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Dynamic Adjustment of Age Distribution in Human Resource Management by Genetic Algorithms

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Abstract—Adjustment of a given age distribution to a desired age distribution within a required time frame is dynamically performed for the purpose of Human Resource (HR) planning in Human Resource Management (HRM). The adjustment process is carried out by adding the adjustment magnitudes to the existing number of employees at the selected age groups on the yearly basis. A model of a discrete dynamical system is employed to emulate the evolution of the age distribution used under the adjustment process. Genetic Algorithms (GA) is applied for determining the adjustment magnitudes that influence the dynamics of the system. An interesting aspect of the problem lies in the high number of constraints; though the constraints are fundamental, they are considerably higher in number than in many other optimization problems. An adaptive penalty scheme is proposed for handling the constraints. Numerical examples show that GA with the utilized adaptive penalty scheme provides potential means for HR planning in HRM.

Key Words: Human Resource Management (HRM), Human Resource (HR) planning, Genetic Algorithms (GA), multiple constraints, dynamic systems, age distribution.

I. INTRODUCTION

Since Drucker [1] first put forward the concept of human resource (HR), great progress has been made in the theory of human resource management (HRM). The competitiveness of an organization essentially depends on HR. For middle to large scale organization, a medium to long term planning of HR is crucial to the overall success of the organization. One issue of interest in the HR planning is relevant to the age distribution. In general, young workers or employees are good at their energetic characteristics, fresh knowledge from academic institutions, less expenditure in terms of salary, etc. Senior employees, though not physically strong as the young employees, have advantages in their long experiences and capability in solving critical situations. However, the senior employees put harder burdens on the organization expenditures. Regarding to the intellectual capital, the experiences of the knowledge workers at the senior levels are indispensable to the competitiveness of the organization (see [2] and [3]). To be sustainable and viable, the organization needs to have an age distribution in a balanced aspect, i.e. an appropriate age distribution.

In real situations, it is possible that the age distribution which exists in an organization requires a adjustment into a desired age distribution. The adjustment of the present age distribution to the desired age distribution can be achieved in several ways. This paper introduces a systematic adjustment in the age profile that the HR department may consider as an approach for the modification of an existing or a present age distribution. The adjustment in the age distribution is proposed to be performed in an evolutionary manner in which the number of employees is selectively adjusted from year to year within a prescribed time frame. The adjustment of the number of employees is carried out by adding adjustment magnitudes to the existing number of employees at the selected age groups. The determination of the respective adjustment magnitudes is formulated in forms of a constrained optimization problem, where the adjustment magnitudes are the variables to be optimally determined. It is noted that there exist criteria for recruitment or lay-off at every selected age group. These criteria can be defined in terms of theoretical knowledge, practical knowledge, experiences, etc. In addition, the criteria are naturally not identical for the age groups but they must be considered fair when comparing between different age groups. The definitions of the criteria for the adjustment in each age group is, however, not within the scope of this paper and thus will not be the subject of further discussion hereinafter. Genetic Algorithms (GA) are applied as a search tool for determining the optimal values of the adjustment magnitudes.

The rest of the paper is organized as follows. Section II contains the problem formulation. Solution by GA is then described in Section III. Section IV elaborates the proposed methodology through numerical examples. Finally, the concluding remarks are made in Section V.

II. PROBLEM FORMULATION

List of Symbols

		List of Symbols		
NAge	:	The total number of age groups		
Nyear	:	The year that the adjustment in the age		
distribut	ion	is expected to meet the desired		
age distribution				
P_0	:	The age distribution at present year		
P_{NYear}	:	The age distribution at the <i>NYear</i> th year after		
		the present year		
P_D	:	The desired age distribution		
A_j	:	The <i>j</i> th age group		
t_i	:	The <i>i</i> th year		
$E_{Aj}(t_i)$:	The number of employees in the age group A_j		
at		time t_i		
E_{Aj}^{D}	:	The desired number of employees in the age		
group		A_j		
$\delta_{Aj}(t_i)$:	The adjustment magnitude of the number of		
-		employees in the age group A_j at time t_i		
R	:	The set of real numbers		
ERR	:	The total discrepancy between P_{NYear} and P_D		
max(ER	$R_{inf}(A)$	Δ)): The maximum of the objective function		
	-	values for the current population in the		
		infeasible region		
$v_k(\Delta)$:	The violation magnitude of the k th constraint		
$< v_k(\Delta) >$:	The average of $v_k(\Delta)$ over the population		
c_k	:	The penalty parameter for the k th constraint		

defined at each generation *NCon* : The total number of constraints $F(\Delta)$: The fitness function

