

POWER-INVARIANT MAGNETIC SYSTEM MODELING

A Dissertation

by

GUADALUPE GISELLE GONZALEZ DOMINGUEZ

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

Major Subject: Electrical Engineering

Power-Invariant Magnetic System Modeling

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## ABSTRACT

Power-Invariant Magnetic System Modeling.

(August 2011)

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In all energy systems, the parameters necessary to calculate power are the same in functionality: an effort or force needed to create a movement in an object and a flow or rate at which the object moves. Therefore, the power equation can be generalized as a function of these two parameters: effort and flow,

$$P = \text{effort} \times \text{flow}.$$

Analyzing various power transfer media this is true for at least three regimes: electrical, mechanical and hydraulic but not for magnetic. This implies that the conventional magnetic system model (the reluctance model) requires modifications in order to be consistent with other energy system models.

Even further, performing a comprehensive comparison among the systems, each system's model includes an effort quantity, a flow quantity and three passive elements used to establish the amount of energy that is stored or dissipated as heat. After evaluating each one of them, it was clear that the conventional magnetic model did not follow the same pattern: the reluctance, as analogous to the electric resistance, should be a dissipative element instead it is an energy storage element. Furthermore, the two other

elements are not defined. This difference has initiated a reevaluation of the conventional magnetic model.

In this dissertation the fundamentals on electromagnetism and magnetic materials that supports the modifications proposed to the magnetic model are presented. Conceptual tests to a case study system were performed in order to figure out the network configuration that better represents its real behavior. Furthermore, analytical and numerical techniques were developed in MATLAB and Simulink in order to validate our model.

Finally, the feasibility of a novel concept denominated magnetic transmission line was developed. This concept was introduced as an alternative to transmit power. In this case, the media of transport was a magnetic material.

The richness of the power-invariant magnetic model and its similarities with the electric model enlighten us to apply concepts and calculation techniques new to the magnetic regime but common to the electric one, such as, net power, power factor, and efficiency, in order to evaluate the power transmission capabilities of a magnetic system.

The fundamental contribution of this research is that it presents an alternative to model magnetic systems using a simpler, more physical approach. As the model is standard to other systems' models it allows the engineer or researcher to perform analogies among systems in order to gather insights and a clearer understanding of magnetic systems which up to now has been very complex and theoretical.

## DEDICATION

To my parents, sisters and godmother

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## CHAPTER I

### INTRODUCTION

#### *A. Generalities*

Magnetic materials have become indispensable in modern technology. They are the basis of many electromechanical and electronic devices needed for generation, distribution and conversion of energy. Similarly, in transportation they play an important role as electric motors are fundamental in the operation of hybrid-electric and electric vehicles which are becoming more and more popular. Moreover, information technologies ranging from personal computers to main frames use magnetic materials in tapes, floppy diskettes or hard disks to store information. These applications are just some examples of a \$30 billion annual market industry that keeps increasing and diversifying [1, 2]. As researchers and engineers it is our duty to provide efficient and optimal designs for these devices, in order to do so, we need a deep understanding of magnetic materials and magnetic theory in general.

Since the Greeks first documented the properties of the natural magnetic materials in the 8<sup>th</sup> century B.C., scientists have been interested in its properties and characteristics. Progress in magnetism was made after Oersted discovered in 1820 that a magnetic field could be generated with an electric current. Famous scientists, including Gauss, Maxwell and Faraday, tackled the phenomenon of magnetism from a theoretical

---

This dissertation follows the style and format of the *IEEE Transactions on Power Electronics*.

point of view, however, contributions from 20<sup>th</sup> century physicists like Curie and Weiss with their studies on the phenomenon of spontaneous magnetization and its temperature dependence laid the foundations of modern magnetism [3, 4].

Advances in manufacturing and modeling techniques have improved exponentially the quality of electromagnetic devices. In the last years, magnetic research has been characterized by the development of different models based either on mathematical or physical approach. Improvements in computer technologies have enabled the use of numerical techniques like the finite element method, the finite difference method and the boundary element method, which have been successfully applied to solve several types of engineering problems such as heat conduction, fluid dynamics, electric and magnetic fields, among others [5]. However, in some cases, these numerical methods are used as a last resort technique or as an optimization tool in order to avoid their complexities. Instead, other techniques like the lumped parameter modeling are used as a first design approach.

The use of lumped parameters to model and analyze electric, mechanic and hydraulic systems is very common. Instead of using field equations, as in most of the numerical methods, it uses their integrated effects. For example, in the case of electric systems, it deals with quantities as voltage, current, charge, resistance, capacitance, inductance, etc., rather than field intensity, current density, charge density, resistivity, permittivity, permeability and so on, since they provide a physical meaning of the system that is being analyzed [6]. In the magnetic regime on the other hand, it is more common to use field quantities as described by Maxwell's equations in order to analyze



the systems. Still, using these field quantities might be challenging due to the theoretical level at which the system is analyzed. Nonetheless, the easiness that the lumped parameters provide to the analysis of other regimes systems has been the reason to use them in the magnetic regime as well.

### *B. Lumped Parameter Magnetic Models*

Several lumped parameter models have been developed for magnetic systems. The reluctance model is the oldest and most widely used in magnetic analysis therefore we consider it as the conventional magnetic model. In this section we present a brief description of the reluctance model as well as other lumped parameter models that have been developed in order to provide a solution to the deficiencies of the reluctance model.

#### *1. The Reluctance Model*

The first model developed to represent magnetic circuits using integrated quantities is the reluctance model. This model is based on Hopkinson's law, which is analogous to Ohm's law for electric circuits [7].

In electric circuits, Ohm's law is an empirical relation between the electromotive force (*emf*) or voltage applied across an element and the current (*I*) flowing through that element. It is expressed as

$$emf = IR \quad [\text{V}], \quad (1)$$

where *R* is the electric resistance of the material [ $\Omega$ ].

Similarly, for magnetic circuits, Hopkinson's law is an empirical relation between the magnetomotive force (*mmf*) and the magnetic flux ( $\varphi$ ), given by

$$mmf = \varphi \mathcal{R} \quad [\text{t}\cdot\text{A}], \quad (2)$$

where  $\mathcal{R}$  is the magnetic reluctance [ $H^{-1}$ ].

When the reluctance model was defined, (2) was considered analogous to (1) in both form and functionality, but it has been well established that this is not the case. The main difference between these expressions is that in electric circuits, the resistance is a measure of how much energy can be dissipated as heat while current flows in the material. Meanwhile, in magnetic circuits, the reluctance is a measure of magnetic energy storage rather than being a measure of magnetic energy dissipation.

## *2. The Permeance-Capacitor Model*

In 1969, Dr. R.W. Buntenbach from the University of California stated in [8] that there are several stumbling blocks that inhibit the progress of magnetic circuit theory, namely, 1) failure to recognize that electric and magnetic quantities are dimensionally amenable to identical mathematical treatment; 2) the absence of a magnetic impedance concept; 3) the erroneous reluctance-resistance analogy; 4) the inductance concept; and 5) the lack of satisfactory energy transducing element to interconnect the electric and magnetic circuits of the inductor.

In an effort to correct some of these statements, Buntenbach proposed a different model also based on the analogy between electrical and magnetic quantities. In Table I, we can see how in this model the flux is analogous to electric charge, contrary to the reluctance model where flux was analogous to electric current. Also, the terms flux rate, magnetic resistance, magnetic inductance and magnetic impedance were introduced. However, the main contribution of this model is that it presents the Permeance-Capacitor analogy. This model suggests that the magnetic circuit is better represented as an

TABLE I  
ANALOGS OF ELECTRIC AND MAGNETIC QUANTITIES

ELECTRIC			MAGNETIC		
Quantity	Units	Symbols	Quantity	Units	Symbols
Charge	Coulombs	Q	Flux	Webers	$\phi$
Current	Amperes	$I = dq/dt$	Flux rate	Webers/sec	$d\phi/dt$
Electromotive force	Volts	$E = N d\phi/dt$	Magnetomotive force	turns·A	NI
Electrical resistance	Ohms	$R = E/I$	Magnetic resistance	1/ohms	$R_m = NI/(d\phi/dt)$
Capacitance	Farads	$C = Q/E$	Permeance	Henrys	$P = \phi/NI$
Capacitance reactance		$1/j\omega C$	Permeant reactance		$1/j\omega P$
Electrical inductance		$L = N\phi/I$	Magnetic inductance		$L_m = N^2P$
Inductive reactance		$j\omega L = j\omega N^2P$			
Electrical impedance		$Z = E/I$	Magnetic impedance		$Z_m = NI/(d\phi/dt)$

analogue to an electric capacitance rather than an electric resistance, which is true in functionality and can be validated by examining energy relations.

As we know, the energy of an electric capacitor can be expressed as function of the capacitance ( $C$ ) and the voltage ( $V$ ) as

$$E = \frac{1}{2} C \cdot V^2 \quad [\text{J}]. \quad (3)$$

Using a similar relation for the magnetic circuit, where energy is a function of the permeance ( $P$ ) and the magnetomotive force ( $mmf$ ) then,

$$E = \frac{1}{2} P \cdot mmf^2. \quad (4)$$

If we perform a dimensional analysis of (4) we can see that the energy units are Joules as expected.

$$E = \frac{1}{2} P \cdot mmf^2$$

$$E = \left( \frac{\text{webers}}{\text{turns} \cdot A} \right) (\text{turns} \cdot A)^2$$

$$E = \text{webers} \cdot \text{turns} \cdot A$$

$$E = \text{turns} \cdot \frac{V}{s} \cdot A$$

$$E = \text{turns} \cdot \text{Joules}. \quad (5)$$

### 3. The Gyrator-Capacitor Model

In 1993, Dr. D. Hamill applied a method similar to [8] to model magnetic circuits. He applied the gyrator-capacitor approach to model: a gapped-core inductor, a two-winding transformer and a Ćuk converter with satisfactory results [9, 10].

In the Gyrator-Capacitor model, the permeance is used analogous to the electric capacitance, but as seen in Table II, it is the only circuit quantity of the model, contrary to the Permeance-Capacitor model which includes expressions for magnetic resistance and inductance. However, in later publications appear extensions to the original gyrator-capacitor model where a magnetic resistance and magnetic inductance are defined but there is neither a comprehensive description nor an application of them [11, 12].

The quantities of the gyrator-capacitor model are based on the gyrator theory [13,

TABLE II  
GYRATOR-CAPACITOR MODEL ANALOGIES

Magnetic Circuit			Electrical Circuit		
Quantity	Units	Symbols	Quantity	Units	Symbols
MMF	A	$mmf$	Voltage	V	$v$
Flux rate	V	$d\phi/dt$	Current	A	$i$
Permeance	H	P	Capacitance	F	C
Flux	Wb	$\phi$	Charge	C	$q = \int idt$
Permeability	H/m	$\mu = \mu_o\mu_r$	Permittivity	F/m	$\epsilon = \epsilon_o\epsilon_r$
Power	W	$P = Fd\phi/dt$	Power	W	$P = vi$
Energy	J	$E = \int Fd\phi$	Energy	J	$E = \int v dq$

14]. The gyrator, first introduced by Tellegen in 1948, is a two-port network passive element that links the electric and magnetic regimes.

In Chapter III, we will explain in detail the gyrator as it will be used to define the quantities that we propose for our model.

### C. Problem

According to the law of conservation of energy, energy can neither be created nor destroyed only transformed. If we analyze the electric model, for example, there are three quantities that represent the transformation of energy: the resistance, which indicates the energy that is transformed into heat, the capacitance, which indicates the amount of energy stored in the electric field and the inductance which indicates the energy that is stored in the magnetic field. In Table III, we can see that equivalent to these quantities can be found in other systems except for the magnetic one. Furthermore, losses which are of great importance in any design cannot be calculated either by using the power equation or by an analogy of the electric Joules law because they do not apply in the conventional magnetic model, the reluctance model. Instead, they are calculated

TABLE III  
DIFFERENT SYSTEM'S ANALOGIES

Electric	Mechanic	Hydraulic	Magnetic
Charge {Q} [C]	Distance {x} [m]	Volume {V} [m <sup>3</sup> ]	-----
Current {I} [A]	Velocity {v} [m/s]	Volume flow rate {Q} [m <sup>3</sup> /s]	Flux {φ} [Wb]
Voltage {V} [V]	Force {F} [N]	Pressure {P} [Pa]	mmf {F} [t·A]
Resistance {R} [Ω]	Friction {B}	Friction	Reluctance {R} [H <sup>-1</sup> ]
Capacitance {C} [F]	Compliance {1/k}	Capacitance {C <sub>i</sub> } [m <sup>3</sup> /Pa]	-----
Inductance {L} [H]	Mass {M} [kg]	Mass {M} [kg]	-----

using empirical equations or most commonly, by using the area of hysteresis loops as in the case of the hysteresis loss calculation.

In this dissertation, we will present an alternative model to magnetic systems that seeks to standardize the models in all regimes and show an elegant solution to the power calculation problem. Our approach is different to the models discussed earlier because we believe that the macroscopic magnetic phenomena can be represented by three quantities (magnetic resistance, inductance and capacitance) of magnetic origin, not just as quantities gyrated from the electric to the magnetic regime. Nevertheless, we understand that this model has limitations and we must be very careful in its characterization to avoid any misconceptions.

#### *D. Research Objectives*

Using gyrator theory and knowing that power is invariant across different energy regimes, we will:

- Standardize the magnetic system model;
- Develop a method of modeling magnetic systems using lumped parameters;
- Use techniques, well known in other regimes, to analyze magnetic systems;
- Use our new models to facilitate the design, study, and simulation of complex magnetic systems;
- Use our new modeling insights to invent new magnetic systems.

### *E. Dissertation Organization*

The next chapter includes a literature review of magnetic materials and electromagnetic theory as these are fundamental for the development of the theory that we are proposing.

In Chapter III, we present our theory. It offers a comprehensive description of all the quantities used in our model which are defined based on an analogy between electric and magnetic regimes.

In Chapter IV we present a conceptual test that was designed with the purpose of finding the configuration of magnetic elements that represents the behavior of an iron core coil used as case study. Also, we elaborate a conceptual experiment using Simulink® in order to validate our model.

The core of Chapter V is the application of our theory. We explore the idea of transmitting power in the magnetic regime using a novel concept that we have denominated a magnetic transmission line. We design a case study system where the electric terminals parameters and the magnetic characteristics of the line are given. The optimal design is chosen based on how the efficiency and the power factor of the system are affected due to variations in geometry, frequency and the number of turns of the windings.

Finally, Chapter VI will summarize the work accomplished and propose future research in this area of knowledge.

## CHAPTER II

### LITERATURE REVIEW

Since the discovery of the loadstone and its application in devices like the navigational compass, invented around 1088 in China, different theories have been developed in order to explain the magnetic phenomena and the properties of magnetic materials. This chapter presents a literature review of the most significant contributions made to this area of knowledge.

#### *A. Fundamentals of Electromagnetism*

##### *1. The Law of Force between Particles*

In physics, the interactive forces are the ways that the simplest particles in the universe interact with one another. The four known interactive forces are strong interaction, weak interaction, gravitation and electromagnetism [15].

Gravitation was the first force to be known that affect all material particles. In 1687, Sir Isaac Newton postulated the inverse-square law of universal gravitation which states that the gravitational attraction between two masses ( $m_1$ ,  $m_2$ ) varies inversely as the square of the distance ( $r$ ) between them,

$$F_g \propto \frac{m_1 m_2}{r^2} \quad [\text{N}]. \quad (6)$$

In the 18<sup>th</sup> century, when electromagnetism was considered as two different forces: electricity and magnetism, it occurred to the pioneers of electricity that the force between electrically charged particles might obey the same rules as gravitation. Experimental verification of this theory was announced in 1785 by Charles Coulomb



[16]. The Coulomb's inverse square law, also known as Coulomb's law, states that the force between two particles at rest is proportional to the product of their charges ( $q_1, q_2$ ), and inversely proportional to the square of the distance ( $r$ ) between them,

$$F_e \propto \frac{q_1 q_2}{r^2} \quad [\text{N}]. \quad (7)$$

Charges with same polarity repel each other while charges with different polarities attract each other. A similar behavior was seen in magnetism.

Almost everyone as a child has played with magnets and felt the mysterious forces of attraction and repulsion between them. These forces appear to originate in regions called poles, located near the ends of the magnet. Two independent studies, one made by John Michell in England in 1750 and the other by Charles Coulomb in France in 1785, showed that the forces between the extremities or poles of two magnets obeyed the inverse square law [17, 18].

In this case, the force between two magnetic poles is proportional to the product of their pole strengths ( $p_1, p_2$ ) and inversely proportional to the square of the distance ( $r$ ) between them,

$$F_m \propto \frac{p_1 p_2}{r^2} \quad [\text{N}]. \quad (8)$$

Nevertheless, in (8) different to (7) a single magnetic pole ( $p_i$ ), which should be analogous to an electric charge ( $q_i$ ), does not exist. Physicist that tried to separate the poles of a magnet obtained, not a pair of single poles, but a pair of magnets. It became apparent that the ultimate indivisible unit of magnetism was not the magnetic charge but a magnetic dipole, which might consist of a pair of equal and opposite magnetic charges indissolubly, fixed together. However, despite the fact that magnetic poles represent a

mathematical concept rather than reality, the concept is useful in visualizing many magnetic interactions, and helpful in the solution of magnetic problems.

## 2. *Electric and Magnetic Fields*

In the same way as the Earth is considered to be surrounded by a gravitational field in which heavy bodies experience forces, the electric charges ( $q_i$ ) are said to be surrounded by an electric field, which is a region in space or matter in which a charged particle experiences a force. This force is directly proportional to the product of the charge ( $q$ ) and the electric field intensity ( $\mathbf{E}$ ),

$$\mathbf{F}_e = q\mathbf{E} \quad [\text{N}]. \quad (9)$$

The electric field intensity ( $\mathbf{E}$ ) is one of the vector quantities used to describe the electric field. The second vector quantity that is used to describe the electric field is the electric flux density or displacement field ( $\mathbf{D}$ ). The primary function of the vector  $\mathbf{D}$  is to give a point-by-point description of the field that is independent of the character of the medium, different to the vector  $\mathbf{E}$  which is dependent of the medium,

$$\mathbf{D} = \varepsilon\mathbf{E} \quad [\text{C/m}^2], \quad (10)$$

where  $\varepsilon$  is the dielectric constant of the medium [F/m].

The electric field intensity, the electric flux density and the electric force are vectors that possess the same direction. As we cannot see the electric field, we form a mental picture of it using the idea of lines of force. A line of force is a continuous curve, whose direction at every point is that of the electric field. For example, in Fig. 1a, the electric field is depicted as lines of force flowing from two opposite charges.

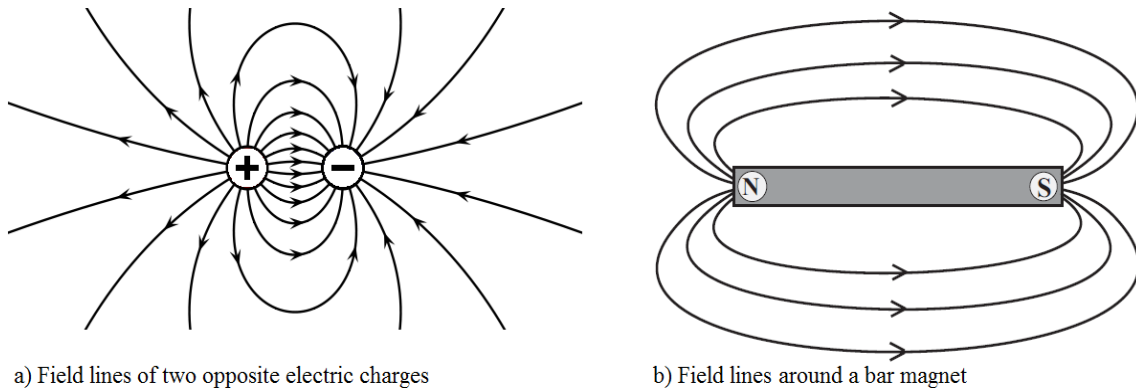


Fig. 1. Electric and magnetic fields depicted by lines of force.

Similar to the electric regime, in the magnetic regime the magnetic pole creates a magnetic field, and is this field which produces a force on a second pole nearby [18]. Experiment shows that this force is directly proportional to the product of the pole strength ( $p$ ) and the field intensity ( $H$ ),

$$F_m = pH \quad [\text{N}]. \quad (11)$$

Fig. 1b shows the magnetic field as lines of force emanating from a bar magnet. By convention the lines originate at the north pole and end at the south pole.

The magnetic field intensity ( $H$ ) is one of the vector quantities used to describe the magnetic field. The second vector quantity that is used to describe the magnetic field is the magnetic flux density or magnetic induction ( $B$ ). In this case, the vector  $H$  is independent of the character of the medium while the vector  $B$  is dependent of the medium,

$$B = \mu H \quad [\text{Wb/m}^2], \quad (12)$$

where  $\mu$  is the permeability of the medium [H/m].

The magnetic field intensity, the magnetic flux density and the magnetic force are vectors that possess the same direction. In cases where the material is anisotropic, the permeability is considered as a vector as well, and the magnetic field intensity and the magnetic flux density do not share the same direction.

In 1865, James Clerk Maxwell presented a set of differential equations that described the relationship that exists between the electric and magnetic fields,

$$\begin{aligned}\nabla \cdot E &= \frac{\rho}{\epsilon} & \nabla \cdot B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} & \nabla \times B &= \mu_0 J + \frac{\mu_0 \epsilon \partial E}{\partial t}.\end{aligned}\quad (13)$$

Basically, they describe how electric charges and electric currents are the sources of electric and magnetic fields. These equations are a compendium of Gauss's, Faraday's and Ampere's laws and are the core of electromagnetism.

### 3. *Electric and Magnetic Moments*

When positive and negative charges in a material are in the presence of an electric field, they tend to be displaced a finite distance ( $\mathbf{d}$ ) relative to each other creating an electric dipole moment ( $\mathbf{p}_e$ ) as depicted in Fig. 2. This moment can be expressed as,

$$\mathbf{p}_e = q\mathbf{d}.\quad (14)$$

The macroscopic dipole moment density ( $\mathbf{P}_e$ ) is given by

$$\mathbf{P}_e = n\mathbf{p}_e,\quad (15)$$

where  $n$  is the number of microscopic electric dipole moments/volume.

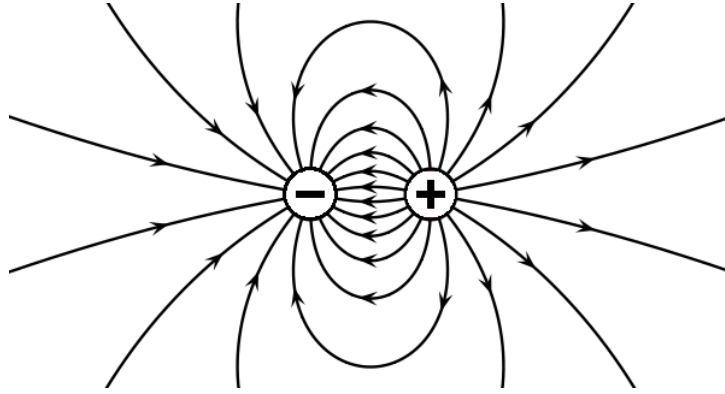


Fig. 2. Electric dipole: two opposite charges separated by a finite distance.

The dipole moment density ( $\mathbf{P}_e$ ) is related to the electric field through the electric susceptibility ( $\chi_e$ ), which is a measure of how easily a material is polarized in the presence of an electric field [19]. This relation can be expressed as,

$$\mathbf{P}_e = \chi_e \mathbf{E}. \quad (16)$$

The relationship between the electric field, the dipole moment density and the electric flux density, in this case also known as electric displacement field, can be expressed in terms of the permittivity of the medium ( $\epsilon$ ) as,

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}_e = (\epsilon_0 + \chi_e) \mathbf{E} = \epsilon \mathbf{E}. \quad (17)$$

On the other hand, in the magnetic regime, the magnetic moment ( $\mathbf{m}$ ) is considered the elementary quantity in solid-state magnetism and can be defined as a quantity that determines the force that a magnet can exert on either a bar magnet or a current loop when it is in an applied field. It can be expressed in terms of a magnetic pole as,

$$\mathbf{m} = p \mathbf{d}, \quad (18)$$

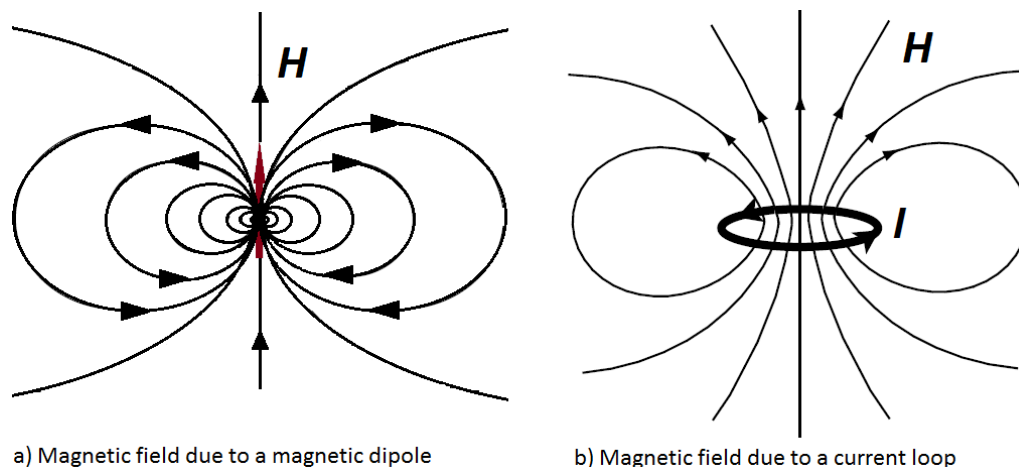


Fig. 3. Sources of magnetic moment.

where  $m$  is the magnetic moment,  $p$  is the magnetic pole strength and  $d$  is the distance between the poles.

In 1850 Hans Cristian Oersted discovered that an electric current produced a magnetic field in its neighborhood. The nature of this phenomenon was fully explored by André Ampère and Wilhelm Eduard Weber. They showed that a current loop was magnetically equivalent to a dipole, fixed at the center of the circuit and perpendicular to its plane (see Fig. 3). This hypothesis agrees with modern physical theory that believes that an electron spin, a nuclear spin and atomic orbits with their circling electrons may be considered as tiny loops of currents. Therefore, the ultimate source of any magnetic field most truly be regarded, not as a small bar magnet, but as a small loop of current [1]. This implies that the magnetic moment can also be described in terms of a current ( $I$ ) flowing in a tiny loop of area  $A$  as,

$$\mathbf{m} = AI. \quad (19)$$

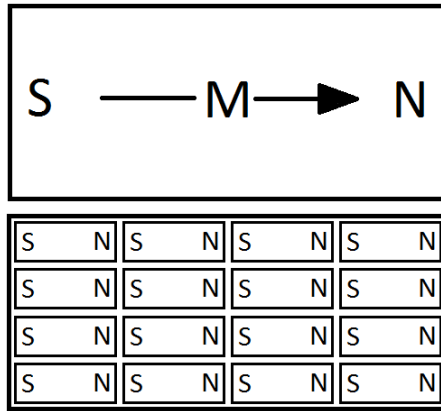


Fig. 4. Schematic of magnetized sample as a composite of microscopic dipoles.

Fig. 4 depicts the macroscopic magnetic dipole density or magnetization ( $\mathbf{M}$ ) of a magnetic material. It is given by,

$$\mathbf{M} = n\mathbf{m}, \quad (20)$$

where  $n$  is the number of microscopic magnetic dipole moments/volume.

In the presence of a magnetic field with a magnetic induction  $\mathbf{B}$ , a torque acts on the magnetic dipole of moment  $\mathbf{m}$ ,

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}. \quad (21)$$

This means, that a magnetic induction  $\mathbf{B}$  tries to align the dipole so that the moment  $\mathbf{m}$  lies parallel to the induction. The energy of the magnetic dipole is,

$$w = -\mathbf{m} \cdot \mathbf{B}. \quad (22)$$

In free space, this energy can be formulated as  $w = -\mu_0 \mathbf{m} \mathbf{H}$ , where  $\mu_0$  is the permeability of the vacuum.

The magnetization ( $\mathbf{M}$ ) is related to the magnetic field through the magnetic susceptibility ( $\chi_m$ ), which is a measure of how easily a material is polarized in the presence of a magnetic field. This relation can be expressed as,

$$\mathbf{M} = \chi_m \mathbf{H}. \quad (23)$$

The relationship between the magnetic field intensity, the magnetic flux density and the dipole moment density can be expressed in terms of the permeability of the medium ( $\mu$ ) by,

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} = (\mu_0 + \chi_m) \mathbf{H} = \mu \mathbf{H}. \quad (24)$$

### B. Magnetization and Magnetic Materials

There are four types of magnetic materials: paramagnetic, antiferromagnetic, ferromagnetic, and ferrimagnetic. The difference between them is the macroscopic magnetic effect experienced due to the orientation of the atomic magnetic moments in the presence of a magnetic field (see Fig 5).

#### 1. Paramagnetic Materials

According to [20], a magnetic material can be imagined as an assembly of

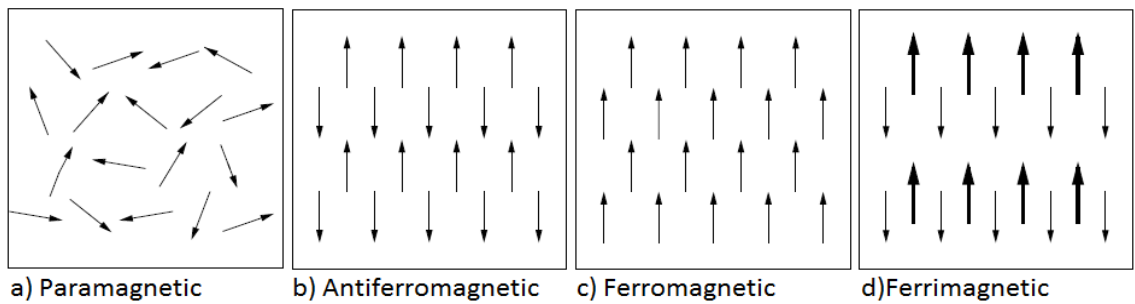


Fig. 5. Ordering of the magnetic dipoles in magnetic materials.



permanent magnets moments ( $m_i$ ) of quantum-mechanic origin. The simplest situation is the ideal paramagnet (see Fig. 5a). This is a system where individual  $m$  vectors do not interact with each other, but are independently shaken by thermal agitation.

As a consequence, they take random orientations in space, which gives zero net magnetization in any macroscopic piece of material.

## 2. *Antiferromagnetic Materials*

In some magnetically ordered materials, the atomic moments couple in antiparallel arrangements with zero net moment rather than parallel as in ferromagnetic materials. Such materials are called antiferromagnetic materials (see Fig. 5b).

In antiferromagnetic materials there is a temperature at which above that value the material loses all its properties and behave as a paramagnetic material. This temperature is known as the Néel temperature in honor to the French physicist Louis Néel who was the first to identify this type of magnetic ordering.

Antiferromagnetic materials have limited technical applications because their net magnetization is zero; they produce no external field and the direction of the atomic moments is not easily affected by an external field.

## 3. *Ferrimagnetic Materials*

When two antiferromagnetically coupled sublattices in a material have unequal moments, the net moment is not zero. Such materials are called ferrimagnetic materials (see Fig. 5d). Ferrimagnetic materials are very important in technical applications because of their high-frequency properties. Some examples of ferrimagnetic materials are the ferrites and the magnetic garnets [21].

#### 4. Ferromagnetic Materials

The most common type of magnetic material used in industry nowadays are ferromagnetic materials. In ferromagnetic materials (Fig. 5c) a nonzero magnetization can be induced by an external field ( $H_a$ ).

According to [20], the potential energy of a single moment in the field is equal to

$$U_p = -\mu_0 m_i H_a. \quad (25)$$

This energy term favors the alignment of the moments along the direction of the field direction. Conversely to the tendency of thermal agitation that just destroys any order possibly present.

An appreciable net magnetization component along the field is obtained when the field energy is at least comparable in order of magnitude to the thermal energy. This can be expressed as,

$$\mu_0 \mu_B H_a \sim k_B T, \quad (26)$$

where  $\mu_0 \mu_B H_a$  represents the field energy and  $k_B T$  represents the thermal energy. The constant  $\mu_B$  is the natural atomic unit of magnetic moments and is called the Bohr magneton.

At room temperature, the net magnetization would take place for fields  $H_a \approx 3 \times 10^8$  A/m which is about  $10^6$  times bigger than the normally applied ( $10$ - $10^2$  A/m). This raises the question on how we overcome thermal agitation and produce any appreciable magnetization.

In 1907, a French physicist Pierre-Ernest Weiss suggested that ferromagnetic materials could exhibit a large spontaneous magnetization even at low fields because the

elementary magnetic moments were by no means independent, as assumed in the previous picture of paramagnets, but were strongly coupled by an internal field proportional to the magnetization itself,  $H_w \propto M$ , which he termed molecular field. The molecular field produces a positive feedback mechanism, because the presence of a net magnetization produces a nonzero molecular field, which in turn acts on all moments trying to further align them. The result is that, below a critical temperature, the Curie temperature ( $T_c$ ), the magnetic moments spontaneously attain long-range order and the material acquires a substantial spontaneous magnetization ( $M_s$ ). The Curie point of a ferromagnetic material is the temperature above which it loses its characteristic ferromagnetic ability. Above the Curie point, the material is purely paramagnetic and there are no magnetized domains of aligned moments. The knowledge of the experimental value of  $T_c$  permits one to estimate the order of magnitude of  $H_w$ , because at the Curie point one expects  $\mu_0 \mu_B H_w \sim k_B T_c$ . In the case of iron, where  $T_c \sim 10^3$  K, one finds  $H_w \sim 10^9$  A/m. Well below  $T_c$ , the moments are nearly perfectly aligned. In iron at room temperature, this gives a spontaneous magnetization of the order of 2 T.

The molecular field hypothesis explains the main aspects of the temperature dependence of the spontaneous magnetization but raises other conceptual difficulties. If there exists an internal field as enormous as the estimate of  $10^9$  A/m seems to indicate, then one would expect to find ferromagnetic materials spontaneously magnetized to saturation under all circumstances. Thus how can a hysteresis loop exist at all? It would seem impossible that fields of the order of 10 A/m can reduce to zero the magnetization of a ferromagnet if they are confronted with internal fields of the order of  $10^9$  A/m.

Weiss proposed to resolve this difficulty by postulating that a ferromagnetic material is subdivided into regions called magnetic domains. In each domain, the degree of magnetic moment alignment is dictated by the molecular field, but the orientation of the spontaneous magnetization can vary from domain to domain. As a consequence, when the magnetization is averaged over volumes large enough to contain many domains, one obtains values mainly determined by the relative orientation and volume of domains. The result can be very different from the spontaneous magnetization and even close to zero. The variety of observed hysteresis loop shapes is the direct consequence of the variety of possible magnetic domain structures.

