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The Flow Field Upstream Of A Horizontal Axis Wind Turbine

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THE FLOW FIELD UPSTREAM OF A HORIZONTAL AXIS WIND TURBINE

Technical Report

by

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ABSTRACT

A mathematical model is developed for a steady-state axi-symmetric upstream flow of a porous disc, in a uniform flow field. The special case of the upstream flow of a windmill, with and without a nacelle, is treated. First, the windmill is considered as a uniform distribution of sources and then as a linear distribution of sources. Solutions for the blade disc of the wind field upstream are obtained in the form of streamlines and velocity vector components.

Sample flow patterns upstream of the blade disc of the UMass 25 kW wind turbine are presented for several power levels. Documented computer programs applicable to any wind turbine are appended.

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INTRODUCTION

The problem of the exact solution of the flow through a porous disc has been of interest since the original works of G.I. Taylor [1]. This solution can have two major applications:

a) Flow through screens (power absorbing devices)

b) Flow through power developing devices.

The first point of view has to do with porous bodies which have been of practical importance in the past. Most of these bodies, which are like parachutes, fish nets, wind breaks, slotted injection discs in oil combustion chambers, and fire developments in forest, can be modeled by a screen.

The second point of view considered in this work deals with the wind power developing machines. These devices can be modeled by a very porous disc in the wind field.

The first comprehensive analysis of flow through screens was carried out by Taylor and Batchelor (1949), [1], [2]. They were interested in the effect of the screen on the turbulence, and therefore their study was oriented towards the non-uniform channel flow, passing through a flat screen. In 1959, Elder considered the more general case of an irregular-shaped and non-uniform screen in a two dimensional channel flow [3].

The problem of a finite plane screen in an infinite flow field was first considered by Küchemann and Weber (1953) [4]. Later, in 1963, Taylor considered the problem in a two-dimensional case [5]. This was done by replacing the screen with uniformly-distributed sources.

Finally, Koo and James considered the more general case of the two-dimensional flow around a submerged screen [2].

Following the idea of modeling any wind machine by the combination of sources, sinks, or vorticles, it was proposed that a windmill can be modeled by a porous screen. This work was oriented towards solving the problem of a three-dimensional porous disc in a steady-state, axi-symmetric, uniform flow. The screen was modeled by a distribution of sources and the problem was divided into two cases. First, the simple case of modeling the windmill by a uniformly-distributed disc of sources. Second, a more realistic model was considered. Taking into consideration the fact that the development of power is higher in the outer region of the windmill blades, the blade disc was modeled by a linearly-distributed disc of sources. The effect of a nacelle and its relative orientation to the blade disc was studied in both cases.

The velocity field and the streamlines were constructed for some numerical examples in association with the 25 kW windmill at the University of Massachusetts Solar Habitat I.

THEORETICAL ANALYSIS

Introduction to the Modeling Idea

The well-known idea of modeling flow fields through a combination of individual factors is used in order to model a porous disc as a distribution of sources in space within a uniform flow field.

J.K. Koo and D.F. James [2] developed G.I. Taylor's [5] idea of modeling a two-dimensional screen by a distribution of sources, by modeling the screen in a duct. This work is oriented to find a general solution for a three-dimensional screen, using a distribution of sources on a disc, in an axi-symmetric uniform flow.

The development of the basic equations is based on the following procedure: first, the potential of a source located at an arbitrary point in space is determined, second, based on this potential, the cases of a disc with a uniform or linear distribution of sources are considered and the potential on the axis of the disc founded, and third, based on the harmonic and more particularly the symmetric properties of the potential function and the solution on the disc's axis by the use of zonal harmonies, the general solution of the function is constructed.

The model is completed by the superposition of a uniform flow on the potential of the disc.

In the case of modeling a windmill, the effect of the nacelle on the disc's axis can be modeled by a single source, which can result in different body shapes.

The solution is in the form of an infinite series of the Legendre and Associated Legendre polynomials. The velocity field and streamlines are constructed by a computer program, and will be described later in this report.

Potential of a Source at an Arbitrary Point

The potential of a point source at the origin can be written as [6]:

$$\Phi = \frac{k}{4\pi} \frac{1}{|\vec{r}|}$$

where k is the source strength.

The potential at a point P of a point source at S is: (Fig. 1)

$$\Phi = \frac{k}{4\pi} \frac{1}{|\vec{r}_1|} = \frac{k}{4\pi} \frac{1}{|\vec{r} - \vec{s}|}$$

knowing

$$\vec{r} = \vec{r}(x, y, z)$$

$$\vec{s} = \vec{s}(\xi, \eta, \zeta)$$

the potential is:

$$\Phi = \frac{k}{4\pi} \frac{1}{\left[(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2\right]^{1/2}} \tag{1}$$

By using a coordinate transformation as shown in Fig. 2, the potential can be transformed to spherical coordinate as:

$$\begin{aligned} \xi &= \rho \cos\beta \\ \eta &= \rho \sin\beta \cos\gamma \\ \zeta &= \rho \sin\beta \sin\gamma \end{aligned} \quad \text{and} \quad \begin{aligned} x &= R \cos\theta \\ y &= R \sin\theta \cos\alpha \\ z &= R \sin\theta \sin\alpha \end{aligned} \tag{1 A}$$

then,

$$\Phi = \frac{k}{4\pi} \frac{1}{\left[(R\cos\theta - \rho\cos\beta)^2 + (R\sin\theta\cos\alpha - \rho\sin\beta\cos\gamma)^2 + (R\sin\theta\sin\alpha - \rho\sin\beta\sin\gamma)^2 \right]^{1/2}} \quad (2A)$$

If the source is in y - z plane, where $\beta = \pi/2$, then

$$\Phi = \frac{k}{4\pi} \frac{1}{\left[R^2\cos^2\theta + (R\sin\theta\cos\alpha - \rho\cos\gamma)^2 + (R\sin\theta\sin\alpha - \rho\sin\gamma)^2 \right]^{1/2}} \quad (2)$$

Model for the Body of a Windmill

The nacelle of a windmill can be approximated as a parabolid of revolution. This is modeled by a point source in a uniform flow [7]. (See Fig. 3).

Taking r_1 and η from the geometry of the nacelle, and assuming a ν_0, the source strength k and its position r_2 can be found by:

$$r_o = \sqrt{\frac{k}{4\pi U_o}} \quad \text{and} \quad \eta = \sqrt{\frac{k}{\pi U_o}}$$

Therefore, the potential for the nacelle in a uniform flow can be written as:

$$\Phi = \frac{k}{4\pi} \frac{1}{|\vec{r}_o|} + U_o r \cos\theta \quad (3)$$

Potential of the Distributed Disc of Sources

Introduction. The total potential Φ of disc's sources is the solution to the Laplace Equation ($\nabla^2\Phi = o$), with the appropriate boundary conditions.

Since the flow is axi-symmetric, that is, independent of α, (See

Fig. 2) the solution of $\nabla^2 \Phi = 0$ can be written as [8]:

$$\Phi = \sum_{n=0}^{\infty} \left(A_n r^n + \frac{B_n}{r^{n+1}} \right) P_n(\cos\theta) \qquad (4)$$

where: Pn(x) = Legendre Polynomial of the first kind and nth order.

Knowing that on the x-axis θ is zero and that Pn (o) = 1 [8]; the solution on the x-axis can be written as:

$$\Phi_x = \sum_{n=0}^{\infty} \left(A_n r^n + \frac{B_n}{r^{n+1}} \right) \qquad (5)$$

Hence, to determine A_n and B_n, the solution on the x-axis should be found, expanded in a power series of r, and then equated to equation (5). Consequently, the first step is to find the disc's source potential on the x-axis.

Consider a disc of distributed sources with the radius A, and source strength per unit area k as shown in Fig. 4.

The element of area dA is equal to $\rho\, d\gamma\, d\rho$ for: $o < \rho \leq A$, and $o \leq \gamma \leq 2\pi$. Using equation (2) the differential potential at any field point (r,θ) can be written in terms of the differential source distribution, as:

$$d\Phi = \frac{k'}{4\pi} \frac{1}{\left[R^2\cos^2\theta + (R\sin\theta\cos\alpha - \rho\cos\alpha)^2 + (R\sin\theta\sin\alpha - \rho\sin\gamma)^2\right]^{1/2}} \qquad (6)$$

To get the solution on the x-axis, equation (6) can be restricted to the x-axis, that is $\theta = o$. The differential potential on the x-axis is:

$$d\Phi_x = \frac{k'}{4\pi} \frac{1}{\left[R^2 + \rho^2\cos^2\gamma + \rho^2\sin^2\gamma\right]^{1/2}}$$

and,

$$d\Phi_x = \frac{k'}{4\pi} \quad \frac{1}{[R^2 + \rho^2]^{1/2}} \tag{7}$$

Integration of equation (7) depends on k. The problem can be divided into two cases: a) k is constant or a uniformly-distributed source disc and b) k is linear in ρ or a linearly-distributed source disc.

Uniformly distributed disc of sources.

(k = const)

If k is constant, equation (7) can be integrated as:

$$\Phi_x = \frac{k'}{2}\left[(R^2+A^2)^{1/2} - R\right] \tag{8}$$

Using a bionomial expansion it can be shown that, for R smaller than A,

$$\left(1+\left(\frac{R}{A}\right)^2\right)^{1/2} = \sum_{n=0}^{\infty}\left[P_n(0) + P_{n-2}(0)\right]\left(\frac{R}{A}\right)^n \tag{9}$$

and for R greater than A:

$$\left(1+\left(\frac{A}{R}\right)^2\right)^{1/2} = \sum_{n=0}^{\infty}\left[P_n(0) + P_{n+2}(0)\right]\left(\frac{A}{R}\right)^{n+2} \tag{10}$$

Therefore Φ_x in equation (8) can be expanded as [9]:

$$\Phi_x = \frac{k'A}{2}\sum_{n=0}^{\infty}\left[P_n(0) + P_{n-2}(0)\right]\left(\frac{R}{A}\right)^n - \frac{k'A}{2}\left(\frac{R}{A}\right) \qquad R < A \tag{11}$$

and

$$\Phi_x = \frac{k'A}{2}\sum_{n=0}^{\infty}\left[P_n(0) + P_{n-2}(0)\right]\left(\frac{A}{R}\right)^{n+1} \qquad R > A \tag{12}$$

The equating of equations (11) and (12) to equation (5) allows A_n and B_n to be determined for the general solution Φ. This leads to:

$$A_n = \frac{\acute{k}A}{2}\,[Pn(o) + P_{n-2}(o)]A^{-n} \qquad (*)$$

$$A_1 = \frac{\acute{k}A}{2} \qquad\qquad R<A$$

$$B_n = o$$

and,

$$A_n = o$$

$$B_n = \frac{\acute{k}A}{2}\,[P_n(o) + P_{n+2}(o)]A^{(n+1)} \qquad\qquad R>A$$

Therefore, the general solution becomes:

$$\Phi = \frac{k'A}{2}\sum_{n=o}^{\infty}[P_n(o)+P_{n-2}(o)]\left(\frac{R}{A}\right)^n P_n(\cos\theta) - \frac{k'A}{2}\left(\frac{R}{A}\right)P_1(\cos\theta) \qquad (13)$$

when $R < A$.

and for $R > A$.

$$\Phi = \frac{\acute{k}A}{2}\sum_{n=o}^{\infty}[P_n(o) + P_{n+2}(o)]\left(\frac{A}{R}\right)^{n+1} P_n(\cos\theta) \qquad (14)$$

Linearly distributed disc of sources. For a linearly distributed disc of sources, $k = m\rho$ where m is a constant. Hence equation (7) can be written as

$$d\Phi_x = \frac{m}{4\pi}\,\frac{\rho^2\,d\rho\,d\gamma}{[R^2+\rho^2]^{1/2}}$$

$*P_n(x) = o$, if $n < o$

and integration of this equation will have the following form

$$\Phi_x = \frac{m}{4\pi} \int_0^{2\pi} \int_0^{A} \frac{\rho^2 \, d\rho}{[R^2 + \rho^2]^{1/2}} \, d\gamma$$

which can be reduced to:

$$\Phi_x = \frac{m}{2} \int_0^{2A} \frac{\rho^2}{[R^2 + \rho^2]^{1/2}} \, d\rho \qquad (15)$$

In order to expand Φ_x in a power series of R, the space is divided into two regions: (See Fig. 5)

a) $R < A$

b) $R \geq A$

In region (a), each point Q (which is inside a sphere of radius A) is affected by two kinds of sources: First, those whose distance from the origin (ρ) is less than R (Radius of Point Q), that is $\rho < R$, and second, those with $\rho > R$.

Therefore, for $R < A$, the potential on the x-axis can be written as:

$$\Phi_x = \Phi_1 + \Phi_2$$

where Φ_1 is the potential of the sources with $\rho < R$ (ρ changes from zero to R) and Φ_2 is the potential of the sources with $\rho > R$ (ρ changes from R to A).

From equation (15), Φ_1 may be written as:

$$\Phi_1 = \frac{m}{2}\int_0^R \frac{\rho^2}{R^2}\left[1 + \rho^2/R^2\right]^{-1/2} d\rho$$

$$= \frac{mR^2}{2}\int_0^1 t^2(1+t^2)^{-1/2}\, dt \qquad (15A)$$

where t = ρ/R.

A bionomial expression can be written as:

$$(t^2+1)^{-1/2} = 1 + \sum_{n=1}^{\infty} \frac{-1/2(-1/2-1)(-1/2-2)\cdots\cdots(-1/2-n+1)}{n!}\, t^{2n}$$

$$= 1 + \sum_{n=1}^{\infty} a_n t^{2n} \qquad \text{for } t^2 \leq 1 \qquad (16)$$

where:

$$a_n = \frac{-1/2(-1/2-1)(-1/2-2)\cdots\cdots(-1/2-n+1)}{n!}$$

Using equation (16) it is obvious that:

$$t^2(+1+t^2)^{-1/2} = t^2 + \sum_{n=1}^{\infty} a_n t^{2n+2}$$

Using this expansion in equation (15A) results in:

$$\Phi_1 = \frac{mR^2}{2}\int_0^1 t^2 dt + \frac{mR^2}{2}\sum_{n=1}^{\infty} a_n \int_0^1 t^{2n+2} dt$$

therefore,

$$\Phi_1 = \frac{mR^2}{2}\left[\frac{1}{3} + \sum_{n=1}^{\infty}\frac{a_n}{2n+3}\right] \qquad (17)$$

Now solving for Φ_2, equation (15) can be written as:

$$\Phi_2 = \frac{m}{2}\int_R^A \rho^2 [R^2+\rho^2]^{-1/2} d\rho$$

$$= \frac{mR}{2}\int_R^A \frac{\rho}{R}[R^2/\rho^2 + 1]^{-1/2} d\rho$$

$$= \frac{mR^2}{2}\int_{R/A}^1 \frac{1}{t^3}(1+t^2)^{-1/2} dt \qquad (15\text{B})$$

where, $t = R/\rho < 1$.

Using the binomial expansion shown in equation (16), it can be shown that:

$$\frac{1}{t^3}(1+t^2)^{-1/2} = \frac{1}{t^3} + \sum_{n=1}^{\infty} a_n t^{2n-3}$$

$$= \frac{1}{t^3} - \frac{2}{t} + \sum_{n=2}^{\infty} a_n t^{2n-3}$$

Considering this result, equation (15B) can be written as:

$$\Phi_2 = \frac{mR^2}{2}\left[\int_{R/A}^1 \frac{1}{t^3} dt - \frac{1}{2}\int_{R/A}^1 1/t \, dt + \sum_{n=2}^{\infty} a_n \int_{R/A}^1 t^{2n-3} dt\right]$$

therefore;

$$\Phi_2 = \frac{mR^2}{2}\left[-\frac{1}{2} + \sum_{n=2}^{\infty} \frac{a_n}{2n-2} + \sum_{n=2}^{\infty} \frac{a_n}{2n-2}\left(\frac{R}{A}\right)^{2n-2} + \frac{1}{2}\left(\frac{R}{A}\right)^{-2} + \frac{1}{2}\ln(R/A)\right] \qquad (18)$$

It was shown that

$$\Phi_x = \Phi_1 + \Phi_2 \qquad \text{for } R<A$$

So the combination of equations (17) and (18) will result in Φ_x for $R < A$, that is:

$$\Phi_x = \frac{mR^2}{2}\left[\frac{1}{3} - \frac{1}{2} + \sum_{n=1}^{\infty}\frac{a_n}{2n+3} + \sum_{n=2}^{\infty}\frac{a_n}{2n-2} + \frac{1}{2}t_1^{-2} - \sum_{n=2}^{\infty}\frac{a_n}{2n-2}\,t_1^{2n-2} + \frac{1}{2}\ln t_1\right]$$

where, $t_1 = (R/A) < 1$

Rearranging this formula results in:

$$\Phi_x = \frac{mA^2}{2}\,t_1^2\left[\frac{1}{3} - \frac{1}{2} - \frac{1}{10} + \sum_{n=2}^{\infty}\frac{(4n+1)a_n}{(2n+3)(2n-2)} + \frac{1}{2}t_1^{-2} - \sum_{n=2}^{\infty}\frac{a_n}{2n-2}\,t_1^{2n-2} + \frac{1}{2}\ln t_1\right]$$

$$= \frac{mA^2}{2}\left[\left(\left(\sum_{n=2}^{\infty}\frac{(4n+1)\,a_n}{(2n+3)(2n-2)}\right) + \frac{8}{30}\right)t_1^2 + \frac{1}{2} - \sum_{n=2}^{\infty}\frac{a_n}{2n-2}\,t_1^{2n} + \frac{1}{2}\,t_1^2\ln t_1\right]$$

The first three terms of the expansion of $\ln t_1$ for the case of $t_1 \leq 1$ are

$$\ln t_1 \cong (t_1 - 1) - \frac{1}{2}(t_1 - 1)^2 + \frac{1}{3}(t_1 - 1)^3 = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 3t - \frac{11}{6}$$

Substitution of this in Φ_x will give the final result of Φ_x for $R < A$.

$$\Phi_x = \frac{mA^2}{2}\left[\frac{1}{2} + \left(\left(\sum_{n=2}^{\infty}\frac{(4n+1)\,a_n}{(2n+3)(2n-2)}\right) - \frac{76}{60}\right)\left(\frac{R}{A}\right)^2 + \frac{3}{2}\left(\frac{R}{A}\right)^3 - \frac{3}{4}\left(\frac{R}{A}\right)^4 + \frac{1}{6}\left(\frac{R}{A}\right)^5 - \sum_{n=2}^{\infty}\frac{a_n}{2n-3}\left(\frac{R}{A}\right)^{2n}\right] \qquad (19)$$

Equation (19) shows the potential of the x-axis for the linearly-distributed disc of sources when $R < A$. For the other region ($R > A$), equation (15) can be written as

$$\Phi_x = \frac{mR}{2}\int_0^A \frac{\rho^2}{R^2}\left[1 + \rho^2/R^2\right]^{-1/2} d\rho \qquad (15C)$$

If $t = \frac{\rho}{R} \leq 1$ and $t_2 = \frac{A}{R}$, equation (15C) can be rewritten as:

$$\Phi_x = \frac{mR^2}{2}\int_0^{t_2} t^2\left[1 + t^2\right]^{-1/2} dt$$

It was previously shown that

$$t^2(1+t^2)^{-1/2} = t^2 + \sum_{n=1}^{\infty} a_n t^{2n+2}$$

Hence,

$$\Phi_x = \frac{mR^2}{2}\int_0^{t_2} t^2\,dt + \frac{mR^2}{2}\sum_{n=1}^{\infty} a_n \int_0^{t_2} t^{2n+2}\,dt$$

A simple integration results in:

$$\Phi_x = \frac{mA^2}{2}\left[\frac{1}{3}\left(\frac{A}{R}\right) + \sum_{n=1}^{\infty}\frac{a_n}{2n+3}\left(\frac{A}{R}\right)^{2n+1}\right] \qquad (20)$$

Equations (19) and (20) are the potential of a linearly-distributed disc of sources of radius A, on the x-axis, for R smaller than A, and R greater than A, respectively.

To find the general solution, as was pointed out previously, these equations should be equated to equation (5). The result of the comparison determines A_n and B_n for both cases of $R > A$, and $R < A$.

