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Ginzburg-Landau-Wilson Aspects of Deconfined Criticality

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Monte Carlo study of the deconfined critical action phase diagram reveals a region where spinon deconfinement occurs through a weak first-order phase transition, in agreement with Ginzburg-Landau theory. Wilson renormalization argument in combination with the absence of the data collapse even in the regime of weak interaction between the spinons casts a serious doubt on the possibility of the continuous deconfinement transition. We also argue that if a continuous deconfined criticality does exist on the phase diagram, its nature is analogous, in a certain precise sense, to that of the XY universality class.

Critical properties of systems described by one or several complex fields coupled to one gauge field have a long history of studies and apply to numerous problems in physics which include normal-superfluid transitions in multi-component neutral or charged liquids (see, e.g. [1]), superfluid–valence-bond solid (SF–VBS) transitions in lattice models ([2, 3, 4]), Higgs mechanism in particle physics [5], etc. Recently, the authors of Refs. [2, 3] argued that the SF–VBS transition in a $(2+1)$-dimensional system is an example of a qualitatively new type of quantum criticality that does not fit the Ginzburg-Landau-Wilson (GLW) paradigm. The discussion of this claim is the main focus of our Letter.

The theory of deconfined critical point (DCP) establishes a remarkable microscopic picture of how a generic continuous SF–VBS transition may happen. A generic II-order SF–VBS transition can not be derived from naïve GLW expansion in powers of order parameters starting from the quantum disordered groundstate, which has no broken symmetries, simply because such a state is unlikely to be present in most models [4, 6]. It may happen, however, that restrictions imposed on parameters of the GLW action are automatic and originate from hidden (or emerging) symmetries. In the DCP action the crucial symmetry of this kind is that between the spinons (or vortices) in the VBS state. Another alternative is that the DCP action *in fact* predicts that the SF-VBS phase transition is of the I-order.

In what follows we discuss results of the Worm algorithm Monte Carlo simulations performed for the DCP action, construct its phase diagram, and show that the SF–VBS transition is I-order at least for sufficiently strong coupling between the spinon fields. This explains the outcome of recent numerical simulations which report either difficulties with finite-size scaling of the data [7] or weak I-order transitions [6]. It is far more difficult to present a direct evidence of the I-order transition in the weak coupling limit. However, problems with the data collapse in finite-size systems hint at this possibility. The global I-order scenario would not be a surprise in view of a simple renormalization argument that

in the DCP action the weak coupling regime maps onto the intermediate-coupling one, where we find a circumstantial evidence that the II-order scenario fails.

Many features of the DCP action are remarkably similar, both qualitatively and quantitatively, to those of a more conventional two-component XY-model (2XY) [8]. Moreover, we argue that if the line of II-order DCP's does exist, then its critical properties are most close to those of 2XY at the U(1)×U(1) critical point, in the following sense. There exists a self-dual model with *marginal* long-range interactions $\propto 1/r^2$, which continuously interpolates between DCP and U(1)×U(1) criticality by varying amplitudes of $\propto 1/r^2$ terms.

There are many equivalent formulations of the action describing coupled gauge and multi-component complex fields [1, 2, 9]. Here we employ the integer-current lattice representation which can be viewed either as a high-temperature expansion for the XY model in three dimensions, or as a path-integral (world-line) representation of the interacting quantum system in discrete imaginary time in $(d+1) = 3$ dimensions. The XY action reads

$$S = U_{\mathbf{r}-\mathbf{r}'}\mathbf{j}_{\mathbf{r}} \cdot \mathbf{j}_{\mathbf{r}'} , \qquad (1)$$

where summation over repeated lattice sites $\mathbf{r}$ is assumed; $U_{\mathbf{r}-\mathbf{r}'} = U\delta_{\mathbf{r},\mathbf{r}'}$; and $\mathbf{j}_{\mathbf{r}} = (j_{\mathbf{r}})_{\mu}$ with $\mu = x, y, \tau$ are integer, zero-divergence, $\nabla\mathbf{j} = 0$, currents defined on bonds of the simple cubic space-time lattice with periodic boundary conditions. The configuration space of j-currents is that of closed oriented loops. In terms of particle world lines, currents in the time direction represent occupation number fluctuations away from the commensurate filling, and currents in the spatial directions represent hopping events. The state of S with small current loops is normal ("high-temperature"); the corresponding particle order-parameter, ψ, is zero in this phase. When particle world lines proliferate and grow macroscopically large, the system enters into the superfluid state with $\langle\psi\rangle \neq 0$.

