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3D FACIAL DATA FITTING USING THE BIHARMONIC EQUATION

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ABSTRACT
This paper discusses how a boundary-based surface fitting approach can be utilised to smoothly reconstruct a given human face where the scan data corresponding to the face is provided. In particular, the paper discusses how a solution to the Biharmonic equation can be used to set up the corresponding boundary value problem. We show how a compact explicit solution method can be utilised for efficiently solving the chosen Biharmonic equation.

Thus, given the raw scan data of a 3D face, we extract a series of profile curves from the data which can then be utilised as boundary conditions to solve the Biharmonic equation. The resulting solution provides us a continuous single surface patch describing the original face.

KEY WORDS
3D face modelling, Biharmonic equation, PDEs

1. Introduction

The advent of efficient and affordable 3D scanning devices means that facial scanning is increasingly becoming common practise. Such scan data can be utilised for 3D face detection and authentication, facial animation as well as reconstructive facial surgery.

Though the availability of 3D facial data is considerably common, many pre-processing steps may be required before the data can be considered to be useful for practical purposes. Common pre-processing steps include removal of noise, filling missing gaps and holes within the data and simplification of data. Such pre-processing stages are often tedious and time-consuming.

A number of techniques for fitting facial data have been proposed. Commonly used methods include both Parametric [16] and non-parametric [1] deformable techniques. For example, dynamically deformable models were first proposed by Terzopoulos *et al.* [13] where a functional based on energy was proposed. Here the internal energy enables to enforce a given set of continuity constraints while an external energy function controls the closeness of fit to the original data. Other methods such as wire frame deformable model [8] can be used to obtain subject specific 3D face representations.

There exist a variety of techniques for repairing scan data especially that of the face. These methods can be broadly categorized as local and global. Lee *et al.* [9], for instance, proposed techniques for locally repairing the scanned facial data by means of a simple local interpolation of surrounding points. Carr *et. al.* [9] on the other hand proposed techniques which can directly reconstruct an entire facial surface by closely approximating the scan data. This process ensures the resulting face is hole free. Similar technique by Kähler *et. al* [3] are used for reconstructing facial scan data. Their method involves the description of a template face where correspondences are identified with the scan data so that the template model is adjusted accordingly until a close fit obtained.

In this paper we describe a technique by which we can fit a smooth surface to a given 3D facial scan data set. The method we utilise can be considered as a global method where we find a single surface patch which can adequately describe the original facial data. Our technique is based on a boundary-value approach whereby we automatically extract a series of curves from the scan data. We then solve the Biharmonic equation subject to the extracted curves as boundary conditions whereby the solution of the Biharmonic equation provides a smooth surface containing the extracted curve data.

The paper is organized as follows. In Section (2) we describe the technical details of the surface fitting methodology adopted. In particular, we describe how the chosen Biharmonic equation is solved explicitly. In Section (3) we present the results of face fitting through a series of examples. Finally in Section (4) we conclude the paper.

2. Method of 3D Facial Data Fitting

The problem of facial data fitting we address here is dealt by solving the Biharmonic equation within a parametric domain subject to a given set of boundary conditions. Thus, we define a parametric surface patch $\mathbf{X}(u,v)$ as a function of two parameters u and v on a finite domain $\Omega \subset R^2$, by specifying boundary data around the edge region of $\partial\Omega$. Here the boundary data are specified in the form of $\mathbf{X}(u,v)$ at the outer boundary $\partial\Omega$ (or border) as well as within the interior of the domain Ω. Thus, it is assumed that the coordinate of a point u and v in the parametric domain is mapped from that point in Ω to a point in the physical space within which the surface patch lie. To satisfy these requirements the surface $\mathbf{X}(u,v)$ is regarded as a solution of the standard Biharmonic equation,

$$\frac{\partial^4 \mathbf{X}}{\partial u^4} + 2\frac{\partial^4 \mathbf{X}}{\partial u^2 \partial v^2} + \frac{\partial^4 \mathbf{X}}{\partial v^4} = 0 \qquad (1)$$

Note that the motivation for using the above formulation for surface representation is that the partial differential operator associated with the Biharmonic equation (1) is an elliptic operator which posses smoothing properties. Hence, with this formulation one can see that the Biharmonic operator in Equation (1) represents a smoothing process in which the value of the function at any point on the surface is, in some sense, a weighted average of the surrounding values. In this way a surface is obtained as a smooth transition between the chosen set of boundary conditions.

