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A NOVEL RECONFIGURATION SCHEME IN QUANTUM-DOT CELLULAR AUTOMATA FOR ENERGY EFFICIENT NANOCOMPUTING

A Thesis Presented

by

MADHUSUDAN CHILAKAM

Submitted to the Graduate School of the University of Massachusetts Amherst in partial fulfillment of the requirements for the degree of

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A NOVEL RECONFIGURATION SCHEME IN QUANTUM-DOT CELLULAR AUTOMATA FOR ENERGY EFFICIENT NANOCOMPUTING

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ABSTRACT

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Quantum-Dot Cellular Automata (QCA) is currently being investigated as an alternative to CMOS technology. There has been extensive study on a wide range of circuits from simple logical circuits such as adders to complex circuits such as 4-bit processors. At the same time, little if any work has been done in considering the possibility of reconfiguration to reduce power in QCA devices. This work presents one of the first such efforts when considering reconfigurable QCA architectures which are expected to be both robust and power efficient. We present a new reconfiguration scheme which is highly robust and is expected to dissipate less power with respect to conventional designs. An adder design based on the reconfiguration scheme will be presented in this thesis, with a detailed power analysis and comparison with existing designs. In order to overcome the problems of routing which comes with reconfigurability, a new wire crossing mechanism is also presented as part of this thesis.

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CHAPTER 1

INTRODUCTION AND MOTIVATION

It was as early as 1965 when Gordon Moore predicted that the number of transistors that can be integrated on to a single chip will double every 18 months [46]. This law put forth by Moore has been a benchmark for semiconductor scaling for more than four decades. The IC industry which has been primarily driven by CMOS technology scaling is now forced to look into other alternatives as the scaling is fast approaching its fundamental limits. The International Technology Roadmap for Semiconductors (ITRS) has predicted that size limit of CMOS technology will be limited to about 5 nm to 10 nm and believes this limit will be reached as early as 2017 [3]. Shrinking transistors have been helpful in achieving high speed and low power circuits. As the devices are exponentially scaled down various factors including power dissipation, gate leakage current, interconnection noise (introduction of crosstalk and hot electron effect) and stray capacitances have become potential bottlenecks that has led to the degradation of circuit performance.

In the last few years as the technology has scaled down to sub 45nm, power dissipation has been a major area of concern for researchers around the world. Fred Pollack of Intel Corporation was one of the first to note the alarming rate at which power density is increasing with the shrinking geometry [52]. Thus power management is a critical issue which needs to addressed at the earliest.

Nanotechnology is touted to be the solution to the problem of device shrinking where the performance is degraded due to increasing quantum effects and to overcome the existing power dissipation. There are many possible candidates which are being considered as a possible replacement to CMOS such as Quantum Dot Cellular Automata [40], Silicon Nano-wires [18], Carbon Nanotubes based Transistors [5, 61], Spin Wave Transistors [71, 54], Superconducting Electronics [66], Resonant Tunneling devices [45, 47] among others. Fig. 1.1 portrays a critical review of some these emerging devices at the nanoscale level.

Quantum - Dot Cellular Automata (QCA) is one such nano computing paradigm that exploits some of the unavoidable nanoscale issues such as quantum effects and device integration for performing useful computation. Some of the potential advantages of QCA include the lack of interconnects, high clock frequency, and since QCA doesn't involve transfer of electrons or flow of current, it has the potential to perform low power computing. One of the most striking features of this emerging technology is that it has the ability to dynamically reconfigure or redesign the functionality of the system which makes the system more powerful and more efficient in terms of computational speed and power dissipation.

Field Programmable Gate Arrays (FPGAs) have always been an attractive low cost option for designers since it offers flexibility in terms of hardware. Researchers have been very successful in establishing that FPGAs have outperformed the implementation of a number of real time applications in terms of computational performance and cost [68]. Reconfigurability represents an attractive application of QCA technology [21, 50]. With the help of reconfiguration, the inbuilt low power nature of QCA can be exploited to design various low power circuits which are not only efficient in terms of power but are also efficient in terms of area and computation.

In this thesis, we explore one possible approach to realize reconfigurability in QCA that is based on the change in polarization of electrons in a QCA cell. This novel reconfiguration scheme which is based on majority gate voter is best suited for complex circuit design which have high fan-outs and require the intermediate computation results. We introduce a new custom wire crossing technique which is

Table 1. Emerging Research Logic Devices—Projected Parameters.										
Availability Sequen	Availability Sequence 1 2 2–3 2–3 4 5 6									
Device		\bigcirc		J-L-× ×						
	FET ^[2]	RSFQ ^[3-5]	1-D structures	Resonant Tunneling Devices	SET ^[30,31]	Molecular	QCA ^[39,40,49]	Spin transistor		
Types	• Si CMOS	لل ∙	CNT FET NW FET NW hetero- structures Crossbar nanostructure	• RTD-FET • RTT	• SET	 2-terminal 3-terminal FET 3-terminal bipolar transistor NEMS Molecular QCA 	• E: QCA • M: QCA	 Spin FET (SFET) Spin-valve transistor (SVT) 		
Supported Architectures	 Conventional 	 Pulse 	 Conventional Cross-bar 	ConventionalCNN	• CNN	 Memory-based QCA 	• QCA	 Quantum Programmable logic 		
Cell Size (spatial pitch)	150 nm*	0.3 μ m	150 nm*	150 nm*	40 nm	Not known	60 nm	150 nm*		
Density (device/cm ²)	4.5E9	1E6	4.5E9	4.5E9	6E10	1E12	3E10	4.5E9		
Switch Speed	9 THz	1.2 THz	Not known	1 THz [26]	1 GHz	Not known	30 MHz	700 GHz		
Circuit Speed	53 GHz	250–800 GHz	53 GHz	53 GHz	1 GHz	<1 MHz (NEMS)	1 MHz	53 GHz		
Switching Energy, J***	3 × 10 ⁻¹⁸	2 × 10 ⁻¹⁹ [Nb] {> 1.4 × 10 ⁻¹⁷ }	3×10 ⁻¹⁸ **	> 3 × 10 ⁻¹⁸	1 × 10 ⁻¹⁸	Not known {> 1.5 × 10 ⁻¹⁷ }**	E :> 1× 10 ^{_18[48]} M: 10 ^{-17 [49]}	3 × 10 ⁻¹⁸		
Binary Throughput, GBit/ns/cm ²	238	0.4	238*	238*	10	N/A	0.06	238*		
Gain			Must be $\gg 1$ for	all devices. See Ta	ble 2 for experin	nental values				
Operational Temperature	RT	 4 K (Nb) 77 K (HTS) 20 K(MgB₂) 	RT	RT	20 K	RT	E: QCA Cryogenic M: QCA RT	 Cryogenic (SFET) RT (SVT) 		
CD Tolerance	Critical	Not critical	Not critical	Very critical	Very critical	Not critical	Very critical < 2% (M: QCA	Critical)		
Materials System	Si	Nb HTS	CNT Si III–V	III–V Si-Ge	III–V Si	Organic molecules	AI/AI2O3 (E: QCA)	 III-V (SFET) Si/FM (SVT) 		
Advantages		 Very high circuit speed 		Density (smaller cell size)		 Identity of individual switches on sub-nm level Potential solution to interconnect problem 	 Morphologica simplicity 	al		
Challenges		Cryogenic operations		 Stand-by power Process integration 	 Cryogenic operations 			 Low spin injection efficiency Short coherence time 		

Figure 1.1: Performance Evaluation for Emerging Logic Device Technologies. (From. ref. [2])

used to overcome the traditional problems of routing in any reconfiguration based design. The design exploits the inherent pipeline nature of QCA which can lead to an enormous reduction in area since the entire computation can be computed in a single block. One can design highly energy efficient circuits as QCA doesn't involve the physical movement of any charge particles. This design will be highly energy efficient along with added advantages of pipelining and area.

The major contributions of this thesis are

- We present our vision for low power computation based on QCA.
- We introduce the concept of reconfigurability in QCA for constructing energy efficient logic devices.
- Design of simple arithmetic circuits like full adders using the proposed reconfiguration scheme.
- We address the problems in routing with a custom wire crossing technique.
- We evaluate benefits of our designs vs. existing designs.

The rest of the thesis is organized as follows: the background on QCA and reconfigurability in nano computing and in particular in QCA are presented in Chapter 2. The concept of proposed reconfiguration scheme along with the wire crossing techniques are discussed in Chapter 3. Analysis of the proposed designs and the power dissipation models considered in the design analysis is presented in Chapter 4. Chapter 5 outlines the existing adder design and introduces the proposed adder design based on the reconfiguration scheme. A detail analysis of the proposed adder design along with the comparative analysis is presented in Chapter 6 and the thesis is concluded in Chapter 7.

CHAPTER 2 TECHNICAL BACKGROUND

Quantum Cellular Automata are models used in quantum computation which are analogous to conventional models of cellular automata suggested by Von Neumann [48]. The first step towards quantizing the existing models of cellular automata was suggested by Richard Feynman [31, 30]. The word Quantum Cellular Automata was defined by Gerhard Grossing and Anton Zeilinger to a model [34] which they developed in the year 1988. However, the model proposed by them had little or no relation to the concepts developed in quantum computation by David Deutsch, hence their model has not been developed as a model of computation [22]. John Watrous was the first to do an in-depth research on the models based on Quantum Cellular Automata [70]. Craig Lent and Doug Tougaw proposed implementation of systems based on the classical cellular automata designed using quantum dots [40] as a replacement for the classical computation using CMOS. In order to differentiate this proposal and the models of cellular automata which are used for performing quantum computation, many authors refers to this subject as Quantum-dot Cellular Automata (QCA).

2.1 QCA Basics

Quantum-dot Cellular Automata (QCA) is a new nano computing paradigm which encodes binary information by charge configuration within a cell instead of the conventional current switches. There is no current flow within the cells since the coulombic interaction between the electrons is sufficient for computation. This paradigm provides one of many possible solutions for transistor-less computation at the nanoscale.

The standard QCA cells have four quantum dots and two electrons [64]. There are various kinds of QCA cells proposed which include a six-dot QCA cell and an eight-dot QCA cell. In a QCA Cell, two electrons occupy diagonally opposite dots in the cell due to mutual repulsion of like charges. An example of a simple unpolarized QCA cell consisting of four quantum dots arranged in a square is as shown in Fig. 2.1. Dots are simply places where a charge can be localized. There are two extra electrons in the cell those are free to move between the four dots. Tunneling in or out of a cell is suppressed.



Figure 2.1: Simple 4-dot Unpolarized QCA cell. (From. ref. [59])

The numbering of the dots in the cell goes clockwise starting from the dot on the top right. A polarization P in a cell, that measures the extent to which the electronic charge is distributed among the four dots, is therefore defined as:

$$P = \frac{(\rho_1 + \rho_3) - (\rho_2 + \rho_4)}{\rho_1 + \rho_2 + \rho_3 + \rho_4}$$
(2.1)

Where ρ_i is the electronic charge in each dot of a four dot QCA cell. Once polarized, a QCA cell can be in any one of the two possible states depending on the polarization of charges in the cell. Because of coulombic repulsion, the two most likely polarization states of QCA can be denoted as P = +1 and P = -1 as shown in Fig. 2.2. The two states depicted here are called most likely and not the only two polarization states because of the small (almost negligible) likelihood of existence of an erroneous state.



Figure 2.2: Polarization States of a 4-dot QCA cell.(From. ref. [59])

In QCA architecture information is transferred between neighboring cells by mutual interaction from cell to cell. Hence, if we change the polarization of the driver cell (left most cell also know as input cell), first it's nearest neighbor changes it's polarization, then the next neighbor and so on. Fig. 2.3 depicts the transfer of polarization between neighboring QCA cells. When the driver cell (input) is P = -1 (or P = +1), a linear transfer of information amongst it's neighboring cells leads to all of them being polarized to P = -1 (or P = +1).



Figure 2.3: Transfer of Polarization between adjacent QCA cells when the polarization changes from P = +1 to P = -1. (From. ref. [59])

As we can see, a change in polarization of the driver cell prompts all the neighboring cells to change polarization in order to attain the most stable configuration. The example illustrated in Fig. 2.3 shows how information can be transferred in a linear fashion over a line of QCA cells. Such a line of cells is used as interconnects between various QCA logic components that we will see in the following section. The speed of change in polarization of a QCA cell depends on a number of factors such as temperature, kink energy which represents the energy required to place adjacent cells in opposite polarization, clock energy which takes into account the energy from the clock to the signal and vice versa, and the quantum relaxation time which refers the minimum time required for the electrons to overlook their particular spin direction in which they are oriented.

