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Recommended Citation

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Electric Dipole Moment of a BPS Monopole

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Abstract

Monopole “superpartner” solutions are constructed by acting with finite, broken supersymmetry transformations on a bosonic $N = 2$ BPS monopole. The terms beyond first order in this construction represent the backreaction of the fermionic zero-mode state on the other fields. Because of the quantum nature of the fermionic zero-modes, the superpartner solution is necessarily operator valued. We extract the electric dipole moment operator and show that it is proportional to the fermion zero-mode angular momentum operator with a gyroelectric ratio $g = 2$. The magnetic quadrupole operator is shown to vanish identically on all states. We comment on the usefulness of the monopole superpartner solution for a study of the long-range spin dependent dynamics of BPS monopoles.

December, 1998

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1. Introduction

It is well known that BPS monopoles of $N = 2$ Yang-Mills theory are invariant under half the supersymmetry generators and hence form a 4-dimensional, short representation of the supersymmetry algebra [1]³. The fact that monopoles in supersymmetric theories can carry non-zero spin allows for the strong possibility that Montonen-Olive type dualities may actually hold in theories with sufficient amounts of supersymmetry (see [3] for a review). Moreover, the spin-dependent interactions of monopoles in $N = 4$ Yang-Mills theory are crucial for the existence of bound states required by duality [4].

These results motivate gaining as clear as possible an understanding of the spin-dependent physics of monopoles. In this paper we contribute to this understanding by studying the long-range fields of the different states in the $N = 2$ BPS monopole supermultiplet. Following work of Aichelburg and Embacher on $N = 2$ BPS black holes [5], we generate the fields of a monopole “superpartner” solution by acting on the bosonic monopole with an arbitrary, finite, broken supersymmetry transformation.

From the work of Jackiw and Rebbi [6], we know that the angular momentum of spinning monopoles is carried by the quantized states of fermionic zero-modes. For a single BPS monopole, the fermionic zero-modes are generated by *infinitesimal* broken supersymmetry transformations. What we get by acting with a *finite* transformation is then the backreaction of the fermionic zero-modes on the other fields. For example, since the fermionic fields carry electric charge, the fermionic zero-mode state acts as a source at quadratic order for the electric field. Because of the quantized nature of the fermionic zero-mode states [6], the fields of the monopole superpartner solution are necessarily operator valued.

The results we find are interesting in themselves. For example, the operator valued electric dipole moment is proportional to the angular momentum operator with a gyroelectric ratio $g = 2$ and the magnetic quadrupole moment tensor is found to vanish identically for all spin states. We also point out that our results would have a more substantial use in a study of spin-dependent monopole dynamics, which unfortunately must await further technical developments.

There are two different ways to study the low energy dynamics of bosonic monopoles. The first, due to Manton [7], postulates that at low energies monopoles follow geodesics on the moduli space of static multi-monopole configurations. For the case of two monopoles,

³ See *e.g.* [2] for a good review of this subject.

Atiyah and Hitchin [8] were then able to construct the exact moduli space metric. The second approach, also due to Manton [9], employs a monopole test-particle or probe propagating in the long-range background $U(1)$ fields of another monopole. This second approach, while less powerful than the first because of its restriction to large separations, gives a physically intuitive derivation of the Taub-NUT limit of the moduli space metric.

The low energy spin-dependent dynamics of $N = 2$ monopoles have also been studied in a moduli space approximation [10]. Monopole bound states in this treatment are related to normalizable harmonic forms on the multi-monopole moduli space. In the $N = 4$ case, the explicit construction of such a form on the two monopole moduli space [4] established the existence of a bound state required by S -duality. It seems likely that one could also study the spin-dependent interactions of a pair of monopoles via probe techniques⁴. A necessary ingredient would be a κ -symmetric superparticle Lagrangian for a BPS monopole propagating in the background fields of $N = 2$ $U(1)$ Yang-Mills theory. Such a Lagrangian does not seem to exist in the literature at this point. The appropriate background fields for studying spin-dependent interactions in the $N = 2$ case, in analogy with the gravitating cases studied in [11],[12], would be the long-range $U(1)$ fields of the monopole superpartner configurations presented here. In the $N = 4$ case, for example, one could then hope to identify in a more physically intuitive way the attractive channel leading to the bound state found in [4].

