rather than isolation.

Teachers of mathematics need to raise their own levels of professional awareness by reading current literature on the issue, including examination statistics. They need to analyse pupil performance in their own classes, and talk to pupils about their attitudes in mathematics. Teachers must collectively identify the problem of the underparticipation and underachievement of girls in mathematics.

In a classroom situation, teachers should consciously try to balance the attention they give, to each of the sexes. It is very easy to answer the loudest or most demanding pupil, usually a boy. If the boys are constantly demanding the teacher's attention in this way, and getting it, the girls will very soon feel left out and discouraged.

Sensitive teaching approaches, with staff fully aware of possible problems, are needed to integrate the girls fully into the learning process.

Not only do teachers have a strong influence on girls' perceptions of mathematics, but so do parents. Their stereotyped attitudes and expectations can have a crucial effect on girls' performance in the subject (Education Observed. Girls Learning Mathematics, 1989). There can be a gap between society's image of mathematics - its expectations of what the subject is and how it should be taught - and the broad range of content and approaches necessary for all pupils to develop fully their mathematical understanding and enjoyment.

Adults, whose own experience of mathematics may be confined to the acquisition of computational techniques through repetitive drill, may find it difficult to accept that the calculator and micro computer should be seen as an agent to overcome difficult hurdles. Media presentations illustrating the diversity and fascination of mathematics may help in changing the image of the subject but it is difficult to change deep-seated prejudices or modify long-held views.

It is interesting to note that although a social stigma is attached to the inability to read or write, many people are apparently quite content to admit that they have never been much good at mathematics. It is important that the public image of mathematics as not being 'the thing' for women is changed.

If more able girls show a proportionately lower level of confidence than other ability levels as the research shows, then this gives cause for concern. It is from this group that the A-level students will come. If girls are lacking confidence in mathematics studies, then they are not likely to choose to study it further. Greater reassurance is needed to build up the confidence of these girls.

The sixth form timetable should be able to accommodate an A-level mathematics option, with a broad range of A-level subject options. Girls should be encouraged to study mathematics and to take it further into higher education. Indeed, the sixth form A-level girl students are, in themselves, role models for girls lower down the school. This forms a self perpetuating influence.

It is important, too, that all schools ensure that information given about future employment is balanced, and is not perpetuating gender stereotyping. Girls, as well as boys, need to realise that the subject is important to them in whatever sphere of activity they engage beyond school. The message to girls needs to be clear and precise - that mathematics is useful, enjoyable and for them.

So, negative perceptions in schools need to be changed by indicating high expectations of the mathematics performance of girls as well as boys. They need to provide successful role-models for girls among teachers and visiting speakers. They need to keep parents, governors and local employers informed of new initiatives and progress in equal opportunity policies and implementation. They need to show that mathematics is a creative activity for all. In short, there is a need to change the ethos in the classroom, through a well formulated plan of gender equality.

If the recommendations made in this study were implemented, there would be an improvement in the way in which girls view mathematics. These, together with carefully devised intervention programmes, aimed at the specific target areas outlined in the study, would improve the performance of girls in mathematics.

In the world outside school, there are strong pressures in many aspects of life to conform to gender stereotypes. Traditional perceptions of mathematics as a male domain persist, and are likely to change only slowly. Improvement in girls' participation and performance in mathematics will occur only when people perceive the problems, and work towards the same objectives. Focussing on the means of providing girls with opportunities of learning mathematics, in congenial and appropriate conditions, will help to ensure that all pupils will reach their respective potential.

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Appendix A : Table (i)

Percentage of Girls and Boys giving the correct responses to each question in Paper 1

Type	Question No.	F	M	% Diff	Type	Question No.	F	M	% Diff
M	1	74.8	82.2	7.4	M	31	79.9	77.6	-2.3
S	2	65.7	71.7	6.0	A	32	38.9	48	9.1
N	3	48.2	49.4	1.2	A	33	20	24.4	4.4
A	4	63.2	61.4	-1.8	N	34	78.9	84.6	5.7
N	5	58.1	72.5	14.4	N	35	67.1	70.2	3.1
A	6	53.1	60.8	7.7	N	36	29.6	42.6	13.0
A	7	43.1	44.7	1.6	S	37	71.1	78.6	7.5
M	8	60.7	70.8	10.1	S	38	39.6	47.6	8.0
S	9	60.6	72.7	12.1	S	39	42.8	47.6	4.8
N	10	79.1	84.3	5.2	M	40	69.0	80.0	11.0
N	11	67.4	75.9	8.5	M	41	69.8	80.4	10.6
S	12	50.0	47.4	-2.6	M	42	49	69.8	20.8
N	13	59.4	65.3	5.9	S	43	47.4	52.6	5.2
S	14	45.1	42.5	-2.6	S	44	56.4	68.2	11.8
S	15	36.1	53.5	17.4	N	45	41.8	54.0	12.2
D	16	44.6	51.6	7.0	N	46	26.3	34.9	8.6
A	17	61.6	67.0	5.4	N	47	42.7	49.3	6.6
A	18	47.0	56.5	9.5	N	48	45.8	49.1	3.3
A	19	40.4	47.4	7.0	N	49	24.7	29.9	5.2
A	20	70.1	75.8	5.7	S	50	73.7	80.1	6.4
A	21	41.7	46.0	4.3	S	51	22.6	34.8	12.2
N	22	53.1	59.7	6.6	S	52	66.0	64.1	-1.9
N	23	50.8	53.2	2.4	N	53	56.3	57	0.7
N	24	25.6	31.9	6.3	N	54	59.6	54.1	-5.5
S	25	63.3	74.3	11.0	N	55	62	63.9	1.9
S	26	35.1	47.6	12.5	A	56	36.3	37.6	1.3
S	27	40.8	53.2	12.4	S	57	49.5	52.6	3.1
D	28	67.1	79.2	12.1	A	58	71.1	73.7	2.6
D	29	38.8	46.1	7.3	N	59	33.8	42.1	8.3
D	30	40.1	45.8	5.7	A	60	39.2	37.1	-2.1

Appendix A : Table (ii)

Percentage differences between Boys and Girls giving correct responses to each section of Paper 2

	F		M				F		M		
1	92.2	-	92.6	N	0.4	28	37.4	-	38.2	A	0.8
2	91.6	-	92.2	N	0.6	29	60.6	-	70.8	S	10.2
3	6	-	12	N	6.0	30	63.6	-	70.2	S	6.6
4	43.4	-	60.4	N	17.0	31	68.2	-	72.6	S	4.4
5	13	-	21.6	A	8.6	32	65.8	-	63	S	-2.8
6	83.2	-	87.2	S	4.0	33	16.4	-	19.2	S	2.8
7	22.2	-	28	S	5.8	34	31.2	-	36	S	4.8
8	55.2	-	60.2	S	5.0	35	18.4	-	20.4	S	2.0
9	46.8	-	46	A	-0.8	36	45.4	-	50	S	4.6
10	17.2	-	22.8	S	5.6	37	13	-	12.8	A	-0.2
11	39.4	-	61	S	21.6	38	52.8	-	73.6	N	20.8
12	19	-	31.6	S	12.6	39	8	-	17	N	9.0
13	48.6	-	56	M	7.4	40	56.8	-	58.6	A	1.8
14	13.6	-	18	A	4.4	41	37.6	-	38.4	A	0.8
15	3.4	-	6.4	N	3.0	42	15.8	-	20	A	4.2
16	47.2	-	61.2	A	14.0	43	12	-	16	A	4.0
17	77	-	83.2	N	6.2	44	45.2	-	55.6	S	10.4
18	72.4	-	79.2	N	6.8	45	9.8	-	16.4	S	6.6
19	66	-	70.4	A	4.4	46	2.8	-	6	S	3.2
20	60	-	68	A	8.0	47	5.4	-	6.6	S	1.2
21	39.8	-	47	A	7.2	48	15.6	-	21.8	S	6.2
22	26.6	-	33.8	A	7.2	49	17.4	-	24.8	S	7.4
23	20.4	-	26	A	5.6	50	11	-	16.8	S	5.8
24	59.8	-	71.8	S	12.0	51	28.6	-	39	S	10.4
25	43	-	50	S	7.0	52	10.6	-	22.8	S	12.2
26	33.2	-	40	S	6.8	53	8.8	-	14.4	S	5.6
27	19.6	-	25.8	S	6.2						

Appendix A : Table (iii)

Mean Marks of Girls and Boys in Paper 3

		Female	Male	Diff.
A	Question 1	2.8939 (2.89)	2.9442 (2.94)	0.05
S	2	3.1172 (3.12)	2.8564 (2.86)	-0.26
A	3	5.1452 (5.15)	4.9884 (4.99)	-0.16
N	4	3.9012 (3.9)	5.2024 (5.2)	1.3
A	5	4.1837 (4.18)	5.1636 (5.16)	0.98
A	6	1.1875 (1.19)	1.8947 (1.89)	0.7
S	7	4.7480 (4.75)	5.8394 (5.84)	1.09
A	8	8.5269 (8.53)	8.4247 (8.42)	-0.11
S	9	7.6766 (7.68)	7.2673 (7.27)	-0.41
S	10	7.3241 (7.32)	8.3484 (8.35)	1.03
S	11	5.9608 (5.96)	6.4343 (6.43)	0.47
M	12	7.0755 (7.08)	7.4737 (7.47)	0.39
D	13	3.6886 (3.69)	4.8871 (4.89)	1.2
D	14	6.6157 (6.62)	6.6790 (6.68)	0.06
D	15	2.8031 (2.8)	2.9407 (2.94)	0.14
D	16	3.6921 (3.69)	3.9919 (3.99)	0.3
D	17	2.1878 (2.19)	3.4332 (3.43)	1.24
D	18	1.9225 (1.92)	2.7152 (2.72)	0.8
D	19	4.5411 (4.54)	5.8581 (5.86)	1.32
D	20	7.7647 (7.76)	8.9330 (8.93)	1.17
D	21	4.7547 (4.75)	5.6393 (5.64)	0.89
D	22	4.1942 (4.19)	5.2787 (5.28)	1.09
D	23	9.7616 (9.76)	9.8889 (9.89)	0.13
D	24	5.4051 (5.41)	6.8214 (6.82)	1.41

N - Number; A - Algebra; M - Measures;
S - Shape and Space; D - Data Handling.

Appendix A : Table (iv)

Performance of the top and bottom 10% of girls and boys in Paper 1

Quest^n No.	Numbers: Top 10%		Numbers: Bottom 10%		Quest^n No.	Numbers: Top 10%		Numbers: Bottom 10%	
	g(59)	b(59)	g(44)	b(48)		g(59)	b(59)	g(44)	b(48)
1	53	51	20	34	31	58	57	16	17
2	48	57	20	21	32	43	49	8	13
3	52	53	7	4	33	32	37	9	3
4	55	57	7	11	34	57	56	18	31
5	53	54	8	26	35	55	53	10	15
6	53	55	11	16	36	40	51	5	6
7	51	53	3	9	37	58	58	8	16
8	56	57	7	13	38	51	55	11	9
9	59	58	4	11	39	54	52	4	7
10	59	57	18	27	40	56	59	11	18
11	56	59	10	17	41	58	59	11	17
12	56	52	6	8	42	51	58	5	14
13	56	57	7	12	43	43	48	19	23
14	43	46	7	7	44	54	55	10	16
15	52	54	5	5	45	49	51	6	11
16	51	58	4	5	46	34	44	4	8
17	51	56	14	20	47	47	54	4	3
18	55	56	8	12	48	47	54	7	6
19	48	56	11	4	49	25	38	6	7
20	59	59	12	11	50	53	58	16	21
21	53	50	3	13	51	32	44	4	3
22	56	54	25	18	52	53	58	7	10
23	52	54	23	21	53	46	49	18	15
24	31	44	12	8	54	57	57	12	5
25	46	56	26	27	55	55	58	6	6
26	46	54	2	5	56	52	48	1	1
27	43	58	5	9	57	50	47	11	11
28	52	56	14	21	58	56	56	14	23
29	31	44	15	10	59	43	49	2	10
30	46	54	3	9	60	43	50	10	5

Appendix A

Figure (i) Number of successful girls/boys in the top and bottom 10% of the ability range in each question of Paper 1

------ girls
——— boys

Appendix A : Table (v)

Performance of the top and bottom 10% of girls and boys in Paper 2 gaining full marks in each Section

Section No.	Numbers Top 10%		Numbers Bottom 10%	
	g(52)	b(52)	g(50)	b(50)
1	50	52	36	35
2	51	50	38	37
3	6	15	0	3
4	45	47	6	12
5	22	33	0	0
6	49	52	19	23
7	37	46	5	2
8	48	49	6	5
9	49	47	0	3
10	23	26	4	3
11	45	51	4	4
12	36	45	0	0
13	40	45	6	13
14	31	37	0	1
15	13	24	0	0
16	41	46	7	11
17	51	52	10	20
18	50	52	8	15
19	48	52	9	7
20	47	51	4	11
21	46	48	1	3
22	40	40	0	0
23	33	40	1	1
24	44	50	8	14
25	49	49	0	2
26	42	47	0	0
27	34	40	0	0
28	47	46	0	1
29	52	49	7	15
30	47	48	9	13
31	52	51	9	12
32	49	50	6	8
33	41	43	1	0
34	45	47	2	0
35	39	45	0	0
36	49	51	0	1
37	37	40	0	0
38	48	50	8	14
39	22	38	0	0
40	48	52	2	7
41	46	47	0	1
42	33	40	0	0
43	35	36	0	0
44	49	49	2	7
45	30	33	0	0
46	5	14	0	1
47	13	22	0	0
48	36	41	0	0
49	36	46	0	0
50	33	43	0	0
51	39	44	0	2
52	25	39	0	0
53	24	35	0	0

Appendix A Figure (ii) Number of successful girls/boys in the top and bottom 10% of the ability range in each section of Paper 2

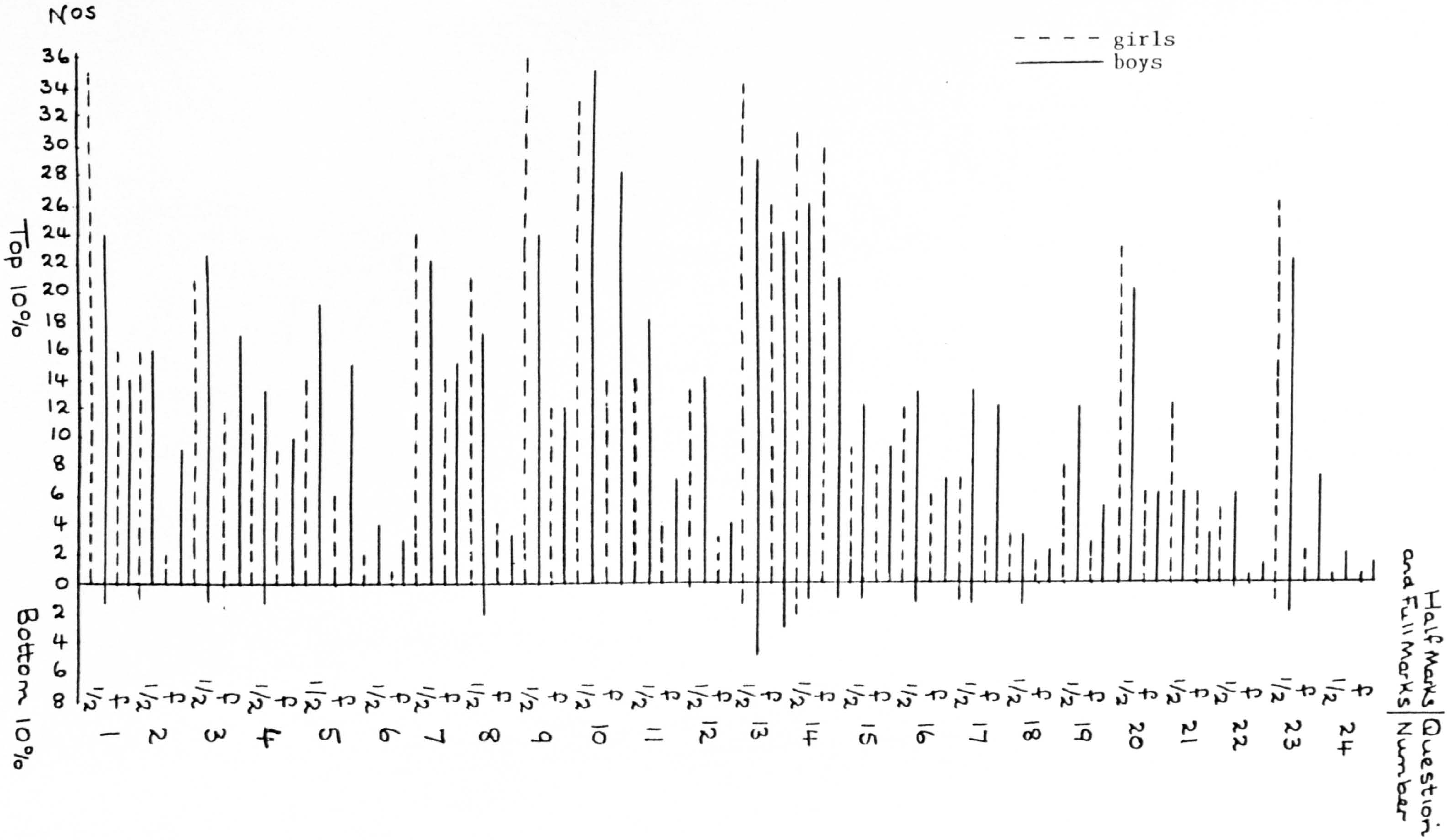
Appendix A : Table (vi)