#### *4.1. B-H Loops and Magnetic Domains*

It has been already explained that a change in magnetic flux density ( $B$ ) depends on the values of the magnetic field intensity ( $H$ ). However, it also depends on the history of the magnetic material that is, the values of  $H$  to which it has been subjected in the past. It is therefore necessary to specify the magnetic history with some exactness.

In a large number of applications such as motors and transformers, the magnetic material is exposed to a magnetizing force which alternates between equal positive and negative values. It is found that after a few of such alternations, the curve connecting  $B$  and  $H$  settles down to a cycle which is exactly reproduced in each successive alternation. The material is then said to be in a cyclic condition and the  $B$ - $H$  curve may take the form shown in Fig. 6 known as hysteresis loop or  $B$ - $H$  loop. The name hysteresis is given due to the lag between cause and effect [22].

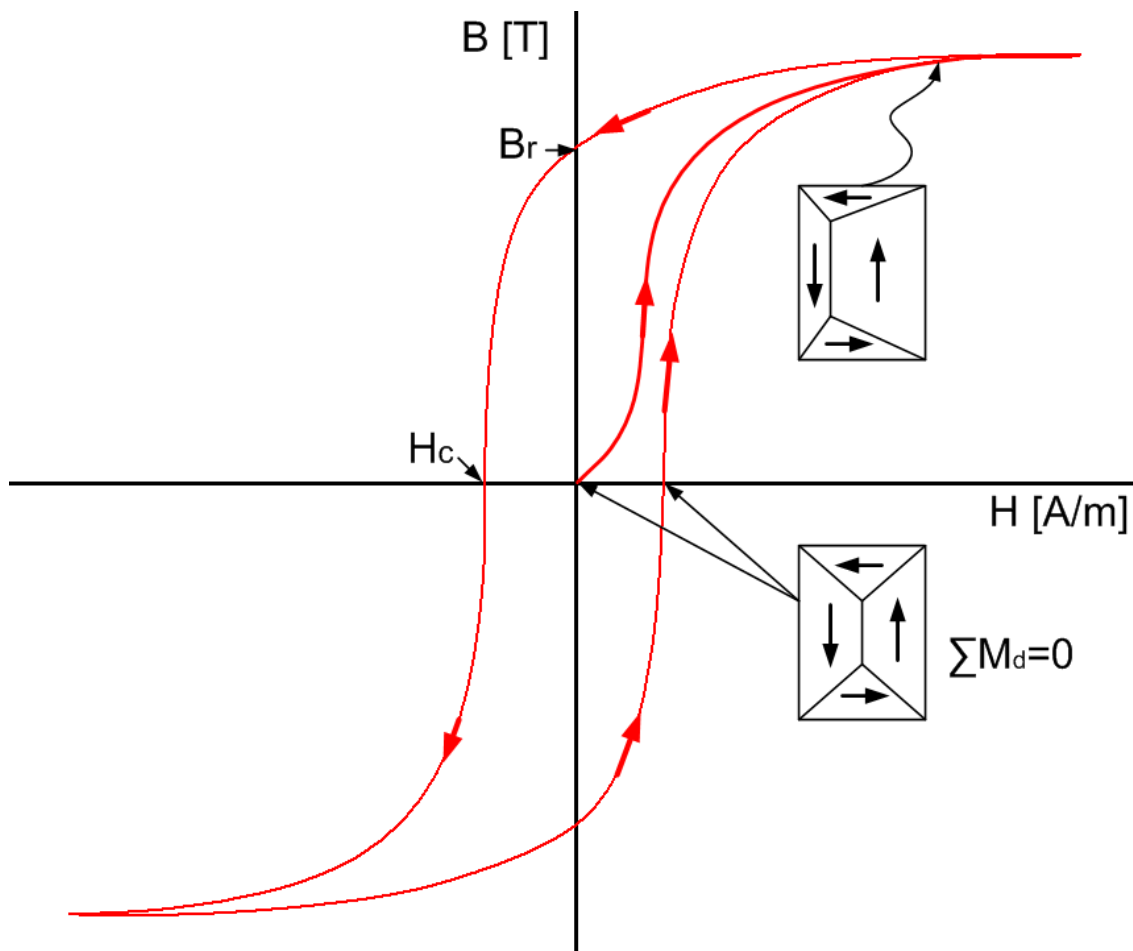


Fig. 6. Hysteresis loop of a magnetic material. The approximate domain structures are indicated at right for demagnetized state and for approach saturation.

The features of the loops may be explained with the help of the domain theory. Consider a magnetic material in the demagnetized state, this is  $B=0$  and  $H=0$ . According to [1], if we apply a field that rises from zero to  $H_{max}$ , the application of the weak field produces motion of domain walls such as to expand the volume of those domains having the largest component of  $M$  along  $H$ . The initial induction  $B$  produced in response to a small field  $H$  defines the initial permeability. At higher fields  $H$ ,  $B$  increases sharply and the permeability increases to its maximum. When most domain wall motion has been

completed, there often remains domains with nonzero components of magnetization at right angles to the applied field direction. The magnetization in these domains must be rotated into the field direction to minimize potential energy  $-\mathbf{M}\cdot\mathbf{B}$ . This process generally costs more energy than wall motion because it involves rotating the magnetization away from an easy direction. When the applied field is of sufficient magnitude that these two processes, wall motion and magnetization rotation are complete, the sample is in a state of magnetic saturation  $B_s = \mu_0(H+M_s)$ .

On decreasing the magnitude of the applied field, the magnetization rotates back towards its easy direction, generally without hysteresis (i.e., rotation is a largely reversible, lossless process). As the applied field decreases further, domain walls begin moving back across the sample. Because energy is lost when a domain wall jumps abruptly from one local energy minimum to the next (Barkhausen jumps), the hysteresis loop opens up. That is, it shows hysteresis.

When the hysteresis loops of different materials are compared, some of the properties are denoted by special terms. The usually accepted definitions of these terms are as follows:

- *Remanence* is the flux density or magnetic induction, which remains in a magnetic material after the removal of an applied magnetizing force.
- *Residual flux density* or *residual induction* ( $B_r$ ), in a magnetic material is the value of the flux density for the condition of zero magnetizing force, when the material is being symmetrically cyclically magnetized. It is distinguished from the *Remanence* by the symmetrically cyclic requirement.

- *Retentivity* is the flux density that remains in the material after a magnetizing force sufficient to cause saturation flux density or saturation induction has been removed.
- *Coercive force ( $H_c$ )* of a magnetic material is the magnitude of the magnetizing force at which the flux density is zero when the material is being symmetrically cyclically magnetized.
- *Coercivity* is the coercive force required to reduce the flux density in the material to zero from a condition corresponding to saturation flux density or saturation induction.

If hysteresis loops are measured for different amplitudes of  $H$ , the locus of the tips of the loops is known as the magnetization curve, saturation curve or  $B$ - $H$  normalized curve (see Fig. 7). The relation between  $B$  and  $H$  is the permeability ( $\mu$ ) and can be calculated using the magnetization curve.

#### 4.2. Magnetic Losses

When an electromagnetic device is excited with a constant electric current, the magnetic flux produces insignificant energy losses unless it is energized and de-energized very frequently. On the other hand, when devices like motors, transformers, reactors, etc., are excited with an alternating electric current, the alternating fluxes in the magnetic material generate heat and therefore, energy losses.

Mainly, there are three types of losses in magnetic materials namely,

- *Hysteresis losses* which are heat losses result of the tendency of the material to retain magnetism or to oppose a change in magnetism.

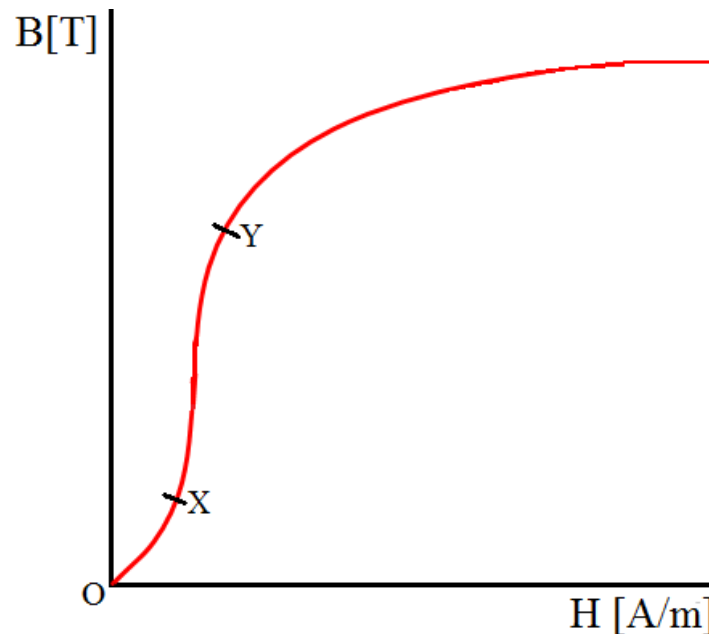


Fig. 7. Magnetization curve of a ferromagnetic material. The first region (OX) consists in the growth in the size of the domains which are favorable to  $H$ . In the second stage (XY) the process continues with the rotation of some domain axes. Around the point Y, called the knee of the curve, the axes of magnetization of the domain all lie as near the direction of the field as possible. After the knee point, for large values of  $H$ , the domain axes are forced into the direction of the field in spite of the crystal forces. The material is said to be saturated.

- *Eddy-current losses* which are the product of the electric currents flowing in the magnetic material. These electric currents are caused by the electromotive forces set up by the varying fluxes.
- *Anomalous losses* which are whatever remains after hysteresis losses and eddy-current losses have been taken into account.

#### 4.2.1. Hysteresis Losses

The presence of hysteresis losses is intimately associated with the phenomenon whereby energy is absorbed by a region which is permeated by a magnetic field. If the



region is other than a vacuum, only a portion of the energy taken from the electric circuit is stored and is wholly recoverable from the region when the magnetizing force is removed. The rest of the energy is converted into heat as a result of work done on the material when it responds to the magnetization [23].

The magnitude of the energy absorbed per unit volume is given as,

$$dw = HdB = \mu_0 HdM \quad [\text{J/m}^3]. \quad (27)$$

The addition of this differential energy represents the area inside a  $B$ - $H$  loop ( $A_{loop}$ ).

The rate at which the energy is dissipated because of hysteresis, also known as hysteresis power loss, can be calculated using (28) if a volume ( $V$ ) of magnetic material has a uniform flux distribution and is subject to a cyclic change at a frequency ( $f$ ) cycles per second.

$$P_h = V \cdot f \cdot A_{loop} \quad [\text{W}]. \quad (28)$$

According to [23], Steinmetz found from a large number of measurements that the area of the normal hysteresis loop of specimens of various irons and steels commonly used in the construction of electromagnetic apparatus of his day was approximately proportional to 1.6<sup>th</sup> power of the maximum flux density throughout the range of flux densities from about 0.1 to 1.2 T. Advances on ferromagnetic manufacturing have widely transformed and improved the material's properties since Steinmetz originally performed his measurements. Therefore, the exponent 1.6 fails nowadays to give the area of the hysteresis loops with a sufficient degree of accuracy to be useful. However, the empirical expression for the energy loss per unit volume per cycle is more properly given as

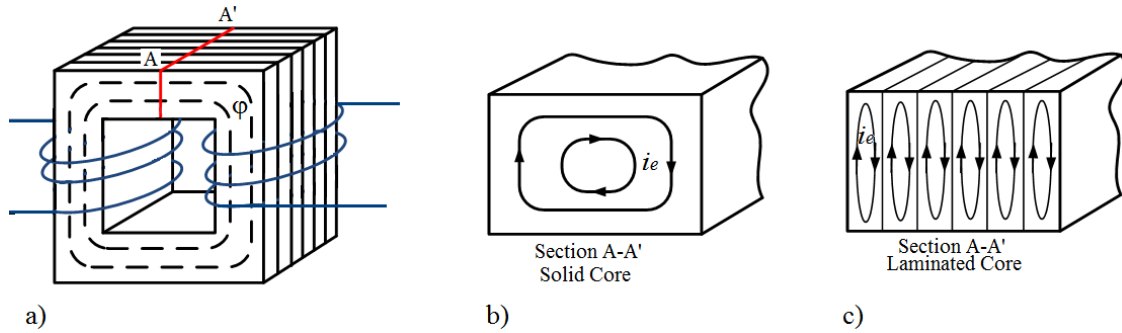


Fig. 8. Eddy-currents in a transformer core, a) representation of a laminated transformer, b) eddy-currents in a solid core, c) eddy-currents in a laminated core.

$$w_h = \eta B_{max}^n \quad [\text{J/m}^3], \quad (29)$$

where  $\eta$  and  $n$  have values that depend on the material. Equation (29) should be used with caution since the value for  $n$ , which may range between 1.5 and 2.5 for present day materials, may not be a constant value for a given material.

The total hysteresis loss in a volume  $V$  in which the flux density is uniform and varying cyclically at a frequency of  $f$  cycles per second can then be expressed empirically as,

$$P_h = \eta V f B_{max}^n \quad [\text{W}]. \quad (30)$$

#### 4.2.2. Eddy-current Losses

According to Faraday, an induced electromotive force (*emf*) results within a magnetic medium due the presence of a time-varying magnetic field. When the medium is conductive, an electric current appears (see Fig. 8). These currents are called eddy-currents ( $i_e$ ) and their presence results in an energy loss in the material proportional to  $i_e^2 R_e$ , called eddy-current loss.

Since the flux density in ferromagnetic materials is usually relatively large and the resistivity is relatively small, the electromotive force, the eddy-currents and the eddy-current loss may become appreciable if means to minimize them are not taken. This loss can be minimized by using thin laminations or highly resistive material.

Eddy-current loss is of considerably importance in determining the efficiency, the temperature rise, and hence the rating of electromagnetic apparatus in which the flux density varies.

In a magnetic circuit containing a volume  $V$  of laminated core material exposed to a sinusoidal flux, the average eddy-current power loss is,

$$P_e = \frac{\pi^2 f^2 \tau^2 B_{max}^2}{6\rho} V \quad [\text{W}], \quad (31)$$

where,  $V$  is expressed in  $\text{m}^3$ ,  $f$  in Hz, the thickness of the lamination ( $\tau$ ) in meters,  $B_{max}$  in Teslas, and  $\rho$  in ohms/ $\text{m}^3$ .

Another phenomenon that is linked to the eddy-currents is skin effect. The effect of such currents is to screen or shield the material from the flux, and to bring about a smaller flux density near the center of the slab than at the surface. This is, similar to the skin effect observed in electric conductors [23].

Since electric and magnetic skin effects are similar in nature, they are subject to the same type of analysis and solution. In electric circuits, the effect is mitigated using a number of insulated wire strands woven together in a carefully designed pattern instead of using a solid conductor. In magnetic circuits, we apply the same strategy. The skin effect is mitigated by laminating the core, thus reducing the eddy-currents.

#### *4.2.3. Anomalous Losses*

According to [2], anomalous losses are the remaining losses after hysteresis and eddy-current losses have been taken into account. They turn out to be comparable in magnitude to eddy-current losses and they arise from extra eddy-current losses due to domain wall motion, non-uniform magnetization and sample inhomogeneity. Essentially, the anomalous losses reflect the broadening of the hysteresis loop with increasing frequency, so the separation of static hysteresis loss and anomalous loss is artificial.

CHAPTER III  
POWER-INVARIANT MAGNETIC MODEL

In all energy systems, the parameters necessary to calculate power are the same in functionality, an effort or force needed to create a movement in an object and a flow or rate at which the object moves [24]. Therefore, we can generalize the power equation as a function of these two parameters: effort and flow,

$$p = \textit{effort} \cdot \textit{flow}. \quad (32)$$

Analyzing various power transfer media this is true for at least three regimes: electrical, mechanical and hydraulic but not for magnetic (see Table IV). This implies that the conventional magnetic system model (the reluctance model) requires modifications in order to be consistent with other energy system models.

Even further, if we do a comprehensive comparison among the systems, each system's model includes an effort quantity, a flow quantity and three passive elements used to establish the amount of energy that is stored or dissipated as heat. After evaluating each one of them, it was clear that the conventional magnetic model did not follow the same pattern: the reluctance, seen as an analogy of the electric resistance,

TABLE IV  
CALCULATION OF POWER IN DIFFERENT REGIMES

Electric	Mechanic	Hydraulic	Magnetic
Voltage x Current	Force x Velocity	Pressure x Flow	MMF x Flux
V x A	N x m/s	N/m <sup>2</sup> x m <sup>3</sup> /s	A x Wb
Watts	Watts	Watts	Joules

TABLE V  
FUNDAMENTAL QUANTITIES OF DIFFERENT SYSTEM'S MODELS

Quantities	Electric	Mechanic	Hydraulic	Magnetic
<b>Conserved Quantity</b>	Charge [C]	Distance [m]	Volume [m <sup>3</sup> ]	-----
<b>Flow</b>	Current [A]	Speed [m/s]	Volume flow rate [m <sup>3</sup> /s]	Flux [Wb]
<b>Effort</b>	Voltage [V]	Force [N]	Pressure [Pa]	MMF [t·A]
<b>Dissipative Element</b>	Resistance [ $\Omega$ ]	Friction	Friction	Reluctance [H <sup>-1</sup> ]
<b>Energy Storage Element</b>	Capacitance [F]	Compliance	Capacitance [m <sup>3</sup> /Pa]	-----
<b>Energy Storage Element</b>	Inductance [H]	Mass [kg]	Mass [kg]	-----

does not dissipate energy, but instead stores energy and the two other elements are not defined (see Table V). This difference has initiated a reevaluation of the conventional magnetic model.

#### A. The Power Equation

As we established earlier, the power equation in the reluctance model is not satisfied. This implies that either the magnetic effort (*mmf*) or the magnetic flow (*flux*) is not properly defined. According to [8], the gyrator is the missing link that can help us find the proper definition of these quantities.

A symbolic representation of the conventional gyrator is presented in Fig. 9. It is

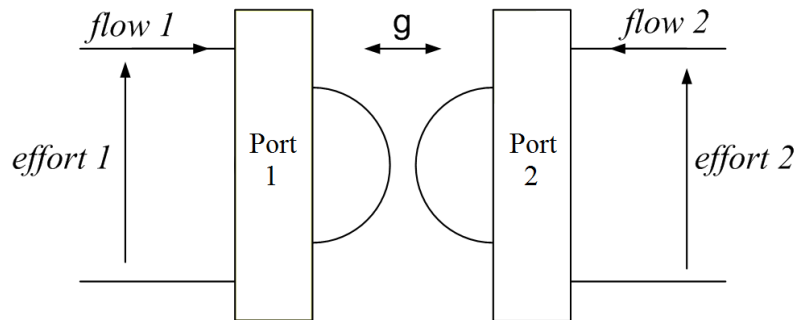


Fig. 9. Symbolic representation of a two-port gyrator.

a two-port network passive element that converts the effort or flow quantity into its dual with respect to its gyration constant ( $g$ ), this is

$$effort_1 = -g \cdot flow_2, \quad (33)$$

$$effort_2 = g \cdot flow_1. \quad (34)$$

This conversion is lossless. Therefore, the power at the terminals of both networks is invariant,

$$P = effort_1 \cdot flow_1 + effort_2 \cdot flow_2 = 0. \quad (35)$$

This characteristic is very important in cases where only one of the two networks is known as it provides valuable information of the unknown network.

The gyrator was developed on the basis that there should exist a complement to the ideal transformer which is a two-port network passive element as well, but in which the effort and flow quantities must follow the reciprocity property. This is,

$$effort_2 = k \cdot effort_1 \quad (36)$$

$$flow_1 = k \cdot flow_2 \quad (37)$$

where  $k$  is a transformation coefficient [25].

Fig. 10 presents an iron core coil. It can be considered a gyrator as it is a two-port network system where the primary network is at the electric regime and the secondary network is at the magnetic regime. Both networks are interconnected through the winding and its number of turns ( $N$ ) serve as the gyrator constant. We will use this case study along this dissertation in order to characterize the magnetic network which at this point we consider unknown.

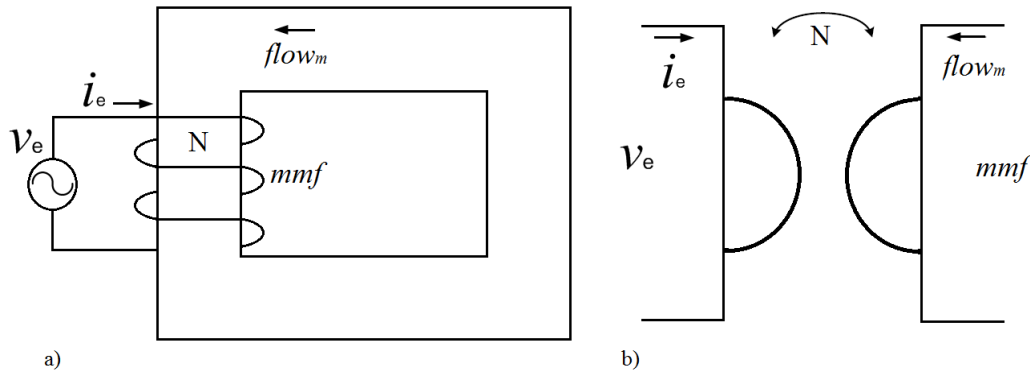


Fig. 10. Representation of an iron core coil a) as a physical system, b) as a two port network system.

From electromagnetic theory we recognize that there are two fundamental laws that relate the electric and magnetic regime, these are, Faraday's and Ampère's law.

- Faraday's law establishes that an electromotive force (*emf*) or voltage ( $v_e$ ) is induced in any close circuit when a magnetic flux ( $\varphi$ ) varies with time,

$$emf = -N \frac{d\varphi}{dt}. \quad (38)$$

- On the other hand, Ampère's law establishes that an electric current ( $i_e$ ) produces a magnetomotive force (*mmf*) in a closed magnetic circuit. It can be expressed as,

$$mmf = N \cdot i_e. \quad (39)$$

Comparing these equations to the gyrator's equations, it is evident that they are similar in form and functionality. Equation (38) and (33) suggests that the electromotive force (*emf*) is the effort quantity of the electric network while the rate of flux ( $d\varphi/dt$ ) is the flow quantity of the magnetic one. Similarly, (39) and (34) presents the magnetomotive force (*mmf*) and the electric current ( $i_e$ ) as the effort and flow quantities of the magnetic and electric regime, respectively.



In order to verify that these quantities satisfy the power equation, we perform a dimensional analysis. In (40) we can see that using the electromotive force ( $mmf$ ) as magnetic effort and the rate of flux ( $d\phi/dt$ ) as magnetic flow in the power equation, it gives watts units;

$$\begin{aligned}
 P &= mmf \cdot \frac{d\phi}{dt} \\
 P &= t \cdot A \cdot Wb/s \\
 P &= t \cdot A \cdot V \\
 P &= t \cdot W, \tag{40}
 \end{aligned}$$

where  $t$  is a dimensionless unit that represents the number of turns of the winding.

Using the rate of magnetic flux ( $d\phi/dt$ ) as flow is the fundamental modification to the reluctance model. It is the key to define the rest of the parameters (magnetic resistance, inductance and capacitance) needed to standardize the magnetic model.

### *B. Magnetic Fundamental Quantities*

After studying different regimes we realized that there are six fundamental scalar quantities needed to fully model any energy transfer process. They can be generalized as, conserved quantity, effort, flow, dissipative element and two energy storage elements, one that stores energy due to effort and the other that stores energy due to flow.

In this section we define these quantities for the magnetic regime. We use an electric analogy as basis to derive the magnetic model, first, because there is an intrinsic relation between both systems and second, we are familiar with electric circuit theory.

### *1. Conserved Quantity*

In general terms we can define the conserved quantity as the fundamental physical entity of a system. Depending on the regime of work it has different designations, for example, in mechanic systems it is the distance ( $m$ ), in fluid dynamics is the mass ( $kg$ ), in hydraulics it is the volume ( $m^3$ ) and in electric systems is the electric charge ( $C$ ). In order to define the magnetic conserved quantity we will present a description of the electric conserved quantity as it will be used as analogous to the magnetic one.

#### *1.1. Electric Conserved Quantity*

In electric systems, the free charge ( $q_e$ ) is usually considered the conserved quantity. However, this is not the general case, depending on the media at which a process takes place the interpretation of the electric conserved quantity may vary. For instance, in free space or in a conductive media, like a metallic conductor, the free charge is considered the conserved quantity. Its interaction with an electromagnetic field or with other charged particles is the main cause of the electric phenomena.

In dielectric materials, on the other hand, due to the tight atom-electron bonds there are no free charges. The electric phenomenon is the result of the interaction of electric fields and the polarized charges of the dielectric material. These charges, which are basically electric dipoles, are known as bound charges to distinguish them from the free charges.

As we explain in Chapter II, the dipoles are two opposite charges. Therefore, the average electric bound charge in a closed surface ( $S$ ) at rest is zero. However, when a

dielectric is excited with an electric field ( $E$ ), these charges get polarized and its effect can be appreciated in the displacement field. The scalar quantity that represents this effect is the electric flux ( $\psi$ ) and it is expressed as,

$$\psi = \oint D dS = \oint (P + \epsilon_0 E) dS. \quad (41)$$

Karl Friedrich Gauss established that the total flux out of any closed surface of area ( $S$ ) is equal to the total charge enclosed by the surface [26]. It is expressed as,

$$\psi = \oint D dS = \sum q_e \quad [\text{C}]. \quad (42)$$

This means that each coulomb of free charge produces a unit of flux ( $\psi$ ). Therefore, as these quantities are equivalent we will consider the electric flux as the conserved quantity when analyzing dielectric materials.

### *1.2. Magnetic Conserved Quantity*

In magnetic materials there are no magnetic free charges since such charges have not been discovered. As we explain earlier, the minimal magnetic quantity is the magnetic dipole. We can infer that there is an analogy between the electric dipoles or bound charges in a dielectric material and the magnetic dipoles of the magnetic material. In order to be consistent with this analogy, we are going to refer to the magnetic dipoles as magnetic bound charges.

Similar to the electric case, the magnetic dipoles have opposite polarities. Therefore, the average magnetic bound charge in a closed surface ( $S$ ) at rest is zero. Nevertheless, when a magnetic material is excited with a magnetic field ( $H$ ), these charges get magnetized and its effect can be appreciated in the magnetic flux density.

The scalar quantity that represents this effect is the magnetic flux ( $\varphi$ ) and it is expressed as,

$$\varphi = \int_S B dS = \int_S \mu_0(M + H) dS \quad [\text{Wb}]. \quad (43)$$

Similar to the electric flux considered as an electric conserved quantity, we will use the magnetic flux ( $\varphi$ ) as the magnetic conserved quantity. In order to be consistent with the electric analogy, we will refer indistinctively to the magnetic conserved quantity in terms of the flux ( $\varphi$ ) or as a bound charge ( $q_m$ ).

It is important to point out that the previous interpretation of magnetic flux as conserved quantity was obtained following an analogy between electric and magnetic systems. Furthermore, it is consistent with the conserved quantity expected from the integration of the magnetic flow obtained using gyrator theory. This is,

$$\varphi = \int i_m dt \quad [\text{Wb}]. \quad (44)$$

As the magnetic flow is the rate of magnetic flux ( $i_m = \frac{d\varphi}{dt}$ ), then the conserved quantity is again the magnetic flux ( $\varphi$ ).

## 2. Flow

The flow can be generalized as the movement of the conserved quantity through certain area per unit of time. Depending on the regime of work it has different designations, for example, in mechanics systems it is known as speed ( $m/s$ ), in fluid dynamics is known as mass flow ( $kg/s$ ), in hydraulics it is called volumetric flow rate ( $m^3/s$ ) and in electric systems it is called electric current ( $C/s$ ). In order to define the magnetic current we will present a description of the electric current as it will be used as analogous to the magnetic one.

### 2.1. Electric Current

Electric currents are divided in two classes, conduction currents and displacement currents. Conduction currents result from the movement of charged particles in a conductive medium. The motion of these particles generates heat and therefore, it is usually associated with energy losses.

Mathematically it is given by,

$$i_{e,cond} = \frac{dq_e}{dt} \quad [\text{A}]. \quad (45)$$

Displacement currents were introduced by James Clerk Maxwell in 1861 in order to satisfy Ampère's circuital law which establishes that currents always flow in a closed path. It answers the current through capacitors as there are no free charges moving inside the dielectric medium. Mathematically it is given by

$$i_{e,disp} = \frac{d\psi}{dt} \quad [\text{A}], \quad (46)$$

where  $\psi$  is used to denominate the electric flux [C].

Displacement currents are not based on the movement of free charged particles but instead in the rate of change of electric fields. However, they are recognized as a moving-charge current related to the bound or polarized charge of a dielectric material.

Displacement currents play little role in metallic conduction. In metals they are dwarf into insignificance by the magnitude of the conductive current; but in insulators where the conduction currents are weak, the displacement current may well play a role even at low frequencies and most certainly when the frequencies are elevated [27].

Another way to express the electric current, both the conduction current and the displacement currents is in terms of its current density ( $\mathbf{J}_e$ ),

$$\mathbf{J}_e = \frac{di_e}{dS} \quad [\text{A/m}^2]. \quad (47)$$

$\mathbf{J}_e$  is a vector quantity that contains microscopic information regarding the current flow.

In case of the conductive current it is given by,

$$\mathbf{J}_{e,cond} = \sigma_e \mathbf{E} \quad [\text{A/m}^2], \quad (48)$$

where  $\sigma_e$  represents the electric conductivity which is the inverse of the electric resistivity  $\rho_e$ . On the other hand, in case of the displacement current it is expressed as,

$$\mathbf{J}_{e,disp} = \frac{d\mathbf{D}}{dt} \quad [\text{A/m}^2], \quad (49)$$

where  $\sigma_e$  is the electric conductivity of the material [ $\Omega^{-1}/\text{m}$ ],  $\mathbf{E}$  is the electric field [ $\text{V/m}$ ] and  $\mathbf{D}$  is the electric flux density [ $\text{C/m}^2$ ].

## 2.2. Magnetic Current

In magnetic circuits, due to the similarity between the dielectric and the magnetic material, this is both need to be polarized, we think that the magnetic current should be defined as a magnetic displacement current. In fact, the expression of flow obtained from the iron core coil as a gyrator suggests that

$$i_{m,disp} = \frac{d\varphi}{dt} \quad [\text{Wb/s}], \quad (50)$$

which is similar in form, functionality and origin to (46) as both are based on flux. In this case  $\varphi$  is used to denominate the magnetic flux.

Similar to electric displacement currents, the magnetic displacement currents are not based on the movement of magnetic free charged particles as the existence of such charges have not been discovered. Instead, they are based in the rate of change of magnetic fields. However, the displacement currents can be recognized as a moving-

charge current related to the movement of the magnetic dipoles during the magnetization of the magnetic material. It is given by (50) or in terms of its current density as,

$$J_{m,disp} = \frac{dB}{dt} \quad [\text{T/s}], \quad (51)$$

Since  $B = \mu_0(M + H)$ , the magnetic displacement current can also be expressed in terms of the magnetization as,

$$J_{m,disp} = \mu_0 \frac{d(M+H)}{dt} \quad [\text{T/s}] \quad (52)$$

### 3. Effort

In general terms, the effort can be defined as the force needed to generate a flow or as the force needed to produce work. Depending on the regime of study it has different designations, for example, in mechanics systems it is known as force ( $N$ ), in fluid dynamics and in hydraulics it is called pressure ( $Pa$ ) and in electric systems it is called electric voltage ( $V$ ). In order to define the magnetic effort we will present a description of the electric voltage as it will be used as analogous to the magnetic one.

#### 3.1. Electric Voltage

In electric systems the effort quantity is most commonly known as voltage ( $V$ ), electric potential ( $U_e$ ) or electromotive force ( $emf$ ). It is the force needed to move a charged particle from one point to another while in the presence of an electric field. In other words, it is the force needed to generate the flow of current. Mathematically,

$$emf = v_{eB} - v_{eA} = - \int_A^B E dl \quad [\text{V}], \quad (53)$$

where  $E$  is the electric field and  $dl$  is an element of the path from point  $A$  to point  $B$ . In case of a close path, the expression of the  $emf$  becomes,

$$emf = v_e = \oint E dl \quad [\text{V}]. \quad (54)$$

### 3.2. Magnetic Voltage

In magnetic systems, the effort quantity has been denominated magnetic potential ( $U_m$ ) or magnetomotive force ( $mmf$ ) analogous to the electromotive force. In this project we will also refer to it as magnetic voltage analogous to electric voltage. According to [17], it may be defined as the force needed to bring a unit magnetic pole from one point to another against the magnetic field. Mathematically,

$$mmf = v_{mB} - v_{mA} = - \int_A^B H dl \quad [\text{t}\cdot\text{A}], \quad (55)$$

where  $H$  is the magnetic field and  $dl$  is an element of the path from point  $A$  to point  $B$ . In case of a close path, the expression of the  $mmf$  becomes,

$$mmf = v_m = \oint H dl \quad [\text{t}\cdot\text{A}] \quad (56)$$

In our model, we maintain the magnetomotive force as magnetic effort because it is consistent with gyrator theory. Nevertheless, in order to complete the analogy, the  $mmf$  must also be defined as the force needed to generate a magnetic current. This statement contradicts the popular knowledge as it is believed that there is no flow of anything in a magnetic circuit corresponding to the flow of charge in an electric circuit. However, we have already established that the magnetic current can be considered product of a moving-charge related to the magnetization of the magnetic dipoles in magnetic materials, similar to the moving-charge current related to the bound or polarized charge of an electric insulator. In both cases, an effort is needed to create the polarization of the bound charges.



#### 4. Resistance

As a standard model element, the resistance can be generalized as the quantity that describes the amount of energy released as heat due to the flow of current in a material. When an effort is applied, the material allows up to certain degree of flow of current, the resistance is a measure of this degree. Its generalized expression is,

$$R = \frac{effort}{flow}. \quad (57)$$

In order to define the magnetic resistance we will present a description of the electric resistance as it will be used as analogous to the magnetic one.

##### 4.1. Electric Resistance

In electric circuits (57) comes from Ohm's law. It relates the electric voltage ( $v_e$ ) and current ( $i_e$ ) flowing through a conductive material,

$$R_e = \frac{v_{Re}}{i_{Re}} \quad [\Omega]. \quad (58)$$

As we explain in the general description, the resistance is a measure of the amount of current the material allows to flow when certain voltage is applied. Therefore, its value fundamentally depends on the intrinsic property of the material (resistivity or conductivity) and the geometry (length and area) of the specimen. This is expressed as

$$R_e = \frac{\oint E dl}{\oint J_e dS} = \frac{\oint E dl}{\oint \sigma_e E dS} = \rho_e \frac{l}{S} \quad [\Omega], \quad (59)$$

where  $l$  [m] and  $S$  [m<sup>2</sup>] represents the length and area of the specimen, respectively and  $\rho_e$  [ $\Omega$  m] and  $\sigma_e$  [ $\Omega^{-1}$ /m] represents the resistivity and conductivity of the material.