2.2 Logical Devices in QCA

As seen in the previous sections, the information in QCA cells is transferred due to coulombic interactions between the neighboring QCA cells, the state of one cell influences the state of the other. The basic logic devices in QCA are:

- Binary Wires.
- Inverter.
- Majority Gate Voter

2.2.1 Binary Wire

A binary wire can be viewed as a horizontal series of cells to transmit information from one cell to another. An example of a QCA wire is as shown in Fig. 2.4. A binary wire is typically divided into various clock zones, to ensure that the signal doesn't deteriorate as signals generally tend to degrade with a long chain of cells in the same clocking zone.



Figure 2.4: A binary wire which propagates information through the line. (From. ref. [44])

2.2.2 Inverter

Two diagonally aligned cells will have the opposite polarization. Henceforth, inverters can be implemented with lines of diagonally aligned cells. An example of a QCA Inverter is as shown in Fig. 2.5.



Figure 2.5: An inverter which uses the interaction of diagonally aligned cells to invert bits. (From. ref. [44])

From Fig. 2.5, we can clearly observe that the signal from a binary wire splits into two parallel wires. The corner cells of the parallel wires are responsible for the change in polarization of the cells diagonal to them in the opposite direction. This causes the signal to be inverted. This anti-aligning behavior of standard cells in diagonal orientation can be useful in the implementation of the large circuits where crossover of wires is unavoidable. One can produce an inverted signal by placing a standard cell and aligning it halfway between an even and odd numbered rotated cell while placing it halfway between an odd and even numbered cell will lead to a buffered signal.

2.2.3 Majority Gate Voter

Majority Gate Voter (MV) is the fundamental logic block in any QCA design. A majority gate can be built with the help of five cells. The top, left and bottom cells are inputs. The device cell in the center interacts with the three inputs and its result (the majority of the input bits) will be propagated to the cell on the right. An example of an MV representation in QCA is as shown in Fig. 2.6. The logic function implemented by the MV is

$$f(A, B, C) = A.B + B.C + C.A$$
(2.2)



Figure 2.6: A three input majority gate. The output is the majority vote of the three inputs. (From. ref. [44])

A majority gate is the basic logic gate in QCA, as it can function as an OR gate with one of the inputs fixed to 1 and function as an AND gate with one of the inputs fixed to 0.

$$MV(A, B, C) = A.B \quad when \quad C = 0 \tag{2.3}$$

$$MV(A, B, C) = A + B \quad when \quad C = 1 \tag{2.4}$$

More complex circuits like full adders and memories can be constructed hierarchically in QCA with appropriate layout.

2.3 Clocking in QCA

Unlike FET based circuits, QCA circuits have no predefined flow of direction of current or electrons, in a QCA circuit information tends to flow in all directions. Thus, clocking plays a very important role in the synchronization and flow of information in a particular direction. QCA circuits typically use a clock that comprises of four clocking phases. It can essentially be viewed as a pump that's constantly pumping out data sequentially. As a result of this, QCA circuits are inherently pipelined. A QCA clock induces four phases in the tunneling barriers of the cells above it. In the first phase, the *switch phase*, the tunneling barriers start to rise. The second phase, the *hold phase* is reached when the tunneling barriers are high enough to prevent electrons from tunneling. The third phase, *release phase* occurs when the high barrier starts to lower, and finally, in the fourth phase, the *relax phase*, the tunneling barriers allow electrons to freely tunnel again. In simple words, when the clock signal is high, electrons are free to tunnel. When the clock signal is low, the cell becomes latched.



Figure 2.7: The QCA clock, it's stages and it's effects on a cell's energy barriers (From. ref. [1])

Fig. 2.7 shows a clock signal with it's four phases and the effects on a cell at each clock phase. A typical QCA design requires four clock phases, each of which is cyclically 90 degrees out of phase with the prior clock phase. The first pair of cells will stay latched until the second pair of cells gets latched and so forth. In this way, data flow direction is controllable through clock phases.

In order, to understand the actual working of a QCA clock, consider the example as shown in Fig. 2.8 where a value is being transmitted across a QCA wire. Initially, let's assume that a frozen input cell polarization with P = -1 is to be propagated through the length of the wire. Such a propagation would take place as follows where in the cells to the left of the input cell (clocking zone 1) would be in the switch phase in the first time step. As seen earlier, in this phase, the tunneling barrier will be raised and the cells will be polarized in accordance to the driver cell, here in this case, it's the input cell with polarization P=-1.



The clock phases to the left of the dark line show the propagation of a binary 0 (polarization P = -1) (assumed to come from an input cell with frozen polarization).

Figure 2.8: Example of QCA Clock Transitions (From. ref. [49])

As we step in to the second time step, we can see a changeover of the phases in the clocking zones. Clocking zone 1 will have a phase change from the switch phase to the hold phase while clocking zone 2 would change over to the switch phase. Once in the hold phase, the tunneling barriers are held high and thus clocking zone 1 will serve as input to clocking zone 2, as a result of which the cells in clocking zone 2 will be polarized in accordance to the cells in clocking zone 1.

In the third time step, the passage of the phases continues and now the clocking zone 1 will be in release phase, clocking zone 2 in hold phase and finally clocking zone 3 in the switch phase. Once in the release phase, the tunneling barrier is lowered for cells in clocking zone 1 and will be in a neutral state while clocking zones 2 and 3 interact with each other in the same manner as clocking zone 1 and 2 in the previous time step.

And in the fourth time step, clocking zone 1 would have witnessed a transition from the release phase to the relax phase, clocking zone 2 to the release phase and clocking zone 3 to the hold phase. The switch phase doesn't follow the release phase instead follows the relax phase due to the fact that the switch phase could affect the cell polarizations of the release phase. Finally, in the fifth time step, the clocking zone 1 returns to the switch phase and re-polarizes such that a new transmission could occur across the wire. At this point we can safely conclude that there is some inherent pipelining built into the QCA technology. After every 4 time steps, it is possible to put a new value onto a QCA wire [49].

There are basically two different clocking mechanism in QCA namely Landauer and Bennett clocking [7, 38, 43, 8]. Fig. 2.9 illustrates Landauer and Bennett clocking of QCA circuits. The figure shows a QCA shift register, implemented by a single line of cells, and a three-input majority gate. The left column $(L1)\rightarrow(L5)$ represents snapshots of the circuit at different times as it is clocked using the Landauer clocking scheme and the right column $(B1)\rightarrow(B7)$ shows snapshots using the Bennett clocking scheme. It is assumed that the input signals come from other QCA circuitry to the left of the circuit shown and that the output signals are transported to the right to other QCA circuits [41].

Bit erasure is the simplest logically irreversible process. It is logically irreversible, in that it requires a one bit input and always returns the null state as the output, so it is impossible to recover the input value from just the output value. It has been experimentally proved that during bit erasure one needs to dissipate some amount of energy to the surroundings, and in case of an irreversible bit erasure where information is lost, then the amount of energy dissipated to the environment is always considerably larger than $k_B Tln(2)$ [63].



Figure 2.9: Landauer and Bennett clocking of QCA circuits. Each figure represents a snapshot in time as the clocking fields move information across the circuit. The left column $(L1)\rightarrow(L5)$ represents Landauer clocking. We can clearly observe the flow of information across the circuit as the clocking field causes different cells to switch from null to active. The circuit shown includes a shift register on top and a three-input majority gate on the bottom. The right column $(B1)\rightarrow(B7)$ represents Bennett clocking for a computational block. Here as the computational edge moves across the circuit intermediate results are held in place. When the computation is complete (B4), there is back tracking of information, undoing the effect of the computation. This approach yield the minimum energy dissipation. (From. ref. [41])

2.3.1 Landauer Clocking

All the cells are initially in the null state(L1). As the clocking signal is activated, information is propagated from left to right(L2). The clocking can be assumed to have a header and a trailer. As information is passed on from left to right, the header copies one bit to the other while the trailer erases the bit to null. Because this erasure is being done in the presence of a copy of the information (i.e., no information is being lost), it can be accomplished without dissipating $k_BTln(2)$. This forms the basis for reversible computation proposed by Landauer [38].

2.3.2 Bennett Clocking

Bennett-clocked operation is shown in Fig. 2.9 (B1) \rightarrow (B7). The difference in the Bennett clocking is the absence of trailer i.e. cells remain held in their respective active states as the information is being passed from left to right. If we consider the example of majority voter, the cells of the loser in majority voter i.e. green signal remains in the active state until the final output is computed. At that time, the output states can be copied to the next stage of computation and the clock begins to lower cells back to the null state from right to left (B4) \rightarrow (B7). In this part of the cycle, erasure of intermediate results does occur but always in the presence of a copy. Thus no minimum amount of energy ($k_BTln(2)$) needs to be dissipated. At the end of the back-cycle the inputs to the computation must either be erased or copied. If they are erased, then an energy of at least $k_BTln(2)$ must be dissipated as heat for each input bit. This is unavoidable but the energetic cost of erasing each of the intermediate results have been avoided [7, 43, 8].

The Bennett clocking scheme has its own benefits and costs which are part of the design of the circuit. The principal benefit is lower power dissipation but the costs include increasing the latency to allow the forward and reverse cycles $(B1)\rightarrow(B4)$ and $(B4)\rightarrow(B7)$. In addition, the amount of pipelining is reduced because for a given

block of computation only one computational edge at a time can be moving across the circuit. In Landauer clocking, by contrast, several computational waves can be traversing the same block at the same time. Finally, the circuitry that provides the clocking signal has to be somewhat more complex to handle the forward and the backward clocking of the cells [41].

2.4 Wire Crossings in QCA

Wire-crossing in traditional QCA cells is done by using a "plus-sign" pattern, as shown in Fig. 2.10. The distances between a plus-sign pattern and a square pattern are exactly the same, allowing for the same Coulombic interactions between electrons in a cell. Thus, when a wire of square cells crosses a wire of plus-sign cells, they do not interact, thus the signals on each wire are preserved.



Figure 2.10: Basic Wire Crossing Technique used in traditional QCA cells (From. ref [64])

The traditional QCA circuits have relied on the coplanar wire crossing model presented but Lent et al [64], where they have proposed a unique combination of 45 degree and 90 degree crossovers. This works fine but fabrication of cells with two different orientations poses fabrication problems and also there are lots of chances of interference or cross talk owing to the weak coupling in the cells. This weak coupling makes the cells sensitive to various physical and environmental parameters such as temperature , etc. Ottavi et al [9] proposed a novel architectural design to overcome the problems posed by this weak coupling such as temperature by coming out with a more thermally robust design. They came up with three designs based on orientation of the cells, the majority gate voting and the interaction between the cells. Even though these proposed techniques was able to solve some of the design issues, there were overheads in area associated with it which is generally not preferred in a computing paradigm tipped to replace CMOS which had an efficient usage of area. Rajeswari et al tried to minimize the area overhead introduced by those complex design flows and presented the first clocking based wire crossings [23] based only on one type of cell. Even though they were successful in implementing their proposed methodology, there were few constraints with regard to the timing, where they proposed a custom eight zones based clocking scheme which reduces the computational speed.

2.5 Reconfiguration in QCA

Reconfigurable computing is a computer paradigm which bridges general purpose microprocessors and application specific integrated circuits with mix of both hardware and software. It uses runtime reconfiguration to perform the intended function. This allows us to configure a hardware system to implement a particular circuit. The underlying hardware functions like an application specific hardware, thereby providing the computational performance of custom hardware. However, since the reconfiguration happens at runtime, reconfigurable computing provides the capability to re-program the underlying hardware to implement different circuits and hence approaches the flexibility of a general purpose microprocessor. This property has been exploited to rake in better performance and lower power dissipation. Before we get into the details of reconfiguration in QCA, we look into reconfiguration in nano computing and how it could help us build energy efficient systems.

2.5.1 Reconfigurability and Nanocomputing

As physical limitations of feature size reduction and heat dissipation in CMOS are reached, nanotechnology provides an approach to overcome these limitation. It has been further envisioned that the amount of power dissipated in a given computational cycle will reduce drastically as we operate our circuits in the nano region. We can exploit this feature of nano-computing to come up with better energy efficient systems. Because nanotechnology could lead to inexpensive production of highly reconfigurable computer hardware, it is natural to exploit the current framework to this emerging technology. Research strongly suggests that reconfigurable architectures, if efficient, will provide a better fit and thus improved performance for general purpose computation [16, 6]. We discuss the various post CMOS and nano - computing paradigms which have been successfully implemented as a programmable device and the outcome of such an implementation in Appendix A.

2.5.2 Application of Reconfiguration in QCA's

From the previous subsection, we have seen that use of programmable logic and reconfigurability in the nano scale level has led to the design of various low power and energy efficient systems which is the need of the hour. Before we get into the application of reconfiguration in QCA, we need to understand what an FPGA is and why it is easier to implement an FPGA in QCA when compared to other nanoscale devices.