Performance of the top and bottom 10% of girls and boys in Paper 3 gaining half marks and full marks for each question

Question No.		Numbers: Top 10%		Numbers: Bottom 10%	
		g	b	g	b
1	½	35	24	0	1
	full	16	14	0	0
2	½	16	16	1	0
	full	2	9	0	0
3	½	21	23	0	1
	full	12	17	0	0
4	½	12	13	0	1
	full	9	10	0	0
5	½	14	19	0	0
	full	6	15	0	0
6	½	2	4	0	0
	full	1	3	0	0
7	½	24	22	0	0
	full	14	15	0	0
8	½	21	17	0	2
	full	4	3	0	0
9	½	36	24	0	0
	full	12	12	0	0
10	½	33	35	0	0
	full	14	28	0	0
11	½	14	18	0	0
	full	4	7	0	0
12	½	13	14	0	0
	full	3	4	0	0
13	½	34	29	1	5
	full	26	24	0	3
14	½	31	26	2	1
	full	30	21	0	1
15	½	9	12	0	1
	full	8	9	0	0
16	½	12	13	0	1
	full	6	7	0	0
17	½	7	13	1	1
	full	3	12	0	0
18	½	3	3	0	1
	full	1	2	0	0
19	½	8	12	0	0
	full	3	5	0	0
20	½	23	20	0	0
	full	6	6	0	0
21	½	12	6	0	0
	full	6	3	0	0
22	½	5	6	0	0
	full	0	1	0	0
23	½	26	22	1	2
	full	2	7	0	0
24	½	0	2	0	0
	full	0	1	0	0

Appendix A

Figure (iii) Number of successful girls/boys in the top and bottom 10% in each question of Paper 3 gaining half marks and full marks



Appendix B

Northern Examining Association

ASSOCIATED LANCASHIRE SCHOOLS EXAMINING BOARD
JOINT MATRICULATION BOARD
NORTH REGIONAL EXAMINATIONS BOARD
NORTH WEST REGIONAL EXAMINATIONS BOARD
YORKSHIRE AND HUMBERSIDE REGIONAL EXAMINATIONS BOARD

Joint GCE O-level and CSE Examinations
MATHEMATICS PAPER 1
Monday 19 May 1986 1.30 p.m.—3.00 p.m.

Answer all 60 questions in this paper on the special answer sheet provided.
One mark will be awarded for each correct answer. Marks will not be deducted for incorrect answers.

For each question there are five responses. When you have selected the response which you think is the best answer to a question, mark this response on the separate answer sheet. Mark all your responses by making a thick pencil stroke under the appropriate letter. Use an HB pencil. Do not use ink or a ball-point pen. If you wish to change your answer to a question, rub out your first mark completely.

Rough work is to be done on this question paper, **not** on the answer sheet.

Diagrams are not necessarily drawn to scale.

A reference material leaflet is provided.

Mathematical tables, slide rules and calculators must not be used in this paper.

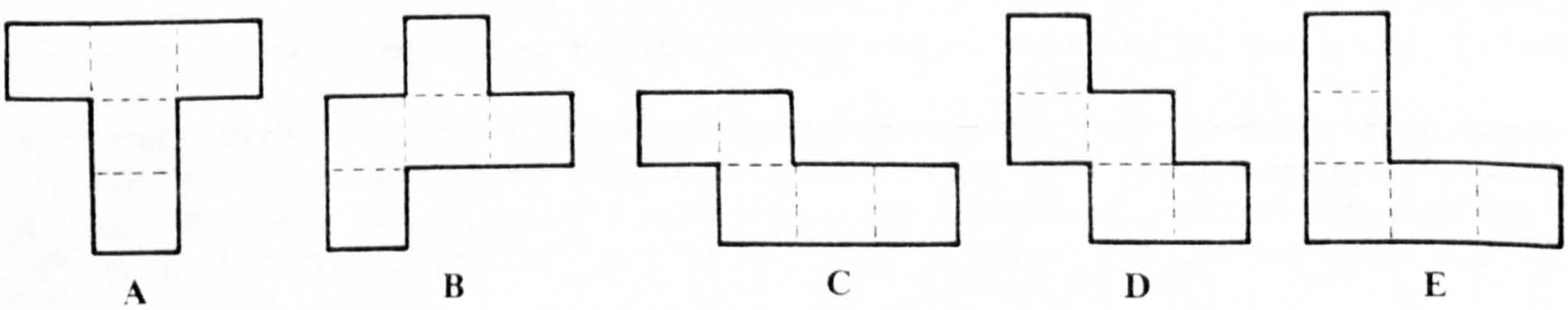
Multiple choice questions

Each of Questions **1** to **16** is followed by five responses, **A**, **B**, **C**, **D** and **E**. For each question select the *best* response and mark its letter on the answer sheet.

1 How many mm are there in 1 m 10 cm?

A 1001
B 1010
C 1100
D 1110
E 1111

2 Which one of the following shapes, each made of five joined squares, will *not* fold up into an open cubical box?



3 The number 108 expressed as a product of prime factors is

A 9×12
B 3×6^2
C $3 \times 4 \times 9$
D $2^3 \times 3^2$
E $2^2 \times 3^3$

4 What is the value of the expression $(x-1)(x+3)$ when $x = -2$?

A -3
B -2
C 1
D 2
E 3

5 The price of a car changed from £800 to £840. What is the increase in price expressed as a percentage of the original price?

A 2%
B 5%
C 20%
D 40%
E 105%

6 What are the values of x and y satisfying the simultaneous equations

$$3x - 2y = 7$$
$$2x + y = 0?$$

A $x = \frac{9}{7}$ $y = -\frac{18}{7}$
B $x = 0$ $y = 0$
C $x = -1$ $y = -2$
D $x = 1$ $y = -2$
E $x = -7$ $y = 14$

7 The expansion of $(a-2b)^2$ is

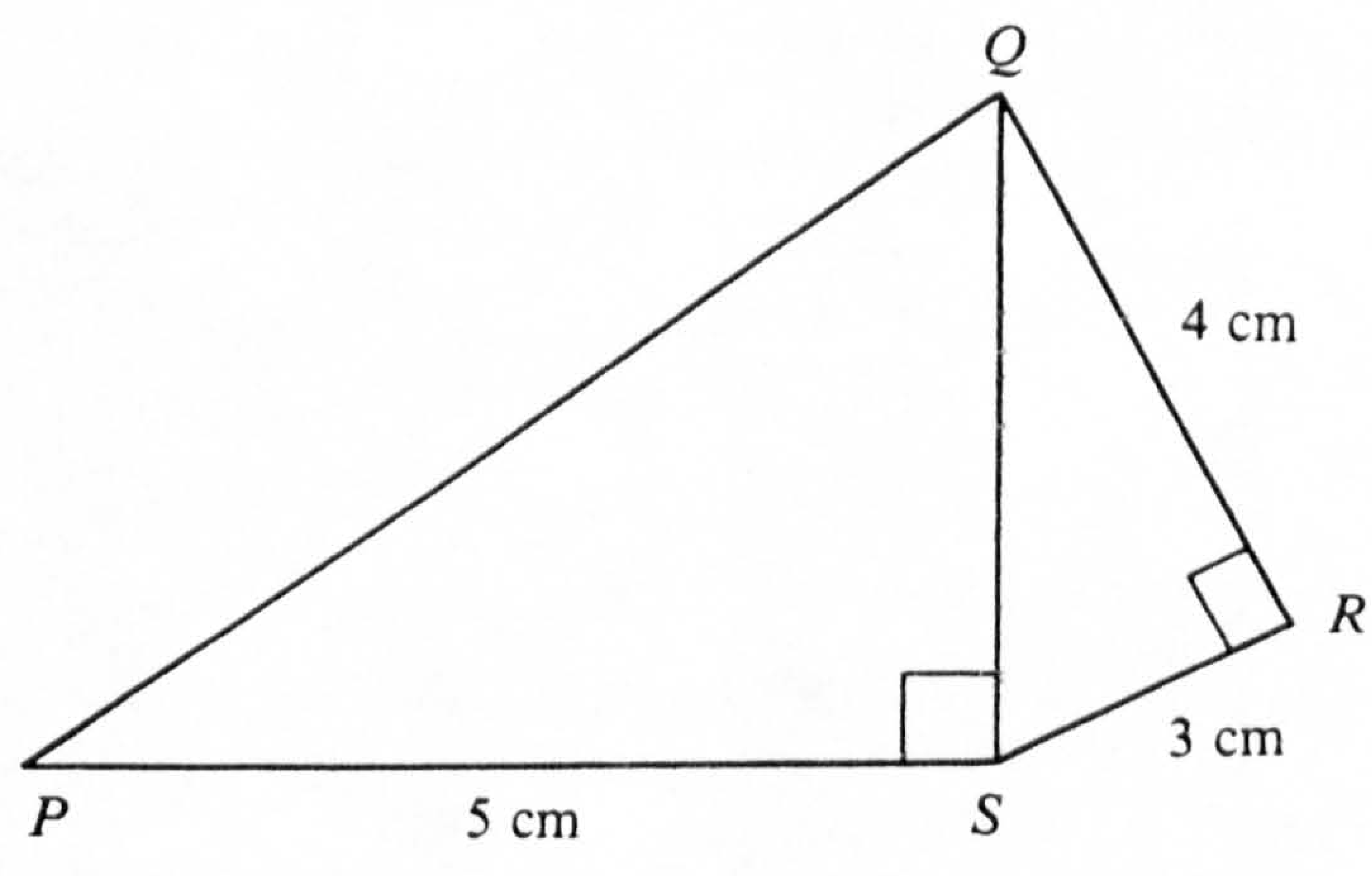
A $a^2 - 4b^2$
B $a^2 - 4ab + 4b^2$
C $a^2 + 2ab + 2b^2$
D $a^2 + 4b^2$
E $a^2 - 4ab - 4b^2$

8 The area of a square is 121 cm^2. What is the perimeter of the square?

A 484 cm
B 121 cm
C 44 cm
D 22 cm
E 11 cm

9 QRS and QSP are right-angled triangles.

QR = 4 cm
RS = 3 cm
PS = 5 cm



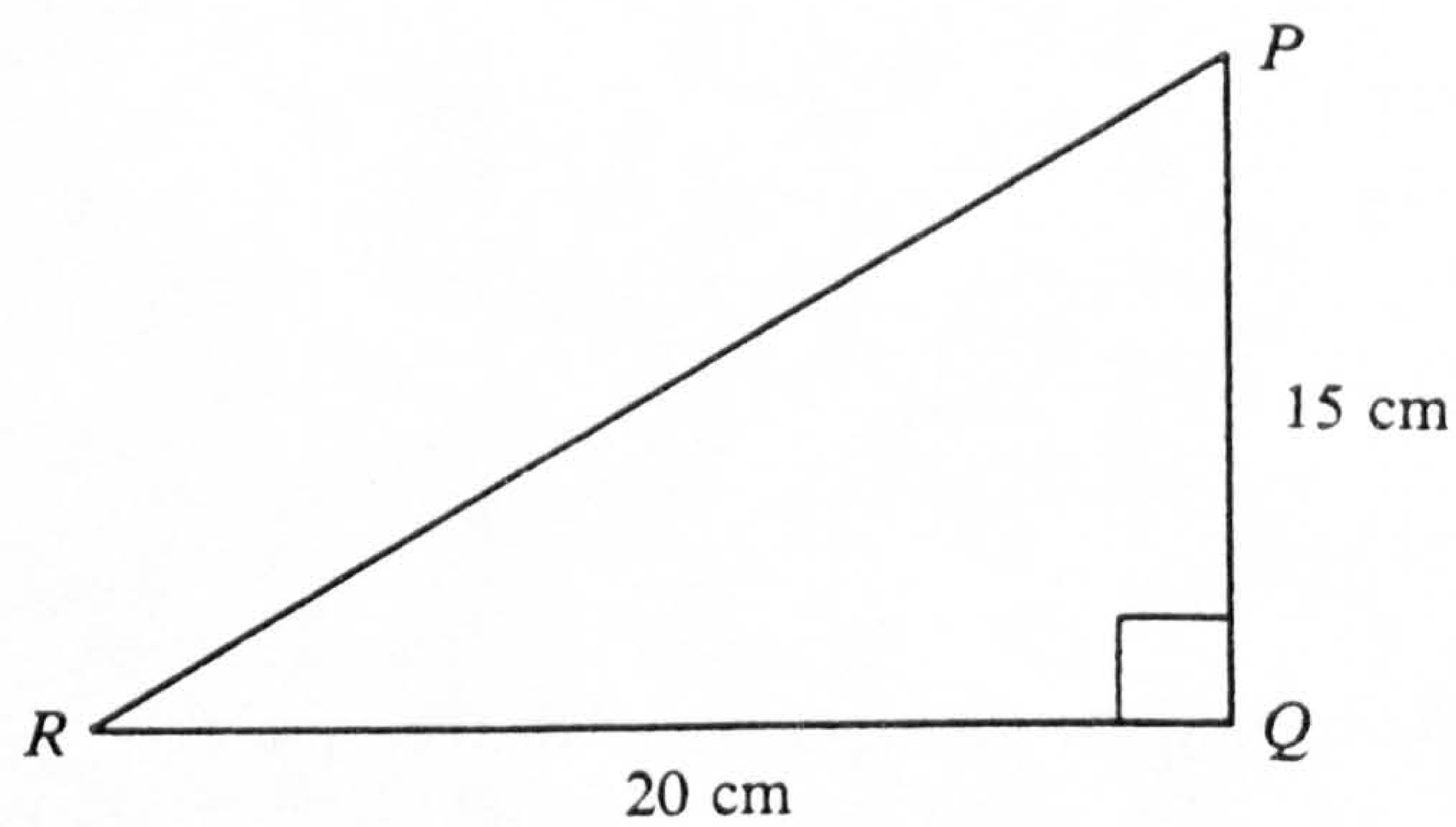
The length of PQ is
A 5 cm
B 6 cm
C $\sqrt{50}$ cm
D $\sqrt{30}$ cm
E 10 cm

10 The ratio of P's share to Q's share of the profits from a business is 3:2. When P receives £225 what is the total profit?
A £375
B £400
C £450
D £675
E £1125