Comparison of the equation (19) with the equation (5) shows that:

$$\left.\begin{aligned}
A_0 &= \frac{mA^2}{2}\left(\frac{1}{2}\right) \\
A_2 &= \frac{mA^2}{2}\left(\left(\sum_{n=2}^{\infty}\frac{(4n+1)\,a_n}{(2n+3)(2n-2)}\right)-\frac{76}{60}\right)\left(\frac{1}{A}\right)^2 \\
A_3 &= \frac{mA^2}{2}\left(\frac{3}{2}\right)\left(\frac{1}{A}\right)^3 \\
A_4 &= \frac{mA^2}{2}\left(\frac{3}{4}\right)\left(\frac{1}{A}\right)^4 \\
A_5 &= \frac{mA^2}{2}\left(\frac{1}{6}\right)\left(\frac{1}{A}\right)^5 \\
A_n &= \frac{a_n}{2n-3}\left(\frac{1}{A}\right)^{2n}\frac{mA^2}{2} \qquad n=2,\infty \\
B_n &= 0 .
\end{aligned}\right\} \quad R<A \qquad (21)$$

Comparison of the equation (20) with the equation (5) shows the following results.

$$\left.\begin{aligned}
A_n &= 0 \\
B_0 &= \frac{mA^2}{2}\left(\frac{1}{3}\right)A \\
B_n &= \frac{mA^2}{2}\,\frac{a_n}{2n+3}\,A^{2n+1}
\end{aligned}\right\} \quad R>A \qquad (22)$$

where:

$$a_n = \frac{-\frac{1}{2}(-\frac{1}{2}-1)(-\frac{1}{2}-2)\cdots\cdots(-\frac{1}{2}-n+1)}{n!}$$

The general solution can be determined by substitution of equations (21) and (22) in equation (4). The following results can thus be derived:

For R < A:

$$\Phi = \frac{mA^2}{2}\Big[\frac{1}{2}P_0(\cos\theta)+\Big(\Big(\sum_{n=2}^{\infty}\frac{(4n+1)a_n}{(2n+3)(2n-2)}\Big)-\frac{76}{60}\Big)\Big(\frac{R}{A}\Big)^2 P_2(\cos\theta)$$

$$+\frac{3}{2}\Big(\frac{R}{A}\Big)^3 P_3(\cos\theta)-\frac{3}{4}\big(R/A\big)^4 P_4(\cos\theta)+\frac{1}{6}\Big(\frac{R}{A}\Big)^5 P_5(\cos\theta)$$

$$-\sum_{n=2}^{\infty}\frac{a_n}{2n-2}\Big(\frac{R}{A}\Big)^{2n} P_{2n}(\cos\theta)\Big] \qquad (21)$$

And, for R > A:

$$\Phi = \frac{mA^2}{2}\Big[\frac{1}{3}\Big(\frac{A}{R}\Big)P_0(\cos\theta)+\sum_{n=1}^{\infty}\frac{a_n}{2n+3}\Big(\frac{A}{R}\Big)^{2n+1}P_{2n}(\cos\theta)\Big] \qquad (22)$$

where:

a_n = binomial coefficients

$$a_n = \frac{-\frac{1}{2}(-\frac{1}{2}-1)(-\frac{1}{2}-2)\cdots\cdots(-\frac{1}{2}-n+1)}{n!}$$

Summary. As a conclusion to part four, it is important to make the following summary. The general solution to the potential of a disc of distributed sources of radius A and source density per unit area k has been found. The solution has been determined for two cases. The solution to the uniform distribution of sources (k = const.) is shown in the equations (13) and (14). For the linear distribution of sources ($k = m\rho$), the potential is established in equations (21) and (22).

Superposition of the Potentials

To complete the flow model for a porous disc and windmill in uniform flow, the necessary potentials should be superimposed.

For the operating ease of the porous disc or a windmill without a nacelle, the potential is:

$$\Phi = \Phi_1 + \Phi_2$$

where:

Φ_1 = potential of the disc

Φ_2 = potential of the uniform flow

In the case of a windmill, the potential can be written as:

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3$$

where:

Φ_1 = potential of the disc

Φ_2 = potential of the single source at the position r_2 on the x-axis as shown in Fig. 3

Φ_3 = potential of the uniform flow

Φ_1 was established in equations, (13), (14), (21) and (22). Φ_2 can be determined from equations (1) and (3) as follows:

$$\Phi_2 = \frac{k}{4\pi} \frac{1}{[(x-\xi)^2+(y-\eta)^2+(z-\varepsilon)^2]^{1/2}} \qquad (1)$$

In this case, $\eta = \varepsilon = 0$. $\xi = r_2$

so Φ_2 can be written as

$$\Phi_2 = \frac{k}{4\pi} \frac{1}{[(x - r_2)^2 + y^2 + z^2]^{1/2}}$$

Using relations (1A), Φ_2 can be transformed to a spherical coordinate.

$$x = R \cos\theta$$

$$y = R \sin\theta \cos\alpha$$

$$z = R \sin\theta \sin\alpha$$

and

$$\xi = r_2 \cos\beta \,, \qquad \beta = 0$$

Hence:

$$\Phi_2 = \frac{k}{4\pi} \frac{1}{[((R\cos\theta) - r_2)^2 + R^2\sin^2\theta)]^{1/2}}$$

Φ_1 is the potential of a uniform flow, and it can be represented by:

$$\Phi_3 = \upsilon R \cos\theta \tag{24}$$

<u>Velocity Field</u>

<u>General Note</u>. The velocity field can be determined by superposition of the velocities. The task of this section is to find the components of the velocity vector for each potential.

The relation between the potential function and the velocity vector is known to be:

$$\vec{\upsilon} = \nabla\Phi \tag{25}$$

The gradient in spherical coordinates can be shown as:

$$\vec{\upsilon} = \frac{\partial\Phi}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\varphi}\hat{a}_\varphi$$

In the case of axi-symmetry the gradient reduces to:

$$\vec{U} = \frac{\partial \Phi}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\hat{a}_\theta \qquad (26)$$

Hence, the components of the velocity vector can be shown as

$$UR = -\frac{\partial \Phi}{\partial r}, \quad \text{and} \quad UT = -\frac{1}{r}\frac{\partial \Phi}{\partial \theta} \qquad (27)$$

where:

υR = Radial Component of the velocity vector

υT = Tangential Component of the velocity vector

Now one can supply this generate note to any special case.

Uniform distribution of Sources. For this case the potential was found and shown in equations (13) and (14) as follows:

$$\Phi_1 = \frac{k'A}{2}\sum_{n=0}^{\infty}\left[P_n(0) + P_{n-2}(0)\right]\left(\frac{R}{A}\right)^n P_n(\cos\theta) - \frac{k'A}{2}\left(\frac{R}{A}\right)P_1(\cos\theta) \quad \text{for } R<A \qquad (13)$$

$$\Phi_1 = \frac{k'A}{2}\sum_{n=2}^{\infty}\left[P_n(0) + P_{n+2}(0)\right]\left(\frac{A}{R}\right)^{n+1} P_n(\cos\theta) \quad \text{for } R>A \qquad (14)$$

a) in the case of R < A, the components of the velocity vector can be derived as follows:

$$UR1 = -\frac{\partial \Phi_1}{\partial R}$$

Therefore, the differentiation of equation (13) yields:

$$UR1 = -\frac{k'A}{2}\left[\sum_{n=0}^{\infty}\frac{n}{A}\left(\frac{R}{A}\right)^{n-1}\left[P_n(0) + P_{n-2}(0)\right]P_n(\cos\theta)\right] + \frac{k'A}{2}\left(\frac{1}{A}\right)P_1(\cos\theta)$$

Hence:

$$UR1 = \frac{k'}{2} P_1(\cos\theta) - \frac{k'}{2}\sum_{n=0}^{\infty} n\left(\frac{R}{A}\right)^{n-1}\left[P_n(0) + P_{n-2}(0)\right] P_n(\cos\theta) \qquad (28)$$

The tangential component can be written as:

$$UT1 = -\frac{1}{R}\frac{\partial \Phi_1}{\partial \theta}$$

Using Φ_1 from equation (13), it can be written that

$$UT1 = -\frac{1}{R}\frac{k'A}{2}\left(\sum_{n=0}^{\infty}\left(\frac{R}{A}\right)^{n}\left[P_n(0) + P_{n-2}(0)\right]\frac{\partial P_n(\cos\theta)}{\partial \theta}\right) + \frac{1}{R}\left(\frac{R}{A}\right)\frac{k'A}{2}\frac{\partial P_1(\cos\theta)}{\partial \theta} \qquad (29)$$

Using the chain rule, the differentials can be changed to

$$\frac{\partial P_n(\cos\theta)}{\partial \theta} = \frac{\partial P_n(\cos\theta)}{\partial(\cos\theta)}\cdot\frac{d\cos\theta}{d\theta} \qquad (30)$$

From the definition of the Associated Legendre polynomials:

$$P_n^m(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

and by rearrangement

$$\frac{d^m P_n(x)}{dx^m} = (1-x^2)^{-m/2} P_n^m(x) \qquad (31)$$

where:

$P_n^m(x)$ = Associated Legendre polynomial of the first kind, the n th order, and the m degree.

Using expression (31) in the equation (30) it can easily be shown that

$$\frac{\partial P_n(\cos\theta)}{\partial\theta} = (1-\cos^2\theta)^{-1/2} \; P_n^1(\cos\theta)\cdot(-\sin\theta)$$

and

$$\frac{\partial P_n(\cos\theta)}{\partial\theta} = -\; P_n^1(\cos\theta)$$

Using the notation AS_n (Cosθ) for P_n^1 (Cosθ), the above equation can be written as:

$$\frac{\partial P_n(\cos\theta)}{\partial\theta} = -\; AS_n(\cos\theta) \qquad (32)$$

Substitution of the equation (32) into the equation (29) yields:

$$UT1 = \frac{k'}{2}\left\{\left[\sum_{n=0}^{\infty}\left(\frac{R}{A}\right)^{n-1}\left[P_n(0)+P_{n-2}(0)\right] AS_n(\cos\theta)\right] - AS_1(\cos\theta)\right] \qquad (33)$$

b) In the case of R > A, following the same procedure for Φ_1 from the equation (14) it can be written that

$$UR1 = \frac{k'}{2}\sum_{n=0}^{\infty}(n+1)\left(\frac{A}{R}\right)^{n+2}\left[P_n(0)+P_{n-2}(0)\right] P_n(\cos\theta) \qquad (34)$$

and

$$UT1 = \frac{k'}{2}\sum_{n=0}^{\infty}\left(\frac{A}{R}\right)^{n+2}\left[P_n(0)+P_{n-2}(0)\right] AS_n(\cos\theta) \qquad (35)$$

Accordingly, the velocity components of the uniformly-distributed disc of sources can be written as equations (29) and (33) for the case of R < A, and as equations (34) and (35) for the case of R > A.

Linearly distributed case. For the case of the linearly-distributed disc of sources, it can be shown that the components of the velocity field are as follows: (See Appendix 1 for the details).

For R < A

$$URI = -\frac{mA}{2}\Big[2\Big(\Big(\sum_{n=2}^{\infty}\frac{(4n+1)\,a_n}{(2n+3)(2n-2)}\Big)-\frac{76}{60}\Big)\Big(\frac{R}{A}\Big)P_2(\cos\theta) + \frac{9}{2}\Big(\frac{R}{A}\Big)^2 P_3(\cos\theta) - 3(R/A)^3 P_4(\cos\theta) + \frac{5}{6}\Big(\frac{R}{A}\Big)^4 P_5(\cos\theta) - \sum_{n=2}^{\infty}\frac{n a_n}{(n-1)}\Big(\frac{R}{A}\Big)^{2n-1} P_{2n}(\cos\theta)\Big] \qquad (36)$$

$$UTI = \frac{mA}{2}\Big(\frac{A}{R}\Big)\Big[\frac{1}{2}AS_0(\cos\theta) + \Big(\Big(\sum_{n=2}^{\infty}\frac{(4n+1)\,a_n}{(2n+3)(2n-2)}\Big)-\frac{76}{60}\Big)\Big(\frac{R}{A}\Big)^2 AS_2(\cos\theta) + \frac{3}{2}\Big(\frac{R}{A}\Big)^3 AS_3(\cos\theta) - \frac{3}{4}\Big(\frac{R}{A}\Big)^4 AS_4(\cos\theta) + \frac{1}{6}\Big(\frac{R}{A}\Big)^5 AS_5(\cos\theta) - \sum_{n=2}^{\infty}\frac{a_n}{(2n-2)}\Big(\frac{R}{A}\Big)^{2n} AS_{2n}(\cos\theta)\Big] \qquad (37)$$

And for the case of R > A, it can be written:

$$UR1 = \frac{mA}{2}\left[\frac{1}{3}\left(\frac{A}{R}\right)^2 P_0(\cos\theta) + \sum_{n=1}^{\infty}\frac{(2n+1)a_n}{2n+3}\left(\frac{A}{R}\right)^{2n+2} P_{2n}(\cos\theta)\right] \quad (38)$$

and

$$UT1 = \frac{mA}{2}\left[\frac{1}{3}\left(\frac{A}{R}\right)^2 AS_0(\cos\theta) + \sum_{n=1}^{\infty}\frac{a_n}{2n+3}\left(\frac{A}{R}\right)^{2n+2} AS_{2n}(\cos\theta)\right] \quad (39)$$

Uniform flow. It is quite simple to show that the components of the velocity field for the uniform flow are as follows:

$$UR3 = -U\cos\theta \quad (40)$$

and

$$UT3 = U\sin\theta \quad (41)$$

Single source at the position r_2 (representing the nacelle). Differentiation of the equation (23) according to equation (27) yields the velocity components due to the body shape of the nacelle as:

$$UR2 = \frac{k}{4\pi}\left(\frac{R - r_2\cos\theta}{[(R\cos\theta - r_2)^2 + R^2\sin^2\theta]^{3/2}}\right) \quad (42)$$

and

$$UT2 = \frac{k}{4\pi}\left(\frac{r_2\sin\theta}{((R\cos\theta - r_2)^2 + R^2\sin^2\theta)^{3/2}}\right) \quad (43)$$

Remarks. In order to utilize the previously derived formulas for the velocity field, two computer programs were written. Both of these programs were written for the general case of the presence of all three potentials.

The first program, Program Project 1 in Appendix 2A, was designed for the uniformly distributed disc of source in general, and its application to the University of Massachusetts windmill in particular.

The second program, Program Project 2 in Appendix 2B, deals with the linearly distributed disc of sources in general, and its application to the University of Massachusetts windmill in particular.

The output of both the programs is the velocity field in cartesian coordinates at each point. The output has the form of: (x, y), the coordinate of the point, (υx, υy), x, and y components of the velocity vector at the point (x,y) and (ETA), the angle between υx and υy.

<u>Streamline construction.</u> The construction of streamlines is based on the fact that no flow crosses a specific stream tube. Once a slection of the streamlines starting point has been made, other points of the same streamline can be found by an application of the conservation of mass.

In reference to Fig. 6, the velocity through the stream tubes is as follows: A_0 is $\upsilon(o)$, A_1 is $\upsilon(1)$, A_2 is $\upsilon(2)$, and so on, where $\upsilon(o)$ is the x-component of the velocity an an x-station and <u>yzo</u>, and the same applies for (1), (2)...., $\upsilon(a)$.

Fig. 7 shows the cross section of the stream tubes.

Accordingly, the area corresponding to any velocity $\upsilon(n)$ can be found as follows:

$$
\left.
\begin{aligned}
A_0 &= (DY/2)^2 \pi \\
A_1 &= (DY/2 + DY)^2 \pi - (DY/2)^2 \pi \\
A_2 &= (DY/2 + 2DY)^2 \pi - A1 \\
&\vdots \\
A_n &= \left(\frac{DY}{2} + nDY\right)^2 \pi - A_{n-1}
\end{aligned}
\right\} \quad (44)
$$

and

Having the point (a), the starting point of the streamline, the mass flow rate or the volumetric flow rate can be found by summing up the flow rate through tubes A_o to A_a as shown in Fig. 8.

When the flow rate through the stream tube (a) is determined at one x station, other points of the same stream tube can subsequently be calculated.

At each station x, the flow rate through the tubes with cross-section A_o, A_1, A_2 . . . and A_a (shown in Fig. 7) should be easily found. The desired stream tube can be determined by summing up these calculated flow rates, F_1, F_2 . . . up to F_n where n indicates the point where this summation is equal to the reference flow rate. If n is known, the y-coordinate can be determined.

The task of the streamline construction is performed by two computer programs. The first one constructs the streamline for a uniformly distributed disc of sources in general, and for the University of Massachusetts windmill, in particular (see Project S1, Appendix 2C). The second program deals with construction of the streamlines for a linearly distributed disc of sources in general, and for the University of Massachusetts windmill in particular, (see Project S2, Appendix 2D).

The input of these programs can be the starting point of the streamline or the flow rate through the stream tube.

The output indicates the form of flow rate, and y coordinate indicates the streamline at each x station.

Applying the one-dimensional momentum theory to the windmills. The source strength density per unit area k can be found in the case of the uniformly distributed source disc and the slope of k, (that is m) for the linearly distributed source disc, by the one dimensional momentum theory and L gally's theorem.

According to the Lagally's theorem [10], the force exerted upon a point source in a uniform flow is $\delta\lambda\upsilon$, where δ is the density of the fluid, λ is the source strength, and υ is the free stream velocity of the uniform flow.

a) Linearly distributed disc.

In this case, the source-strength density is equal to mr, where m is the slope of k as described in section four. According to Lagally's theorem, the force on the disc is:

$$F = \delta k \; \upsilon \tag{45}$$

where k is the total strength of the disc. However, k can be determined as follows:

$$k = \int_0^A mr(2\pi r dr) = \frac{2}{3}\pi m A^2 \quad (46)$$

Substituting equation (46) into equation (45);

$$F = \frac{2}{3}\pi m A^3 \delta U \quad (47)$$

From one dimensional momentum theory it can be shown [11] that the total force on the disc is (See Fig. 9)

$$F = \delta \pi A^2 u (U - U_1) \quad (48)$$

where u is the velocity through the disc and U_1 is the velocity down stream of the disc.

Using the Bernoulli's equation and the fact that $F = \pi A^2 \Delta P$, it can be shown that [8]:

$$F = \delta \frac{S}{2} (U^2 - U_1^2) \quad (49)$$

and

$$u = \frac{U + U_1}{2} \quad (50)$$

where S is the area of the disc.

From the definition of the axial interference coefficient a, it can be written that

$$u = (1-a)U \quad (51)$$

Using equation (50)

$$U_1 = (1-2a)U$$

and

$$U_1 - U = -2aU$$

or

$$U - U_1 = 2aU \quad (52)$$

which can be written as

$$U_1 + U = 2aU + 2U = 2(a+1)U \quad (53)$$

Multiplication of equation (52) by (53) will result in

$$U^2 - U_1^2 = 4a(1+a)U^2$$

substitution into equation (49) leads to;

$$F = 2\delta\pi A^2 a(1+a)U^2$$

The slope m can be determined by equating equations (47) and (54), that is

$$m = \frac{3a(1+a)}{A} U \qquad (55)$$

b) Uniformly distributed disc.

Following the same procedure, the total disc strength in this case is

$$k = \acute{k}\pi r^2 \qquad (56)$$

and the total force on the disc from the Lagally's theorem is:

$$F = U\acute{k}\pi A^2\delta \qquad (57)$$

Equating this formula with the equation (54) will provide k, hence

$$\acute{k} = 2a(1+a)U \qquad (58)$$

It can be shown [12] that the relation between the axial interference factor (a) and the power coefficient Cp is as follows:

$$C_p = 4a(1-a)^2 \qquad (59)$$

The maximum power is developed when a = 1/3 [8].

Thus it has been shown through Lagally's theorem and the simple one-dimensional momentum theory that there is a unique relation between the strength of the distributed source disc k and the power coefficient Cp of the windmill.

RESULTS

The velocity field and some characteristic streamlines have been calculated for the numerical values appropriate to the 25 kW windmill at the University of Massachusetts Solar Habitat I (See Fig. 10).

A characteristic free stream velocity of 38 ft/sec (11.58 m/sec) have been considered. The body of this windmill can be modeled by a single source of K=647.74lbm/sec (294 kg/sec) in a free stream of U = 38 ft/sec.

A sample result of the velocity field upstream the windmill is shown at the end of Project 1 and Project 2 [see Appendix 2A and 2B].

The velocity profile of the uniformly-distributed disc and the linearly-distributed disc model is shown in figs. (11) and (12).

The streamlines constructed for different cases are shown in Fig. 13 through 20.

It is obvious from Figs. 13 through 20 that the effective change in the free stream velocity is almost negligible for more than two radii upstream of the blade disc.

Fig. 13 represents the uniformly distributed disc model without the body for the case of $Cp = Cp_{max} = 0.5$. It is shown that the disc samples almost 57 percent of the volume of the far upstream wind. For the same case, if the body is located at the center of the blade disc (Fig. 15), 51 percent of the flow is sampled. By moving the single source, which forms the body to the position (R_2 = 5.02 ft), to model the University of Massachusetts windmill, the percentage of the flow sampled reduces to 35.

The same analysis for the linearly-distributed disc model (Figs. 14, 16, and 18) indicates similar results. The percentage of the wind sampled changes from 39 to 38 and then to 30.

Comparison of the uniform model and linear model shows that for the same conditions, the linear model samples less flow than the uniform disc. To see this, Figs. 17 and 18 must be compared.

The uniformly-distributed disc samples 35 percent of the flow, while the linearly distributed disc samples 30 percent.

The effect of the body is not restricted to the sampling problem. Although this is the case for the uniformly-distributed model, in the case of the more realistic linearly-distributed model, a significant difference is observed - the problem of leaking.

Figs. 16 and 18 show that due to the higher resistance at the outer region of the blades, some of the sampled flow appears to leak through the central region near the body. This is quite obvious in Fig. 12B, where at the station $x=1^{ft}$ (0.3^{m}), the y component of the velocity is down towards the center between $y=4^{ft}$ (1.2^{m}) and $y-10^{ft}$ (3^{m}).

Using the velocity field the pressure increase in front of the disc can quite easily be calculated.