Generalization to the symmetric multi-component case with short-range interactions is straightforward (in what follows we will concentrate on the two-component case
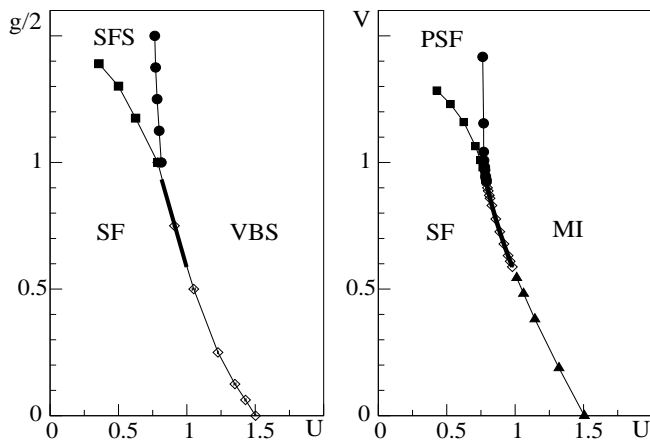
FIG. 1: Phase diagrams of the long-range (left panel), Eq. (3), and short-range (right panel), Eq. (2), actions. Bold solid lines indicate regions where I-order phase transitions were unambiguously verified. Error bars are shown but are typically much smaller than symbol sizes.

FIG. 2: Normalized probability density distributions $P(S)$ of the DCP action for $g = 1.5$ and different system sizes; U(L=8)=0.94375, U(L=12)=0.92345, U(L=16)=0.91667, U(L=22)=0.91274.

which describes coupled XY models)

$$S_2^{(s)} = U\,\mathbf{j}_{1\mathbf{r}}^2 + U\,\mathbf{j}_{2\mathbf{r}}^2 - V\,(\mathbf{j}_{1\mathbf{r}} + \mathbf{j}_{2\mathbf{r}})^2 \ . \tag{2}$$

If particle/XY-spin fields in the original model are coupled through a gauge field, then after the gauge field is integrated out one arrives at the DCP action similar to Eq. (2), which now contains a long-range part [1, 9]

$$S_2^{(l)} = U\mathbf{j}_{1\mathbf{r}}^2 + U\mathbf{j}_{2\mathbf{r}}^2 + gQ_{\mathbf{r}-\mathbf{r}'}\,(\mathbf{j}_{1\mathbf{r}} + \mathbf{j}_{2\mathbf{r}})\cdot(\mathbf{j}_{1\mathbf{r}'} + \mathbf{j}_{2\mathbf{r}'}) \ . \tag{3}$$

The lattice Fourier transform of the interaction potential $Q_{\mathbf{r}-\mathbf{r}'}$ is given by $Q_{\mathbf{q}} = 1/\sum_\mu \sin^2(q_\mu/2)$, which implies the asymptotic form $Q(r \to \infty) \sim 1/r$. In the discussion of the SF–VBS transition, j_1 and j_2 currents in the DCP action represent world lines of spinons which are VBS vortices carrying fractional particle charge $\pm 1/2$ [2, 3].

Phase diagrams for the long- and short-range actions are shown in Fig. 1. Different phases are identified in terms of loop sizes for j_1 and j_2 currents as follows: the Mott insulator (MI) and VBS states are characterized by small j_1- and j_2-loops; in the VBS supersolid (SFS) and paired superfluid (PSF) states only $(j_1 \mp j_2)$-loops grow macroscopically large while single component loops remain small; finally, in the SF state there are macroscopically large j_1- and j_2-loops. The data for the short-range model are reproduced from Ref. [8]. A similar shape of the phase diagram for the DCP action in the angle-gauge field representation was obtained in Ref. [10], though, no I-order transition was identified [11].