2.1 Solution of the Biharmonic Equation

It is noteworthy that the Biharmonic equation in (1) is associated with various physical phenomena in both science and engineering. These include the prediction of stress/strain in physical structures [10], study of fluid dynamics in lungs [6] and image processing [15]. Thus, the solution of the Biharmonic equation is a well studied problem and therefore there exists a variety of techniques to solve Equation (1). These include Eigenfunction expansions, integrals transforms, Green's functions and numerical techniques such as finite difference and finite element method.

For the ease of implementation and efficiency in computation, in this paper we seek an explicit solution of Equation (1) subject to periodic boundary conditions. Note here periodic conditions imply that for the v parameter the condition $\mathbf{X}(u,0) = \mathbf{X}(0,2\pi)$ is satisfied. Thus, for the work described here, we restrict ourselves to periodic conditions and obtain a closed form explicit solution of Equations (1).

The specific form of the boundary conditions for generating a given surface patch can be written as,

$$\mathbf{X}(0,v) = P_0(v)\,, \qquad (2)$$

$$\mathbf{X}(s,v) = P_s(v)\,, \qquad (3)$$

$$\mathbf{X}(t,v) = P_t(v)\,, \qquad (4)$$

$$\mathbf{X}(1,v) = P_1(v)\,, \qquad (5)$$

where $0 < s,t < 1$ and $P_0(v)$, $P_s(v)$, $P_t(v)$ and $P_1(v)$ are given as curves in the physical space.

Now choosing the parametric region to be $0 \le u \le 1$and $0 \le v \le 2\pi$, and assuming that the conditions required to solve Equation (1), are periodic functions, we can use the method of separation of variables to write down the explicit solution of Equations (1) as,

$$\mathbf{X}(u,v) = \mathbf{X}(u)\cos(nv) + \mathbf{X}(u)\sin(nv), \qquad (6)$$

where n is an integer. The form of $\mathbf{X}(u)$ subject to the general boundary conditions given in (2)-(5) can be written as,

$$\mathbf{X}(u) = c_1 e^{nu} + c_2 u e^{nu} + c_3 e^{-nu} + c_4 u e^{-nu}\ , \qquad (7)$$

where c_1, c_2, c_3 and c_4 are given by,

$$\begin{aligned} c_1 = [&-P_0\left(2n^2e^{2n} + 2ne^{2n} + e^{2n} - 1\right) \\ &+ P_1\left(ne^{3n} + e^{3n} + ne^n - e^n\right) \\ &- 2P_s ne^{2n} - P_t\left(e^{3n} - e^n\right)] / d, \end{aligned} \qquad (8)$$

$$\begin{aligned} c_2 = [&P_0\left(2n^2e^{2n} + ne^{2n} - n\right) \\ &- P_1\left(ne^{3n} + 2n^2e^n - ne^n\right) - P_s\left(2ne^{2n} - e^{2n} + 1\right) \\ &+ P_t\left(e^{3n} - 2ne^n - e^n\right)] / d, \end{aligned} \qquad (9)$$

$$\begin{aligned} c_3 = [&P_0\left(e^{4n} - 2n^2e^{2n} + 2ne^{2n} - e^{2n}\right) \\ &- P_1\left(ne^{3n} + e^{3n} + ne^n - e^n\right) + 2P_s ne^{2n} \\ &+ P_t\left(e^{3n} - e^n\right)] / d, \end{aligned} \qquad (10)$$

$$\begin{aligned} c_4 = [&P_0\left(ne^{4n} + 2n^2e^{2n} - ne^{2n}\right) \\ &- P_1\left(2n^2e^{3n} + ne^{3n} - ne^n\right) + P_s\left(e^{4n} - 2ne^{2n} - e^{2n}\right) \\ &+ P_t\left(2ne^{3n} - e^{3n} + e^n\right)] / d, \end{aligned} \qquad (11)$$

where $d = e^{4n} - 4n^2e^{2n} - 2e^{2n} + 1.$

Given the above explicit solution scheme the unknowns c_1, c_2, c_3 and c_4 can be determined by the imposed conditions at u_i where $0 \le u_i \le 1$. Assuming the conditions imposed can be represented by continuous functions, which are also periodic in v, we can write down the Fourier series representation for each boundary condition such that,

$$P_i(v) = \mathbf{C}_0^1 + \sum_{n=1}^{\infty}\left[\mathbf{C}_n^1 \cos(nv) + \mathbf{S}_n^1(v) Sin(nv)\right] \qquad (12)$$

where n is the integer defined in (6) which represents the chosen Fourier mode. Thus, for each Fourier mode representing the boundary condition, an appropriate linear system involving c_1, c_2, c_3 and c_4 can be formulated which can be solved using standard methods such as LU decomposition [12].

