# COMPARATIVE PERFORMANCE ANALYSIS OF THE ALGORITHMS FOR DETECTING PERIODICALLY EXPRESSED GENES

A Thesis

by

## KWADWO SEFA AGYEPONG

#### Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

## MASTER OF SCIENCE

August 2012

Major Subject: Electrical Engineering

Comparative Performance Analysis of the Algorithms for Detecting Periodically Expressed Genes Copyright 2012 Kwadwo Sefa Agyepong

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#### ABSTRACT

Comparative Performance Analysis of the Algorithms for Detecting Periodically Expressed Genes. (August 2012) Kwadwo Sefa Agyepong, B.S., Prairie View A&M University Co–Chairs of Advisory Committee: Erchin Serpedin Edward R. Dougherty

Thus far, a plethora of analysis on genome-wide gene expression microarray experiments on the cell cycle have been reported. Time series data from these experiments include gene expression profiles that might be periodically expressed. However, the numbers and actual genes that are periodically expressed have not been reported with consistency, analysis on similar experiments reports disparate numbers of genes that are periodically expressed with scant overlap. This work ultimately compares the performance of five spectral estimation schemes in their ability to recover periodically expressed genes profiles. Lomb-Scargle (LS), Capon, Missing-Data Amplitude and Phase Estimation (MAPES), Real Value Iterative Adaptive Approach (RIAA) and Lomb-Scargle Periodogram Regression (LSPR) are rigourously studied and pitted against each other in various simulated testing conditions. Results obtained using synthetic and microarray data reveals that RIAA is an efficient and robust method for the detection of periodically expressed genes in short time series data that might be characterized with noisy and irregularly sampled data points. To Leona

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#### CHAPTER I

#### INTRODUCTION<sup>1</sup>

Biological processes undergo rhythms that are dictated by various cell activities such as the cell cycle process. This phenomenon recur at regular intervals and may be annual, seasonal, circadian or even ultradian. These rhythms are controlled by endogenous biological clocks and understanding their molecular basis is of fundamental interest in biology. Knowledge of these rhythms leads to insights into diagnosis and treatment of illness. The rhythmic signals help a living organism to organize its behavior and physiology. Understanding these rhythmic activities has been an important problem in systems biology for many years. Advances in microarray technology equipped us with a means to directly measure and quantify the expressional concentration levels of mRNA, the basic unit structure that encodes chemical instructions for a protein product. These measurements provide a tangible means to characterize regulations in the cell.

Microarray experiments exploit high-throughput gene chips to measure gene expressions at various sampling time points per the suitability of the experimenter and experimental constraints. Experimental constraints [1] lead to scarce sample size, as large sample sizes are not economically feasible due to the cost of gene chips and the maintenance of a conducive ambience for cell cultures over time. The limited data set generally present missing values at random time points. This is due to defective slides

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and the inability of microarrays to cipher non-ideal spots. Experimental noise also corrupts the limited samples, leading to uncertainty that must be addressed within a stochastic framework [1].

The mechanisms of the underlying process is well understood, but the analyses of the datasets led to inconclusive reports on the numbers of periodically expressed genes for many organisms. Work on Saccharomyces cerevisiae [2] [3] has so far reported about 400 to 800 genes that are cell cycle regulated, meaning that they are periodically expressed. For Schizosaccharomyces pombe, about 400 to 700 genes [4][5] have been found to be periodically expressed. In Aradidiopsis thaliana, about 500 to 600 are reported to be cell cycle regulated [6]. The need for an analysis tool that overcomes the innate undesirable characteristics of the microarray data is evident. On experiments that are available to the general public, it is interesting that one cannot get an overlap of more than 400 genes between two different analyses based on similar experimental synchronization designs. There have been many microarray experiments conducted on the budding yeast [2] [4]. The budding yeast in [2] is the most used data source in many analytical experiments for the detection of periodically expressed genes. This is because of its grounding breaking results and the relatively large sample size it provided to literature citeKwadwo08. Spellman [2] analyzed the data on the budding yeast via a scoring criterion where a combination of a correlation score and a Fourier based score were used to rank 800 genes believed to be periodically expressed.

There are basically two main approaches used in the literature to evaluate schemes and models. The norm is to search for hits from a set of 104 genes that are known to be cell cycle regulated [7]. These 104 genes were found from traditional methods where expression profile were visually inspected [2]. The other way of putting a measure of performance on a scheme or statistical test is to combine the results of similar works, by taking a heuristic threshold overlap of results publicly available and counting the overlap of genes between the results of ones model and the overlap of results from other methods.

Our earlier work looked at three spectral analysis tools which could overcome the undesirable characteristics of the microarray experimental data set. The performance of Lomb-Scargle periodogram (LS) [8], Capon (Robust Capon) [9] and Missing data Amplitude and Phase Estimation (MAPES) [10] were compared. Each scheme possesses the ability to detect periodically expressed genes from the expression measurements of mRNA provided that some conditions are met. Lomb-Scargle proved to be the most efficient method when all three schemes were applied on cdc15 dataset from Spellman's experiments [1]. The previous three schemes are included in the present comprehensive study for detailed analysis on a myriad simulated conditions that are semblant to microarray dataset. Stoica's [11] new method called Real Value Iterative Adaptive Approach (RIAA) and a scheme employed by Yang [12] called LSPR have been added to this study. LSPR is a new periodicity detection algorithm that has its foundation built on Lomb-Scargle periodogram and harmonic regression. There have been many methods proposed to detect periodicity in the cell cycle of organisms. Yang [13] used an algorithm which combined time domain and frequency domain analysis to obtain and identify rhythmic expression profiles. It utilizes spectral estimation technique to obtain periodically expressed profile candidates and model these candidates with a time-series model. Giurcaneanu [14] used generalized Gaussian distributions to investigate stochastic complexity inherent in the detection mechanism of genes that are periodically expressed. And esmaki [15] employed a robust periodicity testing procedure that used a non-Gaussian noise assumption and considered a regression method to aide in simulating irregular sampling. Luan [16] used a selection of 'guide' genes and constructed cubic B-spline based periodic functions as a model [1]. The statistical approach by Luan [16] allowed for the identification

of thresholds for false discovery rate. Lu [17] proposed a Bayesian approach to estimate a periodic-normal mixture model from five different experiments. Several additional power spectral density estimation schemes have been used in the literature. Wichert [18] applied the traditional periodogram where any missing data present for all genes were imputed via interpolation. Bowles used synthetic data to compare Capon method and Robust Capon approach[19]. Lichtenberg [20] compared [2], [16] and [17] using a a score obtained via the combination of periodicity and regulation magnitude. Most of the works cited above employed their methods on evenly sampled data. Missing data points were interpolated and in cases where the missing data set were more than 30%, the genes were discarded [1].

Microarray experiments are generally characterized by having datasets that are irregularly sampled. To address the issue of unequally spaced measurements, Lomb [21] and Scargle [22] discovered that a phase shift restores the orthogonality lost by Fourier analysis, due to the unevenness of the data, in the sine and cosine terms. Glynn [8] used the Lomb-Scargle scheme to analyze *Plasmodium falciparum* data set. Stoica [23] modified the Capon method to adapt to irregular sampled data in the field of signal processing. Wang et al. [10] proposed a new approach called missing-data amplitude and phase estimation (MAPES). MAPES estimates any missing data and computes the spectral density estimate iteratively via the Expectation Maximization (EM) algorithm. Real Value Iterative Adaptive Approach (RIAA) [11] induced the present interest to revisit our prior work given the fact that preliminary results show that it presents much promise in being robust to deficiencies in microarray data set. The rest of this work will illustrate the capability of each method while providing a complete review of the work in [1]. The following nested questions are posed and answered in this study: Which scheme performs best in the presence of (1) Noise, (2) Small sample size, (3) Clusters of missing data or irregular sampling? Both synthetic and experimental data are used in this work. The aim of this work is to nominate a scheme that will address the problem of scant overlap in in the existing results assessing periodically expressed genes in the same organism. Results shows that RIAA outperforms the schemes considered in this work on both synthetic and the Cdc 15 yeast data in Spellman's dataset. RIAA is also applied to two different data set, Spellman [2] and Pramila [24] alpha synchronized datasets, to obtain a consistent overlap of results for periodically expressed genes. Full results are provided in the Appendices including Matlab codes, the list of 104 plus 9 new genes provided by Johansson [7] are also included.

#### CHAPTER II

#### METHODS<sup>1</sup>

This section begins by examining RIAA and proceeds with a recapitulation of the existing methods for a proper perspective of the subject. The material of this chapter relies on our previous paper [1]. RIAA belongs to the class of power spectral density estimators that employ least-squares to estimate the spectral density for a sequential data with discrete spectra. Lomb [21] used phase-shift of the sine and cosine functions to restore orthogonality that is lost, due to unevenly sampling, between the cosine and sine harmonics. Scargle [22] extensively reanalyzed Lomb's periodogram to provide derivation of a null hypothesis distribution for the periodogram. The Lomb-Scarlge periodogram has been cited numerous times in many fields and applications including genomics see e.g., [8], [12].. Capon approach represents a filter bank approach for power spectrum density estimation, where a finite-length data spectrum estimator is constructed by estimating the spectral power's distribution over narrow spectral bands. MAPES was developed for regular sampling times with missing data but as mentioned in [10], it belongs to the family of non-parametric spectral estimation techniques. It exploits the expectation maximization (EM) algorithm to estimate missing samples. LSPR is based on Lomb-Scargle periodogram, where inferences made from LS are used as inputs into a harmonic regression model whose output acts as inputs in Akaike's information criterion [25] to obtain a p-value.