11 A straight line is 7.2 cm long. $\frac{3}{8}$ of the length of the line is

A 0.375 cm
B 0.9 cm
C 2.4 cm
D 2.7 cm
E 3.75 cm

12



In this triangle (shown above) the value of sin P is

A $\frac{3}{5}$

B $\frac{3}{4}$

C $\frac{4}{5}$

D $\frac{4}{3}$

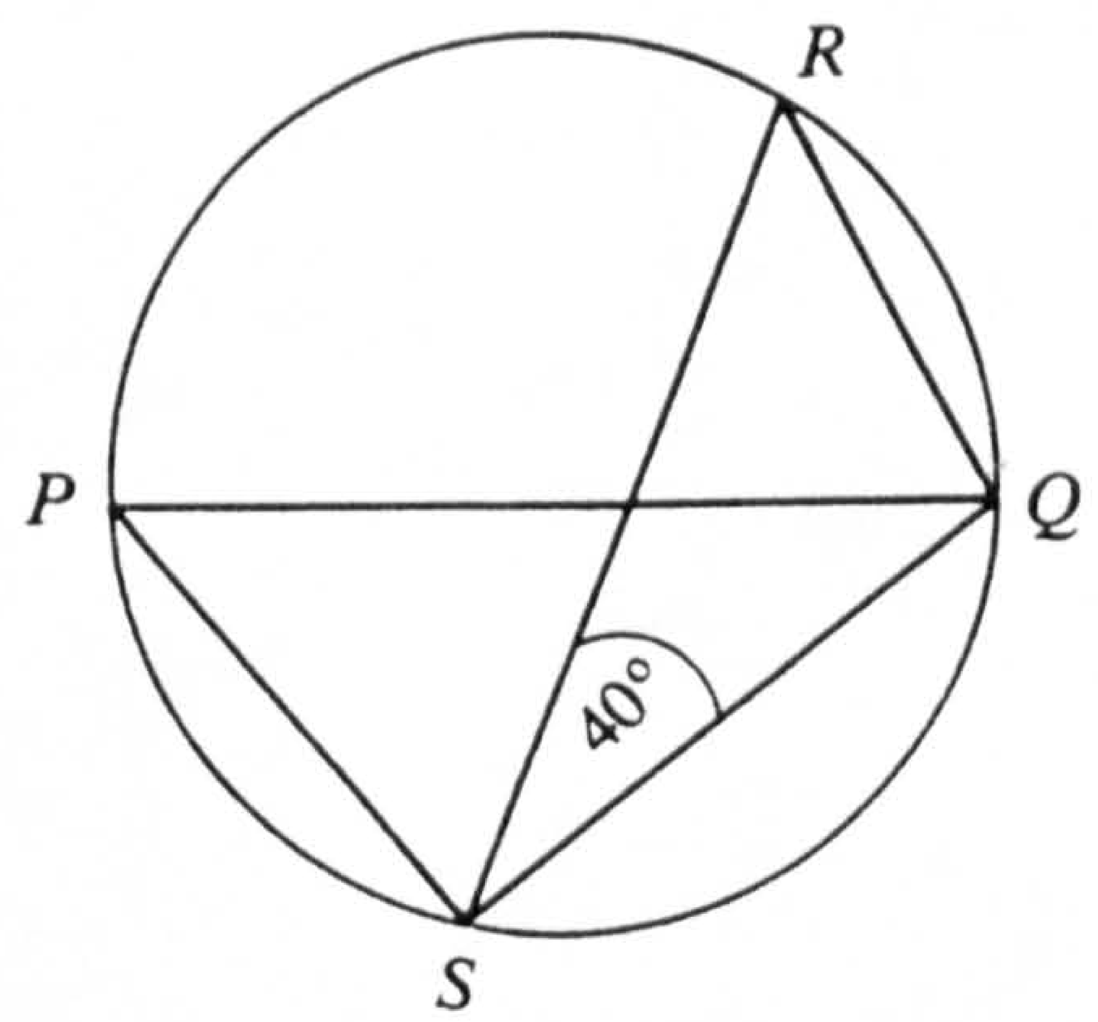
E $\frac{5}{3}$

13 A sum of £1000 was invested for one year, £600 at a rate of 10% per annum and the remainder at a rate of 12% per annum.

How much more interest would have been received if the entire £1000 had been invested at the higher rate of 12%?

A £8
B £12
C £20
D £40
E £72

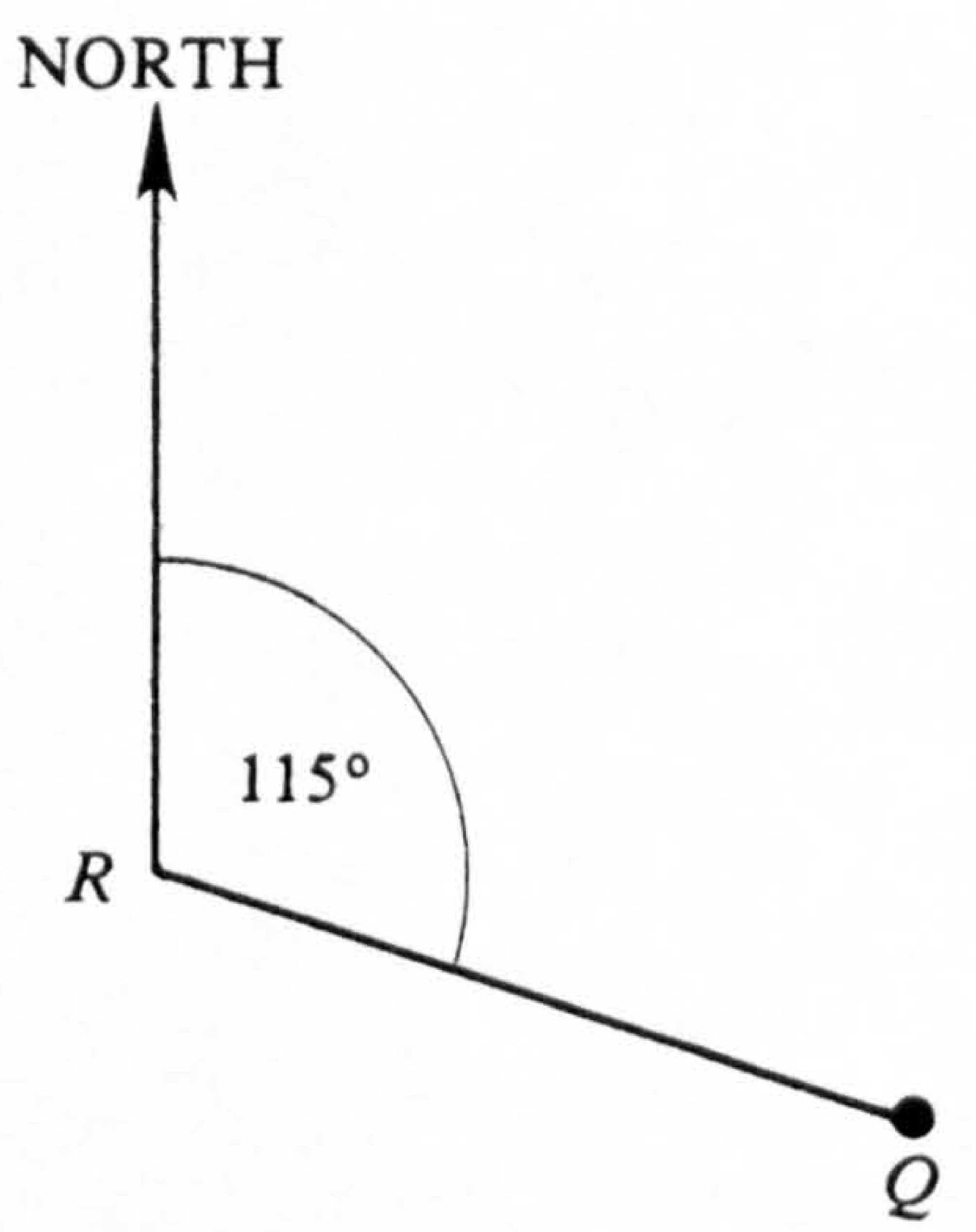
14



PQ is a diameter of the circle and *RS* is a chord. The size of angle *QSR* is 40°.
What is the size of the angle *PQR*?

A 40°
B 45°
C 50°
D 60°
E 80°

15



The bearing of *Q* from *R* is 115°.
What is the bearing of *R* from *Q*?

A 065°
B 115°
C 130°
D 230°
E 295°

16 The arithmetic mean of four numbers is 20, and the arithmetic mean of three of these numbers is 16. What is the fourth number?

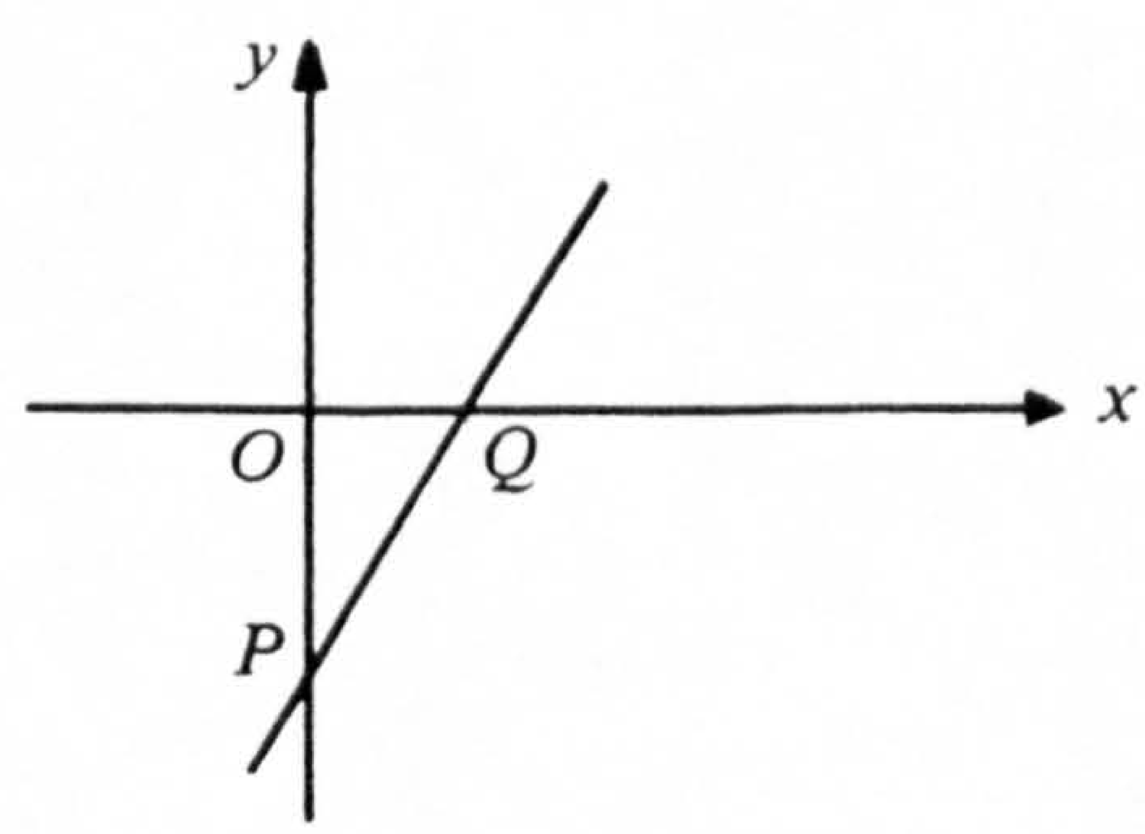
A 4
B 11
C 18
D 28
E 32

Multi-facet questions

In Questions **17** to **44** each group of questions refers to a set of data which may be a diagram, a table or a mathematical expression.
Each of Questions **17** to **44** is followed by five responses, **A**, **B**, **C**, **D** and **E**. For each question select the *best* response and mark its letter on the answer sheet.

Questions 17—19

The graph represents the line $y = 2x - 3$, cutting the y-axis in P and the x-axis in Q.



17 The coordinates of P are

A $(0, -3)$
B $(-3, 0)$
C $(-2, 0)$
D $(0, -2)$
E $(0, -1)$

18 The coordinates of Q are

A $(0, 2)$
B $(1\frac{1}{2}, 0)$
C $(3, 0)$
D $(0, -3)$
E $(0, 3)$

19 The gradient of line PQ is

A -3

B $-\frac{3}{2}$

C $-\frac{2}{3}$

D $\frac{3}{2}$

E 2

Questions 20 and 21

Positive numbers y and x are connected by the equation $y = kx^2$ where k is a constant.

20 When $k = \frac{1}{4}$ and $x = 6$ the value of y is

A $1\frac{1}{2}$
B $2\frac{1}{4}$
C 3
D 6
E 9

21 When $k = \frac{1}{4}$ and $y = 25$, the value of x is

A $2\frac{1}{2}$
B $6\frac{1}{4}$
C 10
D 20
E 100

Questions 22—24

For Questions **22** to **24** you are given the following information about the logarithms of the numbers represented by p and q.

number	logarithm (base ten)
p	1.50
q	0.50

22 What is the logarithm of pq?

A 0.75
B 1.00
C 1.5050
D 1.55
E 2.00

23 What is the logarithm of q^3?

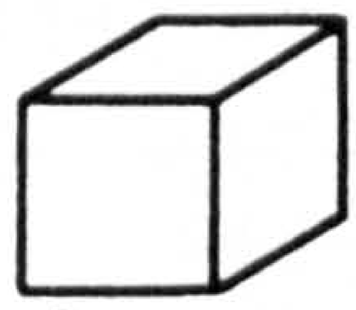
A 0.125
B 0.35
C 0.53
D 1.50
E 3.50

24 The number represented by p lies between

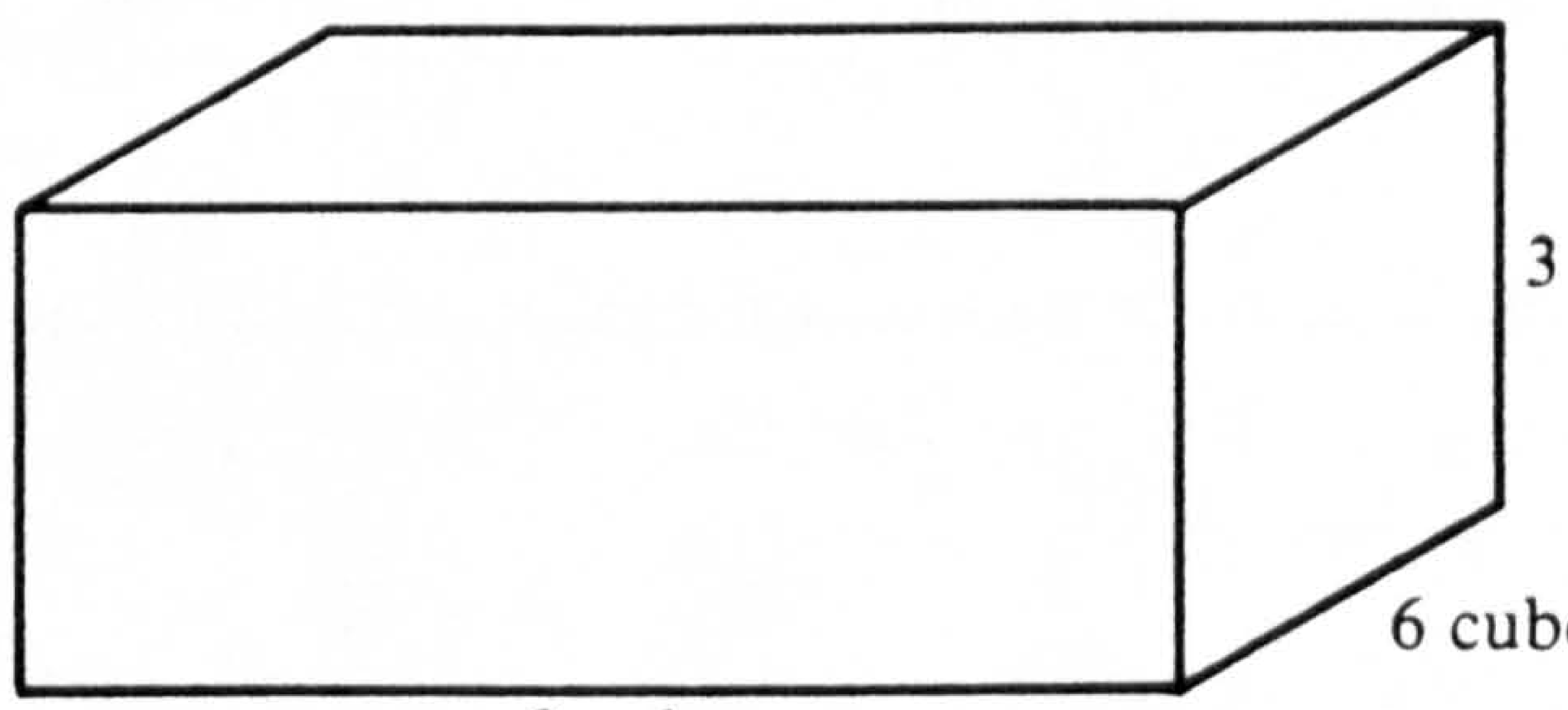
A 0 and 0.1
B 0.1 and 1
C 1 and 2
D 10 and 100
E 100 and 1000

Questions 25—27

A solid rectangular block is built up of cubes of edge 3 cm.



