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Interactive Design of Complex Mechanical Parts using a Parametric Representation

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Summary. In CAD, when considering the question of new designs of complex mechanical parts, such as engine pistons, a parametric representation of the design is usually defined. However, in general there is a lack of efficient tools to create and manipulate such parametrically defined shapes.

In this paper, we show how the geometry of complex mechanical parts can be parameterised efficiently enabling a designer to create and manipulate such geometries within an interactive environment. For surface generation we use the PDE method which allows surfaces to be defined in terms of a relatively small number of design parameters. The PDE method effectively creates surfaces by using the information contained at the boundaries (edges) of the surface patch. An interactively defined parameterisation can then be introduced on the boundaries (which are defined by means of space curves) of the surface. Thus, we show how complex geometries of mechanical parts, such as engine pistons, can be efficiently parameterised for geometry manipulation allowing a designer to create alternative designs.

1 Introduction

There can be of no doubt that in the design and manufacture of complex mechanical parts the trend nowadays is towards the extensive use of computer aided techniques. There exist many Computer-Aided Design (CAD) systems, which use a variety of geometry representation schemes, e.g. B-Rep [8] , CSG [8], feature based modelling [[9] and [11]] and variational methods [[10], [5] and [7]]. Examples of existing commercial CAD systems include CATIA from Dessault Systemme, Pro/Engineer from Parametric Technology and PowerSHAPE from Delcam International. Using standard commercial CAD packages, complex mechanical parts can be built up from an intersecting series of geometric solids which form ‘primary’ surfaces and rolling ball blend surfaces between the primary surfaces [[16] and [17]]. Thus, existing commercial CAD systems can create and manipulate the geometry of complex mechanical parts, although the design process may not always be straightforward.

As Imam [6] notes, when considering the design of engineering surfaces, it is essential that CAD systems, used to create the design, be able to parameterise the geometry of the design efficiently. In parametric design, the basic approach is to develop a generic description of an object or class of objects, in which the shape is controlled by values of a set of design variables or parameters. A new design, created for a particular application is obtained from this generic template by selecting particular values for design parameters so that the item has properties suited to that application. As noted above, it is often the case that commercial systems often fail to generate a parametric model of the complex mechanical part in question. This is often due to the lack efficient tools which enable to define complex geometries in the form of a generic parametric model.

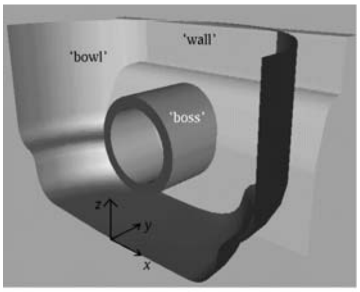

Fig. 1. The basic components of a traditional model of a piston

Consider a typical design for the inside of a piston of an internal combustion engine. The traditional design is constructed from a series of relatively simple geometric parts. Fig. 1 shows a typical design of the piston where the internal 'bowl' and the 'boss' form the major components of the piston. Using a standard CAD package, the geometry of the piston can be generated as follows.

The internal bowl of the piston is constructed by rotating a given profile through 2π about z-axis. This is intersected by two surfaces, formed by translating a second profile in the direction of the x-axis and is referred as the piston wall. The walls are intersected by two cylinders which form the boss. The result of this is shown in Fig. 1, where the other half of the (symmetrical) part is removed for the sake of clarity. The basic primary surfaces then have blends added at all sharp edges and various cuts made into them. The blends are traditionally formed as simple 'rolling ball' blends [12] .

An important point to note in the design process outlined above is the lack of consistency in creating different parts of the design. More importantly, the design process lacks the ability to easily change the design in order to take alternative designs into consideration. In other words, there is a lack of a flexible enough parametric representation to easily create alternative designs.

The aim of this paper is to show how a PDE parametric model [15] can be used to design and manipulate complex mechanical parts. The design parameters of the PDE parametric model are those introduced via the boundary conditions which specify the shape of the surface. Adjustments of the design parameters allow the user to select from a whole range of possible designs once an initial generic parameterisation has been specified. Often the initial design will be 'close' to the desired final design, so that adjustments of the design parameters will allow user to fine tune the surface shape once an initial approximation of the desired surface has been generated.

2 The PDE Method and Geometry Generation

In geometric design, it is common practice to define curves and surfaces using some form which represents the surface parametrically. Thus, surfaces are defined in terms of two parameters u and v so that any point on the surface $\underline{X}$ is given by an expression of the form:

$$\underline{X} = \underline{X}(u, v). \tag{1}$$

Equation (1) can be viewed as a mapping from a domain Ω in the (u, v) parameter space to Euclidean 3-space. In the case of the PDE method this mapping is defined as a partial differential operator:

$$L_{uv}^{m}(\underline{X}) = \underline{F}(u, v), \tag{2}$$

where the partial differential operator L is of degree m. Thus, effectively, surface design is treated as an appropriately posed boundary-value problem with appropriate boundary conditions imposed on $\partial\Omega$, the boundary of Ω. The partial differential operator L is usually taken to be that of elliptic type and the degree m of this operator depends on the level of surface control and continuity required on the shape of the surface. The function $\underline{F}(u, v)$ is included for completeness and is generally taken to be zero.