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#### A. Real Value Iterative Adaptive Approach - RIAA

Real Value Iterative Adaptive Approach (RIAA) is a spectral estimator (periodogram), designed to alleviate undesirable characteristics that arise in the spectral density estimation of non-uniformly sampled data. This method can be thought of as an iterative weighted least-squares method which utilizes an adaptive weighting matrix obtained from the most recent spectral density estimate [11]. Let  $(t_l, y_l), l = 0, \ldots, N - 1$ , denote N time-series observations where  $t_l$  are the observational times or time lag and  $y_l$  is the expression measurement of a gene or time series. RIAA is formulated within the framework of least-squares periodogram and so to explain RIAA, it is prudent to expound on the ordinary least-squares periodogram. The Fourier transform periodogram of the data set will normally be expressed as:

$$\Phi_{FT}(\omega) = \frac{1}{N^2} \left| \sum_{l=0}^{N-1} y_l e^{-j\omega t_l} \right|^2, \qquad (2.1)$$

where  $\omega$  is the angular frequency variable. An equivalent expression for  $\Phi_{FT}(\omega)$  can be obtained via least-squares theory [26] as,

$$\Phi_{FT}(\omega) = |\hat{\alpha}(\omega)|^2,$$
$$\hat{\alpha}(\omega) = \arg\min_{\alpha(\omega)} \sum_{l=0}^{N-1} |y_l - \alpha(\omega)e^{j\omega t_l}|^2.$$
(2.2)

Employing real value signals, Equation(2.2) can be re-written as:

$$\min_{\substack{\Theta \ge 0\\\phi \in [0,2\pi]}} \sum_{l=0}^{N-1} [y_l - \Theta \cos(\omega t_l + \phi)]^2,$$
(2.3)

where  $\Theta$  and  $\phi$  depend on  $\omega$ . Set  $a = \Theta \cos(\phi)$  and  $b = -\Theta \sin(\phi)$  to obtain:

$$\min_{a,b} \sum_{l=0}^{N-1} [y_l - a\cos(\omega t_l) - b\sin(\omega t_l)]^2.$$
(2.4)

The solution to Equation (2.4) is given by:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \mathbf{R}^{-1} \mathbf{r}.$$
 (2.5)

Where,

$$\mathbf{R} = \sum_{l=0}^{N-1} \begin{bmatrix} \cos(\omega t_l)^2 & \cos(\omega t_l)\sin(\omega t_l) \\ \sin(\omega t_l)\cos(\omega t_l) & \sin(\omega t_l)^2 \end{bmatrix}.$$
 (2.6)

and

$$\mathbf{r} = \sum_{l=0}^{N-1} \begin{bmatrix} \cos(\omega t_l) \\ \sin(\omega t_l) \end{bmatrix} y_l.$$
(2.7)

The ordinary least squares periodogram can then be defined as:

$$\Phi_{LSP}(\omega) = \frac{1}{N} \mathbf{r}^{\mathbf{T}} \mathbf{R}^{-1} \mathbf{r}.$$
(2.8)

#### 1. Frequency Window and Grid Size

A spectral window that can resolve spectral tendencies without aliasing from the sampling times is presented in this section. Proceeding with the premise that other sinusoidal components are present in the data, an error term is introduced into the spectral density estimate and taking the spectral norm of this error term as in [11], a solution is obtained that depends on the sampling pattern. From this solution, the spectral window can be derived as a function of  $\omega$ . Stoica [11] approximated this window as  $W(\omega) = |\sum_{l=0}^{N-1} e^{j\omega t_l}|^2$ . This window is used to find the smallest frequency  $\omega_o$  for which the spectral window function is at its next maximum, different from the global maximum obtained at  $\omega = 0$ . If there are no frequencies that have a maximum nearest the peak of  $N^2$ , set  $\omega_o = \infty$  or a value representative for the data under study. Using  $\omega_o$ , the maximum frequency is defined as

$$\omega_{max} = \frac{\omega_o}{2} \tag{2.9}$$

which provides the interval  $[0, \omega_{max}]$ . In this window, care must be taken to ensure that the smallest frequency separation can be adequately detected in choosing a frequency search grid  $\Delta \omega$ . There are many grid size approximations used in the literature [11][27]. However, Equation(2.10) can be used since it is a widely used approximation for irregular sampling:

$$\Delta \omega = \frac{2\pi}{t_{N-1} - t_0}.$$
 (2.10)

The number of grid points is then given by:

$$J = \frac{\lfloor \omega_{max} \rfloor}{\Delta \omega}.$$
 (2.11)

And this leads to a uniform frequency grid as in [28] given by

$$\omega_j = \Delta \omega_j, \quad j = 1, \dots, J. \tag{2.12}$$

Thus far, the ordinary least-squares spectral estimation method has been described. To continue formulating RIAA, there is a need to introduce the following parameters,

$$\mathbf{y} = \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix}, \quad \mathbf{A}_{\mathbf{j}} = \begin{bmatrix} \mathbf{c}_{\mathbf{j}} & \mathbf{s}_{\mathbf{j}} \end{bmatrix}, \quad \mathbf{\Theta}_{\mathbf{j}} = \begin{bmatrix} a(\omega_j) \\ b(\omega_j) \end{bmatrix},$$
$$\mathbf{c}_{\mathbf{j}} = \begin{bmatrix} \cos(\omega_j t_0) \\ \vdots \\ \cos(\omega_j t_{N-1}) \end{bmatrix}, \quad \mathbf{s}_{\mathbf{j}} = \begin{bmatrix} \sin(\omega_j t_0) \\ \vdots \\ \sin(\omega_j t_{N-1}) \end{bmatrix}. \quad (2.13)$$

Re-parametrization of Equation (2.2) presents the following solution,

$$\min_{\Theta_{j}} \|\mathbf{y} - \mathbf{A}_{j} \Theta_{j}\|^{2}$$
$$\hat{\Theta}_{j} = (\mathbf{A}_{j}^{T} \mathbf{A}_{j})^{-1} \mathbf{A}_{j}^{T} \mathbf{y}.$$
(2.14)

The covariance matrix of other possible components in the data other than the component with  $\omega_j$  is defined:

$$\mathbf{Q}_{j} = \sum_{m=1, m \neq j}^{J} (a_{m}^{2} + b_{m}^{2}) \mathbf{A}_{m} \mathbf{A}_{m}^{T}.$$
 (2.15)

At  $\omega_j$ , all other frequency components are considered to be noise and Equation(2.15) carries their contribution. Using Eq.(2.15) if available, the following weighted least squares approach is employed because it is known to be more accurate under general conditions than the ordinary least squares [29].

$$\min_{\alpha_j} \|\mathbf{y} - \mathbf{A}_j \alpha_j\|^2_{\mathbf{Q}_j^{-1}}$$
(2.16)

The solution to the problem above is given as:

$$\hat{\Theta}_j = \frac{\mathbf{A}_j^T \mathbf{Q}_j^{-1} \mathbf{y}}{\mathbf{A}_j^T \mathbf{Q}_j^{-1} \mathbf{A}_j}.$$
(2.17)

Then RIAA also known as the weighted least square periodogram (WLSP) is defined as:

$$\Phi_{WLSP}(\omega_j) = \frac{1}{N} \hat{\Theta}_j^T (\mathbf{A}_j^T \mathbf{A}_j) \hat{\Theta}_j.$$
  
$$\Phi_{WLSP}(\omega_j) = |\alpha_j|^2.$$
(2.18)

**Initialization** Use the ordinary least squares to obtain the initial value of  $\alpha_i^0$ .

**Iteration** At the *kth* iteration, the estimate of  $\hat{\alpha}_j$  i.e., at  $\omega_j$  is  $\alpha_j^k = \frac{\mathbf{A}_j^T(\mathbf{Q}_j^k)^{-1}\mathbf{y}}{\mathbf{A}_j^T(\mathbf{Q}_j^k)^{-1}\mathbf{A}_j}$  for  $k = 1, \ldots, K$  where  $\mathbf{Q}_j^k = \sum_{m=1, m \neq j^J} |\alpha_j^{k-1}|^2 \mathbf{A}_m \mathbf{A}_m^T$ .

**End** Iteration is terminated after 15 iterations or when  $|\alpha_j^{k+1} - \alpha_j^k|^2 < 10^{-4}$ .

After the last iterative step,  $\{\hat{\Theta}_{j}^{K}\}$  is used to compute the power spectral density for RIAA:

$$\Phi_{RIAA}(\omega_j) = \frac{1}{N} (\hat{\boldsymbol{\Theta}}_j^K)^T (\mathbf{A}_j^T \mathbf{A}_j) (\hat{\boldsymbol{\Theta}}_j^K), \quad j = 0, \dots, J.$$
(2.19)

RIAA does not suffer from the global and local leakage that are characteristic for the other methods. Therefore, peaks detected by RIAA have a high probability of denoting cyclicity and simulation results show that RIAA does not suffer from the spurious peaks problem of LS, which leads to false positives.

#### B. Lomb-Scargle Periodogram

In cases where evenly sampled data cannot be obtained, Lomb-Scargle periodogram has been the method of choice when estimating spectral components in the data. Lomb-Scargle periodogram ignores the unevenness of the data by imputing a phaseshift to the sine and cosine harmonic functions. This restores the orthogonality which, otherwise, is lost due to the nature of the data. Given N time-series observations  $(t_l, y_l), l = 0, ..., N - 1$ , where t stands for the time tag and y stands for the value of a time series point or sampled expression of a specific gene, the normalized LombScargle periodogram at an angular frequency  $\omega$  is defined as in [1]

$$\Phi_{LS}(\omega_j) = \frac{1}{2\hat{\sigma}^2} \left( \frac{\left(\sum_{l=0}^{N-1} [y_l - \bar{y}] cos[\omega_j(t_l - \tau)]\right)^2}{\sum_{l=0}^{N-1} cos^2[\omega_j(t_l - \tau)]} + \frac{\left(\sum_{l=0}^{N-1} [y_l - \bar{y}] sin[\omega_j(t_l - \tau)]\right)^2}{\sum_{l=0}^{N-1} sin^2[\omega_j(t_l - \tau)]} \right),$$
(2.20)

for j = 1, ..., J as defined in Equation(2.12) where  $\bar{y}$  and  $\hat{\sigma}^2$  stand for the mean and variance of the sampled data, respectively, and  $\tau$  is defined as:

$$\tau = \frac{1}{2\omega_j} atan\left(\frac{\sum_{l=0}^{N-1} sin(2\omega_j t_l)}{\sum_{l=0}^{N-1} cos(2\omega_j t_l)}\right).$$
(2.21)

The frequency grid defined under RIAA is also applied to the Lomb-Scargle periodogram. Lomb-Scargle periodogram is an efficient solution in estimating the spectra of unevenly sampled data sets especially when the underlying noise assumption is Gaussian.

#### C. Robust Capon Method

The general framework for the Capon method is reproduced from our earlier work [1] Capon method is a filter-bank approach that is based on a data-dependent bandpass filter [9]. It was originally designed for evenly sampled data. It estimates the spectral density of a time series input signal by first passing it through a bank of bandpass filters with varying center frequencies, called the steering frequencies. It then measures and uses the output power of the filter's passband. By dividing the measured power by the passband bandwidth, an estimate of the power spectrum density is obtained. The filter is designed in such a way that it minimizes all the contribution of other frequencies in the input signal except the frequency components at  $\omega$ . In other words, the Capon method seeks to solve the following optimization problem:

$$\mathbf{h} = \underset{\mathbf{h}}{\operatorname{arg\,min}} \mathbf{h}^{H} \mathbf{R} \mathbf{h} \quad \text{subject to} \quad \mathbf{h}^{H} \mathbf{a}(\omega_{j}) = 1, \qquad (2.22)$$

whose solution provides the spectrum estimate at frequency  $\omega_i$  as

$$\Phi_C(\omega_j) = \frac{1}{\mathbf{a}^H(\omega_j \Delta) \mathbf{R}^{-1} \mathbf{a}(\omega_j \Delta)},$$
(2.23)

where matrix  $\mathbf{R}$  stands for the data covariance matrix with a dimension  $N_0$ , which is inversely proportional to the bandwidth of the Capon filter. The steering vector is defined as follows

$$\mathbf{a}(\omega_j) = \left(1 \ e^{j\omega_j} \cdots e^{j\omega_j(N_0 - 1)}\right)^T.$$
(2.24)

To guarantee the existence of inverse  $\mathbf{R}^{-1}$ , the bandwidth parameter  $N_0$  need not exceed  $\lfloor (N-1)/2 \rfloor$ . However, a smaller  $N_0$ , will adversely affect the resolution of the spectral estimates while the accuracy of the estimate of the covariance matrix will increase. Hence,  $N_0$  should be set as a tradeoff between resolution and accuracy of the Capon method [23].