The block is 9 cubes long, 6 cubes wide and 3 cubes high.



25 The number of cubes needed to build the block is

A 6
B 12
C 54
D 81
E 162

26 The area of the base of the block, in cm^2, is

A 30
B 54
C 162
D 256
E 486

27 The number of cubes of edge 9 cm needed to make a block of the same dimensions is

A 3
B 6
C 9
D 12
E 27

turn over

Questions 28—30

The distance from Manchester to Blackpool is 84 km. The cost of a *day return* ticket for this journey is £3.36. An extract from the time-table shows the times for two trains.

Manchester Victoria Depart	13.00	13.45
Bolton	—	14.02
Chorley	—	14.17
Preston	—	14.40
Blackpool North Arrive	14.10	15.02

28 How much longer does the slower train take for the journey than the express?

A 2 minutes
B 7 minutes
C 10 minutes
D 15 minutes
E 17 minutes

29 What is the cost per km travelled on the day return ticket?

A $\frac{1}{4}$p
B $\frac{1}{2}$p
C 1p
D 2p
E 3p

30 The cost of the day return (£3.36) is 60 per cent of the cost of a period return. What is the cost of the period return ticket?

A £2.02
B £4.04
C £4.70
D £5.38
E £5.60

Questions 31—33

The surface area, S, of a sphere is given by the equation

$$S = 4\pi r^2$$

where r is the radius. Take π as $\frac{22}{7}$.

31 What is the surface area of a sphere with radius 7 cm?

A 44 cm^2
B 88 cm^2
C 616 cm^2
D 4312 cm^2
E 379456 cm^2

32 Air is added to a spherical balloon, initially of surface area X, so that its radius is doubled. What is the new surface area of the balloon?

A X
B $2X$
C $3X$
D $4X$
E $8X$

33 If the equation is rearranged, the radius of a sphere is given by

A $\frac{S}{4\pi}$

B $\frac{S}{8\pi}$

C $\frac{1}{2}\sqrt{\frac{S}{\pi}}$

D $\left(\frac{S}{4\pi}\right)^2$

E $\sqrt{S - 4\pi}$

turn over

Questions 34—36

There are 1500 employees in a factory, of whom 600 are female.

34 The percentage of female employees in the factory is

A 6%
B 40%
C 60%
D $66\frac{2}{3}$%
E 90%

35 The ratio of male to female employees expressed in its lowest terms is

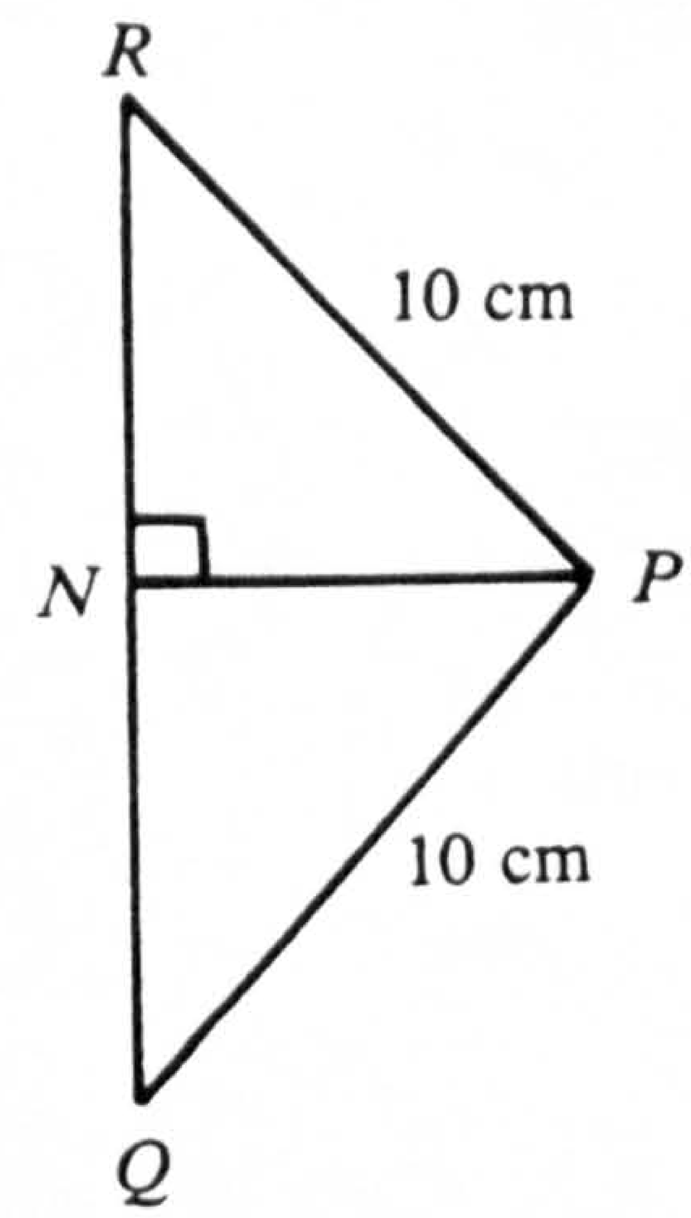
A 5 : 2
B 9 : 6
C 15 : 10
D 3 : 2
E 9 : 1

36 The average weekly wage of the male employees is £60 and of the female employees £45. What is the average weekly wage of all the employees?

A £52.50
B £54
C £63
D £70
E £105

Questions 37—39

In the triangle shown,
$RQ = 16$ cm.



37 The length of PN, in cm, is

A 3
B 4
C 5
D 6
E 8

38 The cosine of angle QRP is

A 0.5
B 0.6
C 0.75
D 0.8
E 1.33

39 The tangent of angle QPN is

A $\frac{3}{5}$
B $\frac{3}{4}$
C $\frac{4}{5}$
D $\frac{6}{5}$
E $\frac{4}{3}$

turn over

Questions 40—42
A moped is travelling at a constant speed of 30 km per hour.

40 How far will the moped travel in 10 minutes?

A $\frac{1}{3}$ km
B 3 km
C 5 km
D 6 km
E 10 km

41 How long will it take to travel 45 km?

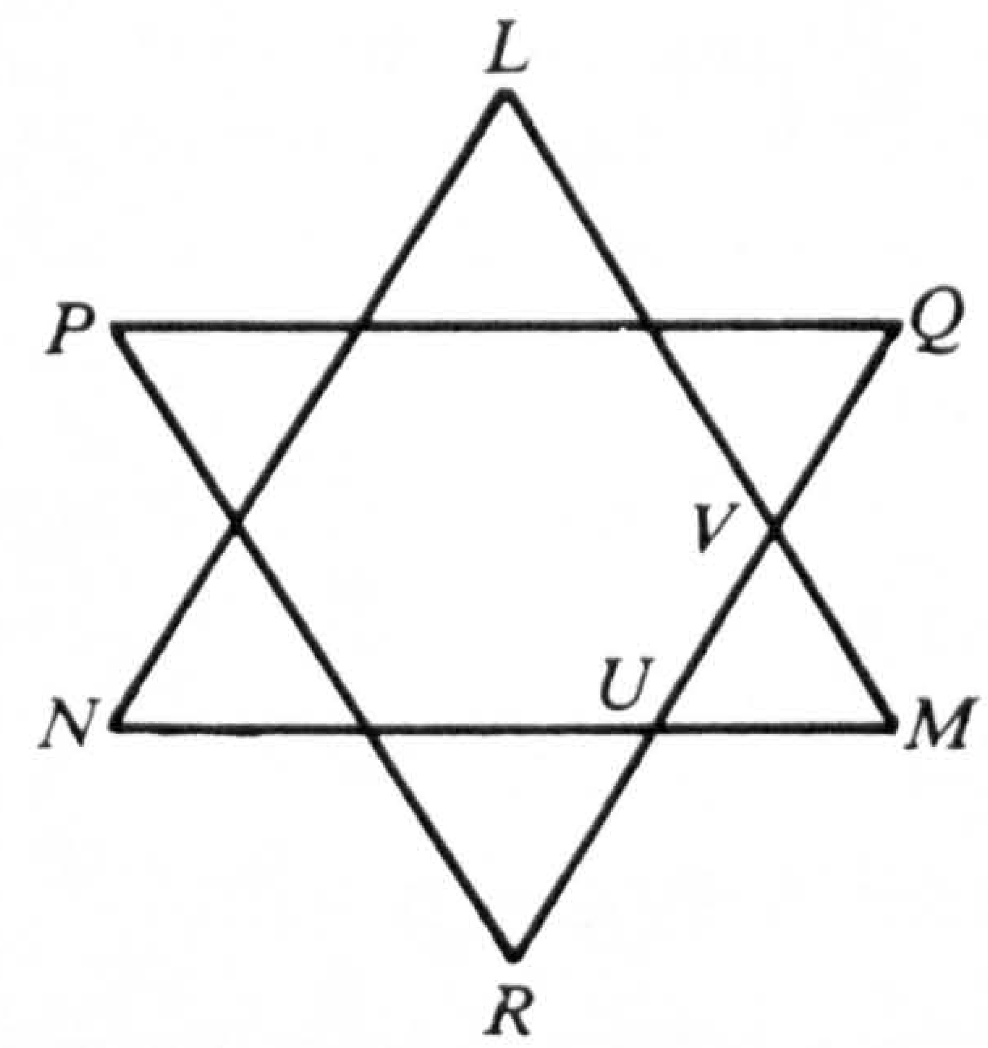
A 30 minutes
B 45 minutes
C $\frac{2}{3}$ hour
D 1 hour 15 minutes
E 90 minutes

42 How long will it take to travel 500 metres?

A 1 minute
B 2 minutes
C $\frac{3}{50}$ hour
D 12 minutes
E $\frac{1}{2}$ hour

Questions 43 and 44

The triangles *PQR* and *LMN* are equilateral. The points *P, L, Q, M, R, N* form a regular hexagon.



43 How many lines of symmetry has this shape?

A 0
B 2
C 3
D 4
E 6

44 The reflection of point *N* in a straight line drawn through *L* and *R* is point

A *U.*
B *L.*
C *V.*
D *M.*
E *Q.*

Matching pairs questions

In Questions **45** to **55** each group of questions has a set of responses, **A**, **B**, **C**, **D** and **E**. In each group each letter may be used once, more than once, or not at all.
For each question select the *best* response and mark its letter on the answer sheet.

Questions 45 and 46

A 0.0484
B 0.242
C 2.42
D 4.84
E none of **A**, **B**, **C** or **D**

The value of $\frac{1}{2.068}$ correct to 4 significant figures is 0.4836.

For each fraction given in Questions **45** and **46** select from **A** to **E** above the appropriate value correct to 3 significant figures.

45 $\frac{10}{2.068}$

46 $\frac{5}{20.68}$

Questions 47—49

A −3
B −2
C −1
D 1
E 2

For each of Questions **47** to **49**, select from **A** to **E** above the correct value of n when the calculation given in the question is written in the form $a \times 10^n$, where a is a number lying between 1 and 10.

47 1.6×100

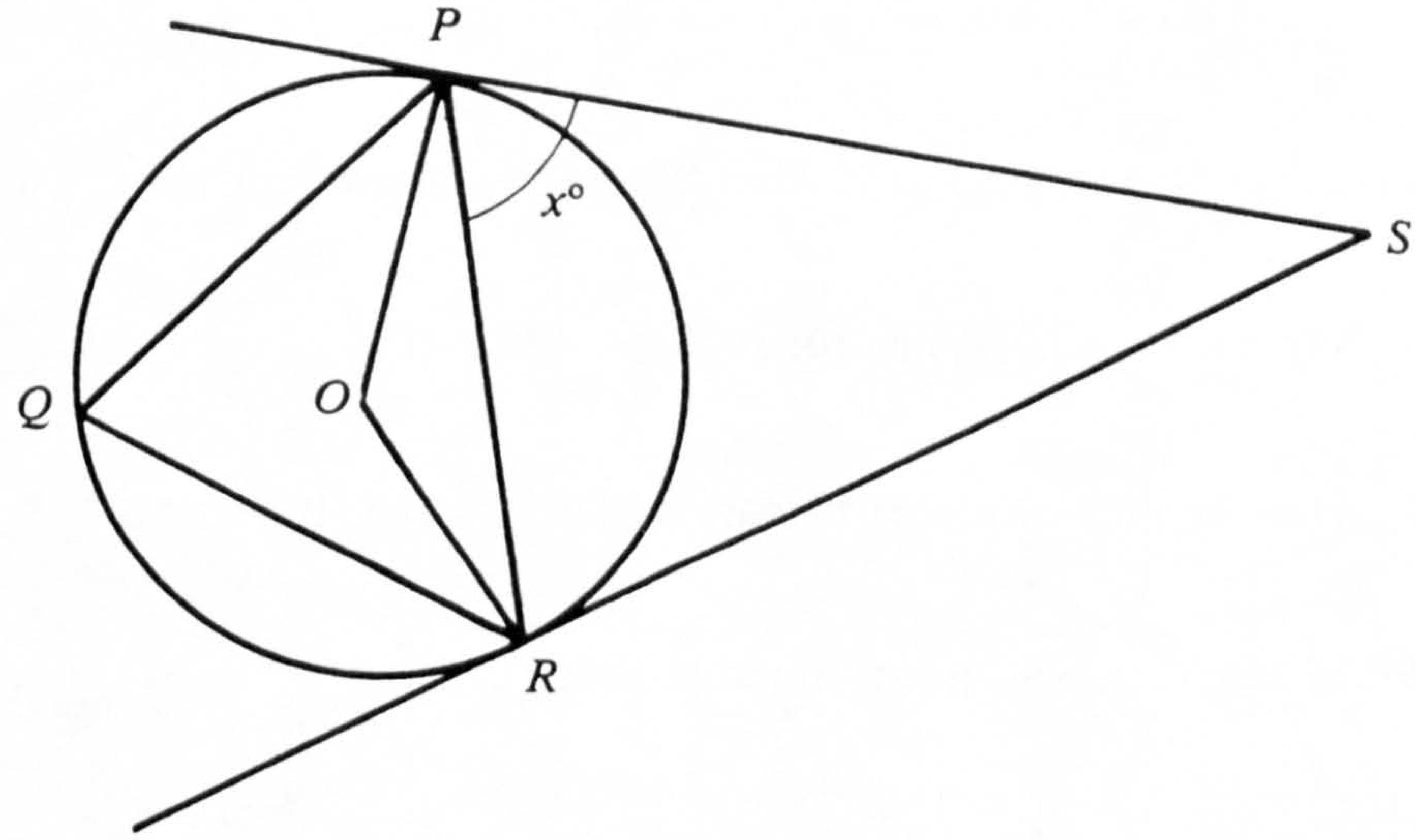
48 $2.4 \div 1000$

49 $(0.3)^2$

Questions 50—52

SP and *SR* are tangents
to the circle centre *O*.

Angle $RPS = x^\circ$.



A $(90-x)^\circ$

B x°

C $\frac{x^\circ}{2}$

D $(180-2x)^\circ$

E $2x^\circ$

For each of Questions **50** to **52** select from the list **A** to **E** above the size of the angle named.

50 angle *PRS*

51 angle *PQR*

52 angle *PSR*

Questions 53—55

A 1001
B 1010
C 1100
D 1111
E 1200

For each of Questions **53** to **55** select from **A** to **E** above the number which answers the question.

53 What is the nearest number to $(34)^2$?

54 How would the number 40 (base TEN) be written in base THREE?

55 In base TWO, what is the sum of 110 and 11?

Multiple completion questions

In each of Questions **56** to **60** *one or more* of the responses is/are correct. Decide which of the responses to the question is/are correct and mark **A**, **B**, **C**, **D** or **E** on the answer sheet as follows.

A if (1) alone is correct.
B if (3) alone is correct.
C if (1) and (2) only are correct.
D if (2) and (3) only are correct.
E if (1), (2) and (3) are correct.