 ε : The tolerance for the 100-percent criteria

The HRM policy for an age distribution is stated as follows. For a given age distribution at present year $P_0 = \{E_{A1}(t_0), \dots, E_{Aj}(t_0), \dots, E_{ANAge}(t_0)\}$, it is desired that the age distribution $P_{NYear} = \{E_{A1} \quad (t_{NYear}), \dots, E_{Aj}(t_{NYear}), \dots, E_{ANAge}(t_{NYear})\}$ in the next *NYear* years after the present year t_0 be close to the desired age distribution $P_D = \{E_{A1}^D, \dots, E_{Aj}^D, \dots, E_{ANAge}^D\}$ as much as possible. The adjustment of the age distributions P_0 to P_{NYear} is achieved via the consecutive adjustment in the number of employees at various ages. Accordingly, a mathematical expression which represents such a procedure can be given as

$$E_{A_{j}}(t_{i}) = E_{A_{j-1}}(t_{i-1}) + \delta_{A_{j}}(t_{i})$$

; $i = 1, ..., NYear$ and $j = 1, ..., NAge$ (1)

$$E_{A_0}(t_i) = 0 \qquad ; \forall i \qquad (2)$$

 $A_j - A_{j-1} = 1 \qquad \qquad ; \forall j \qquad (3)$

and $t_i - t_{i-1} = 1$; $\forall i$ (4)

, where $E_{Aj}(t_i) \in \Re$ and $\delta_{Aj}(t_i) \in \Re$. Note that (3) says the difference between the consecutive age group is equal to 1

year old. The number of employees $E_{Aj}(t_i)$ is taken as percentage of total number of employees, i.e.

$$\sum_{j=1}^{NAge} E_{Aj}(t_i) = 100, \ \forall i$$
(5)

This adjustment has to be decided and implemented by the HRM department and is the subject of the study in this paper. When $\delta_{Aj}(t_i)$ equals to 0's for all A_j 's and t_i 's, (1) is just an age evolution for every passing year. Consequently, (1) describes both the dynamics of age evolution and the process of the age-distribution adjustment. Equation (1) may be extended to the case of resignation. However, it is beyond the scope of this study.

According to the HRM policy for an age distribution above, the following optimization problem is formulated:

$$\begin{array}{l}
\underset{\delta_{Aj}(t_{i})}{Min} ERR = \sum_{j=1}^{NAge} \left(E_{Aj}^{D} - E_{Aj}(t_{NYear}) \right)^{2} \\
; \ i = 1, \dots, NYear \ \text{and} \ \ j = 1, \dots, NAge \\
\text{or} \qquad \underset{\delta_{Aj}(t_{i})}{Min} ERR = \sum_{j=1}^{NAge} \left(E_{Aj}^{D} - E_{Aj}(\delta_{Aj}(t_{i})) \right)^{2} \\
; \ i = 1, \dots, NYear \ \text{and} \ \ j = 1, \dots, NAge \\
\end{cases} (6.1)$$

, subject to $E_{Aj}(t_i) \ge 0$; $\forall i \text{ and } \forall j$ (7)

And
$$\sum_{j=1}^{NAge} E_{Aj}(t_i) = 100 \quad ; \forall i$$
 (8)

in which $\delta_{Aj}(t_i)$, i = 1,...,NYear and j = 1,...,NAge, are the variables to be optimally determined. A set of inequality constraints (7) are required for the non-negativity of the number of employees at all times. The 100-percentage criteria of the number of employees at all times, as given previously by (3), are preserved by a set of equality constraints (8). The optimization problem considered here involves the dynamics of a system subjected to multiple constraints.

III. SOLUTION BY GENETIC ALGORITHMS (GA)

GA is a stochastic search technique based on the mechanism of natural selection. It combines Darwin's principle of survival of the fitter and a structured information exchange using randomized operators to evolve an efficient search mechanism (see e.g. [5],[4],[6],[9], or [10]).

GA is naturally suitable to combinatorial optimization problems. Since the problem considered here is a combinatorial optimization problem in which the combination of $\delta_{Aj}(t_i)$'s that leads to the optimal solution needs to be determined, GA is selected as a tool for searching the optimal values of $\delta_{Aj}(t_i)$'s. A binary coding for

real value (see e.g. [4]) will be used to represent $\delta_{Aj}(t_i)$. The combination of these strings forms a chromosome.