2. Monopole Superpartners

We now turn to the construction of the BPS monopole superpartner solutions. We work in $N = 2$ Yang-Mills theory with gauge group $SU(2)$. The lagrangian is given by

$$\begin{aligned} \mathcal{L}_{N=2} = & \text{Tr} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (D_\mu P)^2 - \frac{1}{2} (D_\mu S)^2 - \frac{e^2}{2} [S, P]^2 \right. \\ & \left. + i\bar{\psi}\gamma^\mu D_\mu\psi - e\bar{\psi}[S, \psi] - e\bar{\psi}\gamma_5[P, \psi] \right), \end{aligned} \quad (1)$$

where all fields are $SU(2)$ Lie algebra valued, *e.g.* $S = S^a T^a$, S and P are two scalar Higgs fields and ψ is a Dirac fermion. The nonabelian electric and magnetic field strengths are defined by $E^{ai} = -F^{a0i}$ and $B^{ai} = -\frac{1}{2}\epsilon^{ijk} F_{jk}^a$. Gauge symmetry breaking is imposed through the boundary condition at infinity $\sum_a S^a S^a = v^2$, which breaks the $SU(2)$ gauge

⁴ The spin-dependent interactions of $N = 2$ BPS black holes [11] and more recently M2-branes [12] have been studied using probe techniques.

symmetry down to a $U(1)$ subgroup. The space of possible vacuum values for the Higgs field S is a two-sphere, leading to the existence of magnetic monopole configurations.

The Lagrangian (1) is invariant, up to a total derivative term, under the global supersymmetry transformations

$$\begin{aligned}\delta A_\mu &= i\bar{\alpha}\gamma_\mu\psi - i\bar{\psi}\gamma_\mu\alpha, & \delta P &= \bar{\alpha}\gamma_5\psi - \bar{\psi}\gamma_5\alpha, & \delta S &= i\bar{\alpha}\psi - i\bar{\psi}\alpha, \\ \delta\psi &= \left(\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu} - i\gamma^\mu D_\mu S + \gamma^\mu D_\mu P\gamma_5 - i[P, S]\gamma_5\right)\alpha,\end{aligned}\tag{2}$$

where the parameter α is a Grassmann valued Dirac spinor⁵. For a static, BPS monopole field configuration with $P = A_0 = \psi = 0$ and

$$D_i S^a = \frac{1}{2}\epsilon_{ijk}F_{jk}^a,\tag{3}$$

only the fermion ψ has a nontrivial supersymmetry variation given by

$$\delta\psi = -2(\gamma^k D_k S)P_-\alpha,\tag{4}$$

where $P_\pm = \frac{1}{2}(1 \pm \Gamma_5)$ are projection operators with $\Gamma_5 = -i\gamma_0\gamma_5$. If we define projected spinors α_\pm satisfying $P_\pm\alpha_\pm = \alpha_\pm$, then α_+ generates unbroken supersymmetry transformations, while α_- generates broken supersymmetry transformations. The variation $\delta\psi$ under a broken supersymmetry transformation gives a zero-mode of the fermion field equation in the monopole background.

Following work of Aichelburg and Embacher [5] on $N = 2$ BPS black holes, we now look at the field configuration generated by acting with an arbitrary finite broken symmetry transformation $\alpha = P_-\alpha$ on a purely bosonic BPS monopole. The finite transformation is obtained by simply iterating the infinitesimal transformations. Schematically representing all the fields by Φ and the original bosonic field configuration by $\bar{\Phi}$, we have the expansion

$$\Phi = e^{\delta}\bar{\Phi} = \bar{\Phi} + \delta\bar{\Phi} + \frac{1}{2}\delta^2\bar{\Phi} + \frac{1}{3!}\delta^3\bar{\Phi} + \frac{1}{4!}\delta^4\bar{\Phi},\tag{5}$$

where, as in [5], the expansion truncates at fourth order because of the Grassmann nature of α . The expansion (5) generates an exact solution to the field equations which is non-linear in the broken supersymmetry parameter α . Since the linear term in α (4) simply gives the fermion zero-modes, the full expansion represents the backreaction of these modes on the