The power dissipated by the resistance is usually calculated using Joule's law. This equation uses the root mean square (RMS) values of the current and voltage to calculate the power and can only be applied when the resistance is a constant value.

$$P_{Re} = R_e \cdot I_{e,rms}^2 = \frac{1}{R_e} \cdot V_{e,rms}^2 \quad [\text{W}]. \quad (60)$$

#### 4.2. Magnetic Resistance

Similarly, in magnetic circuits we can define a magnetic equivalent resistance based on the general definition. Therefore, it is the quantity that measures the amount of magnetic current the material allows to flow when certain voltage is applied. It describes the amount of energy lost as heat.

We can relate the magnetic current ( $i_m$ ) and the magnetic voltage ( $v_m$ ) using,

$$R_m = \frac{v_{Rm}}{i_{Rm}} \quad [\Omega^{-1}]. \quad (61)$$

Nevertheless, its value depends on the intrinsic property of the magnetic material (magnetic resistivity or magnetic conductivity) and the geometry (length and area) of the specimen. This is expressed as

$$R_m = \frac{\oint H dl}{\oint J_m dS} = \frac{\oint H dl}{\oint \sigma_m H dS} = \rho_m \frac{l}{S} \quad [\Omega^{-1}], \quad (62)$$

where  $l$  [m] and  $S$  [m<sup>2</sup>] represents the length and area of the specimen, respectively and  $\rho_m$  [ $\Omega^{-1} \cdot \text{m}$ ] and  $\sigma_m$  [ $\Omega/\text{m}$ ] represents the magnetic resistivity and conductivity of the magnetic material.

It is important to point out that there is a fundamental difference between the electric and magnetic resistance and it has its origins in the mechanism that produces the flow of currents. The electric resistance describes the opposition that the conductive

media presents to the flow of free charges. Therefore, it reflects the heat losses due to the kinetic energy released by the movement of these free charges. However, as we established earlier, the magnetic currents result from the polarization or magnetization of the magnetic dipoles in the magnetic material, similar to the displacement electric currents are the result of the polarization of bound charges in the dielectric material. The power dissipated as heat is the result of the resistance of the magnetic material to be magnetized.

In terms of the field densities the magnetic current can also be expressed similar to (48) as (63) knowing that the magnetic conductivity  $\sigma_m$  [ $\Omega/m$ ] and consequently the magnetic resistivity  $\rho_m$  [ $\Omega^{-1}\cdot m$ ] were introduced in this model to maintain consistency.

$$\mathbf{J}_m = \sigma_m \mathbf{H} \quad [\text{T/s}] \quad (63)$$

The power dissipated by the magnetic resistance can also be expressed in terms of an analogy to the Joule's law when the resistance is a constant value. This is,

$$P_{Rm} = V_{m,rms} \cdot I_{m,rms} = R_m \cdot I_{m,rms}^2 = \frac{1}{R_m} \cdot V_{m,rms}^2, \quad (64)$$

which is the average of the instantaneous power [W].

If the resistance is nonlinear, the power dissipated by the resistance can be calculated directly by calculating the average of the instantaneous power,

$$P_{Rm} = \frac{1}{n} \sum_{i=1}^n (v_{Rm} \cdot i_{Rm}) \quad [\text{W}]. \quad (65)$$

## 5. Capacitance

As a standard model element, the capacitance can be generalized as the quantity that measures the amount of energy stored in the regime of work due to an applied effort. Mathematically it is expressed as,

$$C = \frac{q_{rw}}{effort}, \quad (66)$$

where  $q_{rw}$  stands for conserved quantity in the regime of work.

In order to define the magnetic capacitance we will present a description of the electric capacitance as it will be used as analogous to the magnetic one.

### 5.1. Electric Capacitance

In electric systems, the capacitance measures the ability of a body to hold a charge ( $q_e$ ) due to a given potential ( $v_e$ ). It describes the amount of energy stored in the electric field. It is expressed as,

$$C_e = \frac{q_{ce}}{v_{ce}} \quad [\text{F}]. \quad (67)$$

Similar to the resistance, the capacitance is also a representation of the material. Therefore, its value depends on the intrinsic property of the dielectric and the geometry of the specimen. This is expressed as,

$$C_e = \frac{\oint D dS}{\oint E dl} = \frac{\oint \epsilon E dS}{\oint E dl} = \epsilon \frac{S}{l} \quad [\text{F}]. \quad (68)$$

where  $l$  [m] and  $S$  [m<sup>2</sup>] represents the length and area of the specimen, respectively and  $\epsilon$  [F/m] represents the permittivity of the dielectric material, which determines the ability of a material to polarize in response to an electric field.

To calculate the energy stored by a capacitance of constant value we use,

$$W_{ce} = \frac{1}{2} \cdot C_e V_e^2 \quad [\text{J}]. \quad (69)$$

According to [28], in a more general case, the energy stored can be expressed as,

$$W_{ce} = \int v_{ce} dq_{ce} \quad [\text{J}]. \quad (70)$$

## 5.2. Magnetic Capacitance

In magnetic systems, the capacitance measures the ability of a body to store energy due to a given potential ( $v_{Cm}$ ). It describes the amount of energy stored in the magnetic field. It is expressed as,

$$C_m = \frac{q_{Cm}}{v_{Cm}} \quad [\text{H}]. \quad (71)$$

Similar to the resistance, the capacitance is also a representation of the material. Therefore, its value depends on the intrinsic property of the magnetic material (permeability) and the geometry of the specimen. This is expressed as,

$$C_m = \frac{\oint B dS}{\oint H dl} = \frac{\oint \mu H dS}{\oint H dl} = \mu \frac{S}{l} \quad [\text{H}]. \quad (72)$$

where  $l$  [m] and  $S$  [m<sup>2</sup>] represents the length and area of the specimen, respectively and  $\mu$  [H/m] represents the permeability of the material, which determines the ability of a material to magnetize in response to a magnetic field.

At this point, it is evident that the magnetic capacitance plays the same role as the permeance in the reluctance model. Therefore, we validate the statement that says that the permeance and its inverse, the reluctance, are in fact energy storage elements instead of dissipative elements.

To calculate the energy stored by a capacitance of constant value we use,

$$W_{Cm} = \frac{1}{2} \cdot C_m V_m^2 \quad [\text{J}]. \quad (73)$$

In a more general case, the energy stored can be expressed either by,

$$W_{Cm} = \int v_{Cm} dq_{Cm} \quad [\text{J}], \quad (74)$$

which is proportional to the energy calculated using the area of the B vs. H plot, or by using the power equation as,

$$W_{cm} = \int_0^{T/4} (v_{cm} \cdot i_{cm}) dt \quad [\text{J}] \quad (75)$$

### 6. Inductance

As a standard model element, the inductance can be generalized as the quantity that measures the amount of energy stored in a complementary or dual of the regime of work due to a flow. Mathematically is expressed as,

$$L = \frac{q_{drw}}{\text{flow}}, \quad (76)$$

where  $q_{drw}$  stands for conserved quantity of the dual regime of work.

In order to define the magnetic inductance we will present a description of the electric inductance as it will be used as analogous to the magnetic one.

#### 6.1. Electric Inductance

In electric systems, the inductance measures the amount of energy stored in the magnetic field, represented by the magnetic flux, due to the flow of electric current ( $i_e$ ). It is expressed as,

$$L_e = \frac{N\phi}{i_{Le}}. \quad (77)$$

Similar to the resistance, the inductance is also a representation of the material. Therefore, its value depends on the intrinsic property of the magnetic material (permeability) and the geometry of the specimen. This is expressed as,

$$L_e = N^2 \frac{\oint B dS}{\oint H dl} = N^2 \frac{\oint \mu H dS}{\oint H dl} = N^2 \left( \mu \frac{S}{l} \right) \quad [\text{H}]. \quad (78)$$

where  $l$  [m] and  $S$  [m<sup>2</sup>] represents the length and area of the specimen, respectively and  $\mu$  [H/m] represents the permeability of the material.

To calculate the energy stored by an inductance of constant value we use,

$$W_{Le} = \frac{1}{2} \cdot L_e I_e^2 \quad [\text{J}]. \quad (79)$$

In a more general case, the energy stored can be expressed either by,

$$W_{Le} = \int i_{Le} d\varphi \quad [\text{J}], \quad (80)$$

which is proportional to the energy calculated using the area of the B vs. H plot, or by using the power equation as,

$$W_{Le} = \int_0^{T/4} (v_{Le} \cdot i_{Le}) dt \quad [\text{J}] \quad (81)$$

## 6.2. Magnetic Inductance

In magnetic systems, the inductance measures the amount of energy stored in the electric field, represented by the electric charge, due to the flow of magnetic current ( $i_m$ ).

It is expressed as,

$$L_m = \frac{Nq_e}{i_{Lm}}. \quad (82)$$

It can also be expressed in terms of its geometry, length ( $l$ ) and area ( $S$ ), and its permittivity ( $\epsilon$ ).

$$L_m = N^2 \frac{\oint D dS}{\oint E dl} = N^2 \frac{\oint \epsilon E dS}{\oint E dl} = N^2 \left( \epsilon \frac{S}{l} \right) \quad [\text{F}] \quad (83)$$

To calculate the energy stored by an inductance of constant value we use,

$$W_{Lm} = \frac{1}{2} \cdot L_m I_m^2 \quad [\text{J}]. \quad (84)$$

In a more general case, the energy stored can be expressed either by,

$$W_{Lm} = \int i_{Lm} dq_e \quad [\text{J}], \quad (85)$$

which is proportional to the energy calculated using the area of the D vs. E plot, or by using the power equation as,

$$W_{Lm} = \int_0^{T/4} (v_{Lm} \cdot i_{Lm}) dt \quad [\text{J}] \quad (86)$$

To summarize, Table VI presents a comparison of the magnetic fundamental quantities proposed in this dissertation and the ones presented in the reluctance model.

TABLE VI  
COMPARISON OF THE POWER-INVARIANT AND RELUCTANCE MODELS

Quantities	Power-Invariant	Reluctance
<b>Conserved Quantity</b>	Magnetic Flux [Wb]	-----
<b>Flow</b>	Magnetic Current [Wb/s]	Flux [Wb]
<b>Effort</b>	Magnetic Voltage [t·A]	MMF [t·A]
<b>Dissipative Element</b>	Magnetic Resistance [ $\Omega^{-1}$ ]	Reluctance [ $H^{-1}$ ]
<b>Energy Storage Element</b>	Magnetic Capacitance [H]	-----
<b>Energy Storage Element</b>	Magnetic Inductance [F]	-----



## CHAPTER IV

## VALIDATION OF THE POWER-INVARIANT MAGNETIC MODEL

To validate the existence of the elements described in the power-invariant magnetic model we will use an iron core coil as case study (see Fig. 11). This system can be considered as a two-port network where the electric and magnetic networks are composed by the electric voltage source and the iron core, respectively. We decided to use this system for various reasons, first, its simplicity; second, we are familiar with its physical behavior and third, we want to represent the magnetic network using known material characteristics (B-H Loops, permeability curves, etc.).

It is our purpose to model the magnetic network solely using the quantities described in the power-invariant model, however, because in this case the excitation is originally electrical and there is not a direct method to measure the amount of magnetic current or voltage at the terminal of the magnetic network, the excitation and the

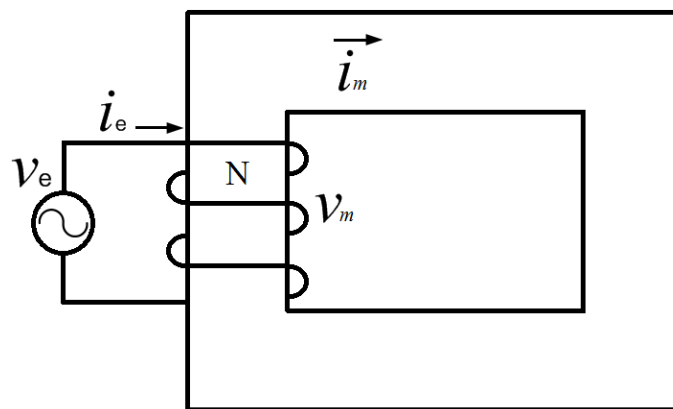


Fig. 11. Representation of an iron core coil.

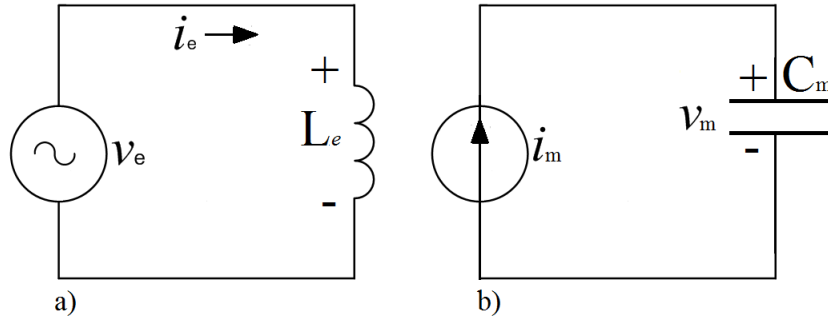


Fig. 12. Representation of an ideal iron core coil a) electric circuit, b) magnetic circuit.

response of the magnetic system have to be measured indirectly through the electric terminal and then gyrated to the magnetic regime in order to analyze it.

#### A. Ideal Iron Core Coil Case Study

When the iron core coil presented in Fig. 11 is considered ideal, this is the copper and iron losses are neglected, the easiest way to characterize the magnetic network is by using the coil's electric circuit model, shown in Fig. 12a. It is an electric voltage source in series with an electric inductance and is mathematically expressed as,

$$v_e = L_e \frac{di_e}{dt} \quad [\text{V}] \quad (87)$$

In this model, the iron core is depicted as a magnetic energy storage element by the electric inductance. The element that performs this same function in the magnetic regime is the magnetic capacitance. Therefore, we can model the ideal iron core coil as a magnetic capacitance in series with a magnetic current source as it is shown in Fig. 12b.

According to gyrator theory, the electric voltage source can be modeled as a magnetic current source of value

$$i_m = \frac{1}{N} v_e \quad [\text{Wb/s}], \quad (88)$$

and the electric current is modeled as a magnetic voltage of value

$$v_m = Ni_e \quad [\text{A}]. \quad (89)$$

Replacing (88) and (89) in (87),

$$\begin{aligned} Ni_m &= \frac{L_e}{N} \frac{dv_m}{dt} \\ i_m &= \frac{1}{N^2} L_e \frac{dv_m}{dt} \\ i_m &= C_m \frac{dv_m}{dt} \quad [\text{Wb/s}]. \end{aligned} \quad (90)$$

This means that the value of the magnetic capacitance is equivalent to the electric inductance and is expressed as,

$$C_m = \frac{1}{N^2} L_e \quad [\text{H}]. \quad (91)$$

This is consistent with gyrator theory [25] and with our expression of magnetic capacitance because,

$$C_m = \frac{1}{N^2} L_e = \frac{1}{N^2} \frac{N\varphi}{i_e} = \frac{\varphi}{Ni_e} = \frac{\varphi}{v_m} \quad [\text{H}]. \quad (92)$$

It is important to point out that even though this model does not represent either the losses or hysteresis, it represents the linearity or nonlinearity of the material and can be used to identify the saturation level of the material (see Fig. 13).

### *B. Real Iron Core Coil Case Study*

The capacitance element alone does not represent the real behavior of an iron core coil. Hysteresis and magnetic losses, among other factors, need to be taken into consideration when modeling the magnetic network.

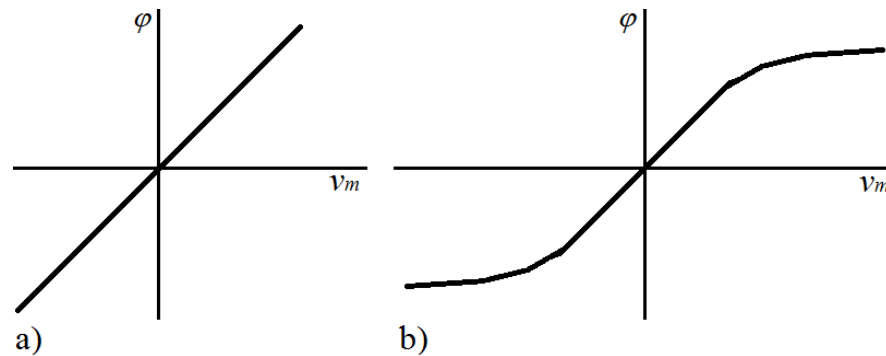


Fig. 13. Characteristic of a magnetic capacitance a) linear, b) nonlinear.

Such network or in this case the magnetic material can be characterized by varying the excitation's magnitude, frequency or waveform and analyzing how the system responds to such variations. Hysteresis loops are good example of this technique.

These loops provide valuable information about the system under study, for instance, hysteresis losses are represented by the area inside each loop and the saturation level of the material can be identified using the tip of the major hysteresis loop. Also, the shape of the loops is used to identify the type of magnetic the material. Fig. 14 shows the hysteresis loops of a Grain-Oriented Electrical Steel material excited at 50 Hz where each loop varies due to an amplitude variation. On the other hand, Fig. 15 shows the hysteresis loops of the Magnetic Industries' Permalloy 80 where each loop varies due to a frequency variation.

According to our theory, in order to represent the real behavior of the magnetic material we should model the network using the magnetic resistance, inductance and capacitance. We understand that the effect of some of those elements is more significant

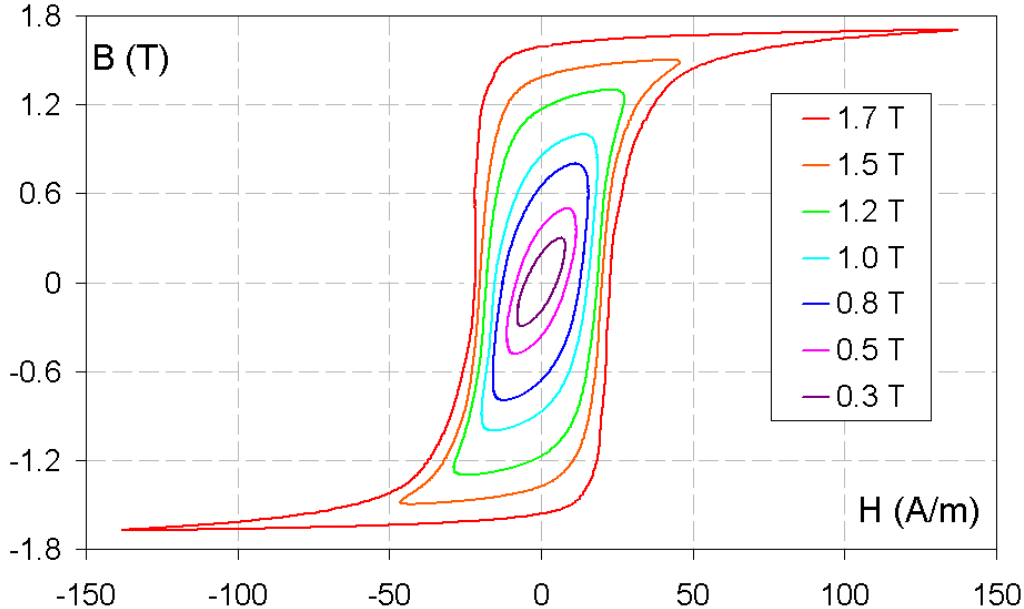


Fig. 14. Family of hysteresis loops at different amplitudes.  
(Source: [http://en.wikipedia.org/wiki/File:B-H loop.png](http://en.wikipedia.org/wiki/File:B-H_loop.png))

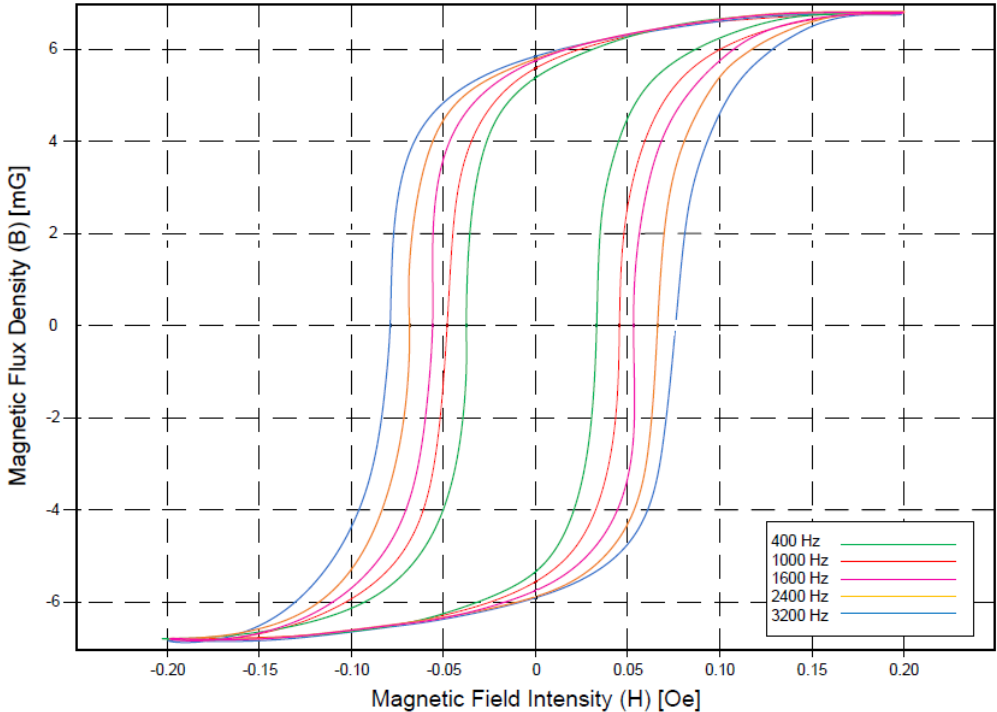


Fig. 15. Family of hysteresis loops at different frequencies.  
(Source: *Magnetic Industries Catalogue*)

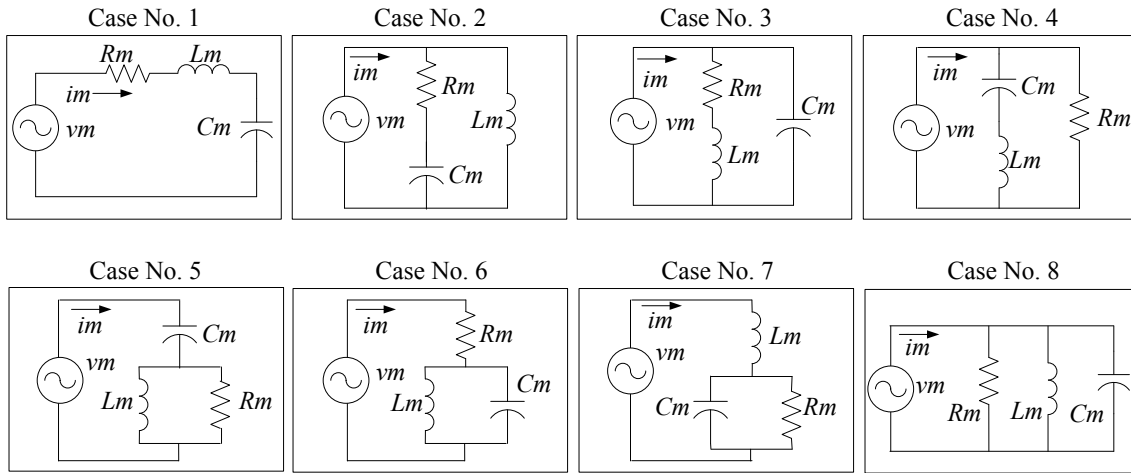


Fig. 16. Magnetic network configurations.

than others in the model however, for completeness we will represent the network based on the three elements. The reason behind this will be clarified in section 3.1.

Fig. 16 shows the eight possible configurations in which the elements could be arranged. Each one of these configurations is a representation of the magnetic network only. The elements can be either linear or nonlinear and the source can be either a magnetic current or voltage source depending on the electric source.

### 1. Determining the Element's Configuration based on Frequency Tests

In order to choose the correct configuration we need to perform a series of conceptual tests. These tests are based on the response that the system shows due to variation in magnitude and frequency of the excitation. In this case we will assume that the system is excited with an electric current source of  $I_e$  value.

### 1.1. Zero Frequency Test

In the electric regime, we know from experimentation that in steady state, an iron core coil excited with an electric current source at zero frequency behaves as a short circuit. In other words, the electric voltage induced in the terminals of the iron core coil is zero. This is,

$$V_{Le} = L \frac{di_e}{dt} = 0. \quad (93)$$

As these are the values that correspond to the terminal of the magnetic network, we could interpret them, for simplicity, as a magnetic voltage source of value

$$V_m = NI_e, \quad (94)$$

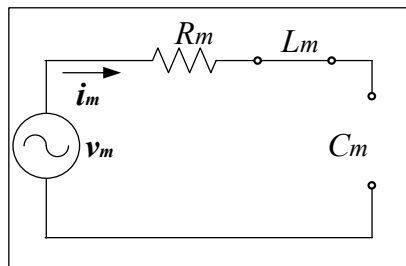
and its response as a magnetic current of value,

$$I_m = 1/N \cdot V_{Le} = 0. \quad (95)$$

This implies that we need to find a network configuration that provides a zero magnetic current when it is excited with a DC magnetic voltage source.

First we will assume that all the elements are linear so that we can represent the circuits' equations in the frequency domain. As we are evaluating the DC response, the angular frequency ( $\omega$ ) is zero. Therefore,

- Case No. 1

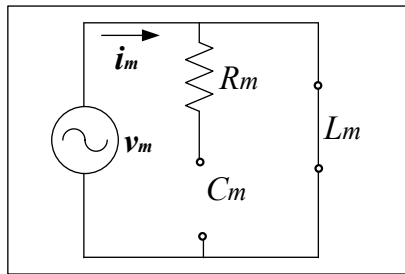


$$V_m = I_m \left( R_m + j\omega L_m + \frac{1}{j\omega C_m} \right)$$

$$V_m = I_m(\infty)$$

$$I_m = 0$$

- Case No. 2

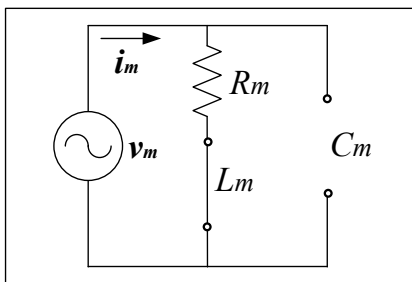


$$V_m = I_m \cdot \frac{(R_m + \frac{1}{j\omega C_m})(j\omega L_m)}{R_m + j\omega L_m + \frac{1}{j\omega C_m}}$$

$$V_m = I_m(0)$$

$$I_m = \infty$$

- Case No. 3

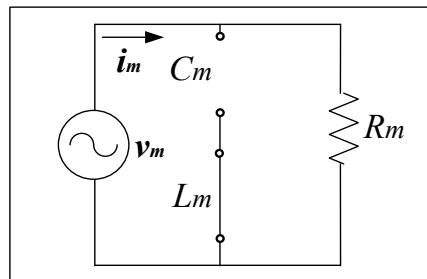


$$V_m = I_m \cdot \frac{(R_m + j\omega L_m)(\frac{1}{j\omega C_m})}{R_m + j\omega L_m + \frac{1}{j\omega C_m}}$$

$$V_m = I_m R_m$$

$$I_m = V_m / R_m$$

- Case No. 4

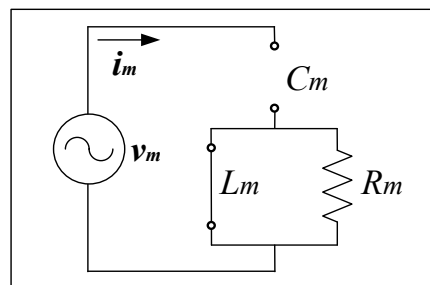


$$V_m = I_m \cdot \frac{(j\omega L_m + \frac{1}{j\omega C_m})(R_m)}{R_m + j\omega L_m + \frac{1}{j\omega C_m}}$$

$$V_m = I_m R_m$$

$$I_m = V_m / R_m$$

- Case No. 5



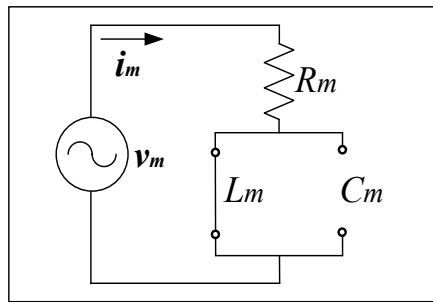
$$V_m = I_m \left( \frac{1}{j\omega C_m} + \frac{j\omega L_m R_m}{R_m + j\omega L_m} \right)$$

$$V_m = I_m(\infty)$$

$$I_m = 0$$



- Case No. 6

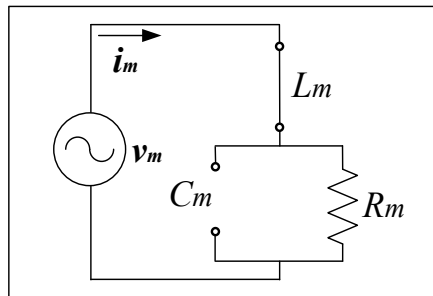


$$V_m = I_m \left( R_m + \frac{L_m/C_m}{j\omega L_m + \frac{1}{j\omega C_m}} \right)$$

$$V_m = I_m R_m$$

$$I_m = \frac{V_m}{R_m}$$

- Case No. 7

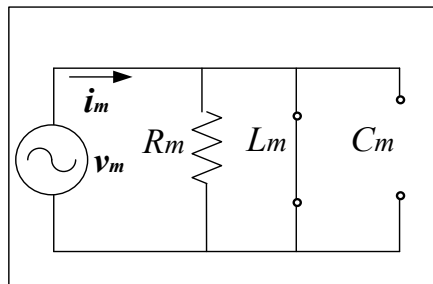


$$V_m = I_m \left( j\omega L_m + \frac{R_m \frac{1}{j\omega C_m}}{R_m + \frac{1}{j\omega C_m}} \right)$$

$$V_m = I_m R_m$$

$$I_m = \frac{V_m}{R_m}$$

- Case No. 8



$$V_m = I_m \left[ \frac{\left( \frac{R_m \cdot j\omega L_m}{R_m + j\omega L_m} \right) \left( \frac{1}{j\omega C_m} \right)}{\frac{R_m \cdot j\omega L_m}{R_m + j\omega L_m} + \frac{1}{j\omega C_m}} \right]$$

$$V_m = I_m(0)$$

$$I_m = \infty$$

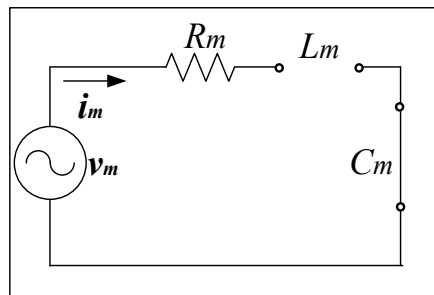
After evaluating these equations we can eliminate six of these networks because they do not respond as expected. The networks that we should continue studying are the ones depicted in cases No. 1 and 5.

### 1.2. Infinite Frequency Test

We cannot actually perform an experiment at infinite frequency but we can perform experiments at very high levels of frequency. Magnetic materials excited at a very high frequency (GHz minimum) lose its magnetic characteristics. The losses are extremely high and it seems more as an electric capacitor than an inductor when it is seen from the electric regime. This behavior has been attributed to the inner-winding capacitances [29]. However, beside the effect provided by these inner winding capacitances, the magnetic material itself has some dielectric properties that are more perceptible at higher frequencies [30].

If this is the case, the magnetic network will behave mostly as a magnetic inductance. Therefore, performing similar tests to the networks of case No. 1 and case No. 5 but assigning an angular frequency ( $\omega$ ) equal to infinite then you will see that case No. 1 is the one that depicts the behavior expected.

- Case 1

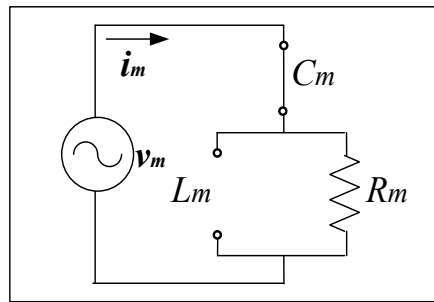


$$V_m = I_m \left( R_m + j\omega L_m + \frac{1}{j\omega C_m} \right)$$

$$V_m = I_m(\infty)$$

$$I_m = 0$$

- Case 5



$$V_m = I_m \left( \frac{1}{j\omega C_m} + \frac{j\omega L_m R_m}{R_m + j\omega L_m} \right)$$

$$V_m = I_m R_m$$

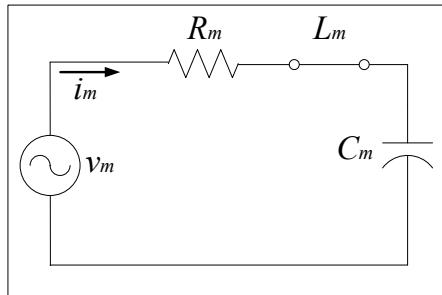
$$I_m = \frac{V_m}{R_m}$$

### 1.3. Medium Frequency Test

Beside the fact that the network of case No. 1 provides the expected solution at zero and infinite frequency, we need to verify that it models the magnetic material at the levels of frequency most commonly used (Hz-MHz). At these frequencies, the dielectric property of the magnetic material is very small and most of the times it is neglected. Therefore, our network configuration should be able to represent the losses and the energy storage capacity of the material.

If we perform a circuit analysis to the case No.1 and case No.5 neglecting the inductance, the circuits need to be modified in order to represent that there is an almost zero voltage drop at the terminals of the magnetic inductance or in other words, the inductance is symbolized by a short circuit. Therefore,

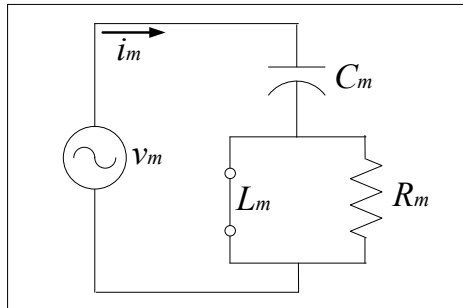
- Case 1



$$V_m = I_m \left( R_m + \frac{1}{j\omega C_m} \right)$$

$$I_m = V_m \left( R_m + \frac{1}{j\omega C_m} \right)^{-1}$$

- Case 5



$$V_m = I_m \left( \frac{1}{j\omega C_m} \right)$$

$$I_m = V_m (j\omega C_m)$$

In case No. 5, no current will flow through the resistance and the losses will not be modeled. On the other hand, case No. 1 still represents the magnetic losses and the energy storage capacity at these levels of frequencies. These results corroborate our decision to use the configuration of case No. 1, depicted in Fig. 17, to describe the behavior of a real iron core coil and consequently, it will be used to validate the elements described in the power-invariant magnetic model.

