INTERFERENCE CHANNEL WITH STATE INFORMATION

A Dissertation

by

LILI ZHANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2012

Major Subject: Electrical Engineering

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Approved by:

Chair of Committee,	Shuguang Cui
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ABSTRACT

Interference Channel with State Information. (August 2012)

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In this dissertation, we study the state-dependent two-user interference channel, where the state information is non-causally known at both transmitters but unknown to either of the receivers. We first propose two coding schemes for the discrete memoryless case: simultaneous encoding for the sub-messages in the first one and superposition encoding in the second one, both with rate splitting and Gel'fand-Pinsker coding. The corresponding achievable rate regions are established. Moreover, for the Gaussian case, we focus on the simultaneous encoding scheme and propose an *active interference cancellation* mechanism, which is a generalized dirty-paper coding technique, to partially eliminate the state effect at the receivers. The corresponding achievable rate region is then derived. We also propose several heuristic schemes for some special cases: the strong interference case, the mixed interference case, and the weak interference case. For the strong and mixed interference case, numerical results are provided to show that active interference cancellation significantly enlarges the achievable rate region. For the weak interference case, flexible power splitting instead of active interference cancellation improves the performance significantly.

Moreover, we focus on the simplest symmetric case, where both direct link gains are the same with each other, and both interfering link gains are the same with each other. We apply the above coding scheme with different dirty paper coding parameters. When the state is additive and symmetric at both receivers, we study both strong and weak interference scenarios and characterize the theoretical gap between the achievable symmetric rate and the upper bound, which is shown to be less than 1/4 bit for the strong interference case and less than 3/4 bit for the weak interference case. Then we provide numerical evaluations of the achievable rates against the upper bound, which validates the theoretical analysis for both strong and weak interference scenarios. Finally, we define the generalized degrees of freedom for the symmetric Gaussian case, and compare the lower bounds against the upper bounds for both strong and weak interference cases. We also show that our achievable schemes can obtain the exact optimal values of the generalized degrees of freedom, i.e., the lower bounds meet the upper bounds for both strong and weak interference cases.

To my Dad

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CHAPTER I

INTRODUCTION

A. Overview of Prior Works

The interference channel (IC) models the situation where several independent transmitters communicate with their corresponding receivers simultaneously over a common spectrum. Due to the shared medium, each receiver suffers from interferences caused by the transmissions of other transceiver pairs. The research of IC was initiated by Shannon [1] and the channel was first thoroughly studied by Ahlswede [2]. Later, Carleial [3] established an improved achievable rate region by applying the superposition coding scheme. In [4], Han and Kobayashi obtained the best achievable rate region known to date for the general IC by utilizing simultaneous decoding at the receivers. Recently, this rate region has been re-characterized with superposition encoding for the sub-messages [5, 6]. However, the capacity region of the general IC is still an open problem [4].

The capacity region for the corresponding Gaussian case is also unknown except for several special cases, such as the strong Gaussian IC and the very strong Gaussian IC [7, 8]. In addition, Sason [9] characterized the sum capacity for a special case of the Gaussian IC called the degraded Gaussian IC. For more general cases, Han-Kobayashi region [4] is still the best achievable rate region known to date. However, for the general Gaussian interference channel, the calculation of the Han-Kobayashi region bears high complexity. The authors in [10] proposed a simpler heuristic coding scheme, for which they set the private message power at both transmitters in a special way such that the interfered private signal-to-noise ratio (SNR) at each receiver is

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The journal model is *IEEE Transactions on Automatic Control.*

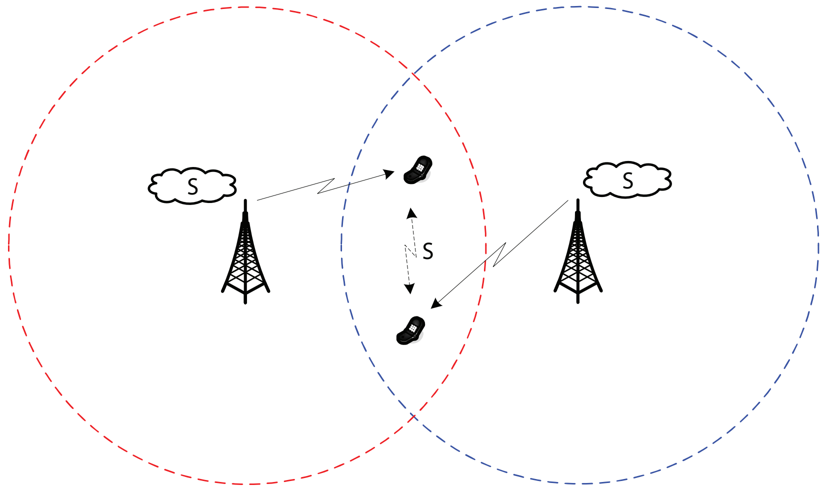

Fig. 1.: A multi-cell downlink communication example, which can be modeled as an interference channel with state information non-causally known at both transmitters.

equal to 1. An upper bound on the capacity was also derived in [10] and it was shown that the gap between the heuristic lower bound and the capacity upper bound is less than one bit for both weak and mixed interference cases.

Many variations of the interference channel have also been studied, including the IC with feedback [11] and the IC with conferencing encoders/decoders [12]. Here, we study another variation of the IC: the state-dependent two-user IC with state information non-causally known at both transmitters. This situation may arise in a multi-cell downlink communication scenario as shown in Fig. 1, where two interested cells are interfering with each other and the mobiles suffer from some common interference (which can be from other neighboring cells and viewed as state) non-causally known at both of the two base-stations via certain collaboration with the neighboring base-station. Notably, communication over state-dependent channels has drawn lots of attentions due to its wide applications such as information embedding [13] and computer memories with defects [14]. The corresponding framework was also initiated by Shannon in [15], which established the capacity of a state-dependent discrete memoryless (DM) point-to-point channel with causal state information at the transmitter. In [16], Gel'fand and Pinsker obtained the capacity for such a point-to-point case with the state information non-causally known at the transmitter. Subsequently, Costa [17] extended Gel'fand-Pinsker coding to the state-dependent additive white Gaussian noise (AWGN) channel, where the state is an additive zero-mean Gaussian interference. This result is known as the dirty-paper coding (DPC) technique, which achieves the capacity as if there is no such an interference. For the multi-user case, extensions of the afore-mentioned schemes appeared in [18–21] for the multiple access channel (MAC), the broadcast channel, and the degraded Gaussian relay channel, respectively.

B. Overview of Contributions

In this dissertation, we study the state-dependent IC with state information non-causally known at the transmitters and develop two coding schemes, both of which jointly apply rate splitting and Gel'fand-Pinsker coding. In the first coding scheme, we deploy simultaneous encoding for the sub-messages, and in the second one, we deploy superposition encoding for the sub-messages. The associated achievable rate regions are derived based on the respective coding schemes. Then we specialize the achievable rate region corresponding to the simultaneous encoding scheme in the Gaussian case, where the common additive state is a zero-mean Gaussian random variable. Specifically, we introduce the notion of *active interference cancellation*, which generalizes dirty-paper coding by utilizing some transmitting power to partially cancel the common interference at both receivers. Furthermore, we propose heuristic schemes for the strong Gaussian IC, the mixed Gaussian IC, and the weak Gaussian IC with state information, respectively. For the strong Gaussian IC with state information, the transmitters only send common messages and the DPC parameters are optimized for one of the two resulting MACs. For the mixed Gaussian IC with state information, one transmitter sends common message and the other one sends private message, with DPC parameters optimized only for one receiver. For the weak interference case, we apply rate splitting, set the private message power at both transmitters to have the interfered private SNR at each receiver equal to 1 [10], utilize sequential decoding, and optimize the DPC parameters for one of the MACs. The time-sharing technique is applied in all the three cases to obtain enlarged achievable rate regions. Numerical comparisons among the achievable rate regions and the capacity outer bound are also provided. For the strong and mixed interference cases, we show that the active interference cancellation mechanism improves the performance significantly; for the weak interference case, it is flexible power allocation instead of active interference cancellation that enlarges the achievable rate region significantly.

Furthermore, we characterize the *theoretical* gap between the achievable rate and the upper bound. We focus on the simplest symmetric case, where the direct link gains are normalized to 1 and the interfering link gains are both g. For the strong interference case ($g > 1$), we use the previously mentioned coding scheme but with different auxiliary random variables, and derive the gap between the achievable symmetric rate and the upper bound, which is shown to be less than 1/4 bit. For the weak interference scenario ($g < 1$), we choose particular auxiliary random variables, set up the power splitting assignment such that the interfering private SNR is equal to 1, and analyze the gap between the achievable symmetric rate of the Gaussian IC with state information and that of the traditional interference channel, which turns out to be less than 1/4 bit. By combining with the results in [10], we conclude that the gap between the achievable symmetric rate and the upper bound is less than 3/4 bit when $g < 1$ (For our AWGN model, all the random variables are defined over the field of real numbers $\mathbb{R}$). Numerical results are provided to validate the theoretical analysis for this symmetric case. Moreover, we define the generalized degrees of freedom for the symmetric Gaussian case and derive the lower bounds corresponding to our achievable schemes, which meet the upper bounds and achieve the exact optimal generalized degrees of freedom.

C. Organization

The rest of the dissertation is organized as following. In Chapter II, the discrete memoryless channel model and the definition of achievable rate region are presented. Then we provide two achievable rate regions for the discrete memoryless IC with state information non-causally known at both transmitters, based on the two different coding schemes, respectively. In Chapter III, we discuss the Gaussian case and present the main idea of active interference cancellation. The strong interference, mixed interference, and weak interference cases are studied. In addition, the numerical results comparing different inner bounds against the outer bound are given. In Chapter IV, the symmetric Gaussian channel model and the definition of symmetric capacity are provided. Then we present the auxiliary random variables and analyze the gap between the achievable symmetric rate and the upper bound for the strong interference case. Afterwards, we focus on the weak interference scenario and present the gap analysis. Numerical comparisons between the achievable symmetric rate and the upper bound are shown. Furthermore, we derive the optimal generalized degrees of freedom for the symmetric Gaussian case. At last, we conclude our work in Chapter V.

CHAPTER II

DISCRETE MEMORYLESS CHANNEL

In this chapter, we first present the discrete memoryless channel model for the state-dependent interference channel. Then we propose two new coding schemes for this DM interference channel with state information non-causally known at both transmitters and quantify the associated achievable rate regions. For both coding schemes, we jointly deploy rate splitting and Gel'fand-Pinsker coding. Specifically, in the first coding scheme, we use simultaneous encoding on the sub-messages, while in the second one we apply superposition encoding.

A. Channel Model

Consider the interference channel as shown in Fig. 2, where two transmitters communicate with the corresponding receivers through a common medium that is dependent on state S. The transmitters do not cooperate with each other; however, they both know the state information S non-causally, which is known to neither of the receivers. Each receiver needs to decode the information from the corresponding transmitter.

We use the following notations for the DM channel. The random variable is defined as X with value x in a finite set $\mathcal{X}$. Let $p_X(x)$ be the probability mass function of X on $\mathcal{X}$. The corresponding sequences are denoted by x^n with length n.

The state-dependent two-user interference channel is defined by $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2, \mathcal{S}, p(y_1, y_2|x_1, x_2, s))$, where $\mathcal{X}_1, \mathcal{X}_2$ are two input alphabet sets, $\mathcal{Y}_1, \mathcal{Y}_2$ are the corresponding output alphabet sets, $\mathcal{S}$ is the state alphabet set, and $p(y_1, y_2|x_1, x_2, s)$ is the conditional probability of $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ given $(x_1, x_2, s) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{S}$. The

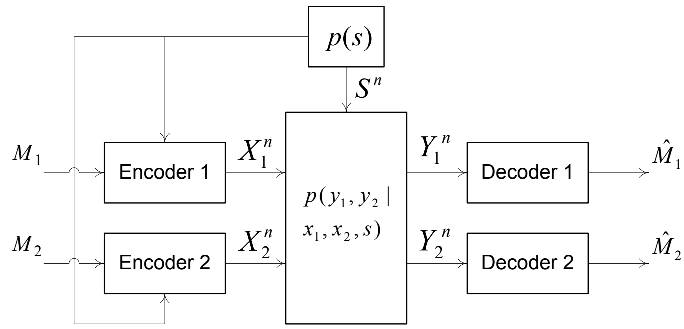

Fig. 2.: The interference channel with state information non-causally known at both transmitters.

channel is assumed to be memoryless, i.e.,

$$p(y_1^n, y_2^n | x_1^n, x_2^n, s^n) = \prod_{i=1}^{n} p(y_{1i}, y_{2i} | x_{1i}, x_{2i}, s_i),$$

where i is the element index for each sequence.

A $(2^{nR_1}, 2^{nR_2}, n)$ code for the above channel consists of two independent message sets $\{1, 2, \cdots, 2^{nR_1}\}$ and $\{1, 2, \cdots, 2^{nR_2}\}$, two encoders that respectively assign two codewords to messages $m_1 \in \{1, 2, \cdots, 2^{nR_1}\}$ and $m_2 \in \{1, 2, \cdots, 2^{nR_2}\}$ based on the non-causally known state information s^n, and two decoders that respectively determine the estimated messages $\hat{m}_1$ and $\hat{m}_2$ or declare an error from the received sequences.

The average probability of error is defined as:

$$P_e^{(n)} = \frac{1}{2^{n(R_1+R_2)}} \sum_{m_1, m_2} \Pr\{\hat{m}_1 \neq m_1 \text{ or } \hat{m}_2 \neq m_2 | (m_1, m_2) \text{ is sent}\}, \tag{2.1}$$

where (m_1, m_2) is assumed to be uniformly distributed over $\{1, 2, \cdots, 2^{nR_1}\} \times \{1, 2, \cdots, 2^{nR_2}\}$.

Definition 1. *A rate pair (R_1, R_2) of non-negative real values is achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes with $P_e^{(n)} \to 0$ as $n \to \infty$. The set of all achievable rate pairs is defined as the capacity region.*

B. Simultaneous Encoding Scheme

Now we introduce the following rate region achieved by the first coding scheme, which combines rate splitting and Gel'fand-Pinsker coding. Let us consider the auxiliary random variables Q, U_1, V_1, U_2, and V_2, defined on arbitrary finite sets $\mathcal{Q}$, $\mathcal{U}_1$, $\mathcal{V}_1$, $\mathcal{U}_2$, and $\mathcal{V}_2$, respectively. The joint probability distribution of the above auxiliary random variables and the state variable S is chosen to satisfy the form $p(s)p(q)p(u_1|q,s)p(v_1|q,s)p(u_2|q,s)p(v_2|q,s)$. Moreover, for a given Q, we let the channel input X_j be an arbitrary deterministic function of U_j, V_j, and S. The achievable rate region of the simultaneous encoding scheme is given in the following theorem.

Theorem 1. *For a fixed probability distribution* $p(q)p(u_1|q,s)p(v_1|q,s)p(u_2|q,s)$ $p(v_2|q,s)$*, let* $\mathcal{R}_1$ *be the set of all non-negative rate tuple* $(R_{10}, R_{11}, R_{20}, R_{22})$ *satisfying*

$$\begin{aligned}
R_{11} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) + I(V_1;Y_1|U_1,U_2,Q) \\
&\quad -I(V_1;S|Q), && (2.2)\\
R_{10} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) + I(U_1;Y_1|V_1,U_2,Q) \\
&\quad -I(U_1;S|Q), && (2.3)\\
R_{10}+R_{11} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) + I(U_1,V_1;Y_1|U_2,Q) \\
&\quad -I(U_1;S|Q) - I(V_1;S|Q), && (2.4)\\
R_{11}+R_{20} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) + I(V_1,U_2;Y_1|U_1,Q) \\
&\quad -I(V_1;S|Q) - I(U_2;S|Q), && (2.5)\\
R_{10}+R_{20} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) + I(U_1,U_2;Y_1|V_1,Q) \\
&\quad -I(U_1;S|Q) - I(U_2;S|Q), && (2.6)\\
R_{10}+R_{11}+R_{20} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) + I(U_1,V_1,U_2;Y_1|Q)
\end{aligned}$$

$$-I(U_1;S|Q)-I(V_1;S|Q)-I(U_2;S|Q), \tag{2.7}$$

$$R_{22} \leq I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(V_2;Y_2|U_2,U_1,Q)$$
$$-I(V_2;S|Q), \tag{2.8}$$

$$R_{20} \leq I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2;Y_2|V_2,U_1,Q)$$
$$-I(U_2;S|Q), \tag{2.9}$$

$$R_{20}+R_{22} \leq I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2,V_2;Y_2|U_1,Q)$$
$$-I(U_2;S|Q)-I(V_2;S|Q), \tag{2.10}$$

$$R_{22}+R_{10} \leq I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(V_2,U_1;Y_2|U_2,Q)$$
$$-I(V_2;S|Q)-I(U_1;S|Q), \tag{2.11}$$

$$R_{20}+R_{10} \leq I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2,U_1;Y_2|V_2,Q)$$
$$-I(U_2;S|Q)-I(U_1;S|Q), \tag{2.12}$$

$$R_{20}+R_{22}+R_{10} \leq I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2,V_2,U_1;Y_2|Q)$$
$$-I(U_2;S|Q)-I(V_2;S|Q)-I(U_1;S|Q). \tag{2.13}$$

Then for any $(R_{10},R_{11},R_{20},R_{22}) \in \mathcal{R}_1$, the rate pair $(R_{10}+R_{11},R_{20}+R_{22})$ is achievable for the DM interference channel with state information defined in Section A.

Remark 1. *The detailed proof is given in Appendix A with the outline sketched as follows. For the coding scheme in Theorem 1, the message at transmitter j ($j=1$ or 2) is splitted into two parts: the public message m_{j0} and the private message m_{jj}. Furthermore, Gel'fand-Pinsker coding is utilized to help both transmitters send the messages with the non-causal knowledge of the state information. Specifically, transmitter j finds the corresponding public codeword u_j and the private codeword v_j such that they are jointly typical with the state s^n. Then the transmitting codeword is constructed as a deterministic function of the public codeword u_j, the private codeword*

v_j, and the state s^n. At the receiver side, decoder j tries to decode the corresponding messages from transmitter j and the public message of the interfering transmitter. The rest follows by the usual error event grouping and error probability analysis.

Remark 2. *The auxiliary random variables in Theorem 1 can be interpreted as follows: Q is the time-sharing random variable; U_j and V_j ($j = 1$ or 2) are the auxiliary random variables to carry the public and private messages at transmitter j, respectively. It can be easily seen from the joint probability distribution that U_j and V_j are conditionally independent given Q and S, which means that the public and private messages are encoded "simultaneously".*

An explicit description of the achievable rate region can be obtained by applying the Fourier-Motzkin algorithm [5] on our implicit description (2.2)-(2.13), as shown in the next corollary.

Corollary 1. *For a fixed probability distribution $p(q)p(u_1|q,s)p(v_1|q,s)p(u_2|q,s)$ $p(v_2|q,s)$, let $\hat{\mathcal{R}}_1$ be the set of all non-negative rate pairs (R_1, R_2) satisfying*

$$R_1 \le \min\{d_1, g_1, a_1+b_1, a_1+f_1, a_1+e_2, a_1+f_2, b_1+e_1, e_1+f_1, e_1+f_2\}, \tag{2.14}$$

$$R_2 \le \min\{d_2, g_2, a_2+b_2, a_2+f_2, a_2+e_1, a_2+f_1, b_2+e_2, e_2+f_2, e_2+f_1\}, \tag{2.15}$$

$$R_1+R_2 \le \min\{a_1+g_2, a_2+g_1, e_1+g_2, e_2+g_1, e_1+e_2, a_1+a_2+f_1, a_1+a_2+f_2, a_1+b_2+e_2, a_2+b_1+e_1\}, \tag{2.16}$$

$$R_1+2R_2 \le \min\{e_1+f_1+2a_2, e_1+2a_2+f_2, e_1+a_2+g_2\}, \tag{2.17}$$

$$2R_1+R_2 \le \min\{e_2+f_2+2a_1, e_2+2a_1+f_1, e_2+a_1+g_1\}, \tag{2.18}$$

where

$$
\begin{aligned}
a_1 &= I(U_1;U_2|Q)+I(U_1,U_2;V_1|Q)+I(V_1;Y_1|U_1,U_2,Q)-I(V_1;S|Q),\\
b_1 &= I(U_1;U_2|Q)+I(U_1,U_2;V_1|Q)+I(U_1;Y_1|V_1,U_2,Q)-I(U_1;S|Q),\\
d_1 &= I(U_1;U_2|Q)+I(U_1,U_2;V_1|Q)+I(U_1,V_1;Y_1|U_2,Q)-I(U_1;S|Q)-I(V_1;S|Q),\\
e_1 &= I(U_1;U_2|Q)+I(U_1,U_2;V_1|Q)+I(V_1,U_2;Y_1|U_1,Q)-I(V_1;S|Q)-I(U_2;S|Q),\\
f_1 &= I(U_1;U_2|Q)+I(U_1,U_2;V_1|Q)+I(U_1,U_2;Y_1|V_1,Q)-I(U_1;S|Q)-I(U_2;S|Q),\\
g_1 &= I(U_1;U_2|Q)+I(U_1,U_2;V_1|Q)+I(U_1,V_1,U_2;Y_1|Q)-I(U_1;S|Q)-I(V_1;S|Q)\\
&\quad -I(U_2;S|Q),\\
a_2 &= I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(V_2;Y_2|U_2,U_1,Q)-I(V_2;S|Q),\\
b_2 &= I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2;Y_2|V_2,U_1,Q)-I(U_2;S|Q),\\
d_2 &= I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2,V_2;Y_2|U_1,Q)-I(U_2;S|Q)-I(V_2;S|Q),\\
e_2 &= I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(V_2,U_1;Y_2|U_2,Q)-I(V_2;S|Q)-I(U_1;S|Q),\\
f_2 &= I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2,U_1;Y_2|V_2,Q)-I(U_2;S|Q)-I(U_1;S|Q),\\
g_2 &= I(U_2;U_1|Q)+I(U_2,U_1;V_2|Q)+I(U_2,V_2,U_1;Y_2|Q)-I(U_2;S|Q)-I(V_2;S|Q)\\
&\quad -I(U_1;S|Q).
\end{aligned}
$$

Then any rate pair $(R_1,R_2)\in\hat{\mathcal{R}}_1$ *is achievable for the DM interference channel with state information defined in Section A.*

C. Superposition Encoding Scheme

We now present the second coding scheme, which applies superposition encoding for the sub-messages. Similar to the auxiliary random variables in Theorem 1, in the following theorem, Q is also the time-sharing random variable; U_j and V_j ($j=1$ or 2) are the auxiliary random variables to carry the public and private messages at transmitter j, respectively. The difference here is the joint probability distribution $p(s)p(q)p(u_1|s,q)p(v_1|u_1,s,q)p(u_2|s,q)p(v_2|u_2,s,q)$, where U_j and V_j are not conditionally independent given Q and S. This also implies the notion of "superposition encoding". The achievable rate region of the superposition encoding scheme is given in the following theorem.