FPGA's can be in general classified as a system consisting of a logic functions which are arranged in a well defined interconnect network. A typical CMOS interconnection scheme involves signals entering an FPGA circuit via some input buffers which are then transferred to horizontal wires. These horizontal wires cross with vertical wires throughout the FPGA and programmable connections can usually be made at crossings to facilitate data routing. While long wires work well in CMOS, the nature of the clock makes them a much more difficult task in QCA. Unlike the standard CMOS clock, the QCA clock is not a signal with a high or low phase which has been discussed in detail in the earlier sections.

Niemer et al [50] have presented a FPGA based on QCA's where in they have tried to adhere to the design of a CMOS FPGA and tried implementing it in QCA's. They have built a logic block based on the NAND gate based Majority voter design and used a programmable multiplexor design for the interconnects instead of the common SRAM based design in CMOS. The basic problem which has been addressed is the complex routing of the clock signals involved in QCA. But this work just presents a simple implementation of QCA FPGA on the lines of CMOS FPGA. Jazbec et al [37] improved the routing and the interconnect network by proposing a Programmable Switching Matrix based on the crossing of the QCA Binary wires. There has been considerable work also done on implementing various QCA based configurable logic blocks which are based on SRAM design, Multiplexor design etc [4, 39]. They have made use of the clocking phase to their advantage to enable the crossing of two QCA binary wires. Recently Devadoss et al [24] have come up with a programmable tile based architecture based on the clocking mechanism. Simple tiles were proposed as the building blocks of this programmable QCA (p-QCA) architecture where in they have retained the 3-input majority gate as the primary logic element unlike existing architectures, which typically use 2-input NAND gates. Any part of the proposed p-QCA device can be programmed to function as a logic element, a routing element, or a memory element. A simple p - QCA device structure is as shown in Fig. 2.11

Thus, we have seen a detailed survey about the different aspects of reconfiguration which has been extended to nano computing in this section and in particular to QCA's which is the filed of interest with regard to this thesis. While we find that most of the work has been targeted on implementing a programmable design, but very few researchers have stressed upon the aspect of saving power through reconfiguration



Figure 2.11: p-QCA device structure: QCA layer with clocking circuitry. (From. ref. [24])

which forms the basis for our research. Some of the work that focussed on saving power [12, 10] and increasing computational speed through reconfiguration in nano - computing has clearly shown that power dissipation can be considerably reduced and there are many avenues available for power saving in reconfiguration. Previous work in QCA have focussed on implementing a FPGA design for QCA but significant amount of work has not been done on exploiting the clocking mechanism or the cell configuration for reconfiguration which could open up several avenues for power savings in QCA.

CHAPTER 3

PROPOSED RECONFIGURATION SCHEME

As seen towards the end of the previous chapter, the number of researchers who have stressed upon the usage of reconfiguration for energy efficient computing are very few but the research that has been carried out by them have given a clear indication that there are avenues for power savings. Our aim is to come upon with a reconfigurable architecture using QCA which makes use of the majority gate logic and the clocking mechanism which reduces the power considerably in comparison to the existing architectures. The major driving force behind the idea of coming up with a reconfigurable architecture in QCA is that it's a relatively unexplored topic, there are strong possibilities that a reconfigurable QCA can lead to even lesser power dissipation since both reduced power dissipation and dynamic reconfigurability is a natural phenomenon that is observed in QCA. Therefore we can quantify the power dissipated during the process of reconfiguration in QCA easily when compared to other paradigms.

As seen earlier, it's feasible to achieve substantially lower power dissipation with the help of reconfigurability in QCA cells. In this chapter, we introduce a custom QCA design based on reconfiguration where we can reconfigure the majority gate voter to perform the functionality of more than one gate i.e. by changing the polarity of the majority gate voter, the same gate can be used either as an AND gate or as an OR gate. If successful, this research could pave way for custom circuit designs which were not feasible in other computing technologies.

3.1 Majority Gate Voter Reconfiguration

By exploiting the majority gate voter in QCA where a majority gate can be made to function as both AND and OR gate by fixing the polarization of one of the cells, we present a reconfigurable scheme where the same QCA cell is being made to function as both AND and OR gate by changing the polarization of electrons. In order to illustrate the proposed idea we use a simple conventional XOR gate which consists of AND and OR gates which are clocked using the Bennett clocking scheme. Such a design is expected to have lower power dissipation when compared to the existing schemes owing to the reversible logical computation offered by Bennett clocking.

An XOR gate can be basically viewed as a series of AND gates whose outputs are given to an OR gate. The schematic and the truth table of the XOR gate is shown Fig. 3.1 and Table 3.1 respectively.

A	B'	Α,	В	C	D	Y
	1	1	0	0	0	-
0	T	T	0	0	0	0
0	0	1	1	1	0	1
1	1	0	0	0	1	1
_1	0	0	1	0	0	0

Table 3.1: Truth Table of XOR Gate







Figure 3.2: Conventional XOR Gate Design Figure 3.3: Proposed XOR Gate Design

QCA implementation of a standard XOR gate is presented in Fig. 3.2. In our proposed methodology (Fig. 3.3), we find that we have only one majority gate as opposed to three majority gates in the conventional approach (Fig. 3.2). The polarization of one the inputs of the majority gate (X) can be switched from +1 to -1 and vice versa in order to switch between an AND and OR gate. The QCA representation of the proposed architecture is as shown in Fig. 3.3.

In our proposed methodology, a single majority gate voter is made to behave as both AND and OR gate. This is achieved by changing the polarity of one of the input cells to +1/-1. We compute the result of the first AND operation (i.e., C is computed as shown in the Fig. 3.4(2)) and store the intermediate results. Once the de-computation of this AND operation is completed, the second AND (i.e., D is computed as shown in the Fig. 3.4(3)) operation is performed whose intermediate results are also stored. Thus the intermediate results are stored in the series of cells forming a shift registers before the output cell Y.

During the de-computation cycle, we activate the feedback path on to the inputs and polarization of the majority gate voter is reversed in order for it to behave as an OR gate(Fig. 3.4(4)). Finally, we start output computation, along with the decomputation of the feedback path. Thus we find that, we have a stored value of the intermediate AND results which can be used as inputs as part of a large and complex circuitries along with the final XOR result (Fig. 3.4(5)).

The Bennett clocking scheme of our proposed reconfiguration design is clearly indicated in Fig. 3.4. The figure represents the final state of the cells where the colors are used to indicate the presence of data in a given cell. Different colors are used for inputs, the shift registers, the majority gates, output and data propagation. The initial state of cells is assumed to be in Null state (Fig. 3.4(1)) and it's assumed that the inputs (both true and complementary form) are available and initially the majority gate voter is polarized to perform AND operation. As the computation



Figure 3.4: Clocking Scheme of Proposed Design XOR Gate Design.

- (1) Initial State of the cells
- (2) Computation and De-computation of the 1st AND operation.
- (3) Computation and De-computation of the 2nd AND operation.
- (4) Feedback and Computation and De-computation of the final result.
- (5) Final State of cells

begins the true form of input A and the complementary form of input B is fed into the cell.

As the computation progresses, the output is stored in the shift registers. During the de-computation, the inputs are restored and just the outputs are stored in the shift register whereby there is no power dissipation (Fig. 3.4(2)). Once the decomputation is completed we repeat the same procedure but with the true form of B and the complementary from of A as the inputs. If one takes a closer look at the proposed design, one can clearly find that true and complementary form of inputs are being written into the same cell. This implies that, there won't be any power dissipation since at any given point of time, we write into the input cell in the presence of a copy. At the end of computation and de-computation cycle the outputs are stored in the bottom shift register as shown in Fig. 3.4(3). Finally during the de-computation cycle, we change the polarization of the majority gate voter so that it can perform as an OR gate now instead of an AND gate. Also, the outputs are fed back as inputs. (Fig. 3.4(4)). During this computation, there is power dissipation when we tend to write the feedback data as inputs and also when we try to change the polarization of the majority gate voter. Finally, we compute the XOR output by computing the OR function of the intermediate results. Thus we find that we have the final output along with the intermediate results which can be used as fan-in to some other circuits when they are part of a large complex circuit as shown in Fig. 3.4(5).

The fundamental assumption in any reconfigurable implementation is the fact that, we have the ability to clock the cells individually. But, keeping in mind the practical constraints of such an assumption we are assuming that we can dynamically cut off the clocking to the feedback path during computation and activate it during the de-computation cycle i.e. we have restricted control over a group of cells and not over individual cells. Future research in this direction could pave way for more realistic and feasible designs.

3.2 Wire Crossing Scheme based on Bennett Clocking

The previous section has established the proposed reconfiguration approach but in any reconfigurable system, routing is a major problem. In a QCA based reconfigurable design also, coplanar wire crossings is an area of major concern since it requires more than one cell. We have discussed in detail with regard to the drawbacks of traditional wire crossing designs in the previous chapter. In this work, we propose coplanar wire crossings based on only one type of QCA cells by taking advantage of dead time during the de-computation cycle of Bennett clocking scheme.

In our proposed clocking scheme, we try to take advantage of the dead time during the de-computation cycle of Bennett Clocking. We propose to break down the entire circuit into different pipeline zones [51]. The pipeline zones are under the control of the designer and may be used only where is wire crossing taking place. One of the fundamental assumptions in our design is the flow of information in both the vertical and the horizontal direction. For simplicity, let us consider a simple design which consist of AND and OR gates as shown in Fig. 3.5.



Figure 3.5: A simple circuit that implements the Boolean equation (A+B)+(B.C)

From the circuit, we can see that input B is being fed into both the AND and OR gate. This is a simple wire crossing problem in QCA. A QCA representation of the same circuit is shown in Fig. 3.6. From the figure, we can clearly see that input B
has to crossover both the inputs A and C in order to reach the other end of circuitry, this is a simple example of wire crossing on which the proposed Bennett clocking with pipeline zones is implemented.



Figure 3.6: QCA representation of a circuit that implements the Boolean equation [(A+B)+(B,C)]

The same QCA structure along with the proposed pipeline zones and the clocking scheme is clearly shown in Fig. 3.7. From the figure, we can see the different clocks controlling the cells, there are 5 different clocks and the timing diagram of the proposed clocking scheme is as shown in Fig. 3.8. We propose to compute and decompute the data through Bennett clocking before and after the pipeline zones. The basic idea behind this implementation is that we try to clock the data till the pipeline zone and we latch it up within the zone. Once it starts to de-compute, we clock the data that needs to be propagated vertically. Thereby, we eliminate the problem of wire crossing. Once the data has crossed over, we start with the computation of the remaining part of the circuit. A detailed explanation of our proposed approach is presented below.

Initially we assume that all the cells are in a null state. In our approach it's assumed that we compute and de-compute on either side of the pipeline zones. In the first stage of our computation, the input data is sent until the pipeline stage where it's latched. Fig. 3.9 gives a clear picture of this implementation where the final state



Figure 3.7: Clocking Scheme of the Proposed Wire Crossing Technique



Figure 3.8: Timing Diagram of the Proposed Wire Crossing Technique



Figure 3.9: Implementation of the Proposed Wire Crossing Technique until the Pipeline Zones

of the cells is shown at the end of every clock cycle while the colors in the figure represent the propagation of data through various cells.

Initially, all the three inputs namely A, B and C are propagated until the pipeline zone which is clearly illustrated in Fig. 3.9(a) and (b). Then the input B which needs to cross over is sent through vertically as shown in Fig. 3.9(d) while the decomputation of inputs takes place (Fig. 3.9(c)). From the Fig. 3.9 we can see that 3.9(c) and 3.9(d) happen simultaneously. Once the input data are latched on in the pipeline zones, the input B begins to propagate in the vertical direction i.e. computation begins along the 2nd clock zone (Clock zones are indicated in Fig. 3.7and Fig. 3.8). During this computation, the de-computation of the inputs takes place since the inputs are already latched on in the pipeline zone which is represented by Fig. 3.9(c) and Fig. 3.9(d). Its assumed that the time taken for the computation of Input B along the vertical direction is less than or equal to the de-computation of the remaining inputs since vertical direction flow is the natural directional flow of data. In case the computation time is more than the de-computation time, then we need to wait until the computation gets over in order to start the next set of computation on the other side of the pipeline zones which are illustrated in Fig. 3.10.