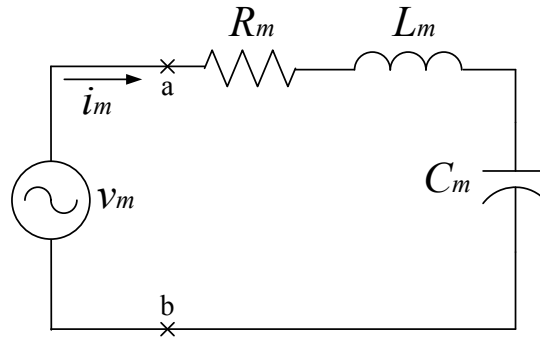


Fig. 17. Magnetic model of an iron core coil.

## 2. Analysis of the Magnetic Circuit Model

As we established earlier, a magnetic material is characterized by its hysteresis loops (see Fig. 14 and Fig. 15). If we perform conceptual tests using the magnetic circuit model shown in Fig. 17 we expect to find similar results.

### 2.1. Linear Elements

First we will explore the response of the circuit in case the elements are linear or constant values. The advantage of using linear elements is the fact that we could find exact solutions to the differential equations using analytical techniques well known in the electrical regime that can be applied to the magnetic regime.

The differential equations that describes this circuit in the time domain are,

$$v_m = v_{R_m} + v_{L_m} + v_{C_m} \quad (96)$$

$$v_m = i_m R_m + L_m \frac{di_m}{dt} + \frac{1}{C_m} \int i_m dt \quad (97)$$

$$v_m = L_m \ddot{q}_m + R_m \dot{q}_m + \frac{1}{C_m} q_m \quad (98)$$

TABLE VII  
MAGNETIC PARAMETERS OF THE CONCEPTUAL TEST

Parameter	Value	Unit
Max. Magnetic Voltage	10.0	t·A
Magnetic Resistance	19.06	$\Omega^{-1}$
Magnetic Inductance	1.05e-9	F
Magnetic Capacitance	1.89e-5	H
Core's Area	1.06	cm <sup>2</sup>
Core's Length	10.1	cm

\* The value of the core's geometry has been based on the geometry of the toroidal core T157-34 of Micrometals, Inc.

The plot of the magnetic voltage at the terminals a-b versus the integral of the magnetic current ( $q_m$ ) is proportional to the B-H hysteresis loop as,

$$H(t) = v_m/l_c \quad [\text{A/m}], \quad (99)$$

$$B(t) = q_m/A_c \quad [\text{T}]. \quad (100)$$

Table VII presents the values used to calculate the magnetic current of this conceptual test. As the excitation of the system has a cosinusoidal waveform and the elements are linear, the differential equations (97) and (98) can be solved easily in the frequency domain instead that in the time domain using Laplace transformations or phasors. The circuit's current equation expressed using phasors is,

$$I_m \angle \theta_{I_m} = V_m \angle \theta_{V_m} \left( R_m + j\omega L_m + \frac{1}{j\omega C_m} \right)^{-1} \quad (101)$$

where the angular frequency  $\omega = 2\pi f$ .

The circuit's equations in the time domain are,

$$v_m = V_m \cos(\omega t + \theta_{V_m}) \quad [\text{t}\cdot\text{A}], \quad (102)$$

$$i_m = I_m \cos(\omega t + \theta_{I_m}) \quad [\text{Wb/s}], \quad (103)$$

$$q_m = \frac{I_m}{\omega} \sin(\omega t + \theta_{I_m}) \quad [\text{Wb}]. \quad (104)$$

Once we have obtained all the equations that describe the system we proceed to find the family of hysteresis loops that result due to variations in amplitudes and frequencies. Fig. 18 presents three hysteresis loops excited with voltage magnitudes of 2.5, 5 and 10 t·A at a frequency of 150 Hz. On the other hand, Fig. 19 presents a family of hysteresis loops excited with a fixed magnetic voltage amplitude of 10 t·A but with frequencies ranging from 50 Hz to 150 Hz. The magnetic field intensity and magnetic flux density correspondent to the magnetic voltage and charge respectively, they are calculated using (99) and (100).

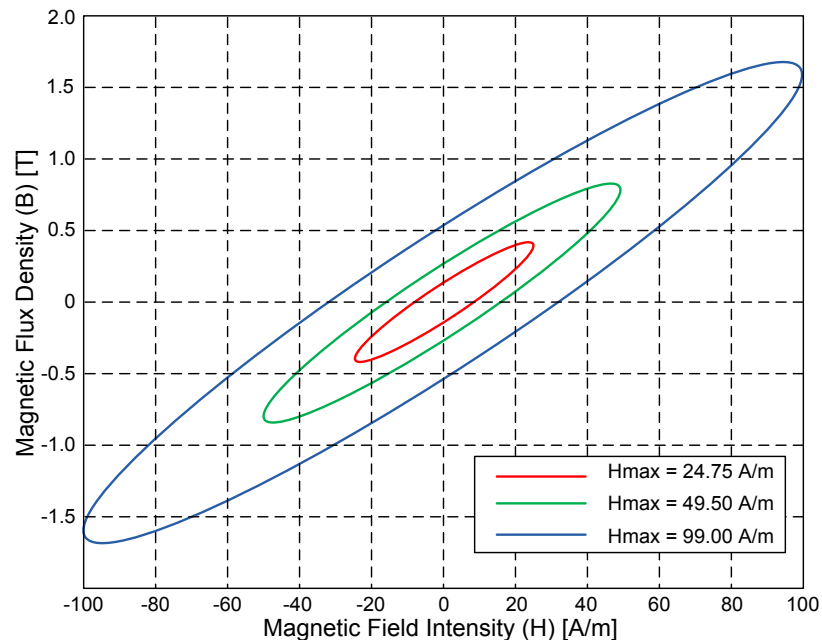


Fig. 18. Family of hysteresis loops at different amplitudes (linear elements).

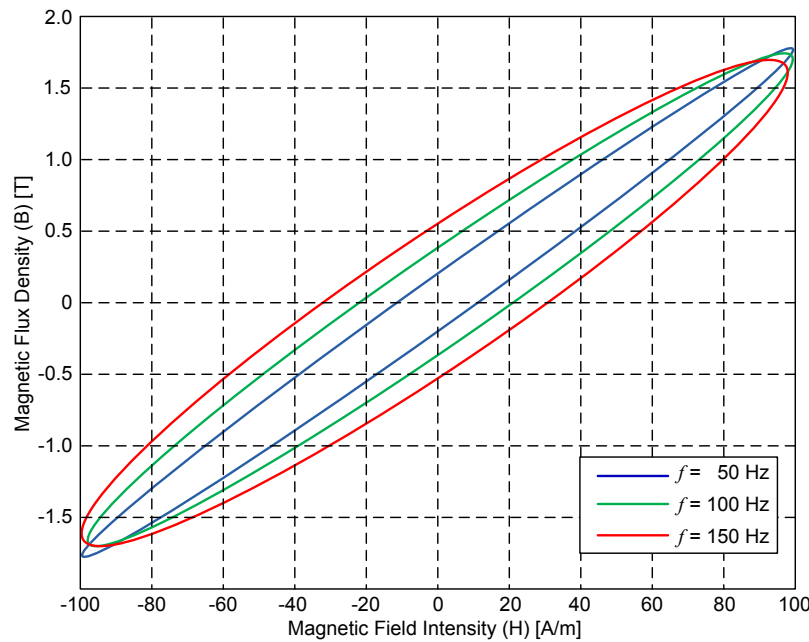


Fig. 19. Family of hysteresis loops at different frequencies (linear elements).

If we perform a qualitative comparison between the loops of Fig. 14 and Fig. 18 which vary with amplitude and between the loops of Fig. 15 and Fig. 19 which vary with frequency, we can see that the circuit satisfies the increase of area experienced by the hysteresis loops when the amplitude and the frequency increase. However, it is evident that the oval shape of the loops does not match the shape of the real loops. This is because we have assumed constant values for the magnetic resistance, inductance and capacitance when we know that at least one of them is nonlinear, the magnetic capacitance. In the next section we will explore how the nonlinearity of the elements affects the shape of the hysteresis loops.



## 2.2. Nonlinear Elements

We have seen that linear elements do not satisfy the hysteresis loop shape of an iron core coil. Therefore, we will implement a similar conceptual test to the one presented in the previous section but with nonlinear elements in order to see if we can model loops with comparable shapes to the ones in Fig. 14 and Fig. 15.

It is important to point out that when we refer to nonlinear elements, they are in fact current-varying or voltage-varying elements. However, the term nonlinear is customary because no matter how simple the circuit is it lead to nonlinear equations.

The nonlinear differential equations of the circuit of Fig. 17 are,

$$v_m = v_{R_m} + v_{L_m} + v_{C_m} \quad (105)$$

$$v_m = i_m R_m(i_m) + L_m(i_m) \frac{di_m}{dt} + \frac{1}{C_m(v_{C_m})} \int i_m dt \quad (106)$$

$$v_m = L_m(i_m) \dot{q}_m + R_m(i_m) q_m + \frac{1}{C_m(v_{C_m})} q_m. \quad (107)$$

Nevertheless, due to the nonlinearity of the elements, the solution of (106) and (107) gets more complicated and requires numerical methods to solve it. Therefore, in order to obtain the hysteresis loops, we have implemented a computer model shown in Fig. 20 using Simulink® (see Appendix A for details).

In this model we introduce nonlinear characteristics to describe each one of these elements. However, as these elements are function of the geometry of the core, which in this case is 1.06 cm<sup>2</sup> for the area and 10.1 cm for the length, we will present the characteristics of the intrinsic properties of a demonstrative material, denominated “material X”, in order to define each element rather than the characteristic of the

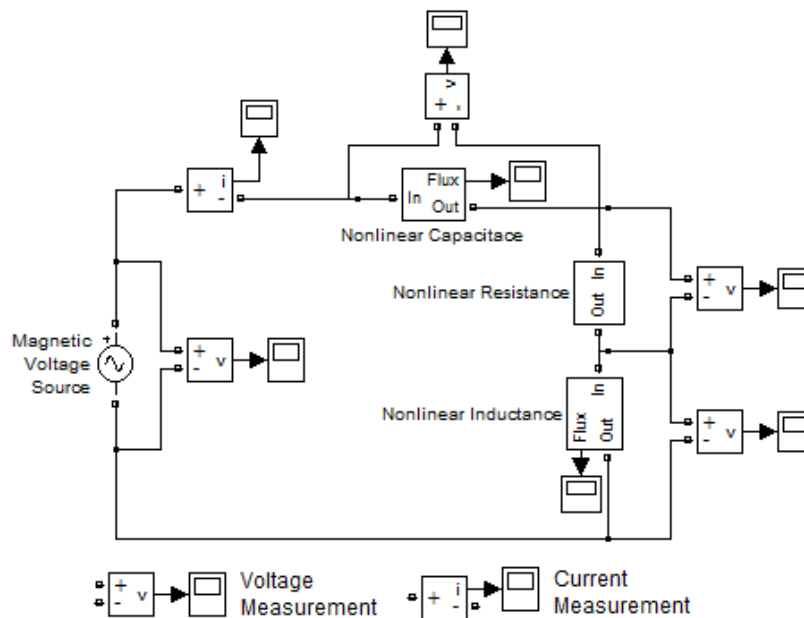


Fig. 20. Simulink model using nonlinear elements.

elements itself; this is the permeability for the magnetic capacitance, the permittivity for the magnetic inductance and the magnetic resistivity for the magnetic resistance. Fig. 21, Fig. 22 and Fig. 23 show the aforementioned characteristics.

The hysteresis loops that we found using these characteristics are presented in Fig. 24 for amplitudes of magnetic voltage ranging from 0.5 to 10 t·A and in Fig. 25 for frequencies ranging from 10 Hz to 100 Hz. The curves at 500 Hz and 1 kHz were calculated varying the magnetic resistivity curve as shown in Fig. 23. This was done in order to simulate the increase in losses due to skin effect.

In order to calculate the magnetic current and charge, Simulink solves the circuit's ordinary differential equations by using the solver ODE45. This solver is based on the Dormand-Prince numerical method which is a member of the Runge-Kutta family

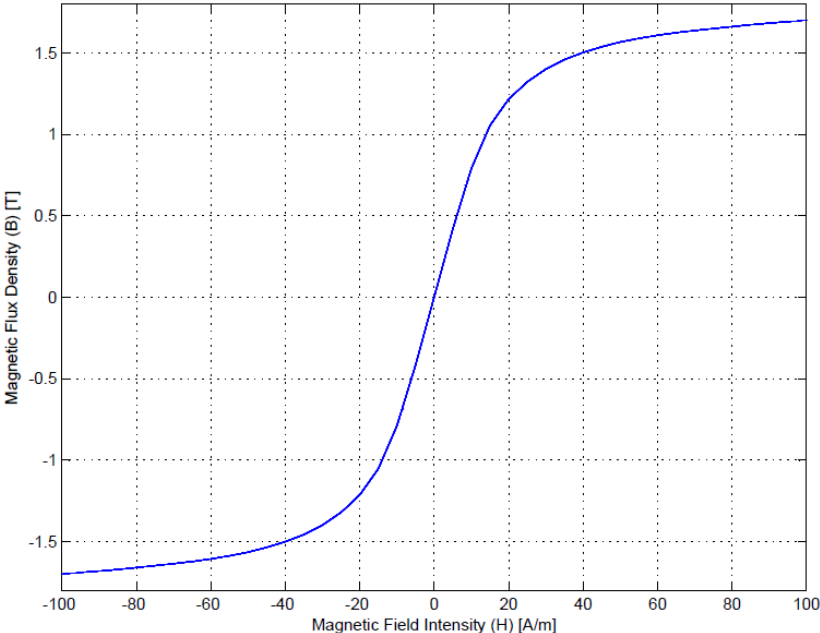


Fig. 21. Permeability's nonlinear characteristic ( $\mu=B/H$ ).

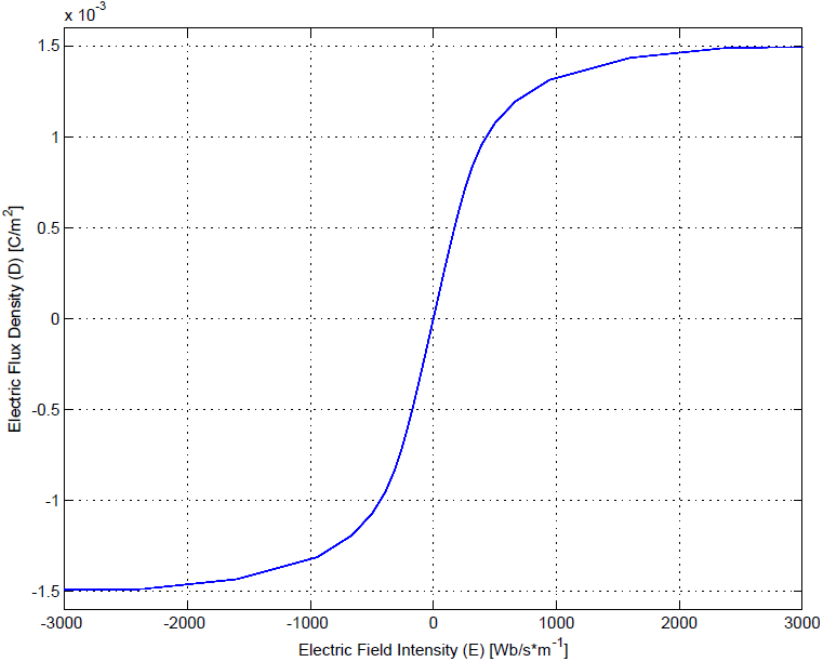


Fig. 22. Permittivity's nonlinear characteristic ( $\epsilon=D/E$ ).

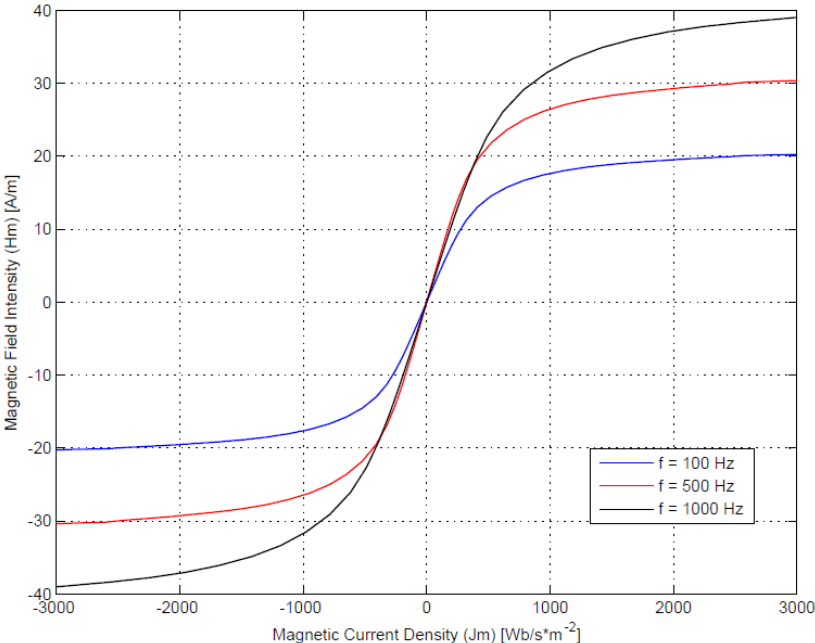


Fig. 23. Magnetic resistivity's nonlinear characteristic ( $\rho_m=H/J_m$ ).

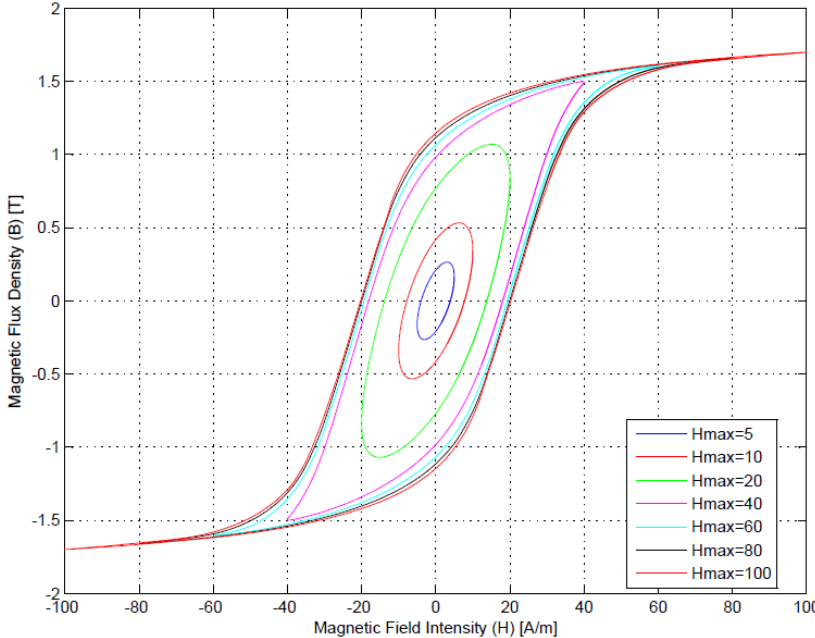


Fig. 24. Family of hysteresis loops at different amplitudes,  $f = 60$  Hz.

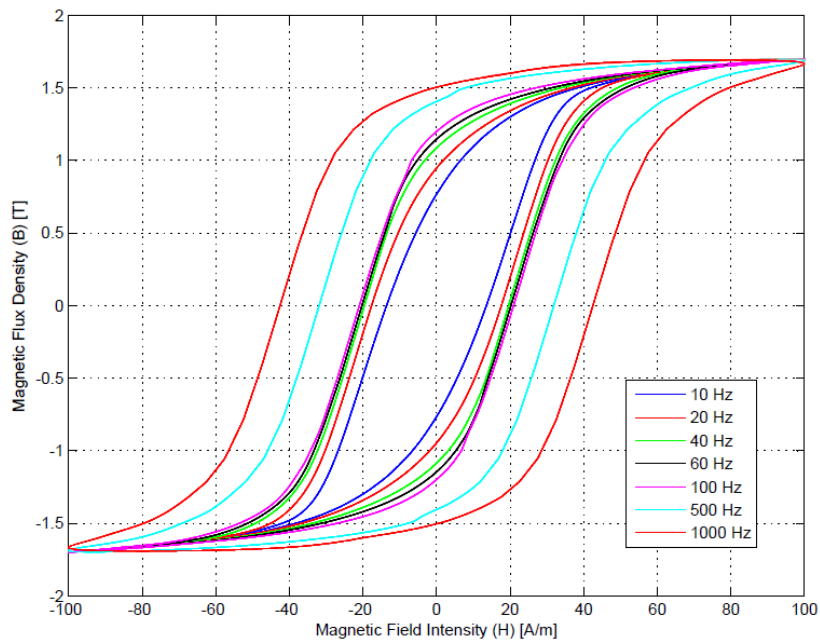


Fig. 25. Family of hysteresis loops at different frequencies.

of ODE solvers. It is a one-step solver. It needs only the solution at the immediately preceding time point to calculate the actual point.

The magnetic field intensity and magnetic flux density correspondent to the magnetic voltage and charge respectively, are calculated using (99) and (100).

As we can see both families of hysteresis loops are comparable in shape and behavior to the real ones presented in Fig. 14 and 15. Therefore, this validates the power-invariant magnetic model proposed in this dissertation.

### 3. Procedure to Determine the Model's Elements Using a Test Specimen

We have established that a magnetic system can be modeled using the elements described in the power-invariant magnetic model. As in the iron core coil case study the magnetic network only consist on the magnetic material of the core, this implies that we

can characterize the material based on these elements and consequently, on their intrinsic properties. In this section we will present the techniques that we have developed in order to find each one of these properties.

### 3.1. Fundamentals of the Test

It has been well established in electric circuits that depending on the value of the system's impedance, the amplitude and phase of the response vary. The same rule applies for magnetic systems and it can be demonstrated if we perform a series of simple waveform analysis using linear elements. The insights obtained from this analysis will be applied to find the characteristics of each element.

- *Resistance Case*

First, we will use a magnetic system where the impedance is only a magnetic resistance. If the system is excited with a voltage source  $v_m = V_m \cos(\omega t + \theta_{V_m})$ , then the response is given by,

$$i_m = \frac{1}{R_m} \cdot V_m \cos(\omega t + \theta_{V_m}) , \quad (108)$$

$$q_m = \frac{1}{R_m} \cdot \frac{V_m}{\omega} \sin(\omega t + \theta_{V_m}) . \quad (109)$$

If we plot the magnetic voltage and currents as shown in Fig. 26, we can see that for a given magnetic voltage the amplitude of the current varies according to the value of the magnetic resistance. Furthermore, the phase angle of the current remains in phase with the phase angle of the voltage. This means that the maximum value of the voltage waveform corresponds to the maximum value of the current waveform.

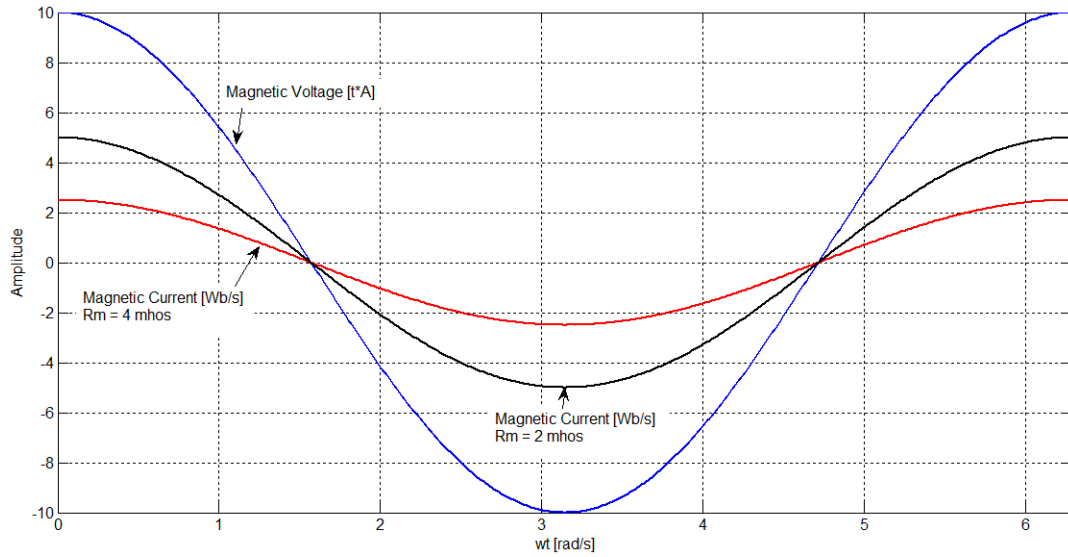


Fig. 26. Magnetic voltage at 60 Hz and currents for different values of resistance.

To emulate the hysteresis loop we plot the instantaneous magnetic voltage versus the instantaneous magnetic charge, as shown in Fig. 27. This plot is a loop with certain area because the magnetic charge is phase shifted  $90^\circ$  with respect to the voltage. Also, we can see that no matter the variation of the resistance the direction of the loop remains parallel to the horizontal axis.

- *Inductance Case*

In this case we will present a magnetic system where the impedance is only a magnetic inductance. The expression of the current if  $v_m = V_m \cos(\omega t + \theta_{V_m})$  is,

$$i_m = \frac{1}{L_m} \cdot \int v_m dt , \quad (110)$$

$$i_m = \frac{1}{L_m} \cdot \frac{V_m}{\omega} \cos(\omega t + \theta_{I_m}) , \quad (111)$$

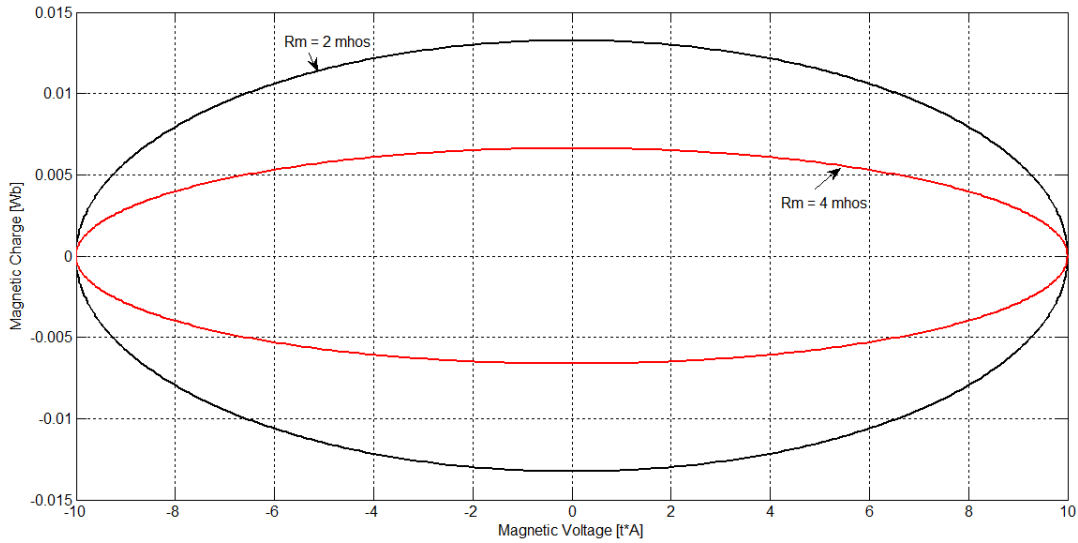


Fig. 27. Magnetic charge vs. voltage for different values of resistance.

where  $\theta_{I_m} = \theta_{V_m} - \frac{\pi}{2}$ ; and,

$$q_m = \frac{1}{L_m} \cdot \frac{V_m}{\omega^2} \sin(\omega t + \theta_{I_m}) . \quad (112)$$

If we plot the magnetic voltage and currents as shown in Fig. 28, we can see that for a given magnetic voltage the amplitude of the current varies according to the value of the magnetic inductive reactance ( $X_{L_m} = \omega L_m$ ). Furthermore, there is a  $90^\circ$  phase shift between the angle of the voltage and the angle of the current. In this case, we commonly say that the current is lagging the voltage, similar to electric systems.

To emulate the hysteresis loop we plot the instantaneous magnetic voltage versus the instantaneous magnetic charge as shown in Fig. 29. We can see that the loop has no area, different to the resistance case; this is because the voltage and the charge are  $180^\circ$  phase shifted. This means the maximum value of the voltage waveform correspond to



the minimum value of the charge waveform. Also, no matter the variation of the inductance the direction of the loop remains being a line with a negative slope.

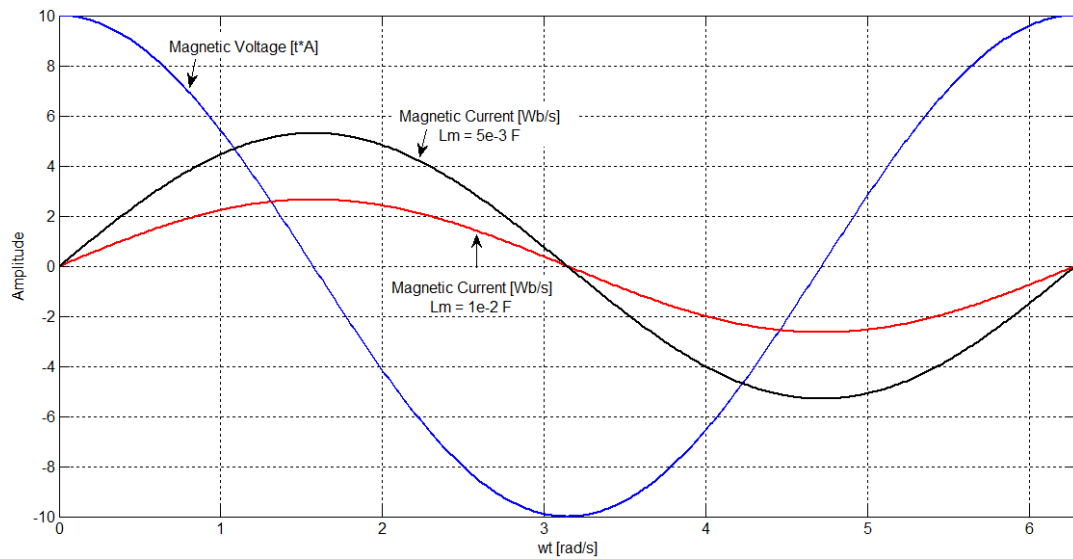


Fig. 28. Magnetic voltage at 60 Hz and currents for different values of inductance.

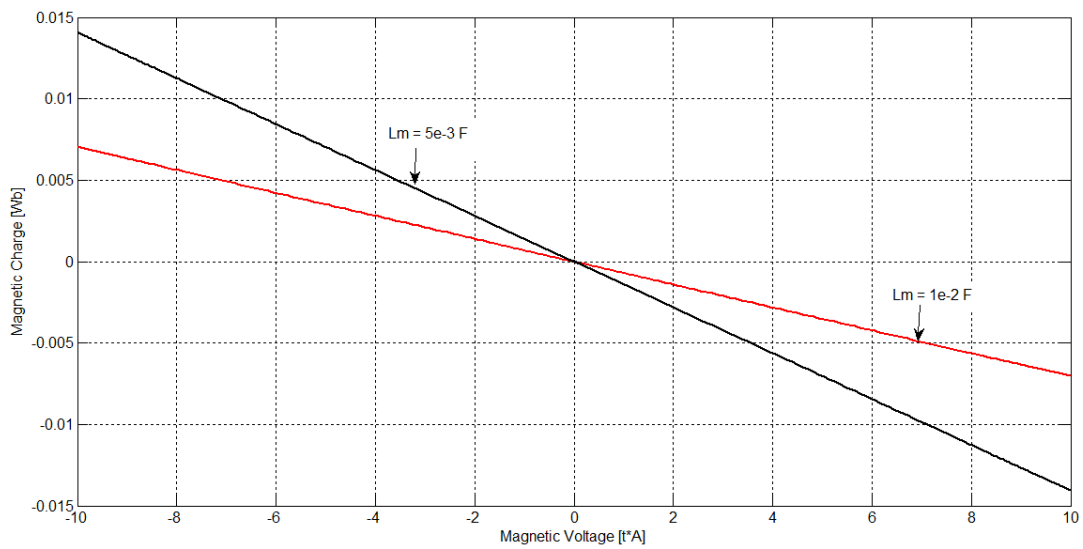


Fig. 29. Magnetic charge vs. voltage for different values of inductance.

- *Capacitance Case*

In this case we will present a magnetic system where the impedance is only a magnetic capacitance. The expression of the response if  $v_m = V_m \cos(\omega t + \theta_{V_m})$  is,

$$i_m = C_m \frac{d(v_m)}{dt}, \quad (113)$$

$$i_m = -\omega C_m V_m \sin(\omega t + \theta_{V_m}), \quad (114)$$

$$i_m = \omega C_m V_m \cos(\omega t + \theta_{I_m}), \quad (115)$$

where  $\theta_{I_m} = \theta_{V_m} + \frac{\pi}{2}$ ; and,

$$q_m = C_m V_m \sin(\omega t + \theta_{I_m}). \quad (116)$$

If we plot the magnetic voltage and currents as shown in Fig. 30, we can see that for a given magnetic voltage the amplitude of the current varies according to the value of

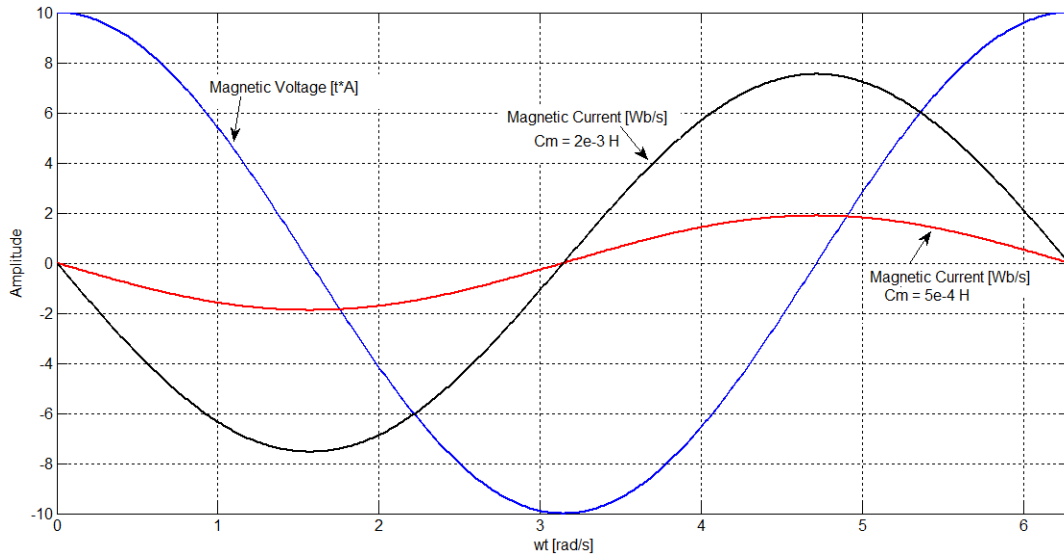


Fig. 30. Magnetic voltage at 60 Hz and currents for different values of capacitance.