It has been proven that given an adequate number of samples, the Capon method yields a better spectral resolution compared with traditional periodogram [9]. The Capon method has been updated to cope with the presence of irregular samples [23]. The same frequency grid denoted in Equation (2.12) is employed. In order to take advantage of the best resolution,  $N_0$  is set to be equal to  $\lfloor (J-1)/2 \rfloor$ , where J is defined in Equation (2.12). In simulation, an estimate of the autocorrelation matrix  $\hat{\mathbf{R}}$  can is obtained from the Lomb-Scargle periodogram, which is represented by

$$\hat{\mathbf{R}} = \frac{1}{J\delta} \sum_{j=1}^{J} \mathbf{a}(\omega_j \delta) \mathbf{a}^H(\omega_j \delta) \Phi_{LS}(\omega_j).$$
(2.25)

The Capon method is slightly more computationally complex than LS and RIAA.

In simulated data, its resolution was better than LS and could rival RIAA if the sample sizes is increased to be greater than 40 samples, but on limited sample size and corrupted biological data, its performance was below a notch compared to LS and RIAA.

#### D. MAPES Method

The general framework for MAPES is also reproduced from our earlier work [1]. Given P time-series observations  $(t_l, y_l), l = 0, \ldots, P - 1$ , the data are assumed to be sampled uniformly. However, only N data points are available and there are P - Nmissing data points. The time-series signal with frequency  $\omega$  is modeled as

$$y_l = \alpha(\omega)e^{j\omega l} + \varepsilon_l(\omega), \quad l = 0, \dots, P - 1, \quad \omega \in [0, 2\pi],$$
(2.26)

where  $\alpha(\omega)$  represents the complex amplitude of the sinusoidal component and  $\varepsilon_l(w)$  denotes a residual term. The same frequency grid as in Equation (2.12) is used. Using the expectation-maximization algorithm, MAPES iteratively estimates the missing data, and while updating the estimates of the spectra by minimizing the mean square error between consecutive estimates.

The data vector  $\mathbf{y} = (y_0, \cdots, y_{P-1})^T$  is partitioned into L overlapping subvectors, each with dimension  $M \times 1$ , and L = P - M + 1. These subvectors constitute the enhanced data vector  $\tilde{\mathbf{y}}$  ( $LM \times 1$ ), which assumes the following expression

$$\tilde{\mathbf{y}} = \begin{pmatrix} \tilde{\mathbf{y}}_0 \\ \vdots \\ \tilde{\mathbf{y}}_{L-1} \end{pmatrix} = \mathbf{U}\boldsymbol{\gamma} + \mathbf{V}\boldsymbol{\mu}, \qquad (2.27)$$

where  $\gamma$  (N × 1) and  $\mu$  ((P - N) × 1) represent the available and missing data, respectively. U (LM × N) and V (LM × (P - N)) denote binary selection matrices for the available and missing data, respectively. The selection matrices are orthogonal to each other:  $U_N^T V_{P-N} = \mathbf{0}_{Nx(P-N)}$ . In other words, given  $\mathbf{U}, \mathbf{V}$  and  $\tilde{\mathbf{y}}$ , the data vectors  $\boldsymbol{\gamma}, \boldsymbol{\mu}$  can be computed in the least-squares (LS) sense as

$$\boldsymbol{\gamma} = (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T \tilde{\mathbf{y}} = \tilde{\mathbf{U}}^{\dagger} \tilde{\mathbf{y}}, \text{ where } \tilde{\mathbf{U}}^{\dagger} = (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T,$$
 (2.28)

$$\boldsymbol{\mu} = (\mathbf{V}^T \mathbf{V})^{-1} \mathbf{V}^T \tilde{\mathbf{y}} = \tilde{\mathbf{V}}^{\dagger} \tilde{\mathbf{y}}, \quad \text{where} \quad \tilde{\mathbf{V}}^{\dagger} = (\mathbf{V}^T \mathbf{V})^{-1} \mathbf{V}^T.$$
(2.29)

The residual vector and its covariance matrix are next defined

$$\mathbf{e}_{l}(\omega) = \left(\varepsilon_{l}(\omega) \ \varepsilon_{l+1}(\omega) \cdots \varepsilon_{l+M-1}(\omega)\right)^{T}, \qquad (2.30)$$

$$\mathbf{Q}(\omega) = E\left(\mathbf{e}_{l}(\omega)\mathbf{e}_{l}^{H}(\omega)\right), \qquad (2.31)$$

where  $E(\cdot)$  denotes the expectation operator, and in practice is replaced by a sample mean estimator. The following two notations are also required by the definition of MAPES power spectral estimator:

$$\boldsymbol{\rho}(\omega) = \begin{pmatrix} e^{j\omega 0} \mathbf{a}(\omega) \\ \vdots \\ e^{j\omega(L-1)} \mathbf{a}(\omega) \end{pmatrix}, \qquad (2.32)$$
$$\mathbf{D}(\omega) = \begin{pmatrix} \mathbf{Q}(\omega) & 0 \\ \ddots \\ 0 & \mathbf{Q}(\omega) \end{pmatrix}. \qquad (2.33)$$

Where  $\mathbf{a}(\omega)$  represents the complex amplitude of the sinusoidal component and  $\mathbf{Q}(\omega)$ is defined as in Equation(2.31). In the *i*th EM iteration, the probability density function (PDF) of the missing data vector  $\boldsymbol{\mu}$  conditioned on the available data  $\boldsymbol{\gamma}$  and other context parameters is complex Gaussian with mean and variance denoted by  $(\mathbf{b}, \mathbf{K})$  as follows

$$\mathbf{b}_{i}(\omega) = \tilde{\mathbf{U}}^{T}\boldsymbol{\rho}(\omega)\alpha_{i}(\omega) + \tilde{\mathbf{U}}^{T}\mathbf{D}_{i}(\omega)\tilde{\mathbf{V}}\left(\tilde{\mathbf{V}}^{T}\mathbf{D}_{i}(\omega)\tilde{\mathbf{V}}\right)^{-1}\left(\boldsymbol{\gamma}-\tilde{\mathbf{V}}^{T}\boldsymbol{\rho}(w)\alpha_{i}(w)\right), \quad (2.34)$$

$$\mathbf{K}_{i}(\omega) = \tilde{\mathbf{U}}^{T} \mathbf{D}_{i}(\omega) \tilde{\mathbf{U}} - \tilde{\mathbf{U}}^{T} \mathbf{D}_{i}(\omega) \tilde{\mathbf{V}} \left( \tilde{\mathbf{V}}^{T} \mathbf{D}_{i}(\omega) \tilde{\mathbf{V}} \right)^{-1} \tilde{\mathbf{V}}^{T} \mathbf{D}_{i}(\omega) \tilde{\mathbf{U}}.$$
 (2.35)

Where  $\tilde{\mathbf{U}}$  and  $\tilde{\mathbf{V}}$  are estimates of the selection matrices at the *i*th EM iteration and  $\mathbf{D}_i(\omega)$  is the estimate of  $\mathbf{D}(\omega)$ , Equation(2.23) at the *i*th EM iteration. Then the estimates for spectral magnitude  $\alpha(\omega)$  and residual matrix  $\mathbf{Q}$  are updated in terms of equations

$$\alpha_{i+1}(\omega) = \frac{\mathbf{a}^{H}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{Z}(\omega)}{\mathbf{a}^{H}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{a}(\omega)},$$
(2.36)

$$\mathbf{Q}_{i+1}(\omega) = \mathbf{S}(\omega) + (\alpha_{i+1}(\omega)\mathbf{a}(\omega) - \mathbf{Z}(\omega)) (\alpha_{i+1}(\omega)\mathbf{a}(\omega) - \mathbf{Z}(\omega))^{H}, \quad (2.37)$$

where the auxiliary matrices are defined as follows

$$\begin{pmatrix} \mathbf{z}_{0} \\ \vdots \\ \mathbf{z}_{L-1} \end{pmatrix} = \mathbf{U}\boldsymbol{\gamma} + \mathbf{V}\mathbf{b}(\omega), \qquad (2.38)$$

$$\mathbf{Z}(\omega) = \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{z}_l e^{-j\omega l}, \qquad (2.39)$$

$$\mathbf{S}(\omega) = \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{\Gamma}_l + \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{z}_l \mathbf{z}_l^H - \mathbf{Z}(\omega) \mathbf{Z}^H(\omega).$$
(2.40)

In Equation(2.40),  $\Gamma_0, \dots, \Gamma_{L-1}$  are  $M \times M$  sub-block matrices located on the main diagonal of matrix **UKU**<sup>T</sup>.

Finally, the MAPES power spectral density estimator is expressed as

$$\Phi_{MAPES}(\omega) = \frac{|\alpha(\omega)|^2}{J}.$$
(2.41)

#### E. LSPR Method

LSPR is not necessary an acronym, however, the LSP stands for Lomb-Scargle periodogram and R stand for regression. As mentioned in Chapter 1, its foundation is built on LS. It uses the output from LS as inputs to a harmonic regression. The algorithm is provided below as shown in [12].

#### LSPR algorithm

- 1. Detrend data and denote it as  $\dot{y}$ .
- 2. Smooth detrended data  $\dot{y}$  with the fourth-order Savitzky-Golay algorithm and denote the resulting data as  $\ddot{y}$ .
- 3. Apply LS on both  $\dot{y}$  and  $\ddot{y}$  and select periods  $\{\dot{T}_j\}$  and  $\{\ddot{T}_j\}$ .
- 4. Use  $\{\dot{T}_j\}$  and  $\{\ddot{T}_j\}$  as inputs into a harmonic regression for  $\{\dot{y}\}$
- 5. Employ Akaike information criterion (AIC) to find the best harmonic regression model and p-value of  $\{\dot{y}\}$  from Step 4.
- 6. Set FDR to be less than 0.05.

Harmonic regression is then used to fit the detrended data  $\dot{y}$  with sinusoidal functions as:

$$\dot{y}_l = \mu + \sum_{j=1}^J \alpha_j \cos(\frac{2\pi}{T}t_l + \phi_j) + \varepsilon_l, t \qquad (2.42)$$

where  $\mu$  is the mean of  $\{\dot{T}_j\}$ ,  $\alpha_j$  are the amplitudes of the predictor trigonometric functions,  $\phi_j$  are the phases of the peaks relative to the time zero,  $\varepsilon_l$  are uncorrelated noise, and  $T_j$  are the periods inferred from LS. The smoothing version of the detrended data produced worse results than the original dataset and hence simulations are limited to the detrended data but the poor performance of the smoothed data are also shown in some pertinent simulations. In Chapter 4, simulations of LSPR are only for  $\dot{y}$ , the detrended data. If it is necessary to compare the performance of the smoothed detrended data, it will be clearly stated. The advantage to this method only serves to reduce the number of false positives that Lomb-Scargle periodogram produces but does not improve on recovering misses that LS failed to observe. LSPR assumes that the trend in the data is known, and by removing it, a limitation of Lomb-Scargle is eliminated, but if the data contained outliers their effect will still be felt and LS will provide spurious peaks which will then propagate through the LSPR algorithm to come to a similar conclusion just like LS.