One should note that the above solution scheme is based on the fact that the boundary conditions can be expressed as a finite Fourier series. However, in practice for facial data this cannot be assumed. We, therefore, adopt a spectral approximation to the Biharmonic equation given in [3] whereby,

$$\mathbf{X}(u,v) \cong \mathbf{X}(u)\cos(nv) + \mathbf{X}(u)\sin(nv) + \mathbf{R}(u,v). \qquad (11)$$

The idea behind the above approximate solution is to truncate the Fourier series at some finite mode M representing the contribution of the high frequency modes to the surface with a remainder function $\mathbf{R}(u,v)$. In practice M is a small integer which is usually taken to be 5. The format of $\mathbf{R}(u,v)$ is somewhat arbitrary and for this work it is taken as an exponential function similar to that given Equation (7).

The main point to bear in mind regarding the above solution method is that it enables us to represent a set of general periodic conditions in terms of a finite M Fourier series, whilst the term $\mathbf{R}(u,v)$, which acts as a correction term, enabling the conditions to be satisfied exactly.

2.2 Example Surface

Consider the curves shown in Figure 1 which represent the conditions for solving Equation (1). It is noteworthy that the requirement here is a surface resulting in a smooth interpolation of the four selected curves.

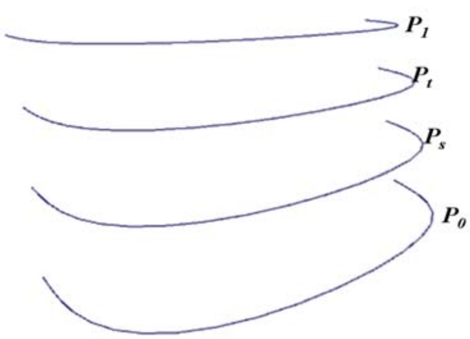

Figure 1. Curves in the physical space representing the necessary condition for the solution of the Biharmonic Equation.

The explicit solution scheme outlined above assumes that the boundary conditions are given as periodic functions. However, one could see that the boundary conditions shown in Figure 1 are not periodic. In solving the Equation (1) for this case (and the subsequent examples which follow) we therefore restrict the domain of the solution to $0 \le u \le 1$ and $0 \le v \le \pi$ which adequately describe the boundary conditions shown in Figure 1.

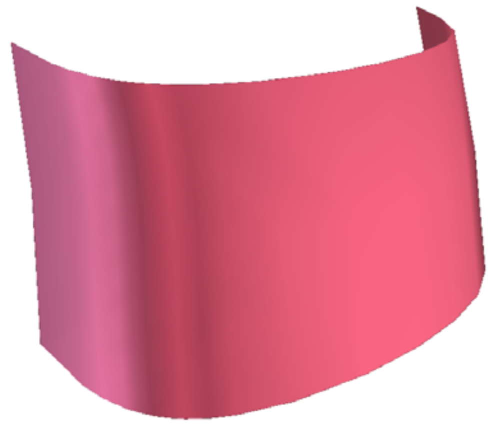

Figure 2. The surface resulting from the solution of the Biharmonic Equation subject to the conditions described in Figure 1.

Figure 2. shows the shape of the surface generated whereby the Biharmonic Equation (1) is solved subject to the conditions shown in Figure 1. One should note that the resulting surface patch contains all the boundary curves shown in Figure 1, thus producing a smooth interpolation of the boundary curves. Furthermore, the resulting surface patch is solely controlled by the four boundary curves.

It is important to point out that consideration should be given in determining the position of curves in the physical space corresponding to the (u,v) parameter space. Note that the curve P_0 corresponds to $u=0$ while the curve P_1 corresponds to $u=1$. The position of u_s and u_t is then determined by taking into consideration the physical separation of the corresponding curves. To do this, we compute the average separation of the curves P_s and P_t in the physical space by means of computing $d_i = \frac{\sum_N d_k}{N}$ where d_k is the distance between individual points on the curves and N is the number of points on each curve. Hence $u_s = \frac{d_s}{D}$ and $u_t = \frac{d_t}{D}$ where D is given to be the average physical separation of the curves P_0 and P_1.