Fig. 3.10 gives us a clear picture about the computation and de-computation that takes place within the proposed pipeline zone and after the proposed pipeline zone until the output. At first, the data that latched in the pipeline zone is propagated till the majority gate voting. Simultaneously the data within the pipeline zone is being de-computed as shown in Fig. 3.10(a) and Fig. 3.10(b). The computation of the data continues till the output after the intermediate results have been computed. Computation of output is done when the de-computation within the pipeline zone is in progress. Thus we have the computed output by the time, the pipeline zone has been de-computed as illustrated in Fig. 3.10(c). Finally, the new input starts propagating towards the pipeline zone while de-computation from the output until the pipeline zone starts. By the time we have latched the new data in the pipeline zone, de-computation would be complete and the next phase of computation beyond the pipeline zone can take place (Fig. 3.10(d)).



Figure 3.10: Implementation of the Proposed Wire Crossing Technique within and after the Pipeline Zones

CHAPTER 4

POWER ANALYSIS AND METHODOLOGY

In this chapter, we present a detailed power analysis of our proposed approach and compare it with the results obtained for the existing designs. We have considered both the upper bound and lower bound limits of power in order to capture the advantages posed by our approach more accurately. Before we get ahead with the comparison of the proposed approach with the existing designs, we need to understand the upper and lower bound limits of power. The fundamental limits of upper and lower bounds of power are explained by taking an example of Bennett clocking based half adder (Fig. 4.1).



Figure 4.1: A Schematic of Half Adder along with it's QCA implementation. The required clocking signals for Bennett Clocking are shown in the graph. (From. ref [29])

4.1 Upper Bound Power Dissipation Model

In thermodynamics, adiabatic process refers to a process in which there is no net transfer of heat to or from the environment. Earlier researchers considered the clocking switching activity to be a quasi-adiabatic process where in a system goes through a sequence of events that are infinitesimally slow such that the entire process is reversible.

Timer et al have proposed a power dissipation model [62, 63] in general to estimate power dissipation in case of a quasi adiabatic switching event. They have presented a detailed quantum mechanical power model, where in they have showed that the power dissipation can be made as low as possible when the clock changes are nearly adiabatic. They identified three components of power: clock power, cell to cell power gain and power dissipation. Even though this model gives us a detailed physical estimates, it's computationally very intensive and difficult to calculate. The power dissipation for a QCA circuit can expressed as the sum of power estimates computed on a per-cell basis. Each cell in a QCA circuit sees three types of events: (i) clock going from low to high i.e. depolarization of the cell, (ii) input or cells in previous clock zone switching states, and (iii) clock changing from high to low, latching and holding the cell state to the new state.

The fundamental power dissipation model which computes the upper bound is based on this quantum mechanical model [58, 57]. Such upper bound represents the worst case power dissipation, which happens in the presence of non-adiabatic clocking. The authors have developed a probabilistic Bayesian model where the probabilities directly maps on to the quantum - mechanical steady state probabilities which are nothing but the density matrix and the cell polarizations. The fundamental upper bound is given by:

$$E_{diss} < \frac{h}{2} \overrightarrow{\Gamma}_{+} \cdot \left(-\frac{\overrightarrow{\Gamma}_{+}}{|\overrightarrow{\Gamma}_{+}|} \tanh\left(\frac{h|\overrightarrow{\Gamma}_{+}|}{k_{B}T}\right) + \frac{\overrightarrow{\Gamma}_{-}}{|\overrightarrow{\Gamma}_{-}|} \tanh\left(\frac{h|\overrightarrow{\Gamma}_{-}|}{k_{B}T}\right) \right)$$

$$(4.1)$$

where $\overrightarrow{\Gamma}$ is the Hamiltonian vector, k_B is Boltzmann Constant, h represent the Planck Constant and T is the temperature.

The entire dissipation bound model has been derived in Appendix B.

Now consider the half adder circuit shown in Fig. 4.1. Since it undergoes reversible computation through Bennett clocking, the information lost in the circuit is only during the switching of inputs. Consider the truth table of the half adder shown in Table. 4.1.

Table 4.1: Truth Table of Half Adder

А	В	S	С
0	1	0	0
0	0	1	0
1	1	1	0
1	0	0	1

Now, from the above derivation we find that, the energy dissipated whenever there is a change in inputs is E_{diss} if there is no copy available. The power dissipation occurs only when we try to rewrite the input cells with the next set of inputs. From the truth table, we find that the power dissipation owing to the switching of inputs is $4E_{diss}$. This is obtained from the fact that the there are 4 sets of inputs possible namely [00, 01, 10, 11]. The values are rewritten as inputs four times while implementing the truth table as shown in Table. 4.1 while the rest of the times, the previous data already exists in the input registers and we don't rewrite. i.e. if there is switching from 1 to 0 or from 0 to 1, then there is power dissipation but it's not the case when there is switching from 1 to 1 or from 0 to 0. Thus we find that the upper bound of Energy Dissipation for the implementation of the presented truth table is $4E_{diss}$.

4.2 Lower Bound Power Dissipation Model

Lower bound power is the minimum amount of power that is dissipated whenever there is a computation taking place in a circuit. There are mainly two types of computation namely reversible and irreversible computation. Lower bound will differ based on the type of computation that is taking place. Any logically irreversible computation such as erasure and rewriting of inputs will have to dissipate certain amount of power. The methodology followed for the estimation of this lower bound is as suggested by Ercan and Anderson[28, 29]. The proposed methodology assumes the circuit to be an ideal circuit and heat dissipation is estimated using the physical information theoretic analysis. The fundamental lower bounds is ideally a four step process which consist of physical decomposition, process abstraction, operational decomposition and cost analysis.

The basic principle behind the computation of the bounds is to decompose the given circuitry and it's surroundings into key circuit elements which consist of information with regard to their physical states as well as their relevant external subsystems. Once the decomposition of the circuit is done, it requires the identification of the circuit states i.e. the circuit elements and the external subsystems, which interact with one another during the computational cycle.

The next step in the process is the process abstraction where we identify the control and the restoration processes. Control processes are those which forces the circuit to change in to a new state leading to an interaction between the circuit and the bath during a well defined time interval. While restoration process restores the heat bath to its original form after the control process.

The third step is the operational decomposition where the computational cycles and the driving force behind these cycles are defined. The computational cycle is the process of computing the outputs for a set of inputs. Since the computational steps depend on the clocking of the circuit, it automatically becomes the driving force.

Now in order to estimate the lower bound, we break down the circuit into data zones and find the information lost in each zone by applying physical information theoretic analysis. The summation of results from each zone will give the total information lost. Data zones are defined by data sub-zones, the inputs to the sub-zones, and the outputs of the sub-zones. Data sub-zones are areas within a single clocking zone where irreversible computing happens. The outputs and inputs to a sub-zone may be in a different clocking zone so data zones may span multiple clocking zones. Once we have broken down the entire circuit into data zones and sub-zones, we can apply information theory to determine how much information is lost within each subzone which can be used to find the data lost in each data zone, which gives the total information lost in the entire circuit. This is done by finding the Shannon entropy at both the input and the output of a sub-zone and taking the difference which gives the information lost in the sub-zone.

Consider two random variables X and Y to be the input and output to a data sub-zone respectively. X can be any value in the set $\{a_1, \ldots, a_m\}$ with probability $P(X = a_i) = p_i$ where *i* is an integer from 1 to m. Y can be any value in the set $\{b_1, \ldots, b_m\}$ with probability $P(X = b_i) = q_i$ where *i* is an integer from 1 to n. If we consider an example of an AND gate. It has two binary inputs and one binary output. We can define the AND gate to be a data sub-zone. Since it has two binary inputs, we have m=4 and we can define the set of values that X can take as $\{00, 01, 10, 11\}$. Now assuming that both the inputs are equally probable, we have $p_1 = p_2 = p_3 = p_4 = 1/4$. The output of an AND gate can either be a 1 or 0, which means that the set of value that Y can take is $\{0, 1\}$. Three of the inputs will result in Y = 0 while the other input results in Y = 1. Hence we can find probability of the outputs by adding the probability of inputs. Therefore $q_0 = p_1 + p_2 + p_3 = 3/4$ and $q_1 = p_4 = 1/4$.

Shannon entropy is the measure of the uncertainty associated with a random variable [55]. This tells us the expected value of the information per generated value within the set. In the case of the random variable X , the input of a sub-zone, the Shannon entropy is defined as:

$$H(X) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$
(4.2)

The Shannon entropy for the random variable Y, the output of a sub-zone, is:

$$H(Y) = -\sum_{j=1}^{n} q_j \log_2(q_j)$$
(4.3)

The information lost from the input to the output of the sub-zone can then be found by

$$-\Delta I = H(X|Y) = H(X) - H(Y) \tag{4.4}$$

which is measured in bits.

Now applying the above approach to the AND gate we have the input entropy as

$$H(X) = -\left[\frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right)\right] = 2$$
(4.5)

and output entropy as

$$H(X) = -\left[\frac{3}{4}\log_2\left(\frac{3}{4}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right)\right] = 0.811$$
(4.6)

Thus the total information lost $-\Delta I = 2 - 0.811 = 1.189 bits$

Finally a space time decomposition all the computational steps involved in the circuit operation is performed. In order to get the bounds, a summation of the bounds of the individual computational steps, which follows from the physical information theoretic analyses of the circuit state changes required. The bound thus obtained will be regarded as the fundamental lower bound since it represents the minimum physical cost required for the circuit to achieve it's computational ends. However, these bounds are specific to a given circuit structure and the mode of operation.

In order to understand the bounds better consider the Bennett clocking based half adder circuit presented in Fig. 4.1. The first step in the process is the physical decomposition of the circuit. By looking at the circuit one can identify the circuit which are contained within the box while the ones outside the box represent the external subsystems. Cells A and B are the input registers while S and C are the output registers. We can define the rest of the system as the thermal bath and rest of the universe. From the circuit clocking, we can clearly see that the switch or relax phases of adiabatic clocking leads to a change in state of the cell. Hence they are the control processes. While the relax or hold phases restores the state of the cell. As shown in Fig. 4.1 the clocking of the circuit is defined by the clocking zones which are separated by different colors which represents the clocking signals. The clock signals which define the computational step is shown by the gray lines in the clocking diagram of Fig. 4.1. All these computational steps make up one computational cycle.

Now, if we consider the half adder circuit, we can clearly see that the only information loss occurs only when the inputs of the circuit, A and B changes its value at the beginning of new computation. So, consider the inputs to the circuit as a data zone, the output of data zone will be current inputs of the circuit, and the input to the data zone will be the next inputs to the circuit. Since, the next set of inputs are independent of the previous set of inputs, we have H(X|Y) = H(X). Therefore, the information lost is solely dependent only on the next set of inputs. Let's assume that all the possible inputs i.e. [00, 01, 10, 11] are equally probable. This implies that we will have H(X) = 2 which means that $-\Delta I = 2bits$ will be lost for the entire circuit. This is the amount of information that is lost on average for a given computational cycle in the Bennett clocked half adder circuit. We can now find the total heat dissipated into the bath by the circuit by using Landauers Principle which states that at least $k_BTln(2)$ must be dissipated when a bit of information is lost from the system. Since we know the circuit loses on average 2 bits, the fundamental lower bound of heat dissipation will be $2k_BTln(2)$ joules for the Bennett clocked half adder in QCA.

4.3 Power Analysis of Proposed Reconfiguration Scheme

A detailed analysis of the proposed scheme and the existing conventional scheme based on the power bound models is presented in this section. We try to compute the upper and lower bounds for both the schemes and compare the results for power efficiency. In the previous chapter, we have understood the working of the proposed scheme. The proposed scheme can be understood from Fig. 3.4. Initially let's compute the upper bound power, as shown in the previous section, the energy dissipated whenever there is a change in inputs is E_{diss} if there is no copy available. From the proposed design (Fig. 3.2), we find that since the true and complementary form of the inputs are being given to the same input cell, there is no power dissipation due to the change in inputs. The power dissipation occurs only when we try to rewrite the input cells with the feedback results and when we switch the polarization of the cell from -1 to +1 and vice versa in order for the majority gate to function as either AND or OR.

From the truth table (Table. 3.1), we find that the power dissipation owing to the switching of inputs due to feedback is $4E_{diss}$ and we add another $2E_{diss}$ owing to the change in polarity. This is obtained from the fact that the feedback results are rewritten as inputs four times while implementing the truth table as shown in Table II while the rest of the times, the previous data already exist in the shift registers and we don't rewrite. i.e. if there is switching from 1 to 0 or from 0 to 1, then there is power dissipation but it's not the case when there is switching from 1 to 1 or from 0 to 0. Thus we find that the upper bound of energy dissipation for the implementation of the presented truth table is $6E_{diss}$.

If we consider the conventional implementation both the true and complementary form of inputs are being fed as inputs to two different majority voter gates, as a result of this, there is power dissipation every time we switch the inputs. As a result of this, we find that the total energy dissipated when executing the given truth table is $14E_{diss}$.