#### CHAPTER III

#### SIGNIFICANCE TESTING<sup>1</sup>

#### A. Periodicity Test

A time series data of length N is used as an input to each of the schemes to obtain power spectral density estimates. Based on peaks from the outputs of the schemes, the data is classified as cyclic or non-cyclic. The null hypothesis is taken to be that the measurements are originated from a Gaussian noise stochastic process [1]. There are a host of tests that can be employed to access the significance of peaks detected by the schemes. Akaike's information criterion (AIC) has been used by [30] to test for periodicity. Stoica employed the Bayesian information criterion (BIC) [9], Glynn used the Fisher test [8] to search for periodicity in *Plasmodium falciparum* microarray gene expression dataset. The likelihood ratio test has been used in [15], Fan [31] showed that  $\chi^2$  test can also be employed to determine the significance of the detected peaks. However, Stoica [11] implied that there was no satisfactory algorithm or approach for testing significance of detected peaks in the case of irregularly sampled, however one can use Fisher's test to determine the significance of peaks detected in a power spectral density estimator  $\Phi(\omega)$  without any drop in performance when compared with other methods [1]. The Fisher's test statistic is defined as

$$T = \frac{\Phi(\omega_{k_{max}})}{N_0^{-1} \sum_{1 \le k \le N_0} \Phi(\omega_k)} , \qquad (3.1)$$

<sup>&</sup>lt;sup>1</sup>Part of this chapter is reprinted with permission from "Detecting periodic genes from irregularly sampled gene expressions: a comparison study" by Zhao, W. and Agyepong, K. and Serpedin, E. and Dougherty, E.R., vol.2008, *EURASIP Journal on Bioinformatics and Systems Biology*, 2008, Copyright 2008 by Zhao and Agyepong and Serpedin and Dougherty. Originally published by SpringerOpen

where  $N_0 = \lfloor (J-1)/2 \rfloor$  for the defined symmetric frequency grid and the highest peak is  $\Phi(\omega_{k_{max}})$ .

Our synthetic data simulations also included testing for multiple peaks. This necessitated the use of Whittle's second peak detection formulation [32], since Fisher's test was only defined for the highest peak. Whittle's second peak detection statistics is defined as

$$T_{2} = \frac{\Phi(\omega_{k_{2}})}{\sum_{k=1}^{N_{0}} \Phi(\omega_{k}) - \Phi(\omega_{k_{max}})},$$
(3.2)

where  $\Phi(\omega_{k_2})$  stands for the second highest peak. The *p*-value for detecting the largest peak is then given as [31]

$$P(T > t) = 1 - e^{-N_0 e^{-t}}.$$
(3.3)

The distribution for Fisher's test Equation(3.1) and Whittle's test Equation(3.2) is similar to that of Equation(3.3). The *p*-value measures the likelihood of obtaining such a peak if the series were generated by noise alone. Whereby a small *p*-value will give the indication that there is a small chance obtaining such a peak if the measurement were of noise alone. A *p*-value threshold serves as a threshold to decide if time series measurement contains any rhythms that are not due to chance. A rejection of the null hypothesis will imply that the magnitude of a frequency in the power spectral density is appreciably bigger than the mean and the time series data are samples from a periodic signal. For more details on the *p*-values, please see Fisher [33] or Brockwell [34].

Once the *p*-values are calculated for each time series or gene, they are ranked in ascending order and the threshold is employed to obtain significant results.

#### B. Multiple Testing Correction

For just one test, a fixed *p*-value is acceptable. For example, if the *p*-value is set to 0.05, the implication is that there is a 5% chance that the results obtained are not true positive. A 5% chance of false positives is high especially when considering over 6000 tests. To overcome the above problem, multiple testing approach must be used to control the results of the tests that were significant and not for all test. As proposed in [35] and [36], multiple testing correction is needed to control the false discovery rate (FDR). For each time series or measured gene, a *p*-value is calculated from the spectral density estimator or periodogram and used to test for periodicity. The *p*-values are ranked in an increasing order with the smallest *i*th *p*-value designated by  $p_{(i)}$  [1]. For real biological data, the estimate for the number of non-cyclic genes among all *n* genes is taken to be  $\hat{n}_0$ ; it is acceptable to take  $\hat{n}_0 = n$ . The testing procedure make inference on the *k* genes with the lowest *p*-values, by using an adjusted *p*-value obtained from the FDR approach defined as

$$\widehat{FDR}_k = \frac{p_{(k)}\widehat{n_0}}{k},\tag{3.4}$$

where  $p_{(k)}\hat{n_0}$  is an estimate of the number of false positives. Estimate of FDR,  $\widehat{FDR}$ , is not a monotonic function of k, the number inferred to be periodic. This makes it hard to choose a p-value threshold [1]. Storey [35] solved this problem by proposing an FDR adjusted p-value called q-value and is given by the following

$$q_k = \min_{k \le j \le n} \widehat{FDR}_j. \tag{3.5}$$

The q-value defined by Equation(3.5) is a monotonically increasing function of k. By specifying a q-value threshold as  $\tau$ , the FDR can be controlled and through that the

number of time series or genes to be inferred as periodic can then be derived as

$$k = \max_{1 \le j \le n} q_j \le \tau. \tag{3.6}$$

#### CHAPTER IV

#### SIMULATION: ARTIFICIAL AND BIOLOGICAL DATA

#### A. Results

A natural query is the question of how to assess the performance of these schemes. The schemes are implemented to investigate the smallest number of samples that each requires to obtain significant results. A purely sinusoidal signal sampled irregularly with a Poisson sampling process was utilized. The schemes were then applied on artificial datasets, obtained from a periodic signal mixed with Gaussian noise and a non-periodic signal, to evaluate their ability to infer periodic signals in the presences of non-idealities. Performance was evaluated based on different p-value thresholds for a fixed sample size. The ability of the schemes under different signal to noise ratio (SNR) was also investigated for a fixed sample size and p-value. The computation time required by each scheme for different sample sizes was also analyzed. An ancillary aim classified the schemes under undesirable characteristics of the microarray dataset, i.e., missing values, sample size, and presence of noise. Finally, the best scheme is applied on two data sets to attempt bridging the gap of disparities in the reported results of periodically expressed genes for yeast, found in literature.

#### 1. Simulation on Artificial Data

A purely sinusoidal signal was irregularly sampled to investigate the minimum number of samples each scheme needed to obtained significant results. For each N in Figure (1), the p-value was calculated as discussed Chapter 3 for each correctly inferred period in our signal, this technique is similar to that performed by Gylnn [8] for Lomb-Scargle periodogram. An approximation to the minimum number of samples that each

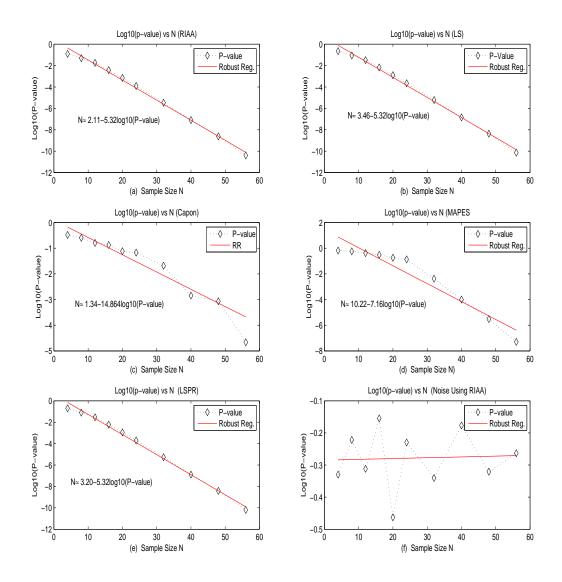


Fig. 1: Determination of sample size.

scheme needs is illustrated in Figure (1) where Matlab's version of robust regression is used to obtain estimates of N . From Figures 1b and 1e, LS and LSPR needed approximately 12 samples and RIAA needed only 9 samples Figure (1a) to produce significant results for a p-value of 0.05. It can be seen from Figure (1c) that the Capon method needed the largest number of samples to obtained significant results. The reason is that the Capon method requires a tradeoff between resolution and statistical accuracy when it comes to the choice of the filter length. Our simulation revealed that choosing the filter length to be approximately equal to one half of the data length, a balance was established for both resolution and accuracy in the estimation of the covariance matrix for the Capon method. It is not surprising that LS and LSPR both needed the same number of samples, as much as LSPR attempts to obtain best fit models from its harmonic regression, as its core is based on LS. Table I shows the number of samples that each scheme needed to show significant results with p-value threshold set at 0.05 and 0.005, respectively. The choice of these p-values is explained later in the chapter.

Table I: Minimum number of samples needed based on p-value thresholds of 0.05 and 0.005

| Method | N <sub>0.05</sub> | N <sub>0.005</sub> |
|--------|-------------------|--------------------|
| RIAA   | 9                 | 14                 |
| LS     | 12                | 19                 |
| MAPES  | 20                | 26                 |
| Capon  | 22                | 26                 |
| LSPR   | 12                | 19                 |

RIAA required the smallest number of samples when the p-value was selected to be more stringent. However, Capon and MAPES approximately needed the same number (26) of samples to obtain significant results. It must again be highlighted that this only provides approximate values for the sample size needed.

#### 2. Artificial Data Model

A modeled to generate artificial data set is given as follows:

$$y_l = \alpha \cos(\omega l + \phi) + \epsilon_l, \tag{4.1}$$

where  $l = 0, ..., N - 1, \phi \in (-\pi, \pi]$  and  $\epsilon_l$  are i.i.d. noise sequence.

Two cases of non-idealities were considered: (1) Addition of Gaussian noise and (2) Addition of non-periodic data and Gaussian noise. Figure (2) shows a signal composed of non periodic pulses and Gaussian noise with zero mean and unit variance which was added to our data model. The pulses represents mRNA bursts that are characteristic for microarray data sets. An experiment similar to [15] was conducted where two thousand time series of length N = 18, 48, and 100 were generated. One hundred of the time series are generated from our data model in Equation (4.1) to be periodic and 1900 non-periodic. For each series, the *p*-value was evaluated and the testing methodology discussed in Chapter 3 was employed for FDR with *q*-values equal to 0.05, 0.01 and 0.005. The sampling was modeled as a Poisson process with parameter  $\lambda$ ; this ensured that sampling was done on an average of every  $\frac{1}{\lambda}$ s. The Poisson process will inherently bring an irregular sampling format that will mimic microarray datasets characterized by uniform sampling, but with ample number of missing values.