The fitness function of a chromosome $F(\Delta)$ is defined as

$$F(\Delta) = \frac{1}{O(\Delta)}$$

; $\Delta = \{\delta_{Aj}(t_i) | i = 1, ..., NY ear \text{ and } j = 1, ..., NAge\}$ (9)

, in which $O(\Delta)$ is defined as

$$O(\Delta) = \begin{cases} ERR(\Delta) & ; \Delta \text{ is feasible} \\ ERR(\Delta) + \sum_{j=1}^{NCon} c_k v_k(\Delta) & ; \Delta \text{ is infeasible} \end{cases}$$
(10)

As seen from (9) and (10), a penalty approach is employed to handle the constraints. The penalty approach is the well-known and widely applied technique for handling constraints in GA [11]. In order to circumvent the difficulty in assigning the penalty factor C_k , the method of adaptive penalty is selected for this study. Different schemes in the method of adaptive penalty are proposed (see, for example, in a recent comprehensive survey by [11]). More specifically, an adaptive penalty scheme which had been proposed by [7] and was then modified by [8] is utilized herein. Such an adaptive penalty scheme has an advantage of its algorithmic simplicity. Accordingly, the penalty factor C_k is given by

$$c_{k} = \left| max \left(ERR_{inf} \left(\Delta \right) \right) \right| \frac{\langle v_{k} \left(\Delta \right) \rangle}{\sum_{l=1}^{NCon} [\langle v_{l} \left(\Delta \right) \rangle]^{2}}$$
(11)

Note that the equality constraint (8) is modified to be an inequality constraint

$$\frac{\left|\sum_{j=1}^{NAge} E_{Aj}(t_i) - 100\right|}{100} \le \varepsilon \qquad ; \forall i \qquad (12)$$

The tolerance ε can be arbitrarily set but is normally a small value.

IV. NUMERICAL EXAMPLES

Three cases of study are considered in this paper. In all cases, the range of age groups includes the age from $A_1 = 25$ years old to $A_{35} = 59$ years old. This implies that the youngest age that will be taken into the organization is 25 and the employee retires from the organization after 59 years old. *NYear* is equal to 5. In other words, the desired age distribution is target at the next 5 years after this present year. The present age distribution P_0 and the desired age

distribution P_5 are shown in Figure 1. The tolerance for the 100-percent criteria ε is set to be 0.01.



Figure 1. The present age distribution P_0 and the desired age distribution P_5 at 5 years.

The first case is considered as follows. The adjustment of the number of employees is performed specifically at the age groups $A_1 = 25$, $A_{11} = 35$, $A_{16} = 40$, and $A_{26} = 50$. The adjustments in these age groups are time-invariant. That is, $\delta_{A1}(t_i) = \delta_{A1}$, $\delta_{A11}(t_i) = \delta_{A11}$, $\delta_{A16}(t_i) = \delta_{A26}$, whereas the other adjustments are equal to null. Total number of constraints *NCon* is equal to 30.

Ten simulations of GA, each of which evolves for 500 generations, are performed. The population size is 100. The best result from all simulations is taken as the adjustment magnitudes (see Figure 2) and is used to compute the age distribution at NYear = 5 (see Figure 3). Figure 4 shows the evolution of the age distribution which indicates that the set of constraints (7) are not violated, i.e. the number of employees is greater or equal to zero. It is noted that all adjustment values are discrete. However, lines are connected between points in the figure in order to facilitate the visualization. Table 1 reports the summation of the number of employees in each consecutive year. It is clearly seen that the 100-percent criteria are satisfied under the prescribed tolerance.





Figure 2. Adjustment magnitudes obtained from GA for study case 1.

A distinct discrepancy between the resulting age distribution and the desired one can be perceived in Figure 3. This suggests that the adjustment at only 4 specific age groups is not sufficient to alter the present age distribution to the desired one within the required time frame of 5 years. A larger number of age groups may be necessary for accelerating the adjustment process of the age distribution. Corresponding to this notion, the following adjustments are introduced: $\delta_{Aj}(t_i) = \delta_{Aj}$, where j = 1,...,35. Thus, the adjustments are performed from the age of 25 years old to that of 59 years old. Consequently, total number of constraints *NCon* is equal to 185.



Figure 3. Age distribution at *NYear* = 5 for study case 1.



Figure 4. Evolution of age distribution for study case 1.

Table 1: Summation of the number of employees in each year for study case 1.

	$\sum_{j=1}^{35} E_{Aj}(t_i)$
Present	100.00
Year 1	100.67
Year 2	100.85
Year 3	100.52
Year 4	99.69
Year 5	99.87

In this second case, the number of simulations of GA, the number of generations, and the population size are the same as in the first case. The best result from all simulations is taken as the adjustment magnitudes (see Figure 5) and is used to compute the age distribution at NYear = 5 (see Figure 6). All the results are shown in Figures 6 and 7, respectively. Figure 7 and Table 2 show that all the constraints are satisfied.

