PROSPECTIVE AND PRACTICING MIDDLE SCHOOL TEACHERS' KNOWLEDGE OF CURRICULUM FOR TEACHING SIMPLE ALGEBRAIC EQUATIONS

A Dissertation

by

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ABSTRACT

Knowledge of curriculum is a significant component of mathematical knowledge for teaching. However, clearly understanding knowledge of curriculum requires further refinement and substantial research. This study consists of three papers that aim to explore prospective and practicing middle school teachers' Knowledge of Curriculum for Teaching Simple Algebraic Equations (KCTE).

The first paper reviews trends in and the evolution of standards and policies and synthesizes significant findings of research on mathematics curriculum and Knowledge of Curriculum for Mathematics Teaching (KCMT). Through this synthesis, the paper examines policy changes and research relevant to mathematics curriculum and KCMT and anticipates future research approaches and topics that show promise.

Building on the context provided by the first paper, the following two papers investigate KCTE from the perspectives of prospective and practicing middle school mathematics teachers. For the second paper, data was collected from a convenience sample of 58 prospective middle school mathematics teachers and a subsample of six participants. The findings of this study identify patterns of key mathematical topics in the teaching sequence of simple algebraic equations, compare the participants' sequences with experts', reveal participants' orientations toward KCTE, draw connections between participants' KCTE and their knowledge of content and teaching, and establish relationships between participants' KCTE and their knowledge of content and students.

Four middle school mathematics teachers participated in the third study. The results indicate that state-level intended curriculum is the most prevailing component of participants' KCTE. Furthermore, from a vertical view of curriculum, participants' awareness of their students' lack of basic mathematical knowledge impacted their KCTE. The paper also identifies the role of the state-level intended curriculum in participants' KCTE, alternative approaches to curriculum implementation that participants used to respond to the multiple intelligences of their students, and the participants' lack of lateral curriculum knowledge in KCTE.

Together, these three papers offer a closer look at KCMT with a focus on simple algebraic equations. This research broadens our understanding of prospective and practicing middle school teachers' KCMT and discusses implications for professional development.

DEDICATION

To my parents

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CHAPTER I

INTRODUCTION

Nationally representative data on the mathematics achievement levels of U.S. fourth and eighth graders mainly come from two sources: the National Assessment of Educational Progress (NAEP) and the Third International Mathematics and Science Study (TIMSS) (The National Center for Education Statistics [NCES], 2007). In cross-national comparative studies such as the TIMSS, the overall performance of U.S. students was mediocre (Kilpatrick, Swafford, & Findell, 2001). On the positive side, between 1995/1996 and 2007, both the TIMSS and the NAEP showed statistically significant improvements in U.S. students' mathematics achievement (NCES, 2007). For instance, on the TIMSS 2007 assessment, U.S. eighth graders performed slightly above the mid-point of the TIMSS achievement scale in the content domain of algebra (Mullis, Martin & Foy, 2008). However, U.S. students' mathematics performance still requires improvement to increase the nation's global competitiveness to improve economy, democracy and equity.

Recent studies indicate that teachers' mathematical knowledge for teaching is associated with student achievement (Hill, Rowan, & Ball, 2005). Increasing student achievement depends upon the improvement of teachers' knowledge (Ball & Bass, 2003). Thus, equipping teachers with mathematical knowledge for effective teaching becomes one significant issue to improve student performance (e.g., Ball, Lubienski, & Mewborn, 2001; National Mathematics Advisory Panel [NMAP], 2008).

Research on teachers' mathematical knowledge for teaching (Ball & Bass, 2000, 2003; Ball, Hill, & Bass, 2005; Ball et al., 2001; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema & Franke, 1992; Hill et al., 2005; Ma, 1999; NMAP, 2008; Shulman, 1986) indicates that teaching requires more than proficiency or knowledge of enacting mathematical tasks (Ball, Sleep, Boerst, & Bass, 2009). Furthermore, Thames and Ball (2010) pointed out that the studies conducted over the past forty years implied that content knowledge is inadequate for teaching mathematics effectively. More efforts have been made to extend our understanding of the knowledge required for mathematics teaching such as the first large-scale international project on elementary and middle school mathematics teacher preparation: the Teacher Education and Development Study in Mathematics (TEDS-M) (Center for Research in Mathematics and Science Education, 2010).

1.1 Rationale for This Study

The rationale for the present study stems from the notion that mathematics teachers need to know more than general pedagogical knowledge and content knowledge to be effective (Ball et al., 2009). Ball et al.'s works are complex, consisting of several categories. Many of the categories have been researched extensively. This study focuses on one category which lacks attention: curriculum knowledge for teaching.

Ball and her colleagues (Ball, 2006; Ball et al., 2005; Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008) have developed a model which can more accurately describe the knowledge required for teaching mathematics effectively. They conceptualized Mathematical Knowledge for Teaching (MKT) as Subject Matter

Knowledge (SMK) and Pedagogical Content Knowledge (PCK). SMK includes Common Content Knowledge (CCK), and Specialized Content Knowledge (SCK). "Knowledge at the Mathematical Horizon" or "Horizon Content Knowledge" was provisionally placed within SMK as a third category (Ball et al., 2008). PCK is composed of Knowledge of Content and Students (KCS), and Knowledge of Content and Teaching (KCT). Knowledge of Content and Curriculum (KCC) was provisionally put within PCK (Ball et al., 2008).

Ball and her colleagues (2005, 2006, 2008, 2009) indicated that KCS, a component of PCK, involves the knowledge of common student errors and difficulties and the impact of students' previous knowledge on what they are currently learning. For example, if given an algebraic problem "find the value of x in $2x+4x=60$," a teacher having KCS should be able to anticipate some common errors and difficulties, such as a misunderstanding of a coefficient or an expression like $2x$. In addition, the teacher should be able to anticipate that students may be challenged by the problem if they did not previously understand the mathematical concept, "coefficient." Such knowledge helps a mathematics teacher to anticipate the outcomes of instructional decisions. KCT refers to the knowledge that "combines knowing about teaching and knowing about mathematics" (Ball et al, 2008, p. 401). To be specific, KCT indicates the sequence of teaching a new mathematical concept and the appropriate use of various representations in introducing different topics. For example, regarding the algebraic problem mentioned above, a teacher with KCT should know the most appropriate materials or tools to introduce the concept of variables. In other words, KCT enables teachers to make the most appropriate instructional decisions to be optimally effective.

Unlike KCS and KCT, which are well-defined and have been studied fairly extensively, KCC, as a significant component of MKT, requires further refinement and lacks substantial research (Ball et al., 2008). Little is known about the perspectives from prospective teachers and practicing teachers on their Knowledge of Curriculum for Mathematics Teaching (KCMT). Furthermore, the connections between KCMT and KCS and the connections between KCMT and KCT are not clear.

In the last few decades, mathematics educators and researchers have increasingly recognized the gatekeeper role of algebra from prekindergarten through grade 12 (e.g., Carraher & Schliemann, 2007; NCTM, 2000; NMAP, 2008; Kaput, 1999). Previous research (Carraher, Schliemann, Brizuela, & Earnest, 2006; Cuevas & Yeatts, 2001; Kaput, 2008; Kaput, Blanton, & Moreno, 2008) reported that the heart of algebra and algebraic thinking is characterized by symbolization processes for generalization based on arithmetic. Fundamental ideas of algebra can be introduced, developed and extended in preK-12 schooling, which include patterns, variables, equations and functions (NCTM, 2000). Despite the debatable assumptions about the validity and utility of cross-national comparisons, large-scale international studies such as the TIMSS revealed the relatively low algebra competence of U.S. students (Smith & College, 2003). Among numerous contributing factors, teachers' MKT might account for the mediocre student achievement in the content domain of algebra. However, relatively little research has been conducted on teachers' MKT in algebra. It is still unclear what middle school prospective and practicing teachers need to know about algebraic content and how teachers can effectively teach algebraic content to students.

In the present study, there are three independent yet related articles. Hereafter, these articles will be referred to as Article One, Article Two and Article Three. Together, the purpose of these three articles is to investigate KCMT with a focus on simple algebraic equations in the Mathematical Knowledge for Teaching (MKT) model. By involving both prospective and practicing teachers, this study presents a grounded theory of KCTE that is based on research guided by the research questions (See next section for details).

1.2 Research Questions

In Article One, the context of examining KCMT is provided as an introduction to the following two papers. Article Two investigates a specific component of prospective teachers' KCMT: knowledge of curriculum on teaching simple algebraic equations (KCTE). Article Three examines practicing teachers' KCTE. The following research questions are addressed in the first article:

1) What standards and principles have been released and developed regarding curriculum and KCMT? What can we learn from the evolution of the standards and principles?
2) What research has been conducted on curriculum and KCMT (Knowledge of Curriculum for Mathematics Teaching)? What can we learn from their findings?
3) What do we need to do next to enhance teachers' KCMT?

In Article Two, the research questions include:

1) What are the features of the participating PTs' Knowledge of Curriculum for Teaching Simple Algebraic Equations (KCTE)?

2) What are PTs' perspectives on KCTE?

3) What connections may exist between PTs' KCTE and their knowledge of content and teaching?

4) What connections may exist between PTs' KCTE and their knowledge of content and students?

The following research questions guided the third article:

1) What are the perspectives on KCTE from participating middle school practicing mathematics teachers?

2) What are the connections between the practicing teachers' KCTE and their knowledge of content and teaching?

3) What are the connections between the practicing teachers' KCTE and their knowledge of content and students?

4) What are the similarities and differences in perspectives on KCTE between practicing teachers and prospective teachers?

1.3 Limitations and Delimitations

Limitations

The present study has at least two limitations. First, questionnaires for prospective and practicing mathematics teachers about their KCTE were used as data for

Articles Two and Three. However, the data from the participants may have reflected only one aspect of their KCTE, and self-report questionnaires have well-known limitations (Podsakoff, MacKenzie, Lee, & Podsakoff, 2003). To compensate for these limitations, in-depth interviews regarding KCTE were also collected from sample participants.

Second, the participants comprised a volunteer convenience sample. The participants' motivations and willingness to participate in the research may have affected the data. To reduce the limitation, during the recruitment, the researcher stated the time limit of both instruments and potential benefits for the participants' professional development.

Delimitations

This research has two delimitations. First, the research is focused on KCMT, only one of the components in prospective and practicing teachers' PCK. The participants' content knowledge (Shulman, 1986) or subject matter knowledge (Ball et al., 2005, 2008) for mathematics teaching was not considered as a part of this research.

Second, this research is concentrated on simple algebraic equations, one of the significant topics in middle school mathematics teaching and learning. Previous research highlighted the importance of fundamental algebra to be introduced and developed in K-12 classrooms (e.g., NCTM, 2000). Therefore, "simple algebraic equations" was chosen as a topic to explore prospective and practicing teachers' KCMT. Other significant algebraic topics in middle school mathematics teaching and learning, such as patterns and functions, were not discussed in this research.

1.4 Definitions

Knowledge of Curriculum for Mathematics Teaching (KCMT): this present research adopts Shulman's (1986) definition of curricular knowledge and the definition of knowledge of curriculum from Ball and Bass (2009). According to Shulman (1986), curricular knowledge has three dimensions: knowledge of alternative available curriculum materials for instructing a subject or content; lateral curriculum knowledge of materials studied simultaneously by students in other subject areas; and vertical curriculum knowledge, which means "familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them" (Shulman, 1986, p. 10). Ball and Bass (2009) defined knowledge of curriculum as composed of educational goals, standards, state assessments, and grade levels where specific topics are taught.

In the three papers of this present research, KCMT is used to indicate a piece of the component of Knowledge of Content and Curriculum (KCC) in the Mathematical Knowledge for Teaching (MKT) framework (Ball et al., 2008). A tentative definition at this time for KCMT is curriculum knowledge required for effective mathematics instruction.

Simple Algebraic Equations: An algebraic equation has different notions (Chazan, 2008). For example, an equation in one variable $3x+2=7$ can be conceptualized as follows: "(1) *a representation of a set*; the solution set to this equation is {5/3}," "(2) *a template for producing sentences about numbers*—sentences that can be true or false depending on the values used to replace x in this template," "(3) *a question about the* *inputs of a function*, as asking for what input(s) will this function produce the output 7?" and "(4) *a comparison of two functions of one variable* where 3*x*+2 is being compared with the constant function *g(x)*=7" (Chazan, 2008, pp. 25-27).

Based on the existing literature, Kieran (2006) summarizes equation solving procedures of beginning algebra students as: 1) intuitive approaches, including using number facts, counting techniques, and cover-up methods; 2) trial-and-error substitution; and 3) formal methods. Using trial-and-error substitution as a procedure in solving equations stems from the *arithmetic approach*; whereas transposing terms to the other side is characterized as the *algebraic approach* (Kieran, 1988). Bernard and Cohen (1988) suggested that a *developmental learning sequence* should be constructed to help students formulate their knowledge and skills of equations and equation solving. In the sequence, the *generate-and-evaluate* or *trial-and-error* method serves as a starting point, followed by *cover-up*, *undoing,* and finally *equivalent equations*.

At this stage in the research, Simple Algebraic Equations will be referred to as equations which involve one variable only and multiple approaches to solving such equations. Examples of a simple algebraic equation are 3*x*+2=7 and *x*-56=341.

Knowledge of Curriculum for Teaching Simple Algebraic Equations (KCTE): In the present research, Knowledge of Curriculum for Teaching Simple Algebraic Equations (KCTE) is used to indicate KCMT focusing on simple algebraic equations. At this stage in the present research, KCTE will be generally defined as a subset of KCC or KCMT in MKT (see Figure 1).

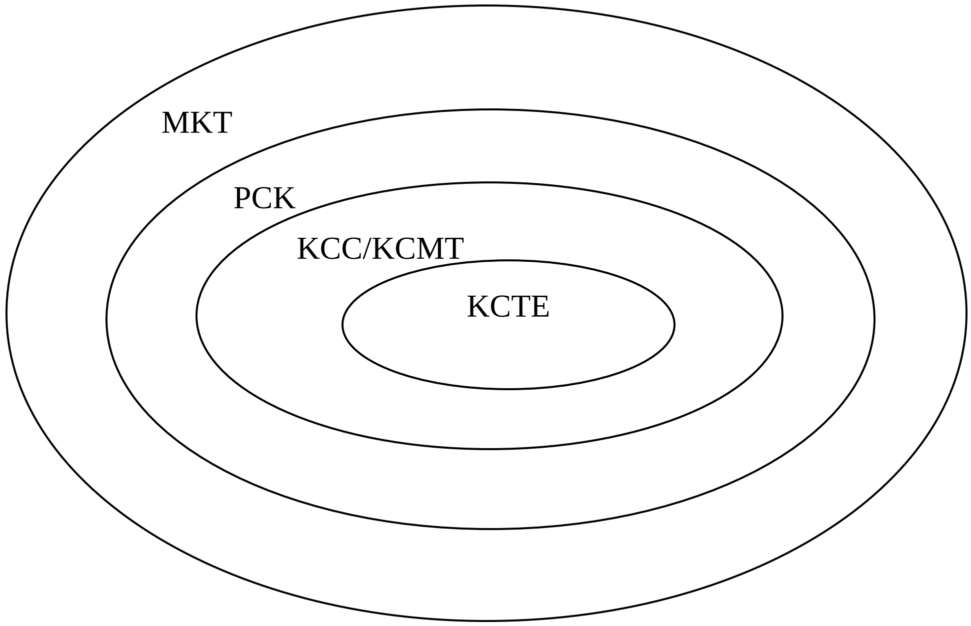

Figure 1. **KCTE in MKT**

1.5 Significance of This Research

This research makes three contributions to mathematics education. It examines the topic of teachers' knowledge of curriculum in a middle school context, it supplements the theoretical framework of MKT, and it suggests practical applications for teaching mathematics.

First, this research extends our understanding of KCMT in two ways—it focuses on the specific content area of middle school mathematics, simple algebraic equations. It also includes the perspectives of both prospective and practicing middle school mathematics teachers.

Second, this research strives to enrich our knowledge of MKT. Many efforts have been aimed at exploring the knowledge base needed for teaching mathematics (Ball & Bass, 2000; Ball et al., 2008; Fennema & Franke, 1992; Hill et al., 2005; Ma, 1999).

However, “incomplete and trivial definitions of teaching held by the policy community comprise a far greater danger to good education than does a more serious attempt to formulate the knowledge base” (Shulman, 1987, p. 20). Through examining a significant component of MKT, this study, to some extent, informs debates about what constitutes expertise in mathematics teaching.

Finally, this study has practical implications for professional development of mathematics teachers. With a focus on one specific and significant mathematical topic, this study may offer insights regarding how prospective teachers can develop their KCMT to be more effective. In addition, this study points to the significant role of KCMT in professional development of practicing teachers. The findings of this study provide implications for those concerned with what prospective and practicing middle school teachers need to learn about KCMT to excel in teaching.

CHAPTER II

KNOWLEDGE OF CURRICULUM FOR MATHEMATICS TEACHING: A REVIEW OF THE CONTEXT

> *The nation can adopt rigorous standards, set forth a visionary scenario, compile the best research about how students learn, change textbooks and assessment, promote teaching strategies that have been successful with a wide range of students, and change all the other elements involved in systemic reform—But without professional development, school reform and improved achievement for all students will not happen. Unless the classroom teacher understands and is committed to standards-based reform and knows how to make it happen, the dream will not be realized.*

—Principles for Professional Development, American Federation of Teachers, 2002, p. 2 (as cited in Sowder, 2007)

2.1 Introduction

It has been widely recognized that teachers are one of the major stakeholders in education reform (e.g., Sowder, 2007). As noted above, the excerpt of "Principles for Professional Development" suggests that without professional development (including teacher preparation for prospective teachers), the utopian ideal of increasing mathematics achievement for ALL students will never happen. Policy makers are committed to providing up-to-date mathematics standards as blue prints in education agenda. Along with policy changes, researchers strive to provide "the best available scientific evidence" to inform teacher education (NMAP, 2008). In the *Second Handbook of Research on Mathematics Teaching and Learning*, Sowder (2007) included developing mathematics teachers' knowledge in six goals of teacher needs as professional development foci. This goal is not a new idea since many researchers have made continuous efforts to promote our understanding of mathematics teachers' knowledge (e.g., Shulman, Ball and their colleagues). However, despite the visions, documents and research papers on mathematics teachers' knowledge, what remains unclear is how to effectively incorporate these ideas into teacher professional development.

The following questions guided this paper:

First, what standards and principles have been released and developed regarding curriculum and knowledge of curriculum for mathematics teaching (KCMT)? What can we learn from the evolution of the standards and principles?

Second, what research has been conducted on curriculum and KCMT? What can we learn from their findings?

Third, what do we need to do next to enhance teachers' KCMT?

2.2 Policy Influences: Standards and Principles Regarding Curriculum and KCMT

NCTM Standards (1989 & 2000)

With the release of the two books *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Principles and Standards for School Mathematics* (NCTM, 2000), the U.S. school mathematics curriculum has been dramatically impacted. NCTM 1989 *Standards* proposed "five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically" (NCTM, 1989, p.5). Following the publication of the 1989 NCTM *Standards*, and after more than 10 years of efforts to reform school mathematics education, *Principles and Standards for School Mathematics* (NCTM, 2000) was published. It served as an updated version of the 1989 standards, as well as incorporating another two earlier NCTM documents, namely, *Professional Standards for Teaching Mathematics* (NCTM, 1991) and *Assessment Standards for School Mathematics* (NCTM, 1995). *Principles and Standards* (NCTM, 2000) focuses on five content standards (i.e., number and operations, algebra, geometry, measurement, data analysis and probability) and five process standards (i.e., problem solving, reasoning and proof, communication, connections, and representation).