Table II on page 27 shows the number of signals inferred to be periodic by each scheme when the number of samples time points N equal to 18 for q-values 0.05, 0.01

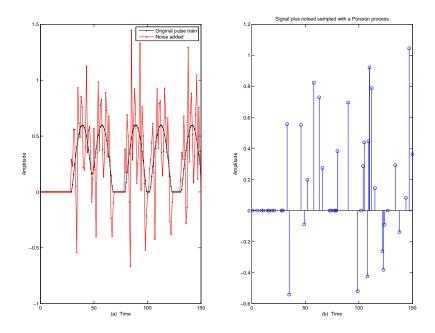


Fig. 2: A non-periodic signal of pulses.

and 0.005. With limited number of samples and SNR 5dB, RIAA was able to detect more periodic components in the data per q-value threshold than any other method. This is important because most microarray datasets have limited number of sample points and a scheme that can detect periodic components with limited resources is of premium. The number in parentheses are true positives. LSPR was employed for only the detrended data.

|        | q-value |        |        |  |  |
|--------|---------|--------|--------|--|--|
| Method | 0.05    | 0.01   | 0.005  |  |  |
| RIAA   | 42(41)  | 29(29) | 15(15) |  |  |
| LS     | 27(21)  | 11(9)  | 1(0)   |  |  |
| MAPES  | 14(11)  | 10(6)  | 3(0)   |  |  |
| Capon  | 9(9)    | 6(0)   | 1(0)   |  |  |
| LSPR   | 23(21)  | 9(9)   | 0(0)   |  |  |

Table II: Inferred number of periodic time series: N=18

Table III: Inferred number of periodic time series: N=48

|        | q-value  |        |        |  |  |
|--------|----------|--------|--------|--|--|
| Method | 0.05     | 0.01   | 0.005  |  |  |
| RIAA   | 103(100) | 76(72) | 65(65) |  |  |
| LS     | 111(89)  | 68(59) | 54(54) |  |  |
| MAPES  | 109(84)  | 72(64) | 53(53) |  |  |
| Capon  | 105(86)  | 66(61) | 54(53) |  |  |
| LSPR   | 111(89)  | 68(59) | 54(54) |  |  |

When the number of samples was increased to 48, Table III shows that RIAA still

outperforms the other three schemes. When the number of samples was changed to 100 time points in Table IV, all the schemes were able to accurately preserve the periodic components in the dataset when the q-value was set as 0.05 and 0.01, respectively. However, the false positives in RIAA and Capon were less than all other schemes the q-value was set as 0.005.

|        | q-value  |          |          |  |  |
|--------|----------|----------|----------|--|--|
| Method | 0.05     | 0.01     | 0.005    |  |  |
| RIAA   | 105(100) | 101(100) | 100(100) |  |  |
| LS     | 111(100) | 107(100) | 104(100) |  |  |
| MAPES  | 117(100) | 113(100) | 101(100) |  |  |
| Capon  | 113(100) | 111(100) | 100(100) |  |  |
| LSPR   | 105(100) | 102(100) | 101(100) |  |  |

Table IV: Inferred number of periodic time series: N=100

The schemes were also compared on their ability to infer closely embedded multiple frequencies in the data set. Gaussian noise was added to sinusoids with frequencies,  $f_1 = 0.29Hz$ ,  $f_2 = 0.32Hz$  and sampled irregularly using the same Poisson process as in Figure (1), the signal to noise ratio was set to 3dB.

With only 16 samples, only RIAA is able to detect the embedded frequencies consistently. LS and LSPR were able to detect the frequencies but based on Figures 3b and 3e, our testing methodology would have resulted in a miss for these frequencies. Capon and MAPES performed poorly for 16 sample points.

However, when the number of samples were increased to 24 points, but with SNR of 2dB, Figure (4) shows the performance of the five scheme with the same frequencies as Figure (3),  $f_1 = 0.29Hz$ ,  $f_2 = 0.32Hz$  and amplitudes 0.45, and 0.35

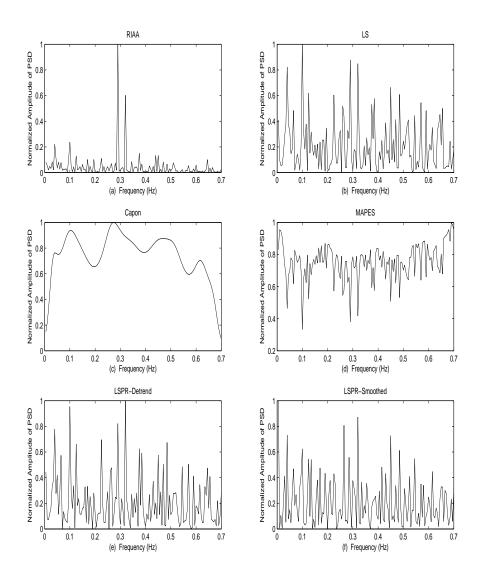


Fig. 3: Performance comparison based on an artificial data set (N=16) with sinusoids with frequencies  $f_1 = 0.29Hz$ ,  $f_2 = 0.32Hz$  (a) RIAA (b) LS (c) Capon (d) MAPES (e) LSPR Detrend (f) LSPR Smoothed

respectively. From the graph, it is obvious to see that RIAA does not suffer from detrimental sidelobes nor mainlobe leakages that LS and LSPR appear to exhibit. Still, both Capon and MAPES are lacking behind in detecting the frequencies. They are able to detect the frequency at 0.29Hz but not the frequency at 0.32Hz.

When the sample size is increased to 100 points, SNR still at 2dB, Figure (5) shows that Capon and MAPES improved dramatically. However, the smoothed version of LSPR could still not detect the two frequencies consistently. The number of samples had to be increased to over 200 samples points before it detected the two frequencies. Such a method is not ideal for microarray data sets where sample size is of premium. An auxiliary interest was to investigate the computational time required by each scheme.

From simulations, Figure (6) shows the disparities in computation time between MAPES and the other schemes. Due to the expectation maximization step in MAPES, it was the only scheme that required noticeable time in computing the power spectral estimates.

The ability of the schemes to detect a periodic signal, sampled with a Poisson process was investigated. With sample times points just 18 and SNR increased from 0 to 3dB, 200 simulations were run for each SNR value and Figure (7) shows the number of times the periodic signal was detected at the exact frequency. In Figure(7), RIAA at SNR=2.7dB was able to detect the embedded frequency out of the 200 simulations runs. It was not after 3dB that the other four schemes were able to detect the frequency for all 200 simulations runs with 18 time points sampled irregularly.

#### 3. Simulation on Spellman's Yeast Data

The schemes were then evaluated on a real biological data set from Spellman's experiment [2]. Performance was judged based on their ability to recover genes from

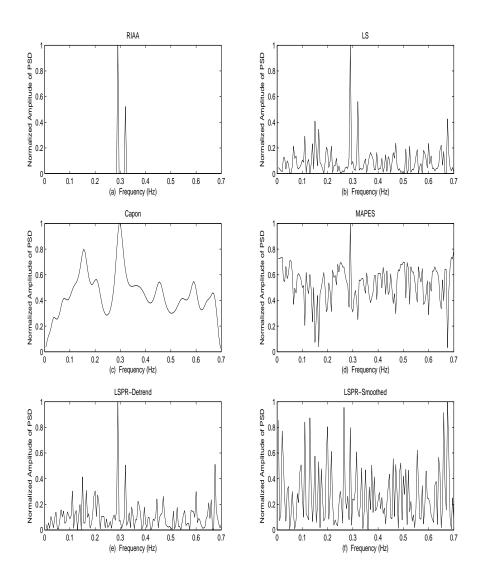


Fig. 4: Performance comparison based on artificial data set (N=24) with sinusoids with frequencies  $f_1 = 0.29Hz$ ,  $f_2 = 0.32Hz$ . (a) RIAA (b) LS (c) Capon (d) MAPES (e) LSPR Detrend (f) LSPR Smoothed

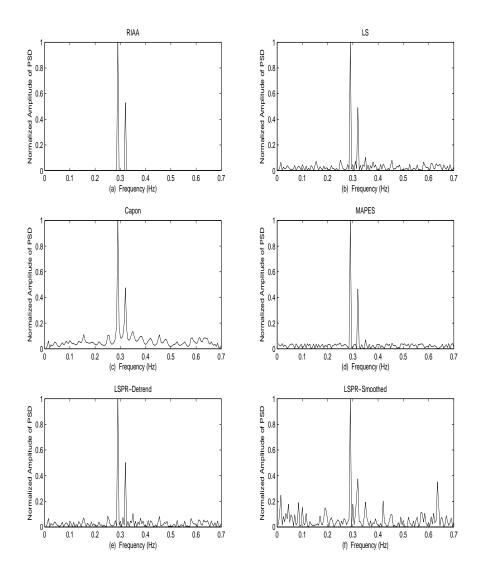


Fig. 5: Performance comparison based on an artificial data set (N=100) with sinusoids with frequencies  $f_1 = 0.29Hz$ ,  $f_2 = 0.32Hz$ . (a) RIAA (b) LS (c) Capon (d) MAPES (e) LSPR Detrend (f) LSPR Smoothed

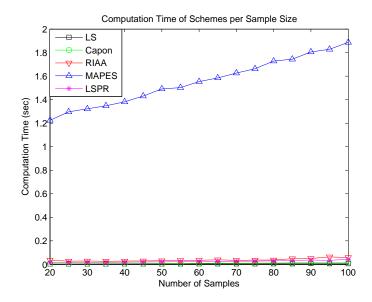


Fig. 6: Computational time for all schemes based on number of sample time points available.

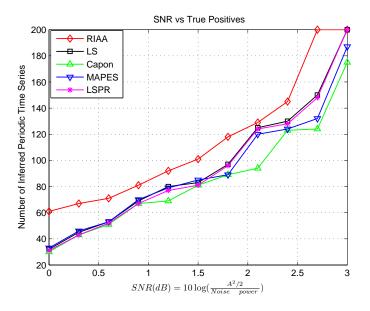


Fig. 7: Performance comparison as SNR is increased from 0 to 3dB.

a set of known periodic genes that were obtained from a small scale experiment. At the time of Spellman's work, there were 104 known periodic genes for the yeast, later in 2003, Johansson [7] added nine genes to provide researchers with 113 cell cycle regulated reference genes. From here on, the 113 cell cycle regulated genes will be referred to as Benchmark set A.

As mentioned in Chapter 1, the standard method in evaluating the performance of schemes that seek to detect periodically expressed genes is to determine the percentage of the reference genes the scheme was able to infer as periodic. The best schemes are expected to have a high number of the reference genes present in the fewest number of inferred genes. For example, Spellman was unable to obtain 92% of the 104 reference genes until 800 genes were inferred or judged to be periodic.

For comparison, the dataset for Cdc15 arrest and Alpha arrest synchronization from the experiment of Spellman [2] were used. Cdc15 data set had 24 sample time points and Alpha data set had 18 time points, there were too few samples for cdc28 and elutriation synchronization data and thus not ideal for Capon as has been demonstrated via artificial data simulation.

The comparison procedure was done as follows, based on the given dataset, each schemes infer a pre-specified number of genes. The inferred genes are designated as periodically expressed genes and are crossed with Benchmark set A. A percentage is obtained from the number of the referenced gene set that are present in the pre-specified number of genes inferred. This is illustrated in Figure (8) where the superiority of RIAA is clearly demonstrated in identifying more known periodically expressed genes than any other scheme for the Cdc15 experiment.

