A Thesis<br>by<br>\section*{CHUNYU TIAN}

Submitted to the Office of Graduate Studies of Texas A\&M University in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE

August 2011

Major Subject: Civil Engineering

# TAXIWAY AIRCRAFT TRAFFIC SCHEDULING: A MODEL AND SOLUTION 

## ALGORITHMS

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ABSTRACT<br>Taxiway Aircraft Traffic Scheduling: A Model and Solution Algorithms. (August 2011)<br>Chunyu Tian, B.S., Shanghai Jiaotong University<br>Chair of Advisory Committee: Dr. Xiu Wang

With the drastic increase in the demand for air travel, taxiway aircraft traffic scheduling is becoming increasingly important in managing air traffic. In order to reduce traffic congestion on taxiways, this thesis proposes a tool for air traffic controllers to use in decision making: a taxiway air traffic model developed using Mixed Integer Programming (MIP) that can be applied to a rolling time horizon.

The objective of this model is to minimize the total taxi time, and the output is a schedule and route for each aircraft. This MIP model assumes that only the origin and destination of each aircraft is fixed; due to some uncertain factors in the air arrival and departure process, it allows for the departure time and arrival time to vary within a certain time window. This MIP model features aircraft type, and also incorporates runway crossings and runway separations.

The model is programmed using C++ and Solved in CPLEX 12.1. Runways 26R and 26L of George Bush International Airport are used to find solutions. The author presents a rolling horizon method by dividing the large scheduling issue into smaller time interval problems according to the scheduled times of departure or arrival. A bound
is also proposed based on the discretized time interval problems. By using partial data from George Bush International Airport (IAH), solutions are obtained. The results are compared with the bound and show fairly high optimality.

Compared with the previous research, this thesis presents a model with more flexibility by considering different operations. By using the rolling horizon method, the problem is broken into smaller units that can be solved efficiently without losing much optimality.

## DEDICATION

To my parents and sister

## ACKNOWLEDGEMENTS

I would like to thank my graduate committee chair, Dr. Xiu Wang, for his guidance and support. He directed me to this interesting topic. I also want to thank my committee members, Dr. Yunlong Zhang and Dr. Natarajan Gautam, for serving on the committee and for their help in the research. Dr. Luca Quadrifoglio offered a research assistantship position in my last semester.

I owe my gratitude to Kai Yin for his kind help. The staff from Houston Airport System provided us data about the George Bush International Airport.

## NOMENCLATURE

| F | The set of all aircraft |
| :---: | :---: |
| D | The set of departure aircraft |
| A | The set of arrival aircraft |
| N | The set of nodes |
| $\mathrm{Nair}_{\text {ar }}$ | A dummy node representing the outside system |
| $\mathrm{n}_{\mathrm{r}}$ | The only runway threshold |
| P | The set of planning period |
| $\mathrm{E}_{\mathrm{p}}$ | The last planning period |
| $\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)$ | A directed link between node $\mathrm{n}_{1}$ and node $\mathrm{n}_{2}$ |
| L | The set of links |
| ORI ${ }_{\text {i }}$ | Origin node of aircraft i |
| DES $_{i}$ | Destination node of aircraft i |
| $\mathrm{L}_{\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)}$ | The length of link ( $\mathrm{n}_{1}, \mathrm{n}_{2}$ ) |
| $\mathrm{C}_{\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)}$ | The connectivity of ( $\left.\mathrm{n}_{1}, \mathrm{n}_{2}\right)$ |
| $\mathrm{t}_{\mathrm{i}}^{\mathrm{j}}$ | The starting time of planning period j for aircraft i |
| $E P T_{i}$ | The earliest pushback time for aircraft i $\in$ D |
| Gap1 | Deviation allowed from EPT $\mathrm{i}_{\mathrm{i}}$ |
| $\mathrm{EAT}_{\mathrm{i}}$ | The estimated arrival time for aircraft i $\in$ A |
| Gap2 | Deviation allowed from EAT ${ }_{\text {i }}$ |

Deviation allowed from EAT $_{\mathrm{i}}$
A binary variable that is 1 if aircraft i arrives at node
n earlier than aircraft j and 0 otherwise
A binary variable that is 1 if departure aircraft i uses runway earlier than crossing aircraft j and 0 otherwise
$R_{\left(i, n_{1}, n_{2}\right)}^{j}$
A binary variable that is 1 if aircraft i moves from node $\mathrm{n}_{1}$ to node $\mathrm{n}_{2}$ at planning period j

Maximum speed for aircraft to taxi
Minimum speed for aircraft to taxi

Wake vortex separations of leading aircraft $\mathrm{i}_{1}$ and following aircrafti ${ }_{2}$

Time separations between a departure aircraft $\mathrm{i}_{1}$ and a crossing aircraft $\mathrm{i}_{2}$

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## CHAPTER I

## INTRODUCTION

## A. Motivations

With the increase of air travel demand during the past few years ${ }^{1}$, many airports are faced with congestion problems. Most airports are operating close to their capacities ${ }^{2}$ Forecast of future air travel demand can be found in massive studies. It is very hard to predict the future air travel demand accurately due to the uncertainty of economy and other factors. However, the trend can still be seen from those forecasts. Figure 1 is from FAA airspace forecast of fiscal year 2008 to $2025^{3}$. In this figure, the annual growth rate of commercial air travel demand is about 3.0\%.

Due to the booming economy of the developing countries, the air traffic is expected to increase significantly. As the markets are correlated between those countries and America, the annual growth rate might be much larger than the expected number. Given a $3.0 \%$ annual growth rate of air travel demand, it would be very difficult for the airports to handle all the aircraft efficiently in the future. As a result, the airports need to take measures to improve its overall capacity.

This thesis follows the style of American Institute of Aeronautics and Astronautics Journal.


Figure 1: The forecast of commercial air travel demand ${ }^{3}$

There are several ways to enlarge the capacity of airports. The first way is to construct new runways and other types of facilities. However, the planners have to take into account the availability of land ${ }^{4}$ and time duration of constructing new runways. For some airports such as LaGuardia Airport, there is no more land for building new runways. In addition, it usually takes more than three years to build new runways. As a result, runway construction can only solve long term problem and has limited meaning to short term problems.

The second way is to improve the efficiency of current airport ground operations. The airport ground network is mainly organized by three parts ${ }^{5}$, which are gate system, taxiway system and runway system. The bottleneck of an airport is the runway. The
sequence of aircraft using a runway ${ }^{6}$ can decide the throughput of a runway. The runways are connected with taxiway system. In order to improve the performance of the runways, the taxiways aircraft traffic need to be scheduled. Thus the runways and taxiways are integrated to get better results.

For most airports, the ground movement of aircraft is controlled by ground controllers. The number of controllers working is related to the volume of traffic that the airport handles. The air traffic control (ATC) tower assigns time windows to the aircraft according to the air controllers' experience. During peak hours, it is difficult for controllers to manage the whole aircraft fleet, which usually leads to substantial delays within the airport.

During these peak times, software with optimization models can assist controllers to better navigate aircraft operations. Such software includes Departure Planner (DP) and Surface Movement Advisor (SMA). The optimization model has to be feasible and incorporate practical computation time.

Extensive research has been conducted in the optimization of airport ground operations. Some of the studies have been focused on runway capacity. ${ }^{6-8}$ The other studies have been based on taxiway scheduling ${ }^{9-10}$ and gate assignment problems. ${ }^{11-12}$ Research has found that the optimization of one system may not guarantee the optimization of another system.

It has been recommended ${ }^{13}$ that the whole airport ground system be studied as an integrated system. However, the problem becomes increasingly complex and impractical for modeling when all the problems are studied together. Therefore, only the issue of
taxiway movement in air traffic scheduling is studied in this thesis. Simplified runway operations are included in the model. In the future, the results of this thesis can be integrated with the gate assignment problem and generate optimal results.

Many airline companies are competing in the airline business today. Each company owns a fleet of aircraft, though it is usually not allowed to manage the movement of its aircraft because that would violate the freedom of using the shared airport facilities. Therefore, all aircraft must follow a first come, first served rule. As a result, airlines race to find a premium position to improve their on-time performance. ${ }^{14}$ However, it is highly undesired to see such a situation arise, because this may lead to decreased efficiency in the whole system.

Lately, however, airlines' demand for time on the ground has been exceeding the capacity of airports. Therefore, it has become necessary for airports to better manage the ground movement of aircraft. As a result, in recent years some policies have been adopted to improve the efficiency of airport ground operations. The most well-known of these policies are the Ground Delay Program and Collaborative Decision Making (CDM). ${ }^{15}$

The Ground Delay Program is currently implemented by the Federal Aviation Administration (FAA) to control air traffic when the acceptance rate of the destination airport is reduced because of bad weather conditions or travel disruptions. Airlines design their schedules according to normal weather conditions. However, when the weather becomes adverse, the acceptance rate can be largely reduced. As a result, the arrival demand exceeds the acceptance rate.

In order to solve this problem, the FAA introduced the Ground Delay Program by assigning a delay to each aircraft at their origin airport. Thus, aircraft can reduce the amount of time they stay in the airspace, which is much more dangerous than staying at the origin airport. Nevertheless, some shortcomings do come part and parcel with the Ground Delay Program. Those shortcomings are mostly due to the lack of data sharing, for not all airlines are willing to update their arrival traffic data. Therefore, the air traffic control tower cannot make accurate plans.

Collaborative Decision Making is a way to integrate the efforts of airlines and air traffic control to improve the efficiency of airport operations. The process can be improved to include pre-departure sequencing, ramp sequencing, and other scheduling intricacies. CDM is based on information sharing and distributed decision making. The most important factor is to collect data from all airspace users, as well as to create common awareness by distributing this information to airspace users and air traffic controllers.