It is hard not to notice a remarkable *quantitative* similarity between the phase diagrams. In particular, the DCP action has VBS to SFS and SFS-SF transitions when coupling between the spinons is strong and they form tightly bound particle states. One immediate prediction based on properties of the short-range model
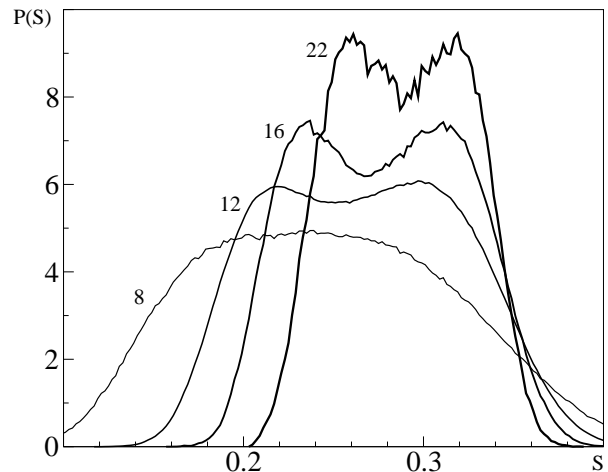
would be that close to the SF-SFS-VBS bicritical point the SF-VBS transition is I-order in nature. Unfortunately, the I-order transition in $S_2^{(s)}$ is very weak and can be clearly seen only at system sizes $L > 24$ by calculating the probability density for the system action, $P(S)$, and observing its double-peak structure. Simulating large L for the long-range action is far more difficult. We were able to collect reliable statistics for $S_2^{(l)}$ only for $L \leq 22$. Still, for $g = 1.5$ we resolved (Fig. 2) the development of the double-peak structure in $P(S)$ starting from an anomalously flat maximum at $L = 8$.

For small g one expects the I-order transition to become weaker (the $g = 0$ point describes two independent XY models) and nearly impossible to study using $P(S)$ functions. However, there are two important circumstances. (i) At $g \to 0$, the initial part of the renormalization flow with $L \to \infty$ simply leads to increasing $g(L) \propto L$ [12] [see also Eq. (5)], thus mapping the weak-coupling regime onto the intermediate-coupling one. [Indeed, $1/r$ interaction is a relevant operator responsible for the spinon confinement.] (ii) For the superfluid stiffness, n_s, a scale invariant II-order criticality implies $n_s L \to$ const as $L \to \infty$. These facts has to be combined with the numerical observation that for $g > 0.1$ the $n_s L$ curves do not intersect at the same U even approximately (see Fig. 3). This strongly suggests the I-order transition scenario for all g (we exclude a *non-scale-invariant* II-order scenario).

Critical self-duality of XY models. Speaking strictly, one cannot rule out that a II-order line actually starts below $g = 0.1$, or, that for some reason the data collapse for $g = 0.5$ sets in only at $L \gg 20$. A possibility of II-order phase transitions in generic multi-component XY models with $1/r$ current-current interactions can hardly be questioned since the duality transformation [13] for the short-ranged action $S_2^{(s)}$ produces a family of such
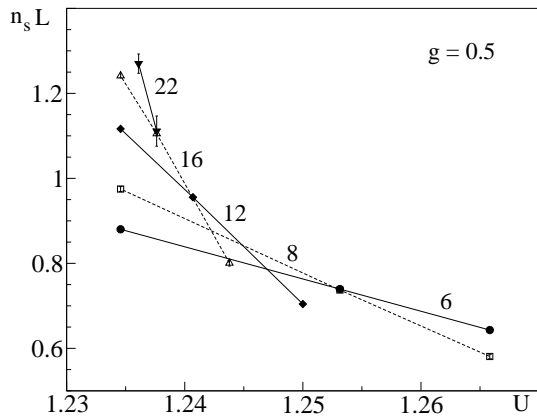
FIG. 3: $n_s(U)L$ curves for $g = 0.5$ and system sizes $L = 6, 8, 12, 16, 22$.

models. To be unambiguous in our further analysis, we adopt the following two definitions.

Def. 1: We call a model self-dual if there exists a duality transformation which at the critical point preserves the form of the Hamiltonian, but not necessarily the values of the critical parameters.

Def. 2: A model $\mathcal{A}$ is critically self-dual if there exists a self-dual model $\mathcal{B}$ such that the critical properties of $\mathcal{A}$ and $\mathcal{B}$ are the same.

XY action can be identically transformed into the dual (inverted-XY) action [13] which has the same functional form as Eq. (1) for integer, zero-divergence currents **l** coupled by the long-range dual interaction potential, $U_{\mathbf{r}\to\infty}^{(d)} \sim 1/r$. In Fourier space, the dual potential is given by $U_{\mathbf{q}}^{(d)} = (\pi^2/4)Q_{\mathbf{q}}/U$. The notion of bond currents also changes from particle to vortex world lines. Now, the proliferation of vortex **l**-lines signals the onset of the transition to the normal state. Formally, long-range interactions in the dual action make the XY model not self-dual in conventional sense, though the configuration space is preserved. Several papers suggested that $1/r$ interactions result in the inverted-XY criticality qualitatively different from the conventional XY point [14, 15].