One of the major features that differentiate the 2000 edition from the 1989 edition is a change in the approach to basic skills, computations and procedures (Latterell, 2004; Willoughby, 2010). The shift is largely due to considerable concerns about a threat to basic skills, computations and procedures because of a lack of emphasis on memorization of facts and rules in the 1989 document (e.g., Kilpatrick, Martin, & Schifter, 2003; Latterell, 2004). The 2000 document clarifies the importance of basic skills, computations and procedures, although problem solving and higher-order thinking such as reasoning and proof remain as standards. For example, the number and operations standard specifies that "instructional programs from prekindergarten through grade 12 should enable all students to compute fluently and make reasonable estimates…developing fluency requires a balance and connection between conceptual understanding and computational proficiency" (NCTM, 2000, p. 35). As Hiebert (2003) argued, skill proficiency does not have to be sacrificed for the emphasis of conceptual understanding. Building on similar ideas, the authors of the 2000 document called attention to both conceptual understanding and computational proficiency including basic skills, computations and procedures. Such a shift in the 2000 document addressed the problem of a possible consequent decline of computational proficiency caused by the recommendations of decreased emphasis on arithmetic operations and skills in the 1989 document.

***Focal Points* (2006)**

As stated in the introduction of *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (or *Focal Points*), the publication of *Focal Points* is meant to "support, expand, and illuminate" *Principles and Standards for School Mathematics* (NCTM, 2006, p.1). More importantly, *Focal Points* was released to answer the question, "What are the key mathematical ideas or topics on which the others build?" (NCTM, n.d.). As Schielack and Seeley (2007) stated, "it involves highlighting a continuum of key ideas across grades and identifying particular grade levels where the emphasis of instruction will be placed for each of these key ideas" (p.79). Furthermore, a mathematical topic may follow the pattern of *a background context—a foreground in-depth instruction—a background context* across grades (Schielack & Seeley, 2007). The publication of *Focal Points* addresses curricular focus and coherence, answering the call for a common and national curriculum. It also intends to arouse interest and encourage discussions on the importance of designing a coherent elementary school mathematics curriculum, focusing upon important mathematical topics for each grade level from Pre-Kindergarten to the 8^{th} grade.

A main driving force for the publication of *Focal Points* was the mediocre performance of U.S. students, especially eighth graders, in cross-national comparative studies such as the *Third International Mathematics and Science Study* (TIMSS) (Kilpatrick et al., 2001). Although the performance differences between U.S. students and their peers in high-achieving countries may result from a combination of different factors, some influential researchers who were involved in the study suggested that the lack of a common and coherent curriculum hampered mathematics teaching and learning in the United States (e.g., Schmidt, Houang, & Cogan, 2002; Schmidt et al., 2001). Compared with the curricula in top-performing countries, the U.S. school mathematics curriculum was described as "underachieving," "a mile wide, an inch deep" and repetitive (McKnight et al., 1987; McKnight & Schmidt, 1998; Schmidt et al., 2001; Silver, 1998). Specifically, four characteristics of U.S. mathematics textbooks in an international curricula comparison were summarized: first, the content of textbooks lacks focus; second, the content is highly repetitive; third, the content is undemanding according to international standards; and fourth, the content is incoherent (Schmidt et al., 2001). For instance, researchers suggested that, in the U.S., curriculum materials lacked guidance and support for placing cognitive demands on teachers and students (Silver, 2009), major textbooks failed to offer students early and coherent algebraic experiences (Cai et al., 2005; Schmidt, McKnight, & Raizen, 1996), and more emphasis should be placed on content connections and cognitive requirements in algebra (Li, 2007).

In contrast to the 2000 document, *Focal Points* provides an approach and a framework for designing, developing and organizing a mathematics curriculum in grades preK-8. The concepts of design, development and organization place emphases on the coherence of significant mathematical topics for each grade and the connections between topics across grades in the curriculum (NCTM, n.d.). *Focal Points* holds the same theoretical assumptions about mathematics as described in the 2000 document. However, it targets a smaller number of key ideas at each grade level instead of simply providing a large collection of general goals, principles, content and process principles or expectations as the 2000 document does. In addition, *Focal Points* addresses the mathematical content that should be taught rather than how to teach it.

Common Core State Standards for Mathematics (CCSSM, 2010)

As a part of the Common Core State Standards Initiative (CCSSI), the Common Core State Standards for Mathematics (CCSSM) was released in 2010 (CCSSI, 2010a). CCSSM builds on the previous NCTM publications including the 1989 NCTM standards, the 2000 NCTM standards, the 2006 *Focal Points*, and the 2009 *Focus in High School Mathematics: Reasoning and Sense Making*. As one of the most recent attempts during "a decades-long journey in developing a national vision for school mathematics that prepares all students for future success" (NCTM, 2010, p. 1), CCSSM envisions what mathematical content and processes or practices should be taught in a *coherent* and *focused* way (NCTM, 2010; Porter, McMaken, Hwang & Yang, 2011). Although this effort was initiated by individual states rather than by the federal government, most states embraced the shared vision of "providing a consistent, clear understanding of what students are expected to learn." By early 2012, 45 states and three territories have adopted CCSSM (CCSSI, 2010b).

Through comparisons between CCSSM and current state standards and assessments, and between CCSSM and standards in benchmarking countries with top student performance, Porter et al. (2011) found CCSSM to be a more focused curriculum with a greater emphasis on higher levels of cognitive demand and with less emphasis on advanced algebra and geometry. In addition to the differences between CCSSM and the state standards, as well as the differences between CCSSM and standards in top-performing countries, CCSSM is different from the current enacted curriculum reported by U.S. in-service teachers (Porter et al., 2011). CCSSM focuses more on “perform procedures,” “demonstrate understanding” and “number sense,” whereas the enacted curriculum emphasizes more on “memorize,” “conjecture,” “solve nonroutine problems” and “geometric concepts” (Porter et al., 2011, p. 114).

In response to Porter et al.’s (2011) article, Cobb and Jackson (2011) asserted that additional conceptualization and perspectives should be taken into account when measuring both the change and quality of curriculum standards and the alignment of the standards with related educational components. Recognizing Porter et al.’s useful methods and valid findings, Cobb and Jackson invited renewed research efforts on CCSSM.

Performance-based Accountability: “Mathematics for all”

The landmark event of the No Child Left Behind (NCLB) Act of 2001 highlighted performance-based accountability in government-funded schools. In 2007, NCLB was reauthorized. According to NCLB, individual states should develop and implement a standards-based accountability (SBA) system. This system consists of grade-specific content and performance standards, assessments to monitor students’ progress, goals to be achieved on student assessments, as well as the Adequate Yearly Progress (AYP) determination criteria (Hamilton et al., 2007; Reys, 2006). AYP identifies schools which require improvement to meet the goals by interventions or sanctions (Hamilton et al., 2007).

Despite varying responses regarding the NCLB Act, the major goals of the Act have been widely accepted—more academically demanding content has been set and implemented, and students are expected to demonstrate satisfying achievement levels in standardized tests that embody performance standards (Goertz, 2010). Both positive and negative impacts of NCLB on schools, teachers and students have been identified. Findings included more focus on student achievement, more rigorous curriculum, and improved student learning. However, previous studies have shown a decreasing morale among school principals and teachers and less positive perceptions from teachers than from administrators on the SBA system's beneficial effects on students (e.g., Hamilton et al., 2007). Among tremendous changes and challenges brought by NCLB, researchers highlighted the disadvantages of high-poverty schools and racially diverse schools (Kim & Sunderman, 2005), a concern of increased dropout rates for underachieving students (Darling-Hammond, 2006), and the lack of intensive support for low-performing schools (Goertz, 2010).

To some extent, NCLB increases the pressure on teachers to closely follow the assessed curriculum and ensure that students can meet the required performance standards. Darling-Hammond (2004, 2009) predicted the failure of the one-way accountability that focused exclusively on high-stake test scores. She recommended a two-way, intelligent accountability with extended visions on such issues as supporting teacher and student learning. Furthermore, the challenges faced by high-poverty schools, racially diverse schools and low-performing schools raise a question of whether NCLB helps close or widen the achievement gap.

Moreover, NCLB promotes the involvement of states in establishing curriculum standards as well as teachers' adherence to the state-level standards (Dingman, 2010). However, the considerable variations in state-level curriculum standards and the varying alignment between textbooks and standards led to diverse and incoherent curriculum materials which "overburden teachers and students both physically and intellectually without improving education" (Willoughby, 2010, p.83) .

2.3 Post-reform Era: States, Textbooks, Teachers and Classroom Instruction

The evolution of the NCTM standards has greatly impacted state mathematics standards and assessments, textbook development, and classroom instruction. At the state level, curriculum standards and school mathematics curriculum frameworks were modified to align with the NCTM standards (Blank & Pechman, 1995). To adhere to the NCTM-initiated mathematics reforms, individual states targeted the recommendations outlined by the NCTM standards. For example, California insisted that the curriculum materials chosen for K-8 should incorporate the standards (Kilpatrick, 2003). Furthermore, some states realigned curriculum frameworks according to *Focal Points* (Usiskin, 2010). As states began supporting standards-based curriculum materials and realigning curriculum frameworks, the National Science Foundation (NSF) funded large-scale projects focusing on mathematics education reform (Stein, Remillard, & Smith, 2007). The NSF funded several Statewide Systemic Initiative projects (SSIs) designed to promote state- and local-level education systems that were aligned with the standards (McCaffrey et al., 2001).

Aligned with the vision of the standards documents, NSF supported programs and projects which produced sets of standards-based mathematics curriculum materials for the elementary, middle, and high school levels (Senk & Thompson, 2003; Stein et al., 2007). Distinct differences in content and approaches distinguish standards-based textbooks from traditional textbooks (National Research Council [NRC], 2004; Robinson, Robinson, & Maceli, 2000; Schoenfeld, 2006). For example, the differences in the standards-based elementary mathematics curricula include: 1) content and pedagogy consistent with the principles of NCTM standards, 2) use of multiple representations, 3) expanded content with new topics (e.g., mental computation and frequent use of calculators in *Everyday Mathematics*), 4) integration of calculators and technology, and 5) emphasis on students' communication and exploration of important mathematics beyond emphasis on computational algorithms (NRC, 2004; Putman, 2003; Stein et al., 2007). According to Stein et al. (2007), on the elementary level, six standards-based textbooks were produced and are currently widely adopted in U.S. public schools; on the middle and high school levels, fourteen standards-based textbooks were published and have been adopted in schools. In addition, commercial publishers adapted their textbooks to reflect the standards, so that by the late 1990s, nearly all mathematics textbooks in U.S. classrooms claimed that they were "standards-based."

In addition to its influence on states and textbooks, the curriculum reform also impacted teachers and classroom instruction. Reform-impacted teachers have the following characteristics in common: 1) they know and understand the mathematics they are going to teach with pedagogical knowledge, focusing on connections among ideas with applications instead of viewing knowledge in an isolated way; 2) they can establish a supportive and challenging learning environment for students; and 3) they emphasize mathematical reasoning and problem solving rather than memorizing facts (McCaffrey et al., 2001; NCTM, 1989, 2000). McCaffrey et al. (2001) found that reformed teachers viewed classrooms as cooperative learning communities, adopt inquiry-based instruction, and focus on reasoning, problem solving and connections among knowledge.

The implementation of the standards in classrooms is another story. Jacob et al. (2006) examined videotapes of nationally representative eighth-grade mathematics classes in TIMSS 1995 and 1999 Video Studies along with questionnaires regarding teachers' knowledge and perceptions of the standards implementation. They claimed that the majority of participating mathematics teachers acknowledged and embraced the standards, connected the standards with their knowledge and reported their lessons as reflective of the principles recommended in the standards (Jacobs et al., 2006). However, the observations obtained from the video studies did not fully support the questionnaire results. Jacobs et al. (2006) suggested that teachers' classroom practices were not well aligned with the standards, with the striking conclusions. Instead of the approaches the teachers claimed they used, the nature of classroom practice was identified as traditional rather than reform-oriented. Teachers did not focus on mathematical conceptual understanding and reasoning as much as expected. Rather, they presented mathematical procedures and then asked students to work on a large amount of similar problems (Jacobs et al., 2006).

Thompson and Senk (2010) discussed four myths about curriculum implementation:

- If a topic is in the mathematics textbook, teachers teach it.
- Teachers who use the same mathematics textbook teach the same content.
- Teachers who use the same textbook offer the same opportunities for students to continue learning mathematics through homework.
- Teachers of the same mathematics course have the same expectations for how their content coverage prepares students for standardized tests. (p.249)

Drawing on data from other studies, Thompson and Senk (2010) provided empirical evidence that all the myths are false. For instance, teachers using NSF-funded standards-based curriculum materials and teachers who adopted publisher-produced curriculum materials covered about 60% and 69% of the lessons, respectively. The results suggested that even if a topic is included in a textbook, teachers may choose to teach it or not. Thompson and Senk (2010) concluded that the variability or the various fidelity of curriculum implementation at the classroom level should be recognized; otherwise such problems as repetitions or gaps of mathematical content may occur across grades or courses. In addition, the results obtained from the assessed curriculum cannot be assumed as valid or appropriate without taking into consideration the implemented curriculum (Thompson & Senk, 2010).

2.4 Post-reform Era: Teachers' Knowledge of Curriculum

The higher expectations of standards-based reform posed new demands for teacher preparation (Darling-Hammond, 2000; Darling-Hammond & McLaughlin, 1995). The *Second Handbook of Research on Mathematics Teaching and Learning* contains a chapter devoted to mathematics teacher development. In that chapter, Sowder (2007) suggested that today's mathematics teachers should be equipped with extensive knowledge, instructional technologies, and the beliefs to meet the needs of planning and instructing mathematics classes for *all* students. She posed and addressed 10 fundamental questions regarding professional development to prepare mathematics teachers to achieve successful teaching and learning in the reform-impacted era. A previous effort addressing teacher preparation changes in the reform was provided by Darling-Hammond and McLaughlin (1995). They argued that the traditional top-down teacher preparation strategies should be changed into expanding the capacity of teachers' bottom-up construction of knowledge. Following the idea of turning teacher education "upside-down," in 2010, the National Council for Accreditation of Teacher Education published a report entitled "Transforming Teacher Education through Clinical Practice" as a national strategy to prepare effective prospective teachers. In the report, a clinically based model for preparing teacher candidates was proposed. The new model indicated "a paradigm shift in the epistemology of teacher education programs…toward more democratic and inclusive ways of working with schools and communities is necessary for colleges and universities to fulfill their mission in the education of teachers" (Zeichner, 2010, p. 89).

No one will deny the substantial role of professional development in the process of curriculum implementation. Researchers suggested that professional development should be provided for teachers to support their learning of curricula, along with the mathematics and pedagogy contained in the curricula (Allen-Fuller, Robinson, & Robinson, 2010). The range of teachers' needs, as well as various needs at different stages of curriculum implementation, should also be taken into account (Allen-Fuller et al., 2010). Through professional development and its alignment with teachers' beliefs, teachers may become capable of expanding their mathematical knowledge and pedagogical skills in addition to reinforcing a shared vision of curriculum improvement (Allen-Fuller et al., 2010).

As implied by Allen-Fuller and her colleagues, the "educative" feature of NSF-funded curricula brought salient benefits for teachers. This feature has aroused increased researchers' interest (Brown, 2009; Davis & Krajcik, 2005; Stein & Kim, 2009; Stein et al., 2007). Through communicating with teachers, educative curricula intend to promote teacher learning in addition to student learning. Stein and Kim (2009) investigated how the educative materials could guide and support teacher learning.

Similar to the notion of helping teachers anticipate students' responses, Project 2061 (2001) adopted criteria to evaluate the quality of instructional guidance. The criteria are related with teachers' curriculum materials under the category of building on students' mathematical ideas: specifying prerequisite knowledge, alerting teachers to students' ideas, assisting teachers in identifying ideas and addressing misconceptions (Kulm, 1999; Project 2061, 2001). First, *specifying prerequisite knowledge* examines the question "Does the material specify and address prerequisite knowledge/skills that are necessary to the learning of the benchmark?" Second, *alerting teachers to student ideas* investigates whether or not the material calls teachers' attention to regularly held student ideas. Third, *assisting teachers in identifying ideas* focuses on the question "Does the material include suggestions for teachers to find out what their students think about familiar situations related to a benchmark before the mathematical ideas are introduced?" Fourth, *addressing misconceptions centered upon whether or not the materials explicitly* addresses commonly held student ideas (Project 2061, 2001).

The emergence of educative curriculum materials is not the only impact of standards on teachers' curriculum knowledge. Breyfogle, McDuffie and Wohlhuter (2010) used a term "curricular reasoning" to refer to the decisions made by teachers regarding curriculum. Curricular reasoning indicates "the thinking processes that teachers engage in as they work with curriculum materials to plan, implement, and reflect on instruction" (p. 308). According to Breyfogle et al. (2010), there have been two major shifts since the reform, impacting mathematics teachers' professional development. One is about curriculum materials and the other is about the Grade Level Expectations (GLEs) of students and related accountability issues. As a consequence of the reform, curriculum materials were developed to foster students' mathematical reasoning and problem solving abilities. Teachers are required to anticipate students' mathematical thinking and learning difficulties; therefore, teachers are expected to demonstrate curricular reasoning in addition to knowing the curriculum (Breyfogle et al, 2010). The other shift is the increased expectation of teachers to "interpret and align their curriculum materials with state GLEs" (p. 309). Teachers struggle more when faced with curriculum materials and students' needs because of the often-conflicting and inconsistent GLEs (Breyfogle et al., 2010). Furthermore, based on the previously proposed frameworks on curricular knowledge, curricular visions and curricular trust, Breyfogle et al. (2010) have established a model to describe curricular reasoning. Building on curricular knowledge, their model produces curricular vision and impacts the development or lack of curricular trust.

2.5 Renewed Research Focuses on Curriculum and KCMT

Linking Curriculum and Students' Learning

Prompted by the development of reform curriculum projects aligned with the visions of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Principles and Standards for School Mathematics* (NCTM, 2000), there is renewed research interest in linking curriculum and students' learning (e.g., NRC, 2004; Senk & Thompson, 2003; Stein et al., 2007).

The results of research conducted on the effects of 12 standards-based school mathematics curriculum materials are promising compared with traditional materials (Senk & Thompson, 2003). Previous studies have indicated that students taught by standards-based curriculum materials demonstrate comparative levels of computational skills and superior performance on tests of mathematical thinking and reasoning (Senk & Thompson, 2003; Stein et al., 2007). In the book edited by Senk and Thompson (2003), there are three commentaries on standards-based elementary mathematics curricula, middle grade mathematics curricula, and high school mathematics curricula respectively.