Theorem 2. *For a fixed probability distribution* $p(q)p(u_1|s,q)p(v_1|u_1,s,q)p(u_2|s,q)$ $p(v_2|u_2,s,q)$*, let* $\mathcal{R}_2$ *be the set of all non-negative rate tuple* $(R_{10}, R_{11}, R_{20}, R_{22})$ *satisfying*

$$\begin{aligned}
R_{11} &\leq I(U_1,V_1;U_2|Q)+I(V_1;Y_1|U_1,U_2,Q) \\
&\quad -I(V_1;S|U_1,Q), && (2.19)\\
R_{10}+R_{11} &\leq I(U_1,V_1;U_2|Q)+I(U_1,V_1;Y_1|U_2,Q) \\
&\quad -I(U_1,V_1;S|Q), && (2.20)\\
R_{11}+R_{20} &\leq I(U_1,V_1;U_2|Q)+I(V_1,U_2;Y_1|U_1,Q) \\
&\quad -I(V_1;S|U_1,Q)-I(U_2;S|Q), && (2.21)\\
R_{10}+R_{11}+R_{20} &\leq I(U_1,V_1;U_2|Q)+I(U_1,V_1,U_2;Y_1|Q) \\
&\quad -I(U_1,V_1;S|Q)-I(U_2;S|Q), && (2.22)\\
R_{22} &\leq I(U_2,V_2;U_1|Q)+I(V_2;Y_2|U_2,U_1,Q) \\
&\quad -I(V_2;S|U_2,Q), && (2.23)\\
R_{20}+R_{22} &\leq I(U_2,V_2;U_1|Q)+I(U_2,V_2;Y_2|U_1,Q) \\
&\quad -I(U_2,V_2;S|Q), && (2.24)\\
R_{22}+R_{10} &\leq I(U_2,V_2;U_1|Q)+I(V_2,U_1;Y_2|U_2,Q) \\
&\quad -I(V_2;S|U_2,Q)-I(U_1;S|Q), && (2.25)\\
R_{20}+R_{22}+R_{10} &\leq I(U_2,V_2;U_1|Q)+I(U_2,V_2,U_1;Y_2|Q)
\end{aligned}$$

$$-I(U_2, V_2; S|Q) - I(U_1; S|Q). \tag{2.26}$$

Then for any $(R_{10}, R_{11}, R_{20}, R_{22}) \in \mathcal{R}_2$*, the rate pair* $(R_{10}+R_{11}, R_{20}+R_{22})$ *is achievable for the DM interference channel with state information defined in Section A.*

The detailed proof for Theorem 2 is given in Appendix B.

Remark 3. *Compared with the first coding scheme in Theorem 1, the rate splitting structure is also applied in the achievable scheme of Theorem 2. The main difference here is that instead of simultaneous encoding, now the private message* m_{jj} *is superimposed on the public message* m_{j0} *for the* j*th transmitter,* $j = 1, 2$*. In addition, Gel'fand-Pinsker coding is utilized to help the transmitters send both public and private messages.*

Remark 4. *It can be easily seen that the achievable rate region* $\mathcal{R}_1$ *in Theorem 1 is a subset of* $\mathcal{R}_2$*, i.e.,* $\mathcal{R}_1 \subseteq \mathcal{R}_2$*. However, whether these two regions can be equivalent is still under investigation, which is motivated by the equivalence between the simultaneous encoding region and the superposition encoding region for the traditional IC [5].*

D. Summary

In this chapter, we considered the interference channel with state information noncausally known at both transmitters. Two achievable rate regions are established based on two coding schemes with simultaneous encoding and superposition encoding, respectively.

CHAPTER III

AWGN CHANNEL

In this chapter, we first present the channel model for the AWGN interference channel with state information. Then we provide the corresponding achievable rate region based on the simultaneous encoding scheme described in Chapter II. In addition to applying dirty paper coding and rate splitting, here we also introduce the idea of active interference cancellation, which allocates some source power to cancel the state effect at the receivers. Finally, we propose heuristic schemes for the strong Gaussian IC, the mixed Gaussian IC, and the weak Gaussian IC with state information, respectively. Numerical comparisons among the achievable rate regions and the capacity outer bound are also provided.

A. Channel Model

The Gaussian counterpart of the previously defined DM channel is shown in Fig. 3, where two transmitters communicate with the corresponding receivers through a common channel that is dependent on state S, which can be treated as a common interference. The corresponding signal structure can be described by the following channel input and output relationship:

$$\begin{aligned} Y_1' &= h_{11}X_1' + h_{12}X_2' + S + Z_1', \\ Y_2' &= h_{22}X_2' + h_{21}X_1' + S + Z_2', \end{aligned}$$

where h_{ij} is the real link amplitude gain from the jth transmitter to the ith receiver, X_i' and Y_i' are the channel input and output, respectively, and Z_i' is the zero-mean AWGN noise with variance N_i, for $i = 1, 2$ and $j = 1, 2$. Both receivers also suffer

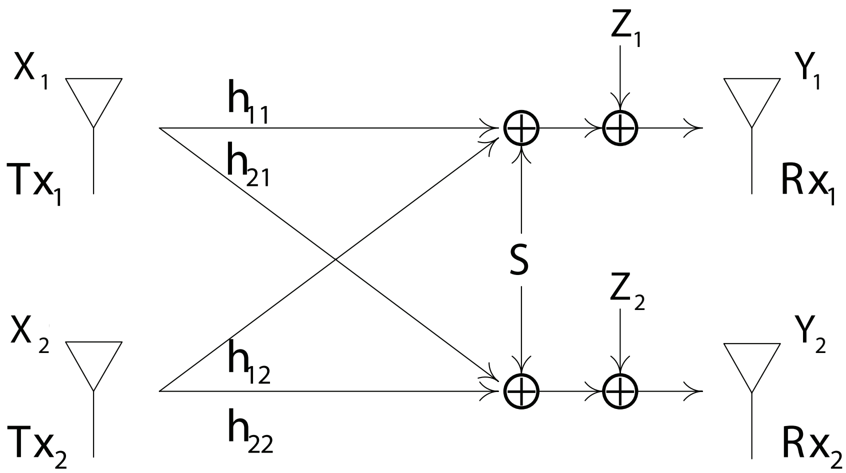

Fig. 3.: The Gaussian interference channel with state information non-causally known at both transmitters.

from a zero-mean additive white Gaussian interference S with variance K, which is non-causally known at both transmitters[1]. Note that for this AWGN model, all the random variables are defined over the field of real numbers $\mathbb{R}$.

Without loss of generality, we transform the signal model into the following standard form [4]:

$$Y_1 = X_1 + \sqrt{g_{12}} X_2 + \frac{1}{\sqrt{N_1}} S + Z_1, \tag{3.1}$$

$$Y_2 = X_2 + \sqrt{g_{21}} X_1 + \frac{1}{\sqrt{N_2}} S + Z_2, \tag{3.2}$$

where

$$\begin{aligned} Y_1 &= \frac{Y_1'}{\sqrt{N_1}},\ X_1 = \frac{h_{11}X_1'}{\sqrt{N_1}},\ g_{12} = \frac{h_{12}^2 N_2}{h_{22}^2 N_1},\ Z_1 = \frac{Z_1'}{\sqrt{N_1}}, \\ Y_2 &= \frac{Y_2'}{\sqrt{N_2}},\ X_2 = \frac{h_{22}X_2'}{\sqrt{N_2}},\ g_{21} = \frac{h_{21}^2 N_1}{h_{11}^2 N_2},\ Z_2 = \frac{Z_2'}{\sqrt{N_2}}. \end{aligned}$$

[1]In general, the additive states over the two links may not be the same, i.e., we may have $Y_1' = h_{11}X_1' + h_{12}X_2' + S_1 + Z_1'$ and $Y_2' = h_{22}X_2' + h_{21}X_1' + S_2 + Z_2'$. However, in this dissertation we only focus on the simplest scenario: $S_1 = S_2 = S$. The more general cases with state (S_1, S_2) and different knowledge levels at the two transmitters will be studied in our future work.

Note that Z_1 and Z_2 have unit variance in (3.1) and (3.2). We also impose the following power constraints on the channel inputs X_1 and X_2:

$$\frac{1}{n}\sum_{i=1}^{n}(X_{1i})^2 \leq P_1, \text{ and } \frac{1}{n}\sum_{i=1}^{n}(X_{2i})^2 \leq P_2.$$

B. Achievable Rate Regions for AWGN Case with Active Interference Cancellation

In the general Gaussian interference channel, the simultaneous encoding over the sub-messages can be viewed as sending $X_j = A_j + B_j$ at the jth transmitter, $j = 1, 2$, where A_j and B_j are independent and correspond to the public and private messages, respectively. Correspondingly, for the Gaussian IC with state information defined in Section A, we focus on the coding scheme based on simultaneous encoding that was discussed in Section B. Specifically, we apply dirty paper coding to both public and private parts, i.e., we define the auxiliary variables as follows:

$$U_1 = A_1 + \alpha_{10}S,\ V_1 = B_1 + \alpha_{11}S, \tag{3.3}$$

$$U_2 = A_2 + \alpha_{20}S,\ V_2 = B_2 + \alpha_{22}S. \tag{3.4}$$

In addition, we allow both transmitters to apply active interference cancellation by allocating a certain amount of power to send counter-phase signals against the known interference S, i.e.,

$$X_1 = A_1 + B_1 - \gamma_1 S, \tag{3.5}$$

$$X_2 = A_2 + B_2 - \gamma_2 S, \tag{3.6}$$

where γ_1 and γ_2 are active cancellation parameters. The idea is to generalize dirty-paper coding by allocating some transmitting power to cancel part of the state effect at both receivers. Assume $A_1 \sim \mathcal{N}(0, \beta_1(P_1 - \gamma_1^2 K))$, $B_1 \sim \mathcal{N}(0, \bar{\beta}_1(P_1 - \gamma_1^2 K))$, $A_2 \sim \mathcal{N}(0, \beta_2(P_2 - \gamma_2^2 K))$, and $B_2 \sim \mathcal{N}(0, \bar{\beta}_2(P_2 - \gamma_2^2 K))$, where $\beta_1 + \bar{\beta}_1 = 1$ and

$\beta_2 + \bar{\beta}_2 = 1$. According to the Gaussian channel model defined in Section A, the received signals can be determined as:

$$
\begin{aligned}
Y_1 &= A_1 + B_1 + \sqrt{g_{12}}(A_2 + B_2) + \mu_1 S + Z_1, \\
Y_2 &= A_2 + B_2 + \sqrt{g_{21}}(A_1 + B_1) + \mu_2 S + Z_2,
\end{aligned}
$$

where $\mu_1 = \frac{1}{\sqrt{N_1}} - \gamma_1 - \gamma_2\sqrt{g_{12}}$ and $\mu_2 = \frac{1}{\sqrt{N_2}} - \gamma_2 - \gamma_1\sqrt{g_{21}}$.

For convenience, we denote $P_{A_1} = \beta_1(P_1 - \gamma_1^2 K)$, $P_{B_1} = \bar{\beta}_1(P_1 - \gamma_1^2 K)$, $P_{A_2} = \beta_2(P_2 - \gamma_2^2 K)$, and $P_{B_2} = \bar{\beta}_2(P_2 - \gamma_2^2 K)$. Also define $G_{U_1} = \alpha_{10}^2 K/P_{A_1}$, $G_{V_1} = \alpha_{11}^2 K/P_{B_1}$, $G_{U_2} = \alpha_{20}^2 K/P_{A_2}$, and $G_{V_2} = \alpha_{22}^2 K/P_{B_2}$.

The achievable rate region can be obtained by evaluating the rate region given in Theorem 1 with respect to the corresponding Gaussian auxiliary variables and channel outputs.

Theorem 3. *Let $\mathcal{R}_1'$ be the set of all non-negative rate tuple* $(R_{10}, R_{11}, R_{20}, R_{22})$ *satisfying*

$$
\begin{aligned}
R_{11} &\le \frac{1}{2}\log\left[\frac{1}{L_1}\left((1 + P_{B_1} + g_{12}P_{B_2})(1 + G_{U_1} + G_{U_2} + G_{U_1}G_{U_2}) \right.\right. \\
&\qquad \left.\left. + K(\alpha_{10} + \alpha_{20}\sqrt{g_{12}} - \mu_1)^2 \left(1 + \frac{G_{U_1}G_{U_2}}{1 + G_{U_1} + G_{U_2}}\right)\right)\right], \\
R_{10} &\le \frac{1}{2}\log\left[\frac{1}{L_1}\left((1 + P_{A_1} + g_{12}P_{B_2})(1 + G_{V_1} + G_{U_2} + G_{V_1}G_{U_2}) \right.\right. \\
&\qquad \left.\left. + K(\alpha_{11} + \alpha_{20}\sqrt{g_{12}} - \mu_1)^2 \left(1 + \frac{G_{V_1}G_{U_2}}{1 + G_{V_1} + G_{U_2}}\right)\right)\right], \\
R_{10} + R_{11} &\le \frac{1}{2}\log\left[\frac{1}{L_1}\left((1 + P_{A_1} + P_{B_1} + g_{12}P_{B_2})(1 + G_{U_2}) \right.\right. \\
&\qquad \left.\left. + K(\alpha_{20}\sqrt{g_{12}} - \mu_1)^2\right)\right],
\end{aligned}
$$

$$\begin{aligned}
R_{11}+R_{20} &\leq \frac{1}{2}\log\left[\frac{1}{L_1}\Bigg(\left(1+P_{B_1}+g_{12}P_{A_2}+g_{12}P_{B_2}\right)\left(1+G_{U_1}\right)\right.\\
&\left.\quad +K(\alpha_{10}-\mu_1)^2\Bigg)\right],\\
R_{10}+R_{20} &\leq \frac{1}{2}\log\left[\frac{1}{L_1}\Bigg(\left(1+P_{A_1}+g_{12}P_{A_2}+g_{12}P_{B_2}\right)\left(1+G_{V_1}\right)\right.\\
&\left.\quad +K(\alpha_{11}-\mu_1)^2\Bigg)\right],\\
R_{10}+R_{11}+R_{20} &\leq \frac{1}{2}\log\left[\frac{1}{L_1}\left(1+P_{A_1}+P_{B_1}+g_{12}P_{A_2}+g_{12}P_{B_2}+\mu_1^2K\right)\right],\\
R_{22} &\leq \frac{1}{2}\log\left[\frac{1}{L_2}\Bigg(\left(1+P_{B_2}+g_{21}P_{B_1}\right)\left(1+G_{U_2}+G_{U_1}+G_{U_2}G_{U_1}\right)\right.\\
&\left.\quad +K(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\mu_2)^2\left(1+\frac{G_{U_2}G_{U_1}}{1+G_{U_2}+G_{U_1}}\right)\Bigg)\right],\\
R_{20} &\leq \frac{1}{2}\log\left[\frac{1}{L_2}\Bigg(\left(1+P_{A_2}+g_{21}P_{B_1}\right)\left(1+G_{V_2}+G_{U_1}+G_{V_2}G_{U_1}\right)\right.\\
&\left.\quad +K(\alpha_{22}+\alpha_{10}\sqrt{g_{21}}-\mu_2)^2\left(1+\frac{G_{V_2}G_{U_1}}{1+G_{V_2}+G_{U_1}}\right)\Bigg)\right],\\
R_{20}+R_{22} &\leq \frac{1}{2}\log\left[\frac{1}{L_2}\Bigg(\left(1+P_{A_2}+P_{B_2}+g_{21}P_{B_1}\right)\left(1+G_{U_1}\right)\right.\\
&\left.\quad +K(\alpha_{10}\sqrt{g_{21}}-\mu_2)^2\Bigg)\right],\\
R_{22}+R_{10} &\leq \frac{1}{2}\log\left[\frac{1}{L_2}\Bigg(\left(1+P_{B_2}+g_{21}P_{A_1}+g_{21}P_{B_1}\right)\left(1+G_{U_2}\right)\right.\\
&\left.\quad +K(\alpha_{20}-\mu_2)^2\Bigg)\right],\\
R_{20}+R_{10} &\leq \frac{1}{2}\log\left[\frac{1}{L_2}\Bigg(\left(1+P_{A_2}+g_{21}P_{A_1}+g_{21}P_{B_1}\right)\left(1+G_{V_2}\right)\right.\\
&\left.\quad +K(\alpha_{22}-\mu_2)^2\Bigg)\right],
\end{aligned}$$

$$R_{20} + R_{22} + R_{10} \leq \frac{1}{2}\log\left[\frac{1}{L_2}\left(1 + P_{A_2} + P_{B_2} + g_{21}P_{A_1} + g_{21}P_{B_1} + \mu_2^2 K\right)\right].$$

where

$$L_1 = (1 + g_{12}P_{B_2})(1 + G_{U_1} + G_{U_2} + G_{V_1}) + K(\alpha_{10} + \alpha_{20}\sqrt{g_{12}} + \alpha_{11} - \mu_1)^2,$$

$$L_2 = (1 + g_{21}P_{B_1})(1 + G_{U_2} + G_{U_1} + G_{V_2}) + K(\alpha_{20} + \alpha_{10}\sqrt{g_{21}} + \alpha_{22} - \mu_2)^2.$$

Then for any $(R_{10}, R_{11}, R_{20}, R_{22}) \in \mathcal{R}_1'$, the rate pair $(R_{10} + R_{11}, R_{20} + R_{22})$ is achievable for the Gaussian IC with state information defined in Section A.

Note that the achievable rate region $\mathcal{R}_1'$ depends on the power splitting parameters, the active cancellation parameters, and the DPC parameters. To be clear, we may write $\mathcal{R}_1'$ as $\mathcal{R}_1'(\beta_1, \beta_2, \gamma_1, \gamma_2, \alpha_{10}, \alpha_{11}, \alpha_{20}, \alpha_{22})$.

Remark 5. *It can be easily seen that the above achievable rate region includes the capacity region of the Gaussian MAC with state information, by only using the common messages for both transmitters and optimizing the respective DPC parameters.*

The following corollary gives the achievable rate region for the Gaussian IC with state information when the state power $K \to \infty$.

Corollary 2. *Let $\widetilde{\mathcal{R}}_1'$ be the set of all non-negative rate tuple $(R_{10}, R_{11}, R_{20}, R_{22})$ satisfying*

$$R_{10} + R_{11} \leq \frac{1}{2}\log\left(\frac{(1 + P_{A_1} + P_{B_1} + g_{12}P_{B_2})\frac{\alpha_{20}^2}{P_{A_2}} + (\alpha_{20}\sqrt{g_{12}} - \mu_1)^2}{L_3}\right),$$

$$R_{11} + R_{20} \leq \frac{1}{2}\log\left(\frac{(1 + P_{B_1} + g_{12}P_{A_2} + g_{12}P_{B_2})\frac{\alpha_{10}^2}{P_{A_1}} + (\alpha_{10} - \mu_1)^2}{L_3}\right),$$

$$R_{10} + R_{20} \leq \frac{1}{2}\log\left(\frac{(1 + P_{A_1} + g_{12}P_{A_2} + g_{12}P_{B_2})\frac{\alpha_{11}^2}{P_{B_1}} + (\alpha_{11} - \mu_1)^2}{L_3}\right),$$

$$
\begin{aligned}
R_{10}+R_{11}+R_{20} &\leq \frac{1}{2}\log\left(\frac{\mu_1^2}{L_3}\right),\\
R_{20}+R_{22} &\leq \frac{1}{2}\log\left(\frac{(1+P_{A_2}+P_{B_2}+g_{21}P_{B_1})\frac{\alpha_{10}^2}{P_{A_1}}+(\alpha_{10}\sqrt{g_{21}}-\mu_2)^2}{L_4}\right),\\
R_{22}+R_{10} &\leq \frac{1}{2}\log\left(\frac{(1+P_{B_2}+g_{21}P_{A_1}+g_{21}P_{B_1})\frac{\alpha_{20}^2}{P_{A_2}}+(\alpha_{20}-\mu_2)^2}{L_4}\right),\\
R_{20}+R_{10} &\leq \frac{1}{2}\log\left(\frac{(1+P_{A_2}+g_{21}P_{A_1}+g_{21}P_{B_1})\frac{\alpha_{22}^2}{P_{B_2}}+(\alpha_{22}-\mu_2)^2}{L_4}\right),\\
R_{20}+R_{22}+R_{10} &\leq \frac{1}{2}\log\left(\frac{\mu_2^2}{L_4}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
L_3 &= (1+g_{12}P_{B_2})\left(\frac{\alpha_{10}^2}{P_{A_1}}+\frac{\alpha_{20}^2}{P_{A_2}}+\frac{\alpha_{11}^2}{P_{B_1}}\right)+(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}+\alpha_{11}-\mu_1)^2,\\
L_4 &= (1+g_{21}P_{B_1})\left(\frac{\alpha_{20}^2}{P_{A_2}}+\frac{\alpha_{10}^2}{P_{A_1}}+\frac{\alpha_{22}^2}{P_{B_2}}\right)+(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\mu_2)^2.
\end{aligned}
$$

As the state power $K \to \infty$, for any $(R_{10}, R_{11}, R_{20}, R_{22}) \in \widetilde{\mathcal{R}}'_1$, the rate pair $(R_{10} + R_{11}, R_{20} + R_{22})$ is achievable for the Gaussian IC with state information defined in Section A.

Remark 6. *It can be easily seen that due to the special structure of DPC [20], a nontrivial rate region can be achieved even when the state power goes to infinity, as long as the state is non-causally known at the transmitters.*

In the following sections, we will consider several special cases of the Gaussian IC with state information: the strong interference case, the mixed interference case, and the weak interference case, respectively.