It is also noteworthy to point out that similar surface generation schemes already exist in the literature. For example, Bloor and Wilson [2] originally proposed the method for surface generation based on the Biharmonic equation. Their original work as well as the subsequent work included solving the Biharmonic equation in a classical way whereby the usual function and derivative boundary conditions are taken at the edges of the surface patch. Thus, in these earlier schemes only the edge boundary curves are satisfied whilst the interior curves are utilised for defining the normal boundary condition at the edges of the surface patch. Detailed discussion on this method can be found in [2, 14].

3 Results

In this section we discuss facial data fitting using the above described technique.

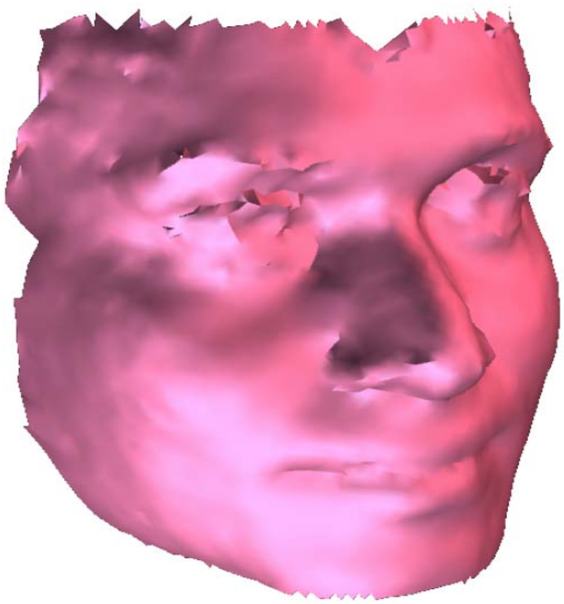

Figure 3. Scan data corresponding to a 3D face.

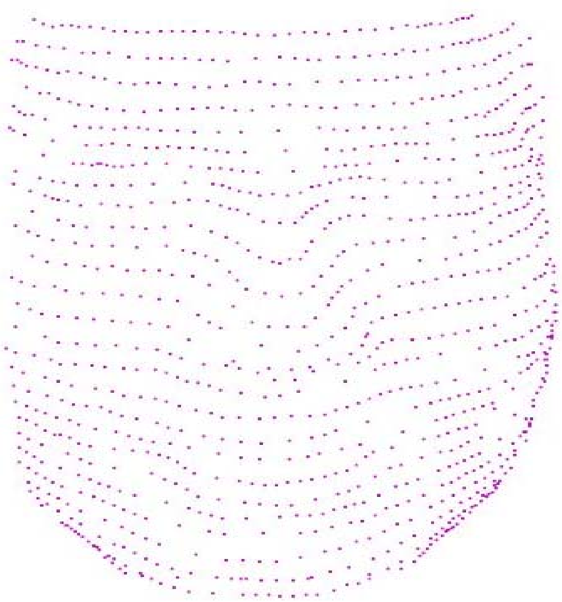

Figure 4. Data points corresponding to an ordered set of curves.

Figure 3. shows a typical facial data set which is obtained from a 3D scanner. The raw data from the 3D scanner are available as .obj files which has the information on the location of data points in the physical space and a triangular connectivity defined between these points. Thus, data from the scanner are readily viewable using existing standard 3D geometry viewers.

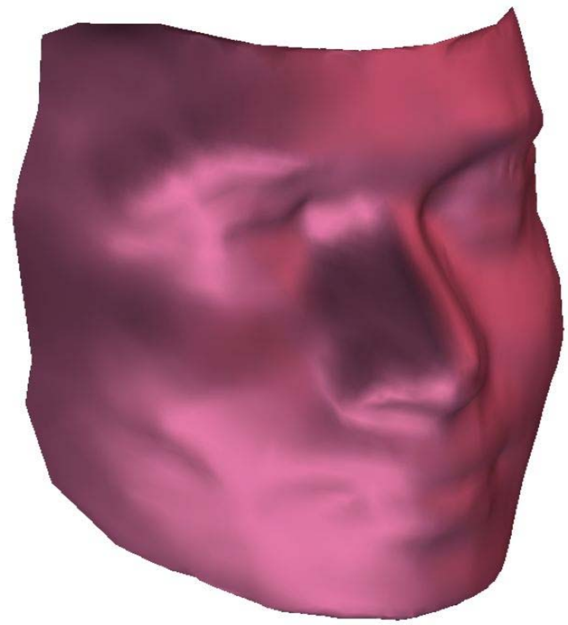

Figure 5. Reconstructed 3D face.