⁵ Our conventions for the Minkowski metric are “mostly minus” $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ and $\gamma_5 = +i\gamma_0\gamma_1\gamma_2\gamma_3$.

other fields. Following the terminology of [5], we call this the monopole “superpartner” solution.

As discussed in [12] an interpretational issue arises because the spacetime fields S , P , ψ and A_μ of the superpartner solution appear to be Grassmann valued. The resolution is to recall the work of [6] and note that, since the nonzero components of α generate fermion zero-modes, they necessarily satisfy a non-trivial algebra of anti-commutation relations. The nonzero components of α must therefore be represented as operators acting on a space of quantum mechanical spin states [6], which in the present case is simply the BPS monopole supermultiplet. The monopole superpartner solution is then seen to be operator valued. To get actual numerical values for the fields expectation values must be taken in specific BPS spin states.

Calculation of the different terms in the expansion (5) is straightforward. At first order, the only nonzero term is the variation of the fermion ψ already given above in (4). At second order, the variations $\delta^2 S$ and $\delta^2 A_k$ vanish and we find only nonzero variations for P and A_0 given by

$$\delta^2 A_0 = -\delta^2 P = -4i (\alpha^\dagger \gamma^k \alpha) D_k S. \quad (6)$$

We will see below that these reduce to dipole fields in the long range limit. Interestingly, the third and fourth order variations of all the fields turn out to vanish. In particular, the third order variation of ψ is found to be

$$\delta^3 \psi^a = 8i (\alpha^\dagger \gamma^k \alpha) \{ \gamma^0 \gamma^l D_l D_k S^a + e \gamma^0 \epsilon^{abc} (D_k S^b) S^c \} P_+ \alpha, \quad (7)$$

which vanishes because $P_+ \alpha = 0$ for the broken supersymmetries. The fourth order variations of the bosonic fields then vanish because they are each proportional to $\delta^3 \psi$. Note, the vanishing of the third and fourth order variations found here is in contrast with the results of [5] on $N = 2$ black hole superpartners, for which these variations are nonzero.

3. The Angular Momentum Operator

The long range limits of the superpartner fields A_0 and P turn out to be related in a simple way to the angular momentum operators for the fermion zero-mode states, which are constructed in the following way. The fermionic fields ψ^a may be expanded in the monopole background as

$$\psi^{a\rho} = -2(\gamma^k)^\rho{}_\sigma \alpha^\sigma D_k S^a + \text{nonzero-modes}, \quad (8)$$

where ρ, σ are spinor indices and we have explicitly displayed only the zero-mode part of the expansion. Using the orthogonality of zero-modes and nonzero-modes, we can then express the spinorial parameters α^λ and α_λ^\dagger as

$$\alpha^\lambda = +\frac{1}{2M} \int d^3x (\gamma^l)^\lambda{}_\rho \psi^{a\rho} D_l S^a, \quad \alpha_\lambda^\dagger = -\frac{1}{2M} \int d^3x \psi_\rho^{a\dagger} (\gamma^l)^\rho{}_\lambda D_l S^a, \quad (9)$$

where $M = 4\pi v/e$ is the mass of the monopole⁶. Making use of the canonical anti-commutation relation for the fermions $\{\psi^{a\sigma}(\vec{x}), \psi_\eta^{b\dagger}(\vec{y})\} = \delta^{ab} \delta_\eta^\sigma \delta(\vec{x} - \vec{y})$, we arrive at anti-commutation relations for α^λ and α_λ^\dagger

$$\{\alpha^\rho, \alpha_\lambda^\dagger\} = +\frac{1}{4M} \delta_\lambda^\rho. \quad (10)$$

Because of the projection condition $P_- \alpha = \alpha$ satisfied by the broken supersymmetries, the anti-commutation relations (10) can be interpreted as the algebra of two sets of fermionic creation and annihilation operators, giving a total of four states, the states of the short BPS supermultiplet. The fields of monopole superpartner solutions constructed above are operator valued in this space of states.