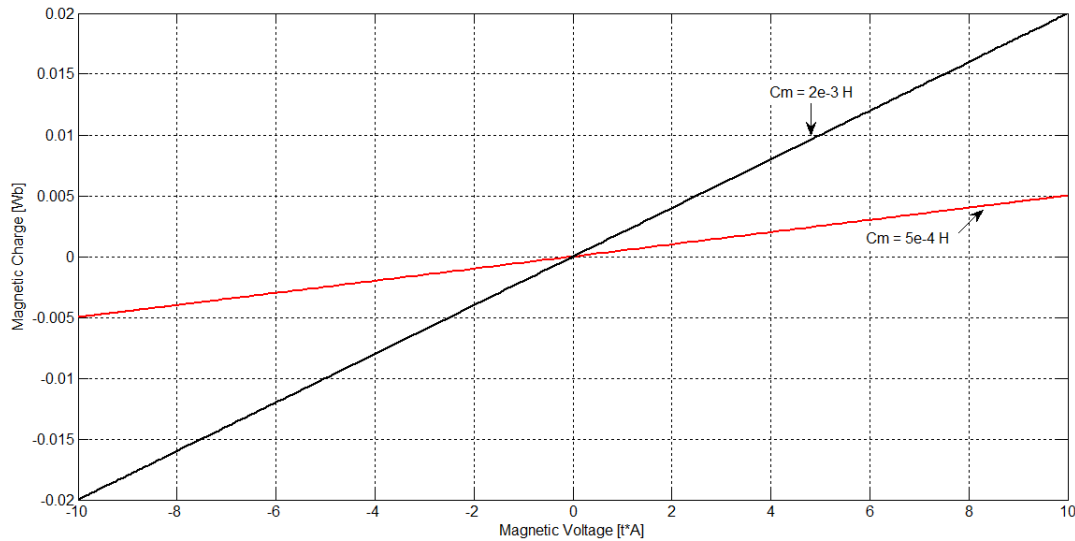


Fig. 31. Magnetic charge vs. voltage for different values of capacitance.

the magnetic capacitive reactance ( $X_{C_m} = (\omega C_m)^{-1}$ ). Furthermore, there is a  $90^\circ$  phase shift between the angle of the voltage and the angle of the current. In this case, we commonly say that the current is leading the voltage, similar to electric systems.

To emulate the hysteresis loop we plot the instantaneous magnetic voltage versus the instantaneous magnetic charge as shown in Fig. 31. We can see that the loop has no area, as in the inductance case, this is because the voltage and the charge are in phase. This means that the maximum value of the voltage waveform correspond to the maximum value of the charge waveform. Also, no matter the variation of the capacitance the direction of the loop remains being a line with a positive slope.

- *Series  $R_m$ ,  $L_m$  and  $C_m$  Case*

In this case we will present a magnetic system where the impedance has the configuration of the three elements in series. This is the configuration we are using to represent the iron core coil (see Fig. 17).

The expression of the magnetic current and charge due to a voltage excitation  $v_m = V_m \cos(\omega t + \theta_{V_m})$  is,

$$i_m = I_m \cos(\omega t + \theta_{I_m}) \quad (117)$$

where 
$$I_m \angle \theta_{I_m} = \left| \frac{v_m}{Z_m} \right| \angle (\theta_{V_m} - \theta_{Z_m}) \quad (118)$$

$$Z_m = \sqrt{R_m^2 + [\omega L_m + 1/(\omega C_m)]^2}; \quad \theta_{Z_m} = \tan^{-1} \left( \frac{[\omega L_m + 1/(\omega C_m)]}{R_m} \right) \quad (119)$$

As we will see in Fig. 32, in this case the current is phase shifted with respect to the voltage as well. However, the value of this phase shift is neither  $0^\circ$  nor  $90^\circ$  making difficult the identification of the elements individually.

When the impedance is a combination of the three elements, the hysteresis loops display an area due to the presence of the resistance and a direction due to the inductive and capacitive reactance. Depending on which element is more relevant in the circuit, the loop presents a positive, negative or no slope at all, as shown in Fig. 33.

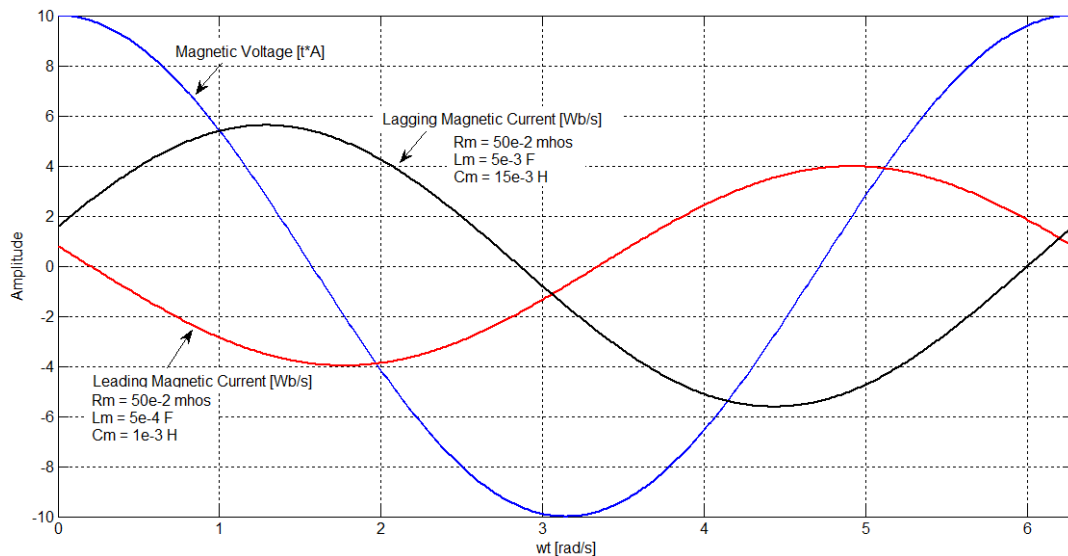


Fig. 32. Magnetic voltage at 60 Hz and currents for different values of impedance.

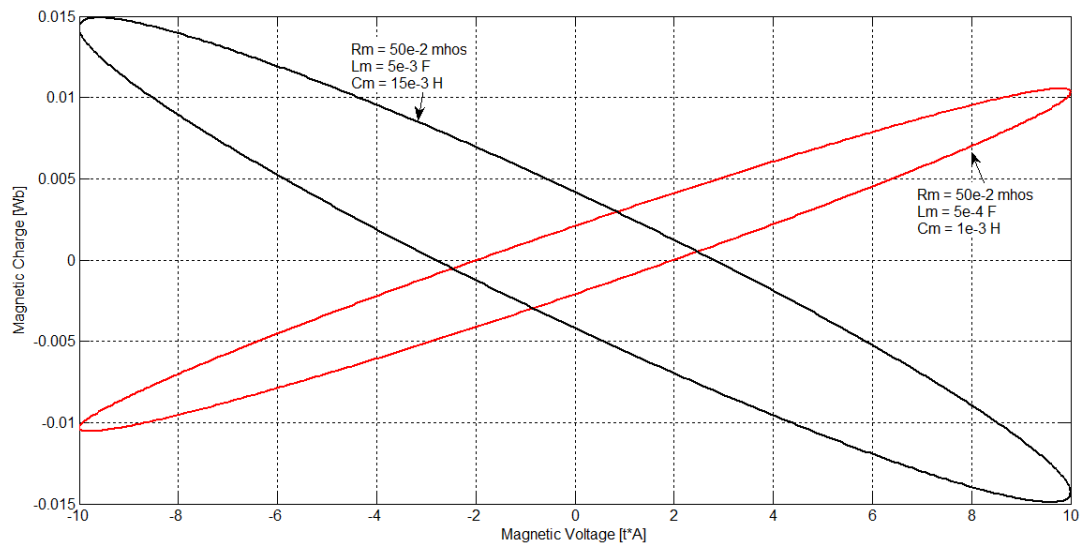


Fig. 33. Magnetic charge vs. voltage for different values of impedance.

This same analysis has been performed using nonlinear elements yielding to similar results (see Appendix B). Therefore, we will base our tests in the behavior of the steady-state currents waveforms, charge waveforms and hysteresis loops, when the system is excited with a magnetic voltage source at different frequencies.

### 3.2. The Experiment Test

Most magnetic industries and test laboratories based the characterization of their magnetic materials (hysteresis loops, permeability curves, etc.) on the test procedures described by the American Society for Testing and Materials (ASTM).

We suggest following their recommendations on the preparation of the test specimen as appears in the standard A: 34/A34M-01 denominated *Standard Practice for Sampling and Procurement Testing of Magnetic Materials*. This practice describes sampling procedures and test specimens for determination of various magnetic

properties of both soft and hard magnetic materials [31]. However, we suggest using the schematic shown in Fig. 34 to build the apparatus of this experiment.

The system is excited with an AC current source. The oscilloscope will be used to capture instantaneous value of the electric current and voltage induced at the electric terminals of the core as these values are manipulated in order to find the hysteresis loops that characterize the magnetic material.

The procedures developed to find the parameters of the magnetic model are based on the family of hysteresis loops at different frequencies and amplitudes. However, as those loops are product of the manipulation of the measured electric current and voltage, these waveforms can be used as well. They are gyrated into the magnetic regime and directly used as terminal values of the magnetic model of Fig. 17.

At this point, we have designed and presented a general description of the

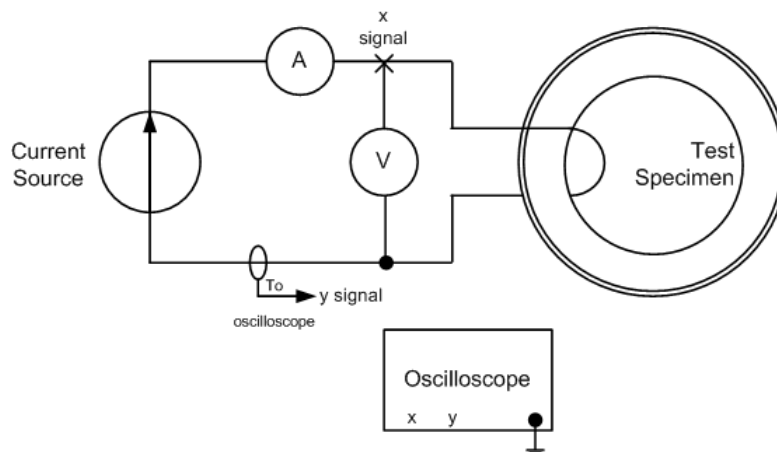


Fig. 34. Schematic illustration of the test apparatus.

experiment settings. However, in order to avoid measurement inaccuracies we decided to use published conventional data, this is manufacturer's hysteresis loops, instead of performing our own experimentation. Unfortunately, the data that we obtained to characterize a magnetic material was insufficient. Nevertheless, as our objective is to explain the procedures developed to find the magnetic elements, we decided to avoid the experiments complications and use the hysteresis loops that result from the computer model that we developed in Simulink as we already show that this model generates hysteresis loops with similar behavior to the real ones.

### *3.3. The Conceptual Test*

In this conceptual test we will perform the same tests we would have implemented in the experiment test. As we will use the model developed in Simulink, depicted in Fig. 20, we expect to find the same material characteristics that we introduced to proof the functionality of the model in the first place. However, because we want to emulate the experiment test, we will only use the waveforms of the magnetic circuit terminals in order to find the elements' characteristics. This is done because these magnetic current and voltage are a representation of the electric voltage and current that we would have measured using the oscilloscope.

#### *3.3.1. Quasi-static Frequency Test*

The first test that we will implement is exciting the model at the lowest none zero frequency possible. In other words, we have to run the model at a quasi-static mode. If the system is excited with a zero frequency magnetic voltage, the steady state magnetic current will be zero as explained in the zero-frequency test. However, we need to

analyze the phase angle, amplitude and shape of the steady state response and the quasi-static frequency allows us to do that.

Fig. 35 shows the waveform of the magnetic voltage and current at steady state. The voltage applied is a cosinusoidal waveform at 0.5 Hz with an amplitude equal to 10 t·A. It is evident that the magnetic current is leading the voltage exactly  $90^\circ$ ; this implies that under this quasi-static condition the magnetic capacitance is the element that predominates in the circuit.

In Appendix C we present magnetic current waveforms for different amplitudes of magnetic voltage, you will see that regarding the nonlinearity of the magnetic current, the phase angle remains almost  $90^\circ$  shifted. This implies that we can characterize the nonlinearity of the magnetic capacitance following this method.

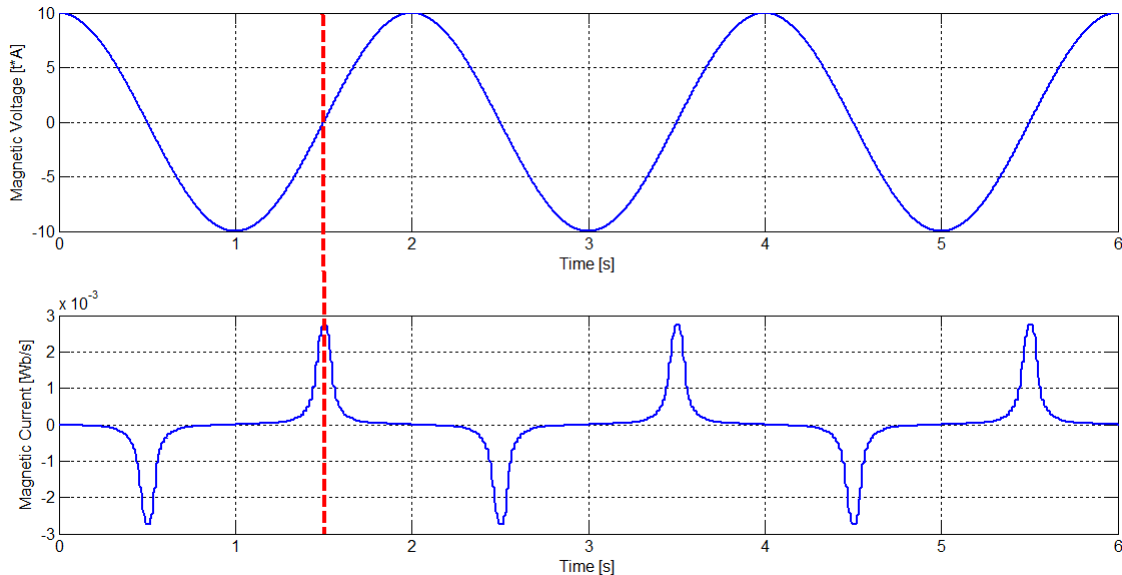


Fig. 35. Magnetic voltage and current at 0.5 Hz.



The capacitance equation (120) expresses a point by point relation between the magnetic flux or magnetic charge ( $\varphi$ ,  $q_m$ ) and the magnetic voltage ( $v_m$ ). Therefore, the characteristic of the magnetic capacitance can be found by plotting the magnetic charge versus the magnetic voltage at a low frequency.

$$C_m = \frac{q_m}{v_m} \quad (120)$$

Fig. 36 displays a family of hysteresis loops, each one result of an increase in amplitude of the magnetic voltage. As (120) is a point by point relation, we expected loops with zero area but instead these plots present loops with very small area as result. This happens because another element is starting to affect the response. In order to reduce the area of the loops the frequency of operation should be lower. However, if the value of this area is very small, the capacitance characteristic can be approximated by a line connecting the tips of each loop as shown in Fig. 36.

It is known that the characteristic of the magnetic capacitance depends on the geometry of the specimen therefore, it is better to provide a more general description of this element in terms of its intrinsic property, the permeability. Fig. 37 presents the circuit's hysteresis loops but in terms of the field intensities. The permeability is the point by point relation between the magnetic flux density and the magnetic field intensity. It can also be displayed as shown in Fig. 38, where the left and right side of the curve represent the saturation of the material and the line at the tip of the curve represents the state at which the capacitance behaves as a linear element.

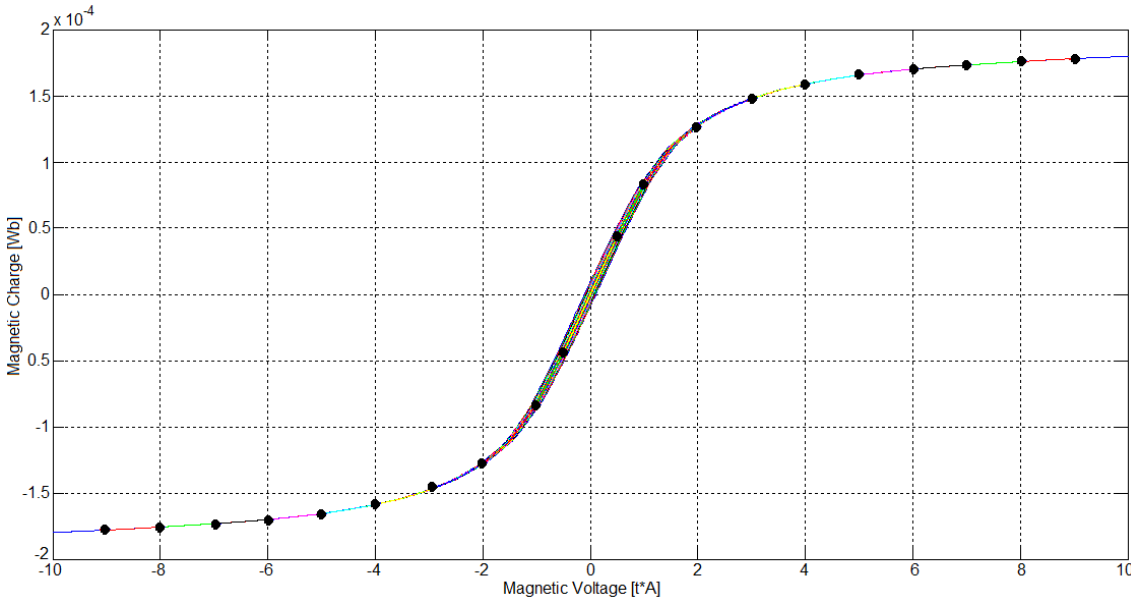


Fig. 36. Magnetic capacitance characteristic (Family of hysteresis loops at 0.5 Hz).

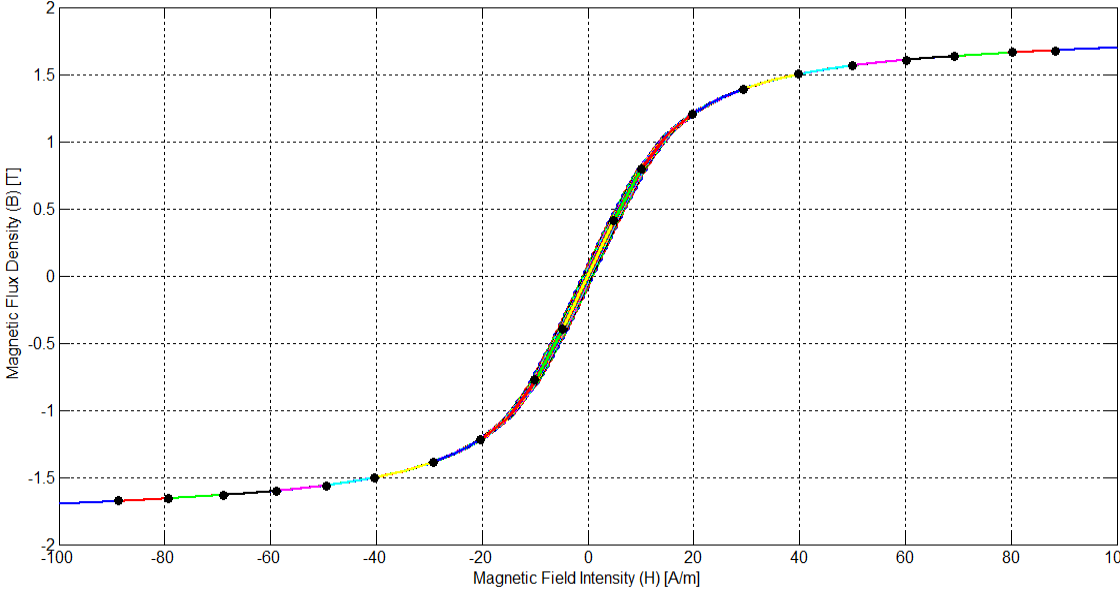


Fig. 37. Permeability characteristic (Family of hysteresis loops at 0.5 Hz).

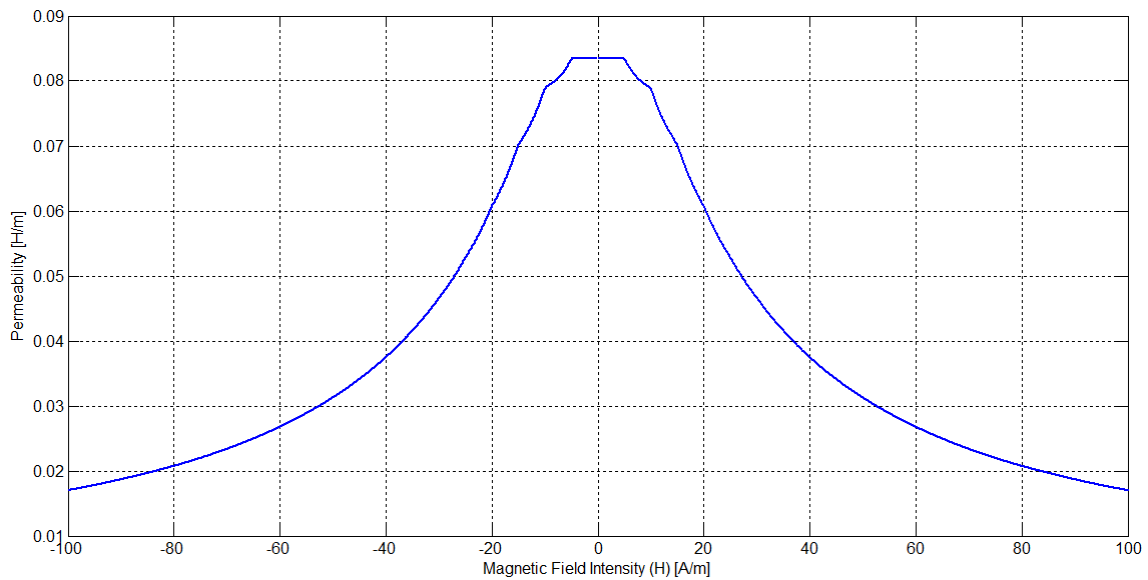


Fig. 38. Permeability vs. magnetic field intensity.

### 3.3.2. Medium Frequency Test

The second test that we will implement is exciting the model at a medium frequency. Fig. 39 shows the waveform of the input magnetic voltage and current at steady state. The voltage applied is a cosinusoidal waveform at 60 Hz with an amplitude equal to 10 t·A. In this case it is not so evident the phase shift between the magnetic current and voltage. This implies that under this condition, more than one element affects the response of the circuit. In Appendix C we present magnetic current waveforms for different amplitudes of magnetic voltage. The nonlinearity of the elements is reflected in the current as the amplitude of the magnetic voltage increases.

As we already obtained the characteristic of the capacitance we can calculate the magnetic voltage of this capacitance. From “measurement” we have the instantaneous magnetic current of the circuit and consequently the magnetic charge of the capacitance.

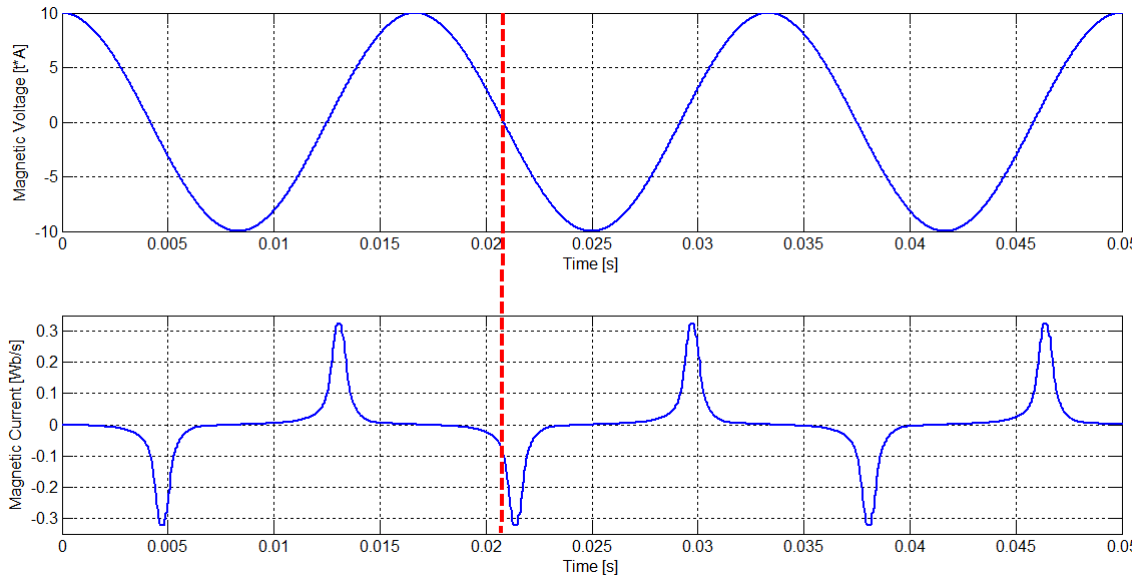


Fig. 39. Magnetic voltage and current at 60 Hz.

The values of the instantaneous magnetic charge are interpolated into the characteristic of the capacitance allowing us to find the correspondent instantaneous magnetic voltage. Once the magnetic voltage of the capacitance have been calculated, we can find the voltage of the branch containing the residual elements by,

$$v_m - v_{C_m} = v_{L_m} + v_{R_m}. \quad (121)$$

If we compare the calculated branch voltage against the magnetic current of the circuit we can see that even though the current is nonlinear, the current waveform seems in phase with the voltage waveform, as shown in Fig. 40. As we studied before the element that generates this behavior is the magnetic resistance. Therefore, we can obtain the characteristic of the magnetic resistance by plotting the branch's magnetic voltage versus the magnetic current as shown in Fig. 41. It is important to point out that because these loops involve two elements, they should display certain area. However, in this particular

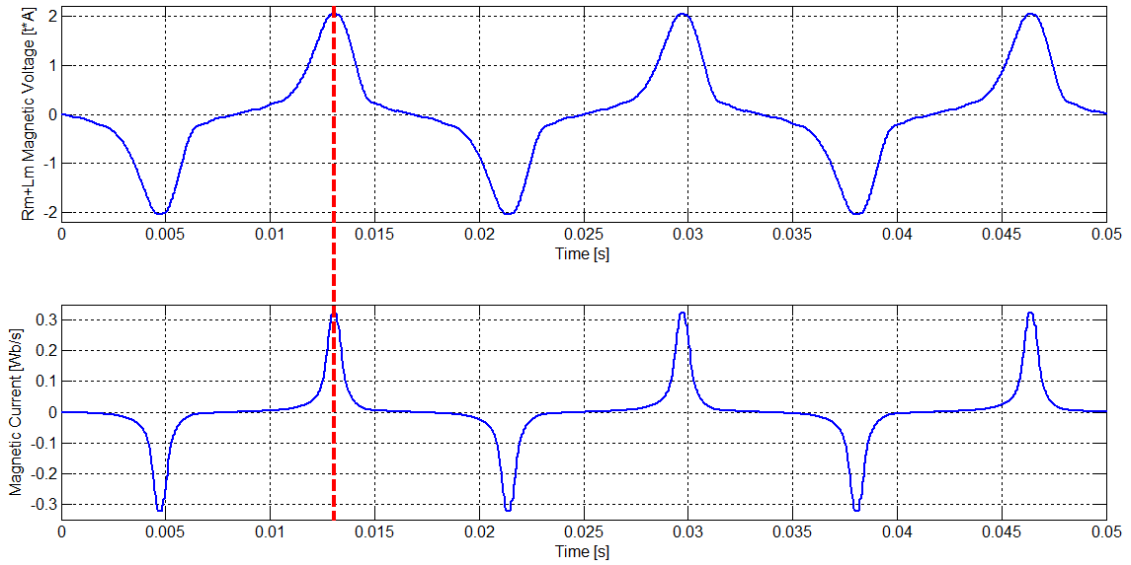


Fig. 40. Magnetic voltage at  $R_m+L_m$  branch and current at 60 Hz.

case, the reactance of the inductance is so insignificant that the voltage of the branch can be considered as the voltage of the resistance,

$$R_m \approx \frac{v_{R_m} + v_{L_m}}{i_m} = \frac{v_{R_m}}{i_m}. \quad (122)$$

Nevertheless, if the frequency of operation were higher such that an area would become visible in these loops then, the resistance characteristic could be approximated by a line connecting the tips of each loop as shown in Fig. 41.

Similar to the capacitance, the magnetic resistance is dependent on the geometry. Therefore, in order to provide a description of the material without depending on the geometry, we will present the characteristic of the resistance in terms of its magnetic resistivity as shown in Fig. 42. The magnetic resistivity is the point by point relation between the magnetic field intensity and the magnetic current density. It can also be displayed as shown in Fig. 43, where the left and right side of the curve represent the

saturation of the material and the line at the tip of the curve represents the state at which the resistance behaves as a linear element.

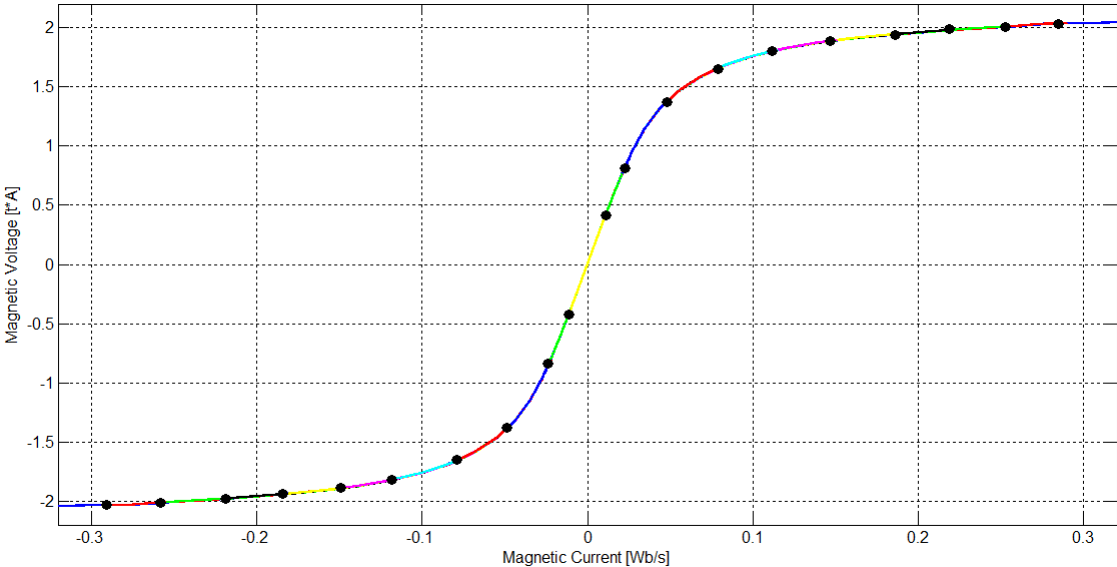


Fig. 41. Rm+Lm branch's magnetic voltage vs. current at 60 Hz.

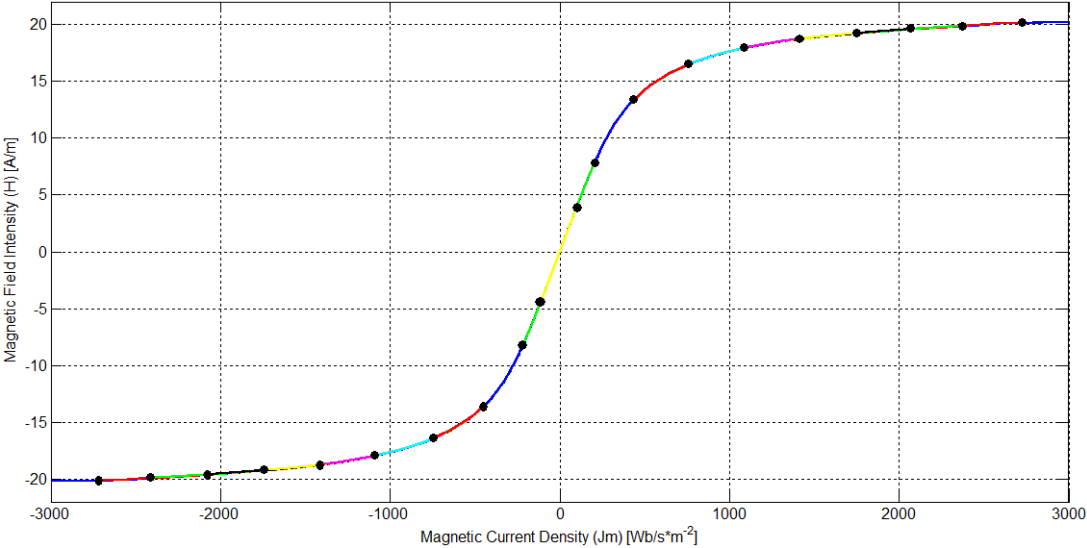


Fig. 42. Magnetic field intensity vs. magnetic current density.

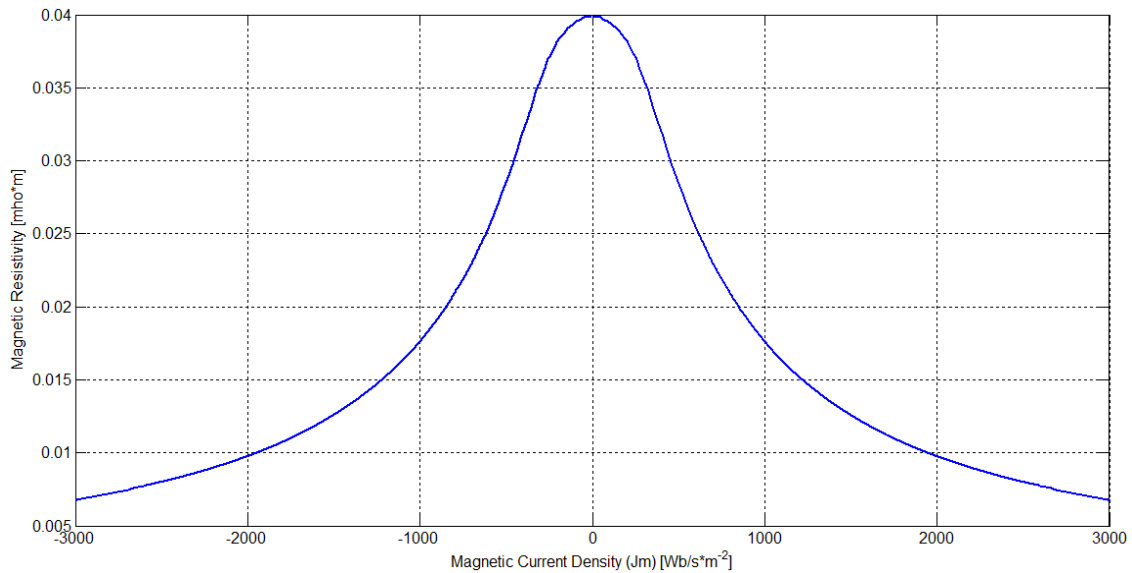


Fig. 43. Magnetic resistivity vs. magnetic current density.