Till now, we formulated the upper bound for both the proposed and the conventional implementation of the XOR gate. Now, let's consider the lower bound which tells us the minimum amount of power dissipated by the circuit. For the calculation of the lower bound, we follow the procedure highlighted in the previous section. As seen in the previous section, we have identified the control and the restoration processes and have clearly defined the computational steps. Now we define the data zones and the sub-zones. During the calculation of the upper bound, we have clearly seen that there is no power dissipation due to change of inputs. However, there is power dissipation when the feedback is being fed as inputs. Thus the total energy dissipated will be equal to the sum of the energies dissipated in the two feedback paths. Let us define the feedback paths to be the data zone. Since the next set of inputs is independent of the previous inputs we have H(X|Y) = H(X). Therefore, the information lost is solely dependent only on the next set of inputs. Let's assume that all the possible inputs i.e. [00, 01, 10] are equally probable. We have only 3 sets of inputs for the feedback path since the input [11] is never encountered as seen from Table. 3.1. This implies that we will have H(X) = ln(3) which means that $-\Delta I = 1.098 bits$ will be lost for the entire circuit. Since we know the circuit loses on average 1.098 bits,

the fundamental lower bound of heat dissipation will be $1.098k_BTln(2)$ joules for the Bennett clocked proposed XOR gate in QCA. A similar calculation was performed for the conventional implementation and the regions of irreversible computation in this case was the switching of the inputs. As a result of this, the minimum power dissipated is given by $2k_BTln(2)$.



Figure 4.2: Timing Diagram of the XOR gate Figure 4.3: Timing Diagram of the XOR based on proposed Reconfiguration Scheme gate based on conventional design methodology

Timing diagram for the proposed scheme and the standard design is presented in Fig. 4.2 and Fig. 4.3. From these two figures, we can see that output is available at the end of 6th cycle in case of the standard design while it's not available until the 12th cycle in the proposed scheme. Thus, computation time is the trade-off in order to achieve low power but the advantage of the proposed scheme can be clearly seen in the reduction in area and also we can feed in the next set of inputs as early as the 12th cycle, while the next set of inputs can start computation in case of the standard design in the 10th cycle. Even though there is a difference of 2 cycles with regard to the latency, in complex circuits we can pipeline data and achieve better results.

CHAPTER 5 QCA ADDERS

In this chapter, we present the work that's being carried out as part of this thesis. Addition is a basic and a fundamental operation involved in any digital logic design or a control system design. The performance of any digital system is characterized and heavily influenced by the performance of the adders in that design. Various designs of fast adders have been proposed by various authors in every possible technology starting from CMOS to nanoscale implementations such as spin wave functions, QCA's to name a few. In this work, we intend to present an adder design based on the reconfiguration scheme presented in the previous chapter which will be robust and power efficient in comparison to the existing designs. Before we get into details of the proposed adder and the metrics to be used for comparison, we will get an insight into the existing adder designs based on QCA.

5.1 Background on QCA Adders

It was as early as 1994, when Lent and co [64] came up with a set of logical devices based on QCA and adder was one of them. They built a full adder based on five majority gates and three inverters. A carry look ahead adder was realized by connecting n such one - bit QCA full adders [67]. Such a design was possible because in a QCA based design, the carry is generated before the sum. The above mentioned adder design were improvised by Wang et al [69] who designed one-bit QCA adder based on only three majority gates and two inverters. A bit serial adder which uses a carry feedback which also uses only three majority gates and two inverters was proposed by Fijany et al [32] but it came with a complex clocking scheme. Srivastava [59] studied full adders based on Bayesian probabilistic modeling while Cho et al proposed modular adder and multiplier design based on QCA architecture and did a thorough analysis of the same [13, 14]. The scheme proposed by Cho was further improvised by Bruschi et al [11] who exploited minority gates in addition to majority ones. A minority based full-adder is presented in [53], but it is not modular. Walus et al proposed a modular ripple carry structure for adders. Modular designs of conditional sum adder was studied in detail in [15] while probabilistic analysis of molecular QCA based adders was presented in [27].

5.2 Proposed Adder Designs

From the above section, we find that there are numerous adder designs which have been presented by researchers around the world. Zhang et al [73] did a thorough comparative study on the performance of QCA based adders and found that QCA ripple-carry adder and bit-serial adder designs actually outperforms carry-look-ahead and carry-select adder designs. Hence we present a 1-bit carry ripple adder and a 1-bit carry look ahead adder based on the proposed reconfiguration scheme which can be extended to n-bits. While most of the work concentrated on improving the performance, the work done so far on reducing the power dissipation is negligible. Hanninen et al [36] were one of the first people to do a power analysis on adders based on QCA. They have evaluated the general heat densities and expected logic for various arithmetic circuits based on Launder's principle. Srivastava et al [59] studied the upper bound power dissipation impacts on various adders and have given a heat dissipation chart for each cell.

Thus, we have clearly seen that there is very little or no work done with regard to estimation and reduction of power dissipation in adders. From the previous chapters, we have clearly seen that built in dynamic reconfigurability of QCA's have not been exploited properly in designing adders which can prove to be highly power efficient. Hence our research will pave way for more adder design which will be power efficient.

In this section, we propose the design of a 1-bit Ripple Carry Adder and 1-bit Carry Look Ahead Adder based on the proposed reconfiguration scheme and extend it to 4-bits. The benefit of such a design is that, the design is inherently pipelined and doesn't require the duplication of the 1-bit adder to build a 4-bit adder. A 1-bit adder design will be capable of performing the operations of a 4-bit adder. One of the major reasons for choosing a ripple carry adder is the fact that one needs to pipeline n-1-bit adders in order to get an n-bit adder, since our design is inherently pipelined and one need not have n-1-bit adders for n-bit additions, which is power efficient and could pave way for more designs in future which could adopt this scheme.

Reconfigurability adds another dimension to the adders since the design can be reused in future which can prove to be effective in terms of cost cutting. The proposed adder design is expected to have very low power dissipation and a significant reduction in area with a slight overhead in the form of latency. The overhead in the form of latency will become significantly less and will become negligible as we scale the adder design from 1-bit to n-bits.

Thus, we intend to use the inbuilt dynamic reconfigurability and exploit the clocking schemes to come up with a more robust, area and power efficient design which will be could lead to inexpensive hardware in the near future.

5.2.1 Carry Ripple Adder

A full adder is an arithmetic circuit which is used for adding binary numbers. A one bit full adder adds three one bit numbers to produce two one bit outputs namely sum and carry. A full adder could be implemented in many ways, one such implementation is the computation of sum using the 3-bit XOR of the inputs and the computation of carry using a 3-bit majority gate function of the inputs. The one

bit full	adder's	truth	table	and	block	diagram	is a	as	shown	in	Table.	5.1	and	Fig.	5.1
respect	ivelv.														

A	В	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Table 5.1: Truth Table of Full Adder

Figure 5.1: Block Diagram of Full Adder

In order to create a logical circuit capable of adding N-bit numbers one can stack a series of N 1-bit full adders where the carry out of the previous adder becomes the carry in of the current adder. This kind of adder is known as a carry ripple adder since each carry bit ripples into the next adder. A block diagram of such an adder is as shown in Fig. 5.2.



Figure 5.2: Block Diagram of Carry Ripple Adder

Even though the design of carry ripple adder is quite simple, it's relatively slow since it has to wait for the carry bit to be calculated from the previous adder before which it can't proceed any further in the computation. Our proposed design takes care of this problem where the carry is computed in a separate block and is available for the computation of the next set of bits as soon as the next set of bits becomes available. The block diagram of our proposed carry ripple adder is as shown in Fig. 5.3.



Figure 5.3: Block Diagram of Proposed Carry Ripple Adder

Our proposed design consists of three blocks which operate asynchronously with respect to one another. All of them perform their own operations and clocking has been designed in such a way that they don't have to wait for data from other blocks. The carry out is generated by the majority voter design block (MVB) while the sum is computed using the two custom reconfigurable XOR blocks (CRXB). The proposed 1-bit Carry Ripple Adder based on the custom reconfiguration scheme is as shown in Fig. 5.4.

The working of the carry ripple adder is quite simple and it's just an extension of the custom reconfiguration scheme where in we have an additional majority gate voter in the design. Therefore as seen from Fig. 5.3 and Fig. 5.4 we can clearly see that we have two blocks of CRXB and a block of MVB. Initially let's assume that all the cells are unpolarized except for the majority gate voter cells and the inputs are readily available as shown in Fig. 5.5

Once the inputs are readily available, the inputs propagate through to the first CRXB block and the MVB block (Fig. 5.6(a)). In the CRXB block, the computation proceeds in the exact same way as the computation of XOR shown in previous



Figure 5.4: QCA Representation of Proposed Carry Ripple Adder



Figure 5.5: Initial State of Proposed Carry Ripple Adder

chapters. It begins with the computation of the first AND operation followed by the computation of the second AND operation once the de-computation of first AND finishes. Finally we propagate the data through the feedback path and change the polarity of the majority gate voter in order to compute the final output of the CRXB block which is sum of the inputs from the feedback (Fig. 5.6(b) - (f)). One can easily observe that Fig. 3.4 and Fig. 5.6 are exactly the same with regard to the CRXB block. One additional computation that is taking place in this case is the computation and de-computation of the MVB block along with the back propagation of the carry which serves one of the input in the next computation cycle.

Once we have computed the first XOR output i.e. $A \otimes B$, we proceed into the computation of the next CRXB block which computes the sum. One can see that by the end of the 6th clock cycle, the initial set of inputs have been de-computed back and the next set of inputs can be fed in and the inputs for the second CRXB block is also readily available (Fig. 5.6(f)). Thus so far we have seen the computation of data in the first CRXB and MVB blocks, now let's take a look at the computation in the second CRXB block (Fig. 5.7).

The computation of the second CRXB block proceeds in the exact same way as that of the first CRXB block, just that the inputs are P and C computed from the first CRXB block and MVB block respectively. If one takes a closer look at Figs. 5.6 and 5.7, the computation of data in the first CRXB and MVB blocks are exactly the same, just that we have new set of inputs after every 6 clock cycles (Fig. 5.6(f) and Fig. 5.7(a)). We can clearly find that the sum output is available in the 12th clock cycle or more precisely at the end of the 11th clock cycle. If we pipeline the data through the adder, we can clearly see that the sum for the second set of inputs will be available at the end of the 17th clock cycle or in the 18th clock cycle.

Thus, from Figs. 5.6 and 5.7 we can clearly see that if we pipeline data into the adder, we get the first output in the 12th clock cycle and successive outputs every



De-computation of Inputs Propagat

(f) Computation of $A \otimes B$ along with Propagation of P and C

Figure 5.6: Clocking Scheme of the Carry Ripple Adder until computation of $\mathbf{A} \otimes \mathbf{B}$



(e) Computation of Sum

(f) Cycle repeats for next set of inputs

Figure 5.7: Clocking Scheme of the Carry Ripple Adder until computation of Sum

6 cycles. Therefore, a single bit adder can perform 4-bit addition which saves a lot of area. If we were to perform n-bit addition using this adder, we would get the nth output in [12+((n-1)*6)] clock cycles or at the end of [11+((n-1)*5)]th clock cycle.

5.2.2 Carry Look Ahead Adder

Carry look ahead adders were designed in order to reduce the computation time. Two signals Propagate (P) and Generate(G) was created exclusively for the purpose reducing the amount of time required to determine the carry bits. P and G is based on whether the carry is being propagated from a least significant bit position (at least one input is '1'), generated in that bit position (both inputs are '1') or killed in that bit position (both inputs are '0'). In general one can conclude that P is sum of the half adder which is nothing but $A \otimes B$ while generate is the carry out which is A.B. The block diagram of the carry look ahead adder is as shown in Fig. 5.8.



Figure 5.8: Block Diagram of Carry Look Ahead Adder

Our proposed carry look ahead adder is not different from the carry ripple adder, just that MVB block is replaced by the propagate generate block (PGB). Our design inherently calculates the propagate (P) since it's the output of the first CRXB block. The block diagram of our proposed carry look ahead adder is as shown in Fig. 5.9.

The working of the proposed carry look ahead adder is quite simple and similar to the working of the carry ripple adder seen in the previous section. The proposed carry look ahead adder is as shown in Fig. 5.10. Therefore as seen from Fig. 5.9 and



Figure 5.9: Block Diagram of Proposed Carry Look Ahead Adder

Fig. 5.10 we can clearly see that we have two blocks of CRXB and a block of PGB. Initially let's assume that all the cells are unpolarized except for the majority gate voter cells and the inputs are readily available as shown in Fig. 5.11.