C. The Strong Gaussian IC with State Information

For the Gaussian IC with state information defined in Section A, the channel is called strong Gaussian IC with state information if the interference link gains satisfy $g_{21} \geq 1$ and $g_{12} \geq 1$. In this section, we propose two achievable schemes for the strong Gaussian IC with state information, and derive the corresponding achievable rate regions. An enlarged achievable rate region is obtained by combining them with the time-sharing technique.

1. Scheme without Active Interference Cancellation

We first introduce a simple achievable scheme without active interference cancellation, which is a building block towards the more general schemes coming next. It is known that for the traditional strong Gaussian IC, the capacity region can be obtained by the intersection of two MAC rate regions due to the presence of the strong interference. However, for the strong Gaussian IC with state information, the two MACs are not capacity-achieving simultaneously since the optimal DPC parameters are different for these two MACs. Here we propose a simple achievable scheme, which achieves the capacity for one of the MACs and leaves the other MAC to suffer from the non-optimal DPC parameters. Note that now all the source power is used to transmit the intended message at both transmitters instead of being partly allocated to cancel the state effect as in Section B.

Theorem 4. *Let $\mathcal{C}_{s1}$ be the set of all non-negative rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} R_1 \leq \min\Bigg\{ & \frac{1}{2}\log\left(1+P_1\right), \\ & \frac{1}{2}\log\left(\frac{(1+g_{21}P_1)\left(1+\frac{\alpha_{20}^2 K}{P_2}\right)+K\left(\alpha_{20}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{20}^2 K}{P_2}+\frac{\alpha_{10}^2 K}{P_1}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}\right)\Bigg\}, \end{aligned}$$

$$R_2 \leq \frac{1}{2}\log\left(\frac{(1+P_2)\left(1+\frac{\alpha_{10}^2 K}{P_1}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{20}^2 K}{P_2}+\frac{\alpha_{10}^2 K}{P_1}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}\right),$$

$$R_1+R_2 \leq \min\left\{\frac{1}{2}\log\left(1+P_1+g_{12}P_2\right),\right.$$

$$\left.\frac{1}{2}\log\left(\frac{1+P_2+g_{21}P_1+\frac{K}{N_2}}{1+\frac{\alpha_{20}^2 K}{P_2}+\frac{\alpha_{10}^2 K}{P_1}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}\right)\right\},$$

where $\alpha_{10} = \frac{P_1}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$ *and* $\alpha_{20} = \frac{\sqrt{g_{12}}P_2}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$*, which are optimal for the MAC at receiver* 1*. Then any rate pair* $(R_1, R_2) \in \mathcal{C}_{s1}$ *is achievable for the strong Gaussian IC with state information.*

Similarly, let $\mathcal{C}_{s2}$ *be the set of all non-negative rate pairs* (R_1, R_2) *satisfying*

$$R_1 \leq \frac{1}{2}\log\left(\frac{(1+P_1)\left(1+\frac{\alpha_{20}^2 K}{P_2}\right)+K\left(\alpha_{20}\sqrt{g_{12}}-\frac{1}{\sqrt{N_1}}\right)^2}{1+\frac{\alpha_{10}^2 K}{P_1}+\frac{\alpha_{20}^2 K}{P_2}+K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\frac{1}{\sqrt{N_1}}\right)^2}\right),$$

$$R_2 \leq \min\left\{\frac{1}{2}\log\left(1+P_2\right),\right.$$

$$\left.\frac{1}{2}\log\left(\frac{(1+g_{12}P_2)\left(1+\frac{\alpha_{10}^2 K}{P_1}\right)+K\left(\alpha_{10}-\frac{1}{\sqrt{N_1}}\right)^2}{1+\frac{\alpha_{10}^2 K}{P_1}+\frac{\alpha_{20}^2 K}{P_2}+K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\frac{1}{\sqrt{N_1}}\right)^2}\right)\right\},$$

$$R_1+R_2 \leq \min\left\{\frac{1}{2}\log\left(1+P_2+g_{21}P_1\right),\right.$$

$$\left.\frac{1}{2}\log\left(\frac{1+P_1+g_{12}P_2+\frac{K}{N_1}}{1+\frac{\alpha_{10}^2 K}{P_1}+\frac{\alpha_{20}^2 K}{P_2}+K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\frac{1}{\sqrt{N_1}}\right)^2}\right)\right\},$$

where $\alpha_{10} = \frac{\sqrt{g_{21}}P_1}{\sqrt{N_2}(1+P_2+g_{21}P_1)}$ *and* $\alpha_{20} = \frac{P_2}{\sqrt{N_2}(1+P_2+g_{21}P_1)}$*, which are optimal for the MAC at receiver* 2*). Then any rate pair* $(R_1, R_2) \in \mathcal{C}_{s2}$ *is achievable for the strong Gaussian IC with state information.*

Proof. We only give the detailed proof for $\mathcal{C}_{s1}$ here. Similarly, $\mathcal{C}_{s2}$ can be obtained by achieving the MAC capacity at receiver 2 and letting the MAC at receiver 1 suffer from the non-optimal DPC parameters.

Due to the presence of the strong interference, we only send common messages at both transmitters instead of splitting the message into common and private ones. Accordingly, both receivers need to decode the messages from both transmitters. For the MAC at receiver 1, the capacity region is given as:

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(1+P_1\right), \\ R_2 &\leq \frac{1}{2}\log\left(1+g_{12}P_2\right), \\ R_1+R_2 &\leq \frac{1}{2}\log\left(1+P_1+g_{12}P_2\right), \end{aligned}$$

where DPC is utilized at both transmitters and the optimal DPC parameters are $\alpha_{10} = \frac{P_1}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$ and $\alpha_{20} = \frac{\sqrt{g_{12}}P_2}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$. However, the MAC for receiver 2 suffers from the non-optimal DPC parameters and has the following achievable rate region:

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(\frac{(1+g_{21}P_1)\left(1+\frac{\alpha_{20}^2K}{P_2}\right)+K\left(\alpha_{20}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{20}^2K}{P_2}+\frac{\alpha_{10}^2K}{P_1}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}\right), \\ R_2 &\leq \frac{1}{2}\log\left(\frac{(1+P_2)\left(1+\frac{\alpha_{10}^2K}{P_1}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{20}^2K}{P_2}+\frac{\alpha_{10}^2K}{P_1}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}\right), \\ R_1+R_2 &\leq \frac{1}{2}\log\left(\frac{1+P_2+g_{21}P_1+\frac{K}{N_2}}{1+\frac{\alpha_{20}^2K}{P_2}+\frac{\alpha_{10}^2K}{P_1}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}\right). \end{aligned}$$

Consequently, we have the achievable region $\mathcal{C}_{s1}$ for the strong Gaussian IC with state information, which is the intersection of the above two rate regions for the two MACs. □

2. Scheme with Active Interference Cancellation

For the strong Gaussian IC with state information, now we propose a more general achievable scheme with active interference cancellation, which allocates part of the source power to cancel the state effect at the receivers. Specifically, DPC is used to achieve the capacity for one of the MACs as shown in Section 1, and active interference cancellation is employed at both transmitters to cancel the state effect at the receivers. The corresponding achievable rate regions are provided in the following theorem.

Theorem 5. *For any* $\gamma_1^2 < P_1/K$ *and* $\gamma_2^2 < P_2/K$*, let* $\mathcal{C}_{s3}(\gamma_1, \gamma_2)$ *be the set of all non-negative rate pairs* (R_1, R_2) *satisfying*

$$\begin{aligned}
R_1 &\leq \min\Bigg\{\frac{1}{2}\log\left(1+P_1-\gamma_1^2K\right), \\
&\qquad \frac{1}{2}\log\left(\frac{(1+g_{21}(P_1-\gamma_1^2K))\left(1+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}\right)+K\left(\alpha_{20}-\mu_2\right)^2}{1+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\mu_2\right)^2}\right)\Bigg\}, \\
R_2 &\leq \frac{1}{2}\log\left(\frac{(1+P_2-\gamma_2^2K)\left(1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\mu_2\right)^2}{1+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\mu_2\right)^2}\right), \\
R_1+R_2 &\leq \min\Bigg\{\frac{1}{2}\log\left(1+P_1-\gamma_1^2K+g_{12}\left(P_2-\gamma_2^2K\right)\right), \\
&\qquad \frac{1}{2}\log\left(\frac{1+P_2-\gamma_2^2K+g_{21}(P_1-\gamma_1^2K)+\mu_2^2K}{1+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\mu_2\right)^2}\right)\Bigg\},
\end{aligned}$$

where $\alpha_{10} = \frac{\mu_1(P_1-\gamma_1^2K)}{1+P_1-\gamma_1^2K+g_{12}(P_2-\gamma_2^2K)}$ *and* $\alpha_{20} = \frac{\mu_1\sqrt{g_{12}}(P_2-\gamma_2^2K)}{1+P_1-\gamma_1^2K+g_{12}(P_2-\gamma_2^2K)}$*, which are optimal for the MAC at receiver* 1*. Then any rate pair* $(R_1, R_2) \in \mathcal{C}_{s3}(\gamma_1, \gamma_2)$ *is achievable for the strong Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as* $\hat{\mathcal{C}}_{s3}$*) of* $\mathcal{C}_{s3}(\gamma_1, \gamma_2)$ *is also achievable.*

Similarly, for any $\gamma_1^2 < P_1/K$ *and* $\gamma_2^2 < P_2/K$*, let* $\mathcal{C}_{s4}(\gamma_1, \gamma_2)$ *be the set of all* *non-negative rate pairs* (R_1, R_2) *satisfying*

$$\begin{aligned}
R_1 &\leq \frac{1}{2}\log\left(\frac{(1+P_1-\gamma_1^2K)\left(1+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}\right)+K\left(\alpha_{20}\sqrt{g_{12}}-\mu_1\right)^2}{1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}+K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\mu_1\right)^2}\right), \\
R_2 &\leq \min\left\{\frac{1}{2}\log\left(1+P_2-\gamma_2^2K\right),\right. \\
&\qquad \left.\frac{1}{2}\log\left(\frac{(1+g_{12}(P_2-\gamma_2^2K))\left(1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}\right)+K\left(\alpha_{10}-\mu_1\right)^2}{1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}+K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\mu_1\right)^2}\right)\right\}, \\
R_1+R_2 &\leq \min\left\{\frac{1}{2}\log\left(1+P_2-\gamma_2^2K+g_{21}(P_1-\gamma_1^2K)\right),\right. \\
&\qquad \left.\frac{1}{2}\log\left(\frac{1+P_1-\gamma_1^2K+g_{12}(P_2-\gamma_2^2K)+K\mu_1^2}{1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+\frac{\alpha_{20}^2K}{P_2-\gamma_2^2K}+K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\mu_1\right)^2}\right)\right\},
\end{aligned}$$

where $\alpha_{10} = \frac{\mu_2\sqrt{g_{21}}(P_1-\gamma_1^2K)}{1+P_2-\gamma_2^2K+g_{21}(P_1-\gamma_1^2K)}$ *and* $\alpha_{20} = \frac{\mu_2(P_2-\gamma_2^2K)}{1+P_2-\gamma_2^2K+g_{21}(P_1-\gamma_1^2K)}$, *which are optimal for the MAC at receiver* 2*. Then any rate pair* $(R_1, R_2) \in \mathcal{C}_{s4}(\gamma_1, \gamma_2)$ *is achievable for the strong Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as* $\hat{\mathcal{C}}_{s4}$*) of* $\mathcal{C}_{s4}(\gamma_1, \gamma_2)$ *is also achievable.*

The proof is omitted here since it is similar to that of Theorem 4 except for applying active interference cancellation to both users. Moreover, we see that the regions $\mathcal{C}_{s1}$ and $\mathcal{C}_{s2}$ are equivalent to $\mathcal{C}_{s3}(0,0)$ and $\mathcal{C}_{s4}(0,0)$, respectively, which means that the achievable scheme without active interference cancellation is only a special case of the one with active interference cancellation.

Note that an enlarged achievable rate region can be obtained by deploying the time-sharing technique for any points in $\mathcal{C}_{s3}(\gamma_1, \gamma_2)$ and $\mathcal{C}_{s4}(\gamma_1, \gamma_2)$, which is described in the following corollary.

Corollary 3. *The enlarged achievable rate region* $\mathcal{C}_s$ *for the strong Gaussian IC with state information is given by the closure of the convex hull of* $\left(0, \frac{1}{2}\log\left(1+P_2\right)\right)$,

$\left(\frac{1}{2}\log\left(1+P_1\right),0\right)$, *and all* (R_1, R_2) *in* $\mathcal{C}_{s3}(\gamma_1,\gamma_2)$ *and* $\mathcal{C}_{s4}(\gamma_1,\gamma_2)$ *for any* $\gamma_1^2 < P_1/K$ *and* $\gamma_2^2 < P_2/K$.

In Section F, we will numerically compare the above achievable rate regions with an inner bound, which is denoted as $\mathcal{C}_{s\_in}$ and defined by the achievable rate region when the transmitters ignore the non-causal state information. The improvement due to DPC and active interference cancellation is clearly shown there. We also compare the above achievable rate regions with an outer bound (denoted by $\mathcal{C}_{s\_o}$), which corresponds to the capacity region of the traditional strong Gaussian IC [8]. Such a correspondence is due to the fact that the traditional Gaussian IC can be viewed as the idealization of our channel model where the state is also known at the receivers.

D. The Mixed Gaussian IC with State Information

For the Gaussian IC with state information defined in Section A, the channel is called mixed Gaussian IC with state information if the interference link gains satisfy $g_{21} > 1$, $g_{12} < 1$ or $g_{21} < 1$, $g_{12} > 1$. In this section, we propose two achievable schemes for the mixed Gaussian IC with state information, and derive the corresponding achievable rate regions. Similarly, we can enlarge the achievable rate region by combining them with the time-sharing technique. Without loss of generality, from now on we assume that $g_{21} > 1$ and $g_{12} < 1$.

1. Scheme without Active Interference Cancellation

Similar to the strong Gaussian IC with state information, here we first introduce a simple scheme without active interference cancellation, which optimizes the DPC parameters for one receiver and leaves the other receiver suffer from the non-optimal

DPC parameters. Furthermore, receiver 1 treats the received signal from transmitter 2 as noise, and receiver 2 decodes both messages from transmitter 1 and transmitter 2. Note that now all the source power is used to send the intended messages at both transmitters instead of employing active interference cancellation.

Theorem 6. *For any α_{22}, let $\mathcal{C}_{m1}(\alpha_{22})$ be the set of all non-negative rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} R_1 &\leq \min\left\{\frac{1}{2}\log\left(1+\frac{P_1}{1+g_{12}P_2}\right),\right. \\ &\left.\frac{1}{2}\log\left(\frac{(1+g_{21}P_1)\left(1+\frac{\alpha_{22}^2K}{P_2}\right)+K\left(\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{10}^2K}{P_1}+\frac{\alpha_{22}^2K}{P_2}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}\right)\right\}, \\ R_2 &\leq \frac{1}{2}\log\left(\frac{(1+P_2)\left(1+\frac{\alpha_{10}^2K}{P_1}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{10}^2K}{P_1}+\frac{\alpha_{22}^2K}{P_2}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}\right), \\ R_1+R_2 &\leq \frac{1}{2}\log\left(\frac{1+P_2+g_{21}P_1+\frac{K}{N_2}}{1+\frac{\alpha_{10}^2K}{P_1}+\frac{\alpha_{22}^2K}{P_2}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}\right), \end{aligned}$$

where $\alpha_{10} = \frac{P_1}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$ that is optimal for the point-to-point link between transmitter 1 and receiver 1. Then any rate pair $(R_1, R_2) \in \mathcal{C}_{m1}(\alpha_{22})$ is achievable for the mixed Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as $\hat{\mathcal{C}}_{m1}$) of all $\mathcal{C}_{m1}(\alpha_{22})$ is also achievable.

Similarly, let $\mathcal{C}_{m2}$ be the set of all non-negative rate pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(\frac{1+P_1+g_{12}P_2+\frac{K}{N_1}}{(1+g_{12}P_2)\left(1+\frac{\alpha_{10}^2K}{P_1}\right)+K\left(\alpha_{10}-\frac{1}{\sqrt{N_1}}\right)^2}\right), \\ R_2 &\leq \frac{1}{2}\log\left(1+P_2\right), \\ R_1+R_2 &\leq \frac{1}{2}\log\left(1+P_2+g_{21}P_1\right), \end{aligned}$$

where $\alpha_{10} = \frac{\sqrt{g_{21}}P_1}{\sqrt{N_2}(1+P_2+g_{21}P_1)}$ *that is optimal for the MAC at receiver* 2*. Then any rate pair* $(R_1, R_2) \in \mathcal{C}_{m2}$ *is achievable for the mixed Gaussian IC with state information.*

Proof. We only give the detailed derivation for $\mathcal{C}_{m1}$ here. The region $\mathcal{C}_{m2}$ can be obtained in a similar manner by achieving the MAC capacity at receiver 2 and letting receiver 1 suffer from the non-optimal α_{10}.

Since the interference link gains satisfy $g_{21} > 1$ and $g_{12} < 1$, the interference for receiver 1 is weaker than its intended signal and the interference for receiver 2 is stronger than its intended signal. Accordingly, we send common message at transmitter 1 and private message at transmitter 2 instead of splitting the message into common and private messages for both transmitters. For the direct link from transmitter 1 to receiver 1, the capacity is

$$R_1 \leq \frac{1}{2}\log\left(1 + \frac{P_1}{1+g_{12}P_2}\right),$$

where the DPC parameter is $\alpha_{10} = \frac{P_1}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$. However, the MAC at receiver 2 suffers from the non-optimal α_{10} and the achievable rate region is:

$$\begin{aligned}
R_1 &\leq \frac{1}{2}\log\left(\frac{(1+g_{21}P_1)\left(1+\frac{\alpha_{22}^2K}{P_2}\right)+K\left(\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{10}^2K}{P_1}+\frac{\alpha_{22}^2K}{P_2}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}\right),\\
R_2 &\leq \frac{1}{2}\log\left(\frac{(1+P_2)\left(1+\frac{\alpha_{10}^2K}{P_1}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}{1+\frac{\alpha_{10}^2K}{P_1}+\frac{\alpha_{22}^2K}{P_2}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}\right),\\
R_1+R_2 &\leq \frac{1}{2}\log\left(\frac{1+P_2+g_{21}P_1+\frac{K}{N_2}}{1+\frac{\alpha_{10}^2K}{P_1}+\frac{\alpha_{22}^2K}{P_2}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\frac{1}{\sqrt{N_2}}\right)^2}\right),
\end{aligned}$$

for any α_{22}. Therefore, we have the achievable rate region $\mathcal{C}_{m1}(\alpha_{22})$ as the intersections of the above two regions. □

2. Scheme with Active Interference Cancellation

Now we propose a more general scheme with active interference cancellation, which allocates some source power to cancel the state effect at both receivers. Similarly, the DPC parameters are only optimized for one receiver, and the other receiver suffers from the non-optimal DPC parameters. The corresponding achievable rate regions are stated in the following theorem.

Theorem 7. *For any* α_{22}*,* $\gamma_1^2 < P_1/K$*, and* $\gamma_2^2 < P_2/K$*, let* $\mathcal{C}_{m3}(\alpha_{22}, \gamma_1, \gamma_2)$ *be the set of all non-negative rate pairs* (R_1, R_2) *satisfying*

$$\begin{aligned}
R_1 &\leq \min\Bigg\{\frac{1}{2}\log\left(1+\frac{P_1-\gamma_1^2K}{1+g_{12}\left(P_2-\gamma_2^2K\right)}\right),\\
&\quad \frac{1}{2}\log\left(\frac{\left(1+g_{21}\left(P_1-\gamma_1^2K\right)\right)\left(1+\frac{\alpha_{22}^2K}{P_2-\gamma_2^2K}\right)+K\left(\alpha_{22}-\mu_2\right)^2}{1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+\frac{\alpha_{22}^2K}{P_2-\gamma_2^2K}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\mu_2\right)^2}\right)\Bigg\},\\
R_2 &\leq \frac{1}{2}\log\left(\frac{\left(1+P_2-\gamma_2^2K\right)\left(1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\mu_2\right)^2}{1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+\frac{\alpha_{22}^2K}{P_2-\gamma_2^2K}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\mu_2\right)^2}\right),\\
R_1+R_2 &\leq \frac{1}{2}\log\left(\frac{1+P_2-\gamma_2^2K+g_{21}\left(P_1-\gamma_1^2K\right)+K\mu_2^2}{1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}+\frac{\alpha_{22}^2K}{P_2-\gamma_2^2K}+K\left(\alpha_{10}\sqrt{g_{21}}+\alpha_{22}-\mu_2\right)^2}\right),
\end{aligned}$$

where $\alpha_{10} = \frac{\mu_1(P_1-\gamma_1^2K)}{1+P_1-\gamma_1^2K+g_{12}(P_2-\gamma_2^2K)}$ *that is optimal for the point-to-point link between transmitter* 1 *and receiver* 1*. Then any rate pair* $(R_1, R_2) \in \mathcal{C}_{m3}(\alpha_{22}, \gamma_1, \gamma_2)$ *is achievable for the mixed Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as* $\hat{\mathcal{C}}_{m3}$*) of* $\mathcal{C}_{m3}(\alpha_{22}, \gamma_1, \gamma_2)$ *is also achievable.*

Similarly, for any $\gamma_1^2 < P_1/K$ *and* $\gamma_2^2 < P_2/K$*, let* $\mathcal{C}_{m4}(\gamma_1, \gamma_2)$ *be the set of all non-negative rate pairs* (R_1, R_2) *satisfying*

$$R_1 \leq \frac{1}{2}\log\left(\frac{1+P_1-\gamma_1^2K+g_{12}\left(P_2-\gamma_2^2K\right)+K\mu_1^2}{\left(1+g_{12}\left(P_2-\gamma_2^2K\right)\right)\left(1+\frac{\alpha_{10}^2K}{P_1-\gamma_1^2K}\right)+K\left(\alpha_{10}-\mu_1\right)^2}\right),$$

$$R_2 \leq \frac{1}{2}\log\left(1+P_2-\gamma_2^2K\right),$$
$$R_1+R_2 \leq \frac{1}{2}\log\left(1+P_2-\gamma_2^2K+g_{21}\left(P_1-\gamma_1^2K\right)\right),$$

where $\alpha_{10}=\frac{\mu_2\sqrt{g_{21}}(P_1-\gamma_1^2K)}{1+P_2-\gamma_2^2K+g_{21}\left(P_1-\gamma_1^2K\right)}$ *that is optimal for the MAC at receiver* 2*). Then any rate pair* $(R_1,R_2)\in\mathcal{C}_{m4}(\gamma_1,\gamma_2)$ *is achievable for the mixed Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as* $\hat{\mathcal{C}}_{m4}$*) of* $\mathcal{C}_{m4}(\gamma_1,\gamma_2)$ *is also achievable.*

The proof is omitted here since it is similar to that of Theorem 6 except for applying active interference cancellation to both users. Moreover, it is straightforward to see that the regions $\mathcal{C}_{m1}(\alpha_{22})$ and $\mathcal{C}_{m2}$ are equivalent to $\mathcal{C}_{m3}(\alpha_{22},0,0)$ and $\mathcal{C}_{m4}(0,0)$, respectively, which means that the achievable scheme without active interference cancellation is only a special case of the one with active interference cancellation.