The PDE method has been discussed before by a number of different references, e.g. [16], [1] and [2]]. It has been shown how surfaces satisfying a wide range of functional requirements can be created by a suitable choice of the boundary conditions and appropriate values for the various design parameters associated with the method [[15], [13] and [4]].

2.1 Interactive Design

For the work described here, and for the majority of work carried out on the PDE method described elsewhere, the PDE chosen is of the form:

$$\left(\frac{\partial^2}{\partial u^2} + a^2 \frac{\partial^2}{\partial v^2} \right)^2 \underline{X}(u, v) = 0, \tag{3}$$

where the condition on the function $\underline{X}(u, v)$ and its normal derivatives $\frac{\partial \underline{X}}{\partial n}$ can be imposed at the edges of the surface patch. The parameter a is a special design parameter which controls the relative smoothing of the surface in the

u and v directions [2]. For periodic boundary conditions (e.g. $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$), a pseudo-spectral method has been developed for the solution of equation (3) which allows $\underline{X}(u,v)$ to be expressed in closed form [3].

As far as interactive design is concerned the boundary conditions are often defined in terms of curves in 3-space. For example, Fig. 2 shows a typical set of boundary curves and the corresponding PDE surface showing the port of a bifurcated transfer port of a 2-stroke engine. Here the value of a was taken to be 1.0100. Note that the curves marked p_1 and p_2 correspond to the boundary conditions on the function $\underline{X}(u,v)$. A vector field corresponding to the difference between the points on the curves marked p_1 and p_2 and those marked d_1 and d_2 respectively, corresponds to the conditions on the function $\frac{\partial \underline{X}}{\partial n}$ such that

$$\frac{\partial \underline{X}}{\partial n} = \left[\underline{p}(v) - \underline{d}(v)\right] s, \tag{4}$$

where s is a scalar. The conditions defined by p_1, p_2 and d_1, d_2 are known as the 'positional boundary conditions' and 'derivative boundary conditions' respectively [13]. Note that the surface patch will not necessarily pass through the curves which define the derivative boundary conditions.

Fig. 2. Typical PDE surface. (**a**) The boundary curves. (**b**) The corresponding PDE surface patch

2.2 Interactive Design using the PDE Parametric Model

In this work the definition of the shape geometry is carried out using the PDE parameter model discussed in [15], where the parameterised boundary curves are used to define the shape of the surface. Essentially, this parameterisation is defined in such a way that linear transformations, such as translation, rotation and dilation, of the boundary curves can be carried out interactively which in turn result in a change in the shape of the surface. The result of this is that the designer is presented with tools which enable him/her to create and modify the geometry an intuitive manner. For convenience, the parameterisation on the boundary curves are denoted using the notation c_{kP_i} $(k = 1,2)$, $(i = x,y,z)$. Here c defines the curve, with the letter p denoting the position curves and the letter d denoting the derivative curves. The index k ranges from 1 to 2 denoting the $u = 0$ and $u = 1$ boundary edges (respectively) of the surface. The letter P denotes the type of parameter, T for a translation, R for a rotation and D for a dilation. Finally the letter i denotes the coordinate directions relevant to a particular type of parameter. Adjustments to the values of these parameters along with the value of a in equation (3) can be used to create and manipulate complex geometries.

As mentioned earlier, the effect of these parameters on the surface shape is very intuitive. For example, Table 1 shows the values of the chosen parameters for $d = 1$ for the surface shown in Fig. 2.

Table 1. Values for the design parameters for the boundary $d = 1$ of the surface shown in Fig. 2

parameter	value
d_{1T_x}	0.000
d_{1T_y}	-0.850
d_{1T_z}	0.000
d_{1D_x}	0.771
d_{1D_y}	0.792
d_{1D_z}	0.000
d_{1R_x}	3.138
d_{1R_y}	0.000
d_{1R_z}	0.000

In order to show the effect of the design parameters we now choose a different set of values for the parameters for the boundary $d = 1$ of the surface shown in Fig. 2. The new values chosen for the parameters are shown in Table 2 and the resulting surface is shown in Fig. 3. Note the value of the a for the surface shown in Fig. 3 is the same as that shown in Fig. 2, i.e. a=1.0100. Essentially, the new values of the parameters produced a dilation followed by a translation which is followed by a rotation of the boundary curve $d = 1$. The parameters introduced on the boundary curves are varied using a graphical interface where the corresponding surface is visualised simultaneously. The spectral approximation method to the solution of the PDE, mentioned earlier, is fast enough for the surfaces to be created and manipulated in real time.