In the reaction to standards-based elementary mathematics curricula, Putnam (2003) claimed that, "The first striking thing to note…is the overall similarity of their findings. Students in these new curricula generally perform as well as other students on traditional measures of mathematics achievement, including computational skills, and they generally do better on formal and informal assessments of conceptual understanding and ability to use mathematics to solve problems" (p. 161). In the commentary on standards-based middle grade mathematics curricula, Chappell (2003) asserted that "collectively, the evaluation results provide converging evidence that standards-based curricula may positively affect middle school students' mathematics achievement, in both conceptual and procedural understanding" (p. 291). In the reaction to standards-based high school mathematics curricula, Swafford (2003) reported that "reform curricula do not hamper student performance on traditional content" (p. 459). In addition, "students in reform curricula are experiencing and profiting from a broader, richer curriculum" concerning problem solving, reasoning, statistics, probability and discrete mathematics (Swafford, 2003, p. 461).

As discussed above, the evidence of the effectiveness of standards-based curriculum materials is substantial and intriguing. Standards-based curricula lead to "satisfactory student achievement" in mathematics classrooms (Kilpatrick, 2003, p. 472). However, evaluating curricula is complex and difficult, since it is challenging to accomplish reliable and valid comparisons on comprehensive and consensus measures of achievement (Kilpatrick, 2003).

Although the direct relationship between curriculum and student achievement is hard to establish (Reys, Reys, Lapan, Holliday, & Wasman, 2003), recent research has documented the improved mathematics achievement of students who use reform-oriented curricula (e.g., Kulm & Capraro, 2008; Reys et al., 2003). Through a quasi-experimental study, Tarr et al. (2008) provided a model describing the connections among various types of curricula (including intended curriculum, textbook curriculum, assessed curriculum, implemented curriculum and learned curriculum) and forces which impact these curricula. Tarr et al. (2008) acknowledged the complexity of curriculum evaluation research and concluded that curriculum type alone does not determine student mathematical achievements. Instead, a variety of factors affect student mathematics performance. Consistent with the findings of Tarr et al. (2008), Kulm (2008) highlighted how the connections and integration of various components in the educational system work as a whole to impact students' mathematics learning. In the book *Teacher Knowledge and Practice in Middle Grades Mathematics*, Kulm (2008) along with other researchers provided evidence about the extent to which the reform-oriented curricula and classroom instruction fostered middle school students' mathematics learning, as well as the required knowledge and skills during teacher preparation. For example, You and Kulm (2008) explored the impact of prospective teachers' content knowledge of linear functions on their knowledge of content and students as well as on their knowledge of content and teaching. On the high school level, Schoen, Ziebarth, Hirsch and BrckaLorenz (2010) designed and developed the *Core-Plus Mathematics* curriculum in a five-year longitudinal study. They reported the design, methods, tools, results and implications in their book (Schoen et al., 2010). The findings obtained from the study inform the second edition of the curriculum.

As described above, much research has focused on the effects of curriculum materials on student learning. The following topics and issues still require further study: the ways of measuring student learning impacted by curriculum, the ways of measuring curriculum implementation, theoretical frameworks for linking curriculum and student learning, and methodological designs for studying the impact of curriculum on student learning. While researchers have begun to explore these topics, their findings are far from exhaustive, and there remain more opportunities for research.

Teachers' Knowledge of Curriculum: From Static and Isolated to Dynamic and Developing in Context

As a forerunner of teacher knowledge studies, Shulman is widely recognized as one of the first scholars who introduced a notion of pedagogical content knowledge (PCK) through relating teachers' content knowledge and pedagogy. According to Shulman (1987), PCK "represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, p.8). In later researchers' works such as Ball et al.'s research (2005, 2006, 2008), the concept of PCK has been largely adopted and developed.

Shulman (1987) proposed seven categories of teacher knowledge base, and one of these categories is curriculum knowledge. Furthermore, Shulman (1986) divided curriculum knowledge into three types: *alternative* curriculum knowledge, *lateral* curriculum knowledge, and *vertical* curriculum knowledge. By arbitrarily separating teacher knowledge into different categories, Shulman (1986, 1987), to some degree, ignored the context, connectedness and development of curriculum knowledge (Petrou & Goulding, 2011). In more recent publications, Shulman did not continue the research on curriculum knowledge or discuss its categories and implications.

Others followed the research thread initiated by Shulman (1986, 1987). For instance, Mishra and Koehler (2006) integrated technology into teacher knowledge and proposed a framework for technological pedagogical content knowledge (TPCK). Most notably, Ball and her colleagues (2005, 2006, 2008) developed PCK and proposed a model of Mathematical Knowledge for Teaching (MKT). In the model, two components of PCK include Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT). Knowledge of Content and Curriculum (KCC) is provisionally put within PCK as a third category. As a progression from viewing curriculum knowledge as an isolated construct, Ball et al. (2008) started to consider the relationship between curriculum knowledge and other knowledge. They put forward a question of whether curriculum knowledge is interconnected with other knowledge: Does curricular knowledge run across the several categories in the MKT model? (Ball et al., 2008, p. 402). Raising this question indicated that Ball and her colleagues acknowledged the complexity of curriculum knowledge. More importantly, they adopted a dialectical approach to the nature of curriculum knowledge, which contrasted the previous metaphysics approach that treated curriculum knowledge as separate and isolated from other phenomena.

Building on the previous studies, researchers synthesized the teacher mathematical knowledge models and conceptualized curriculum knowledge as a construct that is interconnected with other knowledge and develops in the context (Petrou & Goulding, 2011). Through the lens of dialectical materialism, Petrou and Goulding (2011) developed the concept of curriculum knowledge. Their new model revisited the prior related research and shows a shift—which will likely become a future trend—from viewing curriculum knowledge as static and isolated to regarding it as interconnected, dynamic and developing.

2.6 Where Should We Go Next?

Empirical Studies: Bottom-Up Approaches

Curriculum knowledge for mathematics teaching calls for more empirical studies. The bottom-up approaches may provide more insights into curriculum knowledge for mathematics teaching. As suggested by NMAP (2008), research "carried out in a way that manifested rigor and could support generalization at the level of significance to policy" can provide "the best available scientific evidence" for recommendations and assertions to improve U.S. mathematics education (p. 9). Therefore, more rigorous empirical studies on curriculum knowledge are required to add to the literature and contribute to enhancing teacher education.

Yet, as suggested by NMAP (2008), the percentage of such research which met the standard was relatively small compared with the amount of related studies and documents. In carrying out studies, researchers may set the goal of achieving the standard of providing "the best available scientific evidence" for policy makers.

However, a starting point can be small-scope empirical studies focusing on curriculum knowledge for mathematics teaching. Even if the studies involve a limited number of participants, the evidence collected from empirical studies may also offer alternative views into the research.

Theoretical Conceptualization: Top-Down Approaches

In addition to bottom-up empirical studies, curriculum knowledge for mathematics teaching requires an increased and improved theoretical awareness. The top-down approaches may refresh and refocus our knowledge and understanding of this topic and guide practice in a more effective way. In the theoretical conceptualization process, it is indispensable to revisit the previous research, obtain available sources and build on the literature. Hudson, Lahann and Lee (2010) provided a list of sources for research about mathematics curriculum materials:

- *Standards-Based Mathematics Curricula: What Are They? What Do Students Learn edited by Senk and Thompson (2003)*
- *On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Education by the National Research Council (2004)*
- *Professional research journals*
  - *Journal for Research in Mathematics Education*
  - *Journal for Mathematics Teacher Education*
  - *School Science and Mathematics*
- *Center for the Study of Mathematics Curriculum Web site, http://www.mathcurriculumcenter.org/literature.php*
- *What Works Clearinghouse Web site, http://ies.ed.gov/ncee/wwc/*
- *Publishers' and authors' Web sites*
- *Conferences, workshops, and seminars (e.g., AERA, AMTE, NCTM Research Presessions)*
- *Pilot studies completed by local schools* (p.223).

This list of sources provides fundamental yet vital information for researchers and practitioners who are interested in continuing the effort to increase and update the theoretical awareness of curriculum knowledge for mathematics teaching. In addition, Drake (2010) proposed a framework for understanding how teachers supplement curriculum materials. The framework includes reasons for supplementing curriculum materials, resource selection, methods and duration, and assessment and reflection.

A previous example of the theoretical awareness on teacher knowledge is Ma's (1999) work. Based on a comparative study of U.S. and Chinese elementary teachers' understanding and pedagogical approaches of four topics (i.e., subtraction with regrouping, multi-digit multiplication, division by fractions, and area and perimeter), Ma (1999) proposed "profound understanding of fundamental mathematics (PUFM)":

> the awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual structure and instill those basic attitudes in students. A profound understanding of mathematics has breadth, depth, and thoroughness. Breadth of understanding is the capacity to connect a topic with topics of similar or less conceptual power. Depth of understanding is the capacity to connect a topic with those of greater conceptual power. Thoroughness is the capacity to connect all topics. (p. 124)

Despite the question about whether Ma's findings on differences in U.S. and Chinese elementary teachers' knowledge of school mathematics hold significance for larger population or more topics (Hill, 2010), PUFM does provide a theoretical lens about mathematical knowledge for teaching. In the foreword to Ma's book, Shulman regarded the theoretical awareness of PUFM as the one of the "most important contributions" (Ma, 1999, p.ix) of the work.

International Lessons: Outside-Inside Approaches

Another perspective is to learn from top-performing countries on international assessments. Taking the different stances as an insider as well as an outsider, we may gain a deeper understanding of curriculum knowledge for mathematics teaching. Researchers argued that numerous factors such as different education systems, and cultural and economic variations may cause controversy over the possibilities and constraints of learning from cross-national studies on curriculum (e.g., Li, 2007). Despite the learning possibilities and constraints, investigating alternative curricular practices and approaches in those countries which have undertaken school mathematics reforms may contribute to the improvement of mathematics teaching and learning in the U.S. (Kulm & Li, 2009).

In the past decades, U.S. mathematics researchers and educators have been learning from and implementing international curricula. The most noticeable examples include developing *Mathematics in Context*, adopting Singapore Mathematics textbooks, and following the Japanese professional development model of "Lesson Study." For instance, the Dutch answer of Realistic Mathematics Education (RME) to mathematics education reform needs have exerted an influence on primary schools in the Netherlands (Van den Heuvel-Panhuizen, 1996). Based on the Dutch approach to mathematics education through making sense of real mathematic problems, the U.S. developed and implemented *Mathematics in Context*, a U.S. middle school curriculum for grades 5-8. (Romberg, 2001).

CHAPTER III

PROSPECTIVE TEACHERS' KNOWLEDGE OF CURRICULUM FOR TEACHING SIMPLE ALGEBRAIC EQUATIONS

3.1 Introduction

Based on the best available scientific evidence, researchers found that prospective teachers' mathematical preparation requires improvement to achieve effective teaching in classrooms (NMAP, 2008). Among numerous research efforts, Ponte and Chapman (2008) identified major domains regarding prospective teachers' mathematics knowledge and development in the *Handbook of International Research in Mathematics Education*. Through reviewing recent studies on mathematics teacher education from 1998 to 2005, Ponte and Chapman (2008) suggested that school mathematics, curricula and instructional approaches should be enhanced to address deficiencies in prospective teachers' mathematics knowledge .

The Teacher Education and Development Study in Mathematics (TEDS-M), the first large-scale cross-national project, has investigated the preparation of mathematics teachers in elementary and middle schools (Center for Research in Mathematics and Science Education [CRMSE], 2010). The project aims to learn from approaches adopted in other countries to improve the preparation of future mathematics teachers in the U.S. Its major findings include the following: 1) U.S. prospective teachers enrolled in teacher preparation programs are less competent in mathematics than their peers in other countries; 2) U.S. prospective teachers' mathematical preparation for a demanding curriculum such as "Common Core" standards is insufficient; 3) U.S. prospective teachers' mathematics knowledge requires improvement; and 4) a possible solution is to recruit prospective teachers with higher-level mathematical knowledge and train them with a more demanding curriculum (especially in mathematics) in teacher preparation programs (CRMSE, 2010). Despite the solution proposed by TEDS-M project, Hiebert and Morris (2012) raised questions about exclusively relying on recruiting more talented candidates into the profession. Instead, they claimed that to improve classroom instruction, knowledge products of teaching rather than the teachers themselves should be emphasized in teacher education. Similarly, Stigler and Hiebert (2009) asserted that the focus of education should be shifted from teachers to teaching: recognizing classroom routines and choices made by teachers rather than depending on recruiting talented and qualified teacher candidates alone. The contrasting notions of the focus on teaching or teachers in teacher education suggest different trajectories toward building the competence of prospective teachers.

Prospective teachers are faced with challenges in their future mathematics teaching. Previous research indicated a major disconnect between what has been taught in mathematics courses as well as mathematics methods courses in teacher education programs and what is needed for elementary school teachers in teaching mathematics (Askey, 1999; Even & Lappan, 1994). Teachers are often expected to be autonomic in shaping curriculum (Ball & Cohen, 1996); however, prospective teachers generally have difficulties determining and using curriculum due to their lack of teaching experience. To some extent, prospective teachers should be equipped with curriculum knowledge for mathematics teaching (KCMT) in teacher preparation programs to respond to the curricular challenges in classrooms.

This paper focuses on simple algebraic equations to examine prospective teachers' KCMT because algebra has been widely recognized as "the unique and formidable gatekeeper" for K-12 education (RAND Mathematics Study Panel, 2003, p. 47). According to the National Council of Teachers of Mathematics (2000), instructional programs from Pre-Kindergarten through the 12th grade should enable all students "to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships and analyze change in various contexts" (p.37). The recommendation of NCTM (1997, 2000) to introduce algebra in early grades raises the issue of how to integrate algebra into grades Pre-K to 12.

Previous research has documented the nature of middle school mathematics teachers' knowledge for teaching algebra, especially functions (Huang, 2010; Mohr, 2008; You & Kulm, 2008). However, less is known about prospective teachers' pedagogical content knowledge regarding algebra, let alone their KCMT in the content area of simple algebraic equations.

3.2 Purposes and Research Questions

This study examines prospective teachers' (PTs) perspectives on Knowledge of Curriculum for Mathematics Teaching (KCMT) regarding simple algebraic equations. The study also seeks explanations of KCMT from its connections to participants' knowledge of content and teaching as well as their knowledge of content and students. A tentative definition for KCMT at this stage in the study is knowledge about educational goals, standards, assessments and grade levels where specific topics are taught (Ball & Bass, 2009).

The research questions examined in this study include:

1) What are the features of the participating PTs' Knowledge of Curriculum for Teaching Simple Algebraic Equations (KCTE)?

2) What are PTs' perspectives on KCTE?

3) What connections may exist between PTs' KCTE and their knowledge of content and teaching?

4) What connections may exist between PTs' KCTE and their knowledge of content and students?

3.3 Literature Review

To promote the changes in content and pedagogy required in the reforms initiated by the National Council of Teachers of Mathematics (NCTM, 2000), researchers have identified specific needs for adequately educating teacher candidates. Remillard (2000) indicated the importance of curricular guidance in prospective teachers' preparation for teaching.

Regarding prospective mathematics teachers' KCMT, efforts have been made to provide opportunities for them to gain experience from reform-oriented K-12 curriculum materials (Lloyd, 2002, 2004; Lloyd & Behm 2005; Lloyd & Frykholm, 2000; Nicol & Crespo, 2006). To address the challenge of teaching readiness, researchers examined the role of textbooks as a tool for prompting and assisting prospective teachers in developing their knowledge of curriculum (Lloyd & Behm, 2005; Nicol & Crespo, 2006).

Lloyd and Behm (2005) emphasized that prospective teachers should develop abilities to evaluate advantages and disadvantages of different curriculum materials. They also argued that it is important for prospective teachers to analyze instructional materials because it allows them to reflect on what constitutes effective curriculum materials. Furthermore, their research contained the following suggestions:

> 1) increasing teachers' focus on the depth and type of mathematics understandings that students might gain from different instructional materials,
> 2) improving teachers' analysis of the purpose or quality of student interaction and cooperation in the classroom, and
> 3) developing teachers' sense of themselves as curricular decision makers.
> (p. 59)

In another study, Nicol and Crespo (2006) found that the role of textbooks for prospective teachers is not to answer their frequently raised questions, but to pose questions which help them to consider significant issues such as

> How should a teacher teach from a text when a classroom has only enough texts for some but not all of the students? Why should a particular topic be taught at all? How might students respond if the task is adapted? (p.351)

Furthermore, Remillard (2005) proposed a framework which conceptualizes and characterizes aspects of teachers' relationship with curriculum (see Figure 2). According to Remillard (2005), although recent studies offer insights into the influences underlying curriculum use, little is known about the teacher-mathematics curriculum relationship. Intertwined with teaching practices, the teacher-mathematics curriculum relationship is not a straightforward proposition. Rather, it involves four major components: "(a) the teacher, (b) the curriculum, (c) the participatory relationship between them, and (d) the resulting planned and enacted curricula" (Remillard, 2005, p.236). Even if the last component does not apply to prospective teachers, the other three components, especially the participatory relationship between the teacher and the curriculum, are relevant to teacher preparation.

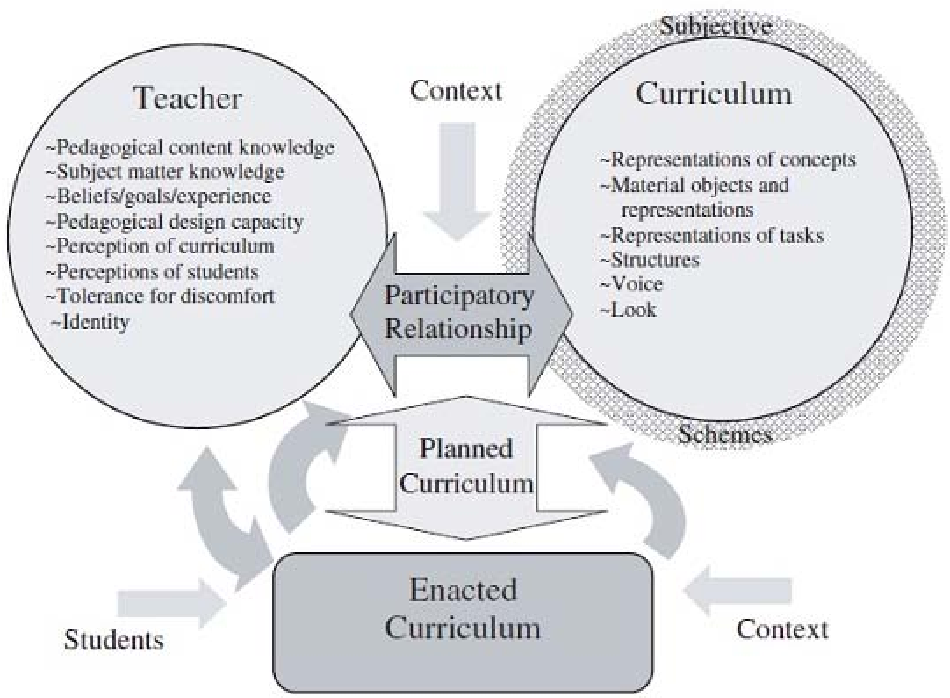

Figure 2. **Framework of components of teacher-curriculum relationship (Remillard, 2005, p. 235).**

3.4 Theoretical Perspectives

The theoretical perspectives for this article were derived from frameworks on mathematical knowledge for teaching (Ball et al., 2005; Ball et al., 2008; Hill et al.,

2008) and various forms of curricula (NCTM, 2010). In recent efforts to provide a map of mathematical knowledge for teaching, Ball and her colleagues (2005, 2006, 2008) proposed a conceptual model (see Figure 3). In the model, they developed Shulman's ideas of Content Knowledge (CK) and Pedagogical Content Knowledge (PCK). According to Shulman (1986), PCK links content and pedagogy, which means subject matter knowledge for teaching (also see Carpenter et al., 1989; Fennema & Franke, 1992). Specifically, PCK includes ways to represent and formulate the content, making it comprehensible to students, and PCK also embodies teachers' understanding of what makes students' learning easy or difficult (Shulman, 1986, p. 9).