Note that an enlarged achievable rate region can be obtained by deploying the time-sharing technique for any points in $\mathcal{C}_{m3}(\alpha_{22},\gamma_1,\gamma_2)$ and $\mathcal{C}_{m4}(\gamma_1,\gamma_2)$, which is described in the following corollary.

Corollary 4. *The enlarged achievable rate region* $\mathcal{C}_m$ *for the mixed Gaussian IC with state information is given by the closure of the convex hull of* $\left(0,\frac{1}{2}\log(1+P_2)\right)$, $\left(\frac{1}{2}\log(1+P_1),0\right)$, *and all* (R_1,R_2) *in* $\mathcal{C}_{m3}(\alpha_{22},\gamma_1,\gamma_2)$ *and* $\mathcal{C}_{m4}(\gamma_1,\gamma_2)$ *for any* α_{22}, $\gamma_1^2<P_1/K$, *and* $\gamma_2^2<P_2/K$.

In Section F, we will numerically compare the above achievable rate regions with an inner bound, which is denoted as $\mathcal{C}_{m\_in}$ and defined by the achievable rate region when the transmitters ignore the non-causal state information. The improvement due to DPC and active interference cancellation is clearly shown there. We also compare the above achievable rate regions with an outer bound (denoted by $\mathcal{C}_{m\_o}$), which is the outer bound derived for the traditional mixed Gaussian IC [10].

3. A Special Case – Degraded Gaussian IC

For the Gaussian IC with state information defined in Section A, the channel is called a degraded Gaussian IC with state information if the interference link gains satisfy $g_{21}g_{12} = 1$, which can be viewed as a special case of the mixed Gaussian IC. For this degraded interference case, we will show the numerical comparison between the achievable rate regions and the outer bound in Section F. Note that the difference from the general mixed interference case is the evaluation of the outer bound $\mathcal{C}_{m\_o}$, which is now equal to the outer bound including the sum capacity for the traditional degraded Gaussian IC [9].

E. The Weak Gaussian IC with State Information

For the Gaussian IC with state information defined in Section A, the channel is called weak Gaussian IC with state information if the interference link gains satisfy $g_{21} < 1$ and $g_{12} < 1$. In this section, we propose several achievable schemes for the weak Gaussian IC with state information, and derive the corresponding achievable rate regions. An enlarged achievable rate region is obtained by combining them with the time-sharing technique.

1. Scheme without Active Interference Cancellation

We first introduce a simple scheme with fixed power allocation and without active interference cancellation. It is shown in [10] that for the traditional weak Gaussian IC, the achievable rate region is within one bit of the capacity region if power splitting is chosen such that the interfered private SNR at each receiver is equal to 1. In our scheme, we set the interfered private SNR equal to 1, utilize sequential decoding, and optimize the DPC parameters for one of the MACs. Note that now the power allocation between the common message and private message is fixed, and all the source power is used to transmit the intended message at both transmitters instead of being partly allocated to cancel the state effect.

Theorem 8. *Let $\mathcal{C}_{w1}$ be the set of all non-negative rate pairs (R_1, R_2) satisfying*

$$\begin{aligned}
R_1 &\leq \frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right)+\min\left\{\frac{1}{2}\log\left(1+\frac{P_{A_1}}{1+P_{B_1}+g_{12}P_{B_2}}\right),\right.\\
&\quad \left.\frac{1}{2}\log\left(\frac{(1+P_{B_2}+g_{21}P_1)\left(1+\frac{\alpha_{20}^2K}{P_{A_2}}\right)+K\left(\alpha_{20}-\frac{1}{\sqrt{N_2}}\right)^2}{L_{w1}}\right)\right\}, &(3.7)\\
R_2 &\leq \frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right)+\min\left\{\frac{1}{2}\log\left(1+\frac{g_{12}P_{A_2}}{1+P_{B_1}+g_{12}P_{B_2}}\right),\right.\\
&\quad \left.\frac{1}{2}\log\left(\frac{(1+g_{21}P_{B_1}+P_2)\left(1+\frac{\alpha_{10}^2K}{P_{A_1}}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}{L_{w1}}\right)\right\}, &(3.8)\\
R_1+R_2 &\leq \min\left\{\frac{1}{2}\log\left(1+\frac{P_{A_1}+g_{12}P_{A_2}}{1+P_{B_1}+g_{12}P_{B_2}}\right), \frac{1}{2}\log\left(\frac{1+P_2+g_{21}P_1+\frac{K}{N_2}}{L_{w1}}\right)\right\}\\
&\quad +\frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right)+\frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right). &(3.9)
\end{aligned}$$

where

$$\begin{aligned}
P_{B_1} &= \min\left\{P_1, \frac{1}{g_{21}}\right\},\\
P_{B_2} &= \min\left\{P_2, \frac{1}{g_{12}}\right\},\\
\alpha_{10} &= \frac{P_{A_1}}{\sqrt{N_1}(1+P_1+g_{12}P_2)},\\
\alpha_{20} &= \frac{\sqrt{g_{12}}P_{A_2}}{\sqrt{N_1}(1+P_1+g_{12}P_2)},\\
L_{w1} &= (1+P_{B_2}+g_{21}P_{B_1})\left(1+\frac{\alpha_{20}^2K}{P_{A_2}}+\frac{\alpha_{10}^2K}{P_{A_1}}\right)+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2.
\end{aligned}$$

Then any rate pair $(R_1, R_2) \in \mathcal{C}_{w1}$ is achievable for the weak Gaussian IC with state *information.*

Similarly, let $\mathcal{C}_{w2}$ be the set of all non-negative rate pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right)+\min\left\{\frac{1}{2}\log\left(1+\frac{g_{21}P_{A_1}}{1+P_{B_2}+g_{21}P_{B_1}}\right),\right.\\ &\left.\frac{1}{2}\log\left(\frac{(1+g_{12}P_{B_2}+P_1)\left(1+\frac{\alpha_{20}^2K}{P_{A_2}}\right)+K\left(\alpha_{20}\sqrt{g_{12}}-\frac{1}{\sqrt{N_1}}\right)^2}{L_{w2}}\right)\right\}, \qquad (3.10)\end{aligned}$$

$$\begin{aligned} R_2 &\leq \frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right)+\min\left\{\frac{1}{2}\log\left(1+\frac{P_{A_2}}{1+P_{B_2}+g_{21}P_{B_1}}\right),\right.\\ &\left.\frac{1}{2}\log\left(\frac{(1+P_{B_1}+g_{12}P_2)\left(1+\frac{\alpha_{10}^2K}{P_{A_1}}\right)+K\left(\alpha_{10}-\frac{1}{\sqrt{N_1}}\right)^2}{L_{w2}}\right)\right\}, \qquad (3.11)\end{aligned}$$

$$\begin{aligned} R_1+R_2 &\leq \min\left\{\frac{1}{2}\log\left(1+\frac{P_{A_2}+g_{21}P_{A_1}}{1+P_{B_2}+g_{21}P_{B_1}}\right), \frac{1}{2}\log\left(\frac{1+P_1+g_{12}P_2+\frac{K}{N_1}}{L_{w2}}\right)\right\}\\ &+\frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right)+\frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right), \qquad (3.12)\end{aligned}$$

where

$$\begin{aligned} P_{B_1} &= \min\left\{P_1, \frac{1}{g_{21}}\right\},\\ P_{B_2} &= \min\left\{P_2, \frac{1}{g_{12}}\right\},\\ \alpha_{10} &= \frac{\sqrt{g_{21}}P_{A_1}}{\sqrt{N_2}(1+P_2+g_{21}P_1)},\\ \alpha_{20} &= \frac{P_{A_2}}{\sqrt{N_2}(1+P_2+g_{21}P_1)},\\ L_{w2} &= (1+P_{B_1}+g_{12}P_{B_2})\left(1+\frac{\alpha_{10}^2K}{P_{A_1}}+\frac{\alpha_{20}^2K}{P_{A_2}}\right)+K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\frac{1}{\sqrt{N_1}}\right)^2. \end{aligned}$$

Then any rate pair $(R_1, R_2) \in \mathcal{C}_{w2}$ is achievable for the weak Gaussian IC with state information.

Proof. We only give the detailed proof for $\mathcal{C}_{w1}$ here. Similarly, $\mathcal{C}_{w2}$ can be obtained by optimizing the DPC parameters for the common messages at receiver 2 and letting the common-message MAC at receiver 1 suffer from the non-optimal DPC parameters.

Due to the presence of the weak interference, we split the message into common and private ones at both transmitters. The sequential decoder is utilized at the receivers, i.e., both receivers first decode both common messages by treating both private messages as noise, and then decode the intended private message by treating the interfered private message as noise. For the common-message MAC at receiver 1, the capacity region is given as follows:

$$\begin{aligned} R_{10} &\leq \frac{1}{2}\log\left(1+\frac{P_{A_1}}{1+P_{B_1}+g_{12}P_{B_2}}\right), \\ R_{20} &\leq \frac{1}{2}\log\left(1+\frac{g_{12}P_{A_2}}{1+P_{B_1}+g_{12}P_{B_2}}\right), \\ R_{10}+R_{20} &\leq \frac{1}{2}\log\left(1+\frac{P_{A_1}+g_{12}P_{A_2}}{1+P_{B_1}+g_{12}P_{B_2}}\right), \end{aligned}$$

where $P_{B_1} = \min\{P_1, 1/g_{21}\}$, $P_{B_2} = \min\{P_2, 1/g_{12}\}$, and DPC is utilized for both common messages with the optimal DPC parameters $\alpha_{10} = \frac{P_{A_1}}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$ and $\alpha_{20} = \frac{\sqrt{g_{12}}P_{A_2}}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$. However, the common-message MAC at receiver 2 suffers from the non-optimal DPC parameters and has the following achievable rate region:

$$\begin{aligned} R_{10} &\leq \frac{1}{2}\log\left(\frac{(1+P_{B_2}+g_{21}P_1)\left(1+\frac{\alpha_{20}^2K}{P_{A_2}}\right)+K\left(\alpha_{20}-\frac{1}{\sqrt{N_2}}\right)^2}{L_{w1}}\right), \\ R_{20} &\leq \frac{1}{2}\log\left(\frac{(1+g_{21}P_{B_1}+P_2)\left(1+\frac{\alpha_{10}^2K}{P_{A_1}}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2}{L_{w1}}\right), \\ R_{10}+R_{20} &\leq \frac{1}{2}\log\left(\frac{1+P_2+g_{21}P_1+\frac{K}{N_2}}{L_{w1}}\right). \end{aligned}$$

where

$$L_{w1} = (1+P_{B_2}+g_{21}P_{B_1})\left(1+\frac{\alpha_{20}^2K}{P_{A_2}}+\frac{\alpha_{10}^2K}{P_{A_1}}\right)+K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\frac{1}{\sqrt{N_2}}\right)^2.$$

Consequently, the IC achievable region for the common messages can be obtained by intersecting the above regions for the two MACs. After decoding the common messages, each receiver is capable of decoding the intended private message with the following rate:

$$\begin{aligned} R_{11} &\leq \frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right), \\ R_{22} &\leq \frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right). \end{aligned}$$

Therefore, after applying the Fourier-Motzkin algorithm, we have the achievable region $\mathcal{C}_{w1}$ for the weak Gaussian IC with state information. □

2. Scheme with Active Interference Cancellation

For the weak Gaussian IC with state information, now we generalize the previous scheme with active interference cancellation, which allocates part of the source power to cancel the state effect at the receivers. Specifically, DPC is used to achieve the capacity for one of the common-message MACs as shown in Section 1, and active interference cancellation is deployed to cancel the state effect at the receivers. The corresponding achievable rate regions are provided in the following theorem.

Theorem 9. *For any $\gamma_1^2 < P_{A1}/K$ and $\gamma_2^2 < P_{A2}/K$, let $\mathcal{C}_{w3}(\gamma_1,\gamma_2)$ be the set of all non-negative rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} R_1 \leq\ & \frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right)+\min\left\{\frac{1}{2}\log\left(1+\frac{P_{A_1}-\gamma_1^2K}{1+P_{B_1}+g_{12}P_{B_2}}\right),\right. \\ & \left.\frac{1}{2}\log\left(\frac{\left(1+P_{B_2}+g_{21}(P_1-\gamma_1^2K)\right)\left(1+\frac{\alpha_{20}^2K}{P_{A_2}-\gamma_2^2K}\right)+K\left(\alpha_{20}-\mu_2\right)^2}{L_{w3}}\right)\right\}, \\ R_2 \leq\ & \frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right)+\min\left\{\frac{1}{2}\log\left(1+\frac{g_{12}(P_{A_2}-\gamma_2^2K)}{1+P_{B_1}+g_{12}P_{B_2}}\right),\right. \end{aligned}$$

$$
\begin{aligned}
& \frac{1}{2}\log\left(\frac{(1+g_{21}P_{B_1}+P_2-\gamma_2^2K)\left(1+\frac{\alpha_{10}^2K}{P_{A_1}-\gamma_1^2K}\right)+K\left(\alpha_{10}\sqrt{g_{21}}-\mu_2\right)^2}{L_{w3}}\right)\Bigg\}, \\
R_1+R_2 \leq\ & \frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right)+\frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right) \\
& \min\left\{\frac{1}{2}\log\left(1+\frac{P_{A_1}-\gamma_1^2K+g_{12}(P_{A_2}-\gamma_2^2K)}{1+P_{B_1}+g_{12}P_{B_2}}\right),\right. \\
& \left.\frac{1}{2}\log\left(\frac{1+P_2-\gamma_2^2K+g_{21}(P_1-\gamma_1^2K)+\mu_2^2KN_2}{L_{w3}}\right)\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
P_{B_1} &= \min\left\{P_1, \frac{1}{g_{21}}\right\}, \\
P_{B_2} &= \min\left\{P_2, \frac{1}{g_{12}}\right\}, \\
\alpha_{10} &= \frac{\mu_1(P_{A_1}-\gamma_1^2K)}{(1+P_1-\gamma_1^2K+g_{12}\left(P_2-\gamma_2^2K\right))}, \\
\alpha_{20} &= \frac{\mu_1\sqrt{g_{12}}(P_{A_2}-\gamma_2^2K)}{1+P_1-\gamma_1^2K+g_{12}(P_2-\gamma_2^2K)}, \\
L_{w3} &= (1+P_{B_2}+g_{21}P_{B_1})\left(1+\frac{\alpha_{20}^2K}{P_{A_2}-\gamma_2^2K}+\frac{\alpha_{10}^2K}{P_{A_1}-\gamma_1^2K}\right) \\
&\quad +K\left(\alpha_{20}+\alpha_{10}\sqrt{g_{21}}-\mu_2\right)^2.
\end{aligned}
$$

Note that here α_{10} *and* α_{20} *are optimal for the common-message MAC at receiver 1. Then any rate pair* $(R_1, R_2) \in \mathcal{C}_{w3}(\gamma_1, \gamma_2)$ *is achievable for the weak Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as* $\hat{\mathcal{C}}_{w3}$*) of* $\mathcal{C}_{w3}(\gamma_1, \gamma_2)$ *is also achievable.*

Similarly, for any $\gamma_1^2 < P_{A1}/K$ *and* $\gamma_2^2 < P_{A2}/K$*, let* $\mathcal{C}_{w4}(\gamma_1, \gamma_2)$ *be the set of all non-negative rate pairs* (R_1, R_2) *satisfying*

$$
R_1 \leq \frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right)+\min\left\{\frac{1}{2}\log\left(1+\frac{g_{21}(P_{A_1}-\gamma_1^2K)}{1+P_{B_2}+g_{21}P_{B_1}}\right),\right.
$$

$$\frac{1}{2}\log\left(\frac{(1+g_{12}P_{B_2}+P_1-\gamma_1^2K)\left(1+\frac{\alpha_{20}^2K}{P_{A_2}-\gamma_2^2K}\right)+K\left(\alpha_{20}\sqrt{g_{12}}-\mu_1\right)^2}{L_{w4}}\right)\Bigg\},$$

$$R_2 \leq \frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right)+\min\Bigg\{\frac{1}{2}\log\left(1+\frac{P_{A_2}-\gamma_2^2K}{1+P_{B_2}+g_{21}P_{B_1}}\right),$$

$$\frac{1}{2}\log\left(\frac{(1+P_{B_1}+g_{12}(P_2-\gamma_2^2K))\left(1+\frac{\alpha_{10}^2K}{P_{A_1}-\gamma_1^2K}\right)+K\left(\alpha_{10}-\mu_1\right)^2}{L_{w4}}\right)\Bigg\},$$

$$R_1+R_2 \leq \frac{1}{2}\log\left(1+\frac{P_{B_2}}{1+g_{21}P_{B_1}}\right)+\frac{1}{2}\log\left(1+\frac{P_{B_1}}{1+g_{12}P_{B_2}}\right)$$

$$\min\Bigg\{\frac{1}{2}\log\left(1+\frac{P_{A_2}-\gamma_2^2K+g_{21}(P_{A_1}-\gamma_1^2K)}{1+P_{B_2}+g_{21}P_{B_1}}\right),$$

$$\frac{1}{2}\log\left(\frac{1+P_1-\gamma_1^2K+g_{12}(P_2-\gamma_2^2K)+\mu_1^2K}{L_{w4}}\right)\Bigg\},$$

where

$$P_{B_1} = \min\left\{P_1, \frac{1}{g_{21}}\right\},$$

$$P_{B_2} = \min\left\{P_2, \frac{1}{g_{12}}\right\},$$

$$\alpha_{10} = \frac{\mu_2\sqrt{g_{21}}(P_{A_1}-\gamma_1^2K)}{1+P_2-\gamma_2^2K+g_{21}(P_1-\gamma_1^2K)},$$

$$\alpha_{20} = \frac{\mu_2(P_{A_2}-\gamma_2^2K)}{1+P_2-\gamma_2^2K+g_{21}(P_1-\gamma_1^2K)},$$

$$L_{w4} = (1+P_{B_1}+g_{12}P_{B_2})\left(1+\frac{\alpha_{10}^2K}{P_{A_1}-\gamma_1^2K}+\frac{\alpha_{20}^2K}{P_{A_2}-\gamma_2^2K}\right) +K\left(\alpha_{10}+\alpha_{20}\sqrt{g_{12}}-\mu_1\right)^2.$$

Note that here α_{10} *and* α_{20} *are optimal for the common-message MAC at receiver 2. Then any rate pair* $(R_1,R_2)\in\mathcal{C}_{w4}(\gamma_1,\gamma_2)$ *is achievable for the weak Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as* $\hat{\mathcal{C}}_{w4}$*) of* $\mathcal{C}_{w4}(\gamma_1,\gamma_2)$ *is also achievable.*

The proof is omitted here since it is similar to that of Theorem 8 except for applying active interference cancellation to both users. Moreover, we see that the regions $\mathcal{C}_{w1}$ and $\mathcal{C}_{w2}$ are equivalent to $\mathcal{C}_{w3}(0,0)$ and $\mathcal{C}_{w4}(0,0)$, respectively, which again implies that the achievable scheme without active interference cancellation is only a special case of the one with active interference cancellation.

As in previous sections, an enlarged achievable rate region can be obtained by employing the time-sharing technique for any points in $\mathcal{C}_{w3}(\gamma_1,\gamma_2)$ and $\mathcal{C}_{w4}(\gamma_1,\gamma_2)$, which is described in the following corollary.

Corollary 5. *The enlarged achievable rate region $\mathcal{C}_w$ for the weak Gaussian IC with state information is given by the closure of the convex hull of $\left(0, \frac{1}{2}\log\left(1+P_2\right)\right)$, $\left(\frac{1}{2}\log\left(1+P_1\right), 0\right)$, and all (R_1, R_2) in $\mathcal{C}_{w3}(\gamma_1,\gamma_2)$ and $\mathcal{C}_{w4}(\gamma_1,\gamma_2)$ for any $\gamma_1^2 < P_{A1}/K$ and $\gamma_2^2 < P_{A2}/K$.*

In Section F, we will numerically compare the above achievable rate regions with an inner bound, which is denoted as $\mathcal{C}_{w\_in}$ and defined by the achievable rate region when the transmitters ignore the non-causal state information. We also compare the above achievable rate regions with an outer bound (denoted by $\mathcal{C}_{w\_o}$), which is the outer bound derived for the traditional weak Gaussian IC [10]. Note that unlike the strong interference case and the mixed interference case, active interference cancellation cannot enlarge the achievable rate region significantly for the weak interference case. Intuitively, the reason is that the source power is too "precious" to cancel the state effect when the interference is weak. Therefore, we next modify the scheme to optimize the power allocation between the common message and the private message at each transmitter.

3. Scheme with Flexible Power Allocation

For the weak Gaussian IC with state information, now we propose a scheme with flexible power allocation. The corresponding achievable rate regions are provided in the following theorem.