### 3.3.3. High Frequency Test

The objective of this test at a high frequency is to find the magnetic inductance. According to [30], magnetic materials do possess dielectric properties therefore we must be able to characterize the magnetic inductance and consequently, its permittivity.

It has been shown that the effect of the magnetic inductance at low levels of frequency may be insignificant, depending on the material under study. However, once the frequency of the system is increased high enough such that the inductive reactance surpasses the effect of the capacitive reactance and the resistance, its effect become more significant. Fig. 44 displays the waveforms of the input magnetic voltage and the circuit's magnetic current. As we can see, the magnetic current is lagging the voltage by  $90^\circ$ . This is consistent with the inductive behavior therefore we will manipulate the input values in order to characterize the inductance.

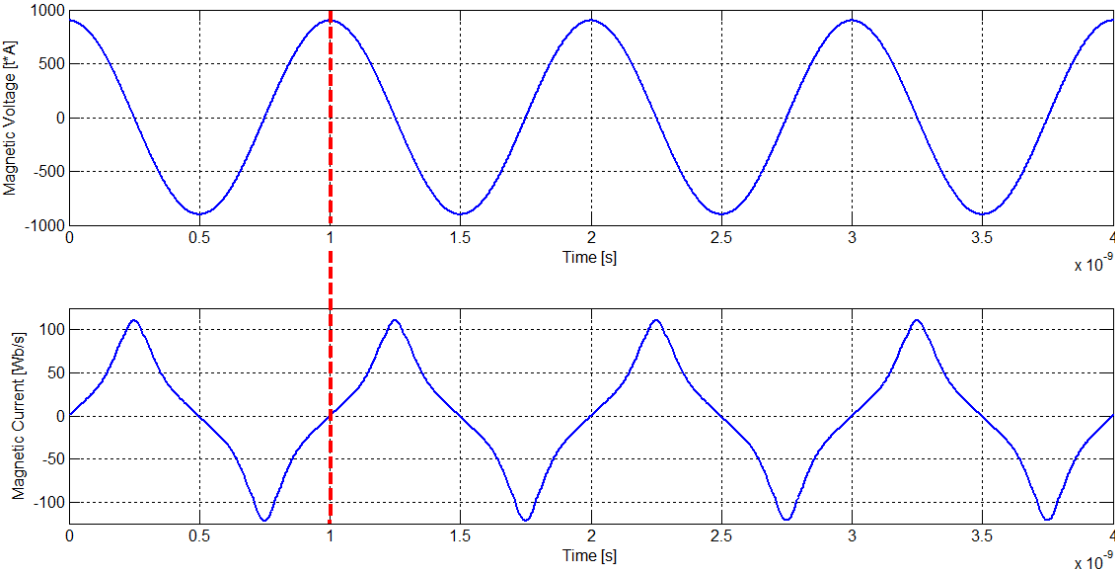


Fig. 44. Magnetic voltage and current at 1 GHz.

Equation (123) expresses that the magnetic inductance is a point by point relation between the electric charge and the magnetic current therefore if we plot this variables we can find the inductance characteristic as shown in Fig. 45.

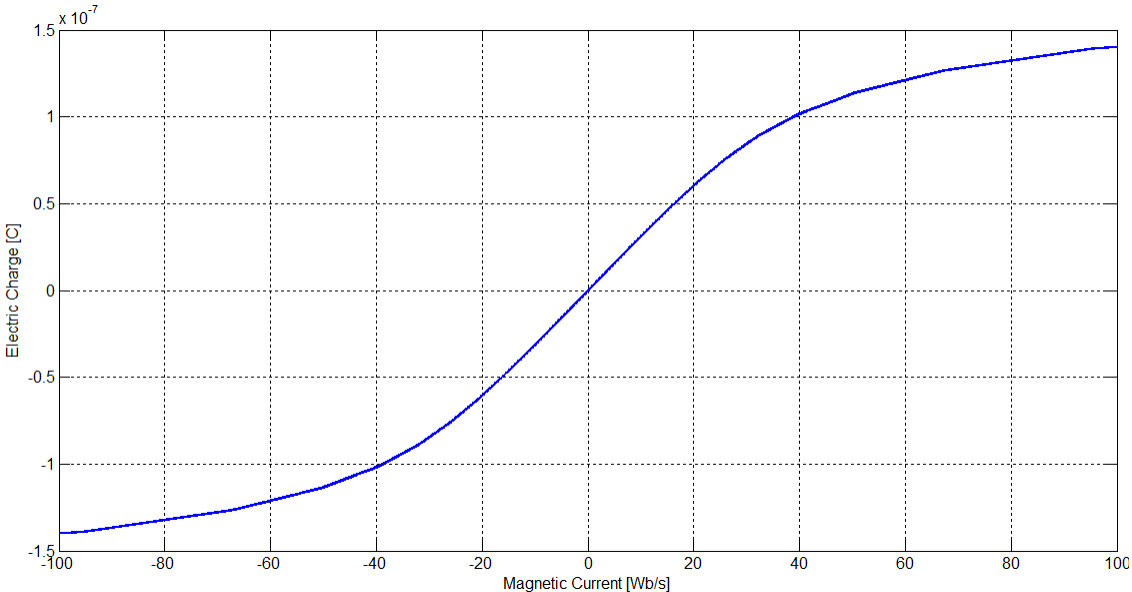


Fig. 45. Electric charge vs. magnetic current at 1 GHz.



$$L_m = \frac{Nq_e}{i_m} \quad (123)$$

In this particular case, the frequency applied is of a value such that no area is seen inside the loop. However if the loop would have shown a small area, it will mean that another element is affecting the response of the system and an approximation similar to the one that we applied to the magnetic capacitance would have been needed. This is, generating a family of loops at different amplitudes and connecting the tips of the loops.

Fig. 46 and Fig. 47 display the characteristic of the permittivity.

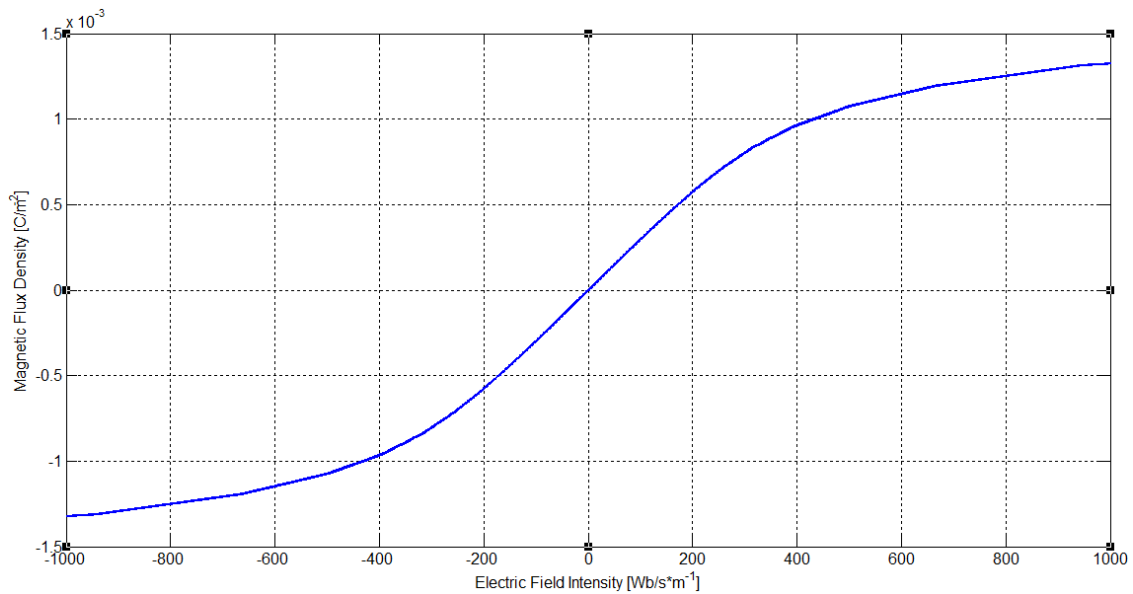


Fig. 46. Electric flux density vs. electric field intensity at 1 GHz.

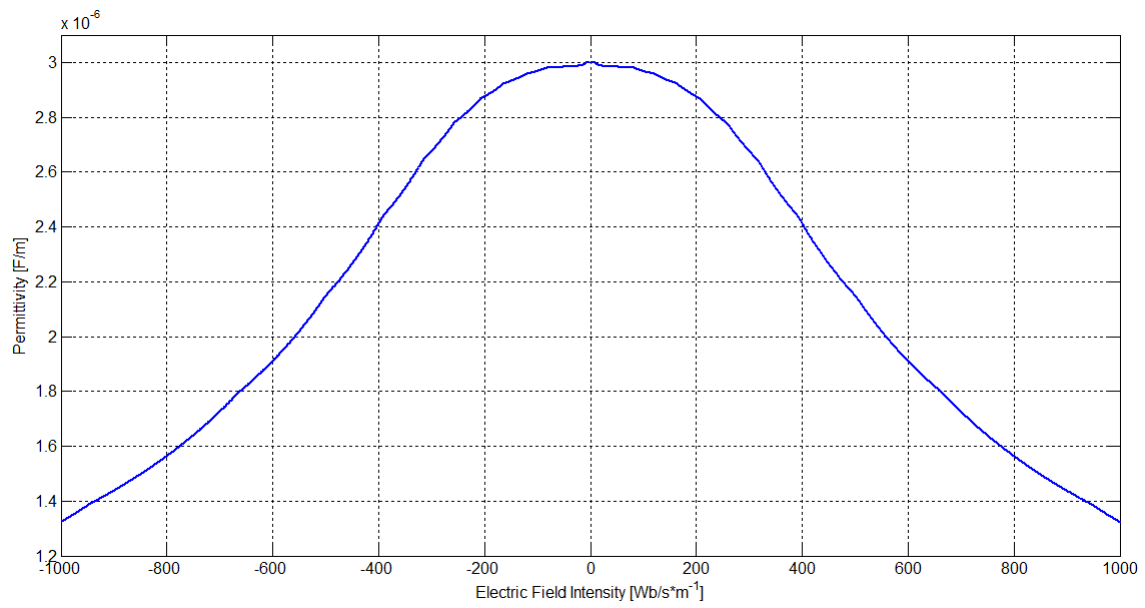


Fig. 47. Permittivity vs. electric field intensity at 1 GHz.

## CHAPTER V

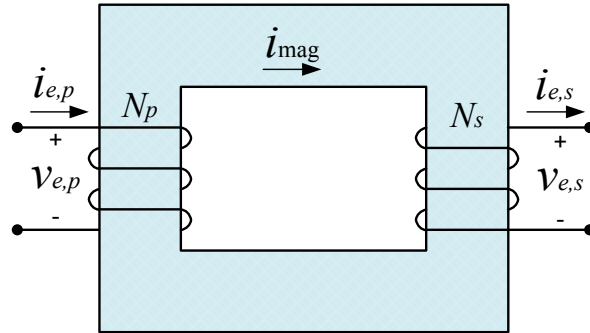
### APPLICATION OF THE POWER-INVARIANT MAGNETIC MODEL

Designing electromagnetic devices is not an easy task. There are many variables that need to be taken into consideration, even compromised, in order to achieve a satisfactory design. Commonly, the designer must make tradeoffs among size, efficiency, weight, price and type of materials before landing a suitable final design. Along the years, engineers have used various approaches in order to reach this goal. We believe that the power-invariant magnetic model is a good candidate as a first approach design tool.

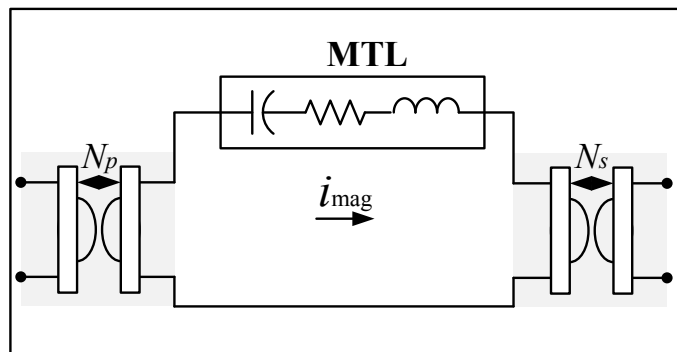
Furthermore, the similarities between the electric and the power-invariant magnetic model enlighten us to introduce analysis techniques, well understood in electric systems but novel in the magnetic ones. These techniques will allow us to perform power transfer analysis in the magnetic regime as well.

In order to demonstrate the capabilities of the power-invariant magnetic model we will design and perform steady-state analysis in a novel concept that we have denominated *magnetic transmission line*.

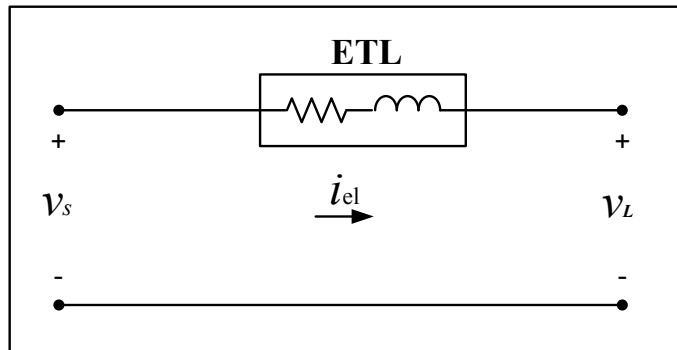
The idea of a magnetic transmission line was born after comparing the model of a short electric transmission line against our model of the transformer (see Fig. 48), which is an extension of the iron core coil model presented in the previous chapter. The objective of this study is to explore the feasibility of using a magnetic material instead of the conventional electric conductor to transport power.



a) Schematic of a transformer/MTL



b) Model of a magnetic transmission line



c) Model of an electric transmission line

Fig. 48. Comparison of the model of a magnetic transmission line and an electric transmission line.

### *A. Designing a Magnetic Transmission Line*

The design of a magnetic transmission line, similar to the design of an electric transmission line includes,

- the material of the conductor,
- the size, weight and cost of the conductor,
- the power loss and temperature rise,
- the frequency of operation,
- efficiency of the system, among others.

The power-invariant magnetic model can be used to design a magnetic transmission line because it includes most of the aforementioned factors. The advantage of this model is that it does not need families of hysteresis loops to model the behavior of the magnetic material. This model characterizes the material based on its intrinsic properties (permeability, permittivity and magnetic resistivity). Therefore, it is easier to find the geometry, frequency, etc., that provides the optimal design for a given condition of operation. However, it is important to point out that this model is still in its first stage of development, therefore some effects like copper losses, eddy current losses, fringing flux and the temperature dependence of the magnetic material, have not been included yet.

In this section we will design a magnetic transmission line using the material's properties described in the previous chapter. Simultaneously, we will perform a power transfer analysis using a Matlab®-based code that we have developed in order to solve the systems' nonlinear differential equations.

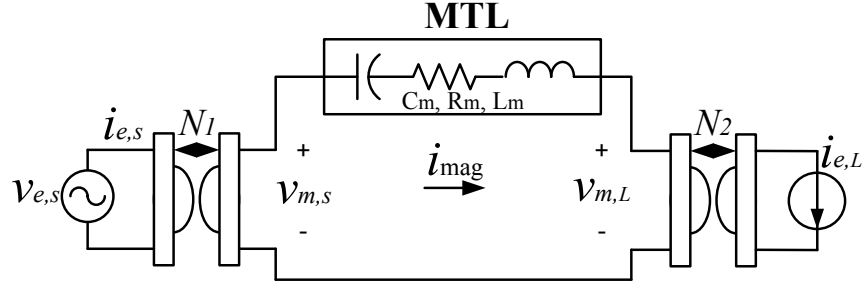


Fig. 49. Model of a magnetic transmission line.

### 1. Matlab-based Code

The code that we have designed in order to evaluate the magnetic transmission line was based on the schematic shown in Fig. 49 and is presented in Appendix D.

In the code, we first introduce lookup tables that represent each one of the intrinsic properties of the material. This is, for the permeability  $B$  vs.  $H$ , for the permittivity  $D$  vs.  $E$  and for the magnetic resistivity  $H$  vs.  $J_m$  for frequencies up to 1 kHz.

Then, we introduce the value of the voltage source and the power of the load, as they are considered known values. Using these values we find the magnetic current of the circuit ( $i_{mag}$ ) and consequently, the electric voltage of the load ( $v_{e,L}$ ) as,

$$i_{mag} = \frac{1}{N_1} v_{e,s}, \quad (124)$$

$$v_{e,L} = N_2 i_{mag}. \quad (125)$$

As we know the power of the electric load ( $P_{e,L}$ ) and the electric voltage of the load ( $v_{e,L}$ ), we can find the electric current at the load terminal ( $i_{e,L}$ ). Then we gyrate it into the magnetic regime to find the magnetic voltage at the magnetic load terminal ( $v_{m,L}$ ). This is,

$$v_{m,L} = N_2 i_{e,L}. \quad (126)$$

As the voltage drop of the magnetic transmission line depends on the characteristics of each element and these elements depend on the geometry of the specimen, we create two loops. The external loop iterates values of area while the inner loop iterates values of length. These values are introduced into the permeability, permittivity and magnetic resistivity tables in order to find the characteristic of the capacitance ( $q_{mag}$  vs.  $v_{cmag}$ ), the inductance ( $q_{elec}$  vs.  $i_{mag}$ ) and the magnetic resistance ( $v_{rmag}$  vs.  $i_{mag}$ ), respectively.

The voltage drop ( $v_{mtl}$ ) is calculated adding the voltage of each one of the elements. In order to do so, the instantaneous values of the magnetic current ( $i_{mag}$ ) are interpolated inside these elements characteristics. The result will give the resistance's magnetic voltage and the inductance's electric charge. This electric charge is differentiated in order to find the inductance's magnetic voltage. Similarly, we interpolate the instantaneous values of the integral of the magnetic current inside the capacitance characteristic in order to find the capacitance's voltage.

After the voltage drop of the magnetic transmission line is calculated, we can find the magnetic voltage at the terminal source ( $v_{m,s}$ ) adding the magnetic voltage at the terminal load and the line's voltage drop. This magnetic voltage is gyrated into the electric regime in order to find the electric current of the source,

$$i_{e,s} = \frac{1}{N_1} v_{m,s}. \quad (127)$$

After the voltages and currents of the system are calculated, we can evaluate the performance of the system using three factors, the losses which is the power dissipated

as heat, the efficiency which is a measure of the amount power transferred from the source to the load and the power factor, which is a measure of the capacity of the line to transmit power. Therefore, we include its calculations in the code as well.

The power dissipated by the magnetic resistance is the average of the instantaneous power at the resistance. This is,

$$P_{mtl} = \sum_{i=1}^n (v_{Rm} \cdot i_{mag}). \quad (128)$$

Because the model is power-invariant, the efficiency of the magnetic line ( $\eta$ ) can be calculated using the net power at either the electric or magnetic regime terminals. The efficiency is given by,

$$\eta = \frac{P_{load}}{P_{source}} \times 100\%. \quad (129)$$

Due to the nonlinearity of the system, we recommend to calculate the net power from the instantaneous power equation as,

$$P_{load} = \sum_{i=1}^n (v_{e,L} \cdot i_{e,L}) = \sum_{i=1}^n (v_{m,L} \cdot i_{mag}), \quad (130)$$

and

$$P_{source} = \sum_{i=1}^n (v_{e,s} \cdot i_{e,s}) = \sum_{i=1}^n (v_{m,s} \cdot i_{mag}). \quad (131)$$

The power factor is the ratio of the power that does work (net power) versus the total power injected into the system (apparent power). As the model is power-invariant the power factor can be calculated either at the electric or magnetic source terminal. It is calculated using,

$$PF = \frac{Net\ Power}{Apparent\ Power} = \frac{P_{source}}{S_{source}}. \quad (132)$$



The apparent power is calculated using the root mean square (RMS) values of the voltage and current waveforms. In the general case, the RMS value of a waveform can be calculated using,

$$x_{rms} = \sqrt{\frac{\sum_{i=1}^n x_i^2}{n}}. \quad (133)$$

Then, the apparent power is calculated based on,

$$S = V_{e,s(rms)} \cdot I_{e,s(rms)} = V_{m,s(rms)} \cdot I_{imag,(rms)}. \quad (134)$$

The calculation of the power factor using this method is consistent with the calculation of the power factor using the distortion factor and the displacement factor of the current waveform used in power electronics [32].

## 2. Case Study

We will design a magnetic transmission line whose source ( $V_{e,s} = 200 \text{ V}_{pk}$ ) and load ( $P_{e,L} = 1 \text{ kW}$ ) are in the electric regime as presented in Fig. 49. We will evaluate how changes in length, area, frequency of operation and number of turns affect the efficiency of the system.

Fig. 50 and Fig. 51 show the efficiency vs. core length profile of a system that is excited at 60 Hz and 1 kHz, respectively. In both cases, the primary and secondary windings ( $N_1, N_2$ ) have a value of 500 turns.

In these figures, each curve represents a different core's area. The minimum practical area (Fig. 50,  $A_{c,min} = 6 \text{ cm}^2$  and Fig. 51,  $A_{c,min} = 36 \text{ mm}^2$ ) depends on the amplitude of the magnetic charge ( $q_m$ ) and the maximum flux level of the material that we want to apply, in this case, the saturation level.

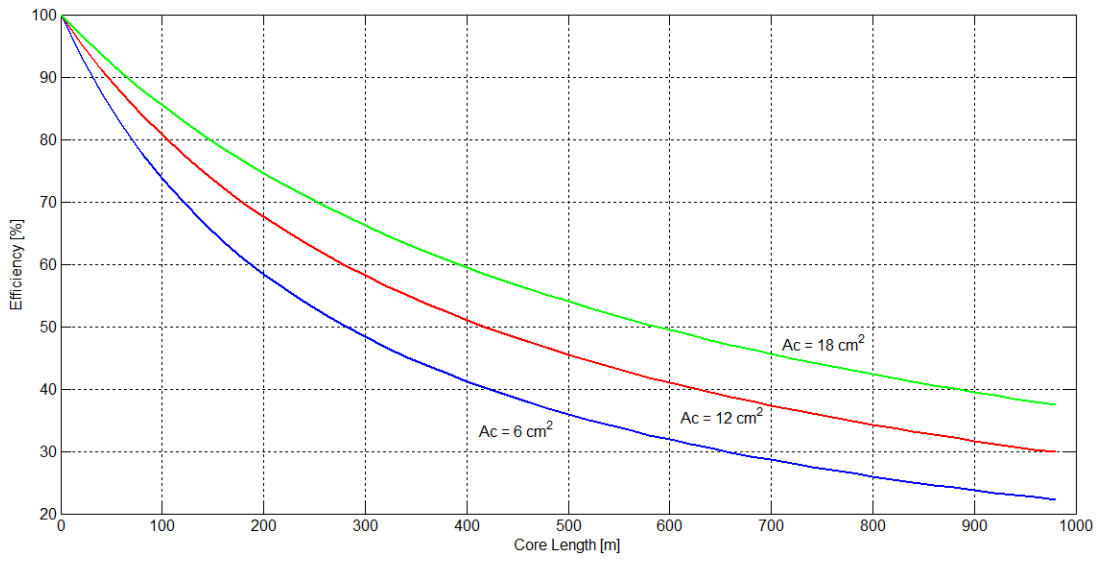


Fig. 50. Efficiency vs. core length for various areas (system at 60 Hz).

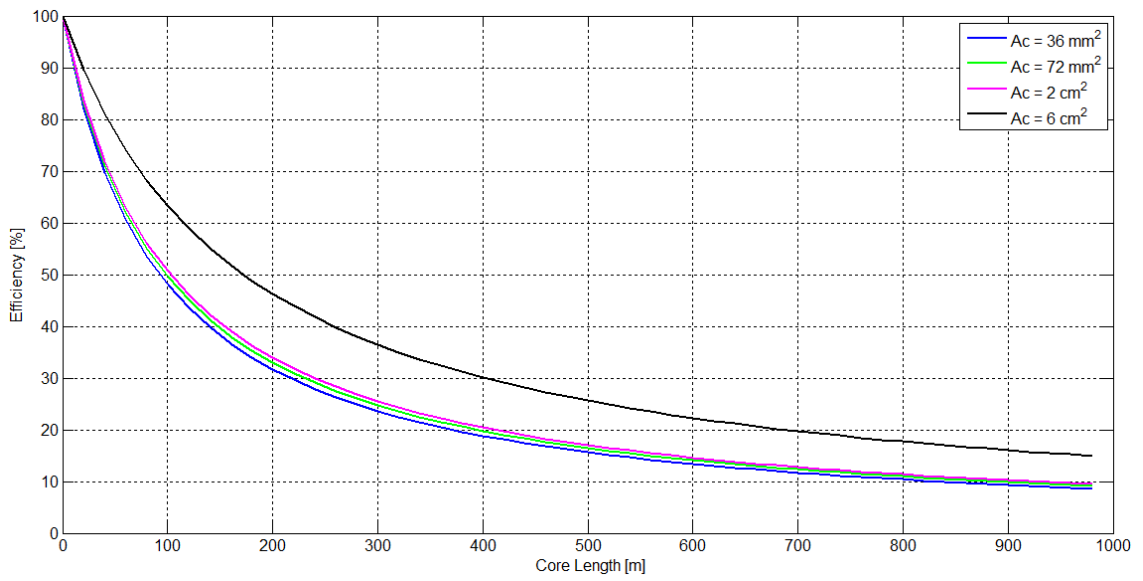


Fig. 51. Efficiency vs. core length for various areas (system at 1 kHz).

The amplitude of the magnetic charge depends on the amplitude of the magnetic current and the frequency of operation. If the amplitude of the magnetic current is fixed and the frequency increases, then the amplitude of the magnetic charge will decrease. Therefore, in order to maintain the maximum flux density, the area has to be reduced as well. This is given by,

$$i_{mag} = I_{mag} \cos(\omega t + \theta_{imag}), \quad (135)$$

$$q_{mag} = Q_{mag} \sin(\omega t + \theta_{imag}), \quad (136)$$

where,

$$Q_{mag} = \frac{1}{\omega} I_{mag}. \quad (137)$$

And as we know,

$$Q_{mag} = B \cdot A_c. \quad (138)$$

Therefore,

$$A_{c,min} = Q_{mag} / B_{sat}. \quad (139)$$

Below this value, the waveform of the magnetic voltage of the line and consequently, the electric current at the source becomes much distorted due to the nonlinearities of the line's capacitive element and it might be unpractical to use.

The advantage of increasing the frequency is that we can transfer power using a magnetic conductor of a small core area. Nevertheless, it also has the disadvantage of increasing the losses and therefore, reducing the efficiency of the system.

Another important factor that needs to be taken into consideration, when designing a magnetic line, is the amount of distortion that the waveform of the electric current is allowed to experience. For instance, Fig. 52 and 53 show the electric current

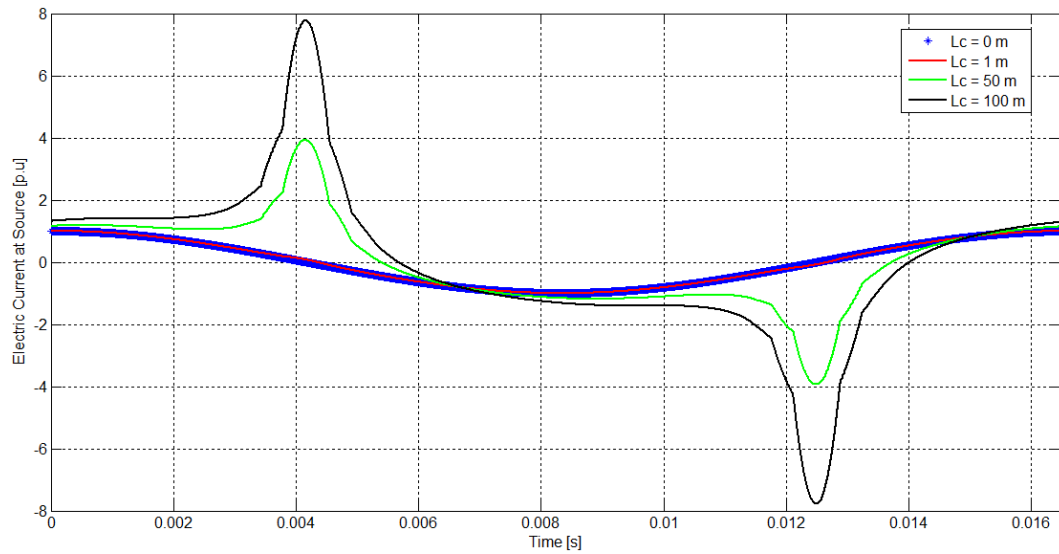


Fig. 52. Normalized electric current vs. time; conductor's core area =  $6 \text{ cm}^2$  and various core's length; frequency = 60 Hz

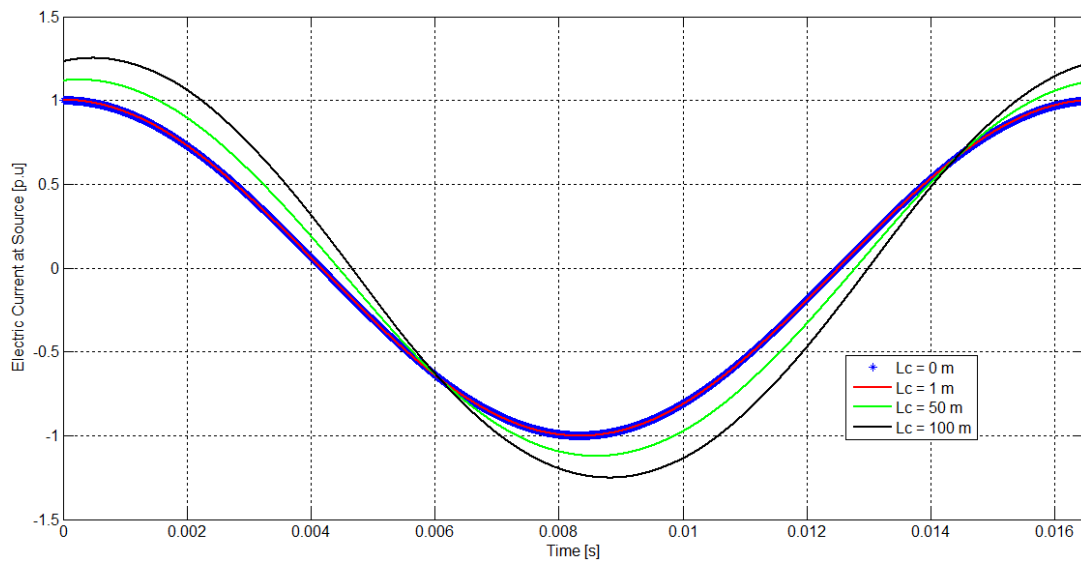


Fig. 53. Normalized electric current vs. time; conductor's core area =  $12 \text{ cm}^2$  and various core's length; frequency = 60 Hz.

waveforms of a magnetic line at 60 Hz, for different core lengths with a core area of 6 cm<sup>2</sup> and 12 cm<sup>2</sup>, respectively. The amplitude of these currents has been normalized with respect to the amplitude of the source current as if the line were ideal (depicted as blue dotted line). This is, the electric current at the source is equal to the electric current at the load.

Analyzing these figures we can see that increasing the area of the core reduces the distortion of the electric current. This happens because the magnetic flux density is reduced and the magnetic transmission line operates near the linear region of the magnetic capacitance characteristic.

The same situation is seen in Fig. 54 and 55 which show the electric current waveforms of a magnetic line at 1 kHz, for different core lengths with a core area of 36 mm<sup>2</sup> and 2 cm<sup>2</sup>, respectively. The amplitude of these currents has been normalized with respect to the amplitude of the source current similar to the previous frequency.

As the distortion of the electric current is dependent on the nonlinearity of the elements of the magnetic transmission line, the power factor is a measure of the distortion of the electric current and consequently, it describes the capacity of the magnetic line to transmit power. Table VIII, presents the power factor of the system which currents are presented in Fig. 52 – Fig. 55.

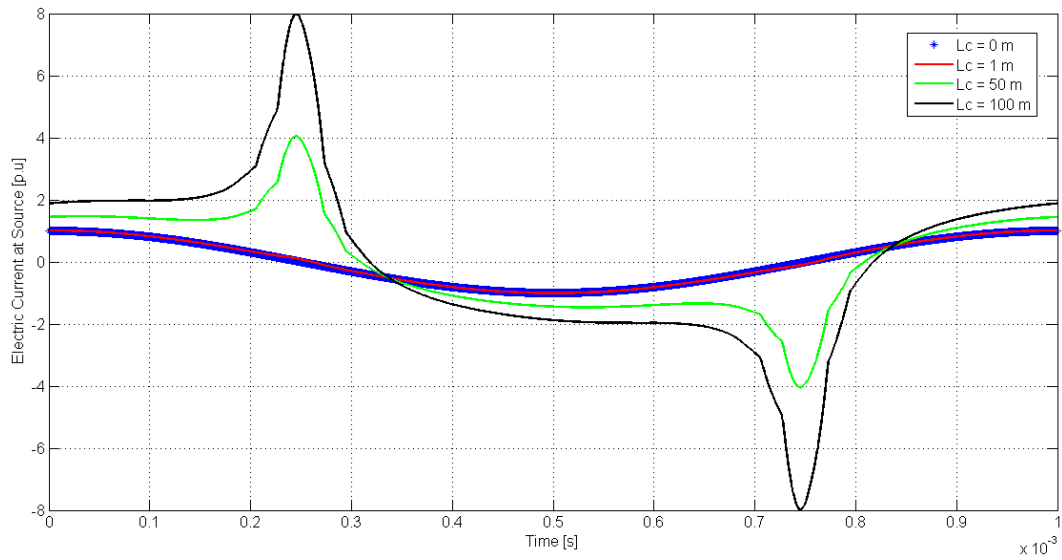


Fig. 54. Normalized electric current vs. time; conductor's core area =  $36 \text{ mm}^2$  and various core's length; frequency = 1 kHz.