Figure 5.10: QCA Representation of Proposed Carry Look Ahead Adder



Figure 5.11: Initial State of Proposed Carry Look Ahead Adder

Once the inputs are readily available, the inputs propagate through to the first CRXB block. Then the computation of first AND operation of the CRXB begins. This can be clearly observed in Fig. 5.12(a) and (b). As seen in the previous sections,

we can clearly observe the availability of both the true and complementary form of the inputs in the CRXB block similar to the carry ripple adder design.

The computation proceeds exactly in the same way as the computation of XOR in CRXB as shown in Fig. 5.12(c) and (d). One can easily observe that Fig. 5.6 and Fig. 5.12 are exactly the same with regard to the CRXB block until the 4th clock cycle. In the 5th clock cycle instead of propagating the feedback and changing the polarity, we just de-compute and proceed to the computation of generate (G) this is nothing but A.B in the 6th clock cycle as seen in Fig. 5.12(e) and (f). One should also note that while computation of generate is going on the inputs are de-computed so that the next set of inputs can be fed in. Then we proceed with the polarity change and de-computation and finally proceed to the computation of first XOR output i.e. $A \otimes B$ similar to the carry ripple adder design as shown in Fig. 5.12(g,h). Thus at the end of the 8th clock cycle, we have both the propagate and the generate which serves as input to the PGB block which computes the next carry. Simultaneously we proceed to compute the sum output in the second CRXB block.

Thus so far we have seen the computation of data in the first CRXB block, now let's take a look at the computation in the second CRXB block and PGB block (Fig. 5.13). The computation in the PGB block is quite straightforward, it's just an OR operation of the two inputs namely P and G which gives the carry out for the next set of inputs. The computation of the second CRXB block proceeds in the exact same way as that of the first CRXB block, just that the inputs are P and C computed from the first CRXB block. One should note that, the input C is fed in for the first bit while it comes from the PGB block for the next set of inputs.

If one takes a closer look at Figs. 5.12 and 5.13, one can clearly notice that the computation of data in the first CRXB block is exactly the same just that we have new set of inputs after every 8 clock cycles (Fig. 5.12(f) and Fig. 5.7(a)). We can clearly find that the sum output is available in the 14th clock cycle or more precisely



Figure 5.12: Clocking Scheme of the Carry Look Ahead Adder until computation of Propagate



Figure 5.13: Clocking Scheme of the Carry Look Ahead Adder until computation of Sum

at the end of the 13th clock cycle. If we pipeline the data through the adder, we can clearly see that the sum for the second set of inputs will be available at the end of the 19th clock cycle or in the 20th clock cycle. Thus, from Figs. 5.12 and 5.13 we can clearly see that if we pipeline data into the adder, we get the first output in the 13th clock cycle and successive outputs every 6 cycles. Therefore, a single bit adder can perform 4-bit addition which saves a lot of area. If we were to perform n-bit addition using this adder, we would get the nth output in [14+((n-1)*6)] clock cycles or at the end of [13+((n-1)*5)]th clock cycle.

Thus in this chapter we have understood the working of our proposed adders. We will look into the advantages of this adder and how it compares with the existing adders in the next chapter.

CHAPTER 6

RESULTS AND COMPARATIVE ANALYSIS

In the previous chapter, we have clearly explained the design and working of our proposed adders. In this chapter, we compute the energy dissipated by the proposed adders based on the methodologies presented in the earlier chapters and compare them with existing adder designs. For comparison we intend to compare our proposed designs with the existing QCA adders that make use of both five majority gates [64, 67] and three majority gates [69] in their design. The QCA representation of the two existing adders which is used for comparison is as shown in Fig. 6.1. The analysis is done for a 1-bit adder and is extended for n-bits.



Figure 6.1: QCA representation of existing adders (From. ref [64, 69])

Some of the metrics used for comparisons are as follows

• Upper Bound and Lower Bound Energy Dissipation

- Area and Cell Count
- Latency
- Speed with respect to Area and Power (SwAP) Analysis

6.1 Upper and Lower Bound Energy Dissipation

As seen from chapter 4, we have clearly established that the upper bound energy dissipated in a single QCA cell in the case of irreversible computation is given by E_{diss} . Similarly, we compute the lower bound based on the methodology discussed earlier where in we divide the circuit into data zones and data sub-zones and computing the information lost as the sum of the information lost in individual data zones. As seen in the previous chapters, we can clearly observe the availability of both the true and complementary form of the inputs in the CRXB block. It must be noted that while the true form of the input is being fed into the input cell of the majority gate of the CRXB block, the complementary form of the same input is being held in the hold phase. The result of such a configuration is that we can feed both the true and complementary form of inputs into the same cell without any loss of energy since at any given point of time there is a copy available. Since our designs can be viewed as three separate blocks, we can compute the energy dissipated in each block and summation of all the energies will give the net energy dissipated. The energy dissipation is based on the giving inputs in the same order as seen in truth table of adder (Table. 5.1).

6.1.1 Analysis of Carry Ripple Adder

The proposed carry ripple adder can be subdivided into three zones namely the two CRXB blocks and one MVB block (Fig. 5.3 and Fig. 5.4). Now let's compute the energy dissipated in the CRXB block. In the CRXB block, the energy dissipation occurs every time we switch the polarity of the majority gate voter to perform both AND and OR operation. Thus, the energy dissipated will be equal to $2E_{diss}$. Other loss occurs whenever the inputs are being fed into the input cell via the feedback path. Thus the energy loss will be $4E_{diss}$ which can be inferred from the truth table (Table. 3.1) since there is dissipation occurring only when we switch inputs from 1 to 0 and vice versa and not from 0 to 0 or 1 to 1. Thus total energy dissipated in the CRXB block is equal to $6E_{diss}$. Now let's compute the energy dissipated in the MVB block. If we consider the MVB block there is energy dissipation only when the inputs switch. There is no other loss of energy. One should note that one of inputs C is being fed only once while the subsequent cycles it's being produced by the MVB block itself which implies that E_{diss} is dissipated when we feed input C the very first time. Therefore if we consider the truth table of a 1-bit full adder as shown in Table. 5.1, we can find that energy dissipated in entire carry ripple adder is given by energy dissipated in the CRXB block + energy dissipated in the MVB block.

$$E_{CRA_{UB}} = E_{CRXB1} + E_{MVB} + E_{CRXB2} + E_{inputs}$$
$$= 6E_{diss} + 0 + 6E_{diss} + 7E_{diss}$$
$$E_{CRA_{UB}} \le 19E_{diss}$$
(6.1)

If we consider an n-bit adder, the energy dissipated in the CRXB and MVB blocks remains the same, the only change will be the energy dissipated by the inputs which is nothing but the switching of inputs. Therefore, in an n-bit adder if we have k input transitions from 0 to 1 or from 1 to 0 we get the total energy dissipated as

$$E_{CRA_{UB_n}} = E_{CRXB1} + E_{MVB} + E_{CRXB2} + E_{inputs}$$
$$= 6E_{diss} + 0 + 6E_{diss} + (k+1)E_{diss}$$
$$E_{CRA_{UB_n}} \le (k+13)E_{diss}$$
(6.2)

Till now, we formulated the upper bound energy, now let's compute the lower bound energy based on physical information theory. The control and restoration process are the same as that of proposed XOR gate design seen in chapter 5. Now, if we consider the adder circuit, the information loss is due to inputs and due to the CRXB blocks. So, consider the inputs to the circuit as a data zone, the output of data zone will be current inputs of the circuit, and the input to the data zone will be the next inputs to the circuit. Since, the next set of inputs are independent of the previous set of inputs, we have H(X|Y) = H(X). Therefore, the information lost is solely dependent only on the next set of inputs. Let's assume that all the possible inputs i.e. [00, 01, 10, 11] are equally probable. This implies that we will have H(X) = 2which means that $-\Delta I = 2bits$ will be lost. Let us define the feedback paths to be another data zone. Since the next set of inputs is independent of the previous inputs we have H(X|Y) = H(X). Therefore, the information lost is solely dependent only on the next set of inputs. Let's assume that all the possible inputs i.e. [00, 01, 10]are equally probable. We have only 3 sets of inputs for the feedback path since the input [11] is never encountered as seen from Table. 3.1. This implies that we will have H(X) = ln(3) which means that $-\Delta I = 1.098bits$ will be lost. An additional 2bits of information is lost due to the changing of polarity. There is no information lost in the MVB block.

$$I_{Total} = I_{CRXB1} + I_{MVB} + I_{CRXB2} + I_{Inputs}$$

= 3.098 + 0 + 3.098 + 2
$$I_{Total} \ge 8.196 bits$$
(6.3)

Since we know the circuit loses on average 8.196 bits, the fundamental lower bound of heat dissipation will be $8.196k_BTln(2)$ joules for the Bennett clocked proposed adder in QCA.

If we consider an n-bit adder, the energy dissipated remains the same since we performing a single n-bit computation in the same circuit and not by a series of n 1-bit adders. But the lower bound will change if we try to perform multiple n-bit computations again and again. For example if we perform a 4-bit computation then the information lost will be just 2bits and the energy dissipated will be equal to $8.196k_BTln(2)$ joules, but if we perform 2 consecutive 4-bit operations then we lose 4bits of information in the input cycle. This is due to the fact that we are inputting a new set of data into the adder. Hence, the energy dissipated depends on the kind of addition one is performing as a 1-bit addition followed by 4-bit addition. Therefore, it's under the discretion of the designer and the energy dissipation depends on the usage of the adder. Therefore, to generalize it and put it in simpler terms we assume that we just perform only one computation i.e. an n-bit adder performs one n-bit addition, we get the total energy dissipated as

$$E_{CRA_{LB_n}} \ge 8.196k_B T \ln(2) joules \tag{6.4}$$

6.1.2 Analysis of Carry Look Ahead Adder

The proposed carry look ahead adder can be subdivided into three zones similar to the carry ripple adder namely the two CRXB blocks and one PGB block (Fig. 5.3 and Fig. 5.4). In the analysis of carry ripple adder, we have clearly seen the energy dissipated in the CRXB block which is equal to $6E_{diss}$. Now let's compute the energy dissipated in the PGB block. If we consider the PGB block there is no energy dissipation since inputs are nothing but the outputs of the previous stage. Thus the energy dissipated is equal to 0. Now let's consider the energy dissipated when we switch the inputs A and B. This is also exactly the same calculation as seen in carry ripple adder which is equal to $7E_{diss}$. Thus the total energy dissipated in entire carry look ahead adder is given by energy dissipated in the CRXB block + energy dissipated in the PGB block + energy dissipated due to switching of inputs.

$$E_{CLA_{UB}} = E_{CRXB1} + E_{PGB} + E_{CRXB2} + E_{inputs}$$
$$= 6E_{diss} + 0 + 6E_{diss} + 7E_{diss}$$
$$E_{CLA_{UB}} \le 19E_{diss}$$
(6.5)

$$E_{CLA_{UB_n}} \le (k+13)E_{diss} \tag{6.6}$$

Similarly the lower bound dissipation can also be calculated by finding out the information lost in the circuit which is nothing but the loss in the CRXB and PGB blocks along with the loss in the inputs.

$$I_{Total} = I_{CRXB1} + I_{PGB} + I_{CRXB2} + I_{Inputs}$$

= 3.098 + 0 + 3.098 + 2
$$I_{Total} \ge 8.196 bits$$
(6.7)

Hence the total energy dissipated is given by

$$E_{CLA_{LB_n}} \ge 8.196k_B Tln(2) joules \tag{6.8}$$

From equations 6.2, 6.4, 6.6 and 6.8 it's clearly visible that the energy dissipated in both the adders is exactly the same which is as expected since both the adders perform the same function and both use the same set of blocks, just that both have different implementations where one uses a majority gate voter design while the other uses an OR gate for computation.