Theorem 10. *For any $\beta_1, \beta_2 \in (0,1)$, let $\mathcal{C}_{w5}(\beta_1, \beta_2)$ be the set of all non-negative rate pairs (R_1, R_2) satisfying (3.7)-(3.9) where $P_{B_1} = \beta_1 P_1$, $P_{B_2} = \beta_2 P_2$, $\alpha_{10} = \frac{(1-\beta_1)P_1}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$, and $\alpha_{20} = \frac{\sqrt{g_{12}}(1-\beta_2)P_2}{\sqrt{N_1}(1+P_1+g_{12}P_2)}$, which are optimal for the common-message MAC at receiver 1. Then any rate pair $(R_1, R_2) \in \mathcal{C}_{w5}(\beta_1, \beta_2)$ is achievable for the weak Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as $\hat{\mathcal{C}}_{w5}$) of $\mathcal{C}_{w5}(\beta_1, \beta_2)$ is also achievable.*

Similarly, for any $\beta_1, \beta_2 \in (0,1)$, let $\mathcal{C}_{w6}(\beta_1, \beta_2)$ be the set of all non-negative rate pairs (R_1, R_2) satisfying (3.10)-(3.12), where $P_{B_1} = \beta_1 P_1$, $P_{B_2} = \beta_2 P_2$, $\alpha_{10} = \frac{\sqrt{g_{21}}(1-\beta_1)P_1}{\sqrt{N_2}(1+P_2+g_{21}P_1)}$, and $\alpha_{20} = \frac{(1-\beta_2)P_2}{\sqrt{N_2}(1+P_2+g_{21})}$, which are optimal for the common-message MAC at receiver 2. Then any rate pair $(R_1, R_2) \in \mathcal{C}_{w6}(\beta_1, \beta_2)$ is achievable for the weak Gaussian IC with state information. Moreover, any rate pair in the convex hull (denoted as $\hat{\mathcal{C}}_{w6}$) of $\mathcal{C}_{w6}(\beta_1, \beta_2)$ is also achievable.

The proof is omitted here since it is similar to that of Theorem 8 except for applying the optimal power allocation between the common and private messages at both transmitters, which is obtained by two-dimensional searching and bears the same complexity as the active interference cancellation scheme in Section 2. Similarly, an enlarged achievable rate region can be obtained by employing the time-sharing technique for any points in $\mathcal{C}_{w5}(\beta_1, \beta_2)$ and $\mathcal{C}_{w6}(\beta_1, \beta_2)$, which is described in the following corollary.

Corollary 6. *The enlarged achievable rate region $\hat{\mathcal{C}}_w$ for the weak Gaussian IC with state information is given by the closure of the convex hull of $\left(0, \frac{1}{2}\log(1+P_2)\right)$,*

$\left(\frac{1}{2}\log\left(1+P_1\right),0\right)$, *and all* (R_1, R_2) *in* $\mathcal{C}_{w5}(\beta_1,\beta_2)$ *and* $\mathcal{C}_{w6}(\beta_1,\beta_2)$ *for any* $\beta_1,\beta_2 \in (0,1)$.

The numerical comparison between the above achievable rate regions with the outer bound $\mathcal{C}_{w\_o}$ [10] is shown in Section F.

4. Scheme with Flexible Sequential Decoder

For the sequential decoder of $\mathcal{C}_{w1}$ in Section 1, each receiver first decodes the common messages by treating the private messages as noise, then decodes the intended private message by treating the interfered private message as noise. Note that we can easily extend the above scheme by changing the decoding order. For example, receiver 1 could also decode the intended common message and private message first, or decode the "interfered" common message and intended private message first. Therefore, each receiver has 3 choices of different sequential decoders, which means that there are 9 different choices with two receivers. Similarly, we could have another 9 choices based on the sequential decoder of $\mathcal{C}_{w2}$, which optimizes the DPC parameter at the MAC for receiver 2. Finally, we can apply Fourier-Motzkin algorithm for each implicit achievable rate region corresponding to each decoder (18 different decoders in total), then obtain the explicit achievable rate regions, and finally deploy the time-sharing technique to enlarge the achievable rate region. The details are omitted here due to its similarity to the previous results.

F. Numerical Results

In this section, we compare the derived various achievable rate regions with the outer bound, which is the same as the outer bound derived for the traditional Gaussian IC [8–10], since the traditional IC can be treated as the idealization of our model where the state is also known at the receivers. We show the numerical results for three cases: the strong interference case, the mixed interference case, and the weak interference case. From the numerical comparison, we can easily see that active interference cancellation significantly enlarges the achievable rate region for the strong and mixed interference case. However, for the weak interference case, flexible power allocation brings more benefit due to the "preciousness" of the transmission power.

In Fig. 4, we compare the achievable rate regions in Section 2 with the outer bound $\mathcal{C}_{s\_o}$, which is the capacity region of the traditional strong Gaussian IC with the state information also known at the receivers [8]. Note that the inner bound $\mathcal{C}_{s\_in}$ is defined as the rate region when the transmitters ignore the non-causal state information. Compared with $\mathcal{C}_{s1}$ and $\mathcal{C}_{s2}$ (only utilizing DPC), we see that the knowledge of the state information at the transmitters improves the performance significantly by deploying DPC. Moreover, it can be easily seen that $\hat{\mathcal{C}}_{s3}$ and $\hat{\mathcal{C}}_{s4}$ (utilizing DPC and active interference cancellation) are much bigger than $\mathcal{C}_{s1}$ and $\mathcal{C}_{s2}$, respectively, which implies that active interference cancellation enlarges the achievable rate region significantly. Finally, we observe that the achievable rate region $\mathcal{C}_s$ is fairly close to the outer bound, even when the state power is the same as the source power.

In Fig. 5, we compare the achievable rate regions in Section 2 with the outer bound $\mathcal{C}_{m\_o}$, which is the same as the outer bound derived for the traditional mixed Gaussian IC [10]. Also we define the inner bound $\mathcal{C}_{m\_in}$ as the achievable rate region when the transmitters ignore the non-causal state information. Compared with $\hat{\mathcal{C}}_{m1}$ and $\mathcal{C}_{m2}$ (only utilizing DPC), we see that the knowledge of the state information at the transmitters enlarges the achievable rate region significantly due to DPC. Furthermore, it can be easily seen that $\hat{\mathcal{C}}_{m3}$ and $\hat{\mathcal{C}}_{m4}$ (utilizing DPC and active interference cancellation) are much larger than $\hat{\mathcal{C}}_{m1}$ and $\mathcal{C}_{m2}$, respectively, which implies that active interference cancellation improves the performance significantly.

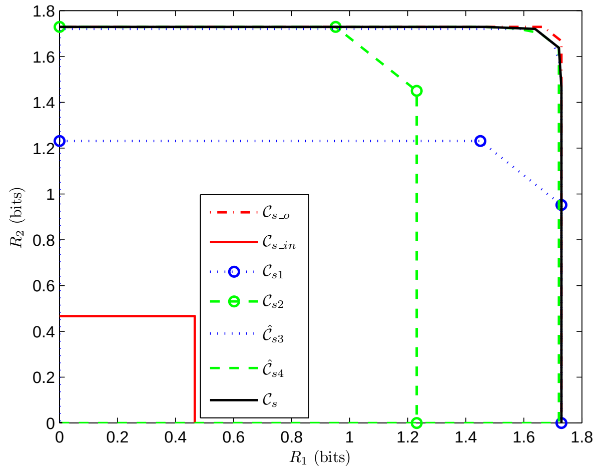

Fig. 4.: Comparison of different achievable rate regions and the outer bound for the strong Gaussian IC with state information. The channel parameters are set as: $g_{12} = g_{21} = 10$, $N_1 = N_2 = 1$, $P_1 = P_2 = K = 10$ dB.

For the degraded Gaussian IC with state information, we compare the achievable rate regions with the outer bound $\mathcal{C}_{m\_o}$ and the inner bound $\mathcal{C}_{m\_in}$ in Fig. 6. Note that the difference from the general mixed interference case is that the outer bound $\mathcal{C}_{m\_o}$ now includes the sum capacity [9]. Similar to the general mixed interference case, active interference cancellation improves the performance significantly when the interference is degraded.

In Fig. 7, we compare the achievable rate regions in Section 2 with the outer bound $\mathcal{C}_{w\_o}$, which is the same as the outer bound derived for the traditional weak Gaussian IC [10]. Also define the inner bound $\mathcal{C}_{w\_in}$ as the achievable rate region when the transmitters ignore the non-causal state information. Compared with $\mathcal{C}_{w1}$ and $\mathcal{C}_{w2}$ (only utilizing DPC), we see that the knowledge of the state information at the transmitters improves the performance significantly due to DPC. However, $\hat{\mathcal{C}}_{w3}$ and $\hat{\mathcal{C}}_{w4}$ (utilizing DPC and active interference cancellation) are only slightly larger than $\mathcal{C}_{w1}$ and $\mathcal{C}_{w2}$, i.e., unlike the strong interference case and the mixed interference case, active interference cancellation cannot enlarge the achievable rate region significantly

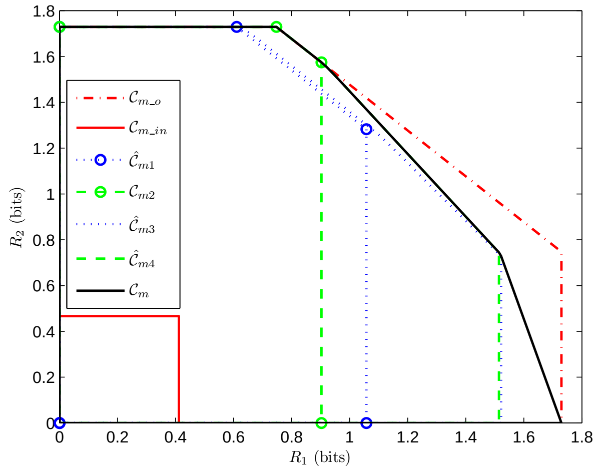

Fig. 5.: Comparison of different achievable rate regions and the outer bound for the mixed Gaussian IC with state information. The channel parameters are set as: $g_{12} = 0.2$, $g_{21} = 2$, $N_1 = N_2 = 1$, $P_1 = P_2 = K = 10$ dB.

for the weak interference case. Intuitively, the reason is that the source power is too "precious" to be used for canceling the state effect if the interference is weak.

In Fig. 8, we compare the achievable rate regions of the flexible power allocation schemes in Section 3 with the outer bound $\mathcal{C}_{w\_o}$ and the inner bound $\mathcal{C}_{w\_in}$. It can be easily seen that $\hat{\mathcal{C}}_{w5}$ and $\hat{\mathcal{C}}_{w6}$ (both utilizing DPC and flexible power allocation) are much larger than $\mathcal{C}_{w1}$ and $\mathcal{C}_{w2}$, respectively, i.e., flexible power allocation between the common and private messages enlarges the achievable rate region significantly for the weak interference case.

G. Summary

In this chapter, we considered the Gaussian interference channel with state information non-causally known at both transmitters. The achievable rate region was established over the simultaneous encoding scheme introduced in Chapter II and the newly proposed active interference cancelation technique. In addition, we proposed heuristic schemes for the strong interference case, the mixed interference case, and the

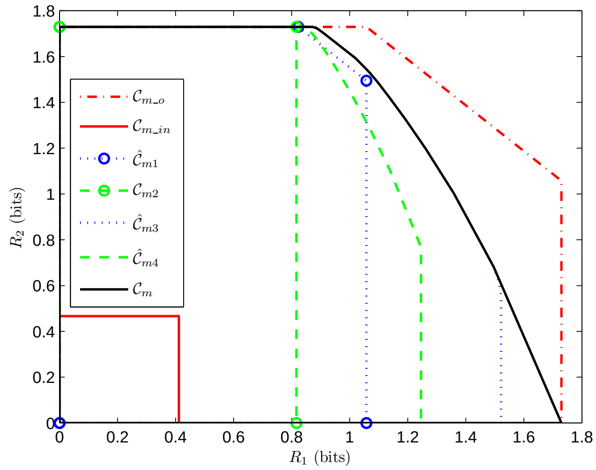

Fig. 6.: Comparison of different achievable rate regions and the outer bound for the degraded Gaussian IC with state information. The channel parameters are set as: $g_{12} = 0.2$, $g_{21} = 5$, $N_1 = N_2 = 1$, $P_1 = P_2 = K = 10$ dB.

weak interference case. The numerical results showed that active interference cancellation significantly improves the performance for the strong and mixed interference case, and flexible power splitting significantly enlarges the achievable rate region for the weak interference case.

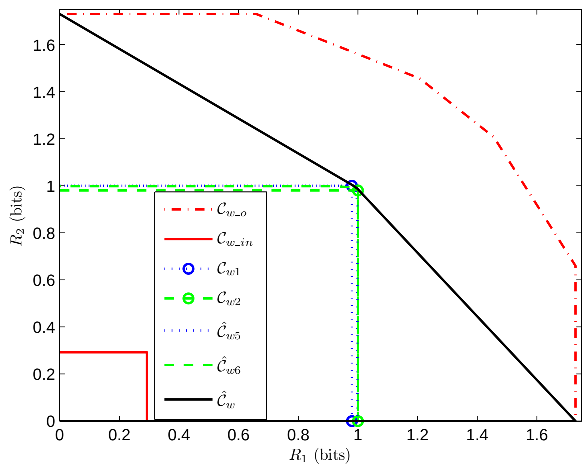

Fig. 7.: Comparison of different achievable rate regions and the outer bound for the weak interference Gaussian IC with state information. The channel parameters are set as: $g_{12} = g_{21} = 0.2,\ N_1 = N_2 = 1,\ P_1 = P_2 = K = 10$ dB.

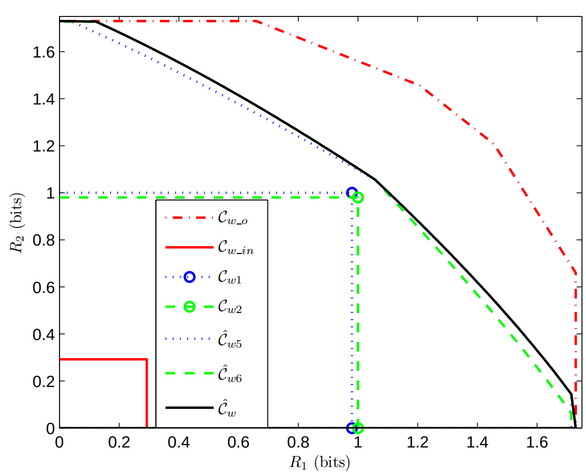

Fig. 8.: Comparison of different achievable rate regions and the outer bound for the weak interference Gaussian IC with state information. The channel parameters are set as: $g_{12} = g_{21} = 0.2,\ N_1 = N_2 = 1,\ P_1 = P_2 = K = 10$ dB.

CHAPTER IV

SYMMETRIC AWGN CHANNEL

In this chapter, we study the state-dependent Gaussian interference channel, where the interfering Gaussian state is non-causally known at both transmitters but unknown to either of the receivers. We focus on the simplest symmetric case, where both direct link gains are the same with each other, and both interfering link gains are the same with each other. We apply the coding scheme in Chapter III with different dirty paper coding parameters. When the state is additive and symmetric at both receivers, we study both strong and weak interference scenarios and characterize the theoretical gap between the achievable symmetric rate and the upper bound, which is shown to be less than 1/4 bit for the strong interference case and less than 3/4 bit for the weak interference case. Then we provide numerical evaluations of the achievable rates against the upper bound, which validates the theoretical analysis for both strong and weak interference scenarios. Finally, we define the generalized degrees of freedom for the symmetric Gaussian case, and compare the lower bounds against the upper bounds for both strong and weak interference cases. We also show that our achievable schemes can obtain the exact optimal values of the generalized degrees of freedom, i.e., the lower bounds meet the upper bounds for both strong and weak interference cases.

A. Channel Model

Consider the symmetric AWGN interference channel as shown in Fig. 9, where two transmitters communicate with the corresponding receivers through a common channel dependent on the additive Gaussian state S. The transmitters do not cooperate with each other; however, they both know the additive state information S non-causally, which is known to neither of the receivers. Each receiver needs to decode the information from the respective transmitter. The channel input and output relationship can be described as follows:

$$\begin{aligned} Y_1 &= h_1 X_1' + h_2 X_2' + S + Z_1, \\ Y_2 &= h_1 X_2' + h_2 X_1' + S + Z_2, \end{aligned}$$

where h_1 is the real link amplitude gain from each transmitter to the intended receiver, h_2 is the link gain from each transmitter to the interfered receiver, X_i' and Y_i are the channel input and output, respectively, and Z_i is the zero-mean unit-variance AWGN noise, for $i = 1, 2$. Both receivers also suffer from the zero-mean additive white Gaussian interference $S \sim \mathcal{N}(0, K)$, which is non-causally known at both transmitters.

Without loss of generality, we transform the above channel model into the following standard form for simplicity:

$$Y_1 = X_1 + gX_2 + S + Z_1, \tag{4.1}$$

$$Y_2 = X_2 + gX_1 + S + Z_2, \tag{4.2}$$

where

$$X_1 = h_1 X_1', \ X_2 = h_1 X_2', \text{ and } g = \frac{h_2}{h_1}.$$

We also assume that the channel inputs X_1 and X_2 satisfy the following symmetric power constraints:

$$\frac{1}{n}\sum_{i=1}^{n}(X_{1i})^2 \leq P, \text{ and } \frac{1}{n}\sum_{i=1}^{n}(X_{2i})^2 \leq P.$$

Here we omit the definitions for the error probability, the achievable rate pair (R_1, R_2), and the capacity region for the above channel. We refer readers to Chapter II

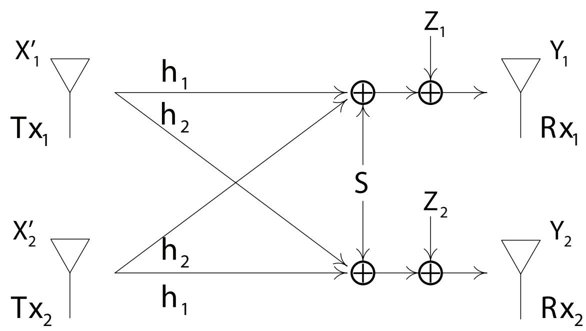

Fig. 9.: The symmetric Gaussian interference channel with state information non-causally known at both transmitters.

and Chapter III for details. Due to the channel symmetry, we define the symmetric capacity [10] as the optimal solution of the following optimization problem:

$$\begin{aligned} C_{\text{sym}} := \quad & \max \quad \min\{R_1, R_2\} \\ & \text{subject to} \quad (R_1, R_2) \text{ is in the capacity region.} \end{aligned}$$

As shown in [10], the symmetric capacity maximizes the sum rate $R_1 + R_2$ since the capacity region is convex and symmetric. Hence, instead of characterizing the inner and outer bounds over the achievable rate region, we will focus on deriving the lower and upper bounds on the symmetric capacity.

B. Strong Interference Case

In this section, we will first present an achievable coding scheme for the channel model in (4.1) and (4.2) with $g > 1$, then calculate the smallest symmetric rate over different state power K, and finally provide an upper bound on the maximum gap between the achievable symmetric rate for the strong Gaussian IC with state information and the symmetric capacity for the traditional strong Gaussian IC without the common interference state, with the latter one providing a capacity outer bound for our channel.

We deploy the simultaneous encoding scheme for the strong interference case in Chapter II, where the transmitters only send common messages and utilize DPC to help. We first give the rate region for the MAC at receiver 1 as follows:

$$R_1 < I(U_1; Y_1|U_2) - I(U_1; S|U_2), \tag{4.3}$$

$$R_2 < I(U_2; Y_1|U_1) - I(U_2; S|U_1), \tag{4.4}$$

$$R_1 + R_2 < I(U_1, U_2; Y_1) - I(U_1, U_2; S), \tag{4.5}$$

where R_1 and R_2 are the achievable rates for transmitters 1 and 2, respectively. Similarly, the rate region for the MAC at receiver 2 can be obtained by substituting Y_1 with Y_2 in (4.3) to (4.5). Therefore, the corresponding achievable rate region for the strong Gaussian IC with state information can be calculated as the intersection of the two MAC regions.

Since the channel is symmetric, we choose the following auxiliary random variables:

$$U_1 = X_1 + \alpha S, \tag{4.6}$$

$$U_2 = X_2 + \alpha S. \tag{4.7}$$

Note that the above auxiliary random variables are different from the ones in Chapter III. It can be easily shown that the two MACs at the two receivers cannot be capacity-achieving simultaneously, since the optimal DPC parameters for the MAC at receiver 1 are different from that for the MAC at receiver 2. The achievable schemes in Chapter III optimize the DPC parameters for one MAC, and make the other MAC suffer from the non-optimal choices, such that the rate region is degraded. To address the above issue, here we choose the same parameter α for both U_1 and U_2 due to the channel symmetry, and present the theoretical comparison between the achievable symmetric rate and the upper bound. The idea is to achieve a larger intersection by balancing the two MACs better than the scheme in Chapter III.

We now characterize an achievable symmetric rate with the above auxiliary random variables in the following Lemma.

Lemma 1. *With different state power K, the smallest achievable rate region for the strong Gaussian IC with state information occurs when $K \to \infty$, and the corresponding achievable symmetric rate is*

$$R_{sym} = \min\left\{\frac{1}{2}\log\left(\frac{(1+P)\alpha^2+(\alpha g-1)^2P}{2\alpha^2+(\alpha+\alpha g-1)^2P}\right), \frac{1}{4}\log\left(\frac{P}{2\alpha^2+(\alpha+\alpha g-1)^2P}\right)\right\}, \tag{4.8}$$

where $0 < \alpha < \frac{2P}{1+2gP}$.