A single-component short-range XY model is critically self-dual. Below we prove this statement numerically. Meanwhile, it is easy to show analytically that XY criticality is *arbitrarily close* to the criticality of self-dual models. First, we introduce XY models with $1/r^2$-potential, which are self-dual. Indeed, the potential with the Fourier transform $U_{\mathbf{q}\to 0} \to A(\pi/2)\sqrt{Q(q)} + B + \dots$ under duality transformation becomes $U_{\mathbf{q}\to 0}^{(d)} \to A^{-1}(\pi/2)\sqrt{Q(q)} - B/A^2 + \dots$. Note that the amplitude in front of the $1/r^2$ term changes. Second, we observe that the $1/r^2$ interaction is *marginal* (this will be explicitly seen below) and thus by adding a $1/r^2$-term with infinitely small amplitude to XY model (1) we do not change its critical behavior and exponents, but formally make it self-dual. It is of crucial importance that XY criticality is mapped onto that of a model with *marginal* in-

teraction; this is required to reconcile critical self-duality with different critical exponents and universal numbers of the XY and inverted XY transitions. These differences naturally follow from different amplitudes of the dual $1/r^2$-terms. The XY and inverted XY critical points turn out to be just two points in the *continuum* of A/r^2-criticalities, corresponding to two special values, A_- and A_+. By changing A one may continuously interpolate between A_- and A_+, gradually transforming XY criticality into inverted XY, and vice versa.

To illustrate the above-mentioned point and, in particular, to extend the proof to the case of finite A, we computed the critical exponent $D_H(A)$ for the interaction potential $U_{\mathbf{q}} = A(\pi/2)\sqrt{Q(q)} + B$. The Hausdorff dimension D_H gives the the average line length of critical loops as a function of system size, $\langle l \rangle \propto L^{D_H}$. Under duality transformation. $D_H(A)$ transforms into $D_H^{(d)}(A)$. As expected, among a continuum of self-dual critical points $B_c(A)$ there is one, $A \approx 3.2$, $B_c(A = 3.2) = -2.8457(10)$ which reproduces Hausdorff dimensions of the inverted-XY and XY systems respectively within the statistical error bars [16].

To understand the origin of critical self-duality, and in particular, the physical meaning of the $1/r^2$ interaction, we start with constructing proper coarse-grained current variables similar to winding numbers in finite-size systems and subject to the real space-time renormalization group (RG) transformations. Crucial for our derivation will be the fact that at criticality there are $\sim \mathcal{O}(1)$ loops of size $\sim r$ in the volume r^3 [17], which is nothing but the requirement of scale invariance.

Imagine a cube of linear size u centered at point $\mathbf{r}$, where $u \gg 1$ is odd integer. Its facets cut bonds of the original lattice a distance $u/2$ away from the center. Integer, zero-divergence currents $\mathbf{J}_{\mathbf{r}}$ are defined as current fluxes through the facets of the cube (we denote the cube by $C_{\mathbf{r}}$ and its facets as $S_{\mathbf{r},\mu}$)

$$(J_{\mathbf{r}})_\mu = \sum_{\mathbf{r}' \in S_{\mathbf{r},\mu}} (j_{\mathbf{r}'})_\mu , \qquad (4)$$

with an additional restriction that only j-loops of size larger than $u/2$ contribute to the sum (4). The last condition is necessary to suppress noise due to small loops winding around the cube edges. The new variables are scale-invariant by construction, i.e. $\langle J^2 \rangle \sim \mathcal{O}(1)$. This fact alone allows us to study the RG flow of the long-range effective interaction potential. Let the original model be long-range with the bare potential decaying as g/r^α, $\alpha \le 2$. After coarse-graining the interaction energy between two cubes $C_{\mathbf{r}_1}$ and $C_{\mathbf{r}_2}$ separated by a distance $R = |\mathbf{r}_1 - \mathbf{r}_2| \gg u$ can be estimated as

$$\frac{g}{\epsilon(u)R^\alpha} \sum_{\mathbf{x} \in C_{\mathbf{r}_1}} \sum_{\mathbf{y} \in C_{\mathbf{r}_2}} \mathbf{j}_{\mathbf{x}} \cdot \mathbf{j}_{\mathbf{y}} \sim \frac{g\, u^2}{\epsilon(u)R^\alpha} \mathbf{J}_{\mathbf{r}_1} \cdot \mathbf{J}_{\mathbf{r}_2} , \qquad (5)$$