Summarised directions for recording responses to multiple completion questions				
A (1) alone	**B** (3) alone	**C** (1) and (2) only	**D** (2) and (3) only	**E** (1), (2) and (3)

56 Which of the following is/are correct?

$(x+2)(x-3)$ is positive when

(1) $x = -1$
(2) $x = -3$
(3) $x = +4$

57 Which of the following statements is/are correct?

(1) The diagonals of any rhombus are perpendicular.
(2) In any square, the diagonals bisect the angles.
(3) In any trapezium, the diagonals are equal.

58 Which of the following is/are true?

(1) $2 \times 3 > 3 \times 2$
(2) $2 - 3 > 3 - 2$
(3) $2^{-3} > 3^{-2}$

59 Which of the following is/are correct?

(1) $0.2 \times 0.04 = 0.08$
(2) $0.2 \div 0.04 = 0.5$
(3) $0.2 - 0.04 = 0.16$

60 For the graph of $y = x^2 + 5x + 4$, which of the following statements is/are correct?

(1) The graph is a straight line.
(2) When $y = 0$, $x = 4$ or 1.
(3) The y intercept has a value of 4.

Appendix B

Northern Examining Association

ASSOCIATED LANCASHIRE SCHOOLS EXAMINING BOARD
JOINT MATRICULATION BOARD
NORTH REGIONAL EXAMINATIONS BOARD
NORTH WEST REGIONAL EXAMINATIONS BOARD
YORKSHIRE AND HUMBERSIDE REGIONAL EXAMINATIONS BOARD

Joint GCE O-level and CSE Examinations
MATHEMATICS PAPER 2
Tuesday 20 May 1986 1.30 p.m. — 3.30 p.m.

Surname ....

Other names ....

Centre names ....

Centre number ....

Attempt all the questions.

Answer each question in the space provided.

All necessary details of working, including rough work, must be shown with the answer.

You will require a ruler and a pair of compasses.

You must have available at least one of the following: a set of three-figure or four-figure mathematical tables; a slide rule; a suitable calculator.

A reference material leaflet is provided.

Supplementary sheets of writing paper and graph paper may be obtained from the supervisor if required. All sheets issued must be tied loosely to the back of this question/answer booklet with string.

Diagrams are not necessarily drawn to scale.

The marks allocated to each question are printed in the right-hand margin.

For examiner's use

Section

1 Evaluate

(i) 5 x (3 + 4),

①

....

.... *(1)*

(ii) (5 x 3) + 4.

②

....

.... *(1)*

2 Given that $3p = 7q$, calculate the value of the ratio $p : q$.

....

③ ....

....

.... *(1)*

3 The mass of a new 2p coin is 7g. Calculate in kg the mass of £35 worth of new 2p coins.

....

④ ....

....

....

....

....

.... *(3)*

4 A woman hires a car for her holidays. The charge is £x for each day the car is hired and y pence for each kilometre driven. Find an expression in x and y for the total cost, in £, if she hires the car for 10 days and drives a total of 1200 km.

⑤

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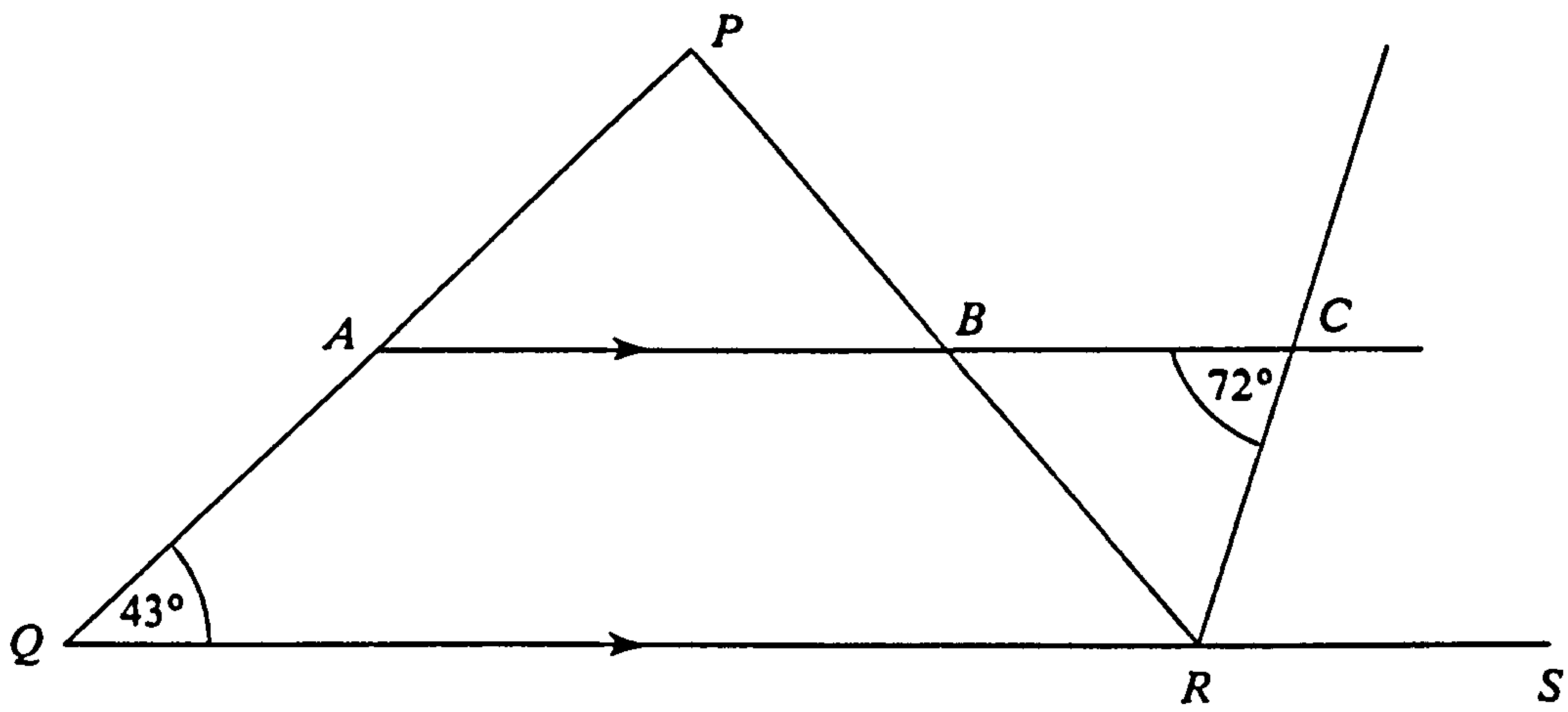
....

....

.... *(3)*

Section

5



In the diagram (not drawn to scale), the straight line ABC is parallel to the straight line QRS, QA and RB produced meet at P and RC is the bisector of the angle BRS. Angle $AQR = 43°$ and angle $BCR = 72°$.
Calculate

(6) (i) the size of angle CRS,

.... (1)

(7) (ii) the size of angle CBR,

....

....

.... (2)

(8) (iii) the size of angle APB.

....

....

....

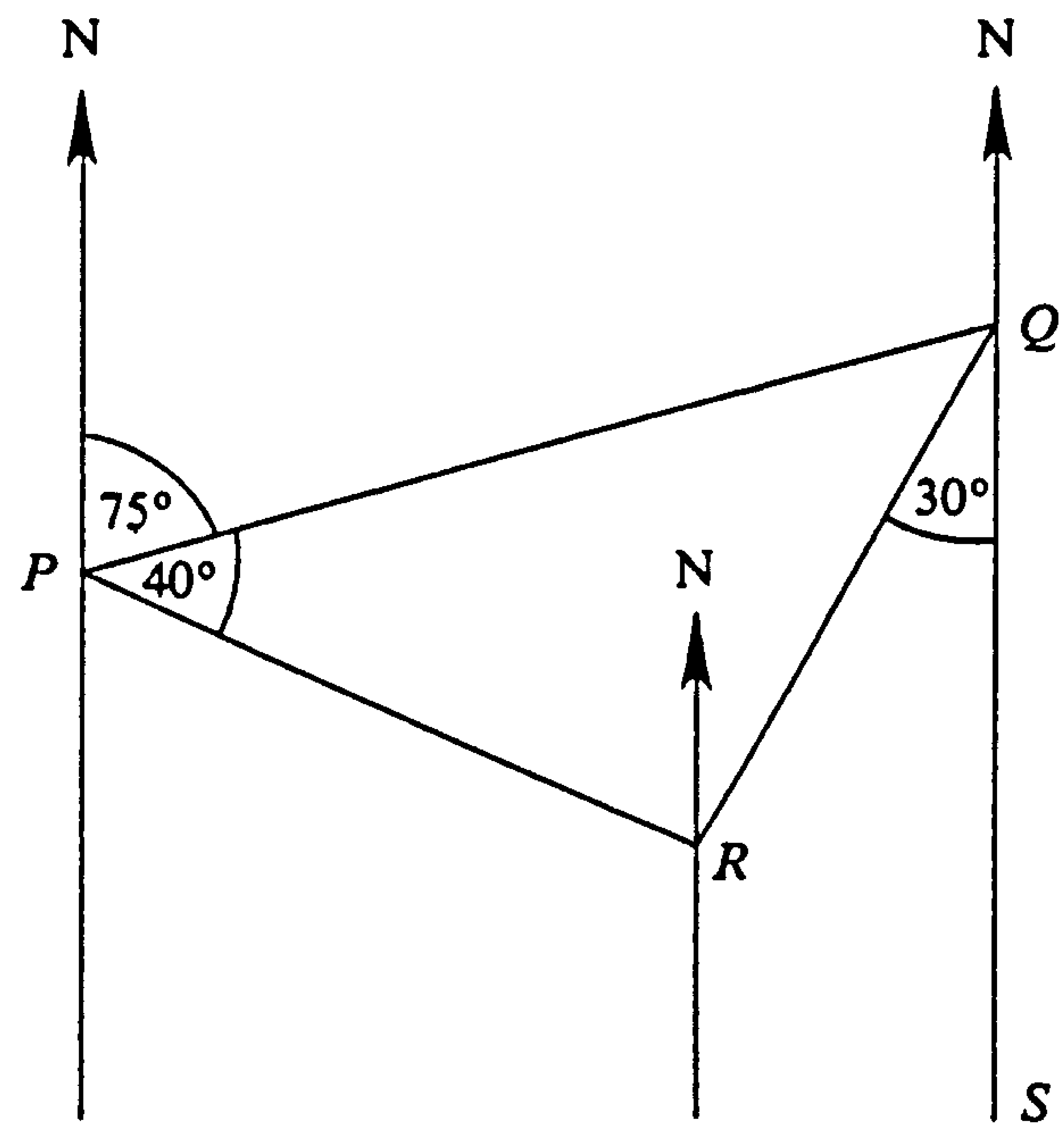
.... (2)

6 Factorise $9y^2 - 25$.

(9) ....

.... (1)

7



In the diagram (not drawn to scale) the bearing of Q from P is 075°. Angle QPR = 40° and angle RQS = 30°.

Calculate

(i) the bearing of Q from R,

(10) ....

.... (1)

(ii) the bearing of R from Q,

(11) ....

.... (1)

(iii) the bearing of P from R.

....

(12) ....

....

.... (1)

8 An athlete runs 800 m in 2 minutes. Calculate her average speed in km per hour.

(3)

9 Given that $9x^2 + 12x + k^2 = (3x + k)^2$, calculate the value of k.

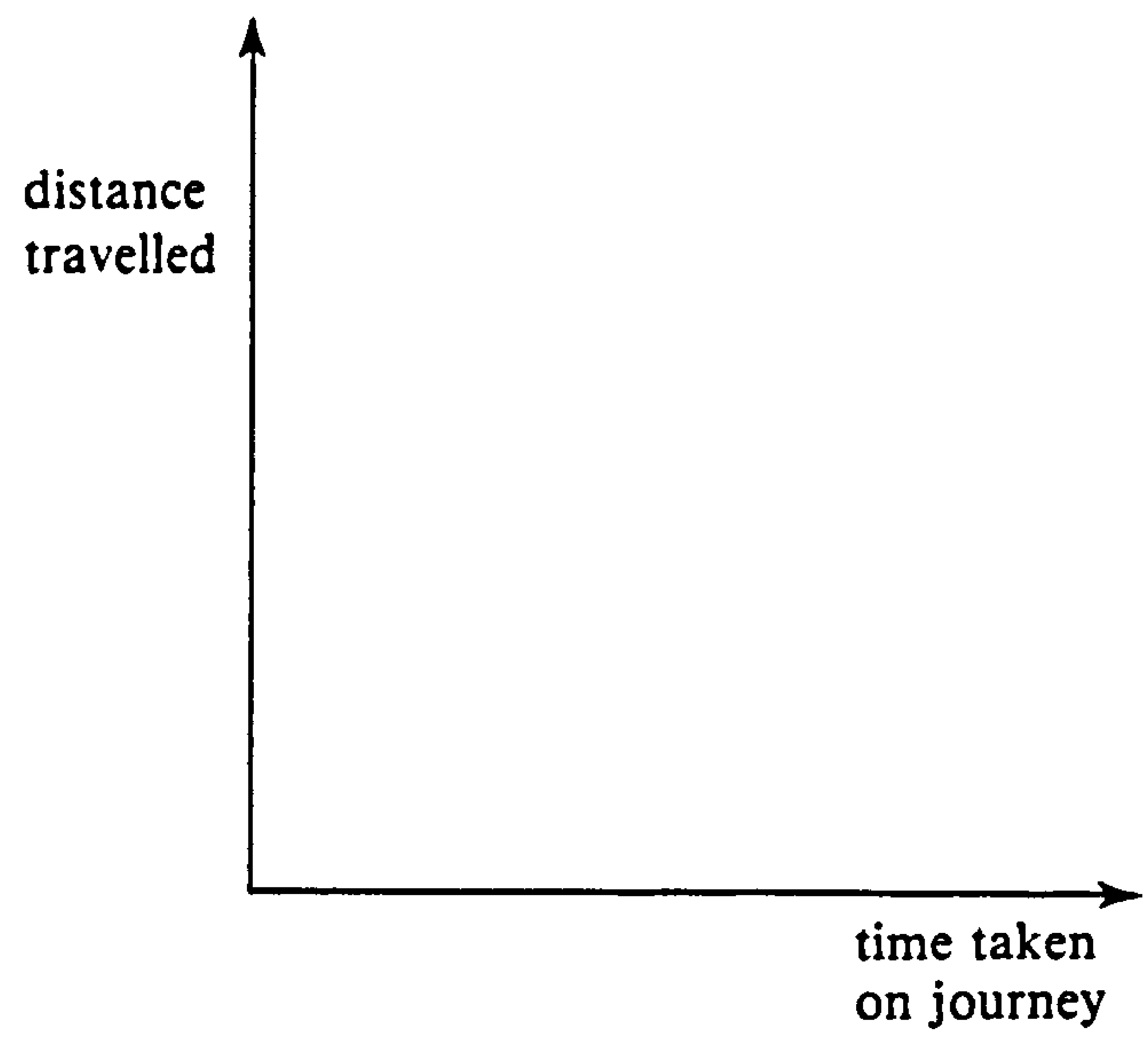
(3)

10 Given that $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, calculate the exact value of c when $a = 2\frac{1}{6}$ and $b = \frac{13}{15}$.

(4)

turn over

11



(16)

A girl leaves home to travel to school. She walks at a constant rate to her friend's home, where she waits until her friend is ready to leave. The two girls are then taken to school by car which travels at a constant speed. Draw three straight lines on the given diagram to illustrate the journey of the first girl. *(3)*

12 A family of four people go to a restaurant with another family of three. The bill is £42 plus VAT at 15%.

(i) Calculate the total bill which is paid.

(17)

....

....

....

....

....

.... *(3)*

(ii) The total bill is divided in proportion to the number of people in each family. Calculate the amount paid by the larger family.

(18)

....

....

....

....

....

.... *(3)*

13 Given that $a = 2$, $b = 0$ and $c = -\frac{1}{8}$,
evaluate, giving each answer in its simplest form,

(i) abc,

(19)

....

.... *(1)*

(ii) $2a - c$,

(20)

....

....

.... *(1)*

(iii) ac^2,

(21)

....

....

.... *(2)*

(iv) $32c^3$,

(22)

....

....

....

.... *(2)*

(v) $c^{\frac{1}{3}}$.

(23)

....