In order to build shapes corresponding to complex mechanical parts, more than one surface patch often needs to be joined together with common boundaries enabling to form a composite surface [[14] and [13]]. The parametric

Fig. 3. The effect on the shape of the surface by changing the design parameters corresponding to the boundary $d = 1$. (**a**) The boundary curves. (**b**) The corresponding PDE surface patch

Table 2. Values for the design parameters for the boundary $d = 1$ of the surface shown in Fig. 3

parameter	value
d_{1T_x}	-0.310
d_{1T_y}	-0.850
d_{1T_z}	0.000
d_{1D_x}	0.371
d_{1D_y}	0.792
d_{1D_z}	0.000
d_{1R_x}	3.138
d_{1R_y}	0.000
d_{1R_z}	0.220

model discussed above has been extended to cater for such composite bodies. For example, Fig. 4 shows a bifurcated port for a modern internal combustion engine. This composite surface has been created using three surface patches, where the final shape resulted in the manipulation of the design parameters introduced onto the boundary curves definining the composite body.

3 Interactive Design and Manipulation of an Engine Piston

In this section we show how a generic shape of a typical piston of an internal combustion engine can be created, i.e. we show how a typical design can be created from 'scratch' and show how such a design can then be modified using the PDE parametric model discussed earlier.

Fig. 4. A bifurcated port for a modern internal combustion engine created interactively.

Fig. 5. Generic geometry of the bowl of the piston. (**a**) The boundary curves. (**b**) The PDE surface patch corresponding to the bowl of the piston

Fig. 5(**a**) shows the boundary curves corresponding to the surface shown in Fig. 5(**b**). This surface defines the internal bowl of the piston. Note that the boundary curve p_1 corresponding to the positional boundary condition at $u = 0$ of the internal bowl is scaled down to a single point. This is carried out by choosing very small values for the dilation parameters p_{1D_x}, p_{1D_y} and p_{1D_z}.

In order to accommodate the boss, part of the internal bowl of the piston needs to be removed. This is carried out by using a curve drawn on the $(u,\ v)$ parameter space corresponding the surface. Since the points in $\Re^3$ corresponding the $(u,\ v)$ points of any curve drawn on the parameter space is guaranteed to lie on the surface, the shape of the portion to be removed from the surface can be determined by choosing an appropriate shape of the curve on the $(u,\ v)$ parameter space. In the case of the shape shown in Fig. 6(**b**), the shape of the corresponding curve on the $(u,\ v)$ parameter space is an ellipse. Once the appropriate shape of the curve on the parameter space is determined, the corresponding shape can be removed. This is carried out by discarding the points which belong to interior of the curve in the $(u,\ v)$ parameter space and solving the original PDE with a reparameterisation accounting for the curve drawn on the (u, v) parameter space. Further details of how this can be carried out is discussed in [14].

Fig. 6. Part of the bowl removed to accommodate boss. (**a**) The boundary curves. (**b**) The corresponding PDE surface patch

Once the desired portion of the surface from the internal bowl is removed, a second surface corresponding to the boss can be created in the usual manner with the curve in $\Re^3$ corresponding to that in the (u, v) parameter space for the portion removed being as one of the positional boundary conditions. Fig. 7(**b**) shows a generic design for the piston where the corresponding boundary curves are shown in Fig. 7(**a**).

Fig. 7. Main components of piston created using two surface patches. (**a**) The PDE boundary curves. (**b**) The PDE surface patches corresponding to piston (the bowl and boss)

Once the initial geometry of the piston is created in the manner described above, the PDE parametric model can be used to manipulate the geometry. This enables one to create alternative designs starting with an initial design. For example, consider the shape of the pistons shown in Fig. 8. Here the geometry of the boss has been changed. This alteration from the shape shown in Fig. 7(**b**) is carried out using the dilation parameters p_{kD_z} and d_{kD_z} with

$k = 1, 2$, i.e. the boundary curves corresponding to the surface patch have been scaled along z-direction. Note that the curve on the $(u,\, v)$ parameter space since the boundary p_3 corresponds to the hole in the surface patch corresponding to the bowl of the piston, the dilation in this case is carried out on the $(u,\, v)$ parameter space. Thus, during the manipulation of the boundary condition p_3, of the piston boss, the surface corresponding to the bowl need to be recalculated.

Fig. 8. Alternative designs of the boss of the piston

4 Conclusion

In this paper we have shown how a parameterisation of a given geometry can effectively make the design and manipulation of complex mechanical parts very intuitive. In particular, we have shown how the PDE parametric model allows complex designs to be created and manipulated in an interactive environment. Since the boundary conditions play a vital role in determining the shape of the surface, the design parameters have been introduced on the boundary curves which define the shape. These parameters are chosen in such a way that simple transformations of the boundary curves can be carried out. The reason for choosing such a model is that the design parameters introduced in this manner allows a designer to easily create and manipulate complex geometries without having to know the mathematical details of the solutions of PDEs.

The PDE parameteric model discussed in this paper mainly allow global maninpulations of the shapes in terms of the boundary conditions which define them. It is an intention to extend the model to cater for local manipulation of the surfaces too.

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