As a first step in fitting a smooth surface to the scan data using the method described above, we automatically extract an ordered set of points from the data. For this purpose we assume the facial data is normalized and appropriately aligned within a Cartesian coordinate system. Note there exist several techniques which enable the automatic pre-processing of data for this purpose. For example the use of PCA algorithms are common in normalizing and aligning facial data. For more detail the interested reader is referred, for example, to [11].

Assuming the data is normalized and properly aligned within a Cartesian coordinate system, we define a series of planes through which facial data passes. For each such plane we then identify the points from scan data which belong to the plane. Hence using this technique a series of profile curves are automatically extracted from the facial data. Figure 4 shows the profile curves which have been automatically extracted for the data corresponding to the face shown in Figure 3.

Once the curves are extracted, for each curve, we then fit a spline of the form $P_i = \sum_i C_i B_i$ where B_i is a cubic polynomial and C_i are the corresponding control points. This process of curve fitting to the extracted discrete curve data enables us to have a smooth curve passing through the discrete data as well as have an equal number of curve points for each profile curve.

Once the profile curves are available in the appropriate format they are then categorized into groups four with common curves in between so that appropriate number of function calls to Equation (1) can be determined.

Figure 6. Scan data corresponding to a 3D face.

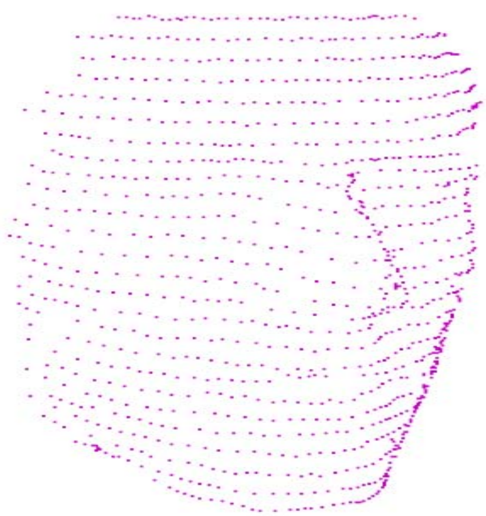

Figure 7. Data corresponding to extracted profile curves.

In order to generate a single surface patch corresponding to a given number of profile curves it is also important that the (u,v) parameter space to be defined as a single mesh entity. For this purpose we undertake a separate triangulation procedure within the 2-dimensional (u,v) parameter space using Delaunay triangulation [5]. This procedure enables us to define a triangulation within the resulting surface patch where there is a one to one correspondence between the (u,v) parameter space and the surface patch in the physical via the Biharmonic operator.

Figure 5 shows the resulting surface patch generated using the curves shown in Figure 4 for the scan data shown in Figure 3. As one can easily visualize, the reconstructed surface clearly represents the original facial shape.

Figure 8. Reconstructed 3D face.

Figure 9. Comparison of scan facial data (shown in mesh format) with the reconstructed face (shown as shaded)

To further demonstrate the techniques utilized here we take another example where the scan data is shown in Figure 6. The curve data that are automatically extracted are shown in Figure 7 while the facial surface generated is shown in Figure 8. Again as can be seen clearly there is good visual correspondence between the original facial data and that generated.

Since we are utilizing a sample of data points to define the profile curves which generates the surface corresponding to the given face, one cannot of course expect the face generated to contain all the data originally available from the scan. Figure 9 shows a comparison between the generated face with that of the original scan data where original data is overlaid (as shown in triangulated form) on the generated surface patch (shown as shaded). Again one can see that there is close agreement between the two data sets. More importantly, it can be seen that the vital features of face are preserved within the reconstructed smooth surface. It is important to highlight that the face generated through Equation (1) is smooth and thus has added benefits.

4. Conclusion

In this paper we have described a technique for fitting smooth surface to scanned facial data. We utilize a boundary value approach whereby the solution of standard Biharmonic equation is solved for appropriately defined curves in the physical space corresponding to the original facial data. The Biharmonic equation is solved explicitly which enables efficient computation of the surfaces and describes the given face as single patch surface. Furthermore, due to the availability of explicit solution we can undertake arbitrary level of surface refinement, should it be necessary.

There are several advantages of having a single patch surface with an analytic representation corresponding to a given facial data. These include efficient visualisation, efficient rendering of the facial data, intuitive manipulation of the data and efficient computation of surface quantities such as curvature. In the case of the latter it can be utilised for efficient face characterization. Apart from the above advantages, the method presented here can be also utilised for automatically repairing holes within scan data.

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