In the case of the linearly-distributed source disc the Bernoulli equation on the streamline ψ_0 (see Fig. 18) can be written as

$$\frac{1}{2} U^2 + P/_S = \text{const.} \qquad (60)$$

At $x = \infty$, the velocity is 38 ft/sec and the pressure is P_∞, with density δ. At x = 1, and y = 16, the velocity is:

$$U = \sqrt{28^2 + 19^2} = 33.8 \text{ ft/sec}$$

then, it is easy to show that

$$P = P_\infty + 149.5\,\delta \qquad (61)$$

or

$$P = 14.801 \text{ psi } (101.84 \text{ kN/m}^2)$$

Following the same procedure for the uniformly-distributed disc of sources, the pressure would be:

$$P = P_\infty + 168\,\rho \qquad (62)$$

or

$$P = 14.802 \text{ psi } (101.85 \text{ kN/m}^2)$$

Comparing equation (60) and (62), it is quite obvious that the pressure on the streamline ψ_o increases more in front of the linear distributed source disc.

Fig. 20 shows the effect of a change in the power coefficient, Cp. The percentage of the flow sampled changes from 57 to 80 percent while the Cp is changed from Cp_{max} to $Cp_{max/2}$. Also, in the limit as $CP \to o$, all the flow would pass through the blade disc uneffected.

The variation of the velocity on the stagnation streamline has been represented in Fig. 21. The diagram shows the velocity variation for the body and the linearly-distributed disc with the body. It also shows that the stagnation point has shifted forwards, and that the velocity decreases more rapidly with the blade disc than for the body alone.

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APPENDIX 1

APPENDIX 1

1) R < A,

The potential Φ was shown in the equation (21).

then
$$UR = -\frac{\partial \Phi}{\partial R}$$

$$UR = -\frac{mA^2}{2}\Big[\Big(\Big(\sum_{n=2}^{\infty}\frac{(4n+1)a_n}{(2n+3)(2n-2)}\Big)-\frac{76}{60}\Big)\Big(\frac{2R}{A^2}\Big)P_2(\cos\theta)$$
$$+\frac{3}{2}\Big(\frac{3R^2}{A^3}\Big)P_3(\cos\theta)-\frac{3}{4}\Big(\frac{4R^3}{A^4}\Big)P_4(\cos\theta)$$
$$-\frac{1}{6}\Big(\frac{5R^4}{A^5}\Big)P_5(\cos\theta)-$$
$$\sum_{n=2}^{\infty}\frac{a_n}{2n-2}\,\frac{2n\,R^{2n-1}}{A^{2n}}\,P_{2n}(\cos\theta)\Big]$$

This can be arranged as:

$$UR = -\frac{mA^2}{2}\Big[\frac{2}{A}\Big(\Big(\sum_{n=2}^{\infty}\frac{(4n+1)a_n}{(2n+3)(2n-2)}\Big)-\frac{76}{60}\Big)\Big(\frac{R}{A}\Big)P_2(\cos\theta)$$
$$+\frac{9}{2A}\Big(\frac{R}{A}\Big)^2P_3(\cos\theta)-\frac{3}{A}\Big(\frac{R}{A}\Big)^3P_4(\cos\theta)+$$

$$\frac{5}{6A}\left(R/A\right)^{4} P_5 \left(\cos\theta\right) -$$

$$\frac{1}{A}\sum_{n=2}^{\infty}\frac{n a_n}{n-1}\left(\frac{R}{A}\right)^{2n-1} P_{2n}\left(\cos\theta\right)\Big]$$

which can be reduced to:

$$UR = -\frac{mA}{2}\Big[2\Big(\Big(\sum_{n=2}^{\infty}\frac{(4n+1)\,a_n}{(2n+3)(2n-2)}\Big) - \frac{76}{60}\Big)\Big(\frac{R}{A}\Big) P_2(\cos\theta)$$

$$+\frac{9}{2}\left(\frac{R}{A}\right)^{2} P_3(\cos\theta) - 3\left(\frac{R}{A}\right)^{3} P_4(\cos\theta)$$

$$+\left(\frac{5}{6}\right)\left(\frac{R}{A}\right)^{4} P_5(\cos\theta) -$$

$$\sum_{n=2}^{\infty}\frac{n a_n}{(n-1)}\left(\frac{R}{A}\right)^{2n-1} P_{2n}(\cos\theta)\Big]$$

which is the equation (36).

Equation (37) can be derived as follows, using the same potential (equation 21) and knowing that

$$UT = -\frac{1}{R}\frac{\partial \Phi}{\partial \theta}$$

it is easy to show that:

$$UT = -\frac{1}{R}\frac{mA^2}{2}\Big[-\frac{1}{2} A S_0 (\cos\theta) +$$

$$\Big(\Big(\sum_{n=2}^{\infty}\frac{(4n+1)\,a_n}{(2n+3)(2n-2)}\Big) - \frac{76}{60}\Big)\Big(\frac{R}{A}\Big)^{2}\big(-A S_1(\cos\theta)\big)$$

$$+ \frac{3}{2}\left(\frac{R}{A}\right)^3(-AS_3(\cos\theta)) - \frac{3}{4}\left(\frac{R}{A}\right)^4(-AS_4(\cos\theta))$$

$$+ \frac{1}{6}\left(\frac{R}{A}\right)^5(-AS_5(\cos\theta)) -$$

$$\sum_{n=2}^{\infty} \frac{a_n}{2n-2}\left(\frac{R}{A}\right)^{2n}(-AS_{2n}(\cos\theta))\Big]$$

By a little algebric multiplication equation (37) can easily be derived.

2) $R > A$.

In this case the equation 22 should be differentiated to produce UR, and UT.

APPENDIX 2: A & B

```
      PROGRAM PROJ1(INPUT,OUTPUT)
      REAL K,L,K1
C***************************************************************
C                                                              *
C                                                              *
C   PROGRAM FOR UNIFORMLY DISRIBUTED DISC OF SOURCES           *
C              WITH THE BODY                                   *
C                                                              *
C                                                              *
C        THIS PROGRAM FINDS THE VELOCITY FIELD(UX,UY,ETA)      *
C         FOR A UNIFORMLY DISTRIBUTED DISC OF SOURCES AND      *
C         A STRONG SOURCE AT POSITION R2 ON THE X-AXIS.        *
C                                                              *
C                                                              *
C  TO RUN THIS PROGRAM:                                        *
C                        ONE SHOUD INPUT THE FOLLOWINGS        *
C                       IN RESPONCE TO THE ASK FOR DATA:       *
C                        NX,NY,DELTX,DELTY,U,A,A1,R2,K1        *
C                        ACORDING TO THE DEFINITION OF         *
C                         THE PARAMETERS GIVEN BELOW:          *
C                                                              *
C                                                              *
C     Y-AXIS                                                   *
C      :                                                       *
C      +--------------------------+ (NY-1)*DELTAY              *
C      :                          :                            *
C      :                          :                            *
C      :      VELOCITY SPACE      :                            *
C      :                          :                            *
C      :                          :                            *
C      +--------------------------+----  X-AXIS                *
C     ORIGIN                    NX*DELTX                       *
C                                                              *
C     PARAMETERS:                                              *
C               NX: NUMBER OF THE INCREMENTS IN X-DIREC.       *
C               NY:NUMBER OF THE INCREMENTS IN Y-DIREC.        *
C               DELTX: INCREMENT IN X-DIRECTION                *
C               DELTY: INCREMENT IN Y-DIRECTION                *
C               A:RADIOUS OF THE DISC                          *
C               XX: VALUE OF THE X-AXIS                        *
C               YY: VALUE OF THE Y-AXIS                        *
C               K1: SRENGTH OF THE BODY SOURCE                 *
C               R2:POSITION OF THE BODY SOURCE                 *
C               U: FREE STREAM VELOSITY                        *
C               A1: AXIAL INTERFERENCE FACTOR                  *
C               K:SOURCE STRENGTH DENSITY ON THE DISC          *
C               R AND T: RADIOUS AND ANGLE OF POLAR            *
C                        COORDINATE IN FT.AND RAD.             *
```

```
C                 UT1: TANGANTIAL VELOSITY  AT POIN (R,T)      *
C                      DUE TO THE DISC.                         *
C                 UR1: RADIAL VELOSITY AT POINT (R,T)           *
C                      DUE TO THE DISC                          *
C                 UT2: TANGANTIAL VELOSITY AT POIN (R,T)        *
C                      DUE TO THE BODY                          *
C                 UR2: RADIAL VELOSITY AT POIN (R,T)            *
C                      DUE TO THE BODY                          *
C                 UT3: TANGANTIAL VELOSITY AT POINT (R,T)       *
C                      DUE TO THE FREE STREAM.                  *
C                 UR3: RADIAL VELOSITY AT POINT (R,T)           *
C                      DUE TO THE FREE STREAM.                  *
C                 P(N,X): LEGENDRE POLYNOMIAL OF THE            *
C                         FIRST KIND AND NTH ORDER DEFINED      *
C                         BY THE FUNCTION P(N,X).               *
C                 AS(N,X): ASSOCIATED LEGENDRE OF THE           *
C                          FIRST KIND ,FIRST DEGREE.            *
C                          AND,NTH ORDER DEFINED BY THE         *
C                          AS(N,X) FUNCTION                     *
C                 UX: X COMPONENT OF THE VELOSITY AT            *
C                     ANY POINT (R,T)OR ITS EQIVALENT(XX,YY)    *
C                 UY: Y COMPONENT OF THE VELOSITY AT            *
C                     ANY POINT(R,T) OR ITS EQUIVALENT(XX,YY)   *
C                 ETA: ANGL  BETWEEN UX AND, UY IN DEG.         *
C                                                               *
C                                                               *
C---------------THE PROGRAM WAS RUN FOR THE FOLLOWING           *
C                 VALUES:NX=20,NY=21,DELTX=1,DELTY=1,U=38       *
C                        ,A=16,A1=1/3(FOR MAX. POWER),R2=0.     *
C                        ,K1=647.78                             *
C                                                               *
C         ALSO SEE THE PROJECT REPORT                           *
C****************************************************************
 READ,NX,NY,DELTX,DELTY,U,A,A1,R2,K1
 IN=50
 PRINT 210,NX,NY,DELTX,DELTY,U,A,A1,R2,K1
 210 FORMAT(///,*NX=*,I5,/,*NY=*,I5,/,*DELTX=*,F5.3,/,*DELTY=*,F5.3,
+/,*U=*,F10.4,/,*A=*,F6.2,/,*A1=*,F10.6,/,*R2=*,F10.5,/,*K1=*,F10.4)
 PRINT 21
 21 FORMAT(//,25X,*VELOSITY FIELD FOR UNIFORM DISC AND THE BODY
+*,/,25X,*---------------------------------------------*)
 PRINT 10
 10 FORMAT(5X,*X IN FT*,9X,*Y IN FT*,6X,*UX IN FT/SEC*,5X,*UY IN FT/SE
+5X,*ETA IN DEG.*,/,5X,*--------*,9X,*--------*,6X,*------------*,5X
+,*-------------*,5X,*-----------*)
 DO 600 III=1,NX
 DO 500 II=1,NY
 YY=(II-1)*DELTY
 XX=FLOAT(III)*DELTX
 K=2*A1*(1.+A1)*U
 R=(((XX**2)+(YY**2))**0.5)
```

```
N0=0
N1=1
T=ATAN((YY/XX))
T1=COS(T)
T2=SIN(T)
T0=0.
IF(R.GE.A) GO TO 5
C
C
C              DISC FOR THE CASE R<A
C
C
AK=K/2.
X=R/A
SUM=0.
DO 1 I=1,IN
N=I-1
M=N-2
SUM=SUM+(N*(X**(N-1))*(P(N,T0)+P(M,T0))*P(N,T1))
1 CONTINUE
UR1=AK*(P(N1,T1)-SUM)
SUM=0.
DO 2 I=1,IN
N=I-1
M=N-2
SUM=SUM+((X**(N-1))*(P(N,T0)+P(M,T0))*AS(N,T1))
2 CONTINUE
UT1=AK*(SUM-AS(N1,T1))
GO TO 200
5 CONTINUE
C
C
C              DISC FOR THE CASE R>A
C
C
X=A/R
SUM=0.
DO 3 I=1,IN
N=I-1
M=N+2
SUM=SUM+((N+1)*(X**M)*(P(N,T0)+P(M,T0))*P(N,T1))
3 CONTINUE
UR1=(K/2)*SUM
SUM=0.
DO 4 I=1,IN
N=I-1
M=N+2
SUM=SUM+((X**M)*(P(N,T0)+P(M,T0))*AS(N,T1))
4 CONTINUE
UT1=(K/2)*SUM
200 CONTINUE
```

```
C
C
C        BODY CALCULATIONS
C
C
 D15=(R-(R2*T1))
 D16=(((R*T1)-R2)**2)
 D17=(R**2)*(T2**2)
 D18=((D16+D17)**(3/2))
 AK1=(K1/(4*3.14))
 UR2=AK1*D15/D18
 D19=R2*T2
 D110=((((R*T1)-R2)**2)+((R**2)*(T2**2)))**(3./2.)
 AK3=K1/(4.*3.14)
 UT2=AK3*D19/D110
C
C
C        UNIFORM FLOW
C
C
 UR3=-U*T1
 UT3=U*T2
C
C
C       SUPERPOSITION
C
C
 UR=UR1+UR2+UR3
 UT=UT1+UT2+UT3
C
C
C      EVALUATION OF UX , UY , AND ETA
C
C
 UX=(UR*T1)-(UT*T2)
 UY=(UR*T2)+(UT*T1)
 XY=UY/UX
 ET=ATAN(XY)*360./(2.*3.14)
 PRINT 300,XX,YY,UX,UY,ET
 300 FORMAT(5(5X,F10.4))
 500 CONTINUE
 PRINT 700
 700 FORMAT(/)
 600 CONTINUE
 END
C
C
C
C
C
C
```

```
C
C
C
C
C
C
C------------------------------------------------------------------*
 FUNCTION P(N,X)
C
C
C THIS IS A FUNCTION TO CALCULATE LEGENDER POLYNOMIALS OF THE
C  FIRST KIND
C
C
 Q0=0.
 P=Q0
 IF(N.LT.0) GO TO 1
 Q0=1.
 P=Q0
 IF(N.EQ.0) GO TO 1
 Q1=X
 P=Q1
 IF(N.EQ.1) GO TO 1
 Q2=((3*(X**2))-1.)/2.
 P=Q2
 IF(N.EQ.2) GO TO 1
 P1=((3*(X**2))-1.)/2.
 P2=X
 I=3
 2 CONTINUE
 P=((((2.*I)-1.)/I)*X*P1)-((I-1.)/I)*P2
 IF(I.EQ.N) GO TO 1
 I=I+1
 P2=P1
 P1=P
 GO TO 2
 1 CONTINUE
 RETURN
 END
C
C
C
C
C
C
C
C
C
C
C
C
```

```
C
C-----------------------------------------------------------------*
 FUNCTION AS(N,X)
 DIMENSION R(1000)
C
C
C  THIS FUNCTION CALCULATES ASSOCIATED LEGENDER POLYNOMIALS
C   OF THE FIRST KIND AND FIRST POWER
C
C
 RO=0.
 RA=RO
 IF(N.LE.0) GO TO 3
 R1=(1-(X**2))**0.5
 RA=R1
 IF(N.EQ.1) GO TO 3
 R2=3.*X*((1-(X**2))**0.5)
 RA=R2
 IF(N.EQ.2) GO TO 3
 P1=X
 P2=((3*(X**2))-1.)/2.
 DO 10 I=3,N
 R(I)=(((2*I)-1)*((1-(X**2))**0.5)*P2)+R1
 P=((((2.*I)-1.)/I)*X*P2)-((I-1.)/I)*P1
 P1=P2
 P2=P
 R1=R2
 R2=R(I)
 RA=R(I)
 10 CONTINUE
 3 CONTINUE
 AS=RA
 RETURN
 END
C
READY.
```

RNN

78/07/04. 22.15.48.
FILE PROJ1

? 16,16,4.,2.,38.,16.,0.33,5.02,647.78

NX= 16
NY= 16
DELTX=4.000
DELTY=2.000
U= 38.0000
A= 16.00
A1= .330000
R2= 5.02000
K1= 647.7800

VELOSITY FIELD FOR UNIFORM DISC AND THE BODY

X IN FT	Y IN FT	UX IN FT/SEC	UY IN FT/SEC.	ETA IN
4.0000	0.	-75.9304	-.0000	.0000
4.0000	2.0000	-30.1489	10.0254	-18.4028
4.0000	4.0000	-22.8738	8.2835	-19.9175
4.0000	6.0000	-23.1659	8.6163	-20.4124
4.0000	8.0000	-24.2123	9.0017	-20.4046
4.0000	10.0000	-25.3713	9.5443	-20.6261
4.0000	12.0000	-26.7800	10.2562	-20.9665
4.0000	14.0000	-28.7235	10.9299	-20.8435
4.0000	16.0000	-31.3540	10.9288	-19.2263
4.0000	18.0000	-33.6426	9.9691	-16.5141
4.0000	20.0000	-35.1916	8.5545	-13.6697
4.0000	22.0000	-36.0882	7.2744	-11.4023
4.0000	24.0000	-36.6225	6.2429	-9.6790
4.0000	26.0000	-36.9612	5.4270	-8.3573
4.0000	28.0000	-37.1885	4.7762	-7.3223
4.0000	30.0000	-37.3484	4.2495	-6.4945
8.0000	0.	-11.4735	-.0000	.0000
8.0000	2.0000	-16.0516	5.3424	-18.4181
8.0000	4.0000	-21.1687	6.4546	-16.9657
8.0000	6.0000	-24.1212	6.7699	-15.6853
8.0000	8.0000	-26.0884	7.0450	-15.1196
8.0000	10.0000	-27.6714	7.3130	-14.8112
8.0000	12.0000	-29.1343	7.5240	-14.4878

8.0000	14.0000	-30.6546	7.7273	-14.1553
8.0000	16.0000	-31.9906	7.4480	-13.1128
8.0000	18.0000	-33.2753	7.0658	-11.9945
8.0000	20.0000	-34.3391	6.5193	-10.7552
8.0000	22.0000	-35.1599	5.9136	-9.5521
8.0000	24.0000	-35.7708	5.3255	-8.4722
8.0000	26.0000	-36.2219	4.7920	-7.5400
8.0000	28.0000	-36.5576	4.3234	-6.7481
8.0000	30.0000	-36.8110	3.9175	-6.0777
12.0000	0.	-23.9398	-.0000	.0000
12.0000	2.0000	-24.4340	1.8019	-4.2199
12.0000	4.0000	-25.6093	3.1907	-7.1055
12.0000	6.0000	-26.9719	4.1169	-8.6829
12.0000	8.0000	-28.2769	4.7155	-9.4724
12.0000	10.0000	-29.4728	5.0455	-9.7192
12.0000	12.0000	-30.5842	5.3377	-9.9048
12.0000	14.0000	-31.6260	5.4193	-9.7285
12.0000	16.0000	-32.5898	5.3807	-9.3800
12.0000	18.0000	-33.4611	5.2281	-8.8848
12.0000	20.0000	-34.2222	4.9879	-8.2967
12.0000	22.0000	-34.8652	4.6934	-7.6707
12.0000	24.0000	-35.3948	4.3755	-7.0507
12.0000	26.0000	-35.8241	4.0575	-6.4651
12.0000	28.0000	-36.1698	3.7539	-5.9283
12.0000	30.0000	-36.4481	3.4723	-5.4447
16.0000	0.	-27.4909	.0000	-.0000
16.0000	2.0000	-28.4354	.8270	-1.6668
16.0000	4.0000	-28.9778	1.7890	-3.5345
16.0000	6.0000	-29.6169	2.5308	-4.8866
16.0000	8.0000	-30.3266	3.0826	-5.8069
16.0000	10.0000	-31.0677	3.4780	-6.3908
16.0000	12.0000	-31.8054	3.7426	-6.7146
16.0000	14.0000	-32.5177	3.8954	-6.8346
16.0000	16.0000	-33.1900	3.9526	-6.7949
16.0000	18.0000	-33.8105	3.9300	-6.6335
16.0000	20.0000	-34.3707	3.8442	-6.3850
16.0000	22.0000	-34.8659	3.7123	-6.0807
16.0000	24.0000	-35.2961	3.5504	-5.7469
16.0000	26.0000	-35.6647	3.3721	-5.4040
16.0000	28.0000	-35.9776	3.1881	-5.0665
16.0000	30.0000	-36.2419	3.0058	-4.7435
20.0000	0.	-30.9024	-.0000	.0000
20.0000	2.0000	-30.9754	.5997	-1.1098
20.0000	4.0000	-31.1848	1.1631	-2.1371
20.0000	6.0000	-31.5059	1.6621	-3.0213

20.0000	8.0000	-31.9068	2.0805	-3.7326
20.0000	10.0000	-32.3569	2.4133	-4.2676
20.0000	12.0000	-32.8305	2.6629	-4.6394
20.0000	14.0000	-33.3077	2.8356	-4.8685
20.0000	16.0000	-33.7734	2.9401	-4.9777
20.0000	18.0000	-34.2167	2.9859	-4.9897
20.0000	20.0000	-34.6297	2.9831	-4.9260
20.0000	22.0000	-35.0076	2.9420	-4.8062
20.0000	24.0000	-35.3482	2.8720	-4.6473
20.0000	26.0000	-35.6514	2.7817	-4.4638
20.0000	28.0000	-35.9187	2.6785	-4.2669
20.0000	30.0000	-36.1529	2.5682	-4.0653

24.0000	0.	-32.4816	-.0000	.0000
24.0000	2.0000	-32.5221	.4046	-.7131
24.0000	4.0000	-32.6404	.7920	-1.3907
24.0000	6.0000	-32.8271	1.1477	-2.0034
24.0000	8.0000	-33.0689	1.4609	-2.5308
24.0000	10.0000	-33.3511	1.7255	-2.9632
24.0000	12.0000	-33.6592	1.9393	-3.2991
24.0000	14.0000	-33.9805	2.1033	-3.5438
24.0000	16.0000	-34.3040	2.2208	-3.7060
24.0000	18.0000	-34.6211	2.2964	-3.7967
24.0000	20.0000	-34.9253	2.3354	-3.8276
24.0000	22.0000	-35.2119	2.3439	-3.8102
24.0000	24.0000	-35.4780	2.3275	-3.7553
24.0000	26.0000	-35.7220	2.2916	-3.6724
24.0000	28.0000	-35.9436	2.2412	-3.5698
24.0000	30.0000	-36.1434	2.1804	-3.4541

28.0000	0.	-33.5582	-.0000	.0000
28.0000	2.0000	-33.5830	.2876	-.4909
28.0000	4.0000	-33.6558	.5660	-.9639
28.0000	6.0000	-33.7724	.8269	-1.4034

TIME LIMIT

SRU 21.302 UNTS.