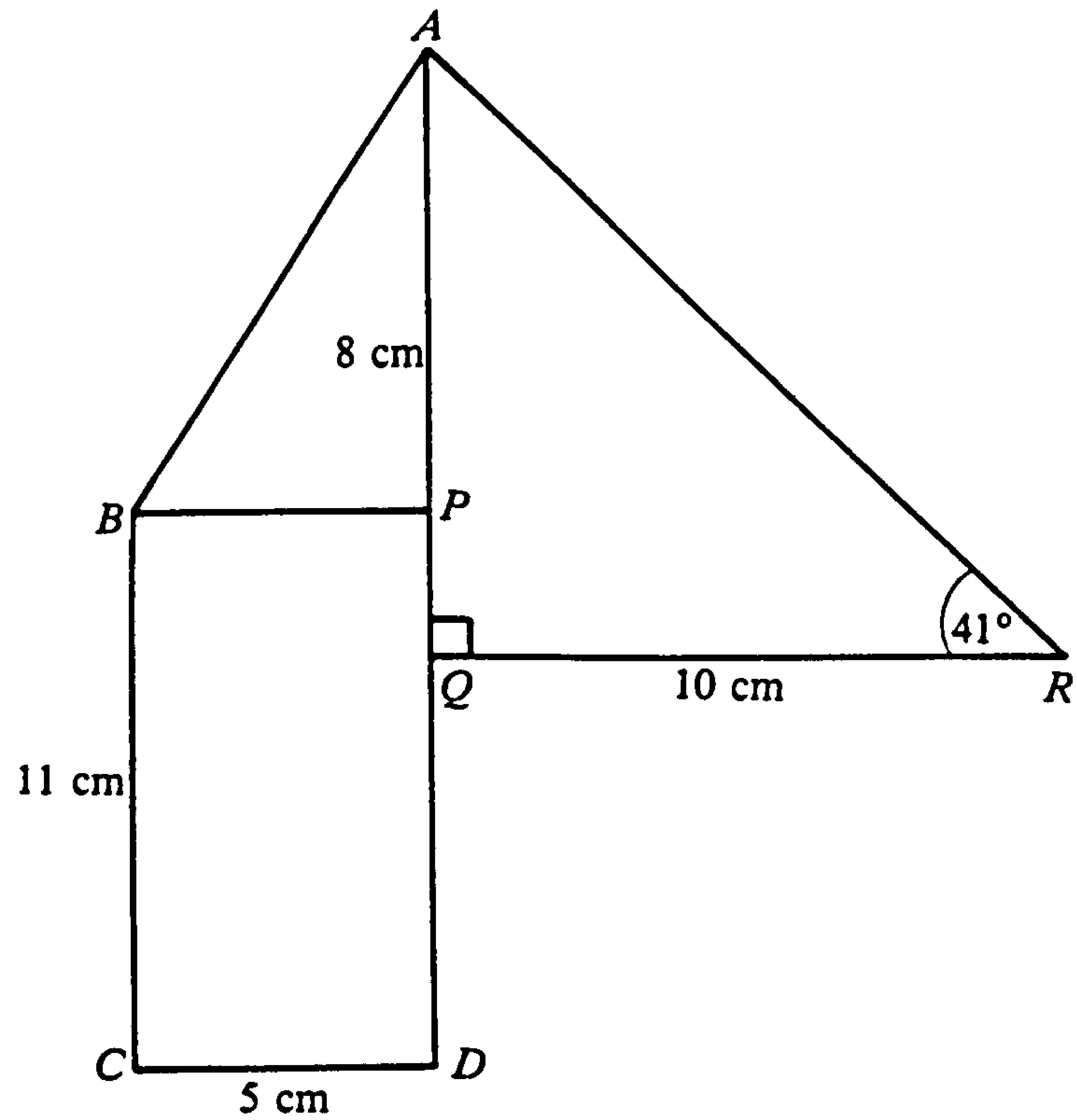
....

....

.... *(2)*

turn over

14



In the diagram (not drawn to scale), *BCDP* is a rectangle, *APQD* is a straight line and angle *PQR* is a right angle.
$AP = 8$ cm, $BC = 11$ cm, $CD = 5$ cm, $QR = 10$ cm and angle $ARQ = 41°$.

Calculate
(i) the area of the trapezium *ABCD*,

(24)

....

....

....

....

....

.... *(3)*

(ii) the length of *AQ*, giving your answer correct to two decimal places,

(25)

....

....

....

....

.... *(2)*

(iii) the length of AR, giving your answer correct to two decimal places,

(26)

....

....

....

....

.... *(3)*

(iv) the size of angle BAP, giving your answer correct to the nearest degree.

(27)

....

....

....

....

.... *(3)*

15 Given that $y = mx + c$, find an expression for x in terms of y, m and c.

(28)

....

....

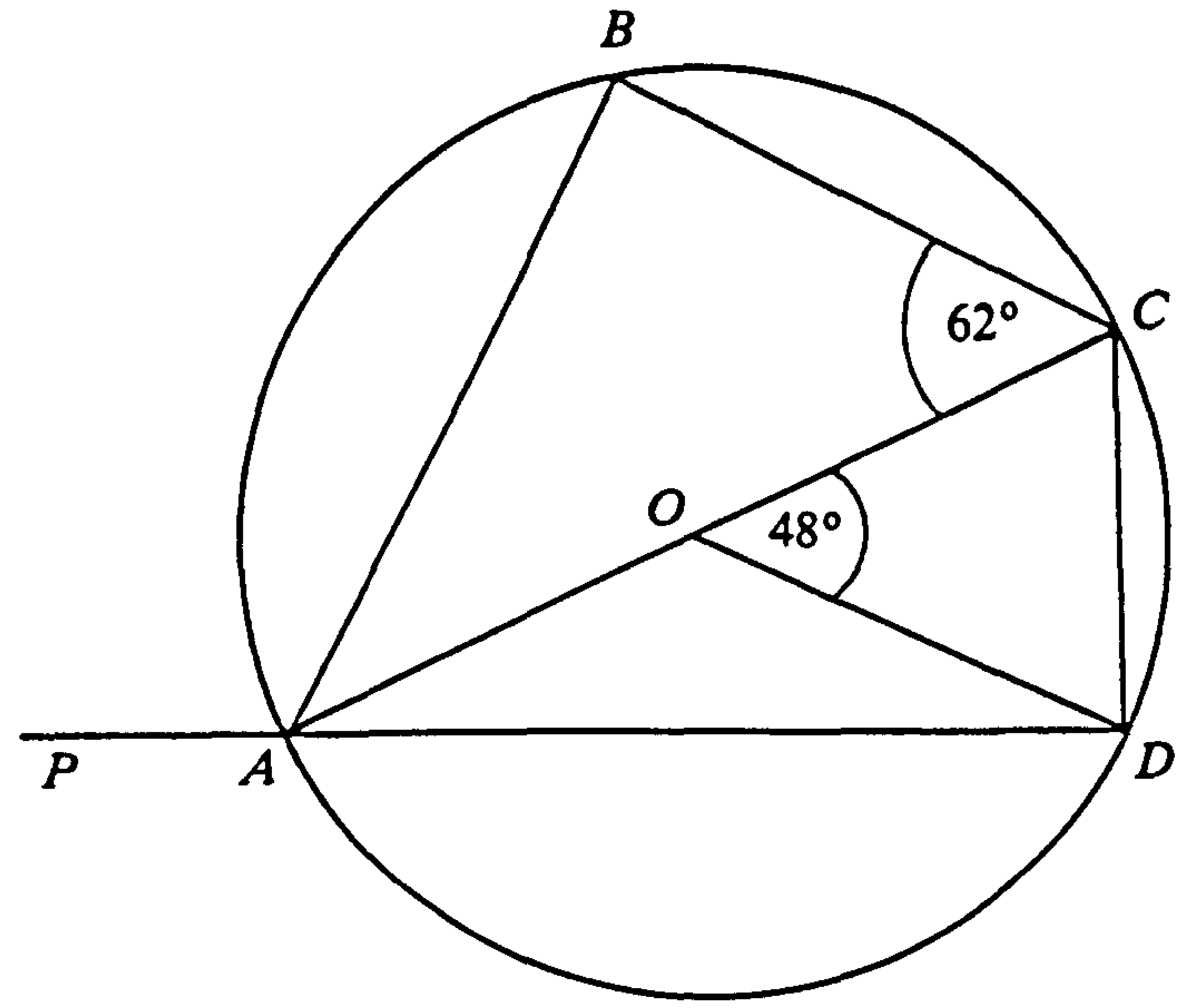
....

....

.... *(2)*

turn over

16



In the diagram (not drawn to scale), O is the centre of the circle and DAP and AOC are straight lines.

Angle $COD = 48°$ and angle $ACB = 62°$.
Calculate

(i) the size of angle ABC,

(29) .... (1)

(ii) the size of angle DAO,

(30) ....

.... (1)

(iii) the size of angle ACD,

(31) ....

....

....

....

.... (1)

(iv) the size of angle BAP.

(32) ....

....

.... (1)

17 A straight stick is placed with its ends on the circumference of a circular hoop of radius 29 cm. The distance of the stick from the centre of the hoop is 20 cm. Calculate the length of the stick.

(33)

....

....

....

....

....

....

.... *(4)*

turn over

18 (i) Calculate the sum of the interior angles of a pentagon.

(34)

....

....

....

....

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.... *(3)*

(ii) The angles of a pentagon measured in degrees are $p - q$, $p + 2q$, $p - 3q$, 190 and 150. Use this information to write down and simplify an equation in terms of p and q.

(35)

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.... *(2)*

(iii) The angles of an isosceles triangle measured in degrees are p, p and q. Use this information to write down and simplify another equation in terms of p and q.

(36)

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.... *(1)*

(iv) Solve your two equations simultaneously to find the values of p and q.

(37)

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.... *(4)*

19 A model of a racing yacht is made to scale of 1:40.

(i) The length of the yacht is 8 m. Calculate, in its simplest form, the length of the model.

(38)

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.... *(2)*

(ii) The area of the sail of the model is $0.04\,m^2$. Calculate the area of the sail of the yacht.

(39)

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.... *(3)*

20 Solve the equations

(i) $2(x-5) = 7x$, **(3)**

(ii) $\frac{a}{2} - \frac{2a}{7} = 6$, **(3)**

(iii) $3p^2 - 9p = 0$, **(3)**

(iv) $11y - 2y^2 = 15$.

(43)

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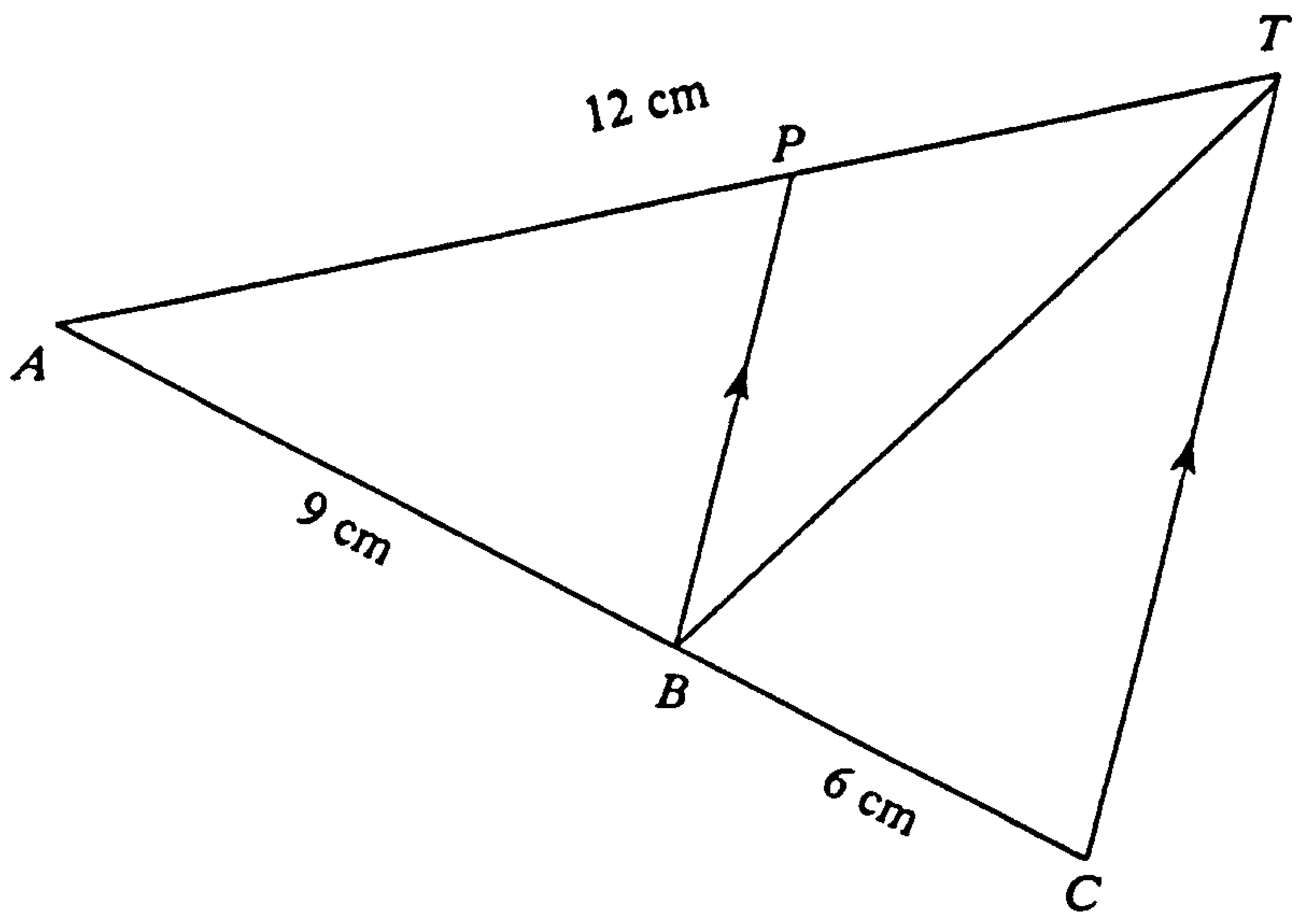
.... *(4)*

21



In the diagram (not drawn to scale), *APT* and *ABC* are straight lines and *BP* is parallel to *CT*. $AB = 9$cm, $BC = 6$cm and $AT = 12$cm.

(44) (i) Name a triangle which is similar to the triangle ABP.

.... *(1)*

(45) (ii) Calculate the length of PT.

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.... *(3)*

(46)

(iii) Calculate, in its simplest form, the ratio $\frac{\text{area of } \Delta\, ABP}{\text{area of } \Delta\, BPT}$.

....

....

....

.... (2)

(47)

(iv) Calculate, in its simplest form, the ratio $\frac{\text{area of } \Delta\, ACT}{\text{area of } \Delta\, ABP}$.

....