In the MKT model, subject matter knowledge indicates the "propositional and procedural knowledge *of* mathematics," covering understanding of content topics, concepts and procedures; PCK means knowledge *about* mathematics (Ball et al., 2005). Following Shulman, Ball and her colleagues define PCK as knowledge of content and pedagogy.

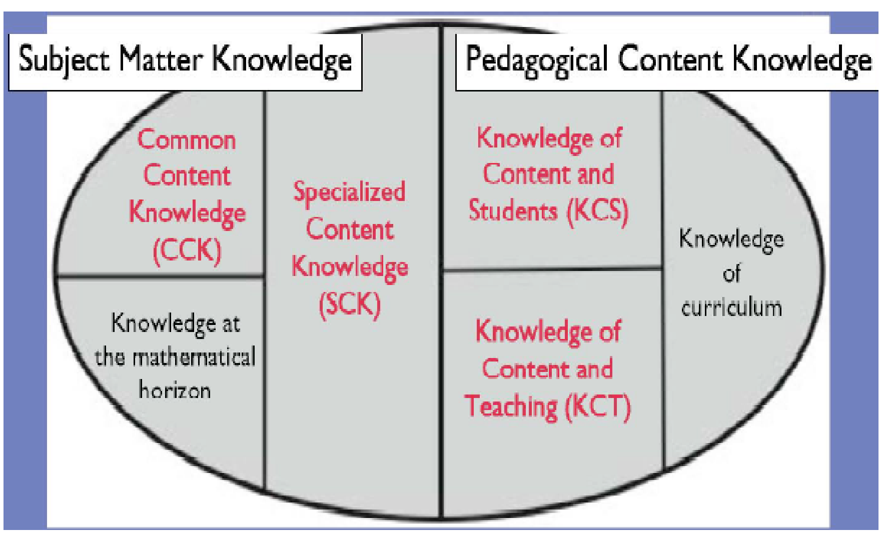

Figure 3. **Mathematical Knowledge for Teaching (MKT) (Ball, 2006, p.15)**

Furthermore, Ball and Bass (2009) viewed knowledge of curriculum as composed of educational goals, standards, state assessments, and grade levels where specific topics are taught. Shulman (1987) claimed that curriculum knowledge serves as a “tool of the trade” for teachers (p.8). With knowledge of curriculum, teachers are able to distinguish and prioritize mathematical goals and topics (Kilpatrick et al., 2001). Knowledge of curriculum, therefore, allows a mathematics teacher to make informed decisions for instruction. Recently, Sullivan (2008) argued that as an integrated part of PCK, KCMT requires teachers to not only study the content of textbooks, but also recognize students’ comprehension of their current classes and choose appropriate topics accordingly.

Given the importance of KCMT, Ball et al. (2008) raised questions about the nature of KCMT:

> We have provisionally placed Shulman’s third category, curricular knowledge, within pedagogical content knowledge … We are not yet sure whether this may be a part of our category of knowledge of content and teaching or whether it may run across the several categories or be a category in its own right. We also provisionally include a third category within subject matter knowledge, what we call “horizon knowledge”…Again we are not sure whether this category is part of subject matter knowledge or whether it may run across the other categories. We hope to explore these ideas theoretically, empirically, and also pragmatically as the ideas are used in teacher education or in the development of curriculum materials for use in professional development. (pp. 402-403)

This excerpt indicates that the categories of KCMT and horizon knowledge are ambiguously defined. Although Ball and Bass (2009) provided a definition for horizon knowledge in a subsequent presentation, they had to clarify that horizon knowledge “is not the same as the detailed curricular knowledge we include in PCK” (p.6). As suggested by Petrou and Goulding (2011), the categories of KCMT and horizon knowledge bear similarities. With changes of foci, the conceptualization of curriculum knowledge requires refinement because the existing research inadequately defines KCMT. Furthermore, the extent of prospective teachers' KCMT remains to be explored.

In addition, according to the 72nd NCTM yearbook (Reys, Reys, & Rubenstein, 2010), curricula have various forms: the intended curriculum, the written curriculum and the implemented curriculum. First, the intended curriculum refers to student learning expectations specified by curriculum authorities. The intended curriculum, or curriculum standards, instructs teachers on what to teach and the degree to which emphasis should be put on the specified content and process standards. Second, the written curriculum includes textbooks and instructional materials that align with curriculum standards. Third, the implemented curriculum is generally shaped by teachers and curriculum materials to provide students' learning opportunities. Each of these three types of curricula plays an important role in determining school mathematics programs and students' learning (Reys et al., 2010). In addition to the above three types of curricula, Thompson and Senk (2010) proposed the concepts of the assessed curriculum and the achieved curriculum. The assessed curriculum is closely related with standardized achievement assessments which determine "what students will have had or should have had an opportunity to learn at a particular grade or from a particular course" (p. 250). The achieved curriculum refers to student achievement assessment results.

The different types of curricula outlined by researchers reveal the meanings of curriculum at multiple levels. The intended curriculum refers to what is determined to be taught at the policy level. The written curriculum primarily regards textbooks and additional curriculum materials. The implemented curriculum describes what happens at the classroom or school-wide level. The assessed and the achieved curricula can be mainly applied to curriculum at the student level.

In sum, this study builds on the theoretical perspectives concerning mathematical knowledge for teaching (Ball et al., 2005; Ball et al., 2008; Hill et al., 2008) and various forms of curricula (NCTM, 2010). From these perspectives, this study strives to fill the research gap in the current literature on KCMT and address the questions posed by Ball et al. (2008) about the nature of KCMT.

3.5 Methods

Participants

Participants for this study were 58 (49 female, 9 male) senior prospective teachers (PTs) enrolled in a middle grades teacher preparation program at a southwestern university. Of the 58 participants, the majority (*n*=47, 81%) identified themselves as White, three (5%) as African American, seven (12%) as Hispanic and one (2%) as Asian American. All the participating PTs had gained field experience in their junior and senior methods courses. They were required to visit their assigned classrooms twice a week for ten weeks. Although they were not required to teach, most PTs voluntarily asked for opportunities to teach or assist with classroom activities. All the participants filled in questionnaires on their KCTE.

These PTs were recruited based on the following criteria. First, they were in the final year of their preparation programs. Upon getting their degrees, participants will have taken seven courses (24 credit hours) in professional development such as planning and development for middle grades curriculum and nine courses of concentration areas on mathematics (27 credit hours) such as structure of Math I and II. In their Junior Methods semester, participants were placed in local public schools for three hours a week for 12 weeks to gain field-based experience. In their Senior Methods semester, they were placed in local public schools for two full, consecutive days each week for 10 weeks. In their assigned classrooms, participants were required to observe and assist their mentors. As a result of this criterion, the participants provided perspectives on KCTE from experience in both course-taking and practicum. Second, they planned to become middle school mathematics teachers after graduation. Third, they were willing to provide data for the study.

Among all participants in this study, the six interviewed PTs were a purposeful sample representing high, medium, and low levels of mathematical performance. In two undergraduate classes designed for prospective teachers to learn methods of middle school mathematics instruction, a mathematics education professor used weekly assignments and mid-term tests to assess her students' mathematical performance. Depending on the professor's assessments on those who volunteered to be interviewed, two PTs were placed at each performance level. All six PTs were female seniors pursuing bachelor of science degrees. Hereafter, the interviewed participants are indicated by PT1, PT2, PT3, PT4, PT5 and PT6.

Instruments and Data Collection

The instruments included a questionnaire and an interview protocol. Both instruments were composed of open-ended questions originating from previous research (Alder & Davis, 2006; Alibali, Knuth, Hattikudur, McNeil, & Stephens, 2007;

Schielack, 2010). A draft of each instrument was developed and provided to selected mathematics education experts for review regarding its feasibility for targeted subject groups and content. After receiving the experts' feedback, the instruments went through multiple reviews before the instruments were finalized and approved. To address Research Question One, participants were asked questions about their selections of the teaching sequence of simple algebraic equations.

As a measure to further investigate PTs' selections, a team made up of the researcher and two experts constructed a version of the sequence. First, the researcher formed a sequence of topics and then consulted one mathematics education researcher and one mathematician on which sequence they would choose. After comparing the sequences of each expert with the researcher's, the researcher discussed the discrepancies with the experts. Finally, the team agreed on an expert version of a teaching sequence. A comparison between PTs' sequences and the expert version was presented in the Results to better answer Research Question One.

The following interview topics correspond to Research Question Two: understanding of intended curriculum and enacted curriculum, understanding of curriculum knowledge and KCTE, familiarity with the sequence of topics and contents of curriculum materials, and advice to peers and professors regarding KCTE learning and teaching. To address Research Question Three, participants were asked to answer questions about 1) representations and instructional techniques when teaching simple algebraic equations, and 2) the role of educational goals, standards, assessments, and textbooks and teacher's manuals in their teaching. In addition, participants answered an interview question about how KCTE helps improve their knowledge of content and teaching. Lastly, the following questions were designed to address Research Question Four: a question about the role of educational goals, standards, assessments, and textbooks and teacher's manuals in anticipating students' difficulties and errors. Additionally, participants answered an interview question about how KCTE helps understand student learning. Moreover, simple algebraic equation problems and scenarios involving students' thinking during equation solving were included in the instruments. The problems and scenarios were designed to provide context for KCTE so that participants have opportunities to connect their KCTE with other components of MKT (Herbal-Eisenmann & Phillips, 2008).

Because this study was built on respondents' experiences, interviews were conducted with six of the subjects (Fontana & Prokos, 2007). According to Charmaz (2002), "qualitative interviewing provides an open-ended, in-depth exploration of an aspect of life about which the interviewee has substantial experience, often combined with considerable insights" (p. 676). Regarding the interviews, most questions were designed beforehand, following the format of grounded theory interview questions (Charmaz, 2000). In addition to the pre-determined questions, follow-up questions relevant to the interviewees' answers were asked to better understand their KCTE. Each of the semi-structured interviews proceeded for approximately 50 minutes. All the interviews were conducted in a quiet, well-maintained office located on a university campus. The researcher recorded the interviews using a digital recorder and took notes on participants' responses that were relevant to KCTE. The interviewees were asked to describe personal views of intended curriculum, enacted curriculum and KCTE. They also shared their perspectives on the connections between KCTE and knowledge of content and teaching as well as between KCTE and knowledge of content and students. At the end of each interview, the participants were asked if they had additional comments or questions about KCTE.

Data Analysis

First, data collected from the questionnaires and interview transcriptions were organized. PTs' answers to Question One in the questionnaire were first analyzed using descriptive statistics. Then the topics selected by PTs and the expert team were compared and assigned with Y or N to show topic placements in the teaching sequence. The topics that were identical between each PT and the expert version were highlighted and calculated at the item level. In the following step, PTs' agreement with experts was determined for each item. Finally, the agreement levels of each item between PTs and experts were grouped into categories of high, medium, fair and poor agreement. Each of these categories contains approximately the same number of percentages. The percentages within each category are similar, falling into a particular range.

The remaining data were coded and analyzed using thematic analysis and content analysis. From a grounded theory perspective (Strauss & Corbin, 1990), the data were coded in two steps: first, initial or open coding, which helped the researcher to discover the various views of participants and decide how to analyze the data; second, selective or focused coding, in which the most frequently occurring initial codes were used to sort, synthesize, and conceptualize the data (Charmaz, 2002). Line-by-line coding was used by the researcher to identify themes and construct conceptual categories. After the data of a first set of participants were coded, the categories were compared and overall patterns were synthesized. Next, the remaining set of participants' data was coded. The researcher compared the themes and patterns obtained from the first set of participants' data with themes and patterns from the second set. In this way, the categories were refined through comparative processes. Finally, the constructed categories were integrated into a descriptive model of participating PTs' KCTE (Charmaz, 2000; Glaser, 1992; Glaser & Strauss, 1967).

3.6 Results

Four major themes that correspond to the four research questions are reported in the summary of results: PTs' KCTE regarding the teaching sequence of simple algebraic equations, PTs' perspectives on KCTE, PTs' KCTE and their knowledge of content and teaching, and PTs' KCTE and their knowledge of content and students. Under these four major themes, a portrait of participating PTs' KCTE is presented.

PTs' KCTE Regarding Teaching Sequence of Simple Algebraic Equations

In addressing Research Question One, concerning the features of the participating prospective teachers (PTs)' Knowledge of Curriculum for Teaching Simple Algebraic Equations (KCTE), an important feature is the sequence of topics in the curriculum. The following results provide an analysis on this feature.

Key Mathematical Topics in Sequence: PTs' Selections

The PTs identified key mathematical topics that should be taught before, during and after teaching simple algebraic equations. These terms (before, during and after) refer to stages in the teaching process that cover relevant and essential mathematical concepts. The questionnaire required the participants either to choose from 14 key mathematical topics related to simple algebraic equations or to fill in the table with their own topics (see Appendix A). Most participants selected from the topics provided instead of adding their own topics.

Table 1
Frequency and Percentage of PTs' Selections of Key Topics in the Teaching Sequence of Simple Algebraic Equations

Item	Frequency and Percentage		
	Before	During	After
Number/visual/word Patterns	54[a] (93.10%[b])	7(12.07%)	1(1.72%)
Linear functions	8(13.79%)	27(46.55%)	27(46.55%)
Number sentences	49(84.48%)	6(10.34%)	4(6.90%)
Representing a number using a letter	22(37.93%)	37(63.79%)	5(8.62%)
Inequalities	32(55.17%)	10(17.24%)	18(31.03%)
Expressions	19(32.76%)	36(62.07%)	4(6.90%)
Negative numbers	55(94.83%)	2(3.45%)	2(3.45%)
Variables	13(22.41%)	45(77.59%)	1(1.72%)
Proportional and other relationships	28(48.28%)	9(15.52%)	24(41.38%)
Linear equations with one unknown	6(10.34%)	44(75.86%)	14(24.14%)
Conceptual understanding of equations	7(12.07%)	23(39.66%)	31(53.45%)
Addition, subtraction, multiplication and division	54(93.10%)	3(5.17%)	1(1.72%)
Properties of Equality	31(53.45%)	13(22.41%)	15(25.86%)
Solving problems using expressions, equations and formulas	2(3.45%)	24(41.38%)	38(65.52%)
Sum of topics	380	290	185
Average number of topics	6.55	5.00	3.19

Note. The three most frequently occurring and illustrative topics for each column have been underlined.
[a]Some participants put a topic into more than one category. For example, one participant put "number/visual/word patterns" in both "before" and "during."
[b]The percentages are out of the total of 58 participating PTs.

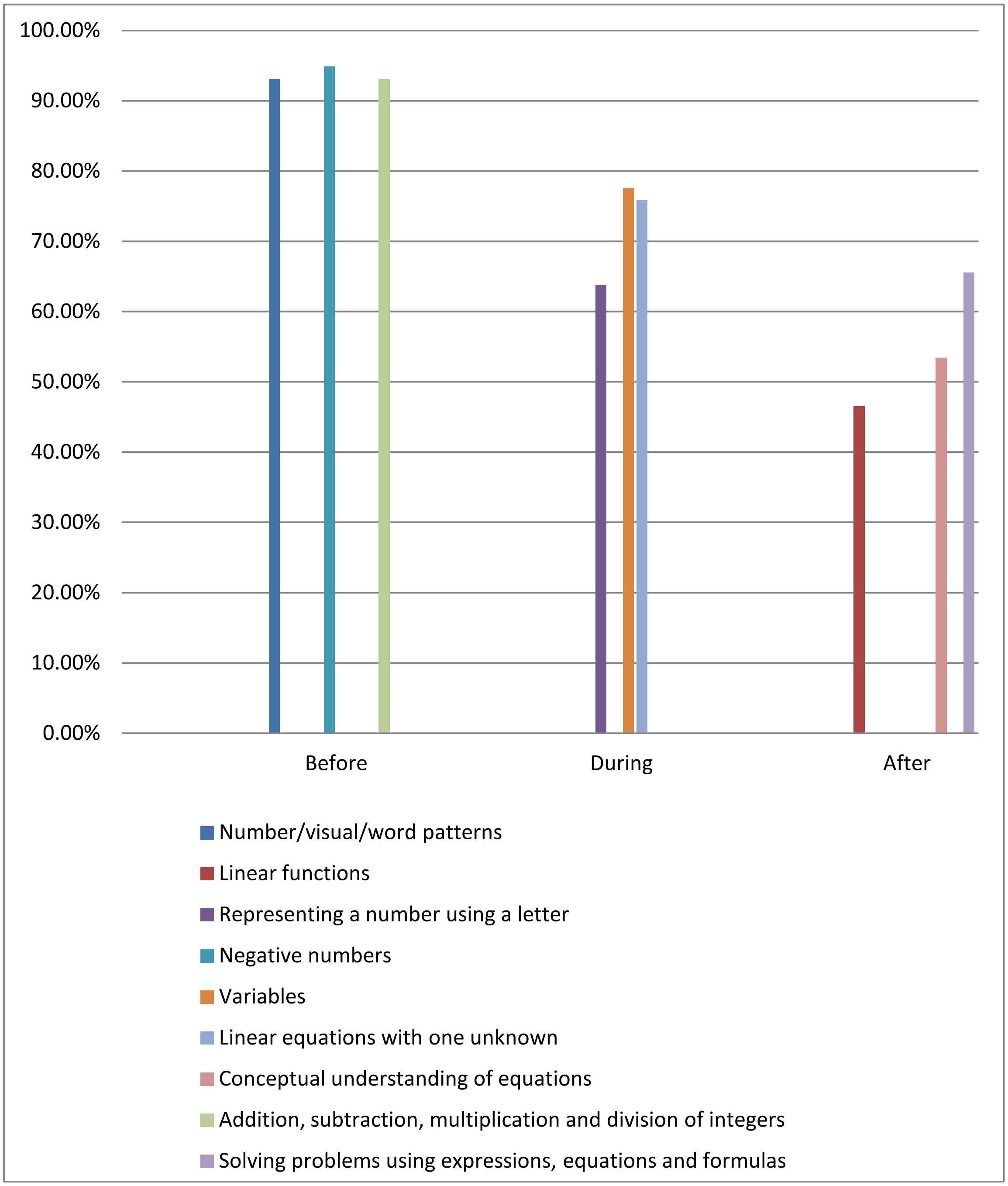

Figure 4. **The most frequently occurring topics in the teaching sequence of simple algebraic equations selected by PTs.**

As shown in Table 1 and Figure 4, participating PTs chose negative numbers (94.83%, n=55), number/visual/word patterns (93.10%, n=54), and addition, subtraction, multiplication and division (93.10%, n=54) as key mathematical topics to be taught before teaching simple algebraic equations. Two of the three choices, namely negative numbers and addition, subtraction, multiplication and division, are basic mathematical knowledge and skills. As for during teaching simple algebraic equations, most PTs selected variables (77.59%, n=45), linear equations with one unknown (75.86%, n=44), and representing a number using a letter (63.79%, n=37) as the top three key mathematical topics. The most frequently occurring topics taught after teaching simple algebraic equations included solving problems using expressions, equations and formulas (65.52%, n=38), conceptual understanding of equations (53.45%, n=31) and linear functions (46.55%, n=27).