T,500

28.0000	8.0000	-33.9266	1.0638	-1.7970
28.0000	10.0000	-34.1108	1.2720	-2.1367
28.0000	12.0000	-34.3169	1.4489	-2.4189
28.0000	14.0000	-34.5373	1.5937	-2.6433
28.0000	16.0000	-34.7649	1.7072	-2.8127
28.0000	18.0000	-34.9936	1.7912	-2.9317
28.0000	20.0000	-35.2185	1.8485	-3.0061
28.0000	22.0000	-35.4357	1.8823	-3.0421

28.0000	24.0000	-35.6424	1.8958	-3.0463
28.0000	26.0000	-35.8366	1.8926	-3.0245
28.0000	28.0000	-36.0174	1.8756	-2.9825
28.0000	30.0000	-36.1842	1.8479	-2.9250
32.0000	0.	-34.3276	-.0000	.0000
32.0000	2.0000	-34.3437	.2126	-.3549
32.0000	4.0000	-34.3913	.4199	-.6999
32.0000	6.0000	-34.4681	.6169	-1.0259
32.0000	8.0000	-34.5710	.7995	-1.3254
32.0000	10.0000	-34.6958	.9643	-1.5928
32.0000	12.0000	-34.8379	1.1093	-1.8247
32.0000	14.0000	-34.9927	1.2333	-2.0195
32.0000	16.0000	-35.1558	1.3361	-2.1776
32.0000	18.0000	-35.3229	1.4183	-2.3005
32.0000	20.0000	-35.4906	1.4812	-2.3910
32.0000	22.0000	-35.6559	1.5262	-2.4523
32.0000	24.0000	-35.8165	1.5555	-2.4880
32.0000	26.0000	-35.9705	1.5708	-2.5018
32.0000	28.0000	-36.1168	1.5743	-2.4972
32.0000	30.0000	-36.2545	1.5680	-2.4777
36.0000	0.	-34.8978	-.0000	.0000
36.0000	2.0000	-34.9087	.1622	-.2664
36.0000	4.0000	-34.9412	.3212	-.5269
36.0000	6.0000	-34.9940	.4738	-.7760
36.0000	8.0000	-35.0653	.6172	-1.0089
36.0000	10.0000	-35.1527	.7493	-1.2217
36.0000	12.0000	-35.2535	.8684	-1.4117
36.0000	14.0000	-35.3649	.9734	-1.5775
36.0000	16.0000	-35.4839	1.0639	-1.7183
36.0000	18.0000	-35.6080	1.1399	-1.8345
36.0000	20.0000	-35.7345	1.2019	-1.9273
36.0000	22.0000	-35.8613	1.2506	-1.9982
36.0000	24.0000	-35.9866	1.2870	-2.0492
36.0000	26.0000	-36.1089	1.3123	-2.0825
36.0000	28.0000	-36.2270	1.3278	-2.1002
36.0000	30.0000	-36.3401	1.3348	-2.1046
40.0000	0.	-35.3327	-.0000	.0000
40.0000	2.0000	-35.3404	.1271	-.2061
40.0000	4.0000	-35.3634	.2520	-.4085
40.0000	6.0000	-35.4010	.3728	-.6036
40.0000	8.0000	-35.4520	.4876	-.7883
40.0000	10.0000	-35.5150	.5948	-.9600
40.0000	12.0000	-35.5884	.6933	-1.1167
40.0000	14.0000	-35.6703	.7822	-1.2569
40.0000	16.0000	-35.7590	.8609	-1.3799

40.0000	18.0000	-35.8525	.9293	-1.4856
40.0000	20.0000	-35.9493	.9874	-1.5742
40.0000	22.0000	-36.0475	1.0356	-1.6465
40.0000	24.0000	-36.1460	1.0744	-1.7034
40.0000	26.0000	-36.2435	1.1045	-1.7463
40.0000	28.0000	-36.3390	1.1265	-1.7765
40.0000	30.0000	-36.4317	1.1414	-1.7954
44.0000	0.	-35.6728	-.0000	.0000
44.0000	2.0000	-35.6784	.1017	-.1634
44.0000	4.0000	-35.6951	.2020	-.3244
44.0000	6.0000	-35.7226	.2995	-.4805
44.0000	8.0000	-35.7601	.3929	-.6298
44.0000	10.0000	-35.8067	.4811	-.7702
44.0000	12.0000	-35.8614	.5633	-.9004
44.0000	14.0000	-35.9230	.6388	-1.0193
44.0000	16.0000	-35.9902	.7070	-1.1260
44.0000	18.0000	-36.0619	.7678	-1.2203
44.0000	20.0000	-36.1368	.8209	-1.3020
44.0000	22.0000	-36.2138	.8666	-1.3716
44.0000	24.0000	-36.2919	.9051	-1.4294
44.0000	26.0000	-36.3701	.9367	-1.4761
44.0000	28.0000	-36.4476	.9619	-1.5125
44.0000	30.0000	-36.5238	.9812	-1.5396
48.0000	0.	-35.9442	-.0000	.0000
48.0000	2.0000	-35.9484	.0829	-.1322
48.0000	4.0000	-35.9609	.1649	-.2628
48.0000	6.0000	-35.9815	.2449	-.3901
48.0000	8.0000	-36.0097	.3220	-.5127
48.0000	10.0000	-36.0449	.3956	-.6291
48.0000	12.0000	-36.0865	.4648	-.7384
48.0000	14.0000	-36.1337	.5292	-.8396
48.0000	16.0000	-36.1856	.5884	-.9321
48.0000	18.0000	-36.2413	.6421	-1.0155
48.0000	20.0000	-36.3001	.6901	-1.0896
48.0000	22.0000	-36.3611	.7324	-1.1545
48.0000	24.0000	-36.4236	.7692	-1.2104
48.0000	26.0000	-36.4868	.8006	-1.2576
48.0000	28.0000	-36.5501	.8269	-1.2966
48.0000	30.0000	-36.6129	.8483	-1.3279
52.0000	0.	-36.1647	-.0000	.0000
52.0000	2.0000	-36.1679	.0687	-.1089
52.0000	4.0000	-36.1775	.1367	-.2166
52.0000	6.0000	-36.1932	.2033	-.3221
52.0000	8.0000	-36.2149	.2679	-.4241
52.0000	10.0000	-36.2421	.3299	-.5219

52.0000	12.0000	-36.2743	.3888	-.6144
52.0000	14.0000	-36.3110	.4442	-.7012
52.0000	16.0000	-36.3517	.4956	-.7815
52.0000	18.0000	-36.3957	.5430	-.8551
52.0000	20.0000	-36.4424	.5861	-.9218
52.0000	22.0000	-36.4913	.6248	-.9814
52.0000	24.0000	-36.5417	.6593	-1.0341
52.0000	26.0000	-36.5932	.6894	-1.0799
52.0000	28.0000	-36.6452	.7155	-1.1192
52.0000	30.0000	-36.6973	.7377	-1.1522
56.0000	0.	-36.3466	-.0000	.0000
56.0000	2.0000	-36.3491	.0577	-.0910
56.0000	4.0000	-36.3566	.1149	-.1812
56.0000	6.0000	-36.3689	.1711	-.2698
56.0000	8.0000	-36.3858	.2259	-.3558
56.0000	10.0000	-36.4071	.2787	-.4388
56.0000	12.0000	-36.4325	.3292	-.5179
56.0000	14.0000	-36.4615	.3770	-.5928
56.0000	16.0000	-36.4939	.4220	-.6629
56.0000	18.0000	-36.5291	.4639	-.7279
56.0000	20.0000	-36.5667	.5024	-.7876
56.0000	22.0000	-36.6063	.5377	-.8419
56.0000	24.0000	-36.6474	.5695	-.8908
56.0000	26.0000	-36.6896	.5980	-.9343
56.0000	28.0000	-36.7327	.6232	-.9725
56.0000	30.0000	-36.7761	.6452	-1.0056
60.0000	0.	-36.4988	-.0000	.0000
60.0000	2.0000	-36.5008	.0491	-.0771
60.0000	4.0000	-36.5067	.0978	-.1536
60.0000	6.0000	-36.5164	.1458	-.2288
60.0000	8.0000	-36.5299	.1926	-.3022
60.0000	10.0000	-36.5469	.2380	-.3733
60.0000	12.0000	-36.5671	.2817	-.4416
60.0000	14.0000	-36.5904	.3234	-.5066
60.0000	16.0000	-36.6165	.3629	-.5680
60.0000	18.0000	-36.6450	.3999	-.6256
60.0000	20.0000	-36.6755	.4345	-.6791
60.0000	22.0000	-36.7079	.4664	-.7284
60.0000	24.0000	-36.7418	.4957	-.7733
60.0000	26.0000	-36.7767	.5223	-.8140
60.0000	28.0000	-36.8126	.5462	-.8505
60.0000	30.0000	-36.8490	.5675	-.8828
64.0000	0.	-36.6276	-.0000	.0000
64.0000	2.0000	-36.6292	.0422	-.0660
64.0000	4.0000	-36.6339	.0841	-.1316

64.0000	6.0000	-36.6418	.1254	-.1962
64.0000	8.0000	-36.6526	.1659	-.2595
64.0000	10.0000	-36.6663	.2053	-.3210
64.0000	12.0000	-36.6827	.2434	-.3803
64.0000	14.0000	-36.7017	.2799	-.4372
64.0000	16.0000	-36.7229	.3147	-.4913
64.0000	18.0000	-36.7462	.3477	-.5424
64.0000	20.0000	-36.7713	.3787	-.5904
64.0000	22.0000	-36.7981	.4076	-.6350
64.0000	24.0000	-36.8261	.4344	-.6762
64.0000	26.0000	-36.8553	.4591	-.7140
64.0000	28.0000	-36.8854	.4816	-.7484
64.0000	30.0000	-36.9161	.5019	-.7794

END.

SRU 52.309 UNTS.

RUN COMPLETE.

APPENDIX 2B

```
LNH

00100 PROGRAM PROJ2(INPUT,OUTPUT)
00110 IN=50
00120*******************************************************************
00130*                                                                 *
00140*                                                                 *
00150*   PROGRAM FOR LINEARLY DISTRIBUTED DISC OF SOURCES              *
00160*                WITH THE BODY                                    *
00170*                                                                 *
00180*                                                                 *
00190*       THIS PROGRAM CALCULATES THE VELOSITY FIELD                *
00200*       (UX,UY,ETA) FOR A LINEARLY DISTRIBUTED DISC               *
00210*       OF SOURCES AND A STRONG SOURCE AT THE POSITION            *
00220*       R2 ON THE X-AXIS.                                         *
00230*                                                                 *
00240*                                                                 *
00250*                                                                 *
00260*  TO RUN THIS PROGRAM:                                           *
00270*                      ONE SHOULD INPUT THE FOLLOWINGS            *
00280*                      IN RESPONCE TO THE ASK FOR DATA            *
00290*                      NX,NY,DELTX,DELTY,U,A,A1,R2,K1             *
00300*                      (ACORDING TO THE DEFINITION OF             *
00310*                      THE PARAMETERS GIVEN BELOW:                *
00320*                                                                 *
00330*                                                                 *
00340*                                                                 *
00350*      PARAMETERS:                                                *
00360*                 SEE PROJ1 PROGRAM DUCUMENTS.                    *
00370*                 ALSO SEE THE PROJECT REPORT                     *
00380*                                                                 *
00390*                                                                 *
00400*                                                                 *
00410*                                                                 *
00420*                                                                 *
00430*******************************************************************
00440 REAL M,K1,L
00450 READ,NX,NY,DELTX,DELTY,U,A,A1,R2,K1
00460 PRINT 210
00470 210 FORMAT(////,10X,*VELOSITY FIELD FOR LINEARLY DISTRIBUTED DISC+BODY*
00480 PRINT 220,NX,NY,DELTX,DELTY,U,A,A1,R2,K1
00490 220 FORMAT(10X,*-----------------------------------------------*,//,
00500+*NX=*,I5,/,*NY=*,I5,/,*DELTX=*,F5.2,/,*DELTY=*,F5.2,/,*U=*,F10.4,/,
00510+*A=*,F10.4,/,*A1=*,F6.4,/,*R2=*,F10.4,/,*K1=*,F10.3,///,)
00520 PRINT 200
00530 200 FORMAT(5X,*X IN FT.*,10X,*Y IN FT.*,7X,*UX IN FT/SEC.*
00531+,7X,*UY IN FT./SEC.*,5X,*ETA IN DEG.*,/,5X,*--------*,10X,
00532+*--------*,7X,*-------------*,7X,*--------------*,5X,
00533+*-----------*)
00550 DO 500 III=1,NX
00560 DO 600 II=1,NY
```

```
YY=(II-1)*DELTY
XX=FLOAT(III)*DELTX
R=(((XX**2)+(YY**2))**0.5)
6 CONTINUE
C              RADIAL COMPONENT OF THE DISC VELOCITY FIELD
N0=0
N1=1
N2=2
N3=3
N4=4
N5=5
X=R/A
PROD=-0.5
M=3*A1*(1.+A1)*U/A
T2=ATAN((YY/XX))
T1=COS(T2)
T3=SIN(T2)
IF(R.GE.A) GO TO 5
SUM1=0.
DO 1 I=2,IN
PROD=PROD*(-0.5-(I-1))/I
SUM1=SUM1+((((4*I)+1.)/(((2*I)+3.)*((2*I)-2.)))*PROD)
1 CONTINUE
S1=SUM1
PROD1=-0.5
SUM2=0.
DO 2 I=2,IN
N=2*I
PROD1=PROD1*(-0.5-(I-1))/I
SUM2=SUM2+(((I*PROD1)/(I-1))*(X**((2*I)-1))*P(N,T1))
2 CONTINUE
S2=SUM2
AK1=-M*A/2.
D0=2*(S1-(76./60.))*X*P(N2,T1)
D1=(9./2.)*(X**2)*P(N3,T1)
D2=3.*(X**3)*P(N4,T1)
D3=(5./6.)*(X**4)*P(N5,T1)
D4=S2
UR1=AK1*(D0+D1-D2+D3-D4)
C                RADIAL COMPONENT OF THE BODY VELOCITY FIELD
D5=(R-(R2*T1))
D6=(((R*T1)-R2)**2)
D7=(R**2)*(T3**2)
D8=((D6+D7)**(3/2))
AK=(K1/(4*3.14))
UR2=AK*D5/D8
UR3=-U*T1
C      TANGANTIAL VELOCITY OF THE FREE STREAM
UT3=U*T3
C        TANGANTIAL VELOCITY OF THE BODY
D9=R2*T3
```

```
D10=((((R*T1)-R2)**2)+((R**2)*(T3**2)))**(3./2.)
AK3=K1/(4.*3.14)
UT2=AK3*D9/D10
C        TANGANTIAL VELOCITY OF THE DISC
PROD2=-0.5
SUM3=0.
DO 3 I=2,IN
N=2*I
PROD2=PROD2*(-0.5-(I-1))/I
SUM3=SUM3+((PROD2/((2*I)-2))*(X**N)*AS(N,T1))
3 CONTINUE
S3=SUM3
AK4=(M*A/2.)*(1./X)
D11=(1./2.)*AS(N0,T1)
D12=(S1-(76./60.))*(X**2)*AS(N2,T1)
D13=(3./2.)*(X**3)*AS(N3,T1)
D14=(3./4.)*(X**4)*AS(N4,T1)
D15=(1./6.)*(X**5)*AS(N5,T1)
D16=S3
UT1=AK4*(D11+D12+D13-D14+D15-D16)
UTB=(UT2+UT3)
UTD=(UT1+UT3)
UT=(UT1+UT2+UT3)
UR=UR1+UR2+UR3
UC=(((UT**2)+(UR**2))**0.5)
UX=(UR*T1)-(UT*T3)
UY=(UR*T3)+(UT*T1)
XY=UY/UX
ET=ATAN(XY)*360./(2*3.14)
URB=(UR2+UR3)/U
URD=(UR1+UR3)/U
PRINT 100,XX,YY,UX,UY,ET
100 FORMAT(5X,F10.4,5X,F10.4,5X,F14.4,5X,F14.4,5X,F10.4)
GO TO 600
C   FOR R GT THAN A
5 X=A/R
C   RADIAL COMPONENTS
D5=(R-(R2*T1))
D6=(((R*T1)-R2)**2)
D7=(R**2)*(T3**2)
D8=((D6+D7)**(3/2))
AK=(K1/(4.*3.14))
UR2=AK*D5/D8
UR3=-U*T1
PROD=1.
SUM1=0.
DO 10 I=1,IN
PROD=PROD*(-0.5-(I-1))/I
N=2*I
SUM1=SUM1+((((2*I)+1)*PROD)/((2*I)+3))*(X**((2*I)+2))*P(N,T1)
10 CONTINUE
```

```
S1=SUM1
S2=(1./3.)*(X**2)*P(NO,T1)
AK1=M*A/2.
UR1=AK1*(S2+S1)
C      TANGANTIAL COMPONENTS
UT3=U*T3
D9=R2*T3
D10=((((R*T1)-R2)**2)+((R**2)*(T3**2)))**(3./2.)
AK3=K1/(4.*3.14)
UT2=AK3*D9/D10
PROD=1.
SUM=0.
DO 30 I=1,IN
N=I*2
PROD=PROD*(-0.5-(I-1))/I
SUM=SUM+(PROD/((2*I)+3))*(X**((2*I)+2))*AS(N,T1)
30 CONTINUE
S3=SUM
S4=(1./3.)*(X**2)*AS(NO,T1)
AK4=M*A/2.
UT1=AK4*(S4+S3)
UTB=UT2+UT3
UTD=UT1+UT3
UT=UT1+UT2+UT3
URB=(UR2+UR3)/U
URD=(UR1+UR3)/U
UR=UR1+UR2+UR3
UC=(((UT**2)+(UR**2))**0.5)
UX=(UR*T1)-(UT*T3)
UY=(UR*T3)+(UT*T1)
XY=UY/UX
ET=ATAN(XY)*360./(2.*3.14)
PRINT 100,XX,YY,UX,UY,ET
600 CONTINUE
800 FORMAT(//)
PRINT 800
500 CONTINUE
END
**********************************************************
FUNCTION P(N,X)
C THIS IS A FUNCTION TO CALCULATE LEGENDER POLYNOMIALS OF THE
C FIRST KIND AND NTH ORDER
Q0=1.
P=Q0
IF(N.EQ.0) GO TO 1
Q1=X
P=Q1
IF(N.EQ.1) GO TO 1
Q2=((3*(X**2))-1.)/2.
P=Q2
IF(N.EQ.2) GO TO 1
```

```
P1=((3*(X**2))-1.)/2.
P2=X
I=3
2 CONTINUE
P=((((2.*I)-1.)/I)*X*P1)-((I-1.)/I)*P2
IF(I.EQ.N) GO TO 1
I=I+1
P2=P1
P1=P
GO TO 2
1 CONTINUE
RETURN
END
C*******************************************************************
FUNCTION AS(N,X)
DIMENSION R(1000)
C THIS IS A FUNCTION TO CALCULATE ASSOCIATE LEGENDER POLYNOMIAL
C OF THE FIRST KIND AND POWER(M).
R0=0.
RA=R0
IF(N.EQ.0) GO TO 3
R1=(1-(X**2))**0.5
RA=R1
IF(N.EQ.1) GO TO 3
R2=3.*X*((1-(X**2))**0.5)
RA=R2
IF(N.EQ.2) GO TO 3
P1=X
P2=((3*(X**2))-1.)/2.
DO 10 I=3,N
R(I)=(((2*I)-1)*((1-(X**2))**0.5)*P2)+R1
P=((((2.*I)-1.)/I)*X*P2)-((I-1.)/I)*P1
P1=P2
P2=P
R1=R2
R2=R(I)
RA=R(I)
10 CONTINUE
3 CONTINUE
AS=RA
RETURN
END
READY.
```

RUN

78/07/04. 22.37.42.
FILE PROJ2

? 8,5,4.,2.,38.,16.,0.33,5.02,647.78

VELOSITY FIELD FOR LINEARLY DISTRIBUTED DISC+BODY

NX= 8
NY= 5
DELTX= 4.00
DELTY= 4.00
U= 38.0000
A= 16.0000
A1= .3300
R2= 5.0200
K1= 647.780

X IN FT.	Y IN FT.	UX IN FT/SEC.	UY IN FT./SEC.	ETA IN DEG.
4.0000	0.	-80.5193	-.0000	.0000
4.0000	4.0000	-25.7175	4.9155	-10.8261
4.0000	8.0000	-22.2953	4.4470	-11.2858
4.0000	12.0000	-18.6772	8.8719	-25.4212
4.0000	16.0000	-30.2627	10.7436	-19.5553
8.0000	0.	-11.3074	-.0000	.0000
8.0000	4.0000	-20.3965	5.2585	-14.4641
8.0000	8.0000	-23.9431	5.8885	-13.8240
8.0000	12.0000	-26.1749	8.7563	-18.5060
8.0000	16.0000	-31.6941	7.0805	-12.5996
12.0000	0.	-21.7867	-.0000	.0000
12.0000	4.0000	-23.4000	2.4939	-6.0864
12.0000	8.0000	-26.2924	3.7251	-8.0681
12.0000	12.0000	-30.7332	4.8705	-9.0097
12.0000	16.0000	-32.5288	5.1147	-8.9404

16.0000	0.	-27.9716	.0000	-.0000
16.0000	4.0000	-29.4585	1.6599	-3.2266
16.0000	8.0000	-30.6262	2.8337	-5.2889
16.0000	12.0000	-31.9344	3.5027	-6.2626
16.0000	16.0000	-33.2036	3.7821	-6.5017
20.0000	0.	-31.1806	-.0000	.0000
20.0000	4.0000	-31.4346	1.0854	-1.9786
20.0000	8.0000	-32.0854	1.9553	-3.4891
20.0000	12.0000	-32.9272	2.5318	-4.3991
20.0000	16.0000	-33.8059	2.8335	-4.7936
24.0000	0.	-32.6392	-.0000	.0000
24.0000	4.0000	-32.7848	.7521	-1.3149
24.0000	8.0000	-33.1794	1.3941	-2.4073
24.0000	12.0000	-33.7283	1.8646	-3.1659
24.0000	16.0000	-34.3369	2.1540	-3.5914
28.0000	0.	-33.6528	-.0000	.0000
28.0000	4.0000	-33.7439	.5442	-.9244
28.0000	8.0000	-33.9976	1.0263	-1.7299
28.0000	12.0000	-34.3659	1.4047	-2.3418
28.0000	16.0000	-34.7930	1.6647	-2.7406
32.0000	0.	-34.3873	-.0000	.0000
32.0000	4.0000	-34.4476	.4074	-.6779
32.0000	8.0000	-34.6182	.7773	-1.2869
32.0000	12.0000	-34.8728	1.0821	-1.7782
32.0000	16.0000	-35.1783	1.3085	-2.1313

END.