Estimated time of arrival (ETA) is very important information for air traffic control. Based on the ETA information provided, ATC can assign runway use time slots to the different aircraft. If the information is accurate, efficient operations are always desirable. In some European airports, such as London's Heathrow Airport, air traffic control adopts a centralized surface management approach to manage the fleet of aircraft.

Based on the Ground Delay Program and CDM, this paper focuses on how to bring about improvements to overall aircraft delays in good weather conditions using a
centralized control policy. Gate holding and pushback sequencing are both important policies ${ }^{14}$ that this paper draws upon. This research aims at providing de-conflicted routing and scheduling plans for all the aircraft moving within an airport. The findings of this research can be helpful for ground controllers to find more efficient movement plans to save taxi time, as well as to reduce noise and fuel consumption on the taxiways. From both short-term and long-term perspectives, this research is meaningful because it adds to the growing understanding of how to make air travel as efficient as possible for passengers, airlines, and airports.

## B. Problem Statement

This paper solves the taxiway air traffic scheduling problem by applying the Mixed Integer Programming (MIP) method to aircraft and airport gates, taxiways, and runways. The aircraft can be classified into departure aircraft and arrival aircraft. The departure aircraft move from their gates to the departure runway. The arrival aircraft come from a runway exit, then enter the taxiway system. After leaving the taxiway system, the arrival aircraft reach their assigned gates. The movement of the departure aircraft, which move from the gate to the runway, is opposite that of the arrival aircraft. Figure 2 shows the movement of aircraft within an airport.

Taxi time is considered to be an evaluation criterion for the performance of an airport. The taxi times for departure and arrival aircraft are defined as follows. ${ }^{16}$ For departure aircraft, taxi out time is calculated by measuring the time from when the aircraft leaves the gate to when it finally takes off from the runway. For arrival aircraft,
taxi in time is calculated by measuring the time from the aircraft lands on the runway to when it finally reaches the gate.


Figure 2: Departure and arrival process within an airport

Figure 3 shows us the change between taxi out time and departure time on July 1, 2010 at IAH. We can see that taxi out time changes with the time of day. An average of 15 minutes for taxi out time is expected. Nevertheless, during the period between 3:00 p.m. and 5:00 p.m., the taxi out time is greater than 25 minutes-much too long by any measure. This indicates that IAH has to take some measures to reduce taxi out time, especially during peak periods. The benefits of lowering taxi time are less fuel consumption, fewer emissions, and less noise, and overall less taxi time is highly desirable from the perspective of economics and environmental protection. To the airlines, it means higher productivity.


Figure 3: Taxi out time of George Bush International Airport on July 1, 2010

Figure 4 illustrates the taxi planning problem. ${ }^{17}$ Here, the airport is modeled as a network of links and nodes. The nodes represent gates, taxiway intersections, runway thresholds, and runway exits. By using the information provided by aircraft and airport configuration, the optimization model tries to calculate the optimal routes and most efficient schedules for each aircraft by minimizing total taxi times. This calculated route is a detailed route that is represented by the nodes through which it travels. The schedule includes the points in time when an aircraft reaches each node along the route. Also, the most efficient ordering of when different aircraft reach each node and runway can be obtained.


Figure 4: Taxi planning problem illustration ${ }^{17}$

In modeling the airport ground network, some constraints ${ }^{13}$ need to be considered. First, two aircraft are traveling on the same link can't overtake each other or taxi side by side. In addition, all the aircraft have to maintain a minimum separation distance to avoid collision. This minimum separation distance can also be replaced by a minimum separation time when studying the scheduling of aircraft. In this thesis, a minimum separation time is used. Another constraint is wake vortex. Wake vortex occurs after an aircraft lands or takes off, and runway operations must guarantee a minimum separation time after wake vortex to ensure the safety of the following aircraft. The minimum separation time necessary is determined by the type of the leading aircraft and the type of the following aircraft.

Another consideration is runway crossing. For large airports with more than one runway, such as George Bush International Airport, an aircraft may have to cross a
runway to reach its taxi destination. However, runway operations do not permit an aircraft to cross when another aircraft is using the runway to take off or land. Runway crossing is also considered in this paper.
C. Summary

This thesis presents a mixed integer linear programming model to optimize the taxiway air traffic scheduling in a continuous timing environment. The objective is to find the optimal routing and scheduling for both departure and arrival aircraft with a minimum total taxi time.

Two parallel runways are used in the model. The one close to the terminal area is used as a departure runway, and the other runway is used as an arrival runway. The model incorporates runway departure, runway arrival, runway crossing, and taxiway scheduling while keeping in mind safety and separation constraints. No more than 20 aircraft can be scheduled at a time due to limitations of computer memory.

CPLEX is applied as a solver, and the network of George Bush International Airport is used. First the author decomposes the network into three sub-networks and chooses one to work with when seeking solutions. By using the simplified network, up to 15 aircraft can be solved within a reasonable timeframe.

In order to solve a large-scale problem, a rolling horizon method is proposed by dividing the long planning period into small, non-overlapping planning periods. In this thesis, a fleet of 46 departure aircraft and 18 arrival aircraft are scheduled within a onehour period. A bound is used for comparison.

## CHAPTER II

## AIRPORT GROUND NETWORK

The airport ground network mainly includes gate and ramp area, taxiway system and runway system.

## A. Terminal Area

The gate and ramp area is where aircraft load and unload. The ramp area connects the gate and the taxiway system. For a departure aircraft, after it leaves a gate, it can wait in the ramp area for pushback to the taxiway system. Gate assignment problem is not studied in this paper. The gate and ramp area is assumed to be a whole system in this paper. The aircraft can hold in the ramp area for pushback after it leaves the gate. As a result, if we consider the gate and ramp area as a single node, the capacity of this node is larger than one. Also it is assumed that gate is always available for arrival aircraft.

There is an earliest pushback time for each aircraft. The departure aircraft can't leave the ramp area until this earliest pushback time. If we hold the aircraft in the gate instead of pushing it into the airport ground network, ${ }^{18}$ the taxi time will be smaller. This is because if there are more aircraft in the ground network, the chance of waiting and queuing is larger. The benefits of saving fuel can be seen from environmental improvement.

## B. Taxiway

A taxiway ${ }^{19}$ is a path connecting the runway and a gate. The pilot is assigned a serious of taxiways before departing from the gate or exiting the runway to return the gate. This set of taxiways is called route. Based on the location of the taxiway, it can be divided into several types: ${ }^{20}$

- Gate access taxiway. This kind of taxiway is directly connected with a gate. Speed limit is lower compared with other taxiways.
- Runway access taxiway. This means the taxiway that has an intersection with runway. As mentioned above, it has to be considered differently because of runway crossing.
- Runway exit taxiway. This is the taxiway an aircraft uses to exit a runway after landing. Some of them allow high-speed and thus called high-speed runway exit.
- Simple taxiway. There is a speed limitation for each simple taxiway.

For safety concerns, any two aircraft on the taxiway must maintain a minimum separation distance or a minimum separation time. In emergency, this distance or time can help the aircraft avoid collision.
C. Runway

Runway is defined as a strip of land ${ }^{3}$ on which an aircraft can take-off and land. Runway capacity, which is defined as the maximum number of aircraft that a runway can handle within a period of time, is being studied by a lot of researchers. It has been proved that
the capacity ${ }^{19}$ is highly related to the mixture of aircraft and also order of using the runway for different aircraft.

An important characteristic with runway is the configuration. According to Alexander ${ }^{3}$ runway configuration can be classified as the following types:

- Single runway. A single runway can accommodate up to 99 operations per hour for small aircraft and approximately 60 operations for large aircraft during fair weather conditions.
- Parallel runways. (a) If the distance between them is larger than 4300 feet, they can be considered as two single runways. (b) If the distance is between 2500 and 4300 feet, they must be highly coordinated, which reduces the capacity significantly. (c) If this distance is less than 2500 ft , they must be considered as a single runway.
- Intersecting runways. When two or more runways cross each other from different directions, they are referred as intersecting runways.

In runway operations, the most important factors that need to be considered include wake vortex separations. Federal Aviation Administration categorizes the aircraft into three classes according to their maximum certificated take-off weight ${ }^{21}$ (MCTW). Those have less than 41000 pounds are called small aircraft. Those with MCTW between 41000 pounds and 255000 pounds are called large aircraft (such as B737, 747). When the MCTW exceeds 255000 pounds, the aircraft is called heavy aircraft.

Wake turbulence forms after an aircraft when it takes off or lands. Wake vortex is the most commonly seen kind of wake turbulence. Wake vortex is very dangerous for the following aircraft. As a result, a time gap must be given for this wake vortex to disappear. Wake vortex separations are generally considered as the most important separation criteria in runway operations. In order to avoid accidents, the aircraft using the runway must obey a set of separations.

Three types of runway operations management (ROM) are analyzed in Wikipedia. They include runway configuration management, runway assignment management and runway operations planning. Runway configuration management depends on the long term planning of an airport. For the second one, it is assumed that the runway assignment doesn't change. This paper mainly studies the runway operations planning. Runway operations include takeoffs, landings and crossings. Depending on the path of an aircraft ${ }^{21}$, there might be a need to cross an active runway. This case is also studied in this paper.

## D. Airport Traffic Control Tower

The airport traffic control tower is responsible for controlling the aircraft movement within the airport and also 5 to 10 nautical miles away ${ }^{17}$ in the airspace. There are usually several controllers working in the airport traffic control tower. Those controllers communicate with pilot using radios to provide information and guidance, which is to avoid collision between the aircraft and also make the traffic flow more efficient. Currently, many aircraft install radar to detect the location of nearby aircraft. However, the radar system still can't replace the function of controllers.