Capon method however performed much better on biological data set than the MAPES based on the criteria used to measure performance. Since only one frequency was believed to be present, resolution for the Capon method was sacrificed in favor of

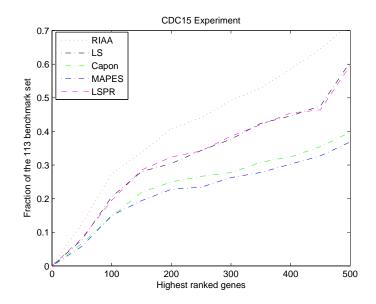


Fig. 8: Performance comparison based on cdc15 arrest data set.

accuracy and this gave the Capon more samples to use within the confines of its methodology. As mentioned previously, the Capon method needs to decide on a tradeoff between resolution and accuracy and the filter length plays a central role in this tradeoff. A small filter length affects the resolution especially in the case when there is a need to differentiate between two closely embedded frequencies. As expected with LS and LSPR, there was no appreciable performance separation between the two.

Applying the schemes on the Alpha data set, RIAA continued to demonstrate its efficacy in matching the referenced genes set per pre-specified inferred genes. With only 18 time points available with some genes having missing data as well, MAPES outperformed the Capon method on this dataset. Again the performance of LS and LSPR were almost identical. As can be seen from Figure (9), there was a slight drop off in the percentage of referenced genes that RIAA and all the other schemes were able to pick, this was expected and understandable with the limited time points available for the Alpha data set. From these figures, it is easy to see that RIAA outperforms the other schemes and should be the analysis tool of choice when the goal of an analysis on a microarray experiment data set is to seek periodically expressed genes.

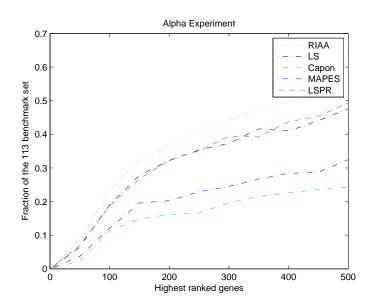


Fig. 9: Performance comparison based on Alpha arrest data set.

### B. Discussion and Conclusions

The datasets of Spellman [2] and Pramila [24] were analyzed using RIAA. Pramila's alpha arrest experiment data set has 25 samples and Spellman's cdc15 experiment has 24 time points. There were numerous missing data points rendering the data set as irregularly sampled. With a q value set to be not more than 0.05, 609 genes were adjudged to be periodic in Spellman's dataset and 596 in Pramila's dataset, the results are shown in Table B1 and B2 respectively in Appendix B. An overlap of 543 genes was obtained between the two data sets. Using RIAA, the results obtained

establishes a better level of clarity in the overlap of periodically expressed genes between two different datasets. The overlap of 543 genes is appreciably more than any two different results reported on the yeast which can be found on [37].

Compared with Spellman [2], there was only an overlap of 357 genes. RIAA using only 550 genes detected 97% out of 104 genes that were known to be cell cycled regulated at the time of Spellman's work while Spellman's method only got 92% out of 800 genes. It must be added that expression profiles of the genes may be more complex than simple sinusoidal curves, however, the visual inspection of the time series profile reveals that the genes inferred to be periodic appeared as sinusoidal in nature and made the assumption sinusoids sound. The earlier work in [1], concluded that LS was effective and an accurate tool to use, but through artificial simulations, it has been seen that LS can be sensitive to large outliers that could be present due to perturbation in the measurement environment. RIAA does not suffer from such sensitivity and is innately designed to limit false discoveries. The Capon filter is a powerful tool, also robust to the presence of noise, but desires a bigger sample size than a typical microarray data sets provides. MAPES is computational expensive and also requires much more data points to be effective in inferring periodically expressed gene from microarray experiments. LSPR turned out not to outperform LS in terms of the actual number of true positives but reduced the number of false positives that LS picks. It is recommended to future researchers seeking to find periodically expressed genes in a microarray experiment to employ RIAA as it has been proven to be an effective tool in identifying periodic gene expression profiles. It is robust to small sample sizes, missing data or clusters of missing data and irregularly sampled data.

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### APPENDIX A

### CODES

### 1. Matlab codes

Lomb-Scargle function

function psd = LombScargle(T,X,W)

% this function is to use loom-scargle

% inputs:

- % T time points
- % X sampled data
- % W frequencies
- % outputs:

% psd - power spectral density corresponds to the frequencies std\_X = std(X);

```
mean_X = mean(X);
```

```
for k = 1:length(W)
tau = 1/2/W(k) * atan(sum(sin(2*W(k)*T))/sum(cos(2*W(k)*T)));
psd(k) = 1/2/std_X^2 * ( sum((X-mean_X).*cos(W(k)*(T-tau)))^2
/sum(cos(W(k)*(T-tau)).^2) ...
+ sum((X-mean_X).*sin(W(k)*(T-tau)))^2/sum(sin(W(k)*(T-tau)).^2) );
end
```

**MAPES** Function

```
function PSD = pmapes(X,T,W)
% T has to be integers
%W = 0.05:0.05:pi;
n = T(end)-T(1);
```

if size(X,1) == 1 % row vector X = X.'; % change it to column vector end if size(T,1) == 1 % row vector T = T.'; % change it to column vector end % ---initilization-----% set 0 to missing data XX = [];avail = []; % availability for k=1:length(T) XX = [XX, X(k)];avail = [avail,1]; if  $k^{-1}(T) \& T(k+1)-T(k)>1$  % not the tail, therefore k+1 is valid XX = [XX, zeros(1, T(k+1)-T(k)-1)]; % set zeros to missing positions avail = [avail,zeros(1,T(k+1)-T(k)-1)]; end

er

end

```
N = length(XX);
miss = ones(1,N) - avail;
g = length(X); % # data available
M = ceil(N/2);
L = N-M+1;
% initilize Q, Sg, Sm
Sg = zeros(L*M,g);
Sm = zeros(L*M,N-g);
for 1 = 0: (L-1)
   for k = 1:M
       if avail(1+k) == 1 % there is a datum here
          Sg(l*M+k,sum(avail(1:l+k))) = 1;
       else
              % there is a miss here
          Sm(l*M+k,sum(miss(1:l+k))) = 1;
       end
   end
end
Sg_tilde = (inv(Sg.'*Sg)*Sg.').';
Sm_tilde = (inv(Sm.'*Sm)*Sm.').';
```

```
for n = 1:length(W)
w = W(n);
% ---initilize alpha-----
```

```
alpha = sqrt(LombScargle(X,T,w));
    a = \exp((0:M-1)*j*w).';
    % initilize Q
    Q = \operatorname{zeros}(M, M);
    for 1 = 0: (L-1)
        yl = XX((1+1):(1+M)).';
        Q = Q + (yl-alpha*a*exp(j*w*l))*(yl-alpha*a*exp(j*w*l))';
    end
    Q = Q/L;
% ---start iterations of EM-----
        % arbitrarily set error to a large value
e = 0;
alpha_old = inf;
rho = [];
for 1=0:L-1
    rho = [rho; exp(j*w*1)*a];
end
loops = 0;
while abs(alpha-alpha_old)/abs(alpha) > 0.1 && loops<100</pre>
    loops = loops+1;
    alpha_old = alpha;
D = Q;
for l=1:L-1
    D = [D,zeros(size(D,2),M);zeros(M,size(D,2)),Q];
```

```
b = Sm_tilde.'*rho*alpha + Sm_tilde.'*D*Sg_tilde*
inv(Sg_tilde.'*D*Sg_tilde)*(X-Sg_tilde.'*rho*alpha);
K = Sm_tilde.'*D*Sm_tilde + Sm_tilde.'*D*Sg_tilde*
inv(Sg_tilde.'*D*Sg_tilde)*Sg_tilde.'*D*Sm_tilde;
```

```
S_tilde = zeros(M,M);
Z = zeros(M, 1);
SmKSm = Sm*K*(Sm.');
SgrSmb = Sg*X+Sm*b;
for l=0:L-1
Gammal = SmKSm((l*M+1):(l*M+1),(l*M+M):(l*M+M));
zl = SgrSmb((l*M+1):(l*M+M));
S_tilde = S_tilde + Gammal + zl*zl';
Z = Z + z l * exp(-j * w * l);
end
Z = Z/L;
S_tilde = S_tilde/L - Z*Z';
S_tilde = S_tilde + 0.01*diag(diag(S_tilde)); %diagnol loading
invS_tilde = inv(S_tilde);
alpha = (a'*invS_tilde*Z)/(a'*invS_tilde*a);
Q = S_tilde + (alpha*a-Z)*((alpha*a-Z)');
end
```

```
PSD(n)=alpha;
```

end

end

```
PSD = abs(PSD).^2;
PSD=circshift(PSD,[0,1]);
```

**RIAA** function

```
function [P_RIAA ] = PSD_RIAA(X, W, T, N, K, s_no,)
```

```
P_RIAA = zeros(K, 1);
Theta = zeros(2, K);
set_A = zeros(N, 2, K);
for j = 1:K
    omega = W(j);
    set_A(:,:,j) = [cos(omega*t_n) sin(omega*t_n)];
```

end

```
% Initialization with Least Squares Periodogram
for k = 1:K
    A = set_A(:,:,k);
    Theta(:,k) = inv(A'*A) * A'*y;
```

end

```
% Power in signal estimation and initiation of iteration
num_o= 0;
flag = 1;
while flag
Theta_tmp = Theta;
```

```
Alpha_tmp = sqrt(Theta_tmp(1,:).^2 + Theta_tmp(2,:).^2);
gam = zeros(N, N);
y_esti = zeros(N, 1);
 for j = 1:K
               % calculate gam
     A = set_A(:,:,j);
    gam = Gam + (Theta(1,j)<sup>2</sup> + Theta(2,j)<sup>2</sup>) / 2 * A * A';
     y_est = y_est+ A * Theta(:, j);
 end
 in_Gam = inv(Gam);
for j = 1:K
     A = set_A(:,:,j);
     Theta(:,j) = inv(A' * in_Gam * A) * (A' * in_Gam * y);
 end
 num_o= num_o+ 1;
 if num_o>= stop_no
     flag = 0;
 end
 Alpha = sqrt(Theta(1,:).^2 + Theta(2,:).^2);
 sentinel = norm(Alpha - Alpha_tmp) / norm(Alpha_tmp);
 if (sentinel < 5e-3)
     flag = 0;
 end
```

 $\operatorname{end}$ 

```
for j = 1:K
A = set_A(:,:,j);
P_RIAA(j) = 1/N * Theta(:,j)' * (A' * A) * Theta(:,j);
```

 $\operatorname{end}$ 

Capon function

function P = pcapon(X,T,m,W)

- % function [P,W] = pcap(X,T)
- % power spectral density estimation by using capon method
- % irregular sampling
- % P power spectral
- % W frequency list
- % X input data sequence

% T - data sampling time points

% m - the order of the filter

X = (X-mean(X))/std(X);

per = LombScargle(T,X,W);
%per = pergram(t,X,W);

R = zeros(m+1,m+1);

wdelta = W(2)-W(1);

for k = 1:length(W)

```
a = exp(-i*wdelta*k*(0:m)).';
R = R + a*a'*per(k);
```

 $\operatorname{end}$ 

```
R = (R/length(W));
%R = R + diag(0.01*diag(R)); % diagnal loading
```

```
% from stoica's Forward-backward
```

```
J = zeros(m+1,m+1);
```

for k = 1:m+1

J(k,m+2-k) = 1;