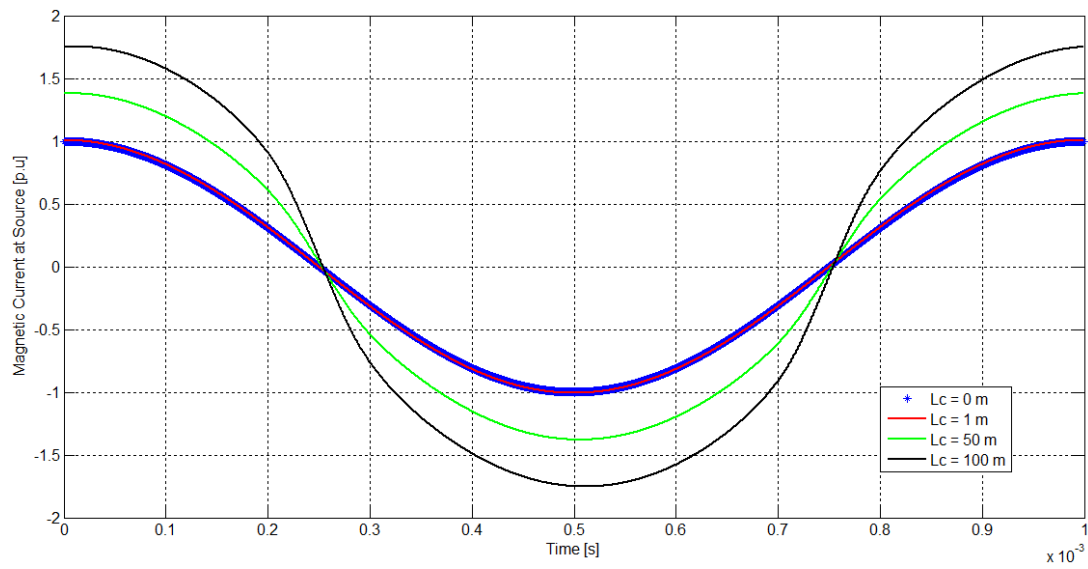


Fig. 55. Normalized electric current vs. time; conductor's core area =  $2 \text{ cm}^2$  and various core's length; frequency = 1 kHz.

TABLE VIII  
POWER FACTOR DUE TO DIFFERENT FREQUENCIES AND GEOMETRIES

Core's Length [m]	$f = 60 \text{ Hz}$		$f = 1 \text{ kHz}$	
	$A_c = 6 \text{ cm}^2$	$A_c = 12 \text{ cm}^2$	$A_c = 36 \text{ mm}^2$	$A_c = 2 \text{ cm}^2$
0	1.00	1.00	1.00	1.00
1	0.99	1.00	0.99	1.00
50	0.58	0.99	0.68	0.99
100	0.38	0.98	0.54	0.99

After evaluating the efficiency, the waveform of the input electric current and the power factor of the system we can choose a geometry and frequency of operation for our magnetic transmission line. This is,

- Frequency of operation = 1 kHz
- Core's length = 25 m
- Core's area =  $36 \text{ mm}^2$

This geometry, using 500 turns in the primary and secondary windings, yield to an efficiency of 78.77% and a power factor of 0.84. If we want to increase the efficiency of the system, we can do so by increasing the number of turns of the windings. This tactic improves the efficiency because an increase in the number of turns reduces the magnetic current, consequently reducing the magnetic losses. We understand that increasing the number of turns increases the electric copper losses however these losses are out of the scope of this analysis. If we tripled the number of turns in both windings, the efficiency rises to 92.5% and the power factor improves to 0.99. This means that the reactive effect of the line is almost imperceptible.

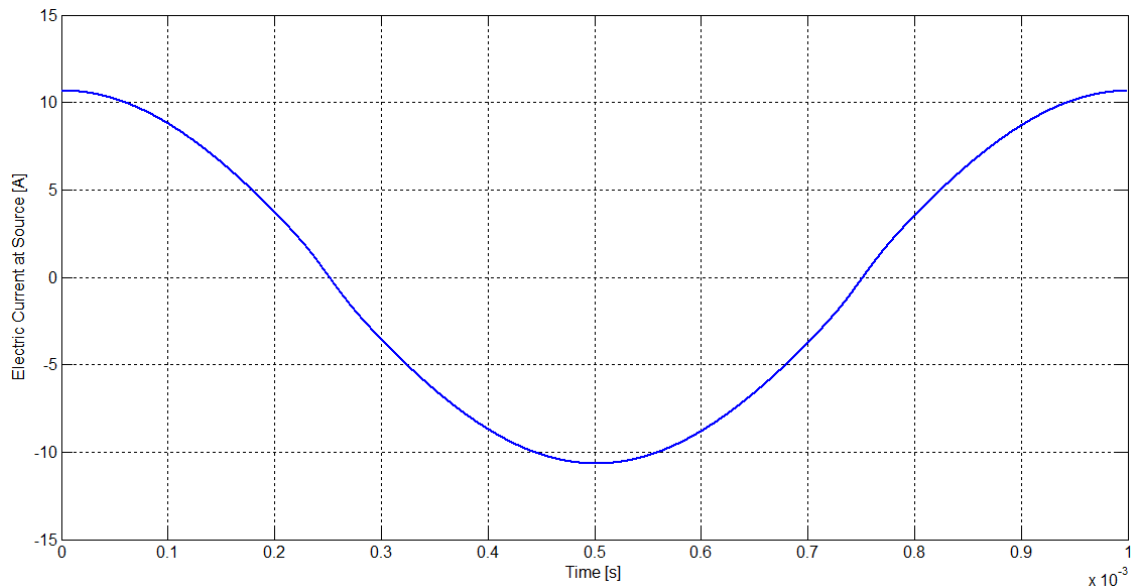


Fig. 56. Electric current ( $i_{e,s}$ ) vs. time.

The electric current at the source side is presented in Fig. 56. The magnetic current and voltages waveform of the magnetic transmission line are presented in Fig. 57 and Fig. 58.

It is important to point out that even though in this case we have chosen a design where the magnetic transmission line possess the same frequency that the rest of the system, we have to evaluate the possibility of using power electronics at the electric source and load side of the magnetic transmission line in order to manipulate the magnetic excitation of the line to seek maximum power transfer.



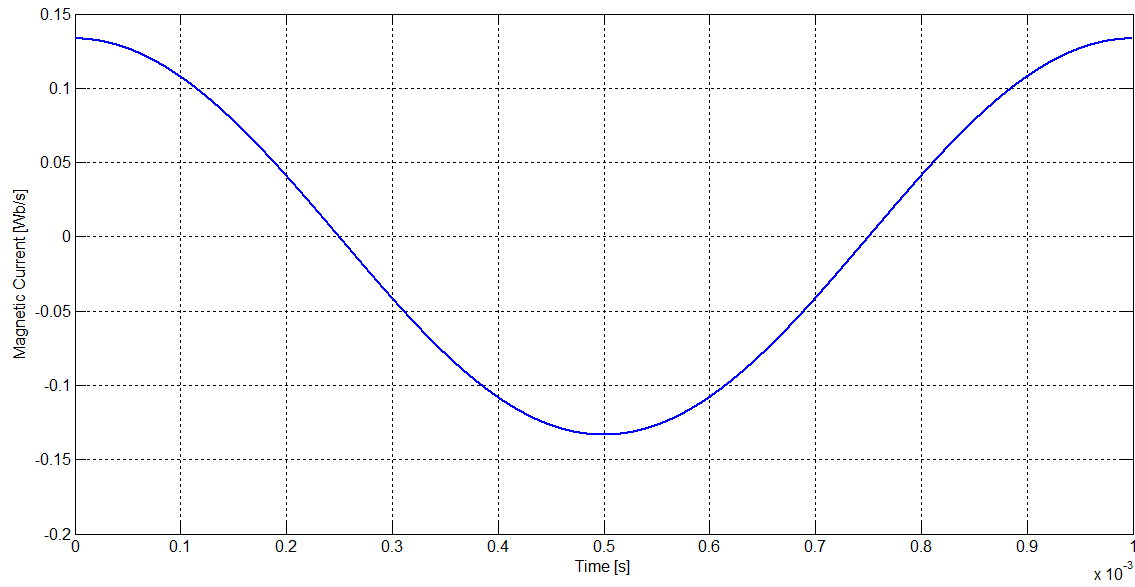


Fig. 57. Magnetic current ( $i_{mag}$ ) vs. time.

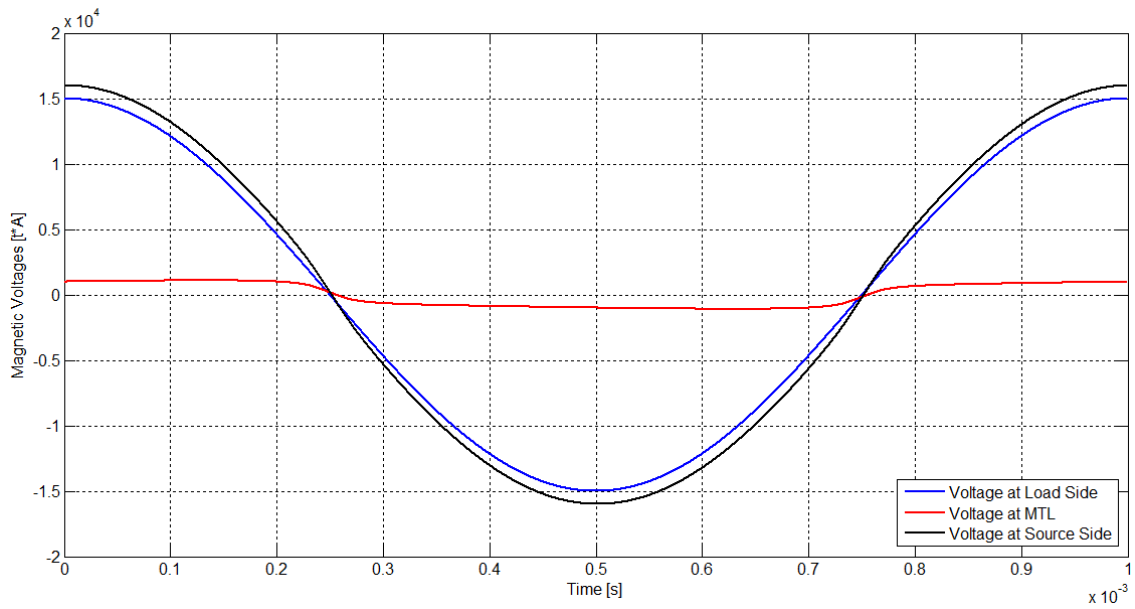


Fig. 58. Magnetic voltages vs. time.

### *2.1. Comparison between a Magnetic and an Electric Conductor*

If we compare the area of the magnetic conductor used in this case study to transmit 1 kW of power,  $A_{c,m} = 36 \text{ mm}^2$ , with an electric conductor used to transmit the same amount of power at 60 Hz. This is an AWG 14 wire with an ampacity up to 20 A and an area,  $A_{c,e} = 2 \text{ mm}^2$ . The magnetic conductor is 18 times bigger than its electric counterpart.

The electric resistance of an AWG 14 wire of 25 m length is approximately 207.15 m $\Omega$ . This will result in copper losses of approximately 10.4 watts which will yield to an efficiency of 98.97%. If we compare this value to the efficiency of the magnetic transmission line system, 92.5%, this implies that transmitting power through an electric media is 1.07 times better than doing so in the magnetic regime.

Even though this comparison suggests that is better to transmit power in the electric regime rather than in the magnetic regime, we believe that the importance of this study lies in the fact that we have established the possibility of transferring power using a magnetic media and also, we have supplied the tools to design and analyze this type of system.

## CHAPTER VI

### SUMMARY AND FUTURE RESEARCH

In this chapter we present the summary of the work accomplished during the course of this research. Also, we establish the contributions of this work and provide some recommendations for future research in this area of study.

#### *A. Summary*

In this dissertation, we have presented a model for magnetic systems that is standard with other regimes models, for instance, electric, mechanic and hydraulic systems. This model has been based on the principle of conservation of energy and consists in describing the energy transfer process in terms of the effort, flow and three elements that describe how the energy is dissipated or stored during the energy transfer process. We have defined these terms for the magnetic regime and used them to model a simple magnetic system, an iron core coil.

As in the iron core coil case study, the process occurs solely in the magnetic material we represented it as a magnetic network composed of three elements, magnetic resistance, capacitance and inductance. However, due to the geometry dependence of these elements and aiming to give a general description of the material, we decided to better describe the magnetic material in terms of the elements' intrinsic properties, magnetic resistivity, permeability and permittivity. Therefore, we have developed and presented a methodology based on a conceptual experiment in order to find the aforementioned material's intrinsic properties.

In order to analyze the iron core coil, we developed analytical and numerical techniques that solved the network's differential equations. Also, we presented the equations used to calculate the power dissipated and the energy stored in the material.

Finally, after evaluating the analysis techniques that this model offers to the modeling of magnetic systems and observing the similarities that it has compared to the electric model, we explored the possibility of transmitting power through the magnetic media using a magnetic transmission line analogue to transmitting power using an electric transmission line.

In Chapter I we addressed the importance of magnetic materials in modern technologies and described how researches in manufacturing and modeling techniques have improved during the years. We presented a series of numerical techniques currently used to model different systems and gave reasons that support why the use of lumped parameters is still a valid option. Also, in this chapter we described three magnetic models that use lumped parameters, these are, the reluctance model, the permeance-capacitor model and the gyrator capacitor model. Finally, we established the problem that we want to solve with this work, the research objectives and the organization of the dissertation.

In Chapter II we included a literature review of electromagnetic theory in order to show the close relation that exists between the electric and magnetic regimes. Also, we presented a description of the different types of magnetic materials, with an especial interest in the properties of ferromagnetic materials as they are the most common magnetic material used in modern applications.

In Chapter III, we presented a description of all the quantities used in our model. Their definitions were based on an analogy between the electric and magnetic regimes.

In Chapter IV we presented how the power-invariant magnetic model can be used to model an iron core coil for either the ideal or real case. A conceptual test was designed in order to find the elements' network configuration that represents the behavior of a real iron core coil. We described the methodology that can be applied to find the magnetic elements based on a conceptual experiment designed in Simulink®. This conceptual experiment was also used to validate our model.

In Chapter V we presented the application of the power-invariant magnetic model. We explored the idea of transmitting power in the magnetic regime using a magnetic transmission line. We designed a case study system and chose the optimal design based on the efficiency of the system and on the level of distortion of the inrush current. We compared the resultant magnetic conductor to an electric one that would have been used to transmit the same amount of power yielding to the conclusion that an electric transmission line is better than a magnetic transmission line in terms of size of conductor and efficiency. Nevertheless, our objective was to demonstrate that the transfer of power through a magnetic media was possible and we presented an alternative to be considered.

### *B. Contributions*

The contributions of the work presented in this dissertation to the state of art can be summarized as follows:

- We have developed a magnetic model, the power-invariant magnetic model, which is standard to other energy regime models. This is important for many reasons. First, following the principle of conservation of energy, all energy systems should behave in a similar pattern during an energy transfer process (some energy is dissipated and some energy is stored to do work later) therefore, new insight and even a different prospective can be achieved to understand magnetic systems by comparing it to other regimes. Second, the same analytical and numerical techniques can be applied across regimes in order to solve systems' differential equations, for instance, when working in the linear region of the characteristics of the magnetic material we could use phasors or Laplace transformations in order to solve the system's equations, this is not common in magnetic system analysis. Third, now we have created a model where we are able to calculate power and energy for magnetic systems in a simpler and more traditional way, using the power equation.
- We have modeled the characteristic of a magnetic material, its hysteresis loops, based on the network configuration and values of the intrinsic properties of the magnetic elements presented in Chapter IV. This particular contribution is very important because modeling hysteresis loops for different frequencies and different amplitudes is not an easy task. Furthermore, nowadays in order to design a magnetic system and perform a dynamic analysis engineers use families of hysteresis loops in order to do the job. This yields in the use of tons of data which can be tedious and confusing for the

engineer and time consuming even when processing the data using a computer. Our approach seeks to reduce the amount of data that we have to manipulate when modeling a magnetic material.

- We presented a basic case study where we analyzed the feasibility of using the magnetic material as a power transfer media. It has been called magnetic transmission line analogous to an electric transmission line and it is an innovative concept because to our knowledge, the magnetic media has not been used specifically for this purpose. It provides an alternative to transfer power in cases where the electric media might not be safe to use or where the magnetic material provides a more efficient transfer of power than the electric one.

### *C. Future Research*

After culminating the research presented in this dissertation, we have identified numerous lines of work related to this topic, for instance:

- Identify the magnetic resistivity, permeability and permittivity of different magnetic materials. In order to do so, we will need to design a test bench where we could recreate the methodology proposed in this dissertation.
- Include the effect of flux leakage, fringing flux and air gaps into the model's network configuration, in order to provide an even more realistic model to magnetic systems.
- Use the power-invariant magnetic model to model other more complex magnetic systems, for instance, solenoids, transformers, motors, etc.,

- Expand the use of this model from lumped parameters to a distributed model based on lumped parameters, similar to the magnetic equivalent circuit (MEC) model [33]. The difference lies in the fact that current MEC models use the reluctance as base element, in our case, we will need to include the three magnetic elements. However, we will need to explore the advantages and disadvantages of expanding the lumped model into a distributed model.
- Keep exploring the feasibility of using the magnetic transmission line to transport power, evaluating the possibility of developing a complete power system in the magnetic regime. This is using magnetic sources and loads.



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## APPENDIX A

### IRON-CORE COIL MODEL

This appendix presents in detail, the model for the iron-core coil developed in Simulink. We programmed our own numerical tool that solves the system's nonlinear differential equations in case the user does not possess Simulink. This code is Matlab-based and is designed to solve the system's differential equations for either a current or voltage excitation. Both programs give the same steady-state results. Therefore, in order to prove it, we compare the system's magnetic current, charge and voltages of the elements of both methods.

#### *A. Simulink's model*

As shown in Fig. A.1, this model mainly consists of the voltage source and three

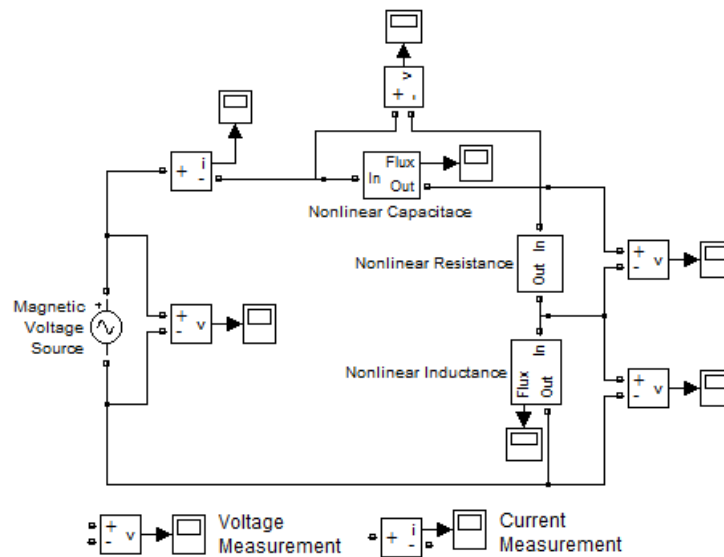


Fig. A.1. Simulink model using nonlinear elements.

subsystems that represent the nonlinear capacitance, resistance and inductance. It also contains voltage and current measurements.

Fig. A.2, Fig. A.3 and Fig. A.4 presents the capacitance, resistance and inductance subsystems, respectively.

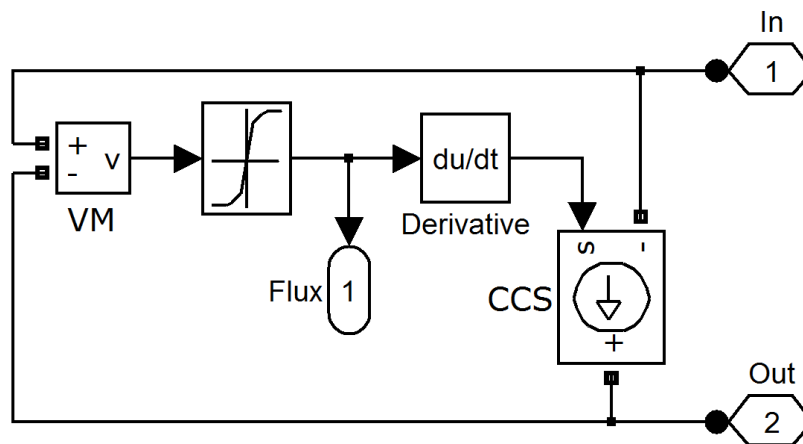


Fig. A.2. Magnetic capacitance subsystem.

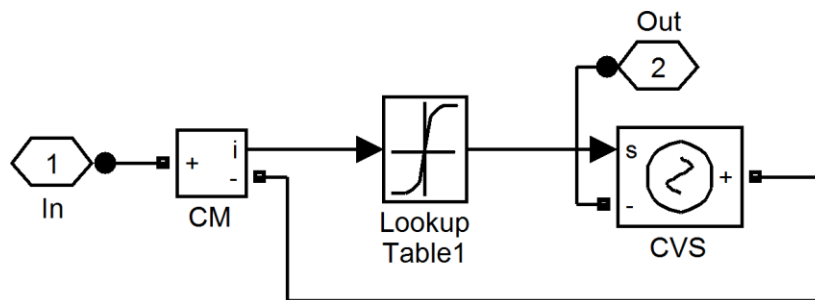


Fig. A.3. Magnetic resistance subsystem.

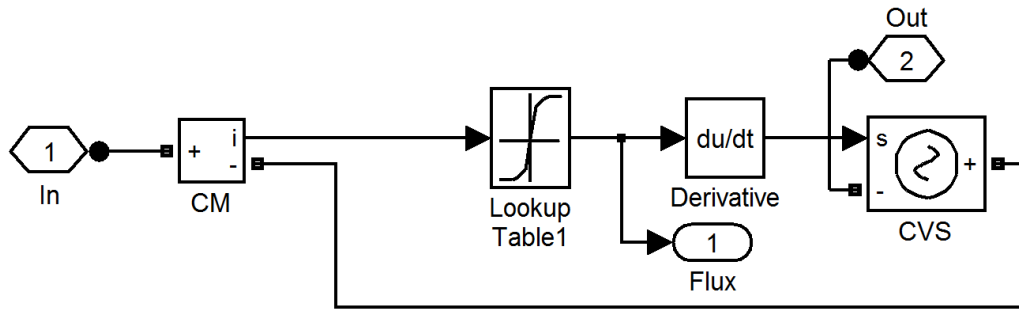


Fig. A.4. Magnetic inductance subsystem.

B. Matlab-based Code

Fig. A.5 presents the block diagram of the code. As you can see, we first

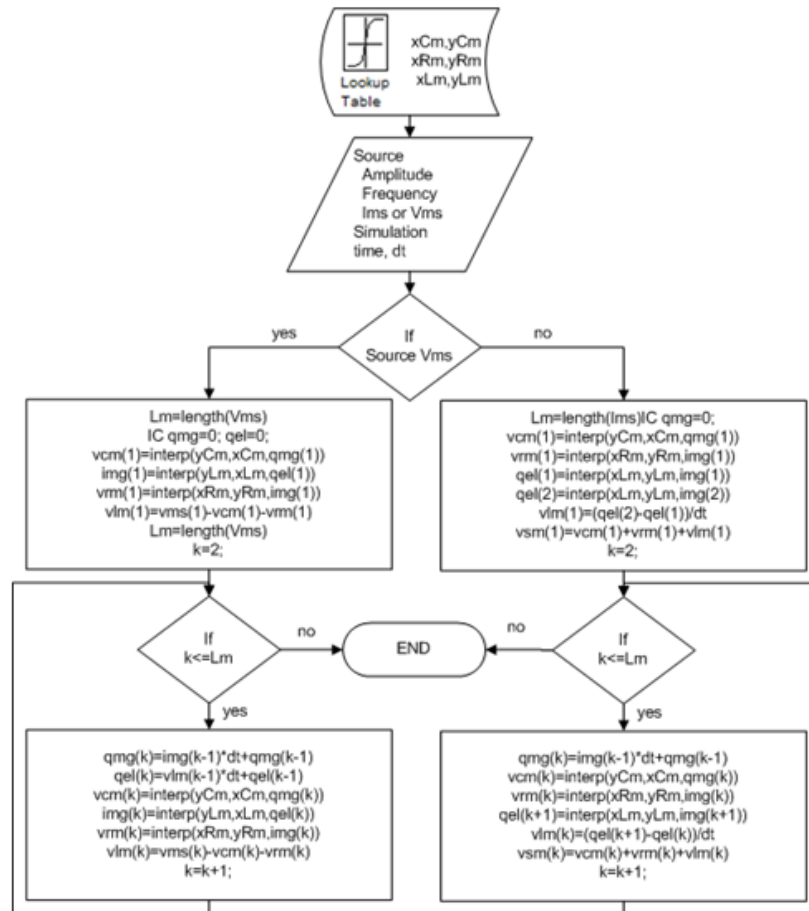


Fig. A.5. Matlab-based code block diagram.

introduce the values correspondent to the elements' data and the source's information. Then, we proceed with the solution of the system. The solving method varies depending on the type of source.

```

%%Global variables
f=60; %Frequency [Hz]
w=2*pi*f; %Angular frequency [rad/s]
Fmax=100; phv=0; %Max.Magnetic effort [t*A]
Imax=600; phi=0; %Max.Magnetic current [Wb/s]
dwt=pi/1e4; dt=dwt/w; %Increments
wt=0:dwt:4*pi-dwt; %Period [rad]
t=wt/w; %Time [s]
Ac=1; %Core's area [m^2]
Lc=1; %Core's length [m]
SOURCE=1; %Definition of source type

%%Elements data
data=importdata('C:\Research\DEq\Parameters2.mat');

if f<100 %Elements data, f<100 Hz
    xC=data.xC; yC=data.yC;
    xR=data.xR; yR=data.yR;
    xL=data.xL; yL=data.yL;
elseif f>100 && f<500 %Elements data, 100<f<500 Hz
    xC=data.xC; yC=data.yC;
    xR=data.xR; yR=data.yR*1.5;
    xL=data.xL; yL=data.yL;
else %Elements data, f<1000 Hz
    xC=data.xC; yC=data.yC;
    xR=data.xR*1.5; yR=data.yR*2;
    xL=data.xL; yL=data.yL;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data improvement
xC=smooth(xC,10,'moving');
yC=smooth(yC,10,'moving');
xR=smooth(xR,10,'moving');
yR=smooth(yR,10,'moving');
xL=smooth(xL,10,'moving');
yL=smooth(yL,10,'moving');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if SOURCE==1 %Loop for voltage source
    mmf=Fmax*cos(wt+phv);
    Lm=length(mmf);
    %%Initial conditions
    qmg(1)=0; flm(1)=0;
    %%%%%%%%%%
    vcm(1)=interp1(yC,xC,qmg(1),'linear','extrap');
    img(1)=interp1(yL,xL,flm(1),'linear','extrap');
    vrm(1)=interp1(xR,yR,img(1),'linear','extrap');

```

```

vlm(1)=mmf(1)-vcm(1)-vrm(1);
for k=2:1:Lm
    flm(k)=vlm(k-1)*dt+flm(k-1);
    img(k)=interp1(yL,xL,flm(k),'linear','extrap');
    qmg(k)=img(k)*dt+qmg(k-1);
    vcm(k)=interp1(yC,xC,qmg(k),'linear','extrap');
    vrm(k)=interp1(xR,yR,img(k),'linear','extrap');
    vlm(k)=mmf(k)-vcm(k)-vrm(k);
end
elseif SOURCE==2 %Loop for current source
    img=Imax*cos(wt+phi);
    Lm=length(img);
    %%Initial conditions
    qmg(1)=0;
    vcm(1)=interp1(yC,xC,qmg(1),'linear','extrap');
    vrm(1)=interp1(xR,yR,img(1),'linear','extrap');
    flm(1)=interp1(xL,yL,img(1),'linear','extrap');
    flm(2)=interp1(xL,yL,img(2),'linear','extrap');
    vlm(1)=(flm(2)-flm(1))/dt;
    mmf(1)=vcm(1)+vrm(1)+vlm(1);
    %%%%%%%%%%
    for k=2:1:Lm-1
        qmg(k)=img(k-1)*dt+qmg(k-1);
        vcm(k)=interp1(yC,xC,qmg(k),'linear','extrap');
        vrm(k)=interp1(xR,yR,img(k),'linear','extrap');
        flm(k+1)=interp1(xL,yL,img(k+1),'linear','extrap');
        vlm(k)=(flm(k+1)-flm(k))/dt;
        mmf(k)=vcm(k)+vrm(k)+vlm(k);
    end
    qmg(Lm)=qmg(2); flm(Lm)=flm(2); mmf(Lm)=mmf(2);
    vcm(Lm)=vcm(2); vrm(Lm)=vrm(2); vlm(Lm)=vlm(2);
end
clc

```

Note: We need to point out that the solution of the system during the current source loop will diverge if the initial value of current produces a charge above the saturation level specified by the capacitance characteristic. In order to solve this, the user can modify the value of the current or the geometry of the system until the charge reaches the limits mentioned.

### C. Comparison of Simulink's and our Matlab-based code system response.

This comparison was made assuming that the system has a cosinusoidal magnetic voltage source of 100 V<sub>pk</sub> at 60 Hz. The elements have the nonlinear intrinsic properties shown in Fig. 21, 22 and 23 and the geometry of the core is unitary. This is  $A_c = 1 \text{ m}^2$  and  $L_c = 1 \text{ m}$ .



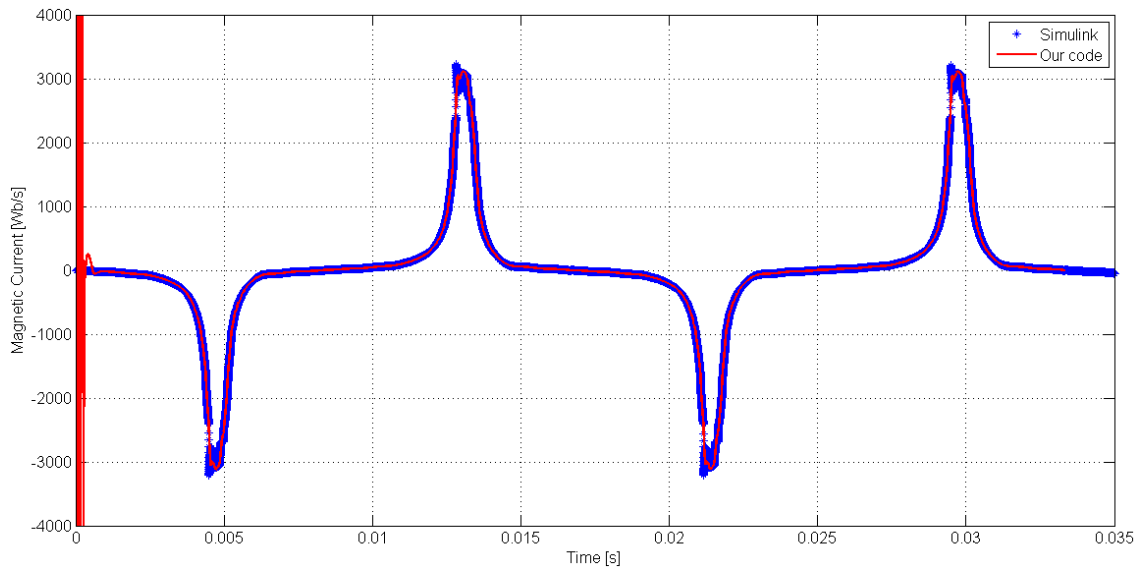


Fig. A.6. Magnetic current vs. time.

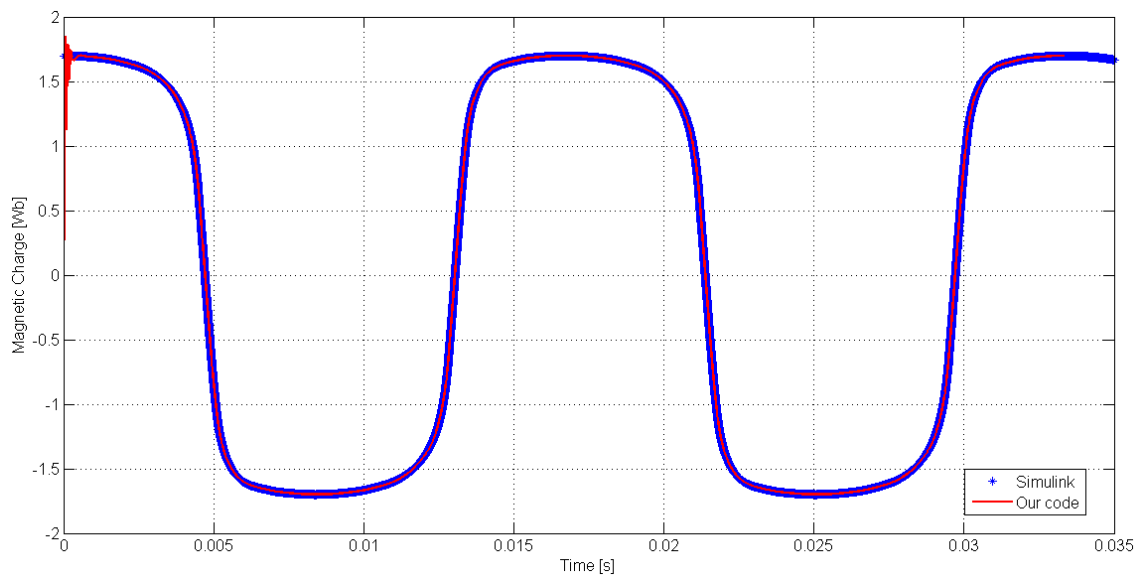


Fig. A.7. Magnetic charge vs. time.

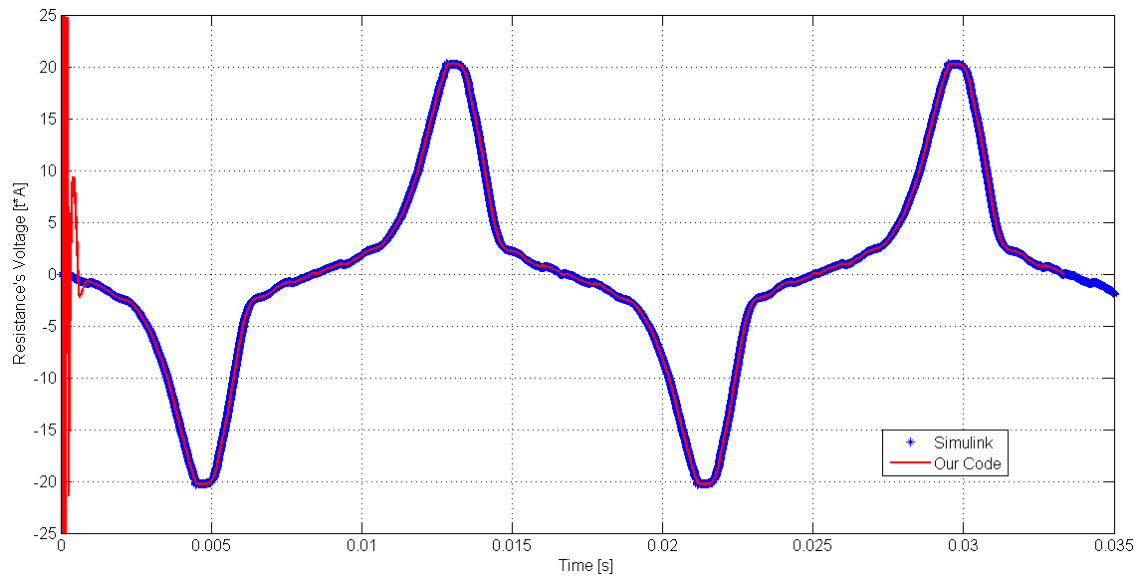


Fig. A.8. Resistance's magnetic voltage vs. time.