6.1.3 Analysis of existing Adders

The adders which are being considered for comparative analysis are as shown in Fig. 6.1. For the purpose of uniformity, we assume that the adders are clocked using
the Bennett clocking scheme. Since it's based on Bennett clocking scheme, the only loss occurs when we switch the inputs. From the truth table, it's clearly evident that A switches twice from a 0 to 1 and vice versa while B switches four times and C switches seven times. Therefore the net energy dissipation is $13E_{diss}$ since there are 13 input transitions taking place to execute the truth table of the adder. A crude method of generalizing it to n-bit adder will be to assume that there are k input transitions in a 1-bit adder. Since the n-bit adder is built as a series of n 1-bit adders (Fig. 6.2) and not in a pipelined manner as in our proposed design, we will have k transitions in every single such block. Therefore total number of input transitions will be sum of individual transitions which is equal to the energy dissipated.

$$E_{EA_{UB_n}} = \left(\sum_{k=1}^n k_i\right) E_{diss} \tag{6.9}$$

$$E_{EA_{UBn}} \approx nk E_{diss} \tag{6.10}$$

Now let's compute the lower bound energy dissipation. Here the loss in information



Figure 6.2: Existing 4-bit adder

is only due to the three inputs. From the truth table of the adder (Table. 5.1), we can clearly see that there are 8 different sets of inputs possible which are equally probable. This implies that we will have H(X) = 3 which means that $-\Delta I = 3bits$ will be lost. Since we know the circuit loses on average 3 bits, the fundamental lower bound of heat dissipation will be $3k_BTln(2)$ joules for the Bennett clocked existing adder in QCA. If we extrapolate it to n-bit adder, we will find that $3k_BTln(2)$ joules will be lost in the 1st adder in series while the remaining n-1 adders will lose $2k_BTln(2)$ joules since the carry gets propagated as a 3rd input from the previous adders. Therefore, the total energy dissipated will be equal to

$$E_{EA_{LB_n}} \ge 3k_B T ln(2) + (n-1) * 2k_B T ln(2) joules$$

$$E_{EA_{LB_n}} \ge (2n+1)k_B T ln(2) joules$$
(6.11)

6.2 Area and Cell Count

In the past few sections, we have clearly explained how the proposed 1-bit adder performs n-bit computation without the need for the stacking of n 1-bit adders in the traditional adders. This gives us an enormous advantage with regard to area as the number of QCA cells remains the same. For our proposed carry ripple adder and carry look ahead adder the number of cells is approximately 200 and 185 cells respectively. QCA cells are extremely area efficient for digital circuits. It typically has an area of $100nm^2$ with a cell dimension of 10nm. A table depicting the area advantage posed by our design in comparison to existing adder is as shown in Table. 6.1.

	Approx. Cell Count / Approx. Area (nm ²)								
	1-bit	4-bit	4-bit 8-bit 16-bit n-bit						
Custom Carry Ripple Adder			200/600						
Custom Carry Look Ahead Adder	185/550								
Existing Adder (3MV Design)	150/400	600/1600	1200/3200	2400/6400	150*n/400*n				
Existing Adder (5MV Design)	180/425	720/1700	1440/3400	2880/6800	180*n/425*n				

Table 6.1: Cell Count per Unit Area Comparison

6.3 Latency

In this section, we try to compare the time taken for the first output to be available along with the time taken for the next set of inputs to be fed into the circuit. Since the proposed designed in a pipelined manner, the next set of inputs can be fed in even before the first output is available. This is an added advantage since the next set of outputs will be available in successive time intervals. A table depicting the advantages of our proposed design is as shown in Table. 6.2. The advantage of the proposed adder is clearly visible as the number of bits is scaled linearly. For example, let's consider a 16-bit adder, even though we get the first 16 bits of output as early as 64th cycle in the existing when compared to 86th or 88th cycle in the proposed adders, we find that, to get the second set of outputs we need to wait until the 192nd cycle whereas we get the output as early as 166th or 168th cycle in our proposed adders. The major reason for such long delays in the existing adders is that, the cells needs to be de-computed before the next set of inputs can be fed into the circuit. As a result of this even though the first set of outputs are available in the 64th cycle, one can't feed the next set of inputs until the 128th cycle whereas in our proposed adders we feed the next set of inputs even before the first set of outputs have arrived.

	Custom Carry Ripple Adder			Custom Carry Look Ahead Adder			Existing Adder (3MV Design)			Existing Adder (5MV Design)						
Number of Bits	1	4	16	n	1	4	16	n	1	4	16	n	1	4	16	n
Time for 1st set of Outputs (Clock Cycles)	11	26	86	5n+6	13	28	88	5n+8	4	16	64	4n	4	16	64	4n
Time for 2nd set of Inputs (Clock Cycles)	5	20	80	5n	5	20	80	5n	8	32	128	8n	8	32	128	8n
Time for 2nd set of Outputs (Clock Cycles)	16	46	166	10n+6	18	48	168	10n+8	12	48	192	12n	12	48	192	12n

Table 6.2: Latency Comparison

6.4 Speed with respect to Area and Power (SwAP) Analysis

Using the results from the previous sections such as performance, speed, area and energy, one single cost comparison can be generated. This is done by combining all the parameters into a single figure of merit. This parameter becomes a measure of operations per cycle per power area. This is basically taking the operations per cycle, this is nothing but the time taken to compute an output for a given set of inputs divided by the minimum power dissipated across the entire circuit and divided by the total area. Minimum power dissipated is nothing but the amount of energy dissipated per unit time. We call this figure of merit as SwAP. SwAP is just a set of bounds which are used for comparison.

$$SwAP = \frac{(Ops/Sec)}{Power * Area}$$
$$= \frac{(Ops/Sec)}{(Energy/Sec) * Area}$$
$$= \frac{Ops}{Energy * Area}$$
(6.12)

	SwAP * 10 ²⁹ (Ops/J cm ²)									
	1-bit 2-bit 4-bit 8-bit 16-bit 32-bit									
Custom CRA				70.84						
Custom CLA		72.81								
Existing Adder (3MV Design)	290.3	87.1	24.18	6.39	1.64	0.417	0.104			
Existing Adder (5MV Design)	273.2	81.96	22.67	6.01	1.54	0.392	0.098			

Table 6.3: SwAP Analysis

The SwAP Analysis for the various adders is as shown in Table. 6.3. From Table. 6.3 it's clearly evident that the SwAP remains a constant for the proposed adders since the energy dissipated and the area remains a constant due to it's inbuilt pipelined nature. If we compare with the existing adders, we can find that SwAP are quadruple times higher than proposed adders for a 1-bit adder, but as we scale the number of bits it's clearly visible that for a 2-bit adder it's just marginally higher and is comparable to the proposed adders. But after 2bits SwAP is very less as the number of bits increases. For example, when we consider a 64-bit adder we can find that the SwAP for the proposed adders is almost 70times better and efficient than the existing adders which is a considerable gain. Even though the latency varies for different adders as seen in previous section, the SwAP for the proposed adders remain a constant because as the computational throughput changes, the power dissipated also changes by the same factor with both energy and area being a constant.

Thus, from the above mentioned sections we have clearly established the advantages of the proposed adders over the existing adders. If we take a look at the existing adders, if it has advantage in terms of power and latency, it loses in terms of area. Similarly if it has area and latency advantage, it loses out on power. So it doesn't have a clear advantage in all the three areas put together for an n-bit adder designs for n values greater than 4 which have been thoroughly established by the SwAP analysis. A graph depicting this comparison is as shown in Fig. 6.3.



Figure 6.3: Graphs comparing the proposed adders with existing adders

CHAPTER 7

CONCLUSION AND FUTURE WORK

The goal of this work was to exploit the inbuilt reconfigurable nature of QCAs and design energy efficient circuits. During the initial phase of this research, a custom reconfigurable technique based on switching the polarity of the majority gates was established. The efficiency and the gains provided by the custom technique was demonstrated with the help of an XOR gate designed based on the custom reconfigurable technique and compared with existing designs. During the next phase, it was decided to design larger circuits and figure out how such a design could be helpful. It started with the designs of a 1-bit carry ripple adder and 1-bit carry look ahead adder. During the final phase, the design was extended to n-bit adders and was compared with existing adders for efficiency in terms of energy and area. A broad conclusion that can be drawn from this work is that reconfigurability does offer a lot of savings in terms of power and area. But if one takes a closer look at the results, it can be said that reconfigurability in QCA adds a new dimension to designing of circuits as the area and power are reduced considerably and it could pave for smaller and power efficient circuit designs in future with a no impact on the computational throughput. The designs considered here may not be practically feasible, unless the underlying clocking network allows control individual cells or small group of cells, it opens up various avenues for further research in reconfigurable QCA. A Summary of the work presented in this thesis is detailed in the undermentioned section.

7.1 Summary

The first chapter gave an insight into what was the major driving force in taking up this thesis and gives an idea to the readers on what to look out for in this thesis. The second chapter gave a detailed technical background on QCA's beginning with the origin of QCA's to existing work on reconfigurability in QCA's. Some of the intricate details such as clocking and wire crossings in QCA was explained in this chapter. The proposed reconfiguration scheme along with the custom wire crossing technique was established in chapter three with supporting figures and timing diagrams. Once the proposed design methodology was clearly established in the third chapter, the power estimation methodologies along with the analysis of the proposed design was explained in the fourth chapter. The fifth chapter consisted of the existing background work on QCA adders and introduces the proposed adder designs based on the reconfiguration scheme established in the previous chapters. The set of metrics used for comparison of the proposed adders and existing adders along with the comparison results was clearly detailed in sixth chapter. Finally the thesis was concluded with future work suggestions in the last chapter.

Some of the main contributions of this work are as follows:

- A novel design technique for low power computation based on reconfiguration in QCA's.
- Routing issues in complex designs addressed through the custom wire crossing technique.
- Design of simple arithmetic circuits like full adders using the proposed reconfiguration scheme.

7.2 Future Work

Even though we have taken a big step forward in the design of energy efficient circuits there are still many open ended questions which needs to be answered. One interesting future work will be to design the underlying clocking network.

Some of the other areas where one can focus their research include the combining of two 32-bit adders to design a single 64-bit adder and compare it with both the existing designs and the designs proposed as part of this thesis.

Research is also needed for using the proposed custom wire crossing techniques in more complex circuits and the principal of using the dead computation time in Bennett clocking scheme should be exploited.

Circuits that make use of both Laundeur and Bennett clocking scheme can be designed and tested for efficiency in terms of power and performance.

APPENDIX A

RECONFIGURABILITY AND NANOCOMPUTING

A.1 Single Electron Transistor

Uchida et al [65] proposed the very first programmable Single Electron Transistor (SET) logic which had a non-volatile memory function. The logic was able to perform different functions based on the status of the memory. A few years later, Hai et al [35] proposed the reconfigurable logic gates based on the single electron spin transistors. This method was further improvised by Sui et al [12] who proposed the reconfigurable SET based on SET inverters without the need for the non-volatile memory function. A thorough investigation of the reconfigurable logic cells based on SET and MOS-FET hybrid circuits have been done by the same authors where in Depending upon the configurable NAND/XNOR cell or a reconfigurable XNOR/XOR cell. A comparative study of the proposed reconfigurable SET design with the traditional MOSFET circuits with regard to power and performance has also been presented in their work where in they have clearly shown the power savings in the reconfigurable design.

A.2 Chemically Assembled Electronic Nanotechnology

Chemically Assembled Electronic Nanotechnology (CAEN) is a promising technology, which uses self-alignment to construct electronic circuits from nano scale devices that takes advantage of quantum mechanical effects. It uses chemical synthesis techniques to construct molecular-sized circuitry elements such as, resistors, transistors, diodes, resonant tunneling diodes (RTDs), and reconfigurable switches. The basic block of this reconfigurable interconnect grid is a molecular diode which can be configured off or on. The following diagram (Fig. A.1) shows how a 3x3 grid is configured to implement an AND gate [33]. A serious drawback to CAEN is the inability to include three-terminal devices (e.g., transistors) in circuits.



Figure A.1: AND gate using reconfigurable CAEN grid (From. ref. [33])

A.3 Nanowire and Carbon Nano Tubes

As VLSI technology continues to scale beyond the end of the roadmap for silicon based technologies, VLSI design is expected to be based on non-silicon nanoscale devices, e.g., carbon nanotubes (CNTs). Such nanoscale devices are expected to be manufactured based on bottom-up self-assembly processes, rather than the traditional top-down lithography based VLSI manufacturing processes [60]. Thus, unconventional architectures are often desirable. Lieber et al have proposed programmable nanowire circuits for nano processors where in they have defined the logic tile consisting of 2 interconnected arrays with 496 functional configuration FET Nodes. With the help of active reprogramming of the nodes, they were able to make the same circuitry work as Full Adder, Multiplexor, De-multiplexor among others [72]. DeHon et. al. [19, 20] have shown how to organize the CNTs, SiNWs and molecular-scale devices that are now being developed into an operational reconfigurable computing system which is commonly referred to as NanoPLA. The molecular-scale wires can be arranged into interconnected, crossed arrays with switching devices at their cross points. A typical Nano wire crossbar architecture is as shown in Fig. A.2 Goldstein et al [17] proposed a similar homogenous reconfigurable architecture known as NanoFabric which makes use of CAEN as discussed earlier. Lieber research group of Harvard University refer to such computing paradigms as the Universal Computing Architecture (UCA). Such nanoscale computer architectures share common characteristics they support unconventional nanoscale manufacturing paradigm via simple homogeneous periodic structures and reconfigurability for post-fabrication design mapping. In general, nanowire-based reconfigurable systems based on the UCA concept are referred to as the Nanowire Reconfigurable Crossbar Architecture (NRCA). Wu et al have proposed an asynchronous nano - architecture which is based on the delay insensitive data encoding and self timed logic known as the Null Convention Logic (NCL), this makes the design totally clock free. In the previous mentioned synchronous design, there is a huge manufacturing difficulty owing to the complex clock distribution schemes which has been overcome in this design which saves on circuit area since we can remove all the hardware components related to the clock.