where $\epsilon(u)$ is the "dielectric" constant at length-scale u. The last relation follows from the continuity of world

lines and the fact that $\mathbf{J}$-currents are facet, not bulk, sums, so that $\sum_{\mathbf{x}\in C_{\mathbf{r}_1}} \mathbf{j_x} \sim u\mathbf{J_{r_1}}$. We now rescale all distances by a factor of u and observe that the effective interaction potential transforms as $gu^{2-\alpha}/\epsilon(u)r^{\alpha}$. This means that (i) long-range interactions with $\alpha < 2$ are screened at the fixed point, $\epsilon(u) \propto u^{2-\alpha} \to 0$, and (ii) the term $\mathbf{J_r} \cdot \mathbf{J_{r'}}/(\mathbf{r} - \mathbf{r'})^2$ is a marginal operator. For $\alpha = 1$ and small initial values of g, the RG flow starts with *epsilon* $= 1$ and leads to $g(u) \sim u$. This result was used to relate the weak- and intermidiate-coupling regimes in the DCP action. This consideration does not depend on the number of components, but it works only as long as $g(u)$ remains small.

The above analysis is supported by microscopic mechanisms leading to the renormalized $1/r^2$-law between microscopic currents. In the SF phase, the $1/r$ potential originates from the kinetic energy of the superflow around vortices. Close to the critical point, the superflow around a vortex line is screened on large distances by small vortex loops, and the kinetic energy integral is transformed into (for simplicity we consider here a straight vortex line) $\int n_s(r)dr/r \to \int dr/r^2$ with scale-dependent superfluid density $n_s \propto 1/r$. This formula implies that at the critical point the interaction energy between the vortex line elements is proportional to $1/r^2$. The emergence of the renormalized potential $\sim 1/r^2$ while approaching the critical point from the normal phase, can be considered as a result of virtual exchange of long-wave sound excitations. Elementary calculations in the hydrodynamical approximation for $d = 2$ superfluid show that such potential scales as $\sim 1/r^2$.

One may also take an alternative, mathematically rigorous, point of view and consider an action describing a *single* j-current line (of arbitrary fractal structure) at the critical point when other degrees of freedom in the system are integrated out

$$S^{(\text{loop})}_{\text{crit}} = \left[(U_{\text{crit}})_{\mathbf{r}-\mathbf{r'}} + \lambda\, \delta_{\mathbf{r}-\mathbf{r'}} \right] \mathbf{j_r} \cdot \mathbf{j_{r'}} , \qquad (6)$$

where $U_{\text{crit}} = A/r^2 +$ short-range terms. It is written in terms of original j- or l-currents and *not* coarse-grained variables. In particular, the statistics of the disconnected loop, formally identical to a polymer of a variable length, must reproduce geometrical exponents at the critical point. It is this critical-loop action that unifies short- and long-range XY models, putting them in a wider context of self-dual $1/r^2$ models.

In conclusion, we present numerical and RG agruments strongly suggesting that the DCP action is the theory of weak I-order SF-S transitions. In the "zoo" of self-dual $1/r^2$ criticalities interpolating between short-range and long-range $1/r$ models, the number of components does not seem to play any qualitative role unless the transition turns into the I-order one.

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[11] Refs. [1, 9] claimed the seocond-order SF-VBS transition with the correlation readius exponent $\nu \approx 0.6$, basing on simulations of system sizes $L \leq 24$ with a straightforward scaling analysis of the data. For comparison, we studied ν of $S_2^{(s)}$ along the SF-MI line up to $L = 128$ using finite-size scaling of the superfluid density derivative, $n_s'(U = U_c) \propto L^{1/\nu-1}$ and found that (i) the scaling curves are nearly perfect straight lines on the log-log plot all the way to the tricritical point, (ii) good scaling with $\nu \approx 0.4$ was observed up to $L = 32$ *along the I-order line(!)*, and (iii) in the upper half of the continuous SF-MI line the slope of the best linear fit produced a set of interaction dependent $\nu(V)$ in the range between 0.67 and 0.5. This is a clear signal that corrections to scaling are important. When they are included into the fit, the data become consistent with $\nu \approx 0.67$ though the error bars increase by an order of magnitude. We conclude that ν can not be reliably determined on system sizes $L \leq 24$. More importantly, one may completely miss the I-order transition.

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