SRU 8.155 UNTS.

RUN COMPLETE.

LNH

```
00100 PROGRAM PROJS1(INPUT,OUTPUT)
00110 DIMENSION UF(50,100),AR(50),YOUT(100)
00120 REAL K,L,K1
00130 IN=50
00140C
00150 READ,NS,NA,NE,DELTX,DELTY,U,A,A1,R2,K1
00160*************************************************************************
00170*
00180*
00190*
00200*    PROGRAM FOR UNIFORMLY DISTRIBUTED DISC OF
00210*    SOURCES AND THE BODY.
00220*
00230*
00240*          THIS PROGRAM CONSTRUCTS THE STREAM LINES OF THE
00250*          FLOW FIELD MENTIONED ABOVE, IN TE FORM OFTHE
00260*          COORDINATES OF THE POINTS ON A SPCIFIC STREAM
00270*          LINE(SPECIFIED BY ITS STARTING POINT OR BY ITS
00280*          MASS FLOW RATE AS WILL BE DESCRIBED LATER.)AS ITS
00290*          Y AND X COORDINATES; THAT IS FOR EACH Y ,THE
00300*          CORESPONDING X VALUE OF THE STREAM LINE IS DETERMINED
00310*
00320*
00330*
00340*          ! Y
00350*          !
00360*          !
00370* (NS-1)*DY!....\       STREAM LINE: SAI=X
00380*          !     .\-------------------------------------.
00390*          !     .                                      .
00400*          !     .                                      .
00410*          !     .                                      .
00420*          !     .                                      .
00430*          !     .                                      .
00440*          !     .                                      .
00450*          !     .                                      .
00460*          !     .                                      .
00470*   ORIGIN +------------------------------------------------- X
00480*              NA*DX                                  NE*DX
00490*
00500*
00510*
00520*   TO RUN THIS PROGRAM:
00530*                      A)HAVING THE STARTING POINT OF THE
00540*                        STREAM LINE; ONE SOULD INPUT THE
00550*                        FOLLOWINGS IN RESPONCE TO THE ASK
00560*                        FOR DATA:NS,NA,NE,DELTX,DELTY,U,A
00570*                        ,A1,R2,K1,ACORDING TO THE DEFINITION
```

```
*                             OF THE PARAMETERS GIVEN BELOW.              *
*                             (NOTE: IN THE PRESENCE OF THE BODY          *
*                                    NA*DX SOULD BE GREATER THAN          *
*                                    THE STAGNATION POINT OF THE          *
*                                    BODY.)                               *
*                           B)HAVING THE MASS FLOW RATE THROUGH           *
*                             THE STREAM TUBE;ONE SOULD FIRST             *
*                             CHANGE THE STATEMENT REFM=SUMZ TO           *
*                             REFM=MASS FLOW INTERESTED IN,(NOTE:         *
*                             SINCE FLOW IS IN MINUS X DIRECTION,         *
*                             THE MASS FLOW WILL HAVE A MINUS             *
*                             SIGN)THEN THE FOLOWING IS INPUT             *
*                             IN RESPONCE TO THE ASK FOR DATA:            *
*                             NS,NA,NE,DELTX,DELTY,U,A,A1,R2,K1.          *
*                             (NOTE: IN THIS CASE  NS SOULD BE            *
*                              SELECTED ON AN ESTIMATE BASE,THAT          *
*                              IS A VALUE THAT ONE IS SUREIS GREATER      *
*                              THAN THE REAL VALUE.ALSO THE SAME          *
*                              RESTRICTION IS VALIED FOR NA AS            *
*                              WAS POINTED OUT IN PART A )                *
*                                                                         *
*                                                                         *
*                                                                         *
*     OUTPUT OF THE PROGRAM:                                              *
*                                VOLUMETRIC FLOW RATE THROUGH THE         *
*                                STREAM TUBE,AND X-Y COORDINATE OF        *
*                                THE STREAM LINE.                         *
*                                                                         *
*                                                                         *
*     PARAMETERS:                                                         *
*              SEE PROJ1 PROGRAM DUCUMENTS                                *
*              SEE THE PROJECT REPORT                                     *
*              SEE THE ABOVE DIAGRAM,                                     *
*              NS: STARTING POINT OF THE STREAM LINE ON                   *
*                  THE Y-AXIS.                                            *
*              NA: STARTING POINT ON THE X-AXIS.                          *
*              NE: ENDING POINT OF THE STREAM LINE ON THE                 *
*                  X-AXIS.                                                *
*                                                                         *
*                                                                         *
*-----------THE PROGRAM WAS RUN FOR:                                      *
*                                     NS=12,NA=3,NE=16,DELTX=1.,          *
*                                     DELTY=1.,U=38.,A=16.,A1=0.33,       *
*                                     R2=0.,K1=647.78                     *
*                                                                         *
*                                                                         *
***************************************************************************
*
 PRINT 201,NS,NA,NE,DELTX,DELTY,U,A,A1,R2,K1
 201 FORMAT(///,20X,*STREAM LINE CONSTRUCTION FOR UNIFORMLY  DISTRIBUTED*
+,/,30X,*DISC OF SOURCES AND THE BODY*,/,20X,*---------------------------*
```

```
+,*-------------------------*,///,*NS=*,I5,/,*NA=*,I5,/,*NE=*,I5,/,*DELTX=*,
+F6.4,/,*DELTY=*,F6.4,/,*U=*,F10.5,/,*A=*F4.1,/,*A1=*,F6.4,
+/,*R2=*,F10.6,/,*K1=*,F10.4)
C
C
C
C
C
 DO 600 III=1,NE
 DO 500 II=1,NS
 YY=(II-1)*1.
 XX=FLOAT(III)
 K=2.*A1*(1.+A1)*U
 R=(((XX**2)+(YY**2))**0.5)
 NO=0
 N1=1
 T=ATAN((YY/XX))
 T1=COS(T)
 T2=SIN(T)
 TO=0.
 IF(R.GE.A) GO TO 5
C*************************        DISC FOR R<A          *******************
 AK=K/2.
 X=R/A
 SUM=0.
 DO 1 I=1,IN
 N=I-1
 M=N-2
 SUM=SUM+(N*(X**(N-1))*(P(N,TO)+P(M,TO))*P(N,T1))
 1 CONTINUE
 UR1=AK*(P(N1,T1)-SUM)
 SUM=0.
 DO 2 I=1,IN
 N=I-1
 M=N-2
 SUM=SUM+((X**(N-1))*(P(N,TO)+P(M,TO))*AS(N,T1))
 2 CONTINUE
 UT1=AK*(SUM-AS(N1,T1))
 GO TO 200
 5 CONTINUE
C****************************     DISC FOR R>A    ********************
 X=A/R
 SUM=0.
 DO 3 I=1,IN
 N=I-1
 M=N+2
 SUM=SUM+((N+1)*(X**M)*(P(N,TO)+P(M,TO))*P(N,T1))
 3 CONTINUE
 UR1=(K/2)*SUM
 SUM=0.
 DO 4 I=1,IN
```

```
N=I-1
M=N+2
SUM=SUM+((X**M)*(P(N,TO)+P(M,TO))*AS(N,T1))
4 CONTINUE
UT1=(K/2)*SUM
200 CONTINUE
C********************      THE BODY    ********************
D15=(R-(R2*T1))
D16=(((R*T1)-R2)**2)
D17=(R**2)*(T2**2)
D18=((D16+D17)**(3/2))
AK1=(K1/(4*3.14))
UR2=AK1*D15/D18
D19=R2*T2
D110=((((R*T1)-R2)**2)+((R**2)*(T2**2)))**(3/2)
AK3=K1/(4*3.14)
UT2=AK3*D19/D110
C****************************  UNIFORM FLOW  **************
UR3=-U*T1
UT3=U*T2
C****************************  SUPERPOSITION  **************
UR=UR1+UR2+UR3
UT=UT1+UT2+UT3
UX=(UR*T1)-(UT*T2)
UY=(UR*T2)+(UT*T1)
XY=UY/UX
ET=ATAN(XY)*360./(2.*3.14)
C
C    VELOCITY FIELD AND STREAM TUBE AREA STORAGE
C
UF(III,II)=UX
500 CONTINUE
600 CONTINUE
AT=0.
DO 601 I=2,NS
AR(1)=3.14*((DELTY/2)**2)
IO=I-1
AT=AT+AR(IO)
AR(I)=(3.14*((((I-1)*DELTY)+(DELTY/2))**2))-AT
601 CONTINUE
C   REFERENCE MASS FLOW CONSTRAUCTION
C
SUMZ=0.
DO 6 I=1,NS
SUMZ=SUMZ+(AR(I)*UF(NA,I))
6 CONTINUE
REFM=SUMZ
PRINT 501,REFM
501 FORMAT(//,20X,*VOLUMETRIC FLOW RATE THROUGH THE STREAM TUBE=*,F14.3)
PRINT 504
504 FORMAT(////,20X,*X IN FT.*,12X,*Y IN FT.*,/,20X,*----------*,10X,*-----
```

```
+------*)
C
C       STREAM LINES CONSTRUCTION
C
 AL=0.1
 DO 7 I=NA,NE
 FLOW=0.
 J=1
 10 CONTINUE
 FLOW1=FLOW
 FLOW=FLOW+(AR(J)*UF(I,J))
 ALLOW=(FLOW-REFM)
 IF(ABS(ALLOW).LE.AL) GO TO 8
 IF(ALLOW.GT.0) GO TO 9
 DELT1Y=(FLOW1/(FLOW1+FLOW))
 YOUT(I)=((J-2)*DELTY)+(DELT1Y*DELTY)
 GO TO 11
 9 J=J+1
 GO TO 10
 8 YOUT(I)=(J-1)*DELTY
 11 CONTINUE
 XXX=I*DELTX
 YOUT(I)=YOUT(I)+(DELTY/2)
 PRINT 505,XXX,YOUT(I)
 505 FORMAT(20X,F10.3,10X,F10.3)
 7 CONTINUE
 END
C
C
C
C
C
C
C
C
C
C
C
C
C
*******************************************************
 FUNCTION P(N,X)
C
C
C THIS IS A FUNCTION TO CALCULATE LEGENDER POLYNOMIALS OF THE
C  FIRST KIND
C
C
 Q0=0.
 P=Q0
 IF(N.LT.0) GO TO 1
```

```
Q0=1.
P=Q0
IF(N.EQ.0) GO TO 1
Q1=X
P=Q1
IF(N.EQ.1) GO TO 1
Q2=((3*(X**2))-1.)/2.
P=Q2
IF(N.EQ.2) GO TO 1
P1=((3*(X**2))-1.)/2.
P2=X
I=3
2 CONTINUE
P=((((2.*I)-1.)/I)*X*P1)-((I-1.)/I)*P2
IF(I.EQ.N) GO TO 1
I=I+1
P2=P1
P1=P
GO TO 2
1 CONTINUE
RETURN
END
FUNCTION AS(N,X)
C***********************************************************
DIMENSION R(1000)
C
C
C  THIS FUNCTION CALCULATES ASSOCIATED LEGENDER POLYNOMIALS
C   OF THE FIRST KIND AND FIRST POWER
C
C
R0=0.
RA=R0
IF(N.LE.0) GO TO 3
R1=(1-(X**2))**0.5
RA=R1
IF(N.EQ.1) GO TO 3
R2=3.*X*((1-(X**2))**0.5)
RA=R2
IF(N.EQ.2) GO TO 3
P1=X
P2=((3*(X**2))-1.)/2.
DO 10 I=3,N
R(I)=(((2*I)-1)*((1-(X**2))**0.5)*P2)+R1
P=((((2.*I)-1.)/I)*X*P2)-((I-1.)/I)*P1
P1=P2
P2=P
R1=R2
R2=R(I)
RA=R(I)
10 CONTINUE
```

```
03130 3 CONTINUE
03140 AS=RA
03150 RETURN
03160 END
READY.
```

```
RUN

 78/07/04. 22.52.15.
   FILE   PROJS1

? 14,7,20,1.,1.,38.,16.,0.33,5.02,647.78
```

STREAM LINE CONSTRUCTION FOR UNIFORMLY DISTRIBUTED
DISC OF SOURCES AND THE BODY

```
NS=    14
NA=     7
NE=    20
DELTX=1.0000
DELTY=1.0000
U=  38.00000
A=16.0
A1= .3300
R2=  5.020000
K1=  647.7800

*TIME LIMIT*

SRU      25.287 UNTS.

T,2000
```

VOLUMETRIC FLOW RATE THROUGH THE STREAM TUBE=   -15675.736

X IN FT.	Y IN FT.
7.000	13.500
8.000	12.957
9.000	12.957
10.000	12.957
11.000	12.958
12.000	12.958
13.000	12.959
14.000	12.959
15.000	12.959
16.000	12.960

17.000	12.960
18.000	12.960
19.000	12.960
20.000	11.957

```
END.

SRU      57.256 UNTS.

RUN COMPLETE.
```

APPENDIX 2D

```
LIST

 78/07/04. 22.56.16.
   FILE   PROJS2

 PROGRAM PROJS2(INPUT,OUTPUT)
 DIMENSION UF(50,100),AR(50),YOUT(100)
 IN=50
C*****************************************************************
C                                                                *
C                                                                *
C    PRGRAM FOR LINEARLY DISTRIBUTED DISC OF                     *
C    SOURCES AND THE BODY.                                       *
C                                                                *
C                                                                *
C                                                                *
C             THIS PROGRAM CONSTRUCTS THE STREAM LINES           *
C             FOR THE LINEARLY DISTRIBUTED DISC OF SOURCES       *
C             IN COMPLETE ACCORDANCE TO PROGRAM PROJS1.          *
C                                                                *
C                                                                *
C*****************************************************************
 REAL K1,M,K,L
C
 READ,NS,NA,NE,DELTX,DELTY,U,A,A1,R2,K1
 PRINT 201,NS,NA,NE,DELTX,DELTY,U,A,A1,R2,K1
 201 FORMAT(///,20X,*STREAM LINE CONSTRUCTION FOR LINEARLY DISTRIBUTED*
+,/,30X,*DISC OF SOURCES AND THE BODY*,/,20X,*-------------------------*,
+,*---------------------*,///,*NS=*,I5,/,*NA=*,I5,/,*NE=*,I5,/,*DELTX=*,
+F6.4,/,*DELTY=*,F6.4,/,*U=*,F10.5,/,*A=*,F4.1,/,*A1=*,F6.4,
+/,*R2=*,F10.6,/,*K1=*,F10.4)
 DO 500 III=1,NE
 DO 600 II=1,NS
 YY=(II-1)*1.
 XX=FLOAT(III)
 R=(((XX**2)+(YY**2))**0.5)
 6 CONTINUE
C             RADIAL COMPONENT OF THE DISC VELOCITY FIELD
 N0=0
 N1=1
 N2=2
 N3=3
 N4=4
 N5=5
 X=R/A
 PROD=-0.5
 M=3*A1*(1.+A1)*U/A
 T2=ATAN((YY/XX))
 T1=COS(T2)
 T3=SIN(T2)
```

```
IF(R.GE.A) GO TO 5
SUM1=0.
DO 1 I=2,IN
PROD=PROD*(-0.5-(I-1))/I
SUM1=SUM1+((((4*I)+1.)/(((2*I)+3.)*((2*I)-2.)))*PROD)
1 CONTINUE
S1=SUM1
PROD1=-0.5
SUM2=0.
DO 2 I=2,IN
N=2*I
PROD1=PROD1*(-0.5-(I-1))/I
SUM2=SUM2+(((I*PROD1)/(I-1))*(X**((2*I)-1))*P(N,T1))
2 CONTINUE
S2=SUM2
AK1=-M*A/2.
D0=2*(S1-(76./60.))*X*P(N2,T1)
D1=(9./2.)*(X**2)*P(N3,T1)
D2=3.*(X**3)*P(N4,T1)
D3=(5./6.)*(X**4)*P(N5,T1)
D4=S2
UR1=AK1*(D0+D1-D2+D3-D4)
C                  RADIAL COMPONENT OF THE BODY VELOCITY FIELD
D5=(R-(R2*T1))
D6=(((R*T1)-R2)**2)
D7=(R**2)*(T3**2)
D8=((D6+D7)**(3/2))
AK=(K/(4*3.14))
UR2=AK*D5/D8
UR3=-U*T1
C    TANGANTIAL VELOCITY OF THE FREE STREAM
UT3=U*T3
C      TANGANTIAL VELOCITY OF THE BODY
D9=R2*T3
D10=((((R*T1)-R2)**2)+((R**2)*(T3**2)))**(3./2.)
AK3=K/(4.*3.14)
UT2=AK3*D9/D10
C       TANGANTIAL VELOCITY OF THE DISC
PROD2=-0.5
SUM3=0.
DO 3 I=2,IN
N=2*I
PROD2=PROD2*(-0.5-(I-1))/I
SUM3=SUM3+((PROD2/((2*I)-2))*(X**N)*AS(N,T1))
3 CONTINUE
S3=SUM3
AK4=(M*A/2.)*(1./X)
D11=(1./2.)*AS(N0,T1)
D12=(S1-(76./60.))*(X**2)*AS(N2,T1)
D13=(3./2.)*(X**3)*AS(N3,T1)
D14=(3./4.)*(X**4)*AS(N4,T1)
```

```
D15=(1./6.)*(X**5)*AS(N5,T1)
D16=S3
UT1=AK4*(D11+D12+D13-D14+D15-D16)
UTB=(UT2+UT3)
UTD=(UT1+UT3)
UT=(UT1+UT2+UT3)
UR=UR1+UR2+UR3
UC=(((UT**2)+(UR**2))**0.5)
UX=(UR*T1)-(UT*T3)
UY=(UR*T3)+(UT*T1)
XY=UY/UX
ET=ATAN(XY)*360./(2*3.14)
URB=(UR2+UR3)/U
URD=(UR1+UR3)/U
GO TO 603
C   FOR R GT THAN A
5 X=A/R
C   RADIAL COMPONENTS
D5=(R-(R2*T1))
D6=(((R*T1)-R2)**2)
D7=(R**2)*(T3**2)
D8=((D6+D7)**(3/2))
AK=(K/(4.*3.14))
UR2=AK*D5/D8
UR3=-U*T1
PROD=1.
SUM1=0.
DO 10 I=1,IN
PROD=PROD*(-0.5-(I-1))/I
N=2*I
SUM1=SUM1+((((2*I)+1)*PROD)/((2*I)+3))*(X**((2*I)+2))*P(N,T1)
10 CONTINUE
S1=SUM1
S2=(1./3.)*(X**2)*P(N0,T1)
AK1=M*A/2.
UR1=AK1*(S2+S1)
C      TANGANTIAL COMPONENTS
UT3=U*T3
D9=R2*T3
D10=((((R*T1)-R2)**2)+((R**2)*(T3**2)))**(3./2.)
AK3=K/(4.*3.14)
UT2=AK3*D9/D10
PROD=1.
SUM=0.
DO 30 I=1,IN
N=I*2
PROD=PROD*(-0.5-(I-1))/I
SUM=SUM+(PROD/((2*I)+3))*(X**((2*I)+2))*AS(N,T1)
30 CONTINUE
S3=SUM
S4=(1./3.)*(X**2)*AS(N0,T1)
```

```
AK4=M*A/2.
UT1=AK4*(S4+S3)
UTB=UT2+UT3
UTD=UT1+UT3
UT=UT1+UT2+UT3
URB=(UR2+UR3)/U
URD=(UR1+UR3)/U
UR=UR1+UR2+UR3
UC=(((UT**2)+(UR**2))**0.5)
UX=(UR*T1)-(UT*T3)
UY=(UR*T3)+(UT*T1)
XY=UY/UX
ET=ATAN(XY)*360./(2.*3.14)
603 CONTINUE
C   VELOCITY FIELD AND STREAM TUBE AREA STORAGE
UF(III,II)=UX
600 CONTINUE
500 CONTINUE
AT=0.
DO 601 I=2,NS
AR(1)=3.14*(((DELTY/2)**2))
IO=I-1
AT=AT+AR(IO)
AR(I)=(3.14*((((I-1)*DELTY)+(DELTY/2))**2))-AT
601 CONTINUE
C
C     REFERENCE MASS FLOW RATE
SUMZ=0.
DO 65 I=1,NS
SUMZ=SUMZ+(AR(I)*UF(NA,I))
65 CONTINUE
REFM=SUMZ
PRINT 562,REFM
562 FORMAT(30X,*REFERENCE FLOW RATE=*,F10.4)
C
C  STREAM LINE CONSTRUCTION
C
PRINT 19
19 FORMAT(//,5X,2X,*X IN FT.*,6X,2X,*Y IN FT.*,/,5X,*----------*,
+5X,*----------*)
AL=0.1
DO 71 I=NA,NE
FLOW=0.
J=1
1011 CONTINUE
FLOW1=FLOW
FLOW=FLOW+(AR(J)*UF(I,J))
ALLOW=(FLOW-REFM)
IF(ABS(ALLOW).LE.AL) GO TO 8
IF(ALLOW.GT.0) GO TO 9
DELT1Y=(FLOW1/(FLOW1+FLOW))
```

```
YOUT(I)=((J-2)*DELTY)+(DELT1Y*DELTY)
GO TO 11
9 J=J+1
GO TO 1011
8 YOUT(I)=(J-1)*DELTY
11 CONTINUE
YOUT(I)=YOUT(I)+(DELTY/2)
XXX=I*DELTX
PRINT 73,XXX,YOUT(I)
73 FORMAT(5X,F10.3,5X,F10.3)
71 CONTINUE
END
C
C
C
*************************************************************
FUNCTION P(N,X)
C THIS IS A FUNCTION TO CALCULATE LEGENDER POLYNOMIALS OF THE
C FIRST KIND AND NTH ORDER
Q0=1.
P=Q0
IF(N.EQ.0) GO TO 1
Q1=X
P=Q1
IF(N.EQ.1) GO TO 1
Q2=((3*(X**2))-1.)/2.
P=Q2
IF(N.EQ.2) GO TO 1
P1=((3*(X**2))-1.)/2.
P2=X
I=3
2 CONTINUE
P=((((2.*I)-1.)/I)*X*P1)-((I-1.)/I)*P2
IF(I.EQ.N) GO TO 1
I=I+1
P2=P1
P1=P
GO TO 2
1 CONTINUE
RETURN
END
C**************************************************************
FUNCTION AS(N,X)
DIMENSION R(1000)
C THIS IS A FUNCTION TO CALCULATE ASSOCIATE LEGENDER POLYNOMIAL
C OF THE FIRST KIND AND POWER(M).
R0=0.
RA=R0
IF(N.EQ.0) GO TO 3
R1=(1-(X**2))**0.5
RA=R1
```

```
IF(N.EQ.1) GO TO 3
R2=3.*X*((1-(X**2))**0.5)
RA=R2
IF(N.EQ.2) GO TO 3
P1=X
P2=((3*(X**2))-1.)/2.
DO 10 I=3,N
R(I)=(((2*I)-1)*((1-(X**2))**0.5)*P2)+R1
P=((((2.*I)-1.)/I)*X*P2)-((I-1.)/I)*P1
P1=P2
P2=P
R1=R2
R2=R(I)
RA=R(I)
10 CONTINUE
3 CONTINUE
AS=RA
RETURN
END
READY.
```

```
RUN

 78/07/04. 23.01.56.
   FILE   PROJS2

? 14,7,20,1.,1.,38.,16.,0.33,5.02,647.78
```