The data show that PTs reached a greater consensus on which topics should be taught before teaching simple algebraic equations and agreed less on which topics should be taught during and after. Specifically, all the PTs' selections for "before" topics reached above 90%. Compared with the "before" topics, the participants' identifications for "during" and "after" topics were less concentrated. For instance, Table 1 shows that the three highest percentages for PTs' selections of the "during" topics were 77.59%, 75.86% and 63.79%.

Average Number of Mathematical Topics in Sequence: PTs' Selections

Figure 5 shows the average number of topics chosen by the participants in the sequence of before—during—after teaching simple algebraic equations. The results indicate a pattern for the average PT in this study. Particularly, the average numbers of topics decrease in the sequence: 6.55 ("before" topics), 5.00 ("during" topics), and 3.19 ("after" topics). This finding shows that PTs covered the greatest number of topics in the "before" phase and the fewest number in the "after" phase.

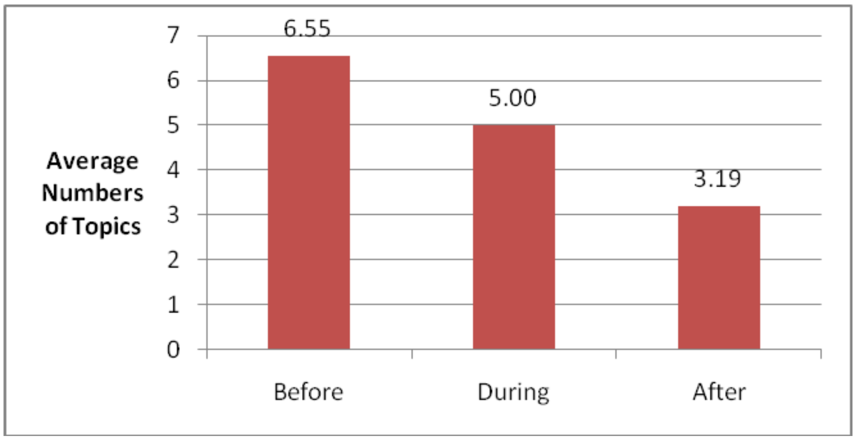

Figure 5. **The average number of mathematical topics chosen by PTs in the teaching sequence of simple algebraic equations.**

Teaching Sequence of Simple Algebraic Equations: Comparisons between PTs' and Experts' Understanding

PTs' selections of mathematical topics in the teaching sequence of simple algebraic equations were compared with experts' selections. This comparison produced percentages and levels of PTs' agreement with experts (see Table 2). Three categories of comparison emerged from data analysis: agreement percentages with each item, agreement levels of the items, and stages of teaching simple algebraic (before, during, and after) within each agreement level.

First, the comparison revealed agreement percentages at the item level. As shown in Table 2, PTs showed decreasing agreement with experts on the following topics: 1) addition, subtraction, multiplication and division, and negative numbers, 2) number/visual/word patterns, 3) number sentences, 4) solving problems using expressions, equations and formulas, 5) linear equations with one unknown, 6) linear functions, 7) conceptual understanding of equations, 8) representing a number using a letter, and inequalities, 9) expressions, 10) properties of equality, 11) variables, and 12) proportional and other relationships.

Second, the comparison indicated agreement levels of the items. The agreement percentages range from 36% to 95%. Descriptive statistical data for the agreement levels include a mean of 66% and a median of 61%. Furthermore, the first four of the above 14 topics fell into the high agreement level with a range from 89% to 95%. The next three topics were grouped into the moderate agreement level, ranging from 63% to 66%. The subsequent four topics were categorized into the fair agreement level with a range from 53% to 58%. The remaining three topics formed the poor agreement level, ranging from 36% to 49%.

Third, the stages of teaching simple algebraic equations (before, during, and after) varied within each agreement level, except for the items at the high agreement level. All the items at the high agreement level come from the “before” stage. The items at the moderate agreement level fell into “during” and “after” stages. The items at the fair and poor agreement levels originated from all three stages.

Table 2
Percentage and Level of PTs' Agreement with Experts on the Teaching Sequence of Simple Algebraic Equations

Item	Stage (Expert Views)	Agreement of PTs' Selections w/Experts	
		Percentage (%)	Level
Addition, subtraction, multiplication and Division	B	95	
Negative numbers	B	95	High
Number/visual/word Patterns	B	93	
Number sentences	B	89	
Solving problems using expressions, equations and formulas	D and A	66	Moderate
Linear equations with one unknown	D and A	64	
Linear functions	A	63	
Conceptual understanding of equations	D	58	
Representing a number using a letter	B	55	Fair
Inequalities	B and A	55	
Expressions	B	53	
Properties of Equality	B and D	49	
Variables	B	47	Poor
Proportional and other relationships	B, D and A	36	

Note. B refers to the stage of before, D refers to the stage of during, and A refers to the stage of after teaching simple algebraic equations.

Perspectives on KCTE from PTs

In this section, two major themes are presented that address Research Question Two: What are PTs' perspectives on KCTE? First, PTs' orientations toward KCTE were mainly focused on assessments. Second, PTs' concerns about curriculum implementation revealed barriers that mainly result from their unfamiliarity with varied school requirements and the complexities of the education system.

PTs' Orientations toward KCTE: Assessment-centered

Remillard and Bryans (2004) suggested that teachers' orientations toward curriculum materials influenced how they used the curriculum. Orientations toward curriculum refers to "a set of perspectives and dispositions about mathematics, teaching, learning, and curriculum that together influence how a teacher engages and interacts with a particular set of curriculum materials and consequently the curriculum enacted in the classroom and the subsequent opportunities for student and teacher learning" (p.364). In this study, "orientations" is used to indicate participants' stances towards KCTE. A stance towards KCTE is composed of perspectives on the concept of simple algebraic equations, teaching simple algebraic equations, and the mathematics curriculum regarding simple algebraic equations.

Through questionnaires and interviews, participants shared their orientations toward KCTE. Most PTs' orientations towards KCTE focused on state assessments. To some degree, state assessments comprise the most important component of participants' KCTE. For instance, two participants shared their stances on teaching to the Texas Assessment of Knowledge and Skills (TAKS):

> The standardized test has been reflective of the teacher…If I was a teacher and only 20% of my students passed our test, they will come back to me and say that I am not teaching them well enough. Maybe the teacher or whoever passed the students onto the next grade level just because they don't want them to be in their classroom for another year and have the same results. The TAKS results reflect the teacher's or administrator's performance or even funding for Title I schools. They have to score high, or they'll lose money. (PT5)

> We are so preoccupied with students passing the tests…We are more worried about them being able to pass the test and move on than really making sure that they know more at a deeper level of why…I will call it 'teaching to the test' and I

> understand why teachers are doing it because the bottom line is they took a look at what percentage of your students that passed the TAKS was, and that is automatically applied to how good a teacher are you. (PT3)

These participants showed a concern for the issue "teaching to the TAKS" with underlying reasons behind the phenomenon. They used the phrases "reflect," "the bottom line," and "automatically applied to" to highlight the dominant role of students' test scores in mathematics teaching. To some extent, students' TAKS scores are closely associated with teachers' and administrators' performance levels and funding availability for their schools. Therefore, mathematics teachers have no other choices but to teach to the state-level tests to guarantee their competitiveness as qualified teachers. It is not surprising that during field experiences the participating PTs developed these orientations towards KCTE as teaching to the TAKS.

Curriculum Implementation: Concerns and Barriers

Concerns and barriers of curriculum implementation were revealed from interviews with PTs. The participants expressed varying degrees of concerns about lacking flexibility or opportunities to make decisions on implementing middle school mathematics curriculum. In other words, curriculum implementation may not be under the control of teachers. The following are responses from two PTs' interviews:

> If my school allowed me to plan the curriculum for my own classroom, you know, is an issue because we don't always get to pick what we teach. Sometimes you have to teach to what your school tells you to teach. (PT4)

> Sometimes it's funny to look at the TEKS because it seems that they might be out of order or they could be paired with different concepts…I guess it's probably important to try to make sure that you follow the TEKS but they may not have to be in that exact order. It would just depend on your students and what you think they'll have the most struggle [with]…I think, normally it depends on which school district because some school districts will give you the curriculum

> for the year and say, 'If I walk in on this day, you should be teaching this TEK.' …You may or may not…be able to stay on curriculum. (PT5)

Both participants used some words or phrases, such as "don't always," "sometimes," and "may or may not," to express uncertainty about their control over curriculum. For example, PT5 recognized that, on some occasions, teachers might need to adjust the curriculum sequence, taking into consideration their students' learning abilities. However, PT5's school district might not allow divergence from the assigned curriculum schedule. The interviews with PT4 and PT5 show that schools may have different requirements for teachers about planning or following mathematics curricula. As PTs, they are less familiar with the curriculum requirements than in-service teachers, who are responsible for implementing the intended curriculum in their classes. Therefore, PTs are faced with the dilemma of not being able to implement the curriculum because they haven't been assigned to teach classes. Even after PTs become in-service teachers, they may remain uncertain about staying on the curriculum due to a complex education system involving the state intended curriculum, school requirements, teachers' expectations, and student learning capabilities.

PTs' KCTE and Their Knowledge of Content and Teaching

In this section, results are presented that address Research Question Three: What connections may exist between PTs' KCTE and their knowledge of content and teaching? In questionnaires and interviews, PTs explained the connections between their KCTE and knowledge of content and teaching. Two major themes were identified from the data analysis: First, a majority of PTs preferred using manipulatives, such as hands-on or virtual manipulatives, in teaching simple algebraic equations. Second, PTs consistently emphasized the importance of computational procedures in teaching simple algebraic equations.

Use of Manipulatives

As shown in Table 3, PTs reported using visual representations as frequent strategies in teaching simple algebraic equations, including manipulatives and models, as well as pictures, graphs and diagrams. Among this group of visual representations, hands-on or virtual manipulatives were used most; approximately 47% of 58 PTs were inclined to use them when teaching simple algebraic equations. Furthermore, most participants regarded visual representations as a helpful and powerful tool for teaching equations.

Table 3
Summary of PTs' Knowledge of Content and Teaching

Category	PTs' Response	
	Frequency*	%**
Instructional Techniques		
Starting with simpler problems	6	15
Connecting with prior knowledge	7	18
Putting the problem in context	1	3
Direct instruction	20	51
Student exploration	4	10
Not demonstrated or specified	1	3
Mathematical Representations		
Manipulatives	27	47
Models	5	9
Pictures, graphs and diagrams	8	14
Not specified	18	31
Reasoning		
No reasoning	28	48
Procedural thinking	20	34
Justifications, explanations or arguments	10	17

Note. *The frequency indicates the occurrence of themes in PTs' answers.
**The percentages are calculated based on the number of occurrence of one theme out of the total number of occurrence of all themes, shown in the left column of the table.

For instance, PT2 reported,

> In an online interactive, you show like each side of an equation using different blocks and pictures, and—I thought that was cool. I don't really know. I probably use online ones and hands-on. I probably have like posters, power points, and something too to show them. Hands-on are not just like worksheets, the same boring things. For equations, you can use things like graphs. I think you can. I don't know, like how to graph paper with different little blocks and pieces of candy or something that you can mark the candy on the graph with, or—I don't know how—what are the other hands-on you can use for solving equations but I am sure there [are]a lot out there.

The participant described the idea of using interactive virtual or hands-on manipulatives to teach simple algebraic equations. However, she used three "I don't know" phrases and unclear sentences with vague meanings. This indicates her uncertainty about the details of what manipulatives she can use and how. More specifically, the participant was unable to explicitly illustrate what manipulatives are available and appropriate to be used in teaching equations, in addition to how to use these manipulatives.

Emphasis on Computational Procedures

Most participating PTs approached simple algebraic equations on the level of operational skills and computational procedures. Rather than understanding the meanings of the concepts and why the algorithms work, they stressed the steps taken to get the answers to mathematical problems. The following response from one PT reflected the major viewpoints of the surveyed and interviewed PTs:

> I would emphasize that when we are solving an algebraic expression for an unknown variable, the goal is to get that variable on one side of the equation itself. In order to do that, we must perform operations of addition, subtraction, multiplication and division to solve for the variable. With this in mind, we must remember that what we do to one side of the equation we must then do to the other side.

The participant focused on the procedure of how to separate a variable from the rest of an algebraic equation. Specifically, *x* should be circled and isolated; operations of addition, subtraction, multiplication and division should be performed; and the rule of "what we do to one side of the equation we must then do to the other side" must be "remembered." However, the participants who held the similar viewpoint did not discuss the underlying reasons that explain why the algorithm works.

Furthermore, PTs reported teaching strategies that are specifically designed for teaching simple algebraic equations. Some PTs used acronyms to help remember the "golden rule" (PT3) of operational skills: "do to one side of the equation what you do to the other." For instance, PT2 showed an overwhelming preference for the acronym "PEMDAS," which outlines the order of operations including parentheses, exponents, multiplication, division, addition and subtraction. Additionally, PT6 recalled the acronym embedded in the phrase of "Please Excuse My Dear Aunt Sally" in her schooling and planned to use the strategy in teaching equations.

The participants regarded these acronyms and mnemonic devices as useful strategies because of the emphasis on operational skills and procedures. The PTs' descriptions stressed the computational procedures in their teaching strategies. This explains why most of them emphasized that "addition, subtraction, multiplication, and division" should have been taught before teaching simple algebraic equations.

PTs' KCTE and Their Knowledge of Content and Students

In this section, evidence is presented to address Research Question Four: What connections may exist between PTs' KCTE and their knowledge of content and students? Two themes that emerged from the data are presented: PTs' awareness of students' foundation of knowledge for learning simple algebraic equations, and PTs' acquisition of knowledge of content and students from in-class experience. By presenting these themes, the connections of PTs' KCTE and their knowledge of content and students are highlighted.

PTs' Awareness of Students' Foundation of Knowledge for Learning Simple Algebraic Equations

Five of the PTs who were interviewed recognized that meeting the needs of all students is difficult. During the interviews, PTs recalled their experience in student teaching related to KCTE. They observed and discerned that students, including high and low achievers, have varying mathematical learning capabilities. Furthermore, some PTs identified students who were poorly prepared for basic mathematics or who had special needs requiring extra attention. For instance, PT2 said,

> Some of the kids don't even know how to do basic multiplication and division and… to solve for x, that kind of stuff. Sometimes they don't know, you know, do it over, bring it over, or carry it over, or just like the basic order of operations… Even some of my students ask, 'What is this plus this' or they have to pull out their facts table for multiplication, 'Are you sure we already know this?' The kids are 5th graders, but I have some kids asking, 'What is 4 times 2?' It's just basic, you know. I was like, that's what you learned before.

Building on the idea that part of a class may be poorly prepared for the new knowledge of simple algebraic equations, PT3 recalled how her mentor asked her to "teach to 50% of the class and catch stragglers later." In that way, her mentor believed that all students at varying levels could get at least a relative grasp of the subject. She continued to comment on the reasons why students lack a solid foundation of basic mathematics knowledge:

> It's been my experience that the teachers who are proficient in math and science don't teach elementary school very often. Those teachers who are in elementary school are generally those who don't like math…I think that math isn't stressed enough in elementary school because a lot of teachers are afraid of math

> themselves. So they [elementary school teachers] will get them[elementary students] through a little bit of what they need to get them through, but the essential skills that they need for the level of math that I am going to be teaching or another colleague who is going to be teaching, they don't have a strong base for that. I don't think it's [because of] their cognitive skills. They [Middle grade students] are ready to know the stuff…to comprehend. It's just…if you don't have a good base to work on…*I mean I can't build a house on a foundation of bricks without any mortar. If I don't have a solid foundation, my house is not going to stand up for very long.* So they might understand the concepts a little bit now, and they can get a few questions right now, but they don't understand the basic skills that the concepts are built upon. Honestly, I think that's why they have such a big problem with it [equations]… You will see kids have a lot of dots on the paper and circle the dots because they're making the groups…*One of the reasons that they are having problems with equations is that they don't know the basic stuff.*[emphasis added]

This long but emotional and powerful description reveals that this PT firmly believed that elementary teachers generally lack proficiency and interest in mathematics. According to this participant, this partly counts for the poor mathematics foundation knowledge of middle school students. Consistent with the responses from other PTs, this participant regarded the ill preparedness of students in basic mathematics as a tremendous obstacle to teaching new mathematical concepts, including simple algebraic equations. Due to the limited sample size, this assertion cannot be generalized to a larger scope of population.

PTs' Acquisition of Knowledge of Content and Students from In-class Experience

Connections between KCTE and knowledge of content and students have been identified as weak by most participating PTs. The PTs' knowledge of content and students mainly comes from their student teaching or class observation experience. For instance, PT4 reported,

> I feel like where I learn the most is actually teaching math, like getting in front of the classroom, or even observing the classroom. I can understand a lot better, you know, how students learn and what difficulties they have because you can read it in the textbook, 'Students are going to have trouble learning this,' but until you actually see it or experience it, it doesn't really stick in your head.

PT2 expressed a similar view:

> I don't know if a curriculum could actually tell you what a student could mess up on or doesn't understand, but like if [I] had prior experiences of classroom observing or I had a teacher in school [who could] tell me a lot of students have trouble with this, then I will be more…likely to look at those kinds of mistakes and be able to help them better if I was taught how to do that. Or if I have experience with prior students or you know, when I was student teaching.

A more detailed account was provided by PT6:

> Observation is a very important thing…being able to observe and see and notice that they [students]are coming up to me with a lot more questions…They are not getting it through assignments, worksheets or quizzes…They are not doing so good. You are just observing…that one or two students are struggling with it [this topic]. Ok, this is what I need to work on with them. But if we notice there is a pattern, a lot of students who are struggling with this topic, then you know that this is the topic…where we need to stop and pause, work on a little bit more specifically, you know, seriously, focus on this a little bit more, maybe push everything off for a couple of days and then just focus on this and maybe create more games and activities for them to master it…

The statements of the participating PTs show that they benefitted more from learning by doing. The PTs learned knowledge of content and students from first-hand experience of observing mathematics classes and discussing issues of teaching and student learning with their mentors. Participants reported that observing classrooms and teaching classes enabled them to notice and gradually identify patterns of frequent questions raised by students. Based on the interview data, the PTs' knowledge of content and students was accumulated more from actual classroom experience than from KCTE.