There are mainly two types of queues in the airport ground network. The first type of queue is push back queue, which is formed when the aircraft wait in the ramp for pushback into the taxiway system. The second kind of queue is departure queue. The departure queue is formed when the departure aircraft wait to takeoff. The controllers have the right to give an order to those aircraft. The control strategy is generally first come first serve (FCFS). However, FCFS might not be efficient. In some cases, the controllers can change the sequence of those aircrafts by allowing some aircraft to leave first. The control strategy can affect the throughput of runway.

## CHAPTER III

## LITERATURE REVIEW

The taxi planning problem (TPP) ${ }^{22-37}$ is very similar to the classic vehicle routing and scheduling problem. The objective can be minimizing the total travel time or total cost consumed by all the aircraft. There are a series of constraints in modeling this problem, which can be analyzed as safety constraints, timing constraints and ordering constraints. In recent years, mixed integer linear programming (MIP) has been widely used by researchers to formulate the airport ground movement problem. A well-developed model may lead to optimal solutions. However, when the problem becomes large, the computation time might not be practical anymore. In this case, a heuristic method can be used to find a feasible solution that is very close to optimal solution.

## A. Vehicle Routing and Scheduling

Scheduling ${ }^{38-45}$ is a decision-making process to optimize the allocation of resources in industries. In this paper, the main resources are taxiways, runways and gates. Runway scheduling has been studied extensively.

The classical vehicle routing problem (VRP) ${ }^{46-53}$ is a combinational optimization and integer programming problem. It can be generalized as finding optimal routes for one or more vehicles through a real network. Extensive research has been done in VRP during the past 50 years.

The first literature ${ }^{46}$ about VRP is proposed by Danzig et al. in 1954. However, the word vehicle Routing first appeared in a paper ${ }^{47}$ published in 1972 by Golden

Lenstra and Rinnooy ${ }^{48}$ proved that the VRP cannot be solved efficiently in polynomial time and thus NP-hard in 1981. Soloman considered time window constraints ${ }^{49}$ in his paper published in 1983. Major problem types, formulations and algorithms can be found in later papers. ${ }^{50-53}$ The theory and algorithms of vehicle routing can be applied to taxi planning problems.

## B. Taxi Planning Problem

There exist differences between the TPP and VRP problems. For the TPP problem, the aircraft does not need to deliver goods to each node it visits. The TPP can be classified into different types of problems according to the route selection, control strategy and the processes. The safety constraints are unique due to the characteristics of airport operations. In this literature, the author will first explain the common points of different TPP problems and then show the differences.

The same points of different TPPs include the safety constraints, link directions, time continuity and objective functions, which is listed from 1 to 5 . The different between different TPPs are shown between 6 and 8, which include control strategy, routing strategy and runway integration. The related problem Gate Assignment Problem (GAP) is briefly mentioned in 9.

## 1. Safety Constraints

In some papers, ${ }^{26,30,32}$ each aircraft is assigned a maximum taxiing speed and minimum taxiing speed based on the types of taxiways they are taxing on. The maximum speed of different runway exits varies because of their physical layout. Another way ${ }^{27,29}$ is to use
a maximum speed only, which is applied to calculate the minimum time required for a certain aircraft to taxi through a taxiway. The last way ${ }^{28}$ is to replace the maximum and minimum speed with an average speed.

In research, a minimum separation constraint is used by all the researchers due to safety considerations. This constraint is necessary because the aircraft needs time to stop. Smeltink et al. ${ }^{26}$ assumes the minimum separation distance to be 200 meters. For timing purposes, this distance is converted into time gap using the speed of the leading aircraft. Rathinam et al. ${ }^{27,32}$ adopts two minimum separation constraints using time space diagram, which the authors name as upstream and downstream constraint. The minimum separation distance is usually set to be 200 meters. ${ }^{26,28,30}$

The minimum separation distance can also be replaced by a minimum separation time, ${ }^{29}$ in which case the speed does not need to be used any more. Another way ${ }^{28,31}$ is to replace the minimum separation constraint by a link occupancy constraint. The minimum separation is achieved by adding a capacity to each link.

## 2. Time Continuity

Time is considered as continuous in some papers. ${ }^{26,27}$ A variable is used to record the time that an aircraft reaches a certain node. As long as the aircraft enters the taxiway system, it has to move along its route. The aircraft is not allowed to hold at the taxiway intersections. The time that an aircraft reaches a node represents the time that this aircraft leaves the node. Accordingly, the time point that an aircraft reaches the runway threshold means it takes off from this runway. A constant number of planning periods ${ }^{29}$ for each aircraft can be used to model the time continuity. Each time point is recorded by
the beginning time of a planning period. The length of the planning period differs from aircraft and differs even for each aircraft. Every time the aircraft enters a new node, the planning period is updated. If an aircraft reaches the destination node within the constant number of planning periods, the rest of the planning periods are used for runway queuing.

Another way is to treat the time as discrete ${ }^{28,31}$ by assigning each aircraft a sequence of planning periods with equal length. The taxiing speed is assumed to be constant for each aircraft. The movement of each aircraft in each planning period is described by a binary variable. The link time is added if the aircraft travels this link in some time period. In this thesis, no planning periods are considered. The time is continuous and the holding is replaced by a lower taxi speed on the taxiway.

## 3. Link Direction

For most of the taxiways, the width can only allow one aircraft to pass. As a result, overtaking constraints and head-on-head constraints are formulated. ${ }^{26-30,} 32$ The overtaking constraint is used to avoid one aircraft to overtake another aircraft when taxing on the same link in the same direction. The head-on-head constraint is used to avoid collision when two aircraft taxi on the same taxiway in opposite directions. Those constraints are necessary and important to airport ground movement. Those onedirectional taxiways are efficient in solving small network examples. However, as for large airport networks, it is crucial to consider bi-directional taxiways because bypassing is permitted and even desired in some situations to improve the overall movement of aircraft. This will be further discussed in the runways integration part.

## 4. Objective Function

Most of the objective functions try to minimize the total travel time because taxi time is considered as an important performance criterion in airport ground movement. For example, the total time of all the aircraft can be minimized. ${ }^{27,30} \mathrm{~A}$ better way is to divide the aircraft into two kinds ${ }^{30}$ based on whether the engine is on or off. Those times when the engines are on are given higher cost. A penalty ${ }^{26}$ for the gap between the actual time and the scheduled time that each aircraft arrives at the destination node can be added to the objective function.

Visser et al. ${ }^{28}$ uses two part in their objective function. The first part is free flow time, which means given a constant speed and no restriction, the time that an aircraft spends on a certain route. The second part is the time that an aircraft spends longer than the free flow time in each sector, which is called delay. Each aircraft is given different priorities by using different weights.

The objective function can also include the travel distance for all the aircraft ${ }^{29}$. When the planning time is assumed to be fixed, some aircraft may fail to reach its destination node. A penalty for not being able to reach the destination node is added. This penalty is calculated by the shortest distance between the position and destination.

## 5. Decision Variables

The decision variables in all those articles can be summarized as two kinds. The first kind is binary variable, which can only be 0 or 1 . The second kind is timing variable, which is used for scheduling purposes and also to calculate the total taxi time. Ordering

Variables are usually used to decide the sequence of using certain resources. Those types of variables are used to decide the order of passing a certain node.

## 6. Routing Strategy

In some papers, ${ }^{26,27}$ routes are pre-assigned. Each aircraft is given a fixed route with sequential nodes before departure or arrival. This problem finally becomes a scheduling problem. The formulations lead to a solution for the best timing point at which each aircraft reaches the node along its route.

However, in real situations, an aircraft is usually allowed to change its route. As long as a controller realizes that some taxiways are not occupied, they try to assign aircraft there to reduce the queue on those congested taxiways. From this prospective, it is better for the aircraft to change its route. For the other papers, ${ }^{29,31}$ the routes are completely unknown when modeling and only the origin and destination of each aircraft is fixed. In this case, there must be some variables to determine the node that an aircraft has visited. The solution is to search optimal routes for all the aircraft and also their schedules of using those routes. This is more flexible compared with pre-assigned routes. Nevertheless, due to the increase of variables and constraints, if there is no efficient way of finding the optimal solution, the computation time can be very large and thus not practical.

Compared with the above routing strategies, some papers ${ }^{28,30,32}$ allow each aircraft to choose from a set of pre-assigned routes. This is more flexible than preassigned routes. In this paper, the origin and destination of each aircraft is fixed. The route of the aircraft is totally unknown before scheduling. A variable is used to decide
the link that an aircraft travels at a certain period. The routing and timing strategy is the output of this model.

## 7. Control Strategy

In airport ground network, the aircraft is sometimes required to hold at some points to relieve congestion. The most commonly used holding point is gate. ${ }^{26-30}$ When the aircraft is held on the gate, the engine is off. From both economic and environmental perspectives, it is desirable. This can be achieved by an earliest push back time, which is the earliest time that an aircraft can leave the gate. Multiple holding points can be considered in some situations. The ramp exit sequence can also be incorporated ${ }^{31}$.

Runway crossing is popular in big airports. For actual operations, arrival aircraft has priority to departure aircraft. Departure aircraft has priority to crossing aircraft. However, when the number of aircraft waiting to cross an active runway is very large, the departure aircraft may give way to those crossing aircraft. In some work, ${ }^{27}$ all the aircraft are given the same priority. However, in some other work, ${ }^{31}$ those crossing aircraft has to wait until there is a large enough gap for it to cross the runway. This gap usually appears between a departure aircraft and an arrival aircraft.

For runway scheduling, usually the aircraft follows a first come first serve rule. However, there might be several queues for departure. For each queue, the first come first serve rule cannot guarantee the optimal throughput of runway. Atkinrt al ${ }^{33}$ uses reordering method to study the sequence of take-off for departure aircraft. This re-ordering can be achieved in a holding area close to the runway threshold. As the demand can be
very high in peak hours, re-ordering is sometimes necessary to improve the performance of runway. In our paper, we allow bypassing in the link next to runway threshold.