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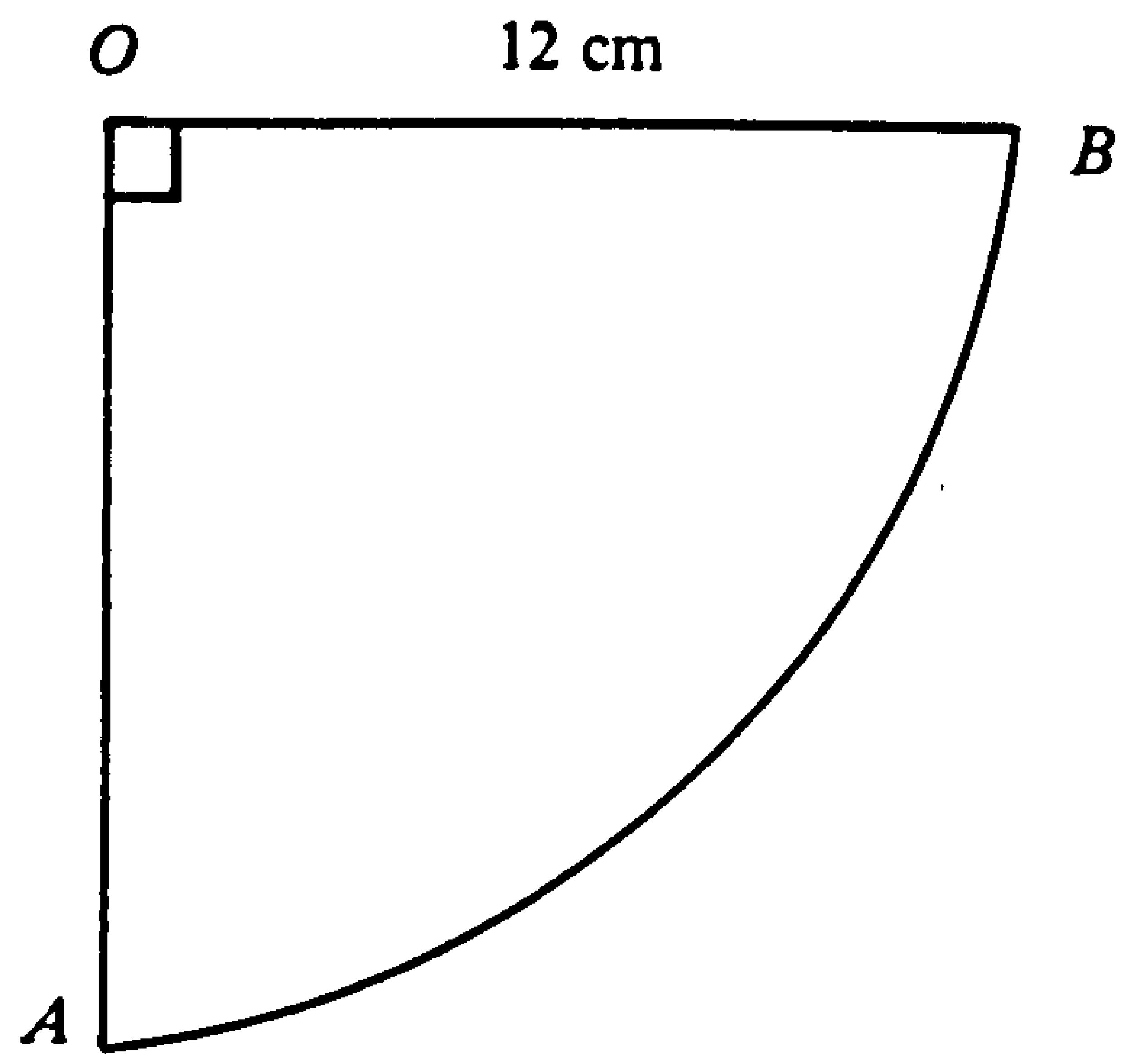
....

.... (2)

turn over

22



The diagram (not drawn to scale), shows a piece of paper cut out in the shape of a quarter of a circle of radius 12 cm. O is the centre of the circle.

The paper is formed into a cone so that OB coincides with OA and the arc AB forms the circumference of the base of the cone.

Calculate

(i) the circumference of the base of the cone leaving your answer in terms of π,

48

(2)

(ii) the radius of the base of the cone,

49

(2)

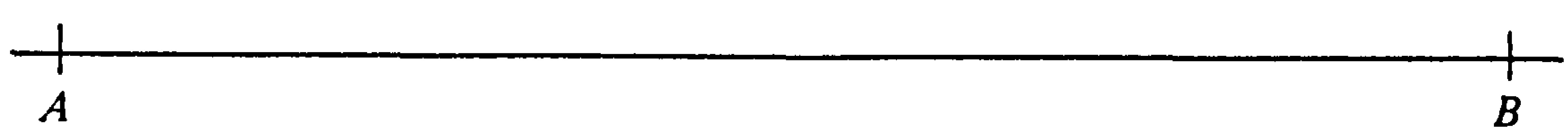
(iii) the area of the base of the cone leaving your answer in terms of π.

50

(1)

23 Use ruler and compasses only in this question and show clearly all your construction arcs.

(i) Use the given line AB to construct the convex quadrilateral $ABCD$ such that angle $BAD = 60°$, $AD = 7.3$cm, $CD = 5.4$cm and $BC = 9.3$cm. *(4)*



On your figure above

(ii) without actual measurement, construct the mid point M of the line AD. Hence mark clearly the positions of the points P and Q which are the centres of the two circles of radius 4cm which touch the line AD at its mid point, *(3)*

(iii) construct the line CX where X is the point on AB such that CX is as short as possible. *(2)*

Appendix B

Northern Examining Association

ASSOCIATED LANCASHIRE SCHOOLS EXAMINING BOARD
JOINT MATRICULATION BOARD
NORTH REGIONAL EXAMINATIONS BOARD
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Joint GCE O-level and CSE Examinations
MATHEMATICS PAPER 3

Topic A Algebra, Trigonometry and Calculus
Topic B Choice, Chance and Statistics

Wednesday 21 May 1986

9.30 am — 9.45 am (reading time)
9.45 am — 11.30 am (examination)

The first 15 minutes should be spent reading the question paper. Do not begin to write until you are told to do so. Answer all questions in the separate answer book or answer sheets provided.

The questions on each Topic appear in two sections.

Answer any combination of questions which gives a total of not more than 12 units.

Each Section 1 question counts as one unit.
Each Section 2 question counts as two units.
You may attempt any questions.

Diagrams are not necessarily drawn to scale.

All necessary details of working, including rough work, must be shown with the answer.

Graph paper may be obtained from the supervisor. All sheets issued must be tied loosely to the back of your script.

You must have available at least one of the following: a set of 3 figure or 4 figure mathematical tables; a slide rule; a suitable calculator.

A reference material leaflet is provided.

A grid showing the questions answered is provided as a separate sheet; at the end of the examination this sheet is to be tied loosely to the front of your script.

TOPIC A
ALGEBRA, TRIGONOMETRY AND CALCULUS

SECTION 1

A1/1 (a) Solve the equation

$$\frac{2}{x} + \frac{3}{2x} = \frac{1}{2}.$$

(b) Express as a single fraction in its lowest terms

$$\frac{2}{x+2} + \frac{3}{x^2-4}.$$

1 unit

A1/2 (a) Find the values of x lying between 0 and 180 for which
$\sin x° = 0.592.$

(b) In the triangle ABC, $AB = 5$ cm, $BC = 6$ cm and $CA = 4$ cm.
Calculate, to the nearest degree, the size of angle BAC.

1 unit

A1/3 P and Q are the two points on the curve $y = x^2$ whose x-coordinates are 2 and 3 respectively.

(i) Calculate the y-coordinate of P and the y-coordinate of Q.

(ii) Calculate the gradient of the line PQ.

(iii) Find the equation of the line PQ in the form $y = mx+c$.

(iv) Write down the equation of the line through the origin parallel to the line PQ.

1 unit

A1/4 The quantity P varies jointly as b and h.
When $b = 10$ and $h = 4$ the value of P is 20.

(i) Find the equation connecting P, b and h.

(ii) Find the value of h when $P = 9$ and $b = 3$.

(iii) When b and h are each trebled in value, by what factor is the corresponding value of P multiplied?

1 unit

A1/5 The equation of a curve is
$y = 4x - x^2.$

(i) Find the value of x at the point on the curve where the gradient of the curve is -2.

(ii) What is the gradient of the curve at the point on it where y has its maximum value? Find this maximum value.

1 unit

A1/6 An elastic band is stretched so that the rate of increase of its length, L cm, after t seconds is given by

$$\frac{dL}{dt} = 0.2 - \frac{t}{100}.$$

The unstretched length of the elastic band is 5 cm.

(i) Find an expression for the length of the elastic band after t seconds.

The stretching continues for 20 seconds.

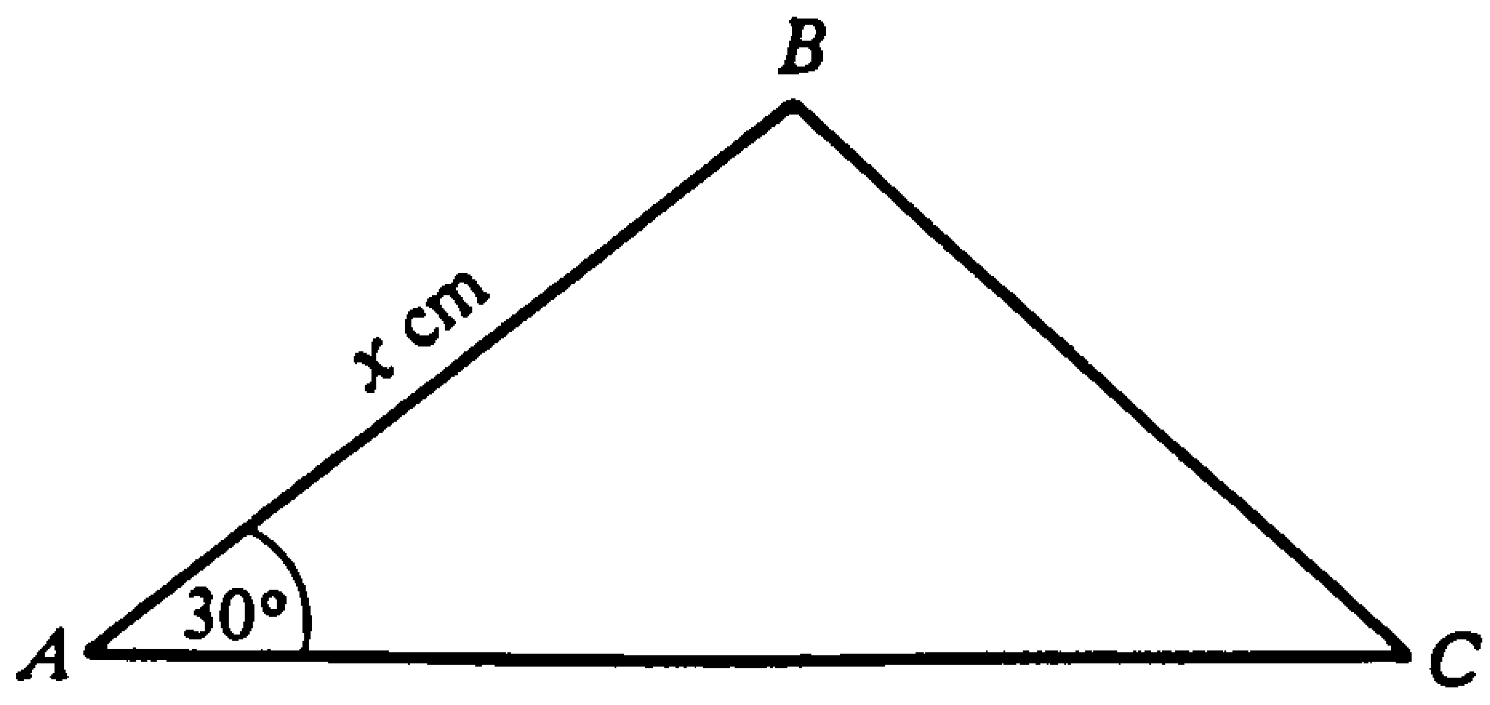
(ii) Calculate the final length of the elastic band.

1 unit

TOPIC A

SECTION 2

A2/1



In triangle ABC, angle $A = 30°$ and side AC is 4 cm longer than side AB. The length of AB is taken as x cm.

(i) Write down the length of AC in terms of x.

(ii) Show that the area of the triangle in cm^2 is given by the expression
$$\tfrac{1}{4}(x^2 + 4x).$$

When the area of the triangle is 15 cm^2, calculate

(iii) the lengths of AB and AC,

(iv) the length of BC, giving the answer correct to the nearest 0.1 cm. *2 units*

A2/2 (i) Copy and complete the table below which gives the values of y for values of x from -1 to 4, where $y = x^2(4 - x)$.

x	-1	0	1	2	3	3.5	4
x^2		0		4		12.25	
$4 - x$		4		2		0.5	
$y = x^2(4 - x)$		0		8		6.1	

(ii) Draw the graph of $y = x^2(4 - x)$ for values of x from -1 to 4. Take 2 cm to represent 1 unit on the x-axis and 2 cm to represent 2 units on the y-axis.

(iii) On the graph shade in the region whose area is represented by
$$\int_2^3 x^2(4 - x)\,dx.$$

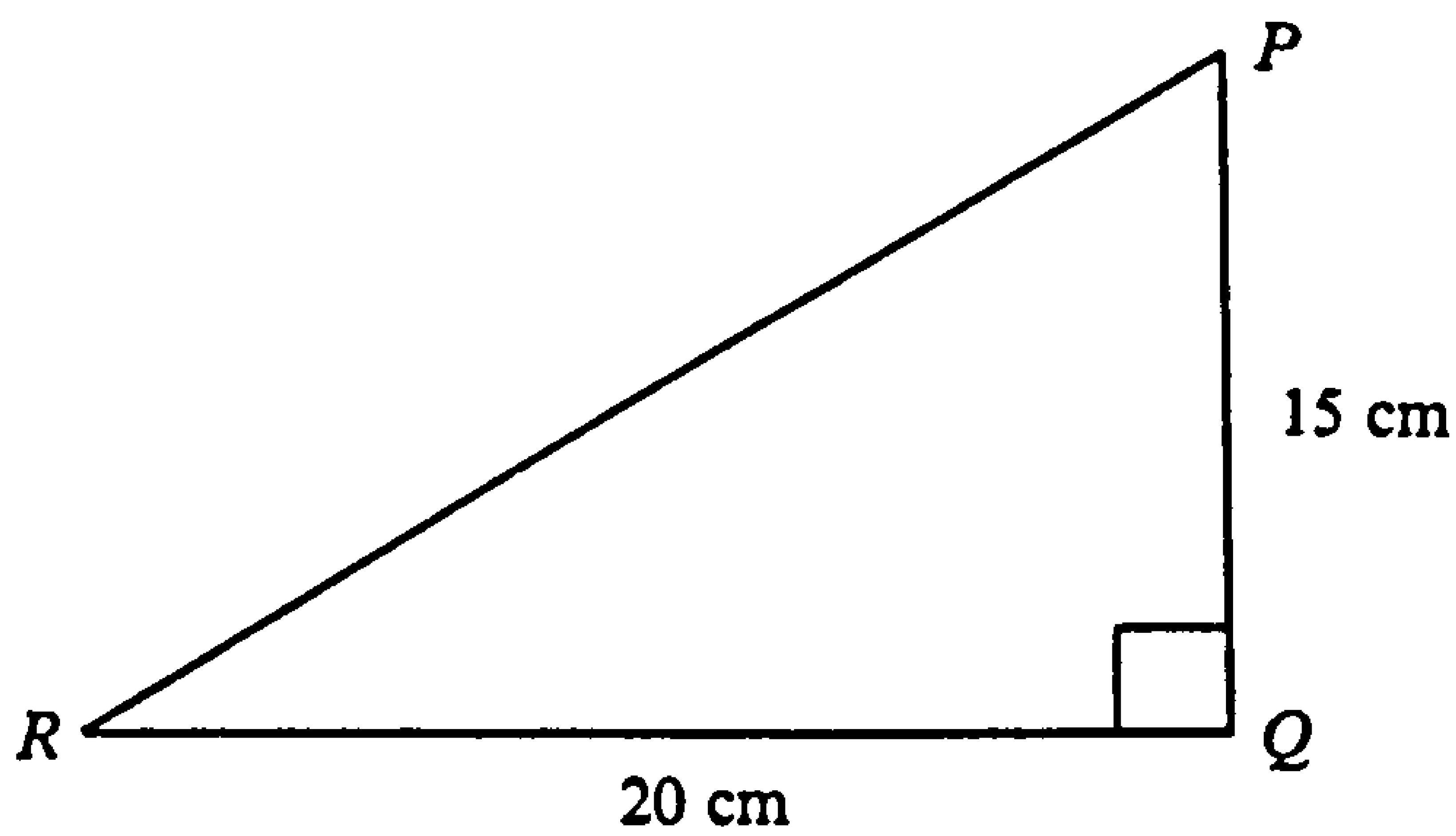
(iv) Find the gradient of the curve whose equation is $y = x^2(4 - x)$ at the point on it where $x = 1$. *2 units*