3.7 Discussion

Through analysis of questionnaires and interview data from PTs, this study generated a description of KCTE and identified its critical features. First, the findings of this study suggested that PTs focused on students' fundamental arithmetic knowledge and skills in the sequence of teaching simple algebraic equations. Data analysis shows a pattern for the sequence: the most topics came from the "before" stage, and the fewest topics from the "after" stage. Second, the comparison between PTs' selections of topics and experts' selections revealed patterns regarding the agreement percentages and levels. Third, this paper demonstrated PTs' orientations toward KCMT. Fourth, the implementations of KCTE were noted. Fifth, connections were drawn between KCTE and Knowledge of Content and Teaching (KCT). Lastly, KCTE and Knowledge of Content and Students (KCS) were found to be weakly connected. Ultimately, this study serves to explore and broaden our understanding of teachers' KCMT as applied to simple algebraic equations.

The findings from this study deserve further discussion. PTs focused on students' fundamental arithmetic knowledge and skills and demonstrated a pattern of decreasing numbers of topics in the teaching sequence of simple algebraic equations. Plausible explanations for this finding include that middle school students may lack a solid foundation in mathematics beyond basic operations and procedures. In addition, the participating PTs may be more familiar with the topics that have been taught before than those that will be taught after. Whether as students or in observations of mathematics teachers, these PTs may never have witnessed conceptual understanding being taught during teaching simple algebraic equations.

The patterns regarding the agreement between PT and expert selections can be illustrated from multiple perspectives. It is interesting to notice that all the items at the high agreement level and all at the low agreement level are those at the “before” stage. Compared with “during” and “after” stages, the items at the “before” stage have the widest range of agreement. The four items with the highest agreement levels are all concerned with numbers. The three items with the lowest agreement levels involve more complex and advanced concepts. This may explain why PTs regarded these items as “during” or “after,” whereas experts categorized them as “before.” In addition, PTs may lack a consciousness of connectedness, coherence and development of topics during different teaching stages; therefore, they preferred placing the same item at one stage instead of across stages. This may account for why those items that go across more than one stage have lower overall agreement levels than those that are categorized in one stage.

State-level examinations and assessments influenced PTs’ orientations towards KCTE. The finding indicated that state assessments were the core component of PTs’ KCTE. This may be partly due to the impact of performance-based accountability. This finding is consistent with a concern raised by researchers and educators that teachers implemented low-level mathematics as they narrowed the curriculum to meet standardized test requirements (Goertz, 2010).

PTs indicated concerns and barriers of curriculum implementation. This finding implies that PTs need more scaffolding to help them face the challenge of implementing the intended curriculum in classrooms. In teacher education programs, PTs should receive more exposure to various curriculum implementations at the administrative level. For example, field trips to vastly different school districts and discussions with administrators and practicing teachers may be helpful for PTs to become more familiar with the impact of school administration on curriculum implementation. In this way, when PTs become novice teachers, they will be better prepared for, or at least be more informed of, curriculum implementation. As a consequence, they will be able to avoid unnecessary struggles.

KCTE and KCT were found to overlap in PTs' preference for using manipulatives and a consistent emphasis on computational procedures in teaching simple algebraic equations. The findings illustrated that PTs demonstrated a preference for manipulatives and may perceive manipulatives in teaching simple algebraic equations as an interesting teaching tool, different from "boring" worksheets. This finding is consistent with Moyer's assertion that the function of manipulatives in classrooms was more focused on fun (2001). Viewing manipulatives in this way, teachers failed to recognize manipulatives as representing mathematical concepts and providing engaging opportunities for students to explore mathematical ideas (Moyer, 2001). To deepen PTs' understanding on how to achieve the effectiveness of the manipulatives in teaching simple algebraic equations, PTs need to be better equipped with KCTE. In teacher preparation classes, they should be taught how to use manipulatives in activities to arouse mathematical thinking and reflection in middle school classes. In addition, in teaching simple algebraic equations, PTs prioritized teaching computational procedures over explaining the underlying reasoning behind the procedures. The findings suggested that PTs focused more on mathematical operations or procedures rather than the underlying reasons of why the knowledge works and the connections between one piece of mathematical knowledge and others. Furthermore, in PTs' answers to the teaching sequence, 53.45% of all PTs' placements of "conceptual understanding of equations" occurred in the "after" stage. It seems that in this study, a large number of PTs perceived conceptual knowledge should come after procedural knowledge instead of the reverse. However, Rittle-Johnson and Alibali (1999) suggested that conceptual knowledge may have a stronger influence on procedural knowledge than vice versa in the causal relations between conceptual and procedural mathematics knowledge. The expert version of the sequence also placed "conceptual understanding of equations" in the "during" stage rather than the "after" stage. Therefore, PTs may need to put more emphasis on conceptual knowledge and reconsider their priority on mathematical operations or procedures over conceptual knowledge in the connections of their KCTE and KCT.

Connections between PTs' KCTE and their KCS are composed of PTs' awareness of students' lack of a solid foundation for simple algebraic equations and PTs' acquisition of KCS from experience as apprentices in mathematics classes. Compared with the revealed connections between PTs' KCTE and their KCT, the connections between PTs' KCTE and their KCS were found to be weaker. The weak connections between KCTE and KCS suggested that KCTE may not have a great influence on KCS.

One plausible explanation is that since KCS involves knowledge of students, PTs probably perceived that their KCS mainly come from their in-class experiences in which they have direct contact with students.

Conceptually, this study contributes to our understanding of KCMT in the MKT framework. It provides some evidence that in the content area of simple algebraic equations, KCMT is connected with other components of MKT. More importantly, this study highlights the importance of KCMT and serves as an initial effort to identify its critical features.

In addition, the findings hold significance for mathematics teacher education. The results obtained from the study offer insights for PTs to improve their KCMT for teaching. As a consequence, more emphasis should be placed on the development of KCMT in mathematics education programs to better equip PTs.

This study also clearly indicates the need for additional research focusing on KCMT. As suggested by Thames and Ball (2010), mathematical knowledge for teaching is complex; it includes mathematical understanding, skill and fluency in helping others learn mathematics. Far from being simple or straightforward, the nature of KCMT requires more research such as studies on interactions between KCMT and other components of MKT.

CHAPTER IV

KNOWLEDGE OF CURRICULUM FOR TEACHING SIMPLE ALGEBRAIC EQUATIONS: PERSPECTIVES FROM FOUR PRACTICING MIDDLE SCHOOL MATHEMATICS TEACHERS

4.1 Introduction

The results from the Trends in International Mathematics and Science Study (TIMSS) 2007 showed an increase in the mathematics performance of U.S. fourth graders since 1995; however, their average performance was still lower than 8 of the 35 participating countries (Gonzales et al., 2008). Even worse, in the 2009 PISA (the Program for International Student Assessment), 15-year-old U.S. students performed below the international average in mathematics literacy (Fleischman, Hopstock, Pelczar, & Shelley, 2010). The most recent results of international assessments imply that U.S. students' mathematical achievements require further improvement to enhance international competitiveness. To increase US students' lackluster mathematics performance, mathematics teachers should enhance their subject matter knowledge and pedagogical content knowledge (Ball & Bass, 2003; Hill, Rowan, & Ball, 2005). As indicated by Ma (1999), the parallel of the achievement gap between students and the knowledge gap between teachers of cross-national studies "is not mere coincidence, it follows that *while we want to work on improving students' mathematics education, we also need to improve their teachers' knowledge of school mathematics*" (p. 144). Equipping teachers with mathematical knowledge for effective teaching becomes a significant issue to improve student performance (e.g., Ball, Lubienski, & Mewborn, 2001; National Mathematics Advisory Panel [NMAP], 2008).

As a critical conceptual construct associated with student achievement, Mathematical Knowledge for Teaching (MKT) has motivated increasing research interest (Ball & Bass, 2000; Fennema & Franke, 1992; Hill et al., 2005; Ma, 1999; NMAP, 2008). MKT includes teachers' subject matter knowledge and pedagogical content knowledge (PCK) (Ball & Bass, 2000). Knowledge of curriculum for mathematics teaching (KCMT), as a part of PCK, lacks substantial research. Little is known about the extent of KCMT for practicing teachers, the connections between teachers' KCMT and their Knowledge of Content and Teaching (KCT), and the connections between teachers' KCMT and their Knowledge of Content and Students (KCS) (see Fig. 3 on Page 55).

According to the RAND Mathematics Study Panel (2003), teachers' knowledge of mathematics has been investigated in several significant content areas, such as fractions, rational numbers, multiplication and division. However, teachers' knowledge in other important areas, particularly algebra, requires more research efforts. What remains unclear is what teachers need to know about algebra and how teachers can effectively teach algebra to help students understand the content.

Despite research efforts to address the international competitiveness of U.S. students, policy makers promoted accountability through an increased focus on high-stakes tests in standards-based reforms. Educational reforms such as the No Child Left Behind (NCLB) Act of 2001 brought considerable impacts on students, teachers, and schools. Darling-Hammond (2004, 2009) suggested that we should not rely on sanctions or rewards to stimulate school or student improvement; instead, a broader notion of two-way and intelligent accountability should be adopted, one that includes the support for teachers to improve knowledge and skills through professional development.

4.2 Purposes and Research Questions

The focus of this study is to describe four middle school mathematics teachers' understanding of the Knowledge of Curriculum for Teaching Simple Algebraic Equations (KCTE). At this stage in the research, KCTE is generally defined as knowledge about the curriculum on simple algebraic equations. Specifically, it concerns knowledge, in vertical, lateral and alternative dimensions (discussed later in detail), that involves one-step, one-variable algebraic equations. The following research questions guided this study:

1) What are the perspectives on KCTE from participating middle school practicing mathematics teachers?

2) What are the connections between the practicing teachers' KCTE and their knowledge of content and teaching?

3) What are the connections between the practicing teachers' KCTE and their knowledge of content and students?

4) What are the similarities and differences in perspectives on KCTE between practicing teachers and prospective teachers?

4.3 Theoretical Perspectives

The theoretical perspectives of this article were guided by frameworks on curricular knowledge (Shulman, 1986; Sullivan, 2008). According to Schoenfeld (2002), teacher professionalism includes curricular knowledge and learning. As a forerunner in curricular knowledge research, Shulman (1987) listed curricular knowledge as one of the categories that form the knowledge base for teachers. Furthermore, Shulman (1986) defined curricular knowledge as knowing the three dimensions of curriculum:

> The curriculum is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (p. 10).

Fennema and Franke (1992) considered knowledge of manipulatives that embody mathematical thinking as one type of curricular knowledge. Furthermore, Shulman (1986) suggested that curricular knowledge is composed of *alternative* curriculum knowledge, *lateral* curriculum knowledge, and *vertical* curriculum knowledge. *Alternative* curriculum knowledge means knowledge of supplemental available materials for instructing a specific subject or content within a grade; *lateral* curriculum knowledge indicates a teacher's ability to connect the content of a specific subject with other subjects being studied simultaneously by students; and *vertical* curriculum knowledge functions as a temporal measurement of prior, current and future knowledge (and associated materials) about one subject. *Vertical* curriculum knowledge has also been defined by Shulman (1986) as "familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them" (p. 10). Understanding these three components of KCMT allows teachers to be able to discriminate and prioritize teaching objectives and mathematical topics (Kilpatrick et al., 2001).

In characterizing categories of the minimum knowledge base for teachers, Shulman (1987) claimed that "curriculum knowledge, with [a] particular grasp of the materials and programs that serve as 'tools of the trade' for teachers" (p.8) is one of seven categories in the knowledge base. Other categories include content knowledge and pedagogical content knowledge. Although Shulman (1987) regarded curriculum knowledge as a distinct knowledge type, separate from pedagogical content knowledge, Ball et al. (2008) suggested that curriculum knowledge may be a component of pedagogical content knowledge. Despite these researchers' claims, further investigation is needed to understand whether curriculum knowledge is a knowledge type by itself or a component of pedagogical content knowledge (Ball et al., 2008). .

In a recent effort to conceptualize teachers' mathematical knowledge for teaching, Petrou and Goulding (2011) emphasized the interplay between the proposed knowledge categories in previous studies. In their synthesized model of teacher mathematical knowledge, curriculum knowledge interacts with pedagogical content knowledge, including knowledge of content and teaching (KCT) and knowledge of content and students (KCS). Curriculum knowledge also interacts with subject-matter knowledge (see Figure 6). Through the theoretical lens of the three components of curriculum knowledge (Shulman, 1986) and the synthesis of models on teacher mathematical knowledge (Petrou & Gouuling, 2011), this study focuses on the components of KCTE and the interplay between KCTE and pedagogical content knowledge.

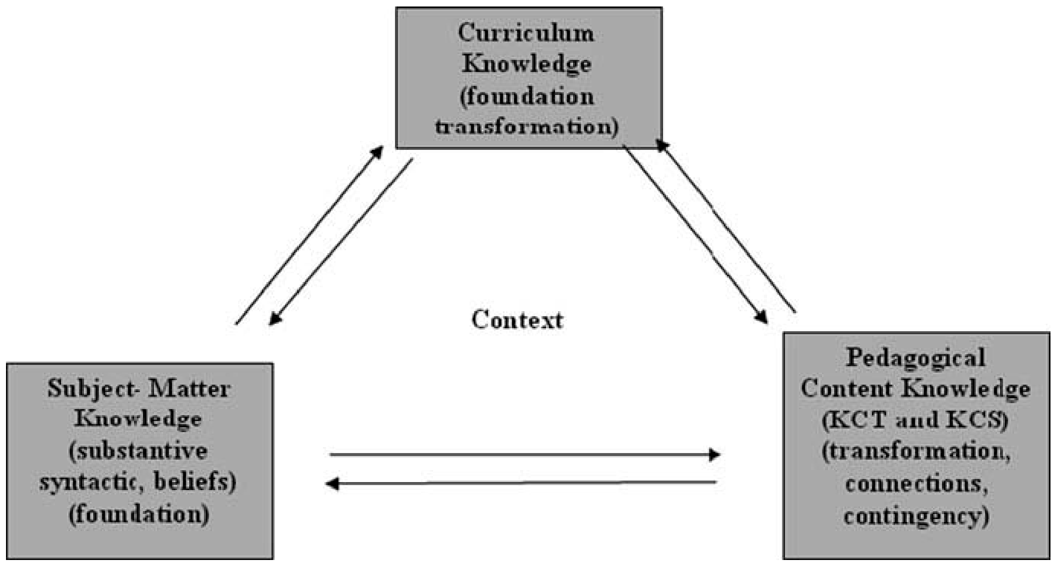

Figure 6. **Synthesis of models on teacher mathematical knowledge (Petrou & Goulding, 2011, p. 21).**

A few studies have focused on how mathematics teachers can acquire curriculum knowledge for successful practice in class. Remillard (2000) investigated how—if at all—a reformed textbook can contribute to mathematics teachers' learning. By examining what and how mathematics teachers learned from using the curriculum materials, Remillard (2000) found that substantial learning did occur through three types of reading by teachers—reading of the text, reading of the tasks and reading of the students. The findings of Remillard's (2000) study provide insight into the ways that curriculum materials can guide and facilitate teachers' learning. This suggests that teachers' KCMT may be developed through their active participation using curriculum materials.

Furthermore, Remillard (2000) suggested that curriculum materials should be designed to support teachers' reading and pedagogical and mathematical decision making. To provide such support, curriculum materials should be written through "speaking *to*" teachers as readers rather than "speaking *through*" teachers (p. 347).

More recently, the "educative" characteristic of curriculum materials have aroused research interests (Brown, 2009; Davis & Krajcik, 2005; Stein & Kim, 2009; Stein et al., 2007). To be specific, educative curricula intend to "communicate directly with teachers" to promote teacher learning besides student learning (Stein et al., 2007, p. 357). Stein and Kim (2009) identified two approaches of sequencing the materials, which impact teacher learning: the first is an *integral* approach, which means the to-be-learned knowledge is "tightly woven into the fabric of the curriculum" and "must be taught in a specified sequence over the years"; the second is a *modular* approach, which has "identifiable and easily articulated student outcomes for each segment that are independent of other segments" (p. 43).

Educative materials intend to direct and support teacher learning during their use of the materials. Stein and Kim (2009) identified the following two aspects of educative materials: (a) making visible curriculum material developers' rationales for including particular tasks, and (b) helping teachers anticipate students' responses. They proposed that *transparency* of curriculum materials refers to the extent to which the curriculum developers make their "rationales, assumptions, or agendas," or "mathematical and pedagogical ideas" underlying the content of curriculum materials "accessible" for teachers using the materials (Stein & Kim, 2009, p. 44).

Remillard (2000) also described the importance of opportunities and supports for teachers to learn and construct their KCMT: "To promote productive use of curriculum materials, professional development opportunities need to foster teachers' teaching and decision making and deepen and broaden their mathematical knowledge" (p. 347).

4.4 Methods

Participants

Four middle school mathematics teachers participated in the study. They are currently teaching in three middle schools in a city of Southern United States. All participants are teaching mathematics for children from various socio-economic backgrounds. The participants were recruited according to the following criteria: first, teaching mathematics at a middle school; second, having taught simple algebraic equations; third, having at least five years of teaching experiences. The following background information was collected (see Table 4). All participants' names are pseudonyms to protect their identity.

Table 4
Background Information of the Participating Practicing Teachers

Pseudonym of Teacher	*Gender*	*Grade(s) Currently Taught*	*Degree*	*Years of Teaching*
Jessica	Female	8^{th}	Bachelor	16
Ashley	Female	7^{th} ,8^{th}	Bachelor	16
Samantha	Female	8^{th}	Master	11
Megan	Female	6^{th}	Master	5

Data Collection

Teaching and research coordinators were contacted to identify potential participants for the research. All participants were first contacted through email and informed about the objectives of the study. Then appointments were made for the participants to complete questionnaires and conduct follow-up interviews.

Except for one participant, three teachers were interviewed immediately after they completed the questionnaires. If the answers to the questionnaire questions were ambiguous, participants were asked to explain their answers prior to interviews. Three of the four interviews were set up in a local restaurant. The participants remained relaxed during the process of filling in the questionnaires and responding to interview questions. During the interviews, mutual trust was established between the researcher and each participant. Individual constructions were elicited and refined by interactions between the researcher and respondents (Guba & Lincoln, 2005).

Data collection techniques include a questionnaire and a semi-structured interview. Individual interviews were conducted to gain insight into respondents' unique experiences and perspectives with KCTE. According to Charmaz (2002), "Qualitative interviewing provides an open-ended, in-depth exploration of an aspect of life about which the interviewee has substantial experience, often combined with considerable insights" (p. 676). Most interview questions were designed beforehand (See Appendix B). However, during the interviews, the participants were asked additional relevant questions to obtain more details from their answers.

Instruments

A questionnaire and an interview protocol were developed for participants to respond to prompts related with their KCTE. The questionnaire includes open-ended questions concerning teachers' KCTE: the teaching sequence of simple algebraic equations, the connections between teachers' KCTE and their knowledge of content and teaching, and the connections between teachers' KCTE and their knowledge of content and students. The interview questions focused on participants' perspectives on KCTE to obtain more in-depth information.