Figure A.2: Typical NanoWire Crossbar Architecture (From. ref. [20])

A.4 Complementary Nano Electromechanical Switch

Bhunia et al [10] have proposed a reconfigurable logic based upon the complementary Nano Electromechanical Switch (CNEMS). The basic structure of these devices consists of three co-planar carbon nanotubes arranged so that the central nanotube can touch the two side carbon nanotubes upon application of a voltage pulse between them. These devices have very low leakage current, low operation voltages, and have built-in energy storage to reduce computation power, resulting in very low overall power dissipation. Once configured in a certain way, the switch remains in the same state until an opposite electric field is applied to reconfigure the switch. Due to this latching mechanism, each switch works as a non-volatile memory element. Therefore, CNEMS is a good candidate for memory implementation and can potentially replace conventional charge-based CMOS static and dynamic RAM since it's immune to soft errors. It has been proposed that CNEMS be used in the conventional FPGA based look up Tables for storing data and these switches can be dynamically reconfigured.

A.5 CMOL

A promising cell based digital logic architecture which has caught the eye of many researchers around the world is the CMOL architecture [42] which combine a semiconductor transistor (CMOS) stack and two levels of parallel nano wires, with molecularscale nano devices formed between the nano wires at every crosspoint. Since this architecture is very similar to the cell based FPGA, the authors have proposed a reconfigurable architecture different from the conventional ones. A field-programmable nanowire interconnect (FPNI) improves on a field-programmable gate array (FPGA) architecture [56] by lifting the configuration bit and associated components out of the semiconductor plane and replacing them in the interconnect with nonvolatile switches, which decreases both the area and power consumption of the circuit. This FPNI architecture has been successfully tested in HP labs. Dong et al [25] proposed a novel reconfigurable architecture 3D nFPGA based on CMOS hybrid circuits and the 3D integration techniques. They also have proposed a dense 2D FPGA architecture based on carbon nanotubes known as FPCNA [26]. Both of these architectures are derived from the popular island-style FPGA architecture. In island-style FPGAs, reprogrammable devices are arranged in regular tiles, where each tile contains one configurable logic block (CLB), two connection blocks (CBs), and one programmable switch block (SB), as shown in Fig. A.3.



Figure A.3: Island Style FPGA Architecture (From. ref. [26])

APPENDIX B

POWER DISSIPATION MODELS

B.1 Upper Bound Power Dissipation Model

The fundamental power dissipation model which computes the upper bound is based on this quantum mechanical model [58, 57]. Such upper bound represents the worst case power dissipation, which happens in the presence of non-adiabatic clocking. The authors have developed a probabilistic Bayesian model where the probabilities directly maps on to the quantum - mechanical steady state probabilities which are nothing but the density matrix and the cell polarizations.

Two possible, orthogonal, eigenstates of a QCA cell is denoted by $|1\rangle$ and $|0\rangle$. The state or wave function at time t, $|\psi(t)\rangle$, evolves according to Schrodinger equation, driven by the underlying Hamiltonian H, which is 2 by 2 matrix using the Hartree approximation [64].

$$H = \left(\begin{bmatrix} -\frac{1}{2} \sum_{i} E_{k} s_{i} f_{i} & -\gamma \\ -\gamma & \frac{1}{2} \sum_{i} E_{k} s_{i} f_{i} \end{bmatrix} \right)$$
$$= \left(\begin{bmatrix} -\frac{1}{2} S & -\gamma \\ -\gamma & \frac{1}{2} S \end{bmatrix} \right)$$
(B.1)

 E_k is the energy cost of two neighboring cells with opposite polarizations; this is also referred to as the kink energy. f_i is the geometric factor capturing electrostatic fall off with distance between cells. s_i is the polarization of the i-th neighboring cell. The tunneling energy between the two states of a cell, which is controlled by the clocking mechanism, is denoted by γ . For notational simplification, we will use S to denote the total kink energy due to the polarized neighbors.

The expected value of any observable, $\langle \hat{A}(t) \rangle$, can be expressed in terms of the wave function as $\langle \hat{A} \rangle = \langle \psi(t) | \hat{A}(t) | \psi(t) \rangle$ or equivalently as $T_r[\hat{A}(t) | \psi \rangle \langle t \rangle \langle \psi(t) |]$, where T_r denotes the trace operation, $T_r[...] = \langle 1|...|1 \rangle + \langle 0|...|0 \rangle$. The term $|\psi(t)\rangle \langle \psi(t)|$ is known as the density operator, $\hat{\rho}(t)$. Expected value of any observable of a quantum system can be computed if $\hat{\rho}(t)$ is known.

Energy (and power) can be estimated by computing the expected Hamiltonian using this density matrix. However, for compact mathematical representation of power dissipation the Bloch formulation of the Schrodinger equation, which expresses the evolution of quantum systems in operator spaces, is used. In this formulation, the expected value of cell energy $\langle \hat{H} \rangle$ at any time is given by:

$$E = \langle \widehat{H} \rangle = \frac{\hbar}{2} \overrightarrow{\Gamma} \cdot \overrightarrow{\lambda}$$
(B.2)

where $\overrightarrow{\Gamma}$ and $\overrightarrow{\lambda}$ are the Hamiltonian and coherence (state) vectors respectively. These are arrived at by expressing the density operator as a linear combination of the Paulis spin operator $\sigma_i : \rho(t) = \sum_{i=1}^3 \lambda_i \sigma_i$, where $\lambda_i = T_r\{\widehat{\rho}\widehat{\sigma}_i\}$. The two state Schrodinger Hamiltonian can be projected onto the Pauli basis of generators to form a real three-dimensional energy vector $\overrightarrow{\Gamma}$, whose components are $\Gamma_i = \frac{T_r\{\widehat{H}\widehat{\sigma}_i\}}{\hbar}$. The explicit form of the Hamiltonian vector corresponding to Hamiltonian is:

$$\overrightarrow{\Gamma} = \frac{1}{\hbar} [-2\gamma, 0, E_k S] \tag{B.3}$$

where S is the sum of neighboring polarizations. The Bloch equation governing the evolution of the coherence vector can be derived from the Liouville equation to be:

$$\frac{d}{dt}\overrightarrow{\lambda} = \overrightarrow{\Gamma} \times \overrightarrow{\lambda} \tag{B.4}$$

This formulation does not account for the effect of dissipative coupling with heat bath. One reasonable approximation is to add an inhomogeneous linear term to this equation to account for damping.

$$\frac{d}{dt}\overrightarrow{\lambda} = \overrightarrow{\Gamma} \times \overrightarrow{\lambda} + \xi \overrightarrow{\lambda} + \overrightarrow{\eta}$$
(B.5)

The parameters ξ and η are chosen so that they represent inelastic dissipative heat bath coupling (open world), with the system moving towards the ground state.

$$\overrightarrow{\eta} = \frac{1}{\tau} \overrightarrow{\lambda}^{ss} \tag{B.6}$$

$$\xi = -\begin{bmatrix} \frac{1}{\tau} & 0 & 0\\ 0 & \frac{1}{\tau} & 0\\ 0 & 0 & \frac{1}{\tau} \end{bmatrix}$$
(B.7)

where $\overrightarrow{\lambda}^{ss}$ is the steady-state coherence vector and τ is the energy relaxation time. If $\tau \to \infty$, it represents the absence of any dissipation. Lower the value of τ , faster the heat dissipation away from the cell. The steady-state coherence vector can be derived from the steady-state density matrix at thermal equilibrium.

$$\rho^{ss} = \frac{e^{-H/k_B T}}{T_r [e^{-H/k_B T}]} \tag{B.8}$$

where k_B is the Boltzmann constant and T is the temperature. The corresponding steady state coherence vector is given by

$$\overrightarrow{\lambda}^{ss} = T_r\{\rho^{ss}\sigma\} = -\frac{\overrightarrow{\Gamma}}{|\overrightarrow{\Gamma}|} \tanh\Delta \tag{B.9}$$

where $\Delta = \frac{\Omega}{k_B T}$, is the thermal ratio, with $\Omega = \sqrt{4\gamma^2 + S^2}$, the energy term (also known as Rabi frequency).

The instantaneous power dissipation in a single QCA cell is:

$$P = \frac{dE}{dt} = \frac{\hbar}{2} \left[\frac{d\overrightarrow{\Gamma}}{dt} \cdot \overrightarrow{\lambda} \right] + \frac{\hbar}{2} \left[\overrightarrow{\Gamma} \cdot \frac{d\overrightarrow{\lambda}}{dt} \right] = P_1 + P_2 \tag{B.10}$$

The term P_1 includes power from clock introduced into the cell P_{clock} and power gain from input to output $(P_{in} - P_{out})$ [6]. We are concerned with $P_2 = P_{diss}$ which represents the instantaneous dissipated power given by

$$P_{diss}(t) = \frac{h}{2} \overrightarrow{\Gamma}(t) \cdot \left(\frac{d}{dt} \overrightarrow{\lambda}(t)\right)$$
(B.11)

Coupling the expression for power dissipation with the damped Bloch equation we see that

$$P_{diss}(t) = -\frac{h}{2\tau} \overrightarrow{\Gamma}(t) \cdot \left(\overrightarrow{\lambda}(t) - \overrightarrow{\lambda}^{ss}(t)\right)$$
(B.12)

If the instantaneous coherence vector tracks the steady state coherence vector for that time instant, i.e., $\overrightarrow{\lambda}(t) \approx \overrightarrow{\lambda}^{ss}(t)$ then the power dissipated is very low.

High dissipation situation arises when $\overrightarrow{\lambda}(t)$ lags the changing $\overrightarrow{\lambda}^{ss}(t)$. From Eq. 10 we see that $\overrightarrow{\lambda}^{ss}(t)$ changes whenever the underlying Hamiltonian changes, which happens when (i) clock goes from low to high $(\gamma_L \to \gamma_H)$ so as to depolarize a cell, (ii) input or cells in previous clock zone switches states $(S_- \to S_+)$, and (iii) clock changes from high to low $(\gamma_H \to \gamma_L)$, latching and holding the cell state to the new state.

The energy dissipated is expressed by integrating P_{diss} over time.

$$E_{diss} = \frac{\hbar}{2} \int_0^\infty \overrightarrow{\Gamma} \cdot \frac{\overrightarrow{d\lambda}}{dt} dt$$
(B.13)

On modeling the above equations using the delta function and integrating them over the above period we can get:

$$E_{diss} = \frac{h}{2} \left(\overrightarrow{\Gamma}_{+} \cdot \overrightarrow{\lambda}_{+} - \overrightarrow{\Gamma}_{-} \cdot \overrightarrow{\lambda}_{-} - \int \overrightarrow{\lambda} \cdot \frac{d\overrightarrow{\Gamma}}{dt} dt \right)$$
(B.14)

$$E_{diss} < \frac{h}{2} \overrightarrow{\Gamma}_{+} \cdot \left(\overrightarrow{\lambda}_{+}^{ss} - \overrightarrow{\lambda}_{-}^{ss} \right)$$
(B.15)

$$E_{diss} < \frac{h}{2} \overrightarrow{\Gamma}_{+} \cdot \left(-\frac{\overrightarrow{\Gamma}_{+}}{|\overrightarrow{\Gamma}_{+}|} \tanh\left(\frac{h|\overrightarrow{\Gamma}_{+}|}{k_{B}T}\right) + \frac{\overrightarrow{\Gamma}_{-}}{|\overrightarrow{\Gamma}_{-}|} \tanh\left(\frac{h|\overrightarrow{\Gamma}_{-}|}{k_{B}T}\right) \right)$$
(B.16)

Table B.1: Bloch Hamiltonian before and after a change in clock or the neighboring polarization

	Clock Up	Driver Polarization	Clock Down
,	$(\gamma_L \to \gamma_H)$	$(S \to S_+)$	$(\gamma_H \to \gamma_L)$
$\overrightarrow{\Gamma}_{-} =$	$\frac{1}{h}[-2\gamma_L, 0, S]$	$\frac{1}{h}[-2\gamma_H, 0, S]$	$\frac{1}{h}[-2\gamma_H, 0, S_+]$
$\overrightarrow{\Gamma}_{+} =$	$\frac{1}{h}[-2\gamma_H, 0, S]$	$\frac{1}{h}[-2\gamma_H, 0, S_+]$	$\frac{1}{h}[-2\gamma_L, 0, S_+]$

Table B.1 summarizes the Bloch Hamiltonian before and after a change in clock or the neighboring polarization. With the help of this data, one could get the accurate values of the energy dissipated per unit time.

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