Proof. With the auxiliary random variables $U_1 = X_1 + \alpha S$ and $U_2 = X_2 + \alpha S$, we can calculate the right-hand sides in (4.3) to (4.5) as follows:

$$\begin{aligned}
I(U_1;Y_1|U_2) - I(U_1;S|U_2) &= h(U_1|S,U_2) - h(U_1|Y_1,U_2) \\
&= h(U_1,U_2,S) - h(U_2,S) - h(U_1,U_2,Y_1) + h(U_2,Y_1) \\
&= \frac{1}{2}\log\left(\frac{(1+P)\left(1+\frac{\alpha^2K}{P}\right)+K\left(\alpha g-1\right)^2}{1+\frac{2\alpha^2K}{P}+K\left(\alpha+\alpha g-1\right)^2}\right), \\
I(U_2;Y_1|U_1) - I(U_2;S|U_1) &= h(U_1,U_2,S) - h(U_1,S) - h(U_1,U_2,Y_1) + h(U_1,Y_1) \\
&= \frac{1}{2}\log\left(\frac{(1+g^2P)\left(1+\frac{\alpha^2K}{P}\right)+K\left(\alpha-1\right)^2}{1+\frac{2\alpha^2K}{P}+K\left(\alpha+\alpha g-1\right)^2}\right), \\
I(U_1,U_2;Y_1) - I(U_1,U_2;S) &= h(U_1,U_2,S) - h(S) - h(U_1,U_2,Y_1) + h(Y_1) \\
&= \frac{1}{2}\log\left(\frac{1+P+g^2P+K}{1+\frac{2\alpha^2K}{P}+K\left(\alpha+\alpha g-1\right)^2}\right).
\end{aligned}$$

Since the channel is symmetric, the intersection of the two MACs can be calculated as:

$$R_1, R_2 < \min\left\{\frac{1}{2}\log\left(\frac{(1+g^2P)\left(1+\frac{\alpha^2K}{P}\right)+K(\alpha-1)^2}{1+\frac{2\alpha^2K}{P}+K(\alpha+\alpha g-1)^2}\right),\right.$$

$$\left.\frac{1}{2}\log\left(\frac{(1+P)\left(1+\frac{\alpha^2K}{P}\right)+K(\alpha g-1)^2}{1+\frac{2\alpha^2K}{P}+K(\alpha+\alpha g-1)^2}\right)\right\}, \qquad (4.9)$$

$$R_1 + R_2 < \frac{1}{2}\log\left(\frac{1+P+g^2P+K}{1+\frac{2\alpha^2K}{P}+K(\alpha+\alpha g-1)^2}\right). \qquad (4.10)$$

It can be easily shown that the first item in (4.9) is larger than the second item, thus we can recast the achievable rate region for the strong Gaussian IC with state information as:

$$R_1, R_2 < \frac{1}{2}\log\left(\frac{(1+P)\left(1+\frac{\alpha^2K}{P}\right)+K(\alpha g-1)^2}{1+\frac{2\alpha^2K}{P}+K(\alpha+\alpha g-1)^2}\right),$$

$$R_1 + R_2 < \frac{1}{2}\log\left(\frac{1+P+g^2P+K}{1+\frac{2\alpha^2K}{P}+K(\alpha+\alpha g-1)^2}\right).$$

Both right-hand sides in the above inequalities are decreasing functions over K, i.e., we can conclude that for different state power K, the smallest achievable symmetric rate occurs when $K \to \infty$:

$$R_{\text{sym}} = \min\left\{\frac{1}{2}\log\left(\frac{\frac{\alpha^2}{P}+\alpha^2+(\alpha g-1)^2}{\frac{2\alpha^2}{P}+(\alpha+\alpha g-1)^2}\right), \frac{1}{4}\log\left(\frac{1}{\frac{2\alpha^2}{P}+(\alpha+\alpha g-1)^2}\right)\right\}. \qquad (4.11)$$

To guarantee that the above achievable symmetric rate is positive, the following inequalities must hold:

$$\frac{2\alpha^2}{P}+(\alpha+\alpha g-1)^2 < \frac{\alpha^2}{P}+\alpha^2+(\alpha g-1)^2,$$

$$\frac{2\alpha^2}{P}+(\alpha+\alpha g-1)^2 < 1.$$

Or equivalently, α must be in $\left(0, \frac{2P}{1+2gP}\right)$. □

Now we will find a heuristic $\alpha \in \left(0, \frac{2P}{1+2gP}\right)$ motivated by the simulations results in Section D, which shows that the optimal α maximizing the first item in the right-hand side of (4.8) is actually very close to the value maximizing the whole right-hand side of (4.8). Then we calculate the corresponding gap between the achievable symmetric rate in (4.8) and the upper bound, which is the symmetric capacity for the traditional strong Gaussian IC.

Theorem 11. *There exists a DPC parameter* $\alpha \in \left(0, \frac{2P}{1+2gP}\right)$ *such that the maximum gap between the achievable symmetric rate for the strong Gaussian IC with state information and the upper bound is less than* $1/4$ *bit.*

Proof. Note that maximizing the achievable symmetric rate in (4.8) over α is indeed a max-min problem and is equivalent to finding the roots of a fourth-order equation, for which we could not find an analytical solution. Hence, we heuristically maximize the single rate item in (4.8):

$$\begin{aligned} \max \quad & \frac{(1+P)\alpha^2 + (\alpha g - 1)^2 P}{2\alpha^2 + (\alpha + \alpha g - 1)^2 P} \\ \text{subject to} \quad & \alpha \in \left(0, \frac{2P}{1+2gP}\right). \end{aligned}$$

It can be easily shown that the optimal α for the above optimization problem is:

$$\alpha^* = \frac{P}{1+P+gP}. \tag{4.12}$$

In Section D, we will show that the above α^* is actually very close to the optimal value which maximizes (4.8).

Now with this α^*, we calculate the achievable symmetric rate in (4.8) as follows:

$$R_{\text{sym}} = \min\left\{\frac{1}{2}\log(1+P), \frac{1}{4}\log\left(\frac{(1+P+gP)^2}{1+2P}\right)\right\}.$$

Moreover, the upper bound on the symmetric capacity of the strong Gaussian

IC with state information is [8]:

$$C_{\text{sym}}^{+} = \min\left\{\frac{1}{2}\log\left(1+P\right), \frac{1}{4}\log\left(1+P+g^2P\right)\right\}.$$

With both the achievable symmetric rate and the upper bound, we split the gap analysis into three cases:

1. If $\frac{(1+P+gP)^2}{1+2P} \geq (1+P)^2$, i.e., $g \geq \frac{1+P}{P}\left(\sqrt{1+2P}-1\right)$, the symmetric capacity is the same as the traditional very strong Gaussian IC $\left(g \geq \sqrt{1+P}\right)$:

$$C_{\text{sym}} = \frac{1}{2}\log\left(1+P\right).$$

2. If $\sqrt{1+P} \leq g < \frac{1+P}{P}\left(\sqrt{1+2P}-1\right)$, the upper bound on the symmetric capacity is still

$$C_{\text{sym}}^{+} = \frac{1}{4}\log\left((1+P)^2\right),$$

and we can achieve the following symmetric rate no matter how large K is:

$$R_{\text{sym}} = \frac{1}{4}\log\left(\frac{(1+P+gP)^2}{1+2P}\right).$$

Due to the monotonic increasing property of the log function, we only need to compare the item inside the log function. Then we see that the gap between the achievable symmetric rate and the upper bound is less than 1/4 bit since:

$$\begin{aligned}\frac{(1+P+gP)^2}{1+2P} - \frac{(1+P)^2}{2} &= \frac{1+2g^2P^2+4gP+4gP^2-2P^3-3P^2}{2(1+2P)} \\ &\geq \frac{1+4gP+4gP^2-P^2}{2(1+2P)} \\ &> 0,\end{aligned}$$

and then

$$\frac{1}{4}\log\left(\frac{(1+P+gP)^2}{1+2P}\right) > \frac{1}{4}\log\left((1+P)^2\right) - \frac{1}{4}.$$

3. If $1 < g < \sqrt{1+P}$, the upper bound on the symmetric capacity is:

$$C_{\text{sym}}^{+} = \frac{1}{4}\log\left(1+P+g^2P\right),$$

and we can achieve the following symmetric rate no matter how large K is:

$$R_{\text{sym}} = \frac{1}{4}\log\left(\frac{(1+P+gP)^2}{1+2P}\right).$$

Similar to the previous case, we only need to compare the item inside the log function. It can be easily shown that the gap between the achievable symmetric rate and the upper bound is still less than 1/4 bit since:

$$\begin{aligned}\frac{(1+P+gP)^2}{1+2P} - \frac{1+P+g^2P}{2} &= \frac{1+P+4gP+4gP^2-g^2P}{2(1+2P)} \\ &> 0, \quad \text{for } 1 < g < \sqrt{1+P}.\end{aligned}$$

□

C. Weak Interference Case

For the traditional symmetric weak Gaussian IC, the authors in [10] proposed a power splitting solution for the Han-Kobayashi scheme, where the private message power levels at both transmitters are chosen such that the interfering private SNR at each receiver is equal to 1. They also showed that the gap between the achievable symmetric rate and the upper bound is less than 1/2 bit (if all the random variables are defined over the field of real numbers $\mathbb{R}$). Here, with similar power assignment for the message splitting, we introduce an achievable coding scheme for the weak symmetric Gaussian IC with state information ($g < 1$), and derive the gap between the achievable symmetric rate and that of the traditional weak Gaussian IC, which turns out to be less than 1/4 bit. Therefore, we conclude that the maximum gap between the achievable symmetric rate of the weak Gaussian IC with state information and the upper bound is less than $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$ bit. Similar to [10], here we focus on the case that the interference power is larger than the noise power, i.e., $g^2P > 1$. Otherwise, the receivers just treat the interference as noise since the channel is noise-limited instead of interference-limited.

The coding scheme can be described as follows. The message is split into common and private parts at each transmitter, and the channel input is shown as follows:

$$\begin{aligned} X_1 &= A_1 + B_1, \\ X_2 &= A_2 + B_2, \end{aligned}$$

where A_i corresponds to the common message part and B_i corresponds to the private message part at transmitter i, for $i = 1, 2$. Here we also set the private message power to ensure that the interfering private SNR at each receiver is equal to 1, i.e., $P_{B_1} = P_{B_2} = \frac{1}{g^2} =: P_B$, $P_{A_1} = P_{A_2} = P - \frac{1}{g^2} =: P_A$. We utilize the sequential decoder, i.e., each receiver first deals with the common message MAC by treating both private messages as noises. Note that here for both MACs, DPC is applied to transmit both common messages. At receiver 1, the achievable rate region of the common message MAC is:

$$R_{10} < I(U_1; Y_1|U_2) - I(U_1; S|U_2), \tag{4.13}$$

$$R_{20} < I(U_2; Y_1|U_1) - I(U_2; S|U_1), \tag{4.14}$$

$$R_{10} + R_{20} < I(U_1, U_2; Y_1) - I(U_1, U_2; S), \tag{4.15}$$

where R_{10} and R_{20} are the achievable rates for the common messages of transmitters 1 and 2, respectively. The corresponding achievable rate region at receiver 2 can be shown similarly by substituting Y_1 with Y_2 in (4.13) to (4.15). Accordingly, the achievable rate regions for the common messages can be obtained as the intersections of the corresponding MAC regions.

After decoding both common messages, each receiver decodes the intended private message by treating the interfering private message as noise. We also use DPC to transmit each intended private message. By setting the DPC parameters for the private messages at the optimal value derived in [17], the following symmetric rate for the intended private messages can be achieved:

$$R^{\text{p}}_{\text{sym}} = \frac{1}{2}\log\left(1+\frac{P_B}{2}\right), \tag{4.16}$$

which is the same as the private message rate in [10]. Thus, in this section we focus on characterizing the gap between the achievable symmetric rate of the common message MAC for the Gaussian IC with state information and that of the traditional Gaussian IC.

Due to the channel symmetry, we choose the following auxiliary random variables, which are different from the choices in Chapter III:

$$U_1 = A_1 + \alpha_1 S, \tag{4.17}$$

$$U_2 = A_2 + \alpha_1 S, \tag{4.18}$$

where both common messages have the same DPC parameter α_1. In the following lemma, we describe the achievable symmetric rate for the common message MAC with the above auxiliary random variables.

Lemma 2. *With different state power K, the smallest achievable rate region for the common message MAC occurs when $K \to \infty$, and the corresponding achievable* *symmetric rate is shown as follows:*

$$R^c_{sym} = \min\left\{\frac{1}{2}\log\left(\frac{(1+g^2Q)\,\alpha_1^2+(\alpha_1-1)^2 Q}{2\alpha_1^2+(\alpha_1+\alpha_1 g-1)^2 Q}\right), \frac{1}{4}\log\left(\frac{Q}{2\alpha_1^2+(\alpha_1+\alpha_1 g-1)^2 Q}\right)\right\}, \tag{4.19}$$

where $Q := \frac{P_A}{2+P_B}$ *and* $0 < \alpha_1 < \frac{2gQ}{1+2gQ}$.

Proof. With the previously-mentioned coding scheme and auxiliary random variables, the intersection of the two MAC regions is calculated as:

$$\begin{aligned} R_{10}, R_{20} &< \min\left\{\frac{1}{2}\log\left(\frac{(1+g^2Q)\,(P_A+\alpha_1^2K)+KQ\,(\alpha_1-1)^2}{P_A+2\alpha_1^2K+KQ\,(\alpha_1+\alpha_1 g-1)^2}\right),\right. \\ &\quad \left.\frac{1}{2}\log\left(\frac{(1+Q)\,(P_A+\alpha_1^2K)+KQ\,(\alpha_1 g-1)^2}{P_A+2\alpha_1^2K+KQ\,(\alpha_1+\alpha_1 g-1)^2}\right)\right\}, \quad (4.20) \\ R_{10}+R_{20} &< \frac{1}{2}\log\left(\frac{(1+P+g^2P+K)\,Q}{P_A+2\alpha_1^2K+KQ\,(\alpha_1+\alpha_1 g-1)^2}\right), \quad (4.21) \end{aligned}$$

where Q is denoted as $Q = \frac{P_A}{2+P_B}$. It can be shown that the first item in (4.20) is always smaller than the second item. Therefore, we rewrite the above region as:

$$\begin{aligned} R_{10}, R_{20} &< \frac{1}{2}\log\left(\frac{(1+g^2Q)\,(P_A+\alpha_1^2K)+KQ\,(\alpha_1-1)^2}{P_A+2\alpha_1^2K+KQ\,(\alpha_1+\alpha_1 g-1)^2}\right), \\ R_{10}+R_{20} &< \frac{1}{2}\log\left(\frac{(1+P+g^2P+K)\,Q}{P_A+2\alpha_1^2K+KQ\,(\alpha_1+\alpha_1 g-1)^2}\right). \end{aligned}$$

Similar to the strong interference case, both right-hand sides in the above inequalities are decreasing functions over K, which means that we can always achieve the following achievable symmetric rate no matter how large K is:

$$R^c_{\text{sym}} = \min\left\{\frac{1}{2}\log\left(\frac{\left(\frac{1}{Q}+g^2\right)\alpha_1^2+(\alpha_1-1)^2}{\frac{2\alpha_1^2}{Q}+(\alpha_1+\alpha_1 g-1)^2}\right), \frac{1}{4}\log\left(\frac{1}{\frac{2\alpha_1^2}{Q}+(\alpha_1+\alpha_1 g-1)^2}\right)\right\}. \tag{4.22}$$

To make sure that the above achievable symmetric rate is positive, we must choose

α_1 such that the following inequalities hold:

$$\begin{aligned}\frac{2\alpha_1^2}{Q} + (\alpha_1 + \alpha_1 g - 1)^2 &< 1, \\ \frac{2\alpha_1^2}{Q} + (\alpha_1 + \alpha_1 g - 1)^2 &< \left(\frac{1}{Q} + g^2\right)\alpha_1^2 + (\alpha_1 - 1)^2,\end{aligned}$$

which means that $\alpha_1 \in \left(0, \frac{2gQ}{1+2gQ}\right)$. □

In the following theorem, we will find a heuristic $\alpha_1 \in \left(0, \frac{2gQ}{1+2gQ}\right)$, and then characterize the corresponding gap between the achievable symmetric rate of the common messages for the Gaussian IC with state information and that of the traditional Gaussian IC.

Theorem 12. *There exists a DPC parameter $\alpha_1 \in \left(0, \frac{2gQ}{1+2gQ}\right)$ for the common messages such that the maximum gap between the symmetric rate of the common messages for the Gaussian IC with state information and that of the traditional Gaussian IC is less than $1/4$ bit.*

Proof. Similar to the strong interference case, maximizing the rate in (4.19) is equivalent to solving a fourth-order equation. Therefore, here we only maximize the first item of the right-hand side in (4.19):

$$\begin{aligned}\max \quad & \frac{(1+g^2Q)\,\alpha_1^2 + (\alpha_1 - 1)^2 Q}{2\alpha_1^2 + (\alpha_1 + \alpha_1 g - 1)^2 Q} \\ \text{subject to} \quad & \alpha_1 \in \left(0, \frac{2gQ}{1+2gQ}\right),\end{aligned}$$

where $Q = \frac{P_A}{2+P_B}$. We can easily show that the optimal α_1 for the above optimization problem is:

$$\alpha_1^* = \frac{gQ}{1+gQ+g^2Q}. \tag{4.23}$$

Now with this α_1^*, the achievable symmetric rate in (4.19) can be written as:

$$R_{\text{sym}}^{\text{c}} = \min\left\{\frac{1}{2}\log\left(1+g^2Q\right), \frac{1}{4}\log\left(\frac{\left(1+gQ+g^2Q\right)^2}{1+2g^2Q}\right)\right\}.$$

Furthermore, it was shown in [10] that the achievable symmetric rate of the common messages for the traditional Gaussian IC is:

$$R_{\text{sym}}^{\text{c+}} = \min\left\{\frac{1}{2}\log\left(1+g^2Q\right), \frac{1}{4}\log\left(1+\left(1+g^2\right)Q\right)\right\}.$$

We calculate the gap between the above two symmetric rates by splitting the analysis into three cases:

1. If $\left(1+g^2Q\right)^2 \leq \frac{\left(1+gQ+g^2Q\right)^2}{1+2g^2Q}$, the achievable symmetric rate in (4.19) is the same as the one for the traditional Gaussian IC:

$$R_{\text{sym}}^{\text{c}} = \frac{1}{2}\log\left(1+g^2Q\right).$$

2. If $\frac{\left(1+gQ+g^2Q\right)^2}{1+2g^2Q} < \left(1+g^2Q\right)^2 \leq 1+\left(1+g^2\right)Q$ (Note that the second inequality is equivalent to $g^4Q \leq 1-g^2$), the achievable symmetric rate of the common messages for the traditional Gaussian IC is:

$$R_{\text{sym}}^{\text{c+}} = \frac{1}{4}\log\left(\left(1+g^2Q\right)^2\right),$$

and the corresponding achievable symmetric rate for the Gaussian IC with state information is:

$$R_{\text{sym}}^{\text{c}} = \frac{1}{4}\log\left(\frac{\left(1+gQ+g^2Q\right)^2}{1+2g^2Q}\right).$$

Similar to the strong interference case, the gap between the two achievable symmetric rates is less than 1/4 bit since:

$$\frac{\left(1+gQ+g^2Q\right)^2}{1+2g^2Q} - \frac{\left(1+g^2Q\right)^2}{2}$$

$$
\begin{aligned}
&= \frac{1 + 4gQ + g^3Q^2 + 3g^3Q^2(1-g) + 2g^2Q^2(1-g^4Q)}{2\left(1+2g^2Q\right)} \\
&> 0,
\end{aligned}
$$

where the last inequality is due to $g < 1$ and $g^4Q \leq 1 - g^2 < 1$ (equivalent to the second inequality of the current constraint on g).

3. If $1 + \left(1+g^2\right)Q < \left(1+g^2Q\right)^2$, or equivalently, $g^4Q > 1 - g^2$, the achievable symmetric rate of the common messages for the traditional Gaussian IC is:

$$
R_{\text{sym}}^{\text{c+}} = \frac{1}{4}\log\left(1 + \left(1+g^2\right)Q\right),
$$

and the corresponding achievable symmetric rate for the Gaussian IC with state information is:

$$
R_{\text{sym}}^{\text{c}} = \frac{1}{4}\log\left(\frac{\left(1+gQ+g^2Q\right)^2}{1+2g^2Q}\right).
$$

Whether the gap between the two achievable symmetric rates is less than 1/4 bit depends on whether the following item is positive or not:

$$
\frac{\left(1+gQ+g^2Q\right)^2}{1+2g^2Q} - \frac{1+\left(1+g^2\right)Q}{2} = \frac{1+(g^2+4g-1)Q+4g^3Q^2}{2\left(1+2g^2Q\right)}.
$$

When $g^2 + 4g - 1 \geq 0$, the ratio above is clearly positive and the gap is less than 1/4 bit. Otherwise, if $g^2 + 4g - 1 < 0$, or equivalently $g < \sqrt{5} - 2$, the ratio in (4.24) is still positive since:

$$
4g^3Q^2 - Q > g^3Q^2 - Q = \frac{Q}{g}\left(g^4Q - g\right) > 0,
$$

where the last inequality is due to $g^4Q > 1 - g^2 > 4g > g$ for $0 < g < \sqrt{5} - 2$.

□

Remark 7. *Note that with DPC for point-to-point channel [17], we can achieve the same private message rates as in [10]. Considering the previous theorem together with*

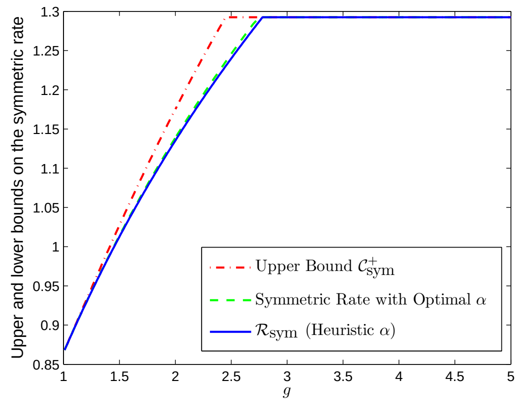

Fig. 10.: Comparison between the achievable symmetric rate with optimal α, the rate with heuristic α, and the upper bound for the strong interference case.

the gap analysis in [10], we conclude that the gap between the achievable symmetric rate for the weak Gaussian IC with state information and the upper bound is less than $3/4$ *bit.*

D. Numerical Results

In this section, we present the comparisons among the achievable symmetric rate with optimal α, the rate with heuristic α, and the upper bound for both strong and weak interference cases. The source power is set as $P = 5$ for the strong interference case, and as $P = 100$ for the weak interference case to satisfy $g^2P > 1$ when $g > 0.1$.

Fig. 10 shows the symmetric rate comparison for the strong interference case with $1 < g < 5$. We can easily see that the gap between the rate with heuristic α and the upper bound is less than $1/4$ bit, which coincides with the theoretical analysis in Section B. Fig. 11 compares the heuristic α and the optimal α obtained by exhausted searching, and demonstrates that the heuristic choice is very close to the optimal value.