## 8. Runway Integration

Ref. 34 studies the key factors that impede the flow of aircraft within an airport and nearby airspace. Runway is concluded as the most important source that results delay in the airport. In big airports, there are usually several runways in operations at the same time. Some runways handle both landings and takeoffs. Some runways are only used for departure aircraft or arrival aircraft. The factors that need to be considered include runway occupancy time, wake vortex separations and runway crossing.

Runway crossing occurs when an aircraft needs to cross an active (meaning there is an aircraft taking off or landing) runway. Only one aircraft can use a runway at the same time. Therefore, if an aircraft is taking off, the other aircraft who want to cross the runway have to wait a certain amount of time. The sequence of using a runway is arrival aircraft, departure aircraft and crossing aircraft.

Due to the separations in arrival and departure aircraft, those aircraft who want to cross can make use of the time separations to cross. An example ${ }^{30}$ is given to explain this. Suppose the runway occupancy time is 55 seconds for a B757. The time separation is 157 seconds. As a result, those aircraft have 102 seconds to cross this runway. It cost the first aircraft 40 seconds to cross. For the following aircraft, they have to maintain a 10 seconds spacing. This means if there are four aircraft waiting to cross, they need 70 seconds, which is less than 102 seconds. This is just a simple case for runway crossing. It is mentioned ${ }^{30}$ that the main contribution to the increase of taxi time is the time spent
on runway crossing. When the runway is busy with aircraft landings and takeoffs, those aircraft who wants to cross may have to wait for a long time.

Some papers ${ }^{26,28}$ only consider the taxiway scheduling problem. The runway scheduling is not mentioned in those two papers. If there is only one runway in the airport, this method is applicable, which means the taxiway planning and runway scheduling can be separated. However, if there are more than two runways in an airport, taxiway scheduling is not practical because even though we maximize the performance of taxiway system, the queue of using runways may still be very large. As a result, the time saved in taxiway planning can be consumed by runway delays.

The sequence of using runways is highly connected with the throughput of runways. In most of the other papers, runway operations are modeled in different ways. Some papers only consider one departure runway. ${ }^{27}$ In addition, there is a point where arrival aircraft can use to cross the departure runway. There is an ordering variable to decide whether a runway is used by a departure aircraft or an arrival aircraft that needs to cross. When the departure queue is very large, sometimes we may have to sacrifice the crossing aircraft to make sure the other aircraft depart as scheduled. Some time periods that can be used for crossing need to be scheduled by the controller. In some work, ${ }^{29}$ only one departure runway is considered and no runway crossing is taken into account. In the other work, ${ }^{30}$ the taxiway operations are incorporated with runway operations and runway crossing is taken into account. Some work also separates the runway operations from the taxiway operations. ${ }^{31}$ The output of taxiway operations is treated as the input of runway operations.

Rathinam ${ }^{32}$ further develops the work ${ }^{27}$ and multiple crossing points are used. Runway crossing is modeled as an ordering problem. The minimum separation distance and minimum speed constraint makes this problem less flexible. In their formulations, the aircraft has to move at certain speed to reach the crossing point and then cross. The following aircraft has to keep a certain separation distance and then decide whether to cross. If the queue is very large, this minimum separation distance can take a significant length of taxiway. This is not practical in real situations. In the author's opinion, it is better to allow the aircraft to queue in front of the crossing point, which can save more space for queuing. In addition, the wake vortex separation or other kinds of separations can be used to schedule the crossing of those queuing aircraft. Those aircraft are allowed to hold in front of the crossing point and cross in batch, which can save the overall crossing time.

## 9. Gate Assignment Problems

Gate assignment problem is highly related to the ground movement problem studied in this paper. As mentioned above, each aircraft is assigned a gate to load or unload passengers and freight. In normal situations, the gate occupancy is assigned as high as $80 \%$ of the gate capacity. ${ }^{32}$ However, there still exist aircraft that have no gate to use when arriving at the apron. This can be attributed by the deviation between the actual arrival time and scheduled arrival time. Most of the time, the arrival aircraft reaches the apron as scheduled. However, if the departure aircraft is still waiting at the gate for pushback, there will be no gate for this arrival aircraft to use. In some cases, even when a gate is available and compatible to a certain aircraft, this aircraft may still need to wait
because the controllers don't want to re-assign the staff needed to serve the passengers. The second reason to assign gate is to minimize the total travel distance of all kinds of embarking, disembarking and transfer passengers. In many airports, transit is available inside the airport terminals that can be used for passengers to travel from one terminal to another. In the current research, gate assignment is not considered in this paper.

## C. Differences between This Thesis and Previous Papers

The basic idea of this thesis is very similar to Richard ${ }^{29}$ However, this thesis has several improvements compared with their work.

First, this thesis consider both departure and arrival aircraft. In their work, only departure aircraft are considered and a single runway is used. In this thesis, two parallel runways are used for solutions. Runway departures, runway arrivals and runway crossings are all taken into account.

Second, the number of variables and constraints are largely reduced. This thesis have two less types of variables, including a two dimensional variable and a five dimensional variable. This thesis also uses more efficient constraint in dealing with the ordering variables.

Third, this thesis makes the time continuous by introducing a maximum speed and a minimum speed. By assuming the length between the destination and the dummy node to be zero, the movement to the dummy node does not take any time. The minimum speed has a very small value.

Last, the departure time and arrival time are allowed to change between time windows. In their work, only the departure aircraft is used and the time length that an aircraft can hold at the gate is arbitrarily long, which is not practical.

## CHAPTER IV

## OPTIMIZATION MODEL

## A. Problem Description

The airport taxiway network is denoted as $\mathrm{G}=(\mathrm{N}, \mathrm{L})$, where N is a set of nodes representing gate, taxiway intersections, runway crossing points, runway threshold (The starting point of a runway) and runway exits and L is a set of directed links representing taxiways or other links connecting those nodes.

The aircraft set F is divided into departure aircraft D and arrival aircraft A. For each aircraft $\mathrm{i} \in \mathrm{F}$, the origin node $\mathrm{ORI}_{\mathrm{i}}$ and destination node $\mathrm{DES}_{\mathrm{i}}$ are fixed. Assume a route for aircraft $i$ is a sequence of nodes denoted by from origin node to destination node. For the departure aircraft, the gate is the origin node and the runway threshold is the destination node. The arrival aircraft has to cross the departure runway to reach the gate. For the arrival aircraft, the origin is the landing runway exit and destination is the gate assigned to it. A dummy node $\mathrm{N}_{\text {air }}$ is used in this paper. This dummy node can be understood as the outside of the airport ground network. After an aircraft takes off, it enters the dummy node. In this paper, gate assignment problem is not considered. It is assumed that the gate is always available. In addition, the turnaround of aircraft is not considered. In this paper, the arrival aircraft leaves the system after it arrives at the gate.

For each aircraft, the number of planning period is fixed, which is $E_{p} . E_{p}$ is chosen to guarantee every aircraft can finish the movement between its origin node to its destination node. Each aircraft moves from the beginning of the first planning period to
the end of its last planning period. All aircraft are required to finish their movements within $\mathrm{E}_{\mathrm{p}}$ planning periods. When an aircraft leaves a certain node, a new planning period begins. It is assumed that all the aircraft enters the dummy node within $\mathrm{E}_{\mathrm{p}}$ planning periods.

This optimization model is about routing/scheduling. In this paper, a route variable is used to find the path that each aircraft takes. There is also a timing variable used to record the time that an aircraft leaves each node. By using some constraints, the starting time of the planning period stays the same as long as an aircraft reaches its destination node. For each aircraft, the time gap between it starts the first planning period and it starts the last planning period is seen as taxi time. The objective of this paper is to minimize the total taxi time of all the aircraft.

## B. Parameters

The most important parameter used in this paper is connection factor. As long as there is a directed link from one node to another node, the connection factor is set to be 1 . In other cases, the connection factor is 0 .This connection factor is related to the order of those two nodes. As the aircraft can only move if the connection factor is 1 , those movements can be achieved by the connection factor. It is assumed that the aircraft after entering the destination node, the aircraft can only stay there. In the real cases, the departure aircraft takes off. In this paper, according to the speed constraints, the time doesn't change any more after the aircraft enters its destination node. As a result, the movement stops when an aircraft enters its destination node.

Other parameters include the maximum and minimum speed for aircraft i to travel on the links. The length of link $\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)$ is denoted as $\mathrm{L}_{\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)}$. The maximum and minimum speed are assumed to be constant for all the links. In an airport, different taxiways have different requirements on the speed. In order to solve this problem, the length is adjusted proportional to the speed. For example, if the maximum speed of ramp is only half of the other taxiways, the length of this ramp area is set up to be twice long. In the end, the outputs are timings. With its real length, the speed can be obtained.

For each aircraft $\mathrm{i} \in \mathrm{D}$, there is an earliest pushback time $\mathrm{EPT}_{\mathrm{i}}$. The departure aircraft has to move later than this time point. For each aircraft $i \in A$, there is an estimated arrival time $E A T T_{i} . W_{i_{1} i_{2}}$ represents the wake vortex separations between two departure aircraft. $\mathrm{Y}_{\mathrm{i}_{1} \mathrm{i}_{2}}$ represents the minimum time separations between a departure aircraft and a crossing aircraft.