Data Analysis

After organizing and transcribing the questionnaires and interviews, the researcher analyzed the data in two steps, using thematic coding. The first step was identifying conceptions and claims relevant to the research questions. Through a process of open coding (Strauss & Corbin, 1990) and focused coding (Charmaz, 2002), coding scheme categories were developed based on the participants' perspectives on KCTE, the connections between their KCTE and knowledge of content and teaching, and the connections between their KCTE and knowledge of content and students. Initial analysis yielded broad categories. The second step was identifying the specific conceptions that each participant used to describe her perspective and connections were identified, and then the claims that she made to explain her choice were examined. The remaining participants' answers to questionnaire and interview questions were examined for similar statements. Upon further analysis, the categories were modified to accommodate the data, while additional categories emerged. As a result, the researcher noticed consistently shared conceptions and claims among the participants. These conceptions and claims were then categorized and sorted into themes through a cross-analysis of participants' stories, which synthesized individual and common perspectives among participants. The purpose of this analysis was to reveal patterns of participants' perspectives on KCTE through "multivoice reconstruction" (Guba & Lincoln, 2005, p. 196). Due to the small convenient sample, the results of this analysis cannot be generalized to a larger population. In addition, the data collected from the participating teachers were compared with the data collected from prospective teachers (See the previous Chapter for details about prospective teachers' data). Through the comparison, the researcher expects to gain more insights on KCTE in and between each group.

4.5 Results

Perspectives on KCTE from Middle School Mathematics Teachers

In this section, two major themes are presented: 1) perspectives on the teaching sequence of simple algebraic equations from practicing teachers, and a comparison of views between practicing teachers and experts, and 2) practicing teachers' perspectives on KCTE regarding state-level curriculum assessments and standards.

Teaching Sequence of Simple Algebraic Equations: Perspectives of Practicing Teachers and a Comparison with Experts' Views

In the questionnaires and interviews, participants were asked to identify and explain the most important mathematical topics that should be taught before, during, and after simple algebraic equations. Hereafter, "before," "during," and "after" are used to indicate the stages at which mathematical topics are taught prior to, at the same point of, and following simple algebraic equations.

The following features are identified through examining practicing teachers' teaching sequences for simple algebraic equations: First, the "before" stage has the most topics in the before-during-after sequence. Samantha and Megan put more topics in "during" than "after". Jessica and Ashley put more topics in "after" than "during." All four participants, however, put the largest number of topics in "before." It seems that all four participants put the most emphasis, and showed the most concern, on the topics that students have already been taught prior to simple algebraic equations. Common topics for "before" included number/visual/word patterns, number sentences, representing a number using a letter, expressions, variables, negative numbers, and addition, subtraction, multiplication and division.

Second, all the participants categorized at least one item into more than one stage. The number of items that were put into more than one stage ranges from 1 to 6 with a mean of 3. For example, Samantha, Ashley and Megan put "proportional and other relationships" and "linear equations with one unknown" into more than one stage. These three participants agreed that linear equations with one unknown should be taught in both "during" and "after." Jessica perceived "solving problems using expressions, equations and formulas" as a topic that should be taught across all three stages.

Comparison between practicing teachers' and experts' views (see the previous Chapter about the construction of the expert version) reveals several themes: First, the participants showed decreasing agreement with experts (See Table 5) on the following items: 1) number/visual/word patterns, number sentences, representing a number using a letter, negative numbers, and addition, subtraction, multiplication and division, 2) variables, and linear equations with one unknown, 3) expressions, 4) solving problems using expressions, equation and formulas, 5) inequalities, proportional and other relationships, and conceptual understanding of equations, and 6) properties of equality, and linear functions. Second, the agreement percentages range from 58% to 100% with a mode of 100%, an average agreement of 83%, and a median of 88%; Third, all participants achieved high percentages (≥ 83%) on the items that are only in the "before" stage. It's interesting that all the highest agreement with experts occurred on items in "before," which include number/visual/word patterns; number sentences; representing a number using a letter; negative numbers; addition, subtraction, multiplication and division.

Table 5
Percentage of Agreement between Practicing Teachers and Experts on the Teaching Sequence of Simple Algebraic Equations

Item	Stage	Agreement w/Experts
Number/visual/word patterns	B	100
Number sentences	B	100
Representing a number using a letter	B	100
Negative numbers	B	100
Addition, subtraction, multiplication and division	B	100
Variables	B	92
Linear equations with one unknown	D and A	92
Expressions	B	83
Solving problems using expressions, equations and Formulas	D and A	75
Inequalities	B and A	67
Proportional and other relationships	B,D, and A	67
Conceptual understanding of equations	D	67
Properties of equality	B and D	58
Linear functions	A	58

Perspectives of Practicing Teachers on State-level Curriculum Assessments and Standards

State-level Assessments: A Major Focus in Curriculum Implementation. During teaching simple algebraic equations, three out of four participating teachers focused on state-level examination requirements in curriculum implementation. Thus, they could ensure that students would learn the necessary knowledge and skills to excel in the examinations. For example, Megan stated:

> The way we prepare a lesson is…we looked at how it has been tested in the past on the previous TAKS tests and the rigor with which it has been tested. And we discussed possible ways and things that we can do to get the kids ready for questions like that…We focus our instruction on that type of questions.

As revealed in the interviews, Megan and her colleagues extensively focused on the state-level assessments in their lesson preparation and classroom practice. Megan used the phrase "ready for questions like that" to explain why they focused on the state-level tests in teaching simple algebraic equations. In their interview responses, teachers gave priority to assessments—one component of KCMT. The main reason why teachers tended to align their curriculum with the state-level assessments was to guarantee students' successful test performance. Moreover, the participants reported how the students' performance on state-level tests directly impacts teachers and schools. Take Megan, for example:

> The TAKS or STAAR or whatever it's going to [be]…If our kids don't perform well on the test, our school loses funding…It loses respect…It looks bad. So in order to idealistically prevent looking bad, we absolutely forget about teaching the content and worry about teaching them to pass the test.

Megan's claim of teaching students to pass the state-level tests while totally forgetting about teaching the content may be an exaggeration. One cannot assume that the examination-oriented instruction can be found in every mathematics classroom in Texas. However, her claim reflects that teachers may be inclined to focus on teaching students how to pass or excel in the state-level mathematics assessments.

Furthermore, participating teachers were fully aware of state-level examination evolution. For instance, Samantha is sensitive to the changes in the examination content. In particular, she described the focal shift in the examinations from algorithms to higher-level mathematical thinking—from one-step mathematics problems to multi-step problems. Samantha's sensitivity and familiarity with the test content illustrate her focus on the state-level assessments.

In sum, participating teachers mainly concentrated on state-level assessments in their curriculum implementation. Sometimes they structured their teaching content based on the examinations, largely due to the decisive role of state-level assessments in their teaching. Teachers' focus on the examination content was reflected in their concerns about the test changes.

State-level Curriculum Standards: Providing General Guidance. According to the participants, intended curriculum and implemented curriculum are somehow disconnected. They all agreed that the state-level curriculum standards provide general guidance for teaching simple algebraic equations. However, the participating teachers claimed that the guidance is too general to be applied in classrooms. For instance, Jessica said:

> If I read the TEKS, they'll say, here is what you should—8th grade students should be able to solve an equation, and then a whole list of that they should be able to solve proportional problems, and they should be able to, you know, do this, this and this. But it doesn't say anything about ways to get the concept across or how to help them learn…TEKS are so general that sometimes it's hard to understand what they intend…They'll give you something broad.

"General" and similar words and phrases such as "nothing in there" and "way too many [topics]" emerged in Jessica's and the other three participants' descriptions about state-level curriculum standards. By using these words and phrases, they expressed a common concern about the massive amount of general guidelines provided by the TEKS. Grasping the specific and essential concepts included in the TEKS seems challenging for the participants. Moreover, they frequently mentioned that the intended curriculum does not explicitly demonstrate "what they intend." In other words, the participants experience difficulty in understanding the underlying intentions of the intended curriculum. Professional development workshops focused on state-level standards and their implementation may help teachers perceive the intentions of the TEKS, the intended curriculum. Through these workshops, teachers may be able to better understand the intended curriculum and successfully implement the curriculum in mathematics classrooms.

Partly due to the insufficient support from the state curriculum standards, participating teachers responded that they seek resources that may help them design and develop their own curriculum. For example, Ashley described the difficulty of trying to teach simple algebraic equations without sufficient curriculum resources:

> It’s very hard. That’s why I go to conferences to get training. I visit with other teachers. I do a lot of planning…It’s just really really hard because unless you have resources out there, you know, teachers start from ground zero.

As a professional with 16 years of mathematics teaching experience, Ashley regards teaching simple algebraic equations as “very hard.” She explained that she has to get additional resources mainly because of the insufficient support from the intended curriculum. In her interview, Ashley used “ground zero” to indicate that she does not know where to start even though the intended curriculum is supposed to provide support for her. Actively seeking and even struggling to find available curriculum resources may be everyday tasks for teachers. Additional assistance would be beneficial for them to overcome the difficulties of this process.

Teachers' KCTE and Their Knowledge of Content and Teaching: Manipulatives and Conceptual Understanding of Equations

The participating teachers claimed that as alternative instructional tools, manipulatives should be provided for students to understand the concept of equations. Samantha shared her viewpoints of building up the concept of equations from concrete manipulatives such as cups and counters, balance scales or algebra tiles. She preferred involving concrete manipulatives in her classroom activities rather than giving verbal step-by-step instruction about how to solve equations. Through allowing students to manipulate concrete objects, Samantha aimed to achieve her teaching goal of "build[ing] from the concrete."

Furthermore, manipulatives are not solely concrete in the participants' viewpoints. They also reported utilizing virtual manipulatives in mathematics teaching. For example, Samantha described teaching simple algebraic equations using the SMART Board interactive whiteboard and web-based activities from the National Library of Virtual Manipulatives. She also discussed how the new generation of students has grown accustomed to digital technologies. Therefore, Samantha adopted technologies like SMART Board in her teaching, and pointed out the need to incorporate technologies in mathematics teaching.

Additionally, participating teachers reported that manipulatives can function to provide opportunities for students to get engaged by seeing, doing and learning in an active way. For instance, Samantha shared her viewpoints: "They [Students] have to do, and they have to think. If they don't think, they are not actively learning. If they are not engaged in their learning, they are not learning." Jessica provided an example of modeling an equation using manipulatives:

> I would give each student manipulatives such as beans and cups to model a simple equation. Cup->variable, pinto beans->positive numbers, kidney beans->negative numbers. We would put the same number of beans on each side to cancel out the number with the variable and find what goes in the cup.

In the follow-up interview, she further explained why she chose to use manipulatives in teaching simple algebraic equations: "When I did the hands-on, like solving the examples with the beans and cups, that helped students understand more." The above interviews show that both Samantha and Jessica believed that manipulatives, as alternative instructional materials, can help students actively learn simple algebraic equations.

Along with other participants, Jessica regarded manipulatives as a bridge leading to conceptual understanding of equations. When asked about her understanding of the concept of equations, Jessica said,

> By conceptual understanding, I guess when [students] look at an x plus 3 equals negative 2, that's not just symbols on the paper. That means something to them. That means some cup, some unknown…They are trying to make a connection to something they've experienced even if it's just touching the beans or something like that.

As said by Jessica, students may be able to "make a connection to something they've experienced" through manipulating concrete objects. Therefore, the concrete meanings of manipulatives can be associated with students' understanding of abstract mathematical symbols.

Teachers' KCTE and Their Knowledge of Content and Students

Teachers' Awareness of Students' Basic Knowledge in Prior Grades: A Vertical View of Curriculum

Participants' vertical curriculum knowledge leads to their concerns about students' basic mathematical knowledge and skills in prior grades. The most frequently expressed concerns regarded mathematical topics that students should have acquired as background knowledge prior to learning equations. Among these topics, they underscored numbers and operations. For example, Samantha described her disappointing experience with pre-AP 8^{th} grade students' fundamental mathematical knowledge: "It's funny that every year you have to go back to review the foundation of knowledge." This description implies that Samantha was unsatisfied with her 8^{th} graders' ill-preparedness for learning simple algebraic equations. Because of their unpreparedness, Samantha had no choice but to re-teach some prerequisite knowledge that students should have already learned in prior grades.

Similarly, Ashley described her students' lack of fundamental knowledge and skills:

> Something that concerns me is when [the students] were actually introduced to the concepts [in previous grades], they got a poor foundation…They need to understand zero characteristics and also the addition-subtraction inverses and the multiplication-division inverses…I try to re-fix…[I] have to re-teach.

In the above remarks, Ashley expressed concerns about her students' fundamental mathematical knowledge prior to teaching simple algebraic equations. She indicated that she had to re-teach those concepts which her class did not completely understand in prior grades. In addition, Ashley pointed to her students' failure to comprehend zero pairs, one of the prerequisites for learning simple algebraic equations. Due to this lack of understanding, Ashley believed that she had to "re-fix" the mathematical deficiencies before she introduced the new topic. Ashley then expressed dissatisfaction with insufficient or inappropriate approaches, as well as informal and imprecise mathematical language used by her students' elementary teachers. Despite all other possible explanations for middle school students' weak mathematical foundation, such as their lack of motivation and parental disinterest in the children's scholastic performance, Ashley implied that one of the major reasons might be the maladaptive approaches used by elementary teachers.

Students' Multiple Intelligences: Alternative Ways to Approach Curriculum

According to the participants, their students possess multiple intelligences, including visual, kinesthetic and auditory proclivities and strengths. To serve various types of learners, the participants adopt multiple strategies in curriculum implementation. For instance, Ashley suggested the importance of utilizing visual instructional tools for teaching simple algebraic equations. By asserting that "they are all visual learners," Ashley emphasized the visual learning style as the most prominent among all her students. This notion implies that there might be an increasing number of visual learners in mathematics classrooms largely due to the impact of our technology-driven society. While Ashley acknowledged the growing population of visual learners, Jessica was more concerned about meeting the requirements of all students with different learning styles:

> I saw students who struggled and really had a hard time…not because these students don't understand math, or they are dumb, or they can't get it, but because we are not

> teaching them the best way that they can learn. For instance, the visual learners can do very well. What about auditory?

It is interesting that Jessica pointed out that teachers should be responsible for students who are struggling with mathematics learning. According to her, students struggle with learning mathematics because teachers do not use the most effective teaching techniques to fulfill students' varied learning styles. To show how she approached auditory learners in teaching simple algebraic equations, Jessica sang a song to the tune of "If you're happy and you know it:"

> An equation must stay balanced to be true.
> What you do to one side, do to two.
> Whether you add, subtract, multiply or divide,
> You must do it to both sides.
> An equation must stay balanced to be true.

Jessica believed that hearing the song repeatedly and singing it while working on equation problems is helpful for auditory learners. Despite her alternative way of approaching curriculum regarding simple algebraic equations, she stressed the difficulty of meeting the needs of learners who have multiple intelligences. Jessica raised an interesting rhetorical question: "How can you meet the needs of all the learners?" Even as a teacher with rich teaching experience, she considered this question a notable challenge.

Similarities and Differences of Perspectives on KCTE between Practicing Teachers and Prospective Teachers

First, practicing teachers' choices for the teaching sequence of simple algebraic equations demonstrated both similar and distinctive features when compared to prospective teachers' selections. The similarities between these two groups of participants included: 1) the highest agreement percentages concentrated on the "before" topics, and 2) poor agreement on the topic of properties of equality. The differences included: 1) practicing teachers showed higher average agreement with a higher median than prospective teachers; 2) all practicing teachers agreed with experts on "representing a number using a letter" as a "before" topic, although prospective teachers' agreement level on the topic was fair; and 3) practicing teachers reached the lowest agreement on "linear functions" as an "after" topic, whereas the agreement level of prospective teachers on the topic was moderate.

Second, both prospective and practicing teachers focused their KCTE on state-level curriculum assessments. Through questionnaires and interviews, a majority of prospective teachers showed a distinct awareness of the central phenomenon of "teaching to the TAKS," which oriented their KCTE. They also provided rationales for this orientation based on their experiences and knowledge. Similarly, most practicing teachers extensively concentrated on the state-level assessments and offered identical explanations for their concentration. The participants' focus on state-level curriculum assessments of KCTE is mainly due to the significant role of student test performance and the consequent performance-based accountability of teachers and schools.

Third, some prospective teachers revealed concerns and barriers in regards to making curriculum implementation decisions. They mainly discussed their unfamiliarity with varied school requirements on curriculum implementation and the complexities of the education system. In contrast, practicing teachers acknowledged guidance from state-level curriculum standards despite their complaint about the insufficient support from the standards in curriculum implementation.

Fourth, in regards to KCTE and knowledge of content and teaching, most prospective teachers preferred adopting manipulatives and consistently emphasized the importance of computational procedures in teaching simple algebraic equations. Although practicing teachers were also inclined to use manipulatives, they demonstrated an overall better understanding of the manipulatives that can be used, in addition to how to use the manipulatives appropriately and effectively. More importantly, some practicing teachers regarded manipulatives as a bridge leading to conceptual understanding of simple algebraic equations. In general, most practicing teachers' understanding of KCTE and knowledge of content and teaching seems more profound than prospective teachers.

Fifth, concerning KCTE and knowledge of content and students, both prospective and practicing teachers demonstrated their acute awareness of students' foundation in mathematics for learning simple algebraic equations. Practicing teachers stressed approaching curricula in alternative ways to meet the needs of varied student learning styles and multiple intelligences. In contrast, PTs reported that their knowledge of content and students mainly came from in-class experience instead of KCTE.

4.6 Discussion

By investigating the participating teachers' KCTE, this exploratory study provides a portrait of middle school practicing teachers' KCMT, focusing on one specific yet significant mathematical topic (see Figure 7). In particular, this study closely examines curriculum knowledge components proposed by Shulman (1986) and Ball & Bass (2009) and the interplay of KCTE and PCK. The findings of this study indicate that “state-level assessments” serve as the most prevailing and prominent factor in teachers’ KCTE. To some extent, the “state-level assessments” component is placed at the center of the participating teachers’ KCTE. In addition, state-level standards are found to provide general guidance for teaching simple algebraic equations. Participating teachers also reported the overwhelmingly massive amount of content contained in the state standards. This result supports the previous research that state standards can help teachers with class preparation and instruction, despite that these standards “included too much content or omitted some important content or both” (Hamilton et al., 2007, p. 59). The mismatch between the state standards and what teachers perceived should be taught implies that professional development regarding standards and the alignment of standards with curricula needs to be further improved.

In addition, since the participants have to re-teach knowledge that students should have learned at the previous grades, the curriculum knowledge component of “the grade levels where specific topics are taught” (Ball & Bass, 2009) is not as distinctly defined as the state standards indicate. Despite the ideal grade-level distinction indicated in the state standards, the content related to simple algebraic equations actually overlaps across the current grade and the prior grades.