 $\operatorname{end}$ 

R = 0.5\*(R+J\*transpose(R)\*J);

invR = inv(R);

```
for k = 1:length(W)
w = W(k);
a = exp(-i*w*(0:m)).';
P(k) = 1/(a'*invR*a);
```

 $\operatorname{end}$ 

Data generator

function [Perodata sampltimes N noise] = datagenerator(f,lamda,N,A)

m=N;

```
times= exprnd(1/lamda,[m 1]);
times = cumsum(times);
```

```
num_sinu=length(A);
```

sampltimes=times;

```
phi = 2*pi * rand([num_sinu,1]);
```

```
variance=0.01;
```

```
noise = sqrt(variance) * randn(m, 1);
```

```
y=(A'*cos(2*pi*f*times' + repmat(phi, [1,m])))';
```

Perodata=y;

## APPENDIX B

# RESULTS

## 2. Spellman's dataset

# Table V: RIAA Cyclic genes from Spellman's dataset

| YMR215W | YIL132C | YCR040W   | YBL009W | YNL044W | YIL052C |
|---------|---------|-----------|---------|---------|---------|
| YBL002W | YBR038W | YLR170C   | YBR189W | YCR084C | YML119W |
| YPL163C | YGR099W | YOL105C   | YGR189C | YIL146C | YDR345C |
| YHR175W | YGL028C | YPR119W   | YPL267W | YOR058C | YGL253W |
| YMR305C | YGL089C | YJR022W   | YNL300W | YJL167W | YNL037C |
| YJL092W | YPL256C | YBR158W   | YDR055W | YPL273W | YDL133W |
| YHR086W | YOR070C | YJL200C   | YOR234C | YJL052W | YNL233W |
| YPL128C | YGL161C | YGL254W   | YOR127W | YBR214W | YJL122W |
| YDL055C | YLR342W | YKL127W   | YNL074C | YLR333C | YER088C |
| YLL028W | YBR071W | YPL090C   | YDR450W | YJR009C | YDR089W |
| YER001W | YJL174W | YER041W   | YDL082W | YHR052W | YFL037W |
| YBR243C | YBR092C | YGR086C   | YNL145W | YOL143C | YCL040W |
| YJL159W | YKL175W | YLR121C   | YPL168W | YFL026W | YEL026W |
| YNL283C | YDL164C | YLR390W-A | YOR025W | YER006W | YJL118W |
| YNL015W | YCL014W | YDR452W   | YJL079C | YNR014W | YKL184W |
| YKR042W | YOR084W | YPL158C   | YGL013C | YGR240C | YCR048W |
| YBR093C | YDL227C | YDL170W   | YHL027W | YHR174W | YNL289W |
| YAR007C | YPL127C | YGR092W   | YIL066C | YKR077W | YGR214W |

| YEL042W | YLR194C | YER026C | YNL162W   | YLR183C | YPL187W   |
|---------|---------|---------|-----------|---------|-----------|
| YGR044C | YMR011W | YJL157C | YDL191W   | YOR312C | YJL181W   |
| YDL224C | YDR097C | YLR325C | YDR224C   | YKL148C | YLR287C-A |
| YGR108W | YML083C | YOR264W | YMR023C   | YKR037C | YCR089W   |
| YML052W | YKL045W | YNR001C | YCR069W   | YML051W | YOR315W   |
| YOL012C | YGR065C | YNL160W | YML085C   | YKL185W | YML102W   |
| YIL123W | YGL200C | YDR302W | YDR146C   | YGR230W | YGR192C   |
| YKL096W | YMR189W | YBL032W | YNL192W   | YJR046W | YJR092W   |
| YLR286C | YOR383C | YMR058W | YER056C-A | YDL018C | YNR019W   |
| YKL066W | YHR005C | YCR067C | YLL041C   | YPR181C | YOR378W   |
| YAR071W | YGL008C | YNL327W | YJL051W   | YDL010W | YGR041W   |
| YKL165C | YMR163C | YMR179W | YBR083W   | YML064C | YKL067W   |
| YHR143W | YEL002C | YAR018C | YGL116W   | YPL242C | YJL187C   |
| YOL007C | YER070W | YJR010W | YAR050W   | YOL091W | YKR024C   |
| YDL003W | YHR211W | YGL154C | YPR032W   | YDR042C | YML099C   |
| YNL176C | YBL102W | YPR120C | YPL188W   | YOR023C | YLR275W   |
| YMR042W | YOR247W | YJL206C | YMR032W   | YNL078W | YOR322C   |
| YPR149W | YPL032C | YOL060C | YHR215W   | YGL255W | YLR180W   |
| YKL164C | YHR061C | YBR049C | YJL185C   | YBR196C | YBR102C   |
| YER095W | YKL001C | YJL137C | YOR009W   | YLR448W | YHR165C   |
| YML058W | YLR190W | YDR033W | YMR078C   | YGL093W | YNL112W   |
| YBR221C | YLR164W | YOR382W | YMR048W   | YLR437C | YPL190C   |
| YJL134W | YJR127C | YLR103C | YHR188C   | YBR181C | YGL090W   |
| YMR307W | YIL056W | YDR488C | YJL074C   | YER065C | YNR009W   |

|           |  |   |   | 1   |
|-----------|--|---|---|---|
| YLR373C   | YKR013W  | YNR068C   | YNL096C   | YPL081W   |
| YDL101C   | YDL155W  | YOL123W   | YBR200W   | YLR182W   |
| YIR018W   | YGL055W  | YPR019W   | YJL078C   | YLR176C   |
| YLR372W   | YOR176W  | YMR019W   | YBR142W   | YER124C   |
| YAL022C   | YOR019W  | YOR044W   | YKL180W   | YMR271C   |
| YDL066W   | YBL103C  | YJR021C   | YDL105W   | YDR297W   |
| YJR098C   | YOR358W  | YPL202C   | YJR150C   | YDR216W   |
| YBR295W   | YGR143W  | YDR534C   | YKR046C   | YDR379W   |
| YEL009C   | YGR029W  | YBR203W   | YOR371C   | YJR051W   |
| YER146W   | YIR038C  | YBL063W   | YDR309C   | YNL312W   |
| YLR048W   | YBR009C  | YEL017W   | YNL298W   | YGL184C   |
| YIL018W   | YKL025C  | YCR061W   | YOR308C   | YLR210W   |
| YER093C-A | YFR053C  | YBR067C   | YLL022C   | YPR030W   |
| YAR073W   | YDR115W  | YNL030W   | YLR005W   | YIL016W   |
| YKR071C   | YPR132W  | YHR149C   | YIL011W   | YDR463W   |
| YMR141C   | YCL067C  | YDR028C   | YGR166W   | YFR016C   |
| YDR436W   | YBR073W  | YNL248C   | YFR034C   | YKL104C   |
| YAL043C   | YIL131C  | YGL030W   | YGR272C   | YDR381W   |
| YML056C   | YBR219C  | YDR356W   | YOR205C   | YHR202W   |
| YHR123W   | YPR106W  | YPL221W   | YEL057C   | YCR065W   |
| YLR274W   | YDR481C  | YDR408C   | YDL179W   | YAR035W   |
| YGR079W   | YLR212C  | YGL237C   | YGL027C   | YGL259W   |
| YIR039C   | YDR085C  | YBR138C   | YAL024C   | YCR024C-A   |
| YBR240C   | YPL061W  | YOR221C   | YDR124W   | YKL008C   |
|           | YDL101CYIR018WYIR018WYLR372WYAL022CYDL066WYDL066WYBR295WYER098CYER146WYER093CYIL018WYER093C-AYAR071CYAR071CYAR071CYMR141CYDR436WYAL043CYAL043CYHR123WYLR274WYIR039CYIR039C | YDL101CYDL155WYIR018WYGL055WYLR372WYOR176WYAL022CYOR019WYDL066WYBL103CYJR098CYGR143WYBR295WYGR143WYER099CYGR029WYER146WYIR038CYLR048WYBR009CYLR048WYFR053CYAR073WYFR053CYAR071CYPR132WYMR141CYCL067CYMR141CYBR073WYAR073WYBR073WYAR043CYBR073WYAL043CYBR1131CYHR123WYPR106WYLR274WYDR481CYIR039CYDR085C | YDL101CYDL155WYOL123WYIR018WYGL055WYPR019WYLR372WYOR176WYMR019WYAL022CYOR019WYOR044WYDL066WYBL103CYJR021CYJR098CYOR358WYPL202CYBR295WYGR143WYDR534CYEL009CYGR029WYBR203WYER146WYIR038CYBL063WYLR048WYBR009CYEL017WYLR048WYFR053CYBR067CYAR073WYDR115WYNL030WYKR071CYPR132WYNL030WYMR141CYCL067CYDR248CYAL043CYBR073WYNL248CYML056CYBR219CYDR356WYHR123WYDR16WYDR408CYHR123WYDR481CYDR408CYIR039CYLR212CYBR138CYIR039CYDR085CYBR138C | YDL101CYDL155WYOL123WYBR200WYIR018WYGL055WYPR019WYJL078CYLR372WYOR176WYMR019WYBR142WYAL022CYOR019WYOR044WYKL180WYDL066WYBL103CYJR021CYDL105WYJR098CYOR358WYPL202CYJR150CYBR295WYGR143WYDR534CYKR046CYEL009CYGR029WYBR203WYOR371CYER146WYIR038CYBL063WYDR309CYLR048WYBR009CYEL017WYNL298WYLR048WYKR025CYBR067CYLR030WYER093C-AYFR053CYBR067CYLR025CYAR071CYPR132WYNL030WYLR05WYMR141CYCL067CYDR03CWYGR166WYMR141CYBR073WYNL248CYFR034CYML056CYBR219CYDR356WYGR205CYLR274WYDR481CYDR408CYDL179WYIR039CYDR085CYBR138CYGL027C |

|         |         | 1       | 1       |         | 1       |
|---------|---------|---------|---------|---------|---------|
| YLR284C | YCL024W | YAR002W | YMR021C | YDR416W | YKL048C |
| YGL037C | YER178W | YML041C | YIL152W | YGR109C | YHR141C |
| YOR095C | YGR034W | YIL133C | YOR120W | YDR386W | YCL027W |
| YMR317W | YDL028C | YLR455W | YIL050W | YBR021W | YDR025W |
| YDR067C | YOR182C | YOR016C | YGR279C | YKL113C | YGR068C |
| YLR367W | YER129W | YHR006W | YLR378C | YLR426W | YMR164C |
| YDL220C | YLR290C | YKR010C | YOR198C | YDL239C | YDR446W |
| YDL142C | YOR288C | YDR047W | YNL002C | YKR019C | YOL036W |
| YMR016C | YLR409C | YER118C | YLR457C | YGR075C | YIR022W |
| YDR528W | YOR204W | YDR425W | YNR047W | YPR001W | YLR326W |
| YBR267W | YPR156C | YJL063C | YLR353W | YGR220C | YDR421W |
| YGR027C | YHR153C | YOR272W | YDR507C | YLR131C | YOL127W |
| YDL194W | YKR099W | YOL090W | YGL062W | YGL207W | YHL047C |
| YBR130C | YER111C | YEL032W | YKL011C | YGR282C | YMR261C |
| YLR288C | YKR094C | YER075C | YOR153W | YJR137C | YBL003C |
| YAL032C | YDL012C | YLR213C | YOR178C | YPL089C | YHR158C |
| YOL070C | YPL014W | YFL033C | YMR199W | YLR049C | YDR255C |
| YHR094C | YGR288W | YKL020C | YMR184W | YEL050C | YBL054W |
| YOR142W | YDL087C | YJL056C | YPL153C | YOR313C | YER167W |
| YDR310C | YIL129C | YJR155W | YHR203C | YAR008W | YLR304C |
| YOR338W | YFL021W | YMR031C | YOR310C | YLR394W | YGR013W |
| YNL103W | YIL122W | YMR145C | YOR122C | YNL069C | YMR070W |
| YLR313C | YCL061C | YER089C | YPL146C |         |         |