STREAM LINE CONSTRUCTION FOR LINEARLY DISTRIBUTED
DISC OF SOURCES AND THE BODY

```
NS=   14
NA=    7
NE=   20
DELTX=1.0000
DELTY=1.0000
U=  38.00000
A=16.0
A1= .3300
R2=  5.020000
K1=  647.7800


*TIME LIMIT*

SRU      24.620 UNTS.

T,5000
```

REFERENCE FLOW RATE=*5390.1555

X IN FT.	Y IN FT.
7.000	13.500
8.000	12.960
9.000	12.959
10.000	12.958
11.000	12.958
12.000	12.959
13.000	11.955
14.000	11.956
15.000	11.957
16.000	11.958

17.000	11.958
18.000	11.958
19.000	11.958
20.000	11.958

END.

SRU 46.583 UNTS.

RUN COMPLETE.

APPENDIX 3

Future Work Recommendations

A more realistic case can be considered in which the strength of the sources on the disc, which falls inside the body of the windmill is put to be zero.

Fig. (22) shows the profile of the source density $\acute{k}$, in the case of linearly distributed source as calculated in this work. Fig. (23) shows the improvement which should be made. This improvement, although physically more realistic, is very cumbersome mathematically.

The potential for the improved model on the x-axis can be found as was described in Chapter II. It can be easily shown that:

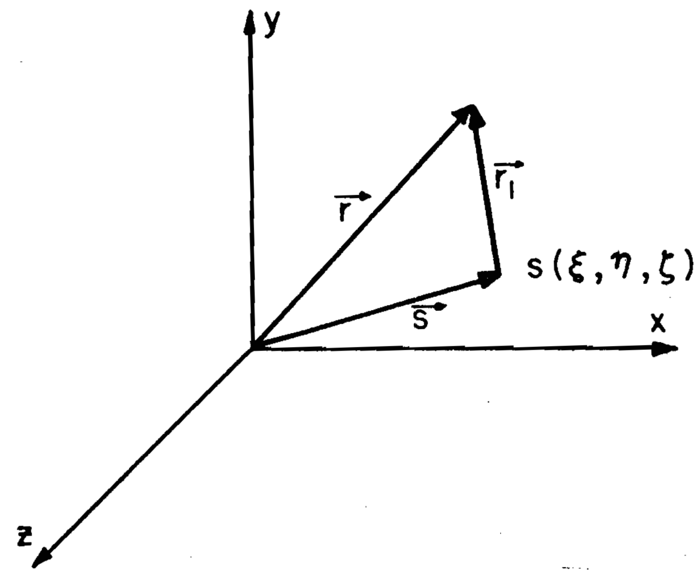
$$\Phi = \frac{m}{2}\left[\left(\frac{A}{2} - b\right)\sqrt{A^2+R^2} - \frac{R^2}{2}\ln\frac{A+\sqrt{A^2+R^2}}{b^2+\sqrt{b^2+R^2}} + \frac{b}{2}\sqrt{b^2+R^2} \right]$$

Using Lagallay's theorem and the one dimensional momentum theory it can be shown that

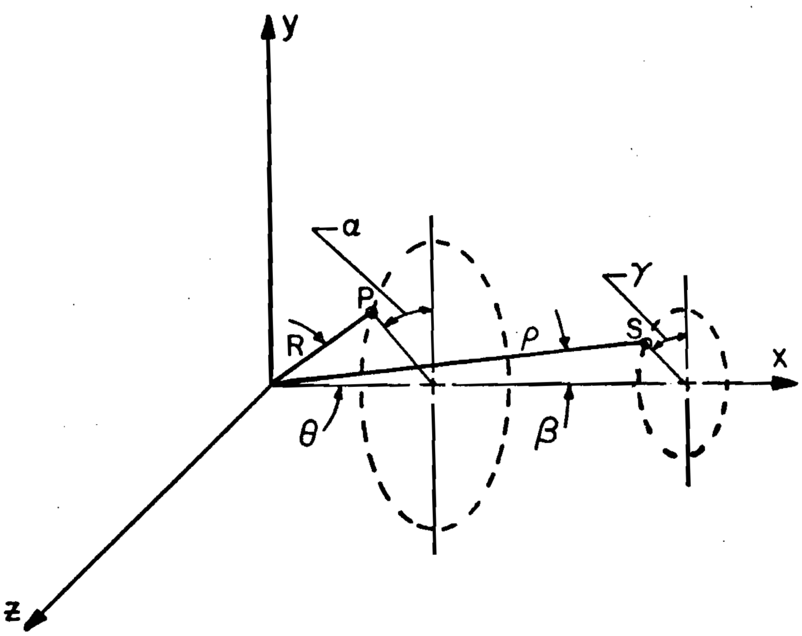
$$m = \frac{3A^2 a(1+a)U}{(A^3 - b^3)}$$

Then what remains to be done is the expansion in zonal harmonics

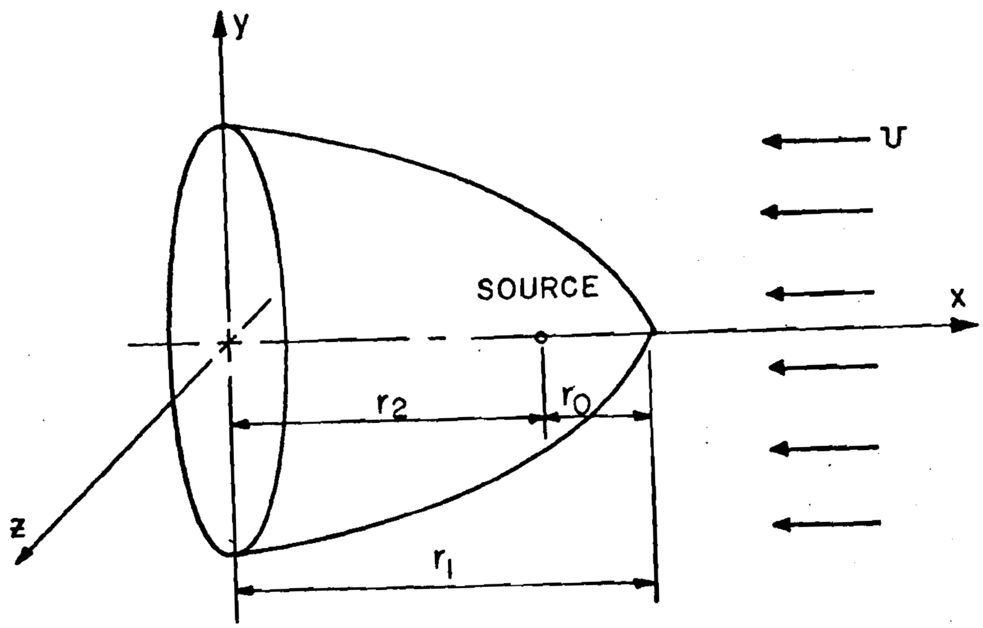
F I G U R E S



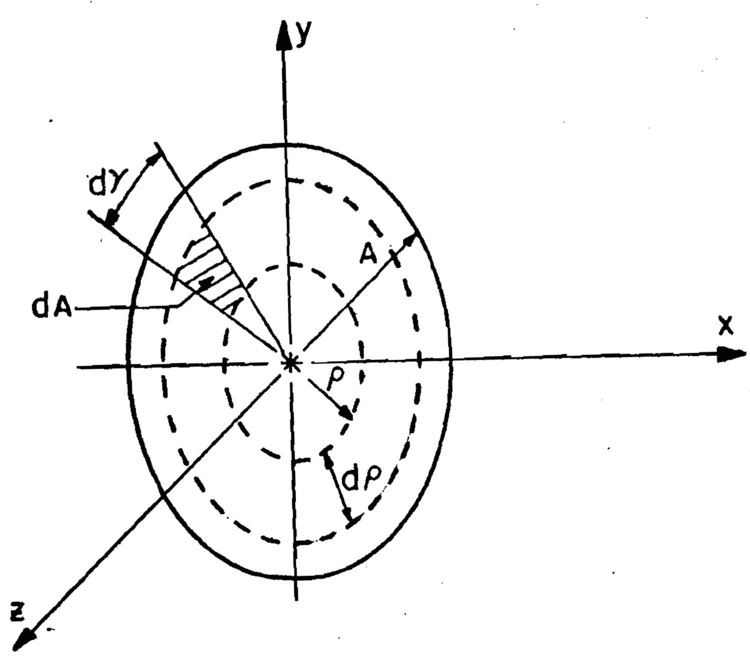
F I G U R E 1.



F I G U R E 2.



F I G U R E 3.



F I G U R E 4.

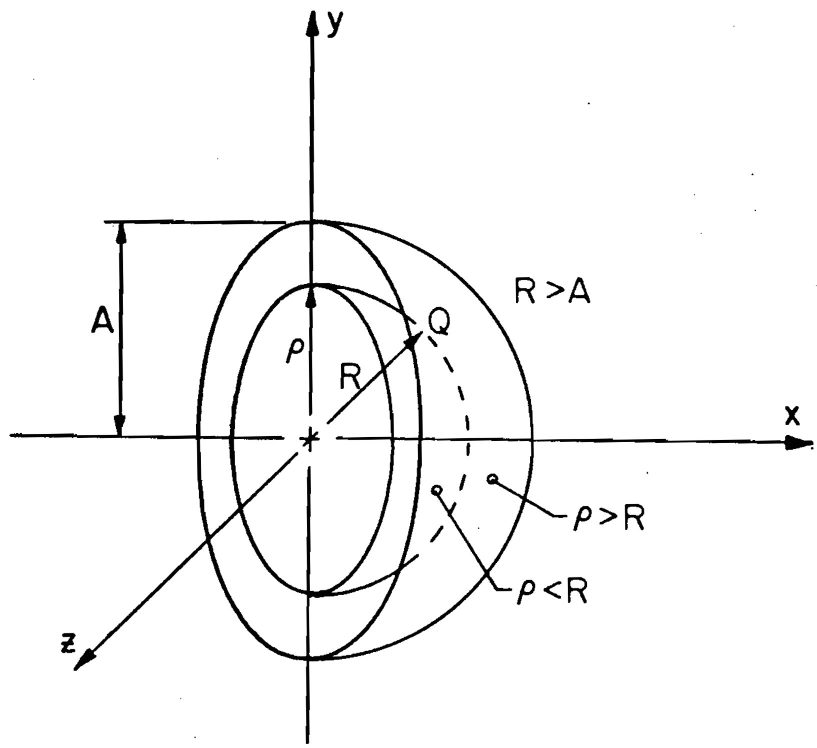


FIGURE 5.

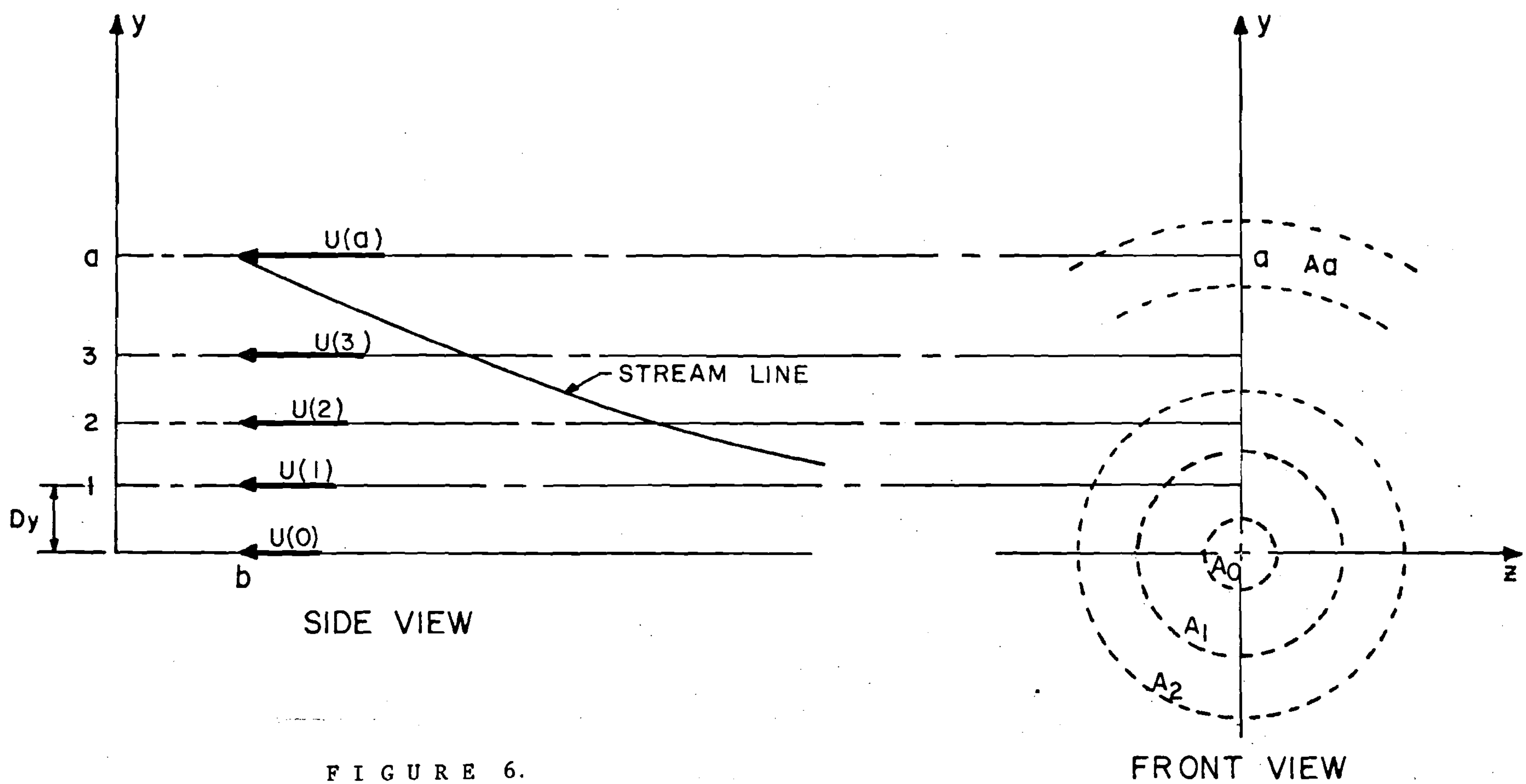
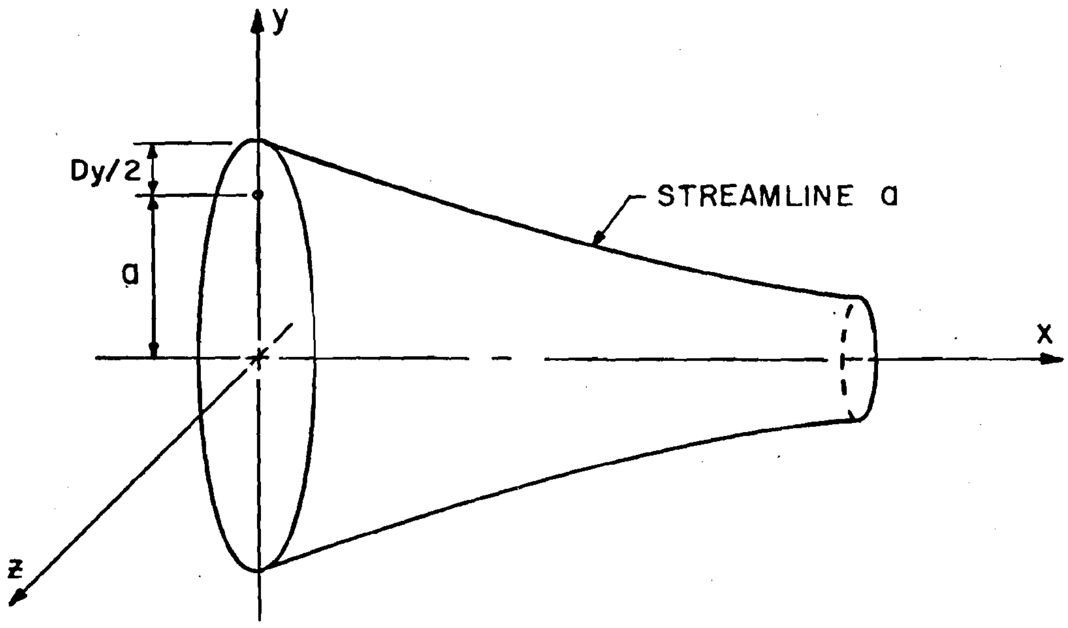
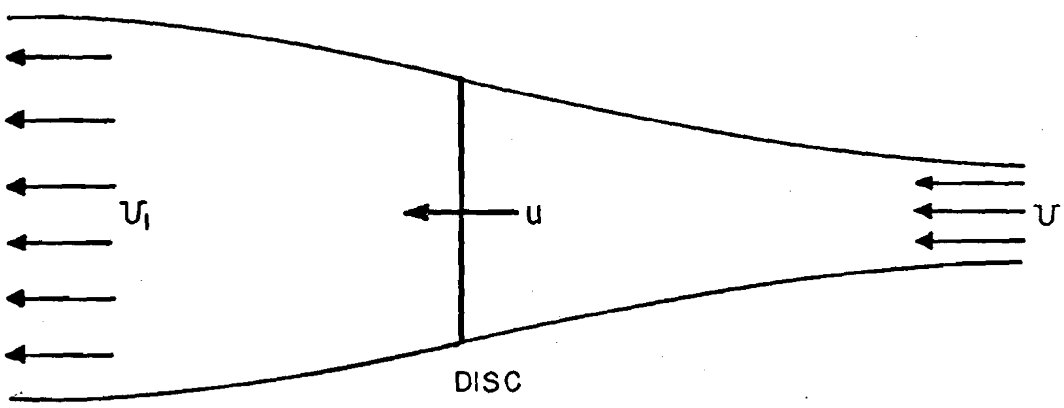


FIGURE 6.