turn over

11 A straight line is 7.2 cm long. $\frac{3}{8}$ of the length of the line is

A 0.375 cm
B 0.9 cm
C 2.4 cm
D 2.7 cm
E 3.75 cm

12



In this triangle (shown above) the value of sin P is

A $\frac{3}{5}$

B $\frac{3}{4}$

C $\frac{4}{5}$

D $\frac{4}{3}$

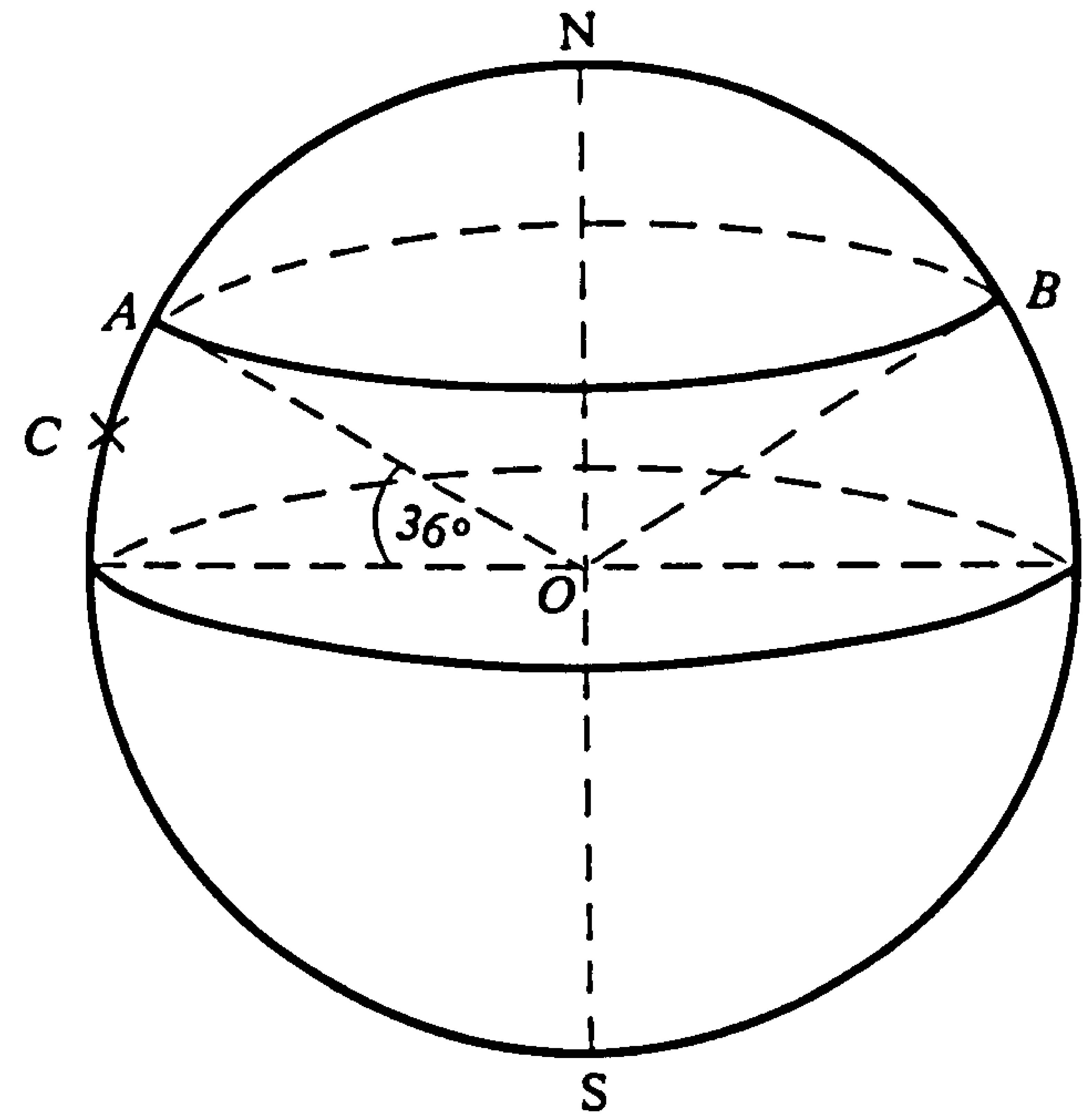
E $\frac{5}{3}$

13 A sum of £1000 was invested for one year, £600 at a rate of 10% per annum and the remainder at a rate of 12% per annum.

How much more interest would have been received if the entire £1000 had been invested at the higher rate of 12%?

A £8
B £12
C £20
D £40
E £72

A2/5



In this question take the Earth as a sphere of radius 6370 km.

Two places A and B both lie on the same circle of latitude 36°N and their longitudes differ by 180°.

(i) Write down the size of angle AOB, where O is the centre of the Earth.

Calculate the distance between A and B

(ii) measured along the circle through the North Pole,

(iii) measured along the circle of latitude.

A third place C is 1550 km due south of A.

(iv) Calculate the latitude of C to the nearest degree. *2 units*

A2/6 (a) A ball is thrown vertically downwards from a height of 20 m above the ground. The distance, s metres, through which it travels in t seconds after being thrown is given by

$$s = 8t + 5t^2.$$

Find

(i) how far the ball is above the ground half a second after being thrown,

(ii) an expression, in terms of t, for its speed t seconds after being thrown,

(iii) the speed with which it was thrown.

(b) Evaluate

$$\int_0^2 (2x - 1)^2 \, dx.$$

2 units

turn over

TOPIC B

CHOICE, CHANCE AND STATISTICS

SECTION 1

B1/1 The results of a class test marked out of 10 are shown in the table below.

Mark	Frequency
0	0
1	1
2	3
3	4
4	3
5	5
6	5
7	0
8	2
9	2
10	0

(i) 5 members of the class were absent when the test was taken. How many pupils are there in the class?

(ii) What was the mean mark achieved by those who took the test?

The 5 absentees took the test at a later date, and their mean mark was 5.4.

(iii) What was the mean mark achieved by the whole class? *1 unit*

B1/2 A survey was made to find the numbers of different types of vehicles passing a certain point on a main road in one hour. The results were then represented on a pie chart, and the information from which the diagram was constructed is given below.

Type of vehicle	Number	Sector angle (degrees)
Private cars	148	b
Buses	16	c
Lorries	d	84
Motor cycles	e	f
Total	240	a

Find the missing numbers represented by the letters a, b, c, d, e and f, and draw the pie chart accurately. *1 unit*

B1/3 In a quiz, 7 competitors gained the scores stated below.

45 21 31 40 54 28 36

Find
(i) the range,
(ii) the median
of these scores.

Calculate
(iii) the mean deviation from the median of these scores. *1 unit*

B1/4 As part of an experiment in Botany the lengths of 40 laurel leaves were measured to the nearest mm. The results obtained are given below.

128	145	156	150	142	135	145	138
135	140	153	135	147	142	173	146
165	154	120	163	176	138	126	168
144	152	148	136	147	140	158	146
157	149	125	144	132	150	164	161

(i) Tally these results to form a grouped frequency distribution, using equal class intervals starting at 120-129.

(ii) Use the grouped frequency distribution obtained to calculate an estimate of the arithmetic mean of these lengths. *1 unit*

B1/5 (a) What is the probability that

(i) on one throw of a fair die it will show an even number,

(ii) two fair dice thrown together will each show a 4?

(b) In a certain town it has been calculated that the probability of a child catching measles is 0.13.

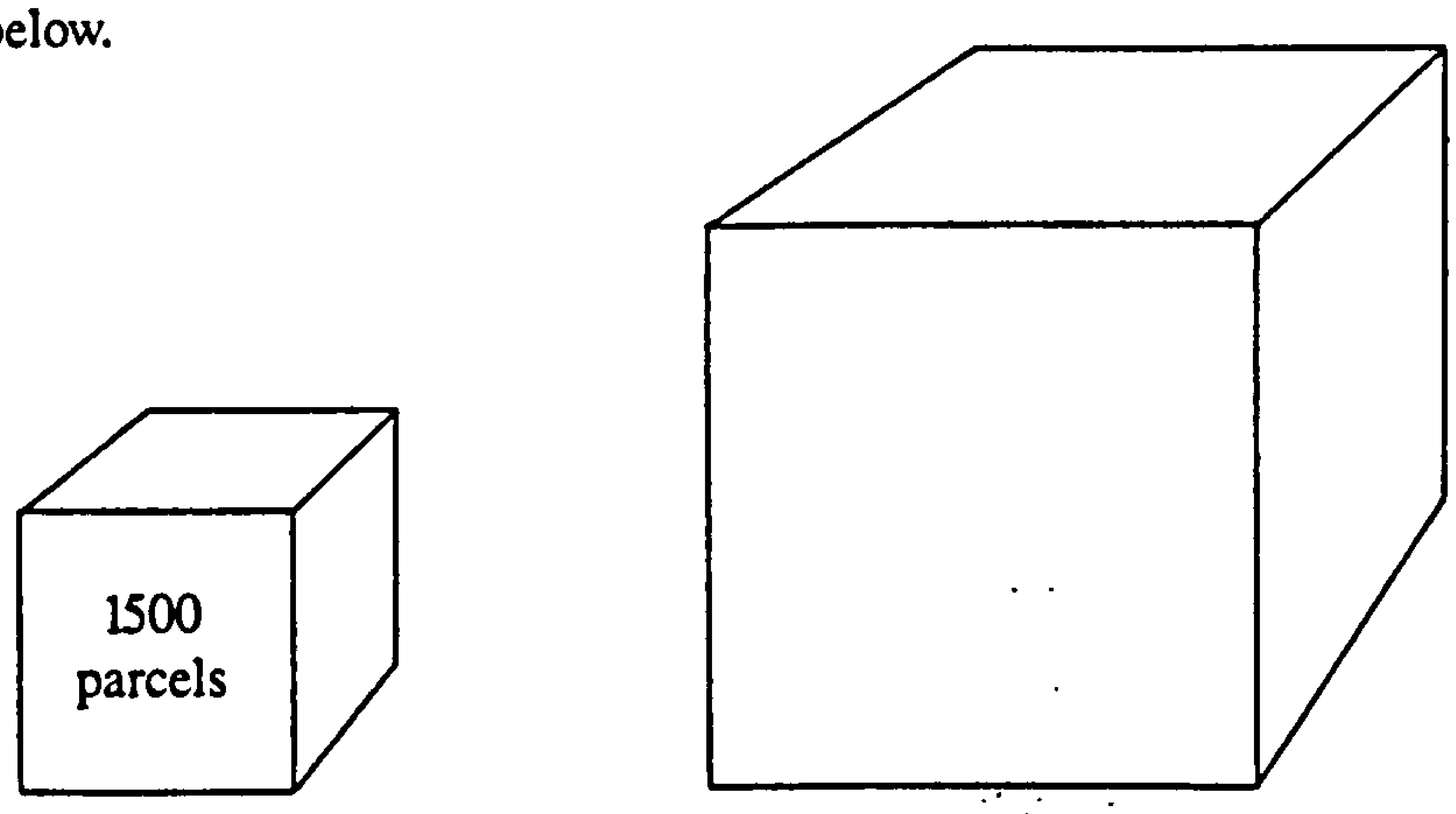
(i) Out of 1000 children in that town how many can be expected to catch measles?

What is the probability that

(ii) a child chosen at random in that town will not catch measles,

(iii) two children chosen at random from that town will both catch measles? *1 unit*

B1/6 A firm of carriers set up business in 1980 and in their first year of trading carried 1500 parcels. By 1982 they had increased their trade and in a report illustrated their progress by diagrams in which the numbers of parcels carried were represented by the volumes of cubes. The number of parcels for 1980 was represented by a cube of length of edge 2 cm. The two diagrams used are reproduced below.



Parcels carried in 1980 | Parcels carried in 1982

(i) Calculate the number of parcels carried in 1982.

In 1984 the number of parcels carried increased to 40 500.

(ii) Illustrate similarly the 1984 trade of the firm.

(iii) Suggest an alternative diagrammatic method of showing all this information, stating briefly one disadvantage of your method. *1 unit*

turn over

TOPIC B

SECTION 2

B2/1 The weekly wages earned by the sixteen employees in a small business are as follows:

Weekly wages (£)	No. of employees
60	2
62	1
64	1
66	1
68	1
70	2
72	0
74	2
76	1
78	1
80	1
82	1
84	2

(i) Calculate the mean and the standard deviation of the weekly wages for these employees.

In an attempt to bring about a new type of wage agreement, the management proposes to the union concerned that the wages should be scaled up so that the mean weekly wage becomes £80 with a standard deviation of £10. If the union agrees to this, what would be

(ii) the new weekly wage of an employee earning £84 per week,

(iii) the increase in the weekly wage of an employee whose new weekly wage would be £75? *2 units*

B2/2 A school entered 50 candidates for an examination and the marks obtained are shown in the grouped frequency table given below.

Marks	No. of candidates
0—10	0
11—20	3
21—30	7
31—35	7
36—40	8
41—45	8
46—50	5
51—60	7
61—70	3
71—80	2

(i) Form a cumulative frequency table for this distribution.

(ii) Draw a cumulative frequency graph using a scale of 2 cm to represent 10 marks and a scale of 2 cm to represent 10 candidates.

Use the graph to estimate

(iii) the median mark,

(iv) the semi-interquartile range of the marks,

(v) the lowest mark for which grade A (the highest grade) was awarded, given that 12% of the candidates achieved that grade,

(vi) the number of candidates who were not awarded a grade, given that a minimum of 24 marks was needed to earn a grade. *2 units*

B2/3 (a) 100 cartons of eggs were checked to find out how many damaged eggs there were in each carton. The results obtained are shown in the table given below.

Number of damaged eggs	No. of cartons
0	80
1	12
2	5
3	3

When one carton is chosen at random from these 100 cartons, what is the probability that it contains

(i) no damaged eggs,

(ii) at least one damaged egg,

(iii) more than one damaged egg?

(b) In a factory, an inspection is made of all the items produced by a machine in one day. It is found that 20% of the items have some defect. An item, chosen at random from those produced by that machine on that day, has a probability p of having no defect and a probability q of having a defect.

(i) Write down the value of p and the value of q, giving each answer as a fraction in its lowest terms.

(ii) Using Pascal's triangle, or otherwise, write down the expansion of $(q + p)^4$.

Four items are chosen at random from all those produced by the machine on that day.

Calculate the probability that

(iii) all four items have no defect,

(iv) exactly three items have no defect. *2 units*

B2/4 (a) A country is divided into four provinces, North, East, South and West. In 1960 the populations, in millions, of these provinces were 10.8, 6.3, 4.7 and 2.3 respectively. From 1960 to 1975 the population of the country as a whole increased by 3.5 millions, while the populations of the East, South and West provinces increased by 0.4 million, 0.3 million and 0.2 million respectively. From 1975 to 1980 the population of North province increased by 1.2 million, but the populations of the other provinces were unchanged.

(i) Tabulate all the information given above showing the populations of each of the four provinces and also the country as a whole in each of the three years 1960, 1975 and 1980.

(ii) Showing all working, show that the percentage of the population of the whole country living in North province increased from 1960 to 1980.

(b) A survey of the ages of pupils at a certain school produced the results given below.

Age (years)	11 but under 12	12 but under 16	16 but under 19
No. of pupils	60	360	90

Illustrate this information by means of a histogram, taking as scales 2 cm to represent 1 year on the age axis and 2 cm to represent 30 on the frequency density (pupils/year) axis. *2 units*

turn over

B2/5 Ten women joined a "weight-watchers" class and details of their weights and average daily food consumption were measured. The results are shown in the table given below.

Weight (kg)	84	93	65	95	72	86	78	70	90	75
Food consumption (100 calories/day)	32	37	26	39	27	35	31	28	35	30

(i) Use these figures to plot a scatter diagram. Take 2 cm to represent 5 units on both axes, starting the "weight" axis at 65 and the "food consumption" axis at 20.

(ii) Calculate the mean weight and mean daily food consumption for these 10 women. Plot clearly the point on the scatter diagram representing these mean values and identify it by the letter *M*.

(iii) On the scatter diagram draw in a line of best fit.

Use the diagram to estimate

(iv) the weight of a person whose food consumption is 3300 calories per day,

(v) the average loss of weight that a reduction in food consumption of 500 calories per day could produce. *2 units*

B2/6 (a) Name three measures of average used in statistics.

Consider the set of seven numbers

5 5 5 9 9 11 12

(i) Which one of the three measures of average for this set of numbers is the smallest?

(ii) The numbers 3 and x are added to the given set of numbers.The three measures of average are unchanged. What is the value of x?

(b) The marks of 400 candidates in an examination are normally distributed. The 10th percentile mark is 24 and the 90th percentile mark is 80.

(i) How many candidates scored more than 24 marks?

(ii) What is the median mark of the distribution?

(iii) At which percentile points would the marks need to be known in order to calculate the semi-interquartile range of the distribution?

(c) State one advantage and one disadvantage of using the range as a measure of dispersion in statistics. *2 units*