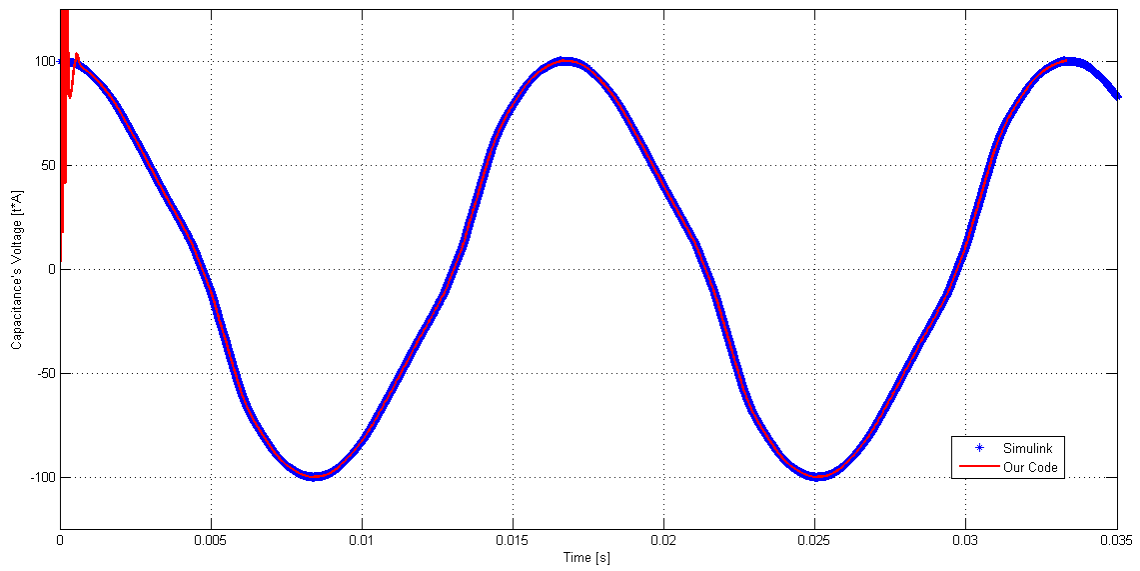


Fig. A.9. Capacitance's magnetic voltage vs. time.

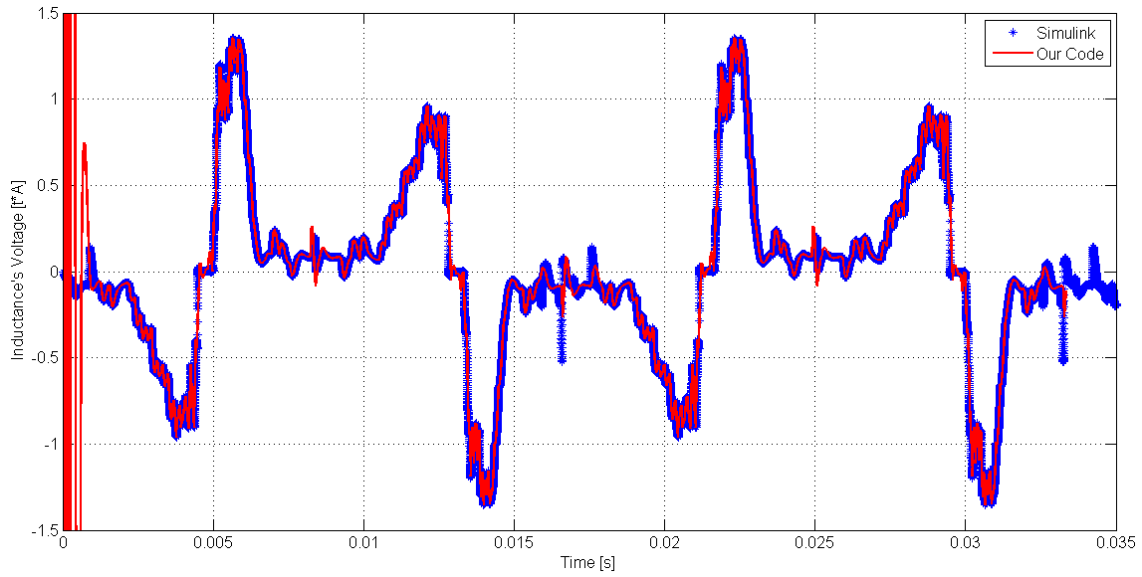


Fig. A.10. Inductance's magnetic voltage vs. time.

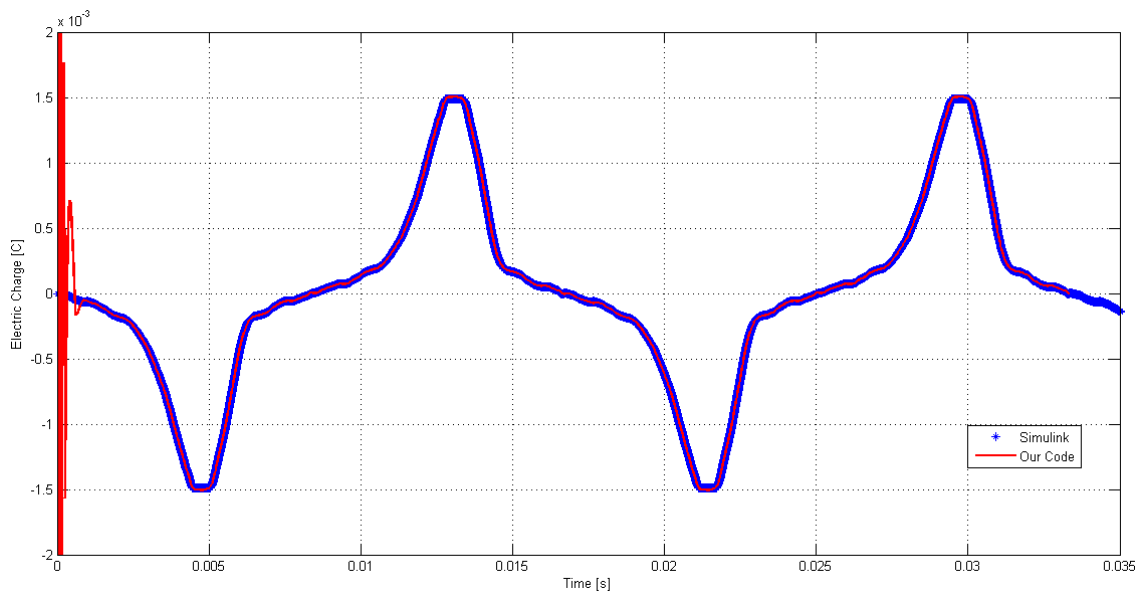


Fig. A.11. Inductance's electric charge vs. time.

## APPENDIX B

## FUNDAMENTALS OF THE TEST: NONLINEAR CASE

This appendix is the extension of the Chapter IV Section 3.1, Fundamentals of the Test.

- *Resistance Case*

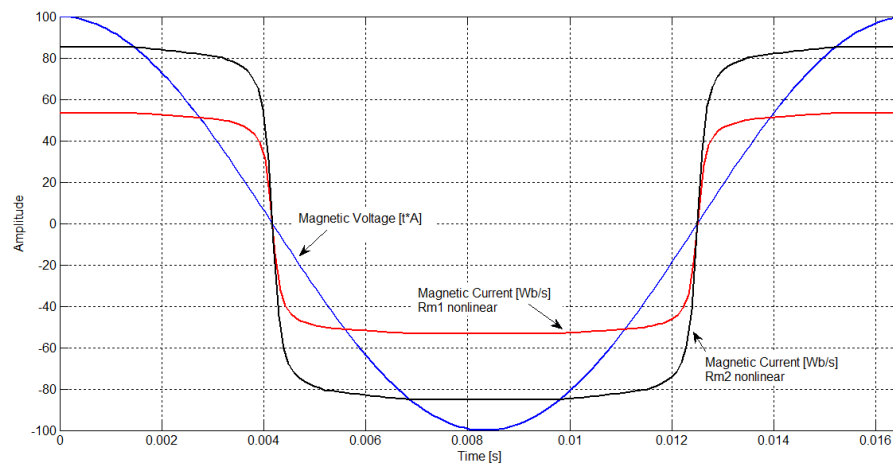


Fig. B1. Magnetic voltage at 60 Hz and currents for different nonlinear resistances.

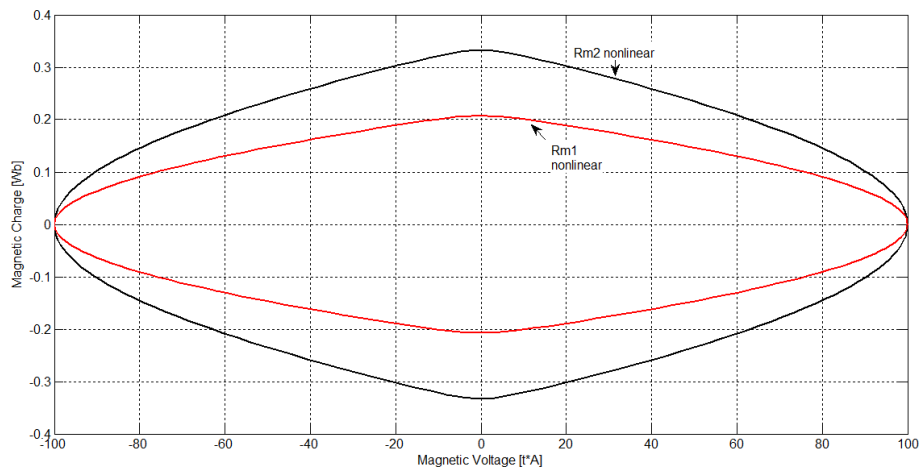


Fig. B2. Magnetic charge vs. voltage for different nonlinear resistances.

- *Inductance Case*

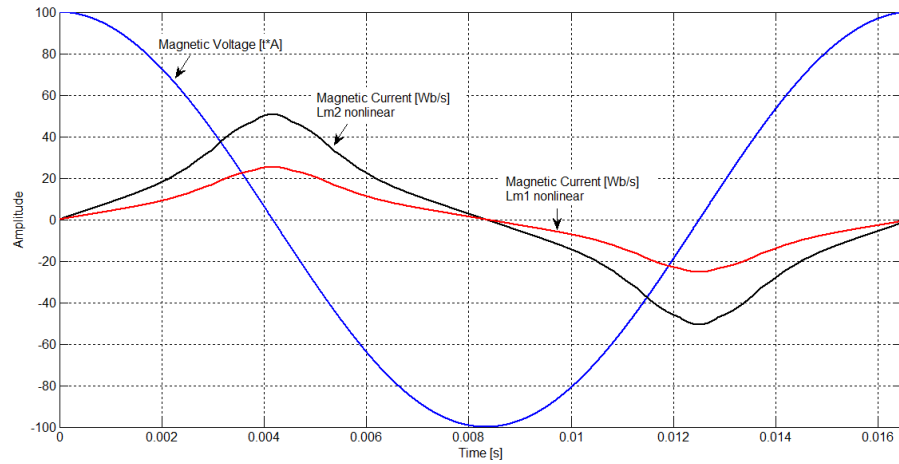


Fig. B3. Magnetic voltage at 60 Hz and currents for different nonlinear inductances.

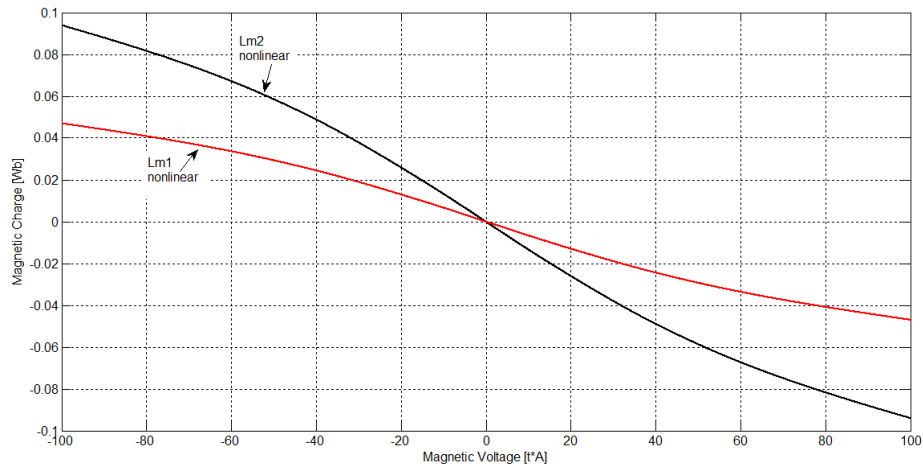


Fig. B4. Magnetic charge vs. voltage for different nonlinear inductances.

- *Capacitance Case*

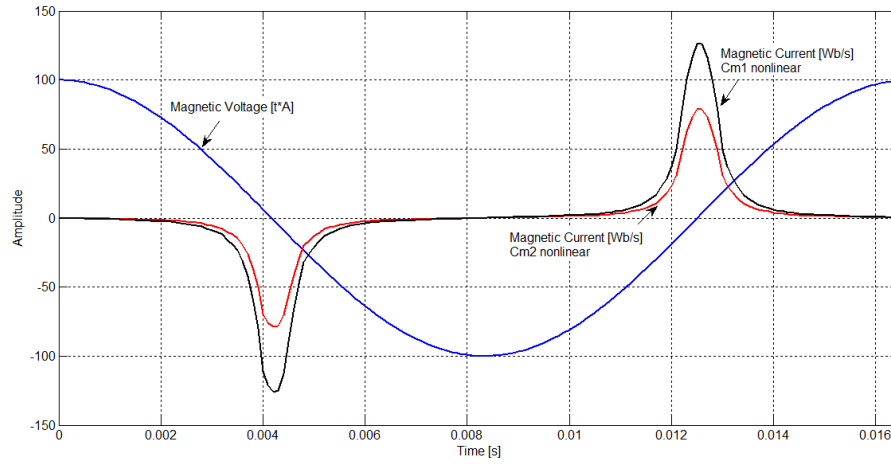


Fig. B5. Magnetic voltage at 60 Hz and currents for different nonlinear capacitances.

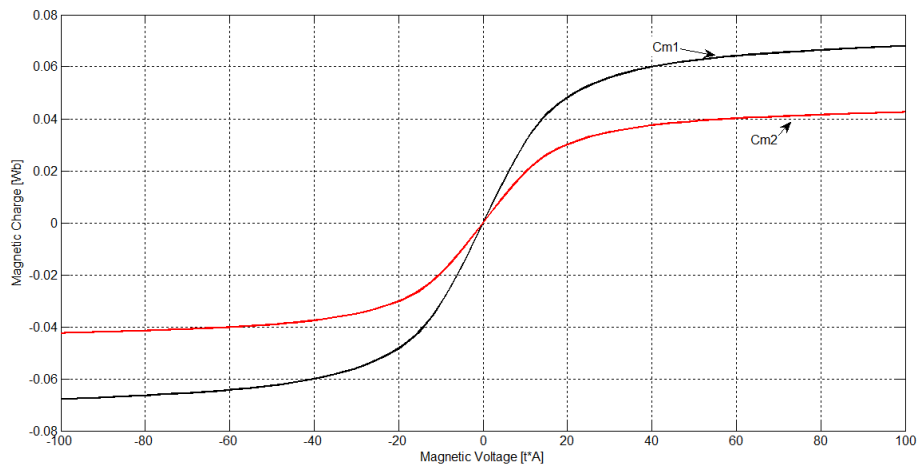


Fig. B6. Magnetic charge vs. voltage for different nonlinear capacitances.

## APPENDIX C

### MAGNETIC VOLTAGES AND CURRENTS

This appendix is an extension of Chapter IV. Section 3.3 and contains the plots of magnetic voltages vs. time and current vs. time when the system is excited at different frequencies. The objective is to demonstrate that the nonlinearity does not affect the phase shift of the element, it only affects the waveform.

- *Case 1: Frequency = 0.5 Hz*

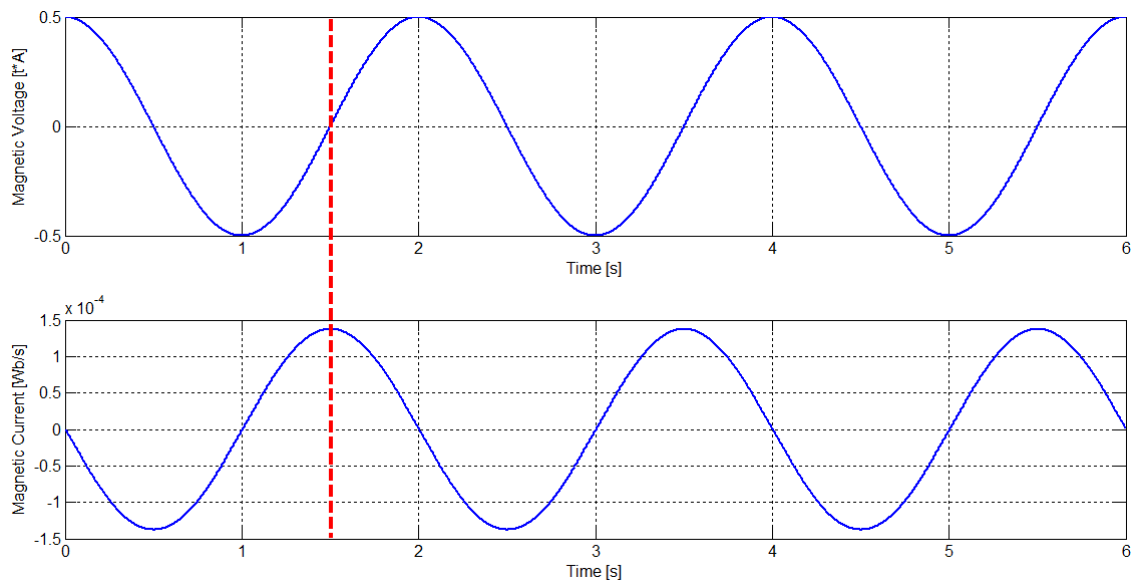


Fig. C.1. Magnetic voltage and current at 0.5 Hz for a voltage amplitude of  $0.5 \text{ A}\cdot\text{t}$ .

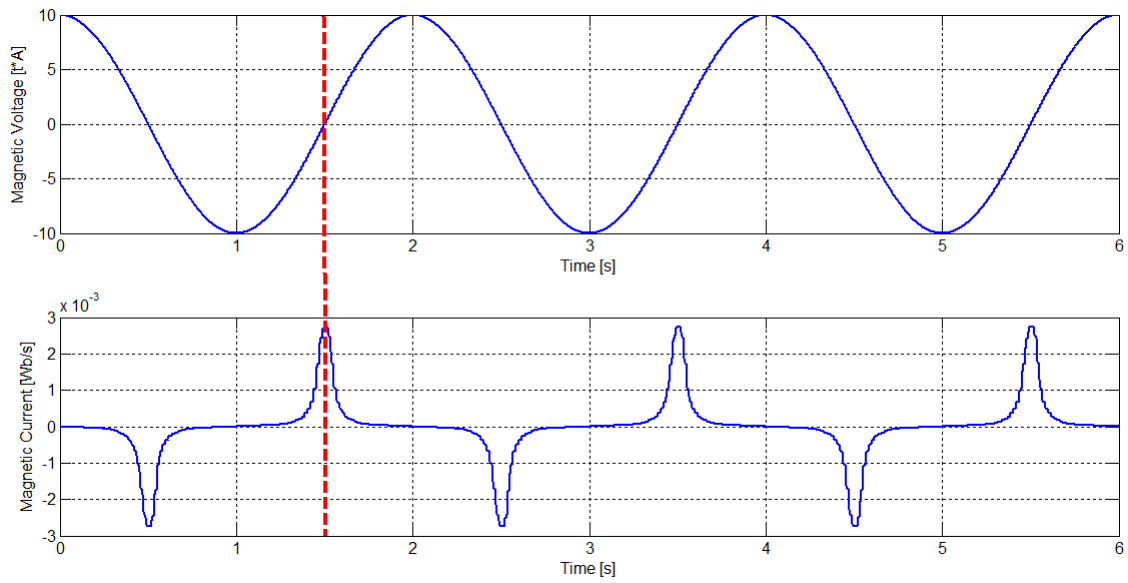


Fig. C.2. Magnetic voltage and current at 0.5 Hz for a voltage amplitude of 10 A·t.

- *Case 2: Frequency = 60 Hz*

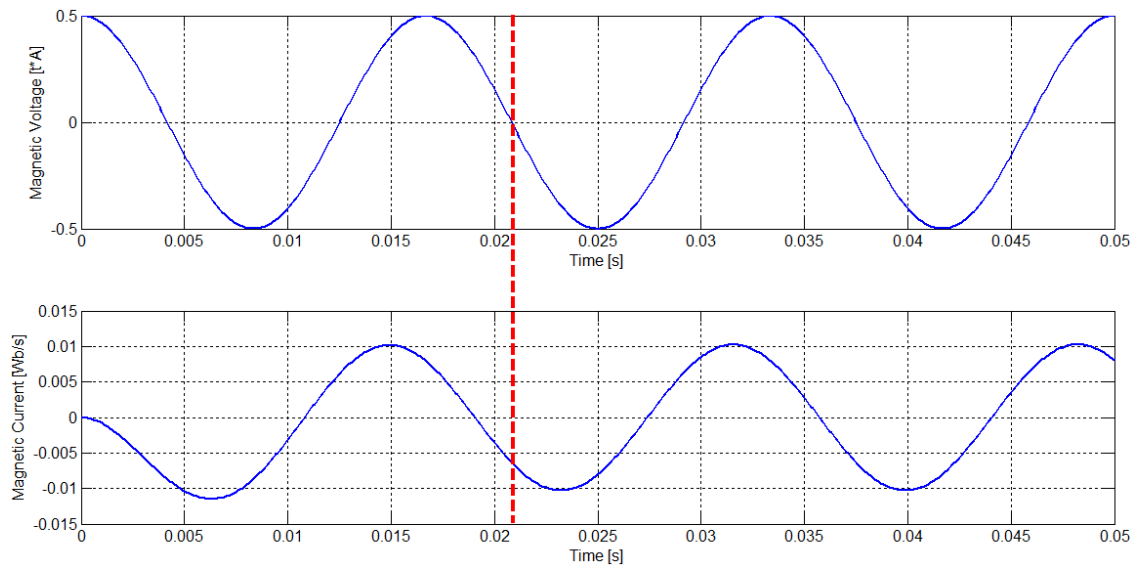


Fig. C.3. Magnetic voltage and current at 60 Hz for a voltage amplitude of 0.5 A·t.



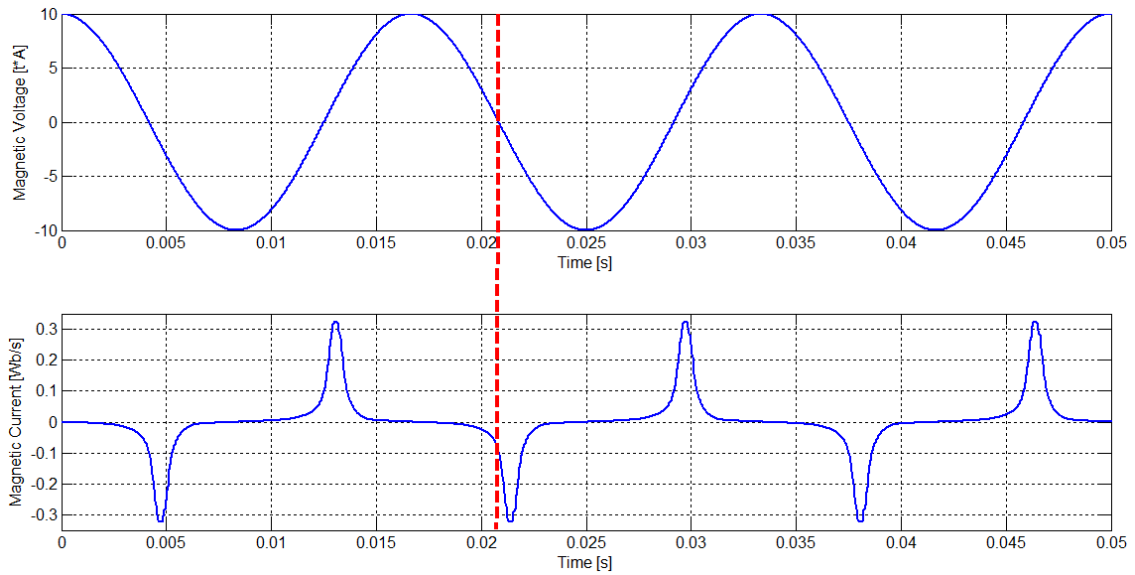


Fig. C.4. Magnetic voltage and current at 60 Hz for a voltage amplitude of 10 A·t.

- *Case 3: Frequency = 1 GHz*

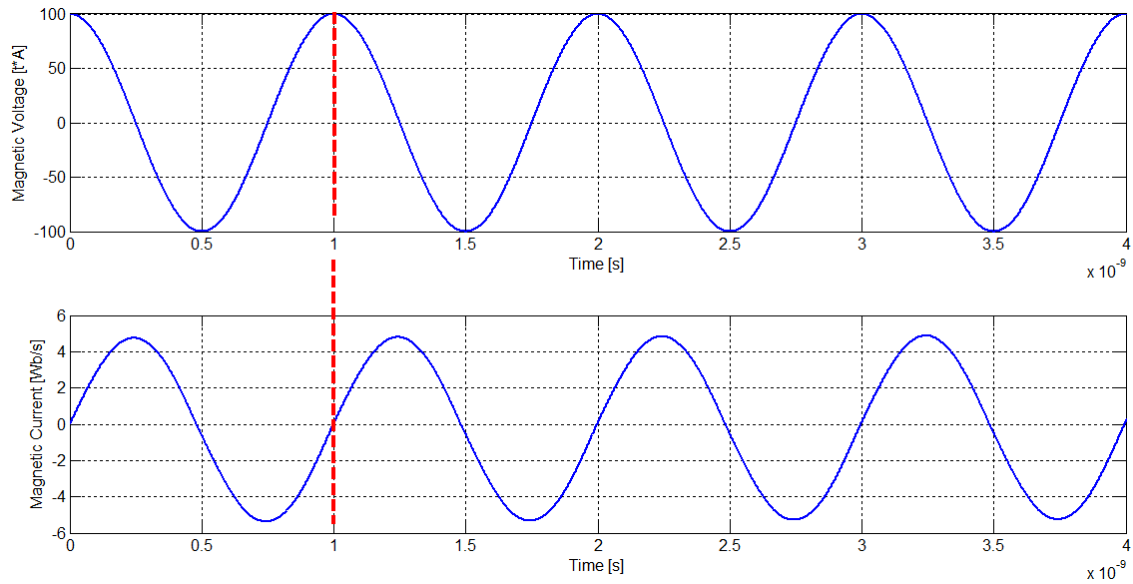


Fig. C.5. Magnetic voltage and current at 1 GHz for a voltage amplitude of 100 A·t.

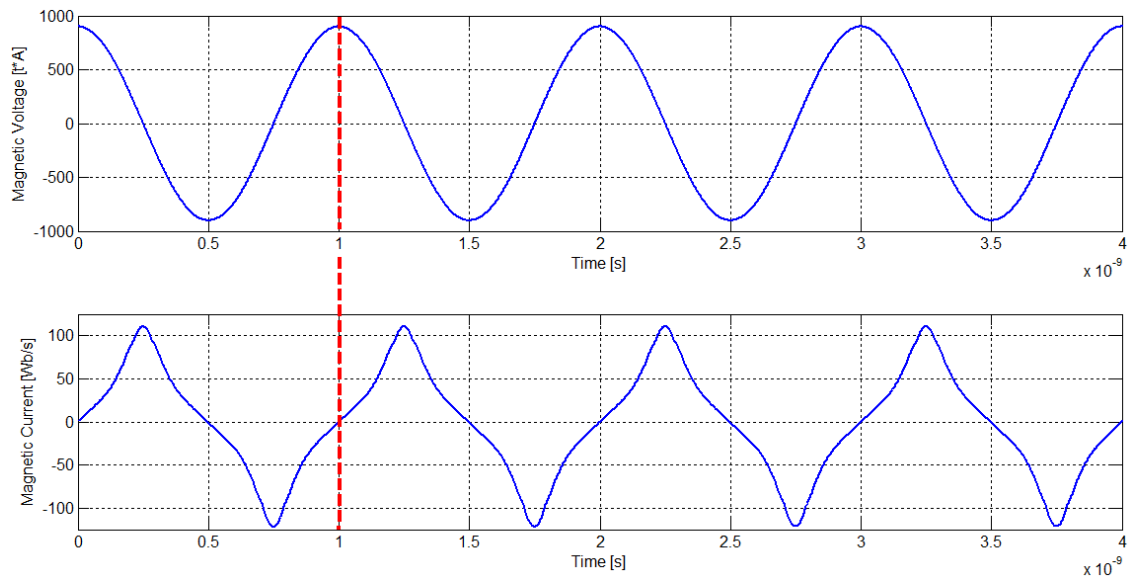


Fig. C.6. Magnetic voltage and current at 1 GHz for a voltage amplitude of 900 A·t..

APPENDIX D  
MAGNETIC TRANSMISSION LINE DESIGN  
(MATLAB CODE)

This appendix contains the Matlab-based code that we programmed in order to solve the nonlinear equations of the system. The fundamental theory behind the programming of this code was explained in Chapter 5.A.1.

```

clear
%%Global variables
f=60; %Frequency of operation [Hz]
w=2*pi*f; %Angular frequency [rad/s]
dwt=pi/1e4; %Ang.Freq increase [rad/s]
wt=0:dwt:2*pi-dwt; %Angular period [rad]
t=wt/w; %Time [s]
data=importdata('C:\Project\Parameters2.mat');
if f<=100
    xmiu=data.xC; ymiu=data.yC; %xmiu=H [A/m]; ymiu=B [T];
    xrho=data.xR; yrho=data.yR; %xrho=Jm [Wb/s/m^2]; %yrho=H [A/m];
    xeps=data.xL; yeps=data.yL; %xeps=E [Wb/s/m]; %yeps=D [C/m^2];
elseif f>100 && f<500
    xmiu=data.xC; ymiu=data.yC; %xmiu=H [A/m]; ymiu=B [T];
    xrho=data.xR; yrho=data.yR*1.5; %xrho=Jm [Wb/s/m^2]; %yrho=H [A/m];
    xeps=data.xL; yeps=data.yL; %xeps=E [Wb/s/m]; %yeps=D [C/m^2];
else
    xmiu=data.xC; ymiu=data.yC; %xmiu=H [A/m]; ymiu=B [T];
    xrho=data.xR*1.5; yrho=data.yR*2; %xrho=Jm [Wb/s/m^2]; %yrho=H [A/m];
    xeps=data.xL; yeps=data.yL; %xeps=E [Wb/s/m]; %yeps=D [C/m^2];
end
N=500; %Number of turns
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Electric Side - Known Values
Vsc=200; %Voltage's amplitude [V]
phvsc=0; %Voltage's phase angle[rads]
Ild=10; %Current's amplitude [A]
phild=0; % Current's phase angle [rads]
vsc=Vsc*cos(wt+phvsc); %Inst. voltage at source [V]
ild=Ild*cos(wt+phild); %Inst. current at load [A]
%%Magnetic Transmission Line
vml=Vsc*Ild; %Magnetic voltage load[t*A]
imag=1/N*vsc; %Magnetic current [Wb/s/t]
qmag=integral(imag,wt,w); %Magnetic charge [Wb]
Bmax=max(ymiu); %Maximum flux density [T]
Acmin=max(qmag)/Bmax; %Minimum core area [m^2]
Ac=6e-4:2e-4:20e-4; %Core's area [m^2]
for k=1:1:length(Ac)
    Bm=max(qmag)/Ac(k); %Flux density [T]
    Lc=10:10:500; %Core's length [m]
    for z=1:1:length(Lc)

```

```

xC=xmiu*Lc(z);yC=ymiu*Ac(k);          %xC=qm [Wb]; yC=vm [t*A]
xR=xrho*Ac(k);yR=yrho*Lc(z);          %xR=im [Wb/s]; yR=vm [t*A]
xL=xeps*Lc(z);yL=yeps*Ac(k);          %xL=im [Wb/s]; yL=qe [C]
vcmag=interp1(yC,xC,qmag,'linear');    %Cap's magnetic voltage [t*A]
vrmag=interp1(xR,yR,imag,'linear');    %Res's magnetic voltage [t*A]
flmag=interp1(xL,yL,imag,'linear');    %Ind's electric charge [C]
vlmag=derivate(flmag,t);               %Ind's magnetic voltage [t*A]
vmtl=vcmag+vrmag+vlmag;                %MTL's voltage [t*A]
vmsc=vmdl+vmtl;                         %Magnetic voltage at source [t*A]
Pmtl=mean(vrmag.*imag);                 %Real power [W]
%%Electric Side
isc=1/N*vmsc;                            %Electric current at source [A]
vld=N*imag;                               %Electric voltage at load [V]
%%Efficiency
Psc=mean(vsc.*isc);                       %Real power at source [W]
Pld=mean(vld.*ild);                       %Real power at load [W]
Ef(k,z)=Pld/Psc*100;                      %Efficiency [%]
%%Power Factor
vscrms=max(vsc)/sqrt(2);                  %Source voltage RMS value [V]
iscrms=sqrt(sum(isc.^2)/length(isc));     %Source current RMS value [A]
Ssc=vscrms*iscrms;                       %Apparent power at source [VA]
PF(k,z)=Psc/Ssc;                          %Power factor
end
end
clc

```

## VITA

Guadalupe Giselle González Domínguez received her Bachelor of Science degree in electromechanical engineering from the Universidad Tecnológica de Panamá, Panama City, Panama in 2004.

Professionally, she has worked in the commercialization department of Bahía Las Minas Corporation, a Panamanian utility company. Since 2006 she has worked as an assistant researcher assigned to the Office for Research at the Universidad Tecnológica de Panamá.

She entered the Department of Electrical Engineering at Texas A&M University in Fall 2006 in order to pursue her doctoral studies and graduated with her Ph.D. in August 2011. Her areas of interest include: electromagnetic theory, design and control of electric machines, computer-aided design, sustainable and renewable energy systems, and power electronics.

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