## C. Decision Variables

(1) For $i \in F$ and $j \in P, n_{1}, n_{2} \in N$ the first variable is the routing variable $R_{\left(i, n_{1}, n_{2}\right)}^{j}$, which is equal to 1 if aircraft i moves from node $n_{1}$ to node $n_{2}$ at planning period $j$ and is equal to 0 otherwise.
(2) For $\mathrm{i}, \mathrm{j} \in \mathrm{F}$ and $\mathrm{n} \in \mathrm{N}$ the second variable is ordering variable $\mathrm{Z}_{(\mathrm{i}, \mathrm{j})}^{\mathrm{n}}$, which is equal to 1 if aircraft i arrives at node n earlier than aircraft j and 0 otherwise. The dummy node is not considered in this variable.
(3) For $\mathrm{i} \in \mathrm{F}$ and $\mathrm{j} \in \mathrm{P}$, the third variable is timing variable $\mathrm{t}_{\mathrm{i}}^{\mathrm{j}}$, which represent the starting time of planning period j for aircraft i. This time variable is continuous and moving forward.
(4) For $i_{1} \in D$ and $i_{2} \in A, C_{(i, j)}^{n_{r}}$ decides the order of using the runway to take off or cross. If this variable is 1 , it means that $i_{1}$ takes off earlier thani ${ }_{2}$ cross the runway, otherwise it is 0 .

## D. Objective Function

The objective function is described as following:

$$
\operatorname{Minimize} \sum_{i \in F} f_{i}\left(t_{i}^{E_{p}}-t_{i}^{1}\right)
$$

$f_{i}$ is a cost indicator. It reflects the relative cost of aircraft. This objective function mainly focuses on minimizing the total taxi time. This taxi time is calculated by using the starting time of the last planning period minus the starting time of the first planning period.

## E. Constraints

The first few constraints are the same Ref.29. The difference is the use of dummy node and the definition of connection factor. It is clearer in this paper that the aircraft can stay in the dummy node after it leaves the system and the time does not change after it leaves the system.

If and only if there is a link between two nodes, an aircraft can move and the routing variable can be 1 . This is guaranteed by constraint (1).

## 1. Taxiway Connection

$\forall \mathrm{i} \in \mathrm{F}$ and $\mathrm{j} \in \mathrm{P}, \mathrm{n}_{1}, \mathrm{n}_{2} \in \mathrm{~N}$,

$$
\begin{equation*}
\mathrm{R}_{\left(\mathrm{i}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}} \leq \mathrm{C}_{\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)} \tag{1}
\end{equation*}
$$

## 2. Origin and Destination

The first planning period is to make sure the aircraft starts from its origin node. $\forall \mathrm{i} \in \mathrm{F}$,

$$
\begin{equation*}
\mathrm{R}_{\left(\mathrm{i}, \mathrm{ORI}_{\mathrm{i}}, \mathrm{ORI}_{\mathrm{i}}\right)}^{1}=1 \tag{2}
\end{equation*}
$$

In order to guarantee an aircraft enters its destination node, there must be a planning period for the aircraft to travel from its destination node to itself. This is guaranteed by the following constraint. $\forall \mathrm{i} \in \mathrm{F}$,

$$
\begin{equation*}
\mathrm{R}_{\left(\mathrm{i}, \mathrm{DES}_{\mathrm{i}}, \mathrm{DES}_{\mathrm{i}}\right)}^{E_{p}}=1 \tag{3}
\end{equation*}
$$

## 3. Continuous Network Flow

For each aircraft $\mathrm{i} \in \mathrm{F}$, there must be exact a movement for it in each period $\mathrm{j} \in \mathrm{P}$.

$$
\begin{equation*}
\sum_{\mathrm{m} \in \mathrm{~N}} \sum_{\mathrm{n} \in \mathrm{~N}} \mathrm{R}_{(\mathrm{i}, \mathrm{~m}, \mathrm{n})}^{\mathrm{j}}=1 \tag{4}
\end{equation*}
$$

The inflow has to be equal to the out flow. $\forall \mathrm{i} \in \mathrm{F}, 1 \leq \mathrm{j} \leq \mathrm{E}_{\mathrm{p}}-1, \mathrm{~m} \in \mathrm{~N}$ if aircraft i moves to node $m$ in period $p$, it must move from $m$ in period $p+1$. The left side in constraint (4) represents that an aircraft moves to node $m$ in a certain period. The right hand side means this aircraft moves from node $m$ in the next planning period. The only exception is when the node $m$ is the dummy node. Constrained by (1), it has to stay in the dummy node.

$$
\begin{equation*}
\sum_{\mathrm{n} \in \mathrm{~N}} \mathrm{R}_{(\mathrm{i}, \mathrm{n}, \mathrm{~m})}^{\mathrm{j}}=\sum_{\mathrm{n} \in \mathrm{~N}} \mathrm{R}_{(\mathrm{i}, \mathrm{~m}, \mathrm{n})}^{\mathrm{j}+1} \tag{5}
\end{equation*}
$$

Even though there is a directed link between two nodes, the aircraft cannot travel reversely in two consecutive periods because an aircraft cannot make a 180 degree turn. This is guaranteed by the following constraint (6). $\forall i \in F, 1 \leq j \leq E_{p}-1, n_{1}, n_{2} \in$ $\mathrm{N}, \mathrm{n}_{1} \neq \mathrm{n}_{2}$,

$$
\begin{equation*}
\mathrm{R}_{\left(\mathrm{i}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}}+\mathrm{R}_{\left(\mathrm{i}, \mathrm{n}_{2}, \mathrm{n}_{1}\right)}^{\mathrm{j}+1} \leq 1 \tag{6}
\end{equation*}
$$

4. Continuous Timing

The first planning period is used to fix an aircraft to its origin node. As a result, the first period does not take any taxi time. Thus for $\forall \mathrm{i} \in \mathrm{F}$, we have the following constraint:

$$
\begin{equation*}
\mathrm{t}_{\mathrm{i}}^{1}=\mathrm{t}_{\mathrm{i}}^{2} \tag{7}
\end{equation*}
$$

The beginning of the second planning period should be later than the earliest pushback time for departure aircraft. $\forall \mathrm{i} \in \mathrm{D}$,

$$
\begin{gather*}
\mathrm{t}_{\mathrm{i}}^{2} \geq \mathrm{EPT}_{\mathrm{i}}  \tag{8}\\
\mathrm{t}_{\mathrm{i}}^{2} \leq \mathrm{EPT}_{\mathrm{i}}+\operatorname{Gap} 1 \tag{9}
\end{gather*}
$$

$\forall \mathrm{i} \in \mathrm{A}$,

$$
\begin{equation*}
\mathrm{EAT}_{\mathrm{i}}-\mathrm{Gap} 2 \leq \mathrm{t}_{\mathrm{i}}^{2} \leq \mathrm{EAT}_{\mathrm{i}}+\text { Gap2 } \tag{10}
\end{equation*}
$$

5. Ordering of Using the Intersection

The order variable of using node $n$ between the same aircraft is set to be 0 .
$\forall \mathrm{i} \in \mathrm{F}$ and $\forall \mathrm{n} \in \mathrm{N}$,

$$
\begin{equation*}
\mathrm{Z}_{(\mathrm{i}, \mathrm{i})}^{\mathrm{n}}=0 \tag{11}
\end{equation*}
$$

$\forall \mathrm{i}_{1}, \mathrm{i}_{2} \in \mathrm{~F}$ and $\mathrm{i}_{1} \neq \mathrm{i}_{2}, \mathrm{n} \in \mathrm{N}$, we have the following constraint:

$$
\begin{align*}
& \mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}}+\mathrm{Z}_{\left(\mathrm{i}_{2}, \mathrm{i}_{1}\right)}^{\mathrm{n}} \leq\left(\sum_{\mathrm{m} \in \mathrm{~N}} \sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{~m}, \mathrm{n}\right)}^{\mathrm{j}}+\sum_{\mathrm{m} \in \mathrm{~N}} \sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{~m}, \mathrm{n}\right)}^{\mathrm{j}}\right) / 2  \tag{12}\\
& \mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}}+\mathrm{Z}_{\left(\mathrm{i}_{2}, \mathrm{i}_{1}\right)}^{\mathrm{n}} \geq \sum_{\mathrm{m} \in \mathrm{~N}} \sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{~m}, \mathrm{n}\right)}^{\mathrm{j}}+\sum_{\mathrm{m} \in \mathrm{~N}} \sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{~m}, \mathrm{n}\right)}^{\mathrm{j}}-1 \tag{13}
\end{align*}
$$

The right hand side of constraint (12) and (13) represents whether aircraft $i_{1}$ and $i_{2}$ pass node $n$. If both aircraft $\mathrm{i}_{1}$ and $\mathrm{i}_{2}$ pass node n , then the right hand side of (12) and (13) are all equal to 1 . As a result, $\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}}+\mathrm{Z}_{\left(\mathrm{i}_{2}, \mathrm{i}_{1}\right)}^{\mathrm{n}}=1$. If only one of them or none of them pass node $n$, then the right hand side of (12) should be equal to 0 or 0.5 . The right hand side of (13) should be equal to 0 or -1 . However, the left hand side variables can only be 0 or 1 . As a result, $\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}}+\mathrm{Z}_{\left(\mathrm{i}_{2}, \mathrm{i}_{1}\right)}^{\mathrm{n}}=0$.