### 3. Pramila's dataset

| Table VI: RIAA | Cyclic genes | from | Pramila's dataset |
|----------------|--------------|------|-------------------|
|                |              |      |                   |

| YDR225W | YMR076C | YDL018C | YPR019W | YBR202W   | YFL008W |
|---------|---------|---------|---------|-----------|---------|
| YBL003C | YPL127C | YOL090W | YPL061W | YPL116W   | YPL255W |
| YNL300W | YKL113C | YMR215W | YDR507C | YBL009W   | YPL124W |
| YBR009C | YNL312W | YMR011W | YMR179W | YGR109C   | YDL093W |
| YER070W | YDR055W | YLR274W | YOR273C | YOR114W   | YAL040C |
| YNL030W | YGR189C | YJL115W | YLR254C | YCR065W   | YLR342W |
| YBL002W | YAR007C | YML058W | YER003C | YML060W   | YML033W |
| YPL163C | YKL101W | YJL074C | YDL101C | YDL156W   | YOR073W |
| YDR224C | YBL035C | YIL026C | YMR003W | YJL157C   | YGR014W |
| YNL289W | YER001W | YOR247W | YDL197C | YLL021W   | YDL096C |
| YBR089W | YIL140W | YDR097C | YNL233W | YER032W   | YOR229W |
| YDL003W | YHR152W | YNL058C | YKL045W | YOR373W   | YBR067C |
| YJL159W | YBR070C | YNL126W | YGR152C | YNL057W   | YKL008C |
| YBR010W | YLR103C | YLR194C | YCR042C | YNL088W   | YLR383W |
| YPL256C | YGL021W | YEL032W | YMR307W | YLL022C   | YGR221C |
| YFL026W | YNL145W | YMR078C | YIL131C | YCR024C-A | YNL166C |
| YOL007C | YHR154W | YLR045C | YBR088C | YKR042W   | YNL192W |
| YNL031C | YPL267W | YGR092W | YNL262W | YOR083W   | YBL111C |
| YLR183C | YDR222W | YPL153C | YLR121C | YJL073W   | YJR030C |
| YNL102W | YMR031C | YIL106W | YDR297W | YJL019W   | YKL104C |
| YOR074C | YGL116W | YER095W | YHR172W | YDL055C   | YGR099W |
| YBR071W | YKL209C | YOR195W | YCL061C | YIL123W   | YNL082W |

|         |           | 1       |         | 1       |         |
|---------|-----------|---------|---------|---------|---------|
| YOR066W | YCL024W   | YDL164C | YER111C | YBR139W | YDR545W |
| YHR005C | YDR113C   | YFL067W | YPR135W | YML061C | YBR073W |
| YJL187C | YKR013W   | YEL061C | YMR199W | YOR321W | YDR400W |
| YFL008W | YEL076C-A | YLR463C | YPL141C | YGR286C | YHR023W |
| YPL255W | YCL040W   | YER037W | YGL225W | YOR127W | YDL155W |
| YPL124W | YML027W   | YJL173C | YPR175W | YOL069W | YDR518W |
| YDL093W | YGR098C   | YBL023C | YJR006W | YNL339C | YHR151C |
| YAL040C | YBR275C   | YEL076C | YGR140W | YCR005C | YJL225C |
| YLR342W | YEL017W   | YML085C | YPL221W | YGR296W | YDR481C |
| YML033W | YLR273C   | YHL026C | YPL057C | YMR001C | YIL158W |
| YOR073W | YEL076W-C | YHR146W | YKL067W | YMR306W | YAR018C |
| YGR014W | YDL103C   | YNL072W | YJR143C | YLL002W | YMR190C |
| YDL096C | YJL051W   | YBR243C | YFL065C | YMR132C | YBR296C |
| YOR229W | YOR058C   | YFL006W | YIL159W | YOL158C | YDR379W |
| YBR067C | YDR077W   | YBL113C | YLR313C | YLR467W | YLR341W |
| YKL008C | YFL037W   | YOR144C | YDR528W | YFL027C | YPR174C |
| YLR383W | YGR279C   | YNR001C | YHR218W | YOR313C | YKL089W |
| YGR221C | YDR191W   | YER118C | YOR246C | YNL338W | YMR006C |
| YNL166C | YNL273W   | YKR010C | YHL021C | YNL150W | YOR288C |
| YNL192W | YPL032C   | YKR098C | YLR032W | YDR146C | YKR037C |
| YBL111C | YML052W   | YPR018W | YMR292W | YOR248W | YMR253C |
| YJR030C | YGL027C   | YKL042W | YFL068W | YHL050C | YBR093C |
| YKL104C | YDR488C   | YBL031W | YLR455W | YDL138W | YDR503C |
| YGR099W | YMR048W   | YGR143W | YIR010W | YLR326W | YPL242C |

|         | 1       |         | 1       |         | 1       |
|---------|---------|---------|---------|---------|---------|
| YNL082W | YKL185W | YDL102W | YIL122W | YBR138C | YDR307W |
| YDR545W | YDR279W | YIL015W | YEL075C | YKL165C | YJR053W |
| YBR073W | YAL039C | YBL109W | YCL012W | YLR234W | YLR049C |
| YDR400W | YFL066C | YJR092W | YLR386W | YML125C | YPR076W |
| YKR090W | YNL111C | YHR086W | YDR501W | YBR140C | YOR111W |
| YGR188C | YOL138C | YIL066C | YOL025W | YOR363C | YDR460W |
| YPL209C | YER114C | YHR158C | YGL207W | YHR159W | YGR075C |
| YOR372C | YJL155C | YDL105W | YDR516C | YDL056W | YOR315W |
| YDR464W | YDR219C | YBR015C | YER190W | YBR028C | YPR139C |
| YGL061C | YEL042W | YOR016C | YPR004C | YDR457W | YOR026W |
| YBR086C | YBR072W | YOL124C | YOR307C | YOR033C | YNL165W |
| YPL283C | YML069W | YDL095W | YER016W | YNL309W | YEL031W |
| YML133C | YNL310C | YPR031W | YIR044C | YCR090C | YBR153W |
| YJL044C | YPR203W | YLR074C | YPL004C | YKL048C | YGR022C |
| YHL049C | YML119W | YAR008W | YGR142W | YML021C | YMR075W |
| YLR247C | YBR087W | YJR076C | YER189W | YDR436W | YDR544C |
| YBR038W | YDR261C | YGL037C | YDR245W | YLR210W | YKL129C |
| YNL334C | YBR042C | YJR054W | YLL067C | YMR117C | YLR457C |
| YLR462W | YBR092C | YFL060C | YBR242W | YAL007C | YJL185C |
| YPR149W | YLR182W | YLL032C | YDR277C | YIL155C | YML034W |
| YLR464W | YAL024C | YHR217C | YLR151C | YLR465C | YER053C |
| YOR176W | YLR380W | YDL248W | YDR147W | YNL335W | YKL052C |
| YEL077C | YOR233W | YBR187W | YPR035W | YKL225W | YIL177C |
| YPL208W | YOL019W | YBL034C | YDL163W | YKL210W | YOL147C |

|         |         | 1       |           |           |           |
|---------|---------|---------|-----------|-----------|-----------|
| YGR292W | YJR043C | YPR202W | YJR112W   | YHR136C   | YOR188W   |
| YBR161W | YDR052C | YGR108W | YGL253W   | YLR372W   | YFL064C   |
| YLR382C | YLR373C | YLR466W | YDR089W   | YDL211C   | YNL176C   |
| YLL066C | YMR160W | YNL263C | YGL065C   | YDL011C   | YPL253C   |
| YJL092W | YLL031C | YEL040W | YMR251W-A | YBL112C   | YBR103W   |
| YOR084W | YGL200C | YPR034W | YMR030W   | YCR023C   | YNL095C   |
| YIL127C | YJR003C | YOR017W | YBR289W   | YAL034W-A | YAL033W   |
| YDR440W | YNL062C | YHR215W | YNL180C   | YPL128C   | YNL291C   |
| YKL004W | YCL001W | YOR162C | YAL023C   | YJL151C   | YGR089W   |
| YOR326W | YOR095C | YNL333W | YHR169W   | YGL163C   | YPL144W   |
| YGR153W | YMR247C | YLR250W | YDR537C   | YNL160W   | YIL156W   |
| YGL216W | YGL013C | YDR227W | YJL034W   | YJL186W   | YDL028C   |
| YML065W | YMR032W | YHR127W | YBL071C   | YER105C   | YLR025W   |
| YMR144W | YPR052C | YNL149C | YBR276C   | YFL044C   | YGL050W   |
| YJL137C | YDR489W | YPL007C | YLR063W   | YMR197C   | YGR026W   |
| YKL160W | YOL030W | YKL049C | YIL144W   | YOR025W   | YMR127C   |
| YGR012W | YLR034C | YDR302W | YNL197C   | YKL161C   | YLR190W   |
| YLL028W | YHR219W | YML012W | YJL176C   | YGL241W   | YCR072C   |
| YBR098W | YLR335W | YDL219W | YDL166C   | YER170W   | YMR163C   |
| YML124C | YPL227C | YNL238W | YDR212W   | YBR133C   | YBR302C   |
| YLR212C | YER044C | YML020W | YDL115C   | YJL029C   | YBR012W-A |
| YOL017W | YER014W | YAL059W | YBR198C   | YPR104C   | YAR003W   |
| YJL080C | YIL007C | YIL047C | YAR071W   | YGR113W   | YBR041W   |
| YFR038W | YKL066W | YCL064C | YDR343C   | YMR258C   | YJL075C   |

| YGL083W | YBR203W | YCR037C | YKL010C | YKL046C | YLR088W |
|---------|---------|---------|---------|---------|---------|
| YDR179C | YNL141W | YOR228C | YGR159C | YNL181W | YIL027C |
| YPL066W | YGL012W | YOL142W | YKR060W | YPL247C | YJL116C |
| YGL101W | YLR188W | YDL030W | YKL136W | YPL212C | YDR085C |
| YNL056W | YOR320C | YLR189C | YJL084C | YIL115C | YLR438W |
| YNR009W | YDR276C | YOR256C | YIL103W | YNL134C | YBL085W |
| YGR250C | YKL151C | YNL296W | YFR028C | YDR189W | YLL004W |
| YGR245C | YJR086W | YDR177W | YJR124C | YDL119C | YER122C |
| YMR274C | YFL017C | YHR170W | YPL058C | YPR075C | YKR050W |
| YDR325W | YGL006W | YJL072C |         |         |         |