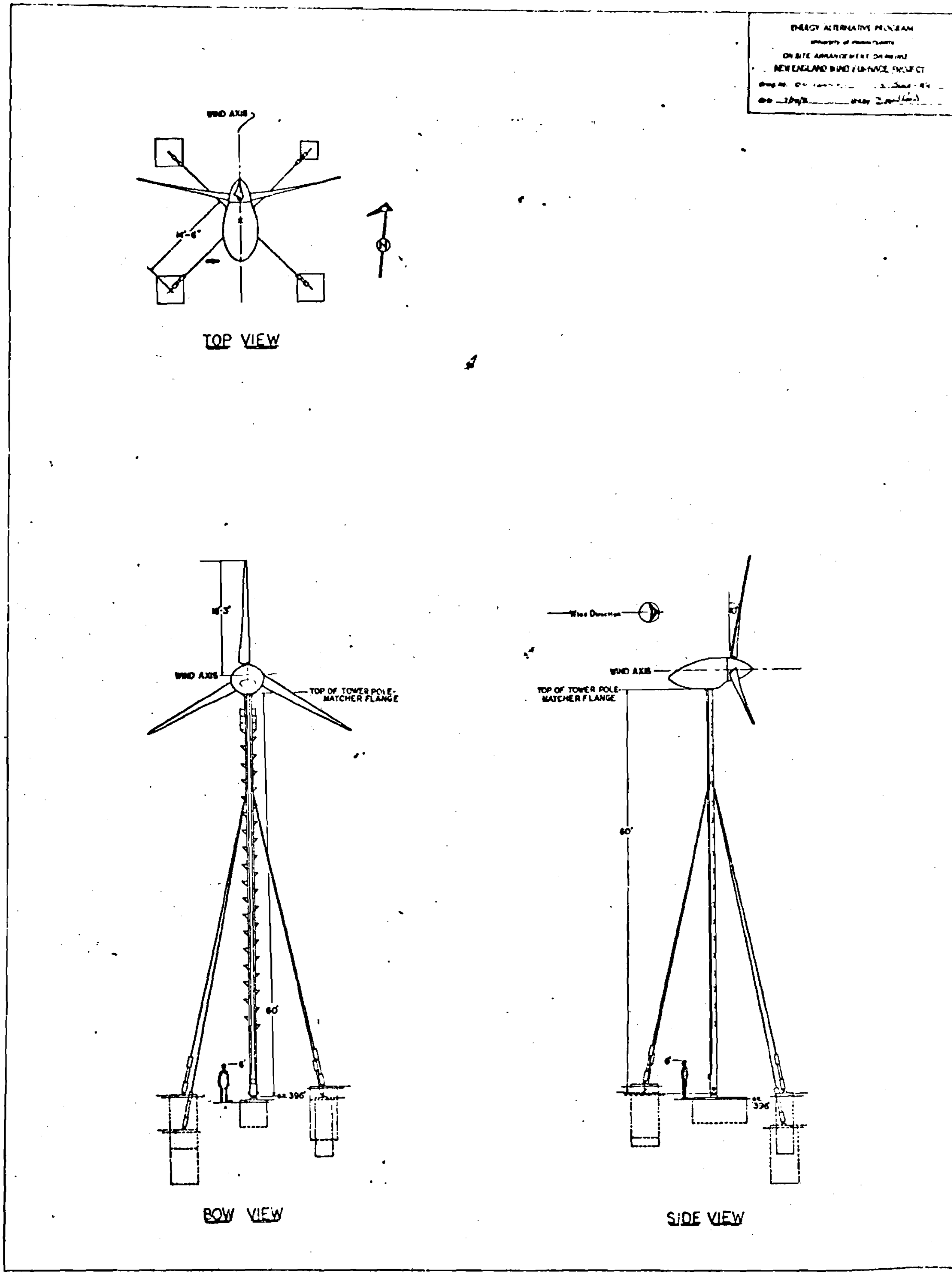
FIGURE 7.



F I G U R E 8.



F I G U R E 9.


NEW ENGLAND WIND FURNACE PROJECT
WIND AXIS
14'-6"
TOP VIEW
16'-3"
WIND AXIS
TOP OF TOWER POLE-MATCHER FLANGE
60'
6'
396'
BOW VIEW
WIND AXIS
TOP OF TOWER POLE-MATCHER FLANGE
60'
6'
396'
SIDE VIEW

F I G U R E 10.

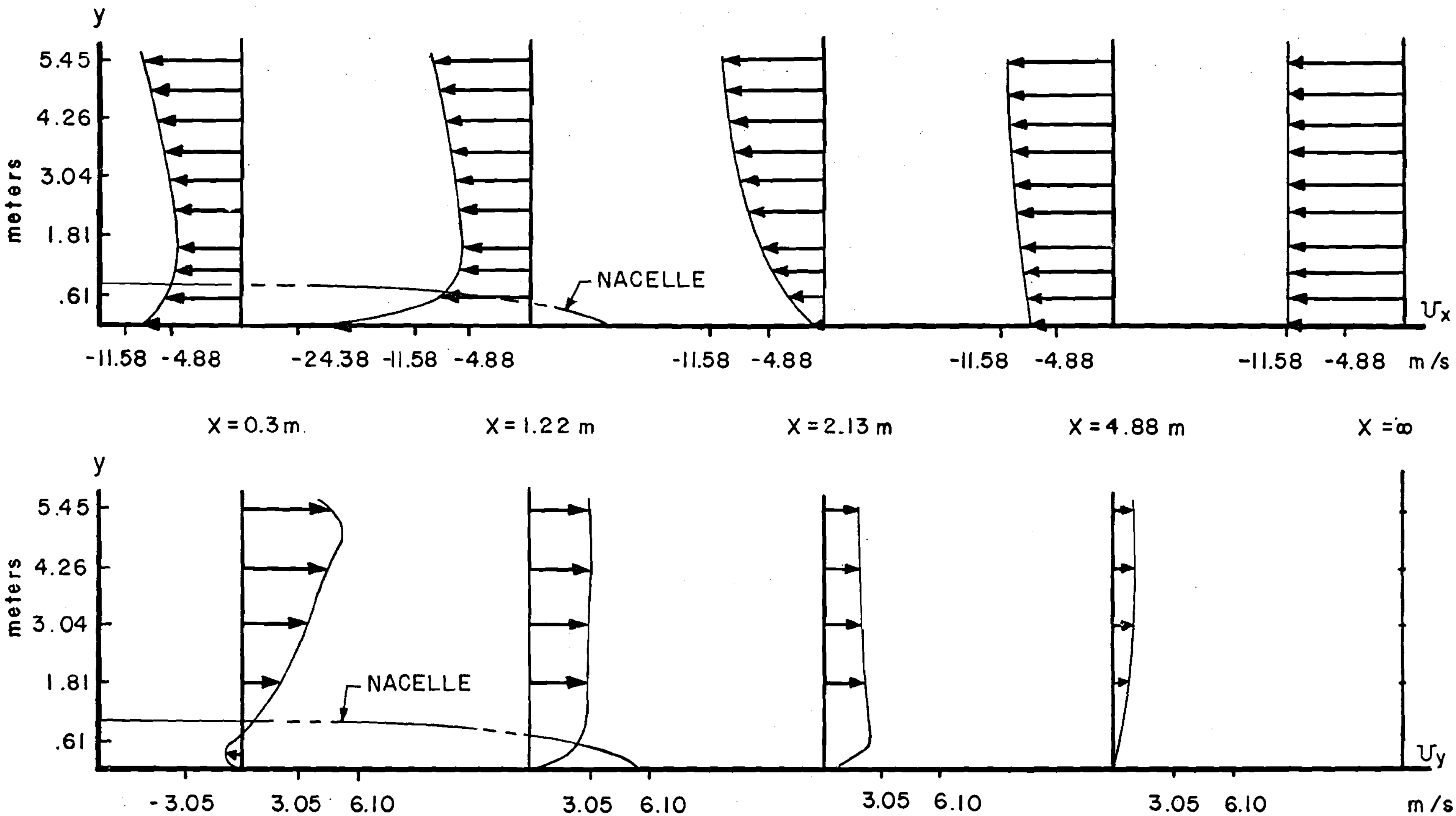

y
meters
5.45
4.26
3.04
1.81
.61
NACELLE
-11.58 -4.88
-24.38 -11.58 -4.88
-11.58 -4.88
-11.58 -4.88
-11.58 -4.88 m/s
Ux
X = 0.3 m
X = 1.22 m
X = 2.13 m
X = 4.88 m
X = ∞
-3.05
3.05 6.10
m/s
Uy

FIGURE 11.

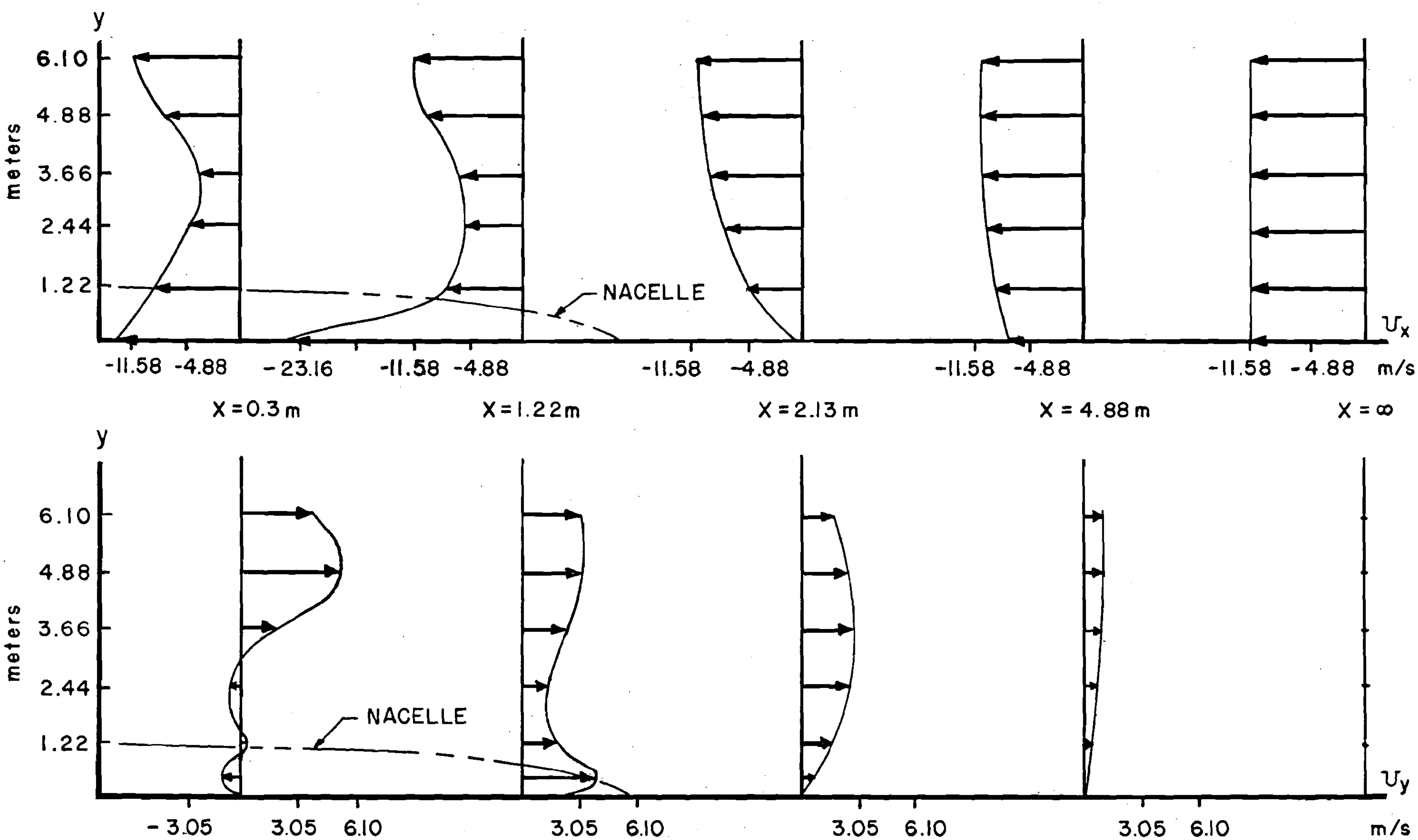

y
meters
6.10
4.88
3.66
2.44
1.22
NACELLE
U_x
-11.58 -4.88
-23.16
-11.58 -4.88
-11.58 -4.88
-11.58 -4.88
-11.58 -4.88
m/s
X = 0.3 m
X = 1.22 m
X = 2.13 m
X = 4.88 m
X = ∞
y
meters
6.10
4.88
3.66
2.44
1.22
NACELLE
U_y
-3.05
3.05 6.10
3.05 6.10
3.05 6.10
3.05 6.10
m/s

FIGURE 12.

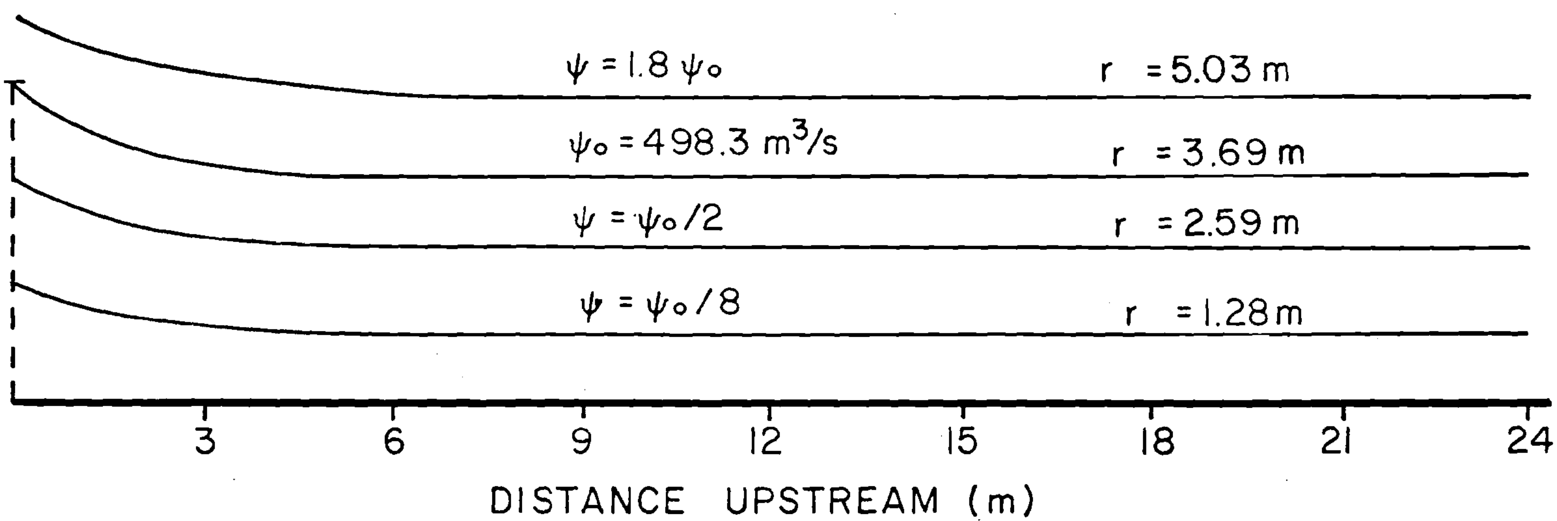
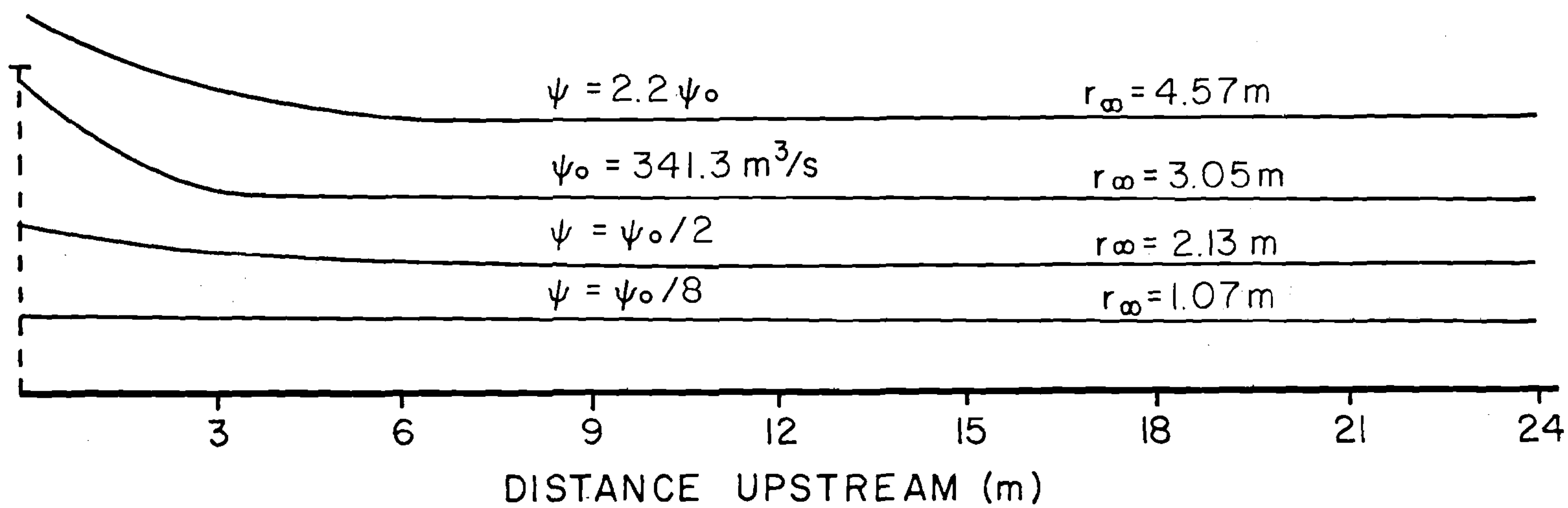

ψ = 1.8 ψ₀
r = 5.03 m
ψ₀ = 498.3 m³/s
r = 3.69 m
ψ = ψ₀/2
r = 2.59 m
ψ = ψ₀/8
r = 1.28 m
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m)

FIGURE 13.



ψ = 2.2 ψo
r∞ = 4.57 m
ψo = 341.3 m³/s
r∞ = 3.05 m
ψ = ψo/2
r∞ = 2.13 m
ψ = ψo/8
r∞ = 1.07 m
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m)

FIGURE 14.

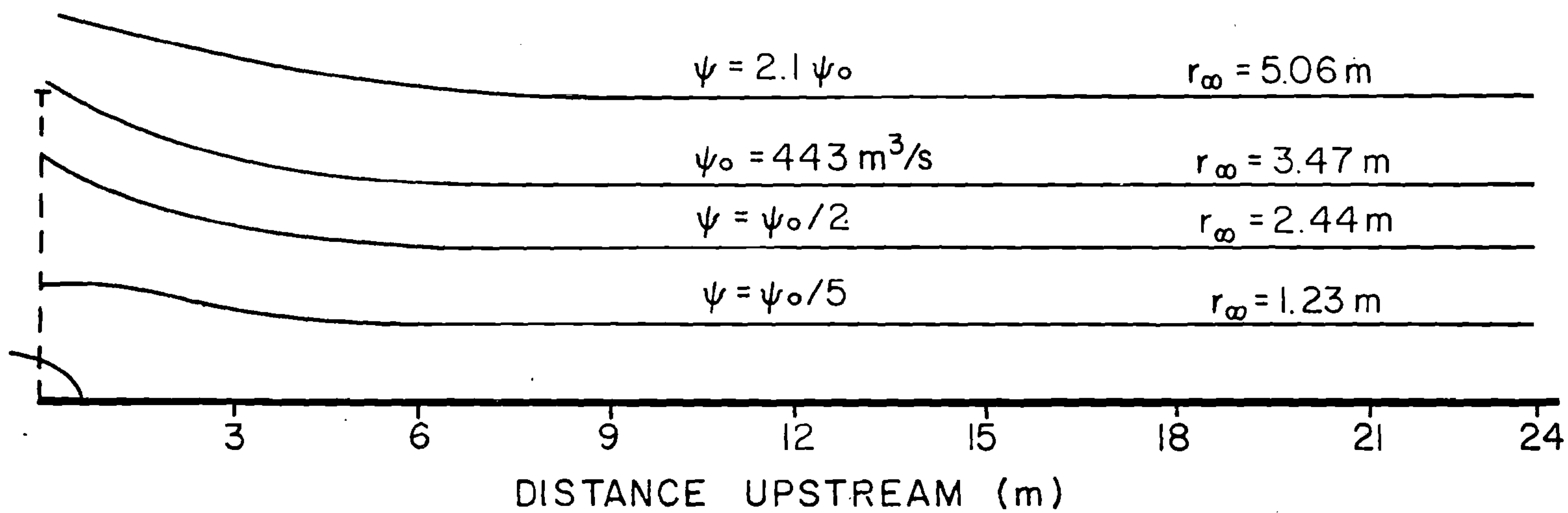

ψ = 2.1 ψo
r∞ = 5.06 m
ψo = 443 m³/s
r∞ = 3.47 m
ψ = ψo/2
r∞ = 2.44 m
ψ = ψo/5
r∞ = 1.23 m
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m)

FIGURE 15.