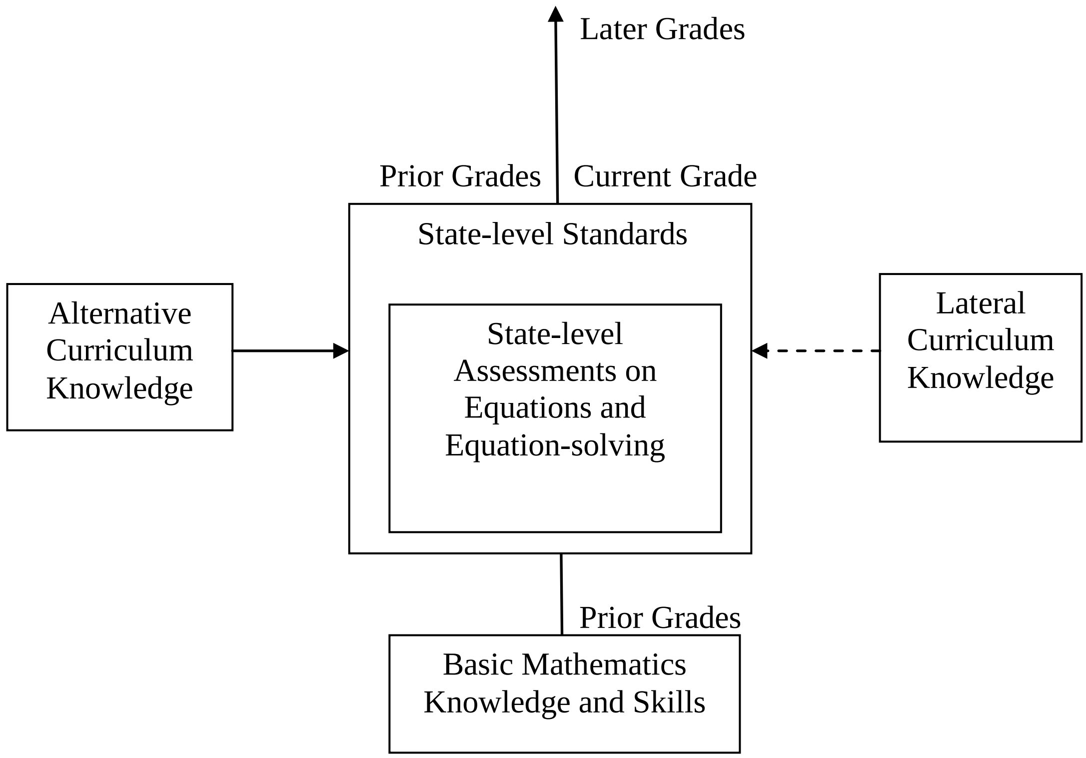

Figure 7. **Identified model of KCTE.**

The findings of this study suggest that the participating teachers acknowledge individual learning styles and prioritize their curriculum accordingly. Recognizing students' multiple intelligences, especially visual and kinesthetic proclivities and strengths, the participants adopt diversified alternative instructional materials, such as hands-on manipulatives, to accommodate students' needs.

In addition, what has been proposed as a theory about KCMT and what has been implemented in practice lacks alignment. Among other researchers, Shulman (1986) and Ball and Bass (2009) have made continuous efforts in revealing multiple dimensions and complexities of curriculum knowledge. Nevertheless, in practice, participants' perspectives on KCTE lack the complexities proposed by the researchers. For instance, almost no participants described their lateral curriculum knowledge that connects KCTE with other subjects, such as science, that are learned simultaneously by students. Up until now, both synthesizing STEM (Science, Technology, Engineering and Mathematics) subjects and identifying appropriate teaching approaches to STEM education have been mainstream topics and widespread concerns. However, the results obtained from this study reveal that the lateral dimension of curriculum knowledge is missing in the participants' KCTE. Therefore, further research is required to investigate how to successfully incorporate the other STEM subjects into mathematics curriculum for the K-12 grades.

Limitations are noted in this study. For example, the small number of participants may not represent the population of practicing mathematics teachers. As claimed by Wang and Lin (2005), more studies are required for greater generalization of the findings obtained from case studies. Although the aim of a small-scope study is not to achieve representativeness and generalization, further study should increase the number of participants for a larger sample. In this way, more perspectives and evidence can be obtained from practicing teachers to further conceptualize KCTE both qualitatively and quantitatively.

Even with these limitations, this exploratory study provides an opportunity to get a closer look into KCMT with a focus on simple algebraic equations. This study also clearly indicates the need for additional research centering on KCMT. As suggested by Fennema and Franke (1992), teacher knowledge is a dynamic construct, interconnected with other components in individual practitioners' classroom contexts. As a starting point, this study may inform further research taking a longitudinal lens and trajectory into KCMT; one potential research topic is the dynamic nature of teachers' KCMT, or its development.

CHAPTER V

SUMMARY AND CONCLUSIONS

This chapter summarizes the three articles and provides the conclusions drawn from them. These three interconnected studies offer a closer look at KCTE, a particular aspect of KCMT, through a sequence of papers. They review the development and implementation of KCMT, and investigate the participants' perspectives on KCTE. Through reviewing the trends in standards, policies, and research on KCMT, the first article provides a context on knowledge of curriculum for mathematics teaching for the following two articles which narrowed the focus to KCTE. By examining prospective teachers' KCTE, the second article identifies a pattern of key mathematical topics in their selections of sequences for teaching simple algebraic equations. In addition, the article reveals participants' orientations toward KCTE and its implementations, and also draws connections between KCTE and KCT, as well as between KCTE and KCS. Through exploring practicing middle school teachers' KCTE, the third article suggests that state-level intended curriculum is the most prevailing component of the participants' KCTE. This article also indicates that middle school students' lack of basic mathematical knowledge and skills impacts participants' KCTE. Other important findings include the participants' alternative curriculum knowledge, the mismatch between the state-level intended curriculum and what teachers perceived should be taught, along with the absence of lateral curriculum knowledge in their KCTE. Together, the results obtained from the three papers serve to provide a more comprehensive perspective on KCMT and broaden our understanding of prospective and practicing middle school teachers' KCTE.

Based on the findings of the three studies, the following conclusions can be drawn: first, state-level assessed curriculum is the core of participants' KCTE. Second, teaching the same mathematics curriculum to all students is a notable challenge for both prospective and practicing teachers. Third, the participants' KCTE, especially practicing teachers' alternative curriculum knowledge, can be explained through a learner-based lens.

5.1 Components of KCTE

These studies with prospective and practicing middle school mathematics teachers provide a deeper understanding of curriculum knowledge components. The components of participating teachers' KCTE are shown in Figure 8.

As the figure illustrates, "State Assessed Curriculum" is the central driving factor in prospective and practicing middle school mathematics teachers' KCTE. The assessed curriculum is a proper subset of "State Intended Curriculum," which is set out in the state standards. In addition, since mathematics teachers have to re-teach knowledge learned at the previous grades, the individual "Grade Levels" are not as distinctively distinguished from one another as the state standards indicate. In other words, "the grade levels where specific topics are taught" (Ball & Bass, 2009) overlap.

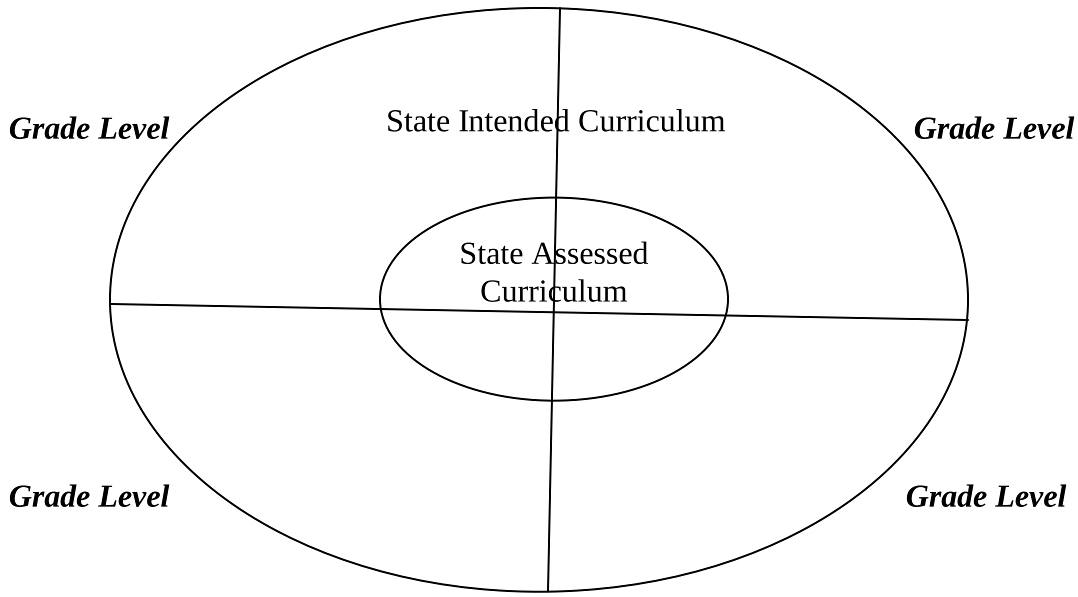

Figure 8. **KCTE components revealed in Chapter III and Chapter IV. When describing knowledge of curriculum, Ball and Bass (2009) have suggested that its components include educational goals, standards, state assessments, and grade levels where specific topics are taught.**

5.2 A Notable Challenge: Teaching the Same Mathematics Curriculum for All

Teaching the same mathematics curriculum for all students is demanding and complex. This research revealed that both prospective and practicing teachers are faced with a notable challenge of teaching the same mathematics curriculum to all students. The U.S. is a multi-ethnic, multi-cultural, and multi-lingual "salad bowl," therefore, culturally relevant or responsive teaching has been advocated (e.g., Ladson-Billings, 2009). In addition to teaching the same mathematics curriculum to students with various ethnic, cultural, and language proficiency backgrounds, American teachers are also required to meet the needs of students with different learning capabilities and styles.

Teacher Education Standards include diversity as one of the essential factors for teacher candidates to acknowledge and promote (National Council for the Accreditation of Teacher Education [NCATE], 2008). Particularly, prospective teachers are required to "operationalize the belief that all students can learn; [and] demonstrate fairness in educational settings by meeting the educational needs of all students in a caring, non-discriminatory, and equitable manner" (NCATE, 2008, p.7). However, "very few teacher education programs have successfully tackled the challenging task of preparing teachers to meet the needs of diverse populations'' (Watson, Charner-Lind, Kirkpatrick, Szczesiul, & Gordon, 2006, p. 396). As a response to this criticism, NCATE (2008) specified that curricula, field experiences and clinical practice should be provided for teacher candidates to improve their knowledge, skills and professional dispositions concerning diversity. The evidence obtained from the research suggests that to better address diversity, more support for teacher preparation and professional development should be provided for both prospective and practicing mathematics teachers.

The findings from this research indicate that both prospective and practicing teachers paid special attention to students with poor basic mathematics knowledge when teaching simple algebraic equations and equation solving. Due to the ill-preparedness of students for the new mathematical content, participants had to teach in a way that allows part or all of the class to learn. Alternatively, participants had to re-teach or review knowledge and skills that students should have already learned in the previous grades. One possible solution to this problem is to enhance mathematics teaching and learning at the elementary level. In addition, curricula are cumulative and become increasingly difficult from the lower grades to the higher grades. Only through grounding the new knowledge of simple algebraic equations on students' prior knowledge, such as inverse operations and properties of basic computations, can students make progress from their previous arithmetic knowledge to the new knowledge of simple algebraic equations.

5.3 KCTE through the Lens of Learners

The most recent focal point in the U.S. curriculum is the nature of individual learners (Brown, 2003; Cullen, Harris, & Hill, 2012; Willis, Schubert, Bullough, Kridel, & Holton, 1994). Rather than focusing on subject matter learning or societal needs, the ultimate goal of education focuses on learners with different cultural and economical identities who can potentially grow into diverse, educated people. The findings of this research support the learner-centered curriculum model.

Article Three on practicing teachers' KCTE identified that the participants acknowledge various learning styles, especially visual and kinetic proclivities and strengths, and prioritize their curricula correspondingly. Additionally, prospective and practicing teachers highlighted the role of manipulatives in teaching simple algebraic equations. In their attempts to meet the needs of different learners, prospective teachers were likely to use manipulatives in future classrooms. Furthermore, as revealed in Article Two, prospective teachers need to be better equipped with why and how to use manipulatives in an appropriate and effective way.

5.4 Disconnect between Theory and Practice

In the past few decades, researchers (e.g., Ball & Bass, 2009; Ball et al., 2008; Shulman, 1986) have made continuous efforts to define curriculum knowledge. Despite the growing body of theoretical awareness, the participants' KCTE in this research is not well aligned with the proposed theoretical models. To be specific, lateral curriculum knowledge (Shulman, 1986) and educational goals (Ball & Bass, 2009) are not revealed in the present research involving prospective or practicing middle school mathematics teachers.

The following questions deserve more attention: How can mathematics teachers/researchers translate theories on KCMT into practice? Do mathematics teachers/researchers need to develop new theoretical models on KCMT? If yes, why and how can the models be developed? Given the disconnect between theory and practice, it is necessary to continue examining prospective and practicing teachers' perspectives on KCMT to obtain more empirical evidence. Furthermore, it is important to initiate a negotiation concerning perspectives on KCMT from politicians, education researchers and practitioners.

5.5 Concluding Remarks

Numerous efforts have been aimed at exploring the knowledge base needed for mathematics teaching (Ball & Bass, 2000; Fennema & Franke, 1992; Grossman, 1990; Hill et al., 2005; Hill et al., 2008; Ma, 1999). However, few studies have attempted to examine the extent of KCMT, the connections between KCMT and knowledge of content and teaching, and the connections between KCMT and knowledge of content and students.

Conceptually, the present study contributes to deepen our understanding of KCMT, focusing on simple algebraic equations. The findings also hold practical significance for professional development of mathematics teachers. Although Ball et al. (2008) provisionally put Knowledge of Content and Curriculum (KCC) within Pedagogical Content Knowledge (PCK), this study shows that teachers' Knowledge of Content and Curriculum, Knowledge of Content and Teaching, and Knowledge of Content and Students are not distinct or independent components of PCK. Instead, these categories of knowledge interact with one another, and sometimes overlap or become integrated. Together, these categories represent an overarching knowledge and conception of mathematical content and pedagogy, serving as

> a "concept map" for instructional decision making, as the basis for judgments about classroom objectives, instructional strategies and student assignments, textbooks and curricular materials, and the evaluation of student learning….Teachers' overarching conceptions are a particularly salient component of the professional knowledge base (Borko & Putman, 1995, p. 47, as cited in Sowder, 2007)

Teacher knowledge cannot be viewed as "an isolated construct…out of context" (Fennema & Franke, 1992, pp.161-162). Instead, "the interactive and dynamic nature" of teacher knowledge should be taken into full consideration. Similarly, teachers' curricular knowledge is a complex construct, which interconnects with other components of teacher knowledge (See *Figure* 9).

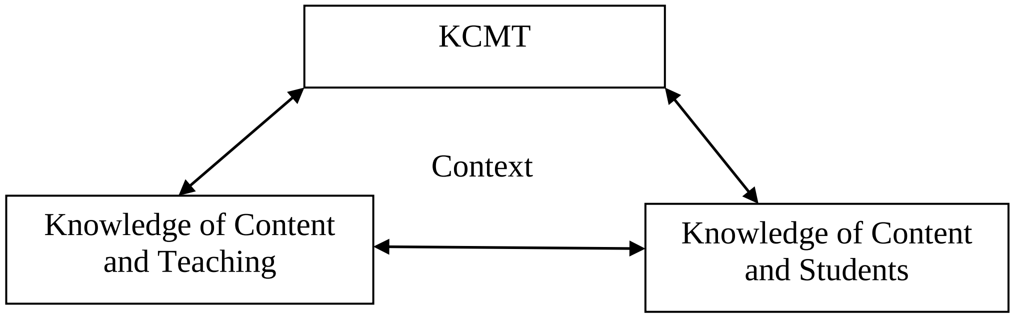

Figure 9. **KCMT in the context.**

The themes that emerged from the research, such as the challenge of teaching the same curriculum for all students, imply the complexities of curricular knowledge. Distinctly separating different categories of mathematical knowledge for teaching into compartments, such as the egg-shaped model proposed by Ball and her colleagues (2005, 2006, 2008), obscures the connections among the categories. Instead, we should break down the divisions between the different knowledge types and shift our vision from investigating what is within each category of knowledge, to the interacting characteristics of mathematics teachers' knowledge base.

In conclusion, the three studies reported in the dissertation add to the growing body of research that focuses on KCMT. However, the dynamic nature of teachers' KCMT, along with what happens in the classroom regarding teachers' KCMT, require further investigation.

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APPENDIX A

Questionnaire: Knowledge of Curriculum for Teaching Simple Algebraic Equations

1. Based on your knowledge of educational goals, standards, state/national assessments, textbooks and teacher's manuals, please fill in the following table using key mathematical topics. In the columns below, please write the mathematical topics you think need to be taught before, during, and after teaching simple algebraic equations. You can choose from the following topics by writing the corresponding numbers, or you can write other topics according to your knowledge:

1. Number/visual/word patterns

2. Linear functions

3. Number sentences

4. Representing a number using a letter

5. Inequalities

6. Expressions

7. Negative numbers

8. Variables

9. Proportional and other relationships

10. Linear equations with one unknown

11. Conceptual understanding of equations

12. Addition, subtraction, multiplication, and division of integers

13. Properties of equality

14. Solving problems using expressions, equations and formulas

Key Mathematical Topics in the Teaching Sequence of Simple Algebraic Equations

Before	During	After

2. Please indicate the resources you used to obtain your knowledge of curriculum for teaching simple algebraic equations. Also, give a brief explanation as to why you chose to follow the resources you did.

3. 1) Different people have different approaches to the same problem. Please solve the following problem and show your work.

435+x=854. What is the value of x?

2) If you were going to teach how to solve the above problem in your class, explain what representations and instructional techniques you would use.

3) How does your knowledge of educational goals, standards, state/national assessments, textbooks and teacher's manuals help you teach?

4) How does your knowledge of educational goals, standards, state/national assessments, textbooks and teacher's manuals help you anticipate possible difficulties and errors your students will encounter?

APPENDIX B

Interview Protocol

We can think about curriculum in two ways: what is intended to be taught and what is actually taught in the classroom. What knowledge about simple algebraic equations is intended to be taught in middle schools? What is actually taught in the classroom?

If you were going to teach simple algebraic equations, what knowledge (e.g., concepts and skills) would you teach?

What representations and instructional techniques will you use to teach simple algebraic equations?

Choose three mathematical topics from the chart of the questionnaire you filled in (one from each column). Explain why you put each of these topics where you did.

If you were going to teach simple algebraic equations, what topics or concepts would have been taught to prepare students? If you have already taught simple algebraic equations, what topics or concepts would you teach next? Give examples.

What is your understanding of knowledge of curriculum? What about knowledge of curriculum for teaching simple algebraic equations?

Does curriculum knowledge of simple algebraic equations help you improve your mathematics knowledge of simple algebraic equations? If so, how?

Does curriculum knowledge of simple algebraic equations help you teach? If so, how?

Does curriculum knowledge of simple algebraic equations help you understand how students learn (e.g., difficulty of topics, common mistakes and confusions)? If so, how?

How familiar are you with the sequence of topics and contents regarding simple algebraic equations?

In terms of simple algebraic equations, how familiar are you with the contents of curriculum materials such as textbooks?

How have you learned about the sequence of topics and contents of curriculum regarding simple algebraic equations?

How have you learned about knowledge of curriculum for teaching simple algebraic equations?

How confident are you that you have enough knowledge of curriculum for teaching simple algebraic equations? Why? What else do you think that you need to know?

What advice would you give to your peers [colleagues] in teacher preparation programs [in your school] concerning what they need to know about knowledge of curriculum for teaching simple algebraic equations?

What advice would you give to professors or lecturers in teacher preparation programs concerning what needs to be taught about knowledge of curriculum for teaching simple algebraic equations?

Is there anything that you might not have thought about before that occurred to you during this interview?