In Fig. 12, we compare the achievable symmetric rate with optimal α_1, the rate

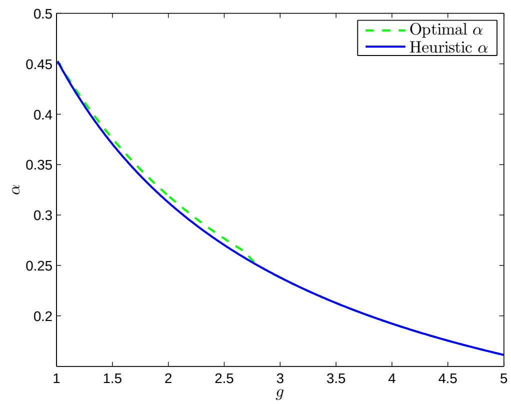

Fig. 11.: Comparison between the heuristic α and the optimal α obtained by exhausted search for the strong interference case.

with heuristic α_1, the symmetric rate for the traditional IC, and the upper bound for the weak interference case with $0.1 < g < 1$. It can be seen that the gap between the rate with heuristic α_1 and the upper bound is less than 3/4 bit, which verifies the theoretical analysis in Section C. In Fig. 13, we show the comparison between the heuristic α_1 and the optimal α_1 calculated by exhausted searching, and we can easily see that the heuristic choice is also very close to the optimal value.

E. Generalized Degrees of Freedom

In this section, we define the generalized degrees of freedom for the symmetric Gaussian IC with state information. We compare the lower bound with the upper bound in both strong and weak interference scenarios, and show that our schemes achieve the optimal degrees of freedom by utilizing symmetric DPC parameters α^* in (4.12) and α_1^* in (4.23), respectively. Now we first characterize the degrees of freedom performance in the high SNR regime for the symmetric strong interference case, i.e., with $g > 1$ and $P \gg 1$. Similar to the generalized degrees of freedom defined in [10], we

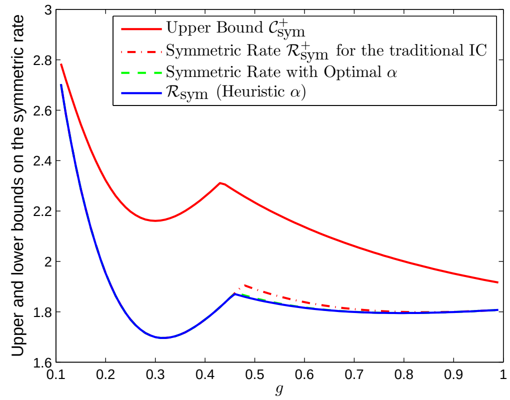

Fig. 12.: Comparison between the achievable symmetric rate with optimal α_1, the rate with heuristic α_1, the achievable symmetric rate for the traditional IC, and the upper bound for the weak interference case.

assume that the interference link SNR satisfies

$$\eta = \frac{\log g^2 P}{\log P}, \tag{4.24}$$

where $\eta > 1$ for the strong interference case. We also assume that the state power K satisfies

$$K = P^{\theta}. \tag{4.25}$$

Then we define the generalized degrees of freedom for the symmetric Gaussian IC with state information as:

$$d(\eta, \theta) := \lim_{P \to \infty : \frac{\log g^2 P}{\log P} = \eta, K = P^{\theta}} \frac{C_{\text{sym}}}{\frac{1}{2}\log(1+P)}, \tag{4.26}$$

where the symmetric capacity C_{sym} is defined in Section A. Based on the above definitions and assumptions, we derive the achievable generalized degrees of freedom for the strong interference case in the following theorem:

Theorem 13. *If $\eta > 1$, the achievable generalized degrees of freedom corresponding*

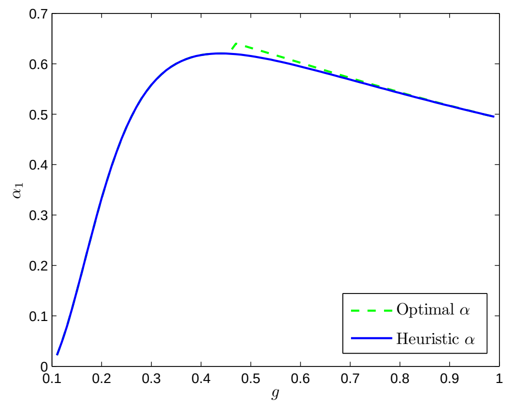

Fig. 13.: Comparison between the heuristic α_1 and the optimal α_1 obtained by exhausted search for the weak interference case.

to the achievable scheme in Section B with DPC parameter α^* *in* (4.12) *are given as:*

$$d_s = \frac{1}{2}\min\{\eta, 2\}. \tag{4.27}$$

Proof. We first recalculate the achievable rate region (4.9) and (4.10) with finite P after substituting the heuristic α^* in (4.12) as follows:

$$R_1, R_2 \quad < \quad \min\left\{R_{1s}, R_{2s}\right\}, \tag{4.28}$$

$$R_1 + R_2 \quad < \quad R_s, \tag{4.29}$$

where

$$\begin{aligned}
R_{1s} &= \frac{1}{2}\log\left(\frac{(1+g^2P)\left((1+P+gP)^2+PK\right)+K\left(1+gP\right)^2}{(1+P+gP)^2+2PK+K}\right), \\
R_{2s} &= \frac{1}{2}\log\left(\frac{(1+P)\left((1+P+gP)^2+PK\right)+K\left(1+P\right)^2}{(1+P+gP)^2+2PK+K}\right), \\
R_s &= \frac{1}{2}\log\left(\frac{(1+P+g^2P+K)\left(1+P+gP\right)^2}{(1+P+gP)^2+2PK+K}\right).
\end{aligned}$$

Then we can derive the achievable generalized degrees of freedom by analyzing how

R_{1s}, R_{2s}, and R_s scale with $\frac{1}{2}\log(1+P)$ as P goes to infinity:

$$\begin{aligned}
\lim_{P\to\infty:\frac{\log g^2 P}{\log P}=\eta, K=P^\theta} \frac{R_{1s}}{\frac{1}{2}\log(1+P)} &= \lim_{P\to\infty} \frac{\frac{1}{2}\log\left(\frac{P^\eta(P^{1+\eta}+P^{1+\theta})+P^{\theta+\eta+1}}{P^{\eta+1}+2P^{1+\theta}+P^\theta}\right)}{\frac{1}{2}\log(1+P)} \\
&= \lim_{P\to\infty} \frac{\frac{1}{2}\log(P^\eta)}{\frac{1}{2}\log(1+P)} \\
&= \eta, \\
\lim_{P\to\infty:\frac{\log g^2 P}{\log P}=\eta, K=P^\theta} \frac{R_{2s}}{\frac{1}{2}\log(1+P)} &= \lim_{P\to\infty} \frac{\frac{1}{2}\log\left(\frac{P(P^{1+\eta}+P^{1+\theta})+P^{\theta+2}}{P^{\eta+1}+2P^{1+\theta}+P^\theta}\right)}{\frac{1}{2}\log(1+P)} \\
&= \lim_{P\to\infty} \frac{\frac{1}{2}\log(P)}{\frac{1}{2}\log(1+P)} \\
&= 1, \\
\lim_{P\to\infty:\frac{\log g^2 P}{\log P}=\eta, K=P^\theta} \frac{R_s}{\frac{1}{2}\log(1+P)} &= \lim_{P\to\infty} \frac{\frac{1}{2}\log\left(\frac{(P+P^\eta+P^\theta)P^{1+\eta}}{P^{\eta+1}+2P^{1+\theta}+P^\theta}\right)}{\frac{1}{2}\log(1+P)} \\
&= \lim_{P\to\infty} \frac{\frac{1}{2}\log(P^\eta)}{\frac{1}{2}\log(1+P)} \\
&= \eta.
\end{aligned}$$

Therefore, we can obtain the following achievable generalized degrees of freedom based on the definition in (4.26):

$$d_s = \frac{1}{2}\min\{\eta, 2\}.$$

□

After comparing the achievable generalized degrees of freedom in Theorem 13 with the upper bound in [10], we have the following corollary:

Corollary 7. *The achievable scheme with the heuristic DPC parameter α^* in* (4.12) *achieves the optimal generalized degrees of freedom, i.e., the lower bound in Theorem 13 coincides with the upper bound in [10].*

Next we characterize the degrees of freedom performance in the high SNR regime for the symmetric weak interference case, i.e., with $g < 1$ and $P \gg 1$. We assume that the interfering link SNR satisfies

$$\eta = \frac{\log g^2 P}{\log P}, \tag{4.30}$$

where $\eta < 1$ for the weak interference case. We also assume that the state power K satisfies

$$K = P^{\theta}. \tag{4.31}$$

Based on the above assumptions and the definition in (4.26), we can show the achievable generalized degrees of freedom for the weak interference case in the following theorem:

Theorem 14. *If $\eta < 1$, the achievable generalized degrees of freedom corresponding to the achievable scheme in Section C with DPC parameter α_1^* in (4.23) are given as:*

$$d_w = \frac{1}{2} \min \{2 - \eta, \max \{2\eta, 2 - 2\eta\}\}. \tag{4.32}$$

Proof. Here we employ the coding scheme and power splitting strategy as described in Section C. Note that the private message can always achieve the following generalized degrees of freedom:

$$d_w^p = 1 - \eta, \tag{4.33}$$

due to the utilization of DPC and the private message power in (4.16) is $P_B = \frac{1}{g^2} = P^{1-\eta}$. Next we will only focus on the generalized degrees of freedom for the common messages.

We first recalculate the achievable rate region (4.20) and (4.21) with finite P after substituting the heuristic α_1^* in (4.23) as follows:

$$R_{10}, R_{20} \quad < \quad \min \{R_{10w}, R_{20w}\}, \tag{4.34}$$

$$R_{10} + R_{20} \quad < \quad R_w, \tag{4.35}$$

where

$$\begin{aligned}
R_{10w} &= \frac{1}{2}\log\left(\frac{(1+g^2Q)\left((2+P_B)\left(1+gQ+g^2Q\right)^2+g^2QK\right)+K\left(1+g^2Q\right)^2}{(2+P_B)\left(1+gQ+g^2Q\right)^2+2g^2QK+K}\right),\\
R_{20w} &= \frac{1}{2}\log\left(\frac{(1+Q)\left((2+P_B)\left(1+gQ+g^2Q\right)^2+g^2QK\right)+K\left(1+gQ\right)^2}{(2+P_B)\left(1+gQ+g^2Q\right)^2+2g^2QK+K}\right),\\
R_w &= \frac{1}{2}\log\left(\frac{(1+P+g^2P+K)\left(1+gQ+g^2Q\right)^2}{(2+P_B)\left(1+gQ+g^2Q\right)^2+2g^2QK+K}\right).
\end{aligned}$$

Note that here $Q = \frac{P_A}{2+P_B}$, $P_B = \frac{1}{g^2} = P^{1-\eta}$, and $P_A = P - P_B$.

Then we can derive the achievable generalized degrees of freedom by analyzing how R_{10w}, R_{20w}, and R_w scale with $\frac{1}{2}\log(1+P)$ as P goes to infinity:

$$\begin{aligned}
&\lim_{P\to\infty:\frac{\log g^2P}{\log P}=\eta,K=P^\theta} \frac{R_{10w}}{\frac{1}{2}\log(1+P)}\\
&= \lim_{P\to\infty} \frac{\frac{1}{2}\log\left(\frac{\left(1+P^{2\eta-1}\right)\left[P^{1-\eta}\left(1+P^{(3\eta-1)/2}\right)^2+P^{2\eta-1+\theta}\right]+P^\theta\left(1+P^{2\eta-1}\right)^2}{P^{1-\eta}\left(1+P^{(3\eta-1)/2}\right)^2+2P^{2\eta-1+\theta}+P^\theta}\right)}{\frac{1}{2}\log(1+P)}\\
&= \begin{cases} 0 & \text{if} \quad 0<\eta\le\frac{1}{2}\\ 2\eta-1 & \text{if} \quad \frac{1}{2}<\eta<1 \end{cases}\\
&\lim_{P\to\infty:\frac{\log g^2P}{\log P}=\eta,K=P^\theta} \frac{R_{20w}}{\frac{1}{2}\log(1+P)}\\
&= \lim_{P\to\infty} \frac{\frac{1}{2}\log\left(\frac{P^\eta\left[P^{1-\eta}\left(1+P^{(3\eta-1)/2}\right)^2+P^{2\eta-1+\theta}\right]+P^\theta\left(1+P^{(3\eta-1)/2}\right)^2}{P^{1-\eta}\left(1+P^{(3\eta-1)/2}\right)^2+2P^{2\eta-1+\theta}+P^\theta}\right)}{\frac{1}{2}\log(1+P)}\\
&= \eta, \quad \text{if} \quad \frac{1}{2}<\eta<1,\\
&\lim_{P\to\infty:\frac{\log g^2P}{\log P}=\eta,K=P^\theta} \frac{R_w}{\frac{1}{2}\log(1+P)}
\end{aligned}$$

$$
\begin{aligned}
&= \lim_{P\to\infty} \frac{\frac{1}{2}\log\left(\frac{\left(P+P^{\eta}+P^{\theta}\right)\left(1+P^{(3\eta-1)/2}\right)^2}{P^{1-\eta}\left(1+P^{(3\eta-1)/2}\right)^2+2P^{2\eta-1+\theta}+P^{\theta}}\right)}{\frac{1}{2}\log(1+P)} \\
&= \eta, \quad \text{if} \quad \frac{1}{2} < \eta < 1.
\end{aligned}
$$

Note that here it is enough to give the scaling result of R_{20w} and R_w when $\frac{1}{2} < \eta < 1$, since the achievable generalized degrees of freedom for the common message would be bounded by 0 when $0 < \eta \leq \frac{1}{2}$ due to the scaling result of R_{10w}. Therefore, we can obtain the following achievable generalized degrees of freedom for the common message based on the definition in (4.26):

$$
d_w^c = \begin{cases} 0 & \text{if} \quad 0 < \eta \leq \frac{1}{2} \\ \min\left\{2\eta - 1, \frac{\eta}{2}\right\} & \text{if} \quad \frac{1}{2} < \eta < 1 \end{cases}
$$

In total, the achievable generalized degrees of freedom can be shown as the sum of the common message and private message parts:

$$
\begin{aligned}
d_w &= d_w^c + d_w^p \\
&= \begin{cases} 1 - \eta & \text{if} \quad 0 < \eta \leq \frac{1}{2} \\ \min\left\{\eta, 1 - \frac{\eta}{2}\right\} & \text{if} \quad \frac{1}{2} < \eta < 1 \end{cases} \\
&= \frac{1}{2}\min\left\{2 - \eta, \max\left\{2\eta, 2 - 2\eta\right\}\right\}.
\end{aligned}
$$

□

After comparing the achievable generalized degrees of freedom in Theorem 14 with the upper bound in [10], we have the following corollary:

Corollary 8. *The achievable scheme with the heuristic DPC parameter α_1^* in* (4.23) *achieves the optimal generalized degrees of freedom, i.e., the lower bound in Theorem 14 coincides with the upper bound in [10].*

F. Summary

In this chapter, we considered the symmetric Gaussian interference channel with state information non-causally known at both transmitters. The coding scheme in Chapter III was deployed with newly defined auxiliary random variables. We showed that the smallest symmetric rate occurs when the state power goes to infinity for both strong and weak interference cases. Theoretical analysis was provided to calculate the gap between the achievable symmetric rate with infinite state power and the upper bound, which was shown to be less than 1/4 bit for the strong interference case and less than 3/4 bit for the weak interference case. Finally, we defined the generalized degrees of freedom for the symmetric Gaussian case, and derived the optimal values in both strong and weak interference scenarios, which are shown achievable with our proposed schemes.

CHAPTER V

CONCLUSION

In this dissertation, we studied the interference channel with state information non-causally known at both transmitters. Two achievable rate regions were established for the general cases based on two coding schemes with simultaneous encoding and superposition encoding, respectively. We also studied the corresponding Gaussian case and proposed the *active interference cancellation* mechanism, which generalizes the dirty paper coding technique, to partially eliminate the state effect at the receivers. Several achievable schemes were proposed and the corresponding achievable rate regions were derived for the strong interference case, the mixed interference case, and the weak interference case. The numerical results showed that active interference cancellation significantly improves the performance for the strong and mixed interference case, and flexible power splitting significantly enlarges the achievable rate region for the weak interference case.

Moreover, we considered the symmetric Gaussian interference channel with state information non-causally known at both transmitters. The coding scheme in Chapter III was deployed with newly defined auxiliary random variables. We showed that the smallest symmetric rate occurs when the state power goes to infinity for both strong and weak interference cases. Theoretical analysis was provided to calculate the gap between the achievable symmetric rate with infinite state power and the upper bound, which was shown to be less than $1/4$ bit for the strong interference case and less than $3/4$ bit for the weak interference case. Finally, we defined the generalized degrees of freedom for the symmetric Gaussian case, and derived the optimal values in both strong and weak interference scenarios, which are shown achievable with our proposed schemes.

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APPENDIX A

PROOF FOR THEOREM 1

The achievable coding scheme for Theorem 1 can be described as follows:

Codebook generation: Fix the probability distribution $p(q)p(u_1|q,s)p(v_1|q,s)$ $p(u_2|q,s)p(v_2|q,s)$. Also define the following function for the jth user that maps $\mathcal{U}_j\times\mathcal{V}_j\times\mathcal{S}$ to $\mathcal{X}_j$:

$$x_{ji} = F_j(u_{ji}, v_{ji}, s_i),$$

where i is the element index of each sequence.

First generate the time-sharing sequence $q^n \sim \prod_{i=1}^{n} p_Q(q_i)$. For the jth user, $u_j^n(m_{j0}, l_{j0})$ is randomly and conditionally independently generated according to $\prod_{i=1}^{n} p_{U_j|Q}(u_{ji}|q_i)$, for $m_{j0} \in \{1, 2, \cdots, 2^{nR_{j0}}\}$ and $l_{j0} \in \{1, 2, \cdots, 2^{nR'_{j0}}\}$. Similarly, $v_j^n(m_{jj}, l_{jj})$ is randomly and conditionally independently generated according to $\prod_{i=1}^{n} p_{V_j|Q}(v_{ji}|q_i)$, for $m_{jj} \in \{1, 2, \cdots, 2^{nR_{jj}}\}$ and $l_{jj} \in \{1, 2, \cdots, 2^{nR'_{jj}}\}$.

Encoding: To send the message $m_j = (m_{j0}, m_{jj})$, the jth encoder first tries to find the pair (l_{j0}, l_{jj}) such that the following joint typicality holds: $(q^n, u_j^n(m_{j0}, l_{j0}), s^n) \in T_\epsilon^{(n)}$ and $(q^n, v_j^n(m_{jj}, l_{jj}), s^n) \in T_\epsilon^{(n)}$. If successful, $(q^n, u_j^n(m_{j0}, l_{j0}), v_j^n(m_{jj}, l_{jj}), s^n)$ is also jointly typical with high probability, and the jth encoder sends x_j where the ith element is $x_{ji} = F_j(u_{ji}(m_{j0}, l_{j0}), v_{ji}(m_{jj}, l_{jj}), s_i)$. If not, the jth encoder transmits x_j where the ith element is $x_{ji} = F_j(u_{ji}(m_{j0}, 1), v_{ji}(m_{jj}, 1), s_i)$.

Decoding: Decoder 1 finds the unique message pair $(\hat{m}_{10}, \hat{m}_{11})$ such that $(q^n, u_1^n(\hat{m}_{10}, \hat{l}_{10}), u_2^n(\hat{m}_{20}, \hat{l}_{20}), v_1^n(\hat{m}_{11}, \hat{l}_{11}), y_1^n) \in T_\epsilon^{(n)}$ for some $\hat{l}_{10} \in \{1, 2, \cdots, 2^{nR'_{10}}\}$, $\hat{m}_{20} \in \{1, 2, \cdots, 2^{nR_{20}}\}$, $\hat{l}_{20} \in \{1, 2, \cdots, 2^{nR'_{20}}\}$, and $\hat{l}_{11} \in \{1, 2, \cdots, 2^{nR'_{11}}\}$. If no such unique pair exists, the decoder declares an error. Decoder 2 determines the unique message pair $(\hat{m}_{20}, \hat{m}_{22})$ in a similar way.

Analysis of probability of error: Here the probability of error is the same for each message pair since the transmitted message pair is chosen with a uniform distribution over the message set. Without loss of generality, we assume $(1,1)$ for user 1 and $(1,1)$ for user 2 are sent over the channel. First, we consider the encoding error probability at transmitter 1. Define the following error events:

$$\begin{aligned}\xi_1 &= \left\{(q^n, u_1^n(1,l_{10}), s^n) \notin T_\epsilon^{(n)} \text{ for all } l_{10} \in \{1,2,\cdots,2^{nR'_{10}}\}\right\},\\ \xi_2 &= \left\{(q^n, v_1^n(1,l_{11}), s^n) \notin T_\epsilon^{(n)} \text{ for all } l_{11} \in \{1,2,\cdots,2^{nR'_{11}}\}\right\}.\end{aligned}$$

The probability of the error event ξ_1 can be bounded as follows:

$$\begin{aligned}P(\xi_1) &= \prod_{l_{10}=1}^{2^{nR'_{10}}} \left(1 - P\left(\left\{(q^n, u_1^n(1,l_{10}), s^n) \in T_\epsilon^{(n)}\right\}\right)\right)\\ &\le \left(1 - 2^{-n(I(U_1;S|Q)+\delta_1(\epsilon))}\right)^{2^{nR'_{10}}}\\ &\le e^{-2^{n\left(R'_{10}-I(U_1;S|Q)+\delta_1(\epsilon)\right)}},\end{aligned}$$

where $\delta_1(\epsilon) \to 0$ as $\epsilon \to 0$. Therefore, the probability of ξ_1 goes to 0 as $n \to \infty$ if

$$R'_{10} \ge I(U_1;S|Q). \tag{A.1}$$

Similarly, the probability of ξ_2 can also be upper-bounded by an arbitrarily small number as $n \to \infty$ if

$$R'_{11} \ge I(V_1;S|Q). \tag{A.2}$$

The encoding error probability at transmitter 1 can be calculated as:

$$P_{\text{enc1}} = P(\xi_1 \cup \xi_2) \le P(\xi_1) + P(\xi_2),$$

which goes to 0 as $n \to \infty$ if (A.1) and (A.2) are satisfied.