It is now straightforward to check that the operators $J^{kl} = 2iM (\alpha^\dagger \gamma^{kl} \alpha)$ satisfy the angular momentum algebra $[J^{kl}, J^{mn}] = i (\eta^{lm} J^{kn} - \eta^{ln} J^{km} - \eta^{km} J^{nl} + \eta^{kn} J^{lm})$, and hence generate rotations on the quantum mechanical zero-mode space of states. Alternatively, we can write down the angular momentum vector $J^k = -\frac{1}{2} \epsilon_{klm} J^{lm}$, which after making use of the identity $\frac{1}{2} \epsilon_{klm} \gamma^{lm} = \gamma^k \Gamma_5$, is given by

$$J^k = 2iM (\alpha^\dagger \gamma^k \alpha). \quad (11)$$

4. The Electric Dipole Moment

We now turn to the long range limit of the electric field for the monopole superpartner solution. In order to extract this from the expression (6) for $\delta^2 A_0^a$, we need to plug in the long-distance limits of the fields of the zeroth order monopole solution, which are given by

$$S^a = \left(\frac{v}{r} - \frac{1}{er^2} \right) x^a, \quad A_i^a = \epsilon_{aij} \frac{x^j}{er^2}, \quad (12)$$

Far from the monopole core, we then have

$$A_0^a = \frac{1}{2} \delta^2 A_0^a = -2i \frac{x^a x^k}{er^4} (\alpha^\dagger \gamma^k \alpha). \quad (13)$$

⁶ Here we have made use of the result $\int d^3x \eta^{kl} (D_k S^a) D_l S^a = -M$

We still need to compute the non-abelian field strength from (13) and extract the $U(1)$ part. The result for the long range abelian electric field obtained in this way is

$$E^i = F_{0i} \equiv \frac{1}{v} S^a F_{0i}^a = -\frac{2i}{e} (\alpha^\dagger \gamma^k \alpha) \left\{ \frac{3x^k x^i}{r^5} - \frac{\delta^{ki}}{r^3} \right\}, \quad (14)$$

which is a dipole field with dipole moment vector $\vec{p} = -\frac{2i}{e} (\alpha^\dagger \vec{\gamma} \alpha)$. The electric dipole moment \vec{p} is clearly proportional to the zero-mode angular momentum operator \vec{J} in (11). If we then define a gyroelectric ratio g for the monopole superpartners via the relation

$$\vec{p} = -(gQ_m/8\pi M)\vec{J}, \quad (15)$$

where $Q_m = 4\pi/e$ is the magnetic charge of the monopole, we find $g = 2$ for the monopole superpartner solution in agreement with general results for a short $N = 2$ multiplet [13],[14]. The minus sign in the relation (15) corresponds to the minus sign in the electromagnetic duality relation

$$\vec{E} \longrightarrow \vec{B}, \quad \vec{B} \longrightarrow -\vec{E}. \quad (16)$$

A current loop of magnetic monopoles has an electric dipole moment which points opposite to the magnetic dipole moment of an electric current loop. We note finally that the vanishing of the fourth order variations of the bosonic fields implies a vanishing quadrupole moment tensor for all states in the monopole BPS multiplet.

Note Added: After this work was completed, a much earlier derivation [15] of the result $g = 2$, for the electric dipole moment of a BPS monopole, was brought to our attention. The result in [15] was obtained by considering the change in energy of a monopole in a weak external electric field. Hence, the present, very different calculation can be considered as offering a complementary perspective.

Acknowledgements: We thank Jerome Gauntlett, Jeff Harvey and Jennie Traschen for helpful discussions and correspondence.

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