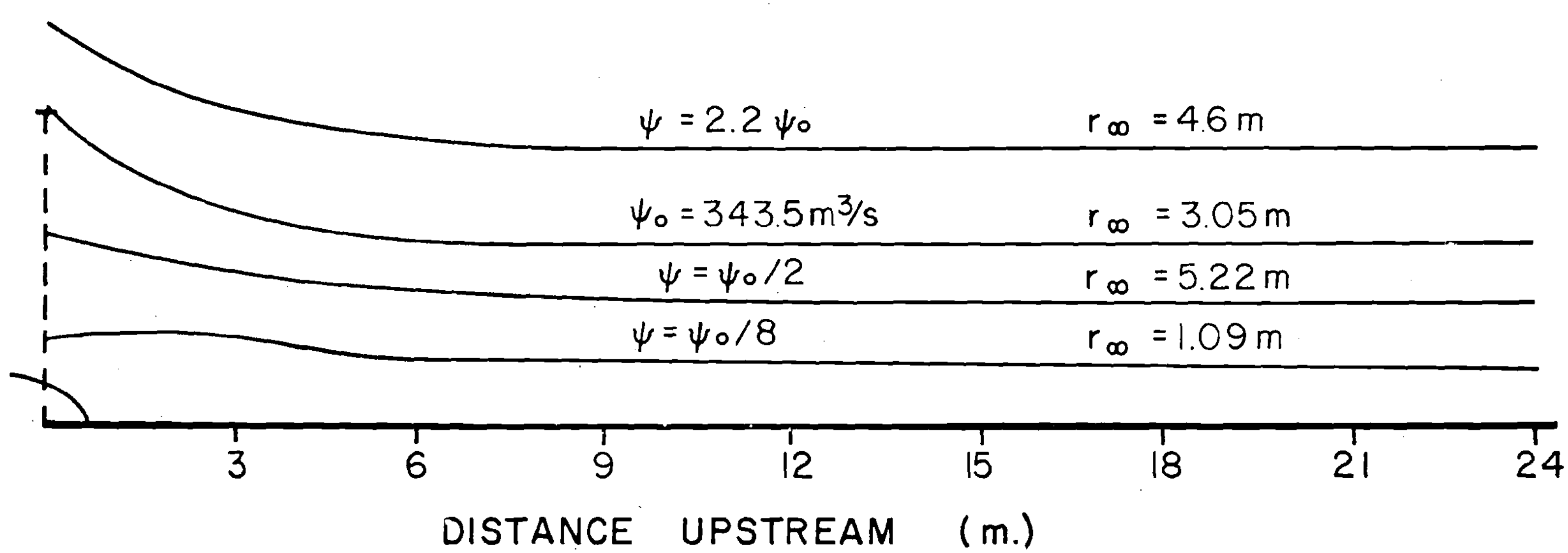

ψ = 2.2 ψo
r∞ = 4.6 m
ψo = 343.5 m³/s
r∞ = 3.05 m
ψ = ψo/2
r∞ = 5.22 m
ψ = ψo/8
r∞ = 1.09 m
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m.)

FIGURE 16.

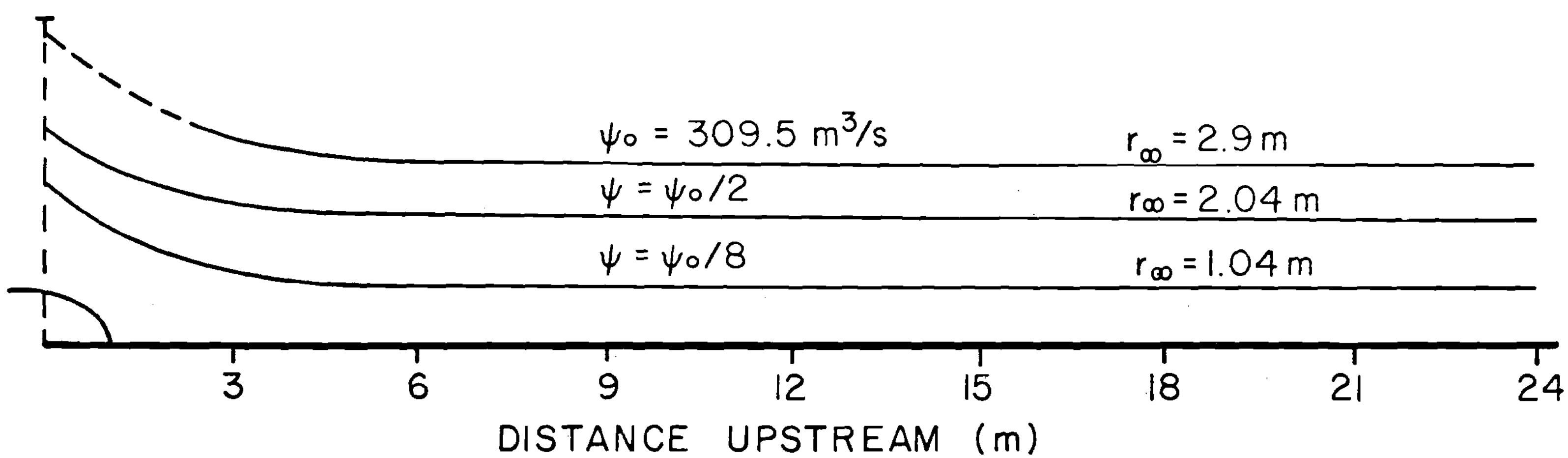

ψo = 309.5 m3/s
ψ = ψo/2
ψ = ψo/8
r∞ = 2.9 m
r∞ = 2.04 m
r∞ = 1.04 m
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m)

FIGURE 17.

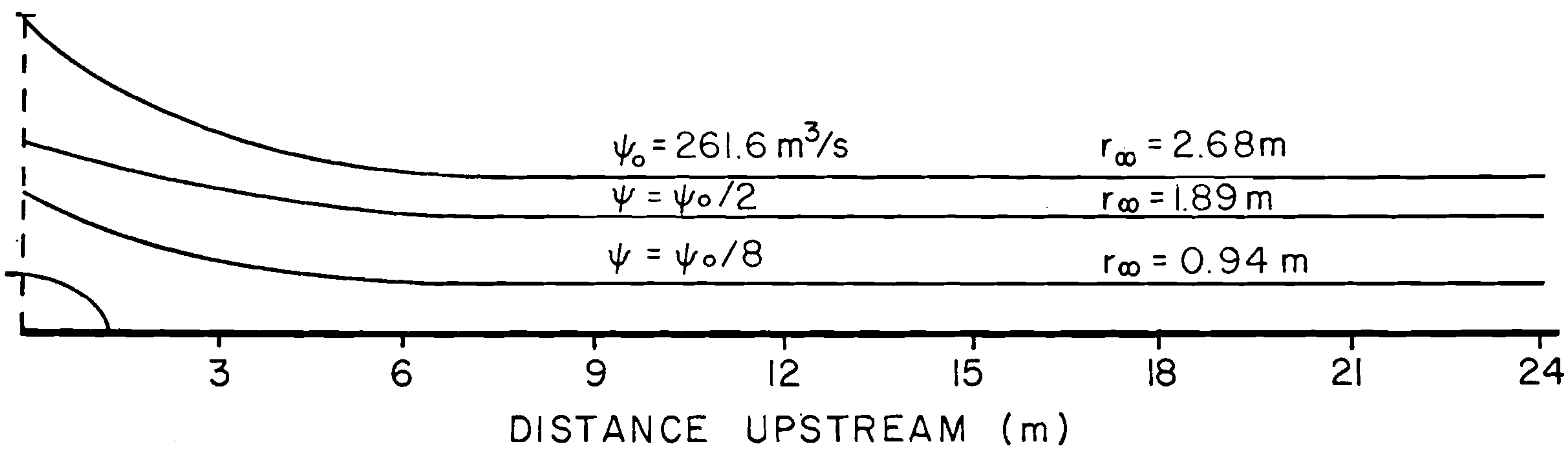

ψo = 261.6 m³/s
ψ = ψo/2
ψ = ψo/8
r∞ = 2.68m
r∞ = 1.89 m
r∞ = 0.94 m
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m)

FIGURE 18.

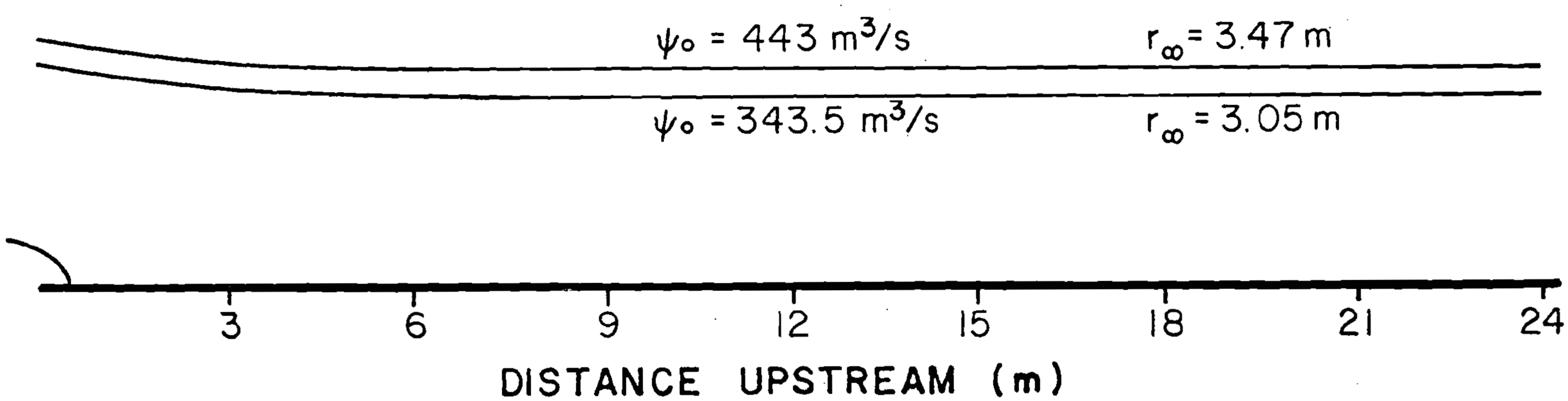

ψ_o = 443 m^3/s
r_∞ = 3.47 m
ψ_o = 343.5 m^3/s
r_∞ = 3.05 m
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m)

FIGURE 19.

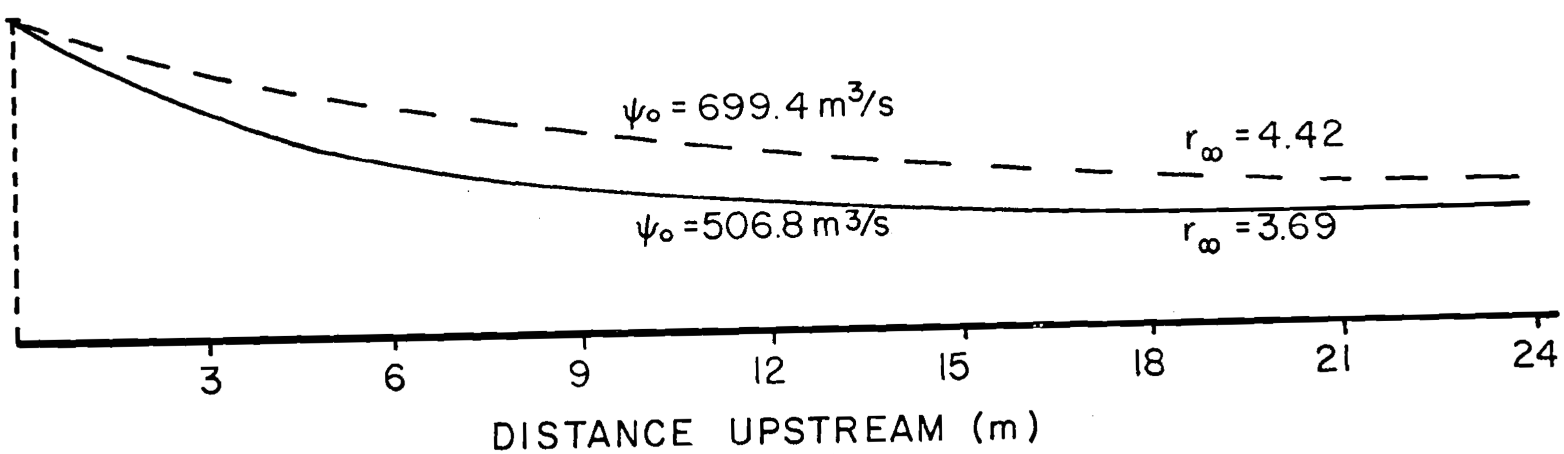
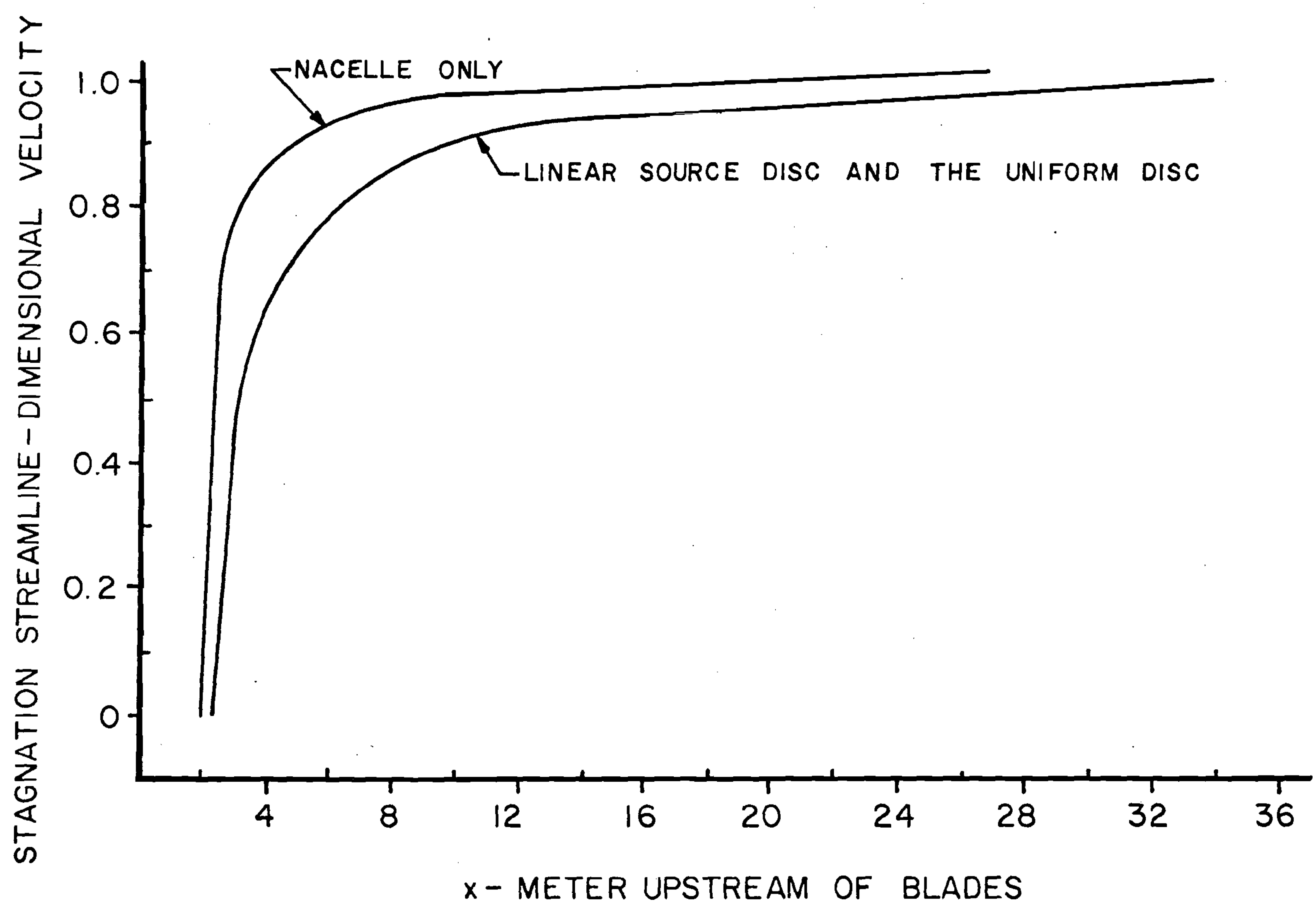
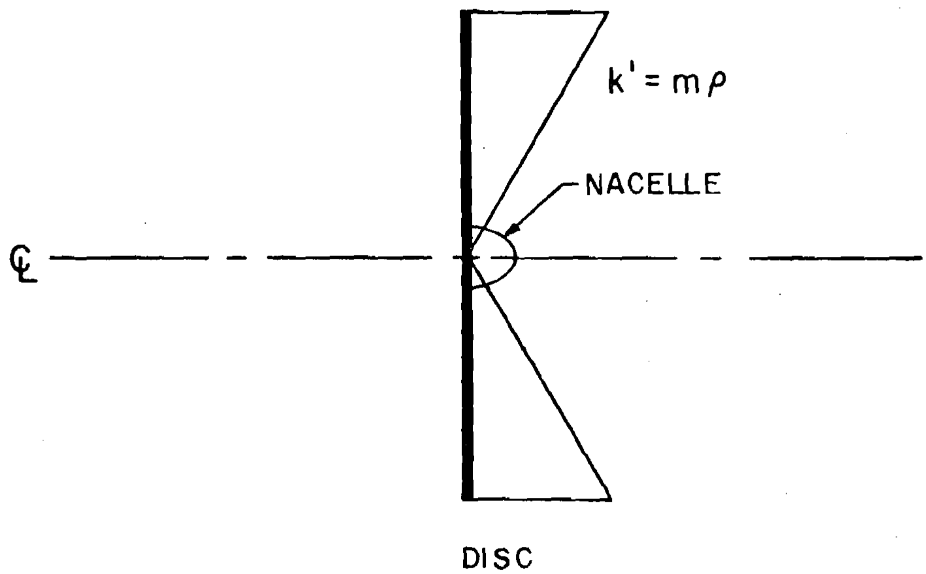

ψ₀ = 699.4 m³/s
r∞ = 4.42
ψ₀ = 506.8 m³/s
r∞ = 3.69
3
6
9
12
15
18
21
24
DISTANCE UPSTREAM (m)

FIGURE 20.

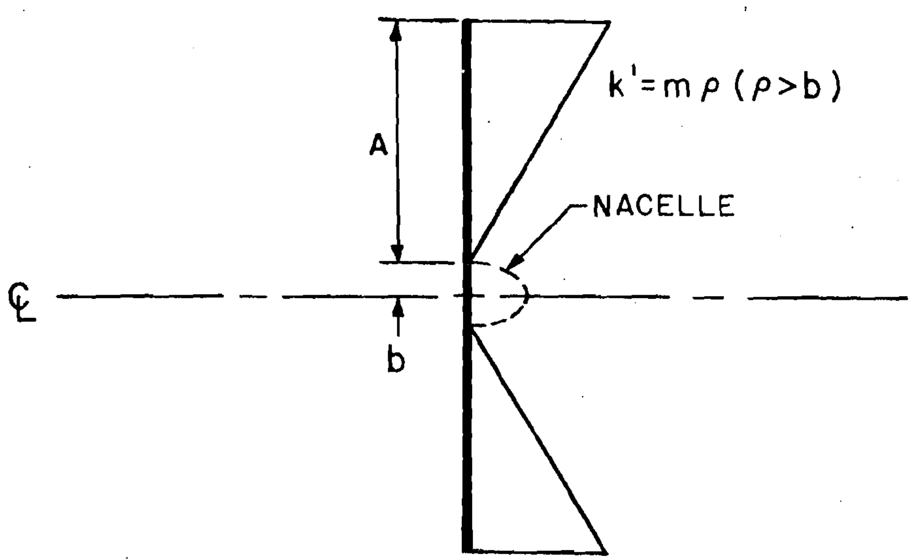


NACELLE ONLY
LINEAR SOURCE DISC AND THE UNIFORM DISC
STAGNATION STREAMLINE - DIMENSIONAL VELOCITY
1.0
0.8
0.6
0.4
0.2
0
4
8
12
16
20
24
28
32
36
x - METER UPSTREAM OF BLADES

FIGURE 21.



F I G U R E 22.



F I G U R E 23.