## 6. Safety of Taxiway

## a. Overtaking Avoidance

In this paper, the taxiway is only wide enough to allow one aircraft to pass. Those can be guaranteed by the following constraints. $\forall \mathrm{i}_{1}, \mathrm{i}_{2} \in \mathrm{~F}, \mathrm{i}_{1} \neq \mathrm{i}_{2}, \mathrm{n}_{1}, \mathrm{n}_{2} \in \mathrm{~N}, \mathrm{n}_{1} \neq \mathrm{n}_{2}$ andn $\mathrm{n}_{2} \neq$ $\mathrm{n}_{\mathrm{r}}$ we have the following constraints:

$$
\begin{align*}
& \mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{1}}-\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{2}} \leq 2-\left(\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}}+\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}}\right)  \tag{14}\\
& \mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{1}}-\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{2}} \geq-2+\left(\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}}+\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}}\right) \tag{15}
\end{align*}
$$

Those constraints mean if $\left(n_{1}, n_{2}\right)$ is on the route of both aircraft, then $Z_{\left(i_{1}, i_{2}\right)}^{n_{1}}=Z_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{2}}$. If one aircraft arrives at one node earlier than the other, it has to arrive at the next node earlier too. The only exception is the runway node.
b. Head on Head Avoidance

In order to avoid head on head collision, some constraints are needed. $\forall \mathrm{i}_{1}, \mathrm{i}_{2} \in \mathrm{~F}$,
$\mathrm{i}_{1} \neq \mathrm{i}_{2}, \mathrm{n}_{1}, \mathrm{n}_{2} \in \mathrm{~N}, \mathrm{n}_{1} \neq \mathrm{n}_{2}$,

$$
\begin{align*}
& \mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{1}}-\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{2}} \leq 2-\left(\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}}+\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{n}_{2}, \mathrm{n}_{1}\right)}^{\mathrm{j}}\right)  \tag{16}\\
& \mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{1}}-\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{2}} \geq-2+\left(\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{n}_{1}, \mathrm{n}_{2}\right)}^{\mathrm{j}}+\sum_{\mathrm{j} \in \mathrm{P}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{n}_{2}, \mathrm{n}_{1}\right)}^{\mathrm{j}}\right) \tag{17}
\end{align*}
$$

Constraints (15) and (16) mean if $\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)$ is on the route of aircraft $\mathrm{i}_{1}$ and $\left(\mathrm{n}_{2}, \mathrm{n}_{1}\right)$ is on the route of aircraft $\mathrm{i}_{2}$, thenZ $\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{1}}=\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{2}}$. This can avoid conflict between those two aircraft.
c. Minimum Separation between Aircraft

In order to make the constraint simpler, we use a minimum separation time instead of a minimum separation distance in Figure 5. Suppose $M$ is a very large number. $\forall \mathrm{i}_{1}, \mathrm{i}_{2} \in \mathrm{~F}$, $\mathrm{i}_{1} \neq \mathrm{i}_{2}, \mathrm{n} \in \mathrm{N}, \mathrm{j}_{1}, \mathrm{j}_{2} \in \mathrm{P}$,

$$
\begin{equation*}
\mathrm{t}_{\mathrm{i}_{1}}^{\mathrm{j}_{1}+1}+\mathrm{t}_{\text {sep }} \leq \mathrm{t}_{\mathrm{i}_{2}}^{\mathrm{j}_{2}+1}+\left(3-\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}}-\sum_{\mathrm{m} \in \mathrm{~N}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{~m}, \mathrm{n}\right)}^{\mathrm{j}_{1}}-\sum_{\mathrm{m} \in \mathrm{~N}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{~m}, \mathrm{n}\right)}^{\mathrm{j}_{2}}\right) * \mathrm{M} \tag{18}
\end{equation*}
$$

This constraint means if aircraft $\mathrm{i}_{1}$ moves to node n during period $\mathrm{j}_{1}$ and $\mathrm{i}_{2}$ moves to node n at planning period $\mathrm{j}_{2}$ and aircrafti ${ }_{1}$ reaches node n earier than $\mathrm{i}_{2}$, then the time gap between the next planning periods begin is at least $t_{\text {sep }}$.


Figure 5: Diagram of minimum separation distance

## 7. Maximum and Minimum Speed

There is a maximum speed and minimum speed for each aircraft in each link, this is expressed by the following constraint, $\forall \mathrm{i} \in \mathrm{F}, \mathrm{j} \in \mathrm{P}$,

8. Runway Separation and Crossing
a. Runway Separation

Runway operation has to be constrained by wake vortex separations, suppose Node $n_{R}$ is the starting point of a runway. $\forall \mathrm{i}_{1}, \mathrm{i}_{2} \in \mathrm{~F}, \mathrm{j}_{1}, \mathrm{j}_{2} \in \mathrm{P}$
$\mathrm{t}_{\mathrm{i}_{1}}^{\mathrm{j}_{1}+1}+\mathrm{W}_{\mathrm{i}_{1} \mathrm{i}_{2}} \leq \mathrm{t}_{\mathrm{i}_{2}}^{\mathrm{j}_{2}+1}+\left(3-\mathrm{Z}_{\left(\mathrm{i}_{1}, \mathrm{i}_{2}\right)}^{\mathrm{n}_{\mathrm{r}}}-\sum_{\mathrm{m} \in \mathrm{N}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{~m}, \mathrm{n}_{\mathrm{r}}\right)}^{\mathrm{j}}-\sum_{\mathrm{m} \in \mathrm{N}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{~m}, \mathrm{n}_{\mathrm{r}}\right)}^{\mathrm{j}_{2}}\right) * \mathrm{M}$
b. Runway Crossing

Assume there is one runway crossing point, before crossing the runway the node is $\mathrm{n}_{\mathrm{cb}}$
(Figure 6). After crossing the runway, the node is $n_{c a}$, then $\forall i_{1} \in D, i_{2} \in A, j_{1}, j_{2} \in P$

$$
\begin{align*}
& \mathrm{t}_{\mathrm{i}_{1}+1}^{\mathrm{j}_{1}+1}+\mathrm{Y}_{\mathrm{i}_{1} \mathrm{i}_{2}} \leq \mathrm{t}_{\mathrm{i}_{2}}^{\mathrm{j}_{2}+1}+\left(3-\mathrm{C}_{\left(\mathrm{i}_{1} \mathrm{i}_{2}\right)}^{\mathrm{n}_{\mathrm{r}}}-\sum_{\mathrm{m} \in \mathrm{~N}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{~m}, \mathrm{n}_{\mathrm{r}}\right)}^{\mathrm{j}_{1}}-\sum_{\mathrm{m} \in \mathrm{~N}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{~m}, \mathrm{n}_{\mathrm{cb}}\right)}^{\mathrm{j}_{2}}\right) * \mathrm{M}  \tag{21}\\
& \mathrm{t}_{\mathrm{i}_{1}}^{\mathrm{j}_{1}+1} \leq \mathrm{t}_{\mathrm{i}_{2}}^{\mathrm{j}_{2}+1}+\left(2+\mathrm{C}_{\left(\mathrm{i}_{1} i_{2}\right)}^{\mathrm{n}_{\mathrm{r}}}-\sum_{\mathrm{m} \in \mathrm{~N}} \mathrm{R}_{\left(\mathrm{i}_{1}, \mathrm{~m}, \mathrm{n}_{\mathrm{ca}}\right)}^{\mathrm{j}_{1}}-\sum_{\mathrm{m} \in \mathrm{~N}} \mathrm{R}_{\left(\mathrm{i}_{2}, \mathrm{~m}, \mathrm{n}_{\mathrm{r}}\right)}^{\mathrm{j}_{2}}\right) * \mathrm{M} \tag{22}
\end{align*}
$$

## Runway Crossing, Rathinam Et al. (2008)



Figure 6: Runway crossing illustration from Rathinam (2008) ${ }^{27}$

## CHAPTER V

## SOLUTIONS

## A. Programming

In order to test the MIP model, ILOG CPLEX Concert technology is used for programming. It allows to embed CPLEX optimizers in C++, JAVA, .NET applications. In this thesis, Visual C++ 2008 is used for programming. There are some other languages such as AMPL and OPL that can be used for programming in CPLEX. However, they are not as flexible as C++. By using C++, the user can apply heuristic methods or simulations by controlling the input.

In this thesis, the network data is written in a .DAT file and read into the program. The data structure of the network is a two dimensional incidence matrix which illustrates the connection of any two nodes.

## B. George Bush International Airport (IAH)

IAH is one of the busiest airports in the United States. There are more than 1000 arrivals and departures everyday. There are five runways in operation, which are 15L, 15R, 26L, 26R, 8-27 (Table 1). The available data includes the AutoCAD diagram of IAH, the flight information of June 2010, the runway usage information and gate usage information. The author chose June 1, 2010 to test the model. The runway 15L and runway 15R are mostly used for departures. Runway 26L and Runway 26R are mostly used for arrivals. There are five terminals in use. In order to test the model by increasing
the frequency of runway crossing, it is assumed that 26L is used for departure and 26R is used for arrival. As we can see from the Figure 7, 26L and 26R are two parallel runways. The distance between those two runways are 4500 ft , which can make those two runways are two independent runways without mutual influence. The departures that were assigned to 15 L are rescheduled to use 26L as departure runway.

| Name | Length(ft) | Length(meter) |
| :--- | :--- | :--- |
| 15L/33R | 12001 | 3658 |
| 15R/33L | 9999 | 3048 |
| 8R/26L | 9402 | 2866 |
| 8L/26R | 9000 | 2743 |
| $9 / 27$ | 10000 | 3048 |

Table 1: Runways of George Bush International Airport


Figure 7: Diagram of George Bush International Airport ${ }^{4}$
C. Simplification of the Network

The network is very large in IAH. Clearly, simplification of the network to find feasible solutions is necessary. The method is to combine some gates, taxiway intersections and other types of nodes. There are nearly 200 gates for the five terminals located on the north side and south side. It is nearly impossible for the model to include all the gates. The gates are combined into nine new nodes, which are displayed in Table 2.

| Gate Number | Node Name |
| :---: | :---: |
| A1-A15 | NA |
| A16-A30 | SA |
| B76-B88 | NB |
| B60-B75 | SB |
| C1-C28 | NC |
| C29-C45 | SC |
| D1-D12 | D |
| E1-E24 | E |

Table 2: Gate simplification of George Bush International Airport

In order to simplify the network, some of the taxiway intersections are considered as part of the links. The number of nodes is reduced by using the simplified network. The length between two nodes is calculated by the shortest path that connects those two nodes.