Now we consider the error analysis at decoder 1. Denote the right Gel'fand-

Pinsker coding indices chosen by the encoders as (L_{10}, L_{11}) and (L_{20}, L_{22}). Define the following error events:

$$\begin{aligned}
\xi_{31} &= \big\{ \left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\quad \text{and some } l_{11}\big\}, \\
\xi_{32} &= \big\{ \left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\quad \text{and some } l_{11}, l_{20} \neq L_{20}\big\}, \\
\xi_{33} &= \big\{ \left(q^n, u_1^n\left(1, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\quad \text{and some } l_{11}, l_{10} \neq L_{10}\big\}, \\
\xi_{34} &= \big\{ \left(q^n, u_1^n\left(1, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\quad \text{and some } l_{11}, l_{10} \neq L_{10}, l_{20} \neq L_{20}\big\}, \\
\xi_{41} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(1, L_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\quad \text{and some } l_{10}\big\}, \\
\xi_{42} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(1, L_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\quad \text{and some } l_{10}, l_{20} \neq L_{20}\big\}, \\
\xi_{43} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(1, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\quad \text{and some } l_{10}, l_{11} \neq L_{11}\big\}, \\
\xi_{44} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(1, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\quad \text{and some } l_{10}, l_{20} \neq L_{20}, l_{11} \neq L_{11}\big\}, \\
\xi_{51} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\quad m_{11} \neq 1, \text{ and some } l_{10}, l_{11}\big\}, \\
\xi_{52} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\quad m_{11} \neq 1, \text{ and some } l_{10}, l_{11}, l_{20} \neq L_{20}\big\}, \\
\xi_{61} &= \big\{ \left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{20} \neq 1,
\end{aligned}$$

$$
\begin{aligned}
& m_{11} \neq 1, \text{ and some } l_{20}, l_{11} \big\}, \\
\xi_{62} &= \big\{ \left(q^n, u_1^n\left(1, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{20} \neq 1, \\
& m_{11} \neq 1, \text{ and some } l_{20}, l_{11}, l_{10} \neq L_{10} \big\}, \\
\xi_{71} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(1, L_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
& m_{20} \neq 1, \text{ and some } l_{10}, l_{20} \big\}, \\
\xi_{72} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(1, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
& m_{20} \neq 1, \text{ and some } l_{10}, l_{20}, l_{11} \neq L_{11} \big\}, \\
\xi_{8} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
& m_{20} \neq 1, \ m_{11} \neq 1, \text{ and some } l_{10}, l_{20}, l_{11} \big\}.
\end{aligned}
$$

The probability of ξ_{31} can be bounded as:

$$
\begin{aligned}
P(\xi_{31}) &= \sum_{m_{11}=2}^{2^{nR_{11}}} \sum_{l_{11}=1}^{2^{R'_{11}}} P\left(\left\{\left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)}\right\}\right) \\
&\leq 2^{n\left(R_{11}+R'_{11}\right)} \sum_{(q^n, u_1^n, u_2^n, v_1^n, y_1^n) \in T_\epsilon^{(n)}} p(q^n)p(u_1^n|q^n)p(u_2^n|q^n)p(v_1^n|q^n)p(y_1^n|u_1^n, u_2^n, q^n) \\
&\leq 2^{n\left(R_{11}+R'_{11}\right)} 2^{-n(H(Q)+H(U_1|Q)+H(U_2|Q)+H(V_1|Q)+H(Y_1|U_1,U_2,Q)-H(Q,U_1,U_2,V_1,Y_1)-\delta_2(\epsilon))} \\
&\leq 2^{n\left(R_{11}+R'_{11}\right)} 2^{-n(I(U_1;U_2|Q)+I(U_1,U_2;V_1|Q)+I(V_1;Y_1|U_1,U_2,Q)-\delta_2(\epsilon))},
\end{aligned}
$$

where $\delta_2(\epsilon) \to 0$ as $\epsilon \to 0$. Obviously, the probability that ξ_{31} happens goes to 0 if

$$
R_{11} + R'_{11} \leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) + I(V_1;Y_1|U_1,U_2,Q). \tag{A.3}
$$

Similarly, the error probability corresponding to the other error events goes to 0, if

$$
\begin{aligned}
R_{11} + R'_{11} + R'_{20} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q) \\
&\quad +I(V_1,U_2;Y_1|U_1,Q), \qquad \text{(A.4)} \\
R_{11} + R'_{10} + R'_{11} &\leq I(U_1;U_2|Q) + I(U_1,U_2;V_1|Q)
\end{aligned}
$$

$$
\begin{aligned}
& & & +I(U_1, V_1; Y_1|U_2, Q), & \text{(A.5)}\\
R_{11} + R'_{10} + R'_{11} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, V_1, U_2; Y_1|Q), & \text{(A.6)}\\
R_{10} + R'_{10} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1; Y_1|V_1, U_2, Q), & \text{(A.7)}\\
R_{10} + R'_{10} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, U_2; Y_1|V_1, Q), & \text{(A.8)}\\
R_{10} + R'_{10} + R'_{11} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, V_1; Y_1|U_2, Q), & \text{(A.9)}\\
R_{10} + R'_{10} + R'_{11} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, V_1, U_2; Y_1|Q), & \text{(A.10)}\\
R_{10} + R_{11} + R'_{10} + R'_{11} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, V_1; Y_1|U_2, Q), & \text{(A.11)}\\
R_{10} + R_{11} + R'_{10} + R'_{11} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, V_1, U_2; Y_1|Q), & \text{(A.12)}\\
R_{11} + R_{20} + R'_{11} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(V_1, U_2; Y_1|U_1, Q), & \text{(A.13)}\\
R_{11} + R_{20} + R'_{10} + R'_{11} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, V_1, U_2; Y_1|Q), & \text{(A.14)}\\
R_{10} + R_{20} + R'_{10} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, U_2; Y_1|V_1, Q), & \text{(A.15)}\\
R_{10} + R_{20} + R'_{10} + R'_{11} + R'_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) \\
& & & +I(U_1, V_1, U_2; Y_1|Q), & \text{(A.16)}
\end{aligned}
$$

$$R_{10} + R_{11} + R_{20} + R'_{10} + R'_{11} + R'_{20} \leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) + I(U_1, V_1, U_2; Y_1|Q). \quad \text{(A.17)}$$

Note that there are some redundant inequalities in (A.3)-(A.17): (A.4) is implied by (A.13); (A.5) is implied by (A.11); (A.8) is implied by (A.15); (A.9) is implied by (A.11); (A.6), (A.10), (A.12), (A.14), and (A.16) are implied by (A.17). By combining with the error analysis at the encoder, we can recast the rate constraints (A.3)-(A.17) as:

$$\begin{aligned}
R_{11} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) + I(V_1; Y_1|U_1, U_2, Q) - I(V_1; S|Q), \\
R_{10} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) + I(U_1; Y_1|V_1, U_2, Q) - I(U_1; S|Q), \\
R_{10} + R_{11} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) + I(U_1, V_1; Y_1|U_2, Q) - I(U_1; S|Q) \\
&\quad - I(V_1; S|Q), \\
R_{11} + R_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) + I(V_1, U_2; Y_1|U_1, Q) - I(V_1; S|Q) \\
&\quad - I(U_2; S|Q), \\
R_{10} + R_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) + I(U_1, U_2; Y_1|V_1, Q) - I(U_1; S|Q) \\
&\quad - I(U_2; S|Q), \\
R_{10} + R_{11} + R_{20} &\leq I(U_1; U_2|Q) + I(U_1, U_2; V_1|Q) + I(U_1, V_1, U_2; Y_1|Q) - I(U_1; S|Q) \\
&\quad - I(V_1; S|Q) - I(U_2; S|Q).
\end{aligned}$$

The error analysis for transmitter 2 and decoder 2 is similar to the above procedures and is omitted here. Correspondingly, (2.8) to (2.13) show the rate constraints for user 2. In addition, the right sides of the inequalities (2.2) to (2.13) are guaranteed to be non-negative when choosing the probability distribution. As long as (2.2) to (2.13) are satisfied, the probability of error can be bounded by the sum of the error probability at the encoders and the decoders, which goes to 0 as $n \to \infty$.

APPENDIX B

PROOF FOR THEOREM 2

The achievable coding scheme for Theorem 2 can be described as follows:

Codebook generation: Fix the probability distribution $p(q)p(u_1|s,q)p(v_1|u_1,s,q)$ $p(u_2|s,q)p(v_2|u_2,s,q)$. First generate the time-sharing sequence $q^n \sim \prod_{i=1}^{n} p_Q(q_i)$. For the jth user, $u_j^n(m_{j0},l_{j0})$ is randomly and conditionally independently generated according to $\prod_{i=1}^{n} p_{U_j|Q}(u_{ji}|q_i)$, for $m_{j0} \in \{1,2,\cdots,2^{nR_{j0}}\}$ and $l_{j0} \in \{1,2,\cdots,2^{nR'_{j0}}\}$. For each $u_j^n(m_{j0},l_{j0})$, $v_j^n(m_{j0},l_{j0},m_{jj},l_{jj})$ is randomly and conditionally independently generated according to $\prod_{i=1}^{n} p_{V_j|U_j,Q}(v_{ji}|u_{ji},q_i)$, for $m_{jj} \in \{1,2,\cdots,2^{nR_{jj}}\}$ and $l_{jj} \in \{1,2,\cdots,2^{nR'_{jj}}\}$.

Encoding: To send the message $m_j = (m_{j0},m_{jj})$, the jth encoder first tries to find l_{j0} such that $(q^n,u_j^n(m_{j0},l_{j0}),s^n) \in T_\epsilon^{(n)}$ holds. Then for this specific l_{j0}, find l_{jj} such that $(q^n,u_j^n(m_{j0},l_{j0}),v_j^n(m_{j0},l_{j0},m_{jj},l_{jj}),s^n) \in T_\epsilon^{(n)}$ holds. If successful, the jth encoder sends $v_j^n(m_{j0},l_{j0},m_{jj},l_{jj})$. If not, the jth encoder transmits $v_j^n(m_{j0},1,m_{jj},1)$.

Decoding: Decoder 1 finds the unique message pair $(\hat{m}_{10},\hat{m}_{11})$ such that $(q^n,$ $u_1^n(\hat{m}_{10},\hat{l}_{10}),u_2^n(\hat{m}_{20},\hat{l}_{20}),v_1^n(\hat{m}_{10},\hat{l}_{10},\hat{m}_{11},\hat{l}_{11}),y_1^n) \in T_\epsilon^{(n)}$ for some $\hat{l}_{10} \in \{1,2,\cdots,$ $2^{nR'_{10}}\}$, $\hat{m}_{20} \in \{1,2,\cdots,2^{nR_{20}}\}$,$\hat{l}_{20} \in \{1,2,\cdots,2^{nR'_{20}}\}$, and $\hat{l}_{11} \in \{1,2,\cdots,2^{nR'_{11}}\}$. If no such unique pair exists, the decoder declares an error. Decoder 2 determines the unique message pair $(\hat{m}_{20},\hat{m}_{22})$ similarly.

Analysis of probability of error: Similar to the proof in Theorem 1, we assume message $(1,1)$ and $(1,1)$ are sent for both transmitters. First we consider the encoding error probability at transmitter 1. Define the following error events:

$$\xi_1' = \left\{(q^n,u_1^n(1,l_{10}),s^n) \notin T_\epsilon^{(n)} \text{ for all } l_{10} \in \{1,2,\cdots,2^{nR'_{10}}\}\right\},$$

$$\xi_2' = \Big\{ (q^n, u_1^n(m_{10}, l_{10}), v_1^n(1, l_{10}, 1, l_{11}), s^n) \notin T_\epsilon^{(n)} \text{ for all } l_{11} \in \{1, 2, \cdots, 2^{nR_{11}'}\} \\ \text{and previously found typical } l_{10} | \bar{\xi}_1' \Big\}.$$

The probability of the error event ξ_1' can be bounded as:

$$\begin{aligned} P(\xi_1') &= \prod_{l_{10}=1}^{2^{nR_{10}'}} \left(1 - P\left(\{(q^n, u_1^n(1, l_{10}), s^n) \in T_\epsilon^{(n)}\}\right)\right) \\ &\leq \left(1 - 2^{-n\left(I(U_1;S|Q)+\delta_1'(\epsilon)\right)}\right)^{2^{nR_{10}'}} \\ &\leq e^{-2^{n\left(R_{10}'-I(U_1;S|Q)+\delta_1'(\epsilon)\right)}}, \end{aligned}$$

where $\delta_1'(\epsilon) \to 0$ as $\epsilon \to 0$. Therefore, the probability of ξ_1' goes to 0 as $n \to \infty$ if

$$R_{10}' \geq I(U_1; S|Q). \tag{B.1}$$

Similarly, for the previously found typical l_{10}, the probability of ξ_2' can be upper-bounded as:

$$\begin{aligned} P(\xi_2') &= \prod_{l_{11}=1}^{2^{nR_{11}'}} \left(1 - P\left(\{(q^n, u_1^n(1, l_{10}), v_1^n(1, l_{10}, 1, l_{11}), s^n) \in T_\epsilon^{(n)}\}\right)\right) \\ &\leq \left(1 - 2^{n\left(H(Q,U_1,V_1,S)-H(Q,U_1,S)-H(V_1|U_1,Q)-\delta_2'(\epsilon)\right)}\right)^{2^{nR_{11}'}} \\ &\leq \left(1 - 2^{-n\left(I(V_1;S|U_1,Q)+\delta_2'(\epsilon)\right)}\right)^{2^{nR_{11}'}} \\ &\leq e^{-2^{n\left(R_{11}'-I(V_1;S|U_1,Q)+\delta_2'(\epsilon)\right)}}, \end{aligned}$$

where $\delta_2'(\epsilon) \to 0$ as $\epsilon \to 0$. Therefore, the probability of ξ_2' goes to 0 as $n \to \infty$ if

$$R_{11}' \geq I(V_1; S|U_1, Q). \tag{B.2}$$

The encoding error probability at transmitter 1 can be calculated as:

$$P_{\text{enc1}} = P(\xi_1') + P(\xi_2'),$$

which goes to 0 as $n \to \infty$ if (B.1) and (B.2) are satisfied.

Now we consider the error analysis at the decoder 1. Denote the right Gel'fand-Pinsker coding indices chosen by the encoders as (L_{10}, L_{11}) and (L_{20}, L_{22}). Define the following error events:

$$\begin{aligned}
\xi'_{31} &= \Big\{ \left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(1, L_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\qquad \text{and some } l_{11} \Big\}, \\
\xi'_{32} &= \Big\{ \left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(1, L_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\qquad \text{and some } l_{11}, l_{20} \neq L_{20} \Big\}, \\
\xi'_{33} &= \Big\{ \left(q^n, u_1^n\left(1, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(1, l_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\qquad \text{and some } l_{11}, l_{10} \neq L_{10} \Big\}, \\
\xi'_{34} &= \Big\{ \left(q^n, u_1^n\left(1, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(1, l_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{11} \neq 1, \\
&\qquad \text{and some } l_{11}, l_{10} \neq L_{10}, l_{20} \neq L_{20} \Big\}, \\
\xi'_{41} &= \Big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(m_{10}, l_{10}, 1, L_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\qquad \text{and some } l_{10} \Big\}, \\
\xi'_{42} &= \Big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(m_{10}, l_{10}, 1, L_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\qquad \text{and some } l_{10}, l_{20} \neq L_{20} \Big\}, \\
\xi'_{43} &= \Big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(m_{10}, l_{10}, 1, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\qquad \text{and some } l_{10}, l_{11} \neq L_{11} \Big\}, \\
\xi'_{44} &= \Big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(m_{10}, l_{10}, 1, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\qquad \text{and some } l_{10}, l_{20} \neq L_{20}, l_{11} \neq L_{11} \Big\}, \\
\xi'_{51} &= \Big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(m_{10}, l_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
&\qquad m_{11} \neq 1, \text{ and some } l_{10}, l_{11} \Big\}, \\
\xi'_{52} &= \Big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(1, l_{20}\right), v_1^n\left(m_{10}, l_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1,
\end{aligned}$$

$$
\begin{aligned}
& \quad m_{11} \neq 1, \text{ and some } l_{10}, l_{11}, l_{20} \neq L_{20}\big\}, \\
\xi'_{61} &= \big\{ \left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(1, L_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{20} \neq 1, \\
& \quad m_{11} \neq 1, \text{ and some } l_{20}, l_{11}\big\}, \\
\xi'_{62} &= \big\{ \left(q^n, u_1^n\left(1, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(1, l_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{20} \neq 1, \\
& \quad m_{11} \neq 1, \text{ and some } l_{20}, l_{11}, l_{10} \neq L_{10}\big\}, \\
\xi'_{71} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(m_{10}, l_{10}, 1, L_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
& \quad m_{20} \neq 1, \text{ and some } l_{10}, l_{20}\big\}, \\
\xi'_{72} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(m_{10}, l_{10}, 1, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
& \quad m_{20} \neq 1, \text{ and some } l_{10}, l_{20}, l_{11} \neq L_{11}\big\}, \\
\xi'_{8} &= \big\{ \left(q^n, u_1^n\left(m_{10}, l_{10}\right), u_2^n\left(m_{20}, l_{20}\right), v_1^n\left(m_{10}, l_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)} \text{ for } m_{10} \neq 1, \\
& \quad m_{20} \neq 1, \; m_{11} \neq 1, \text{ and some } l_{10}, l_{20}, l_{11}\big\}.
\end{aligned}
$$

The probability of ξ'_{31} can be bounded as follows:

$$
\begin{aligned}
P(\xi'_{31}) &= \sum_{m_{11}=2}^{2^{nR_{11}}} \sum_{l_{11}=1}^{2^{R'_{11}}} P\left(\left\{\left(q^n, u_1^n\left(1, L_{10}\right), u_2^n\left(1, L_{20}\right), v_1^n\left(1, L_{10}, m_{11}, l_{11}\right), y_1^n\right) \in T_\epsilon^{(n)}\right\}\right) \\
&\leq 2^{n\left(R_{11}+R'_{11}\right)} \sum_{(q^n, u_1^n, u_2^n, v_1^n, y_1^n) \in T_\epsilon^{(n)}} p(q^n)p(u_1^n|q^n)p(u_2^n|q^n)p(v_1^n|u_1^n, q^n)p(y_1^n|u_1^n, u_2^n, q^n) \\
&\leq 2^{n\left(R_{11}+R'_{11}\right)} 2^{-n\left(H(Q,U_1,V_1)+H(U_2|Q)+H(Y_1|U_1,U_2,Q)-H(Q,U_1,U_2,V_1,Y_1)-\delta'_3(\epsilon)\right)} \\
&\leq 2^{n\left(R_{11}+R'_{11}\right)} 2^{-n\left(I(U_1,V_1;U_2|Q)+I(V_1;Y_1|U_1,U_2,Q)-\delta'_3(\epsilon)\right)},
\end{aligned}
$$

where $\delta'_3(\epsilon) \to 0$ as $\epsilon \to 0$. Obviously, the probability that ξ'_{31} happens goes to 0 if

$$
R_{11} + R'_{11} \leq I(U_1, V_1; U_2|Q) + I(V_1; Y_1|U_1, U_2, Q). \tag{B.3}
$$

Similarly, the error probability corresponding to the other error events goes to 0, respectively, if

$$R_{11} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(V_1, U_2; Y_1|U_1, Q), \quad (B.4)$$

$$R_{11} + R'_{10} + R'_{11} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1; Y_1|U_2, Q), \quad (B.5)$$

$$R_{11} + R'_{10} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q), \quad (B.6)$$

$$R_{10} + R'_{10} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1; Y_1|U_2, Q), \quad (B.7)$$

$$R_{10} + R'_{10} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q), \quad (B.8)$$

$$R_{10} + R'_{10} + R'_{11} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1; Y_1|U_2, Q), \quad (B.9)$$

$$R_{10} + R'_{10} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q), \quad (B.10)$$

$$R_{10} + R_{11} + R'_{10} + R'_{11} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1; Y_1|U_2, Q), \quad (B.11)$$

$$R_{10} + R_{11} + R'_{10} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q), \quad (B.12)$$

$$R_{11} + R_{20} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(V_1, U_2; Y_1|U_1, Q), \quad (B.13)$$

$$R_{11} + R_{20} + R'_{10} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q), \quad (B.14)$$

$$R_{10} + R_{20} + R'_{10} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q), \quad (B.15)$$

$$R_{10} + R_{20} + R'_{10} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q), \quad (B.16)$$

$$R_{10} + R_{11} + R_{20} + R'_{10} + R'_{11} + R'_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q) \quad (B.17)$$

Note that there are some redundant inequalities in (B.3)-(B.17): (B.4) is implied by (B.13); (B.5) is implied by (B.11); (B.7) is implied by (B.9); (B.8) is implied by (B.15); (B.9) is implied by (B.11); (B.6), (B.10), (B.12), (B.14), (B.15), and (B.16) are implied by (B.17). By combining with the error analysis at the encoder, we can recast the rate constraints (B.3)-(B.17) as:

$$R_{11} \leq I(U_1, V_1; U_2|Q) + I(V_1; Y_1|U_1, U_2, Q) - I(V_1; S|U_1, Q),$$

$$R_{10} + R_{11} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1; Y_1|U_2, Q) - I(U_1, V_1; S|Q),$$

$$R_{11} + R_{20} \leq I(U_1, V_1; U_2|Q) + I(V_1, U_2; Y_1|U_1, Q) - I(V_1; S|U_1, Q) - I(U_2; S|Q),$$

$$R_{10} + R_{11} + R_{20} \leq I(U_1, V_1; U_2|Q) + I(U_1, V_1, U_2; Y_1|Q) - I(U_1, V_1; S|Q) - I(U_2; S|Q).$$

The error analysis for transmitter 2 and decoder 2 is similar to the above procedures and is omitted here. Correspondingly, (2.23) to (2.26) show the rate constraints for user 2. Furthermore, the right-hand sides of the inequalities (2.19) to (2.26) are guaranteed to be non-negative when choosing the probability distribution. As long as (2.19) to (2.26) are satisfied, the probability of error can be bounded by the sum of the error probability at the encoders and the decoders, which goes to 0 as $n \to \infty$.

VITA

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