Change Number (%) 2.50 2.00 1.50 0.00 0.50 0.00 0.50 0.00 -0.50 2.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.55 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.00 -0.00 -0.50 -0.00 -0.50 -0.00 -0.00 -0.00 -0.50 -0.00 -0.00 -0.00 -0.00 -0.50 -0.00 -0.00 -0.00 -0.00 -0.50 -0.00 -0.50 -0.00 -0.00 -0.50 -0.00 -0.50 -0.00 -0.50 -0.50 -0.00 -0.50 -0.00 -0.50 -0.50 -0.50 -0.50 -0.50 -0.00 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50 -0.50

Figure 5. Adjustment magnitudes obtained from GA for study case 2.



Figure 6. Age distribution at *NYear* = 5 for study case 2. Employee Number (%)



Figure 7. Evolution of age distribution for study case 2.



	$\sum_{j=1}^{35} E_{Aj}(t_i)$			
Present	100.00			
Year 1	100.83			
Year 2	100.63			
Year 3	100.81			
Year 4	99.58			
Year 5	99.98			

The comparison between both cases of study show that the adjustment process to the age distribution can be significantly improved when the adjustment of the number of employees is applied simultaneously on all age groups under a given time frame. However, the adjustment process according to the second case of study is rather hypothetical in that every age group is modified. Instead, a limited number of age groups should be considered. In the third case of study, the following adjustments are introduced: $\delta_{Ai}(t_i) = \delta_{Ai}$, where $j = 1, \dots, 31$. Thus, the adjustments are performed from the age of 25 years old to 55 years old. The corresponding total number of constraints NCon is equal to 165. GA is then employed to search for the optimal adjustment magnitudes. The number of simulations of GA, the number of generations, and the population size are the same as in the first case. Figures 8 to 10 show the numerical results for the third case of study.



Figure 8. Adjustment magnitudes obtained from GA for study case 3.



Figure 9. Age distribution at *NYear* = 5 for study case 3.



Figure 10. Evolution of age distribution for study case 3.

The performance of each respective HR planning scheme, i.e. the adjustment process, can be visualized using Figures 11 to 13, in which the histogram of the absolute deviation of the computed age distribution and the desire one are depicted, respectively. The histograms confirm that the second and third adjustment processes lead to better age distributions than that in the first case of study. There is no distinct difference in the absolute deviation magnitude between the second and the third case. In addition, the absolute deviation in those two cases lies in the lower order of magnitude when compared with the first case study. The absolute deviation magnitude in the two cases, i.e. second and third ones, is distributed mainly in the range of 0-1 % while the one in the first case is found the range of 0-4%. In this regard, the adjustment process with the adjustment magnitudes applied at a spectrum of age group is an alternative when the rapid convergence to the desired age distribution within a limited time frame is desired.



Figure 11. Histogram of absolute deviation for study case 1.



Figure 12. Histogram of absolute deviation for study case 2.



Figure 13. Histogram of absolute deviation for study case 3.

Although a simple mathematical model is considered in this paper, the following technical aspects should be noted. First, it is shown here that GA reveals satisfactory performance in context of dynamic systems or time-variant problems where the objective function in general can be an implicit function of the variables to be optimized. Second, the total number of variables to be optimized can be higher than that considered in the numerical examples, when non-constant adjustment magnitudes are considered for respective age groups. Third, the utilized adaptive penalty scheme works satisfactorily for sufficiently large numbers of constraints. Interestingly, the number of constraints is relatively high when compared with many other optimization problems. Such a large number of constraints are attributed by the dynamic aspect of the problem, from which the constraints are imposed at every time step. Yet, there are many constraints at each time step. Total number of constraints is even dramatically increased when the number of time steps becomes high. Since the constraints that are considered herein are limited to those relevant to the age only, it is expectable that the number of constraints becomes extremely high in practical HRM. Other kinds of constraint include the financial constraints, performance constraints, merit constraints, etc. Therefore, constraint handling is a critical issue in the application of GA to HRM. Third, the adjustment magnitudes that are used in the determination of the age distribution are selected from the best GA solution. There are other GA solutions that yield the same order of total discrepancy magnitude (ERR). In other words, there are other alternative sets of adjustment magnitudes. The number of alternative sets can be filtered out down to a smaller number of sets by imposing additional constraints. With respect to other possible alternative sets, HR planning for HRM is a multi-modal optimization. Therefore, GA for multi-modal optimization is required when several alternative sets of the adjustment magnitudes are desired.

V. CONCLUSION

Genetic Algorithms (GA) are applied to Human Resource Management (HRM) for Human Resource (HR) planning on the age distribution. Given a present age distribution, the age distribution is dynamically modified by changing the number of employees at pre-determined age groups in order to approach the desired age distribution. The adjustment magnitudes of employee are determined using GA with an adaptive penalty scheme. The numerical studies reveal that GA can be a tool in the area of HRM. However, handling of large numbers of constraint and consideration on multi-modal characteristics should be a point of future investigation.

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