## D. Decomposition of Network and Operations

By using a decomposition method, the airport is divided into several small networks according to the runway configurations. For IAH, three sub networks can be generated.


Figure 8: The simplified network used for numerical tests

The first one includes 15L and 15R, which are parallel runways. The second network includes 26R and 26L. The third network includes 8-27. The network of 26R and 26L is used in this thesis, which is shown in Figure 8. The 26R is used for arrival runway and 26L is used for departure runway. One runway crossing point is used in this network. The corresponding real network for the first sub-network is shown in Figure 9.


Figure 9: The real network corresponding to the network in figure $8^{4}$

The number of variables for the proposed MIP model is very large. The CPLEX uses a branch and cut algorithm to solve it. Limited by the memory of the computer, only a medium-sized network with around 20 nodes and 15 aircraft can be solved efficiently. However, the daily departures and arrivals of IAH are usually around 1500
and the number of nodes is very large if we consider all. In order to solve the above problems, the author will present a rolling horizon method.

## E. Rolling Horizon Scheme

After dividing the network into small networks, the number of aircraft that can be scheduled with the MIP model each time is still small. In order to solve a large fleet of aircraft, a rolling horizon method is proposed. The rolling horizon method is proved to be efficient in soling scheduling problems.

The basic idea of rolling horizon method is to divide a long planning period into several small non-overlapping sub-periods and optimize the schedule within each subperiod. Although original desire is to decide schedule independently within each subperiod, the taxiing time of some aircraft could run through two sub-periods. Here we assume that any two consecutive sub-periods can cover the taxiing time of one aircraft. Figure 10 illustrates an example. Aircraft 1 is scheduled within planning period 1. Only part of the movement of aircraft 2 is finished within planning period 1 , which is aircraft 2-1. The left movement aircraft 2-2 need to be scheduled in planning period 2. The movement of aircraft 3 is totally out of planning period 1.

There are two ways to deal with the movement of aircraft 2 . The first way is to re-schedule the movement of aircraft 2 from the beginning, which is not used in this thesis. The second way is to consider the movement of aircraft 2-1 as fixed when planning the second period. Only the movement of aircraft 2-2 needs to be scheduled. This thesis adopts the second way.


Figure 10: Rolling horizon scheme and bound illustration

In this thesis, the length of each planning period can vary in a way that the number of aircraft scheduled in each period is relatively stable. First the author calculates the taxi time for departure aircraft if they take the shortest path at the maximum speed. Then the authors add this taxi time and add it to the earliest pushback time and get a new time for the departure aircraft. The next step is to sort the departure aircraft according to this new time in ascending order.

For the arrival aircraft, the author sorts them by the estimated arrival time in ascending order. The both the departure and arrival aircraft are sorted and have orders. The information available now is the estimated arrival time and earliest pushback time. If $m$ aircraft are scheduled in the first period, then the staring time of the next period is scheduled time of aircraft $\mathrm{m}+1$. Suppose in this m aircraft, there are n departure aircraft
and (m-n) arrival aircraft. The author finds the number ( $n+1$ ) aircraft in the departure order and number ( $m-n+1$ ) aircraft in the arrival order. Then the author compares the EPT and ETA of those two aircraft and finds the smaller one. Then the smaller one is the starting time of the new planning period.

## F. A Bound for the Test

With the rolling horizon method, we can get a feasible solution for the problem. However, such a method cannot guarantee the optimality of the results. It needs to be compares with a bound of optimal planning time to show to what extents the solutions reach optimality. A bound is proposed to test the optimality of the rolling horizon method.

The rolling horizon method is to schedule the aircraft within each planning period. Those aircraft that cannot finish their movement are continued to be scheduled in the next planning period.

The key of achieving a bound of optimal schedule time relies on how to deal with the aircraft whose taxiing time stretches over two consecutive periods. In such case, the previous period is extended long enough to make sure this aircraft can finish its movement. Then this aircraft does not need to be scheduled again in the later period. Take the Figure 10 again as an example. Aircraft 2 is scheduled in both planning period 1 and 2 in the rolling horizon method. In the bound, aircraft 2 is put in period 1 and period 1 is extended to make sure aircraft 2 finish its movement. However, in the extended time period 2 which starts the same as original period 2, we omit the influence of aircraft 2-2 and only optimize aircraft 3 and aircraft 4.

Based on the above analysis, the bound does not consider the interaction between different periods and optimize the schedule of each aircraft within its period. Thus it can be used as a bound to test the optimality of the rolling horizon method.

## G. A Numerical Test for IAH

The wake vortex separations are decided by the weight class of the aircraft. In this problem, four types of aircraft are taken into account. The separation is analyzed in Table 3.

| Aircraft Type | Small | Large | Heavy |
| :--- | :--- | :--- | :--- |
| Small | 59 | 88 | 109 |
| Large | 59 | 61 | 109 |
| Heavy | 59 | 61 | 90 |

Table 3: Wake vortex separations in seconds

In actual operations, the percentage of large aircraft can be $90 \%$. The percentage of small and heavy are usually 5\% each.

In order to test the optimality of the rolling horizon method, the network in Figure 6 is used. The departure schedule is relatively tight, which is very close to the schedule of IAH from 17:00 PM to 18:00 PM. The arrival schedule is not so tight. There are nine planning periods in this test. For each planning period, 6 arrival aircraft and two departure aircraft are scheduled. There are 46 departure aircraft and 18 arrival aircraft scheduled within a period of time of an hour. The number is not the sum of the number in the table below because some of the aircraft are scheduled in two periods.

There are nine planning periods in this test. For each planning period, 6 arrival aircraft and two departure aircraft are scheduled (Table 4). There are 46 departure aircraft and 18 arrival aircraft scheduled within a period of time of an hour. The number is not the sum of the number in the table below because some of the aircraft are scheduled in two periods.

| Planning Period | NO. of Departure Aircraft | NO. of Arrival Aircraft |
| :---: | :---: | :---: |
| 1 | 6 | 2 |
| 2 | 6 | 2 |
| 3 | 6 | 2 |
| 4 | 6 | 2 |
| 5 | 6 | 2 |
| 6 | 6 | 2 |
| 7 | 6 | 2 |
| 8 | 6 | 2 |
| 9 | 6 | 2 |

Table 4: Number of aircraft scheduled within each period

In Table 5, we can see the solutions from both the bound and the rolling horizon method. Both the bound and the rolling horizon method can generate a complete schedule for all the aircraft. In the bound, the number of aircraft scheduled is integer. In order to compare the results, the total taxi time of the rolling horizon method and the bound are displayed. The optimality is obtained from the ratio of the bound and the rolling horizon method.

| NO. of Planning Period | Number of aircraft |  | Computational Time |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Rolling Horizon Method | Bound | Rolling Horizon Method | Bound |
| 1 | 8 | 8 | 26.36 | 26.36 |
| 2 | 8 | 7 | 174.35 | 11.9 |
| 3 | 8 | 7 | 24.93 | 11.06 |
| 4 | 8 | 7 | 422.58 | 34.54 |
| 5 | 8 | 7 | 208.54 | 20.65 |
| 6 | 8 | 7 | 328.15 | 57.95 |
| 7 | 8 | 7 | 80.93 | 22.43 |
| 8 | 8 | 7 | 46.96 | 12.95 |
| 9 | 8 | 7 | 57.83 | 23.53 |
| Total | 72 | 64 | 1370.63 | 221.37 |
|  | Rolling Horizon Method | Bound | Optimality(Bound/Rolling Horizon Method) |  |
| Total Taxi Time(Seconds) | 20463 | 19185 | 93\% |  |

Table 5: Results of rolling horizon scheme and bound

The ratio of the bound to the rolling horizon method is $93 \%$, which indicates the rolling horizon method is very close to the bound. As the proposed bound is infeasible in actual cases, the real optimal solution should be between the rolling horizon method and the bound. The ratio should be higher than $93 \%$.

The computational time is also compared between the rolling horizon method and the bound in Figure 11. The computational time to get the bound is smaller and more stable than the rolling horizon method. There exist differences in the number of aircraft scheduled using the two methods. In the rolling horizon method, some of the aircraft are
scheduled in two consecutive periods. However, every aircraft is scheduled once in the bound.


Figure 11: Computational time comparison

The gap between the rolling horizon method and the bound can vary from different flight schedules. When the schedule is very loose, nearly all the aircraft can be assigned a time to depart or arrival to make sure it used the shortest path and travel at the maximum speed. As a result, the results from the rolling horizon method are very much close to optimum in this case. However, if the flight schedule becomes tight, which means more aircraft need to use the runway within the same time period, the gap between the rolling horizon method and the bound will increase. The schedule used in
this numerical test is very close to the peak hour schedule in George Bush International Airport daily operations.

## H. Sensitivity Analysis

Some of the parameters are very important for the calculation process. Those parameters include the flexibility of estimated pushback time and the flexibility of estimated arrival time. In order to test the influence of parameters on the results, comparisons are made. Four departure aircraft and four arrival aircraft are used in the tests. The information is displayed in Table 6. The first four aircraft are departure aircraft and the left four are arrival aircraft. The unit of the time is seconds.

| No. of Aircraft | Departure Gate or <br> Destination Gate | Weight Class | Scheduled Time |
| :---: | :--- | :--- | :--- |
| 1 | 18 | 1 | 200 |
| 2 | 18 | 1 | 240 |
| 3 | 20 | 2 | 280 |
| 4 | 17 | 1 | 320 |
| 5 | 1 | 1 | 150 |
| 6 | 2 | 1 | 200 |
| 7 | 18 | 1 | 250 |
| 8 | 18 | 1 | 300 |

Table 6: Aircraft information used in sensitivity analysis

The comparison results are shown in Table 7, Figure 12 and Figure 13. For the departure aircraft, the time to start the second planning period is between EPT and (EPT+ Gap1). For the Arrival aircraft the time to start the second planning period is
between (ETA-GAP2, ETA+Gap2). From the following table, we can see that when Gap2 is 0 or 15 seconds, the problem is infeasible. When Gap2 increases to 30 seconds, the problem is feasible.

| Gap1 (seconds)-Gap- <br> Gap2(Seconds) | Departure Total Taxi Time <br> (Seconds) | Arrival Total Taxi Time <br> (seconds) |
| :---: | ---: | ---: |
| $0-0$ | Infeasible | Infeasible |
| $0-15$ | Infeasible | Infeasible |
| $0-30$ | 1474 | 1758 |
| $0-45$ | 1474 | 1758 |
| $30-30$ | 1393 | 1761 |
| $30-60$ | 1333 | 1761 |
| $30-90$ | 1298 | 1777 |
| $30-120$ | 1262 | 1783 |
| $60-30$ | 1393 | 1759 |
| $60-60$ | 1333 | 1758 |
| $60-90$ | 1255 | 1759 |
| $60-120$ | 1211 | 1768 |

Table 7: Solutions with the change of gap1 and gap2


Figure 12: The change of total taxi time with the change of gap2 (gap1=30 seconds)


Figure 13: The change of total taxi time with the change of
gap2 (gap1=60 seconds)

Further comparison can conclude that the increase of Gap2 can reduce the total taxi time for departure aircraft and Gap1 can make a problem from infeasible to feasible. Based on the above analysis, the parameters used in modeling can decide the final solutions. By increasing Gap1, we make the arrival time of arrival aircraft more flexible and can decide the optimal sequence of landing on the runway. Nevertheless, Gap1 has to be reasonable, which is in a range. This range is not given in this thesis. It is recommended between 30 seconds to 60 seconds. Gap2 is the time allowed for the departure aircraft to hold on the gate. The long Gap2 is, the more possible that the departure aircraft can find the optimal sequence of taking off. However, Gap2 cannot be arbitrarily long. It is recommended that this length should be less than 150 seconds. This is because the other aircraft needs to use the gate. In sum, Gap1 and Gap2 are both very important in searching the solutions.

## I. Discussions

## 1. Limitations from Isolation of Gates and Runways

In this thesis, the gates are assumed to be available all the time. However, in actual situations, sometimes the gates are occupied and thus delays can be incurred to the aircraft. As a result, integrating the gate assignment problem with the model in this thesis will lead to results closer to actual situations. Another issue would be the decomposition of the network. In this thesis, only the selected network is used and the interaction between this network and the other two sub-networks are not taken into
account. If we consider the influence, the taxi time will be increased. Thus this thesis has limitations by neglecting the interactions between different sub-networks.
2. Fairness and Equity to Airlines When This Model Is Applied

The policy used in this thesis may violate the use of facilities for different airlines. In IAH, continental airlines may take $30 \%$ of all operations. This policy can be used for continental airlines to schedule its own aircraft. In addition, with the increase of the congestion, this policy will be feasible in the future. If this policy is used by all the airlines, game theory can be used for them to make their own schedules.

In order to implement this in actual airport operations, a user interface need to be created in software. Then as long as the flight information is available, the optimal schedule can be obtained and output to help the controllers in decision making.

## 3. Comparison with a First Come First Serve Policy

In order to assess the improvement of the optimization model using the rolling horizon method, a First Come First Serve policy (FCFS) is used for comparison. In this FCFS policy, there is no gate holding or arrival time control. All the aircraft depart or arrive as scheduled. In addition, the aircraft uses the runway according to their sequence of reaching the runway.

The same data are used as previous example. The results are obtained and the total taxi time is $6 \%$ more than the rolling horizon method. As a result, the rolling horizon method has improvement on the FCFS policy.

## CHAPTER VI

## SUMMARY AND CONCLUSIONS

The results of the rolling horizon method show $93 \%$ optimality when compared with the proposed bound. In addition, a First Come First Serve (FCFS) policy is also used to test the results of the rolling horizon method. The rolling horizon method offers a $6 \%$ improvement compared with the FCFS policy.

In this thesis, the departure aircraft are allowed to be held at the gate for a maximum time of Gap1, and the arrival aircraft are allowed to arrive within a time window between (ETA-Gap2,ETA+Gap2). When Gap1 and Gap2 are 0 (meaning on time for both arrival and departure), the problem studied is infeasible because the runway separations cannot be satisfied. With the increase of Gap1 and Gap2, the problem becomes feasible and the taxi time shows a decreasing trend. However, Gap1 and Gap2 should still be within a reasonable range.

There is still work to be done on this topic in the future. It would be interesting to consider the stochastic processes and to integrate gate assignment problems with the proposed MIP model in future research. In addition, future research might be able to uncover alternative ways to develop efficient algorithms to solve this large-scale problem.

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## APPENDIX

Note: The units of all the following tables are seconds including Gap1 and Gap2.

Gap1=0,Gap2=30

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 200 | 1 | 525 | 325 |
| 2 | 240 | 2 | 586 | 346 |
| 3 | 280 | 3 | 756 | 476 |
| 4 | 320 | 4 | 647 | 327 |
| 5 | 139 | 5 | 516 | 377 |
| 6 | 200 | 6 | 685 | 485 |
| 7 | 261 | 7 | 709 | 448 |
| 8 | 322 | 8 | 770 | 448 |

Gap1=0 Gap 2=45

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 200 | 1 | 525 | 325 |
| 2 | 240 | 2 | 586 | 346 |
| 3 | 280 | 3 | 756 | 476 |
| 4 | 320 | 4 | 647 | 327 |
| 5 | 105 | 5 | 482 | 377 |
| 6 | 200 | 6 | 685 | 485 |
| 7 | 261 | 7 | 709 | 448 |
| 8 | 322 | 8 | 770 | 448 |
|  |  |  |  | Total:3232 |

Gap1=30 Gap2=30

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 200 | 1 | 525 | 325 |
| 2 | 261 | 2 | 586 | 325 |
| 3 | 310 | 3 | 756 | 446 |
| 4 | 350 | 4 | 647 | 297 |
| 5 | 146 | 5 | 523 | 377 |
| 6 | 207 | 6 | 693 | 486 |
| 7 | 268 | 7 | 717 | 449 |
| 8 | 329 | 8 | 778 | 449 |
|  |  |  |  | Total: 3154 |

Gap1=30 Gap2=60

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | ---: | :--- | :--- |
| 1 | 200 | 1 | 525 | 325 |
| 2 | 261 | 2 | 586 | 325 |
| 3 | 340 | 3 | 756 | 416 |
| 4 | 380 | 4 | 647 | 267 |
| 5 | 146 | 5 | 523 | 377 |
| 6 | 207 | 6 | 693 | 486 |
| 7 | 268 | 7 | 717 | 449 |
| 8 | 329 | 8 | 778 | 449 |
|  |  |  |  | Total:3094 |

Gap1=30 Gap2=90

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 200 | 1 | 526 | 326 |
| 2 | 261 | 2 | 587 | 326 |
| 3 | 359 | 3 | 757 | 398 |
| 4 | 400 | 4 | 648 | 248 |
| 5 | 120 | 5 | 497 | 377 |
| 6 | 208 | 6 | 712 | 504 |
| 7 | 269 | 7 | 717 | 448 |
| 8 | 330 | 8 | 778 | 448 |

Gap1=30 Gap2=120

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 283 | 1 | 609 | 326 |
| 2 | 293 | 2 | 670 | 377 |
| 3 | 280 | 3 | 548 | 268 |
| 4 | 440 | 4 | 731 | 291 |
| 5 | 130 | 5 | 507 | 377 |
| 6 | 208 | 6 | 703 | 495 |
| 7 | 269 | 7 | 717 | 448 |
| 8 | 330 | 8 | 793 | 463 |

Gap1=60 Gap2=120

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 283 | 1 | 609 | 326 |
| 2 | 344 | 2 | 670 | 326 |
| 3 | 280 | 3 | 548 | 268 |
| 4 | 440 | 4 | 731 | 291 |
| 5 | 130 | 5 | 507 | 377 |
| 6 | 260 | 6 | 754 | 494 |
| 7 | 199 | 7 | 647 | 448 |
| 8 | 352 | 8 | 801 | 449 |

Gap1=60 Gap2=90

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 283 | 1 | 609 | 326 |
| 2 | 330 | 2 | 670 | 340 |
| 3 | 280 | 3 | 548 | 268 |
| 4 | 410 | 4 | 731 | 321 |
| 5 | 138 | 5 | 515 | 377 |
| 6 | 260 | 6 | 745 | 485 |
| 7 | 199 | 7 | 647 | 448 |
| 8 | 352 | 8 | 801 | 449 |

Gap1=60 Gap2=60

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 200 | 1 | 525 | 325 |
| 2 | 261 | 2 | 586 | 325 |
| 3 | 340 | 3 | 756 | 416 |
| 4 | 380 | 4 | 647 | 267 |
| 5 | 129 | 5 | 506 | 377 |
| 6 | 260 | 6 | 745 | 485 |
| 7 | 190 | 7 | 638 | 448 |
| 8 | 322 | 8 | 770 | 448 |
|  |  |  |  | Total:3091 |

Gap1=60 Gap2=30

| Number | Origin Time | Number | Destination Time | Taxi Time |
| ---: | ---: | :--- | :--- | :--- |
| 1 | 200 | 1 | 525 | 325 |
| 2 | 261 | 2 | 586 | 325 |
| 3 | 310 | 3 | 756 | 446 |
| 4 | 350 | 4 | 647 | 297 |
| 5 | 138 | 5 | 515 | 377 |
| 6 | 260 | 6 | 745 | 485 |
| 7 | 199 | 7 | 647 | 448 |
| 8 | 329 | 8 | 778 | 449 |

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