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We consider a SQUID tunneling between 2 nearly degenerate flux states. Decoherence caused by paramagnetic and nuclear spins in the low-T limit is shown to be much stronger than that from electronic excitations. The decoherence time τ_ϕ is determined by the linewidth E_o of spin bath states, which can be reduced by a correct choice of ring geometry and isotopic purification. E_o can be measured in either field sweep or microwave absorption experiments, allowing both a test of the theory and design control.

The fundamental importance of decoherence in Nature has been emphasized in fields ranging from quantum gravity and measurement theory to quantum computing and mesoscopic physics. Serious analysis of the *mechanisms* controlling decoherence began with work on dissipative tunneling and on "Macroscopic Quantum Coherence" (MQC) in SQUIDs [1,2]. To see MQC in SQUIDs requires $\tau_\phi(T)|\Delta| \gg 2\pi\hbar$, where τ_ϕ is a "decoherence time", and Δ the tunneling matrix element. However, despite the successful observation of macroscopic tunneling (MQT) in SQUIDs [3], MQC has not been found. The unexpected low-T saturation of $\tau_\phi(T)$ in conductors [4–6], indicates we may not understand decoherence even in metallic systems (let alone in "qubits" or quantum computers!).

Our thesis herein is that at low T in SQUIDs, $\tau_\phi(T)$ is controlled not by electronic or "oscillator bath" environments [2,7,8], but by a "spin bath" [9] of localized modes, including nuclear and paramagnetic spins as well as charge defects. Although spin bath environments have received considerable attention in nanomagnets [10,11] (and now in mesoscopic conductors [6]) their discussion for SQUID tunneling has been only sporadic [9,12]. Here we show how they strongly suppress MQC (but usually with rather weak effects on MQT). Our results should allow experimentalists to both parametrize the spin bath effects, and quantitatively test the theory.

1. *Effective Hamiltonian.* Ignoring electronic and electromagnetic dissipation mechanisms [2,7,8], the "bare" DC SQUID Hamiltonian for flux tunneling is

$$H_0 = \frac{p_\phi^2}{2C} + U_0\left[\frac{(\phi-\phi_x)^2}{2} + g\cos\phi\right] , \qquad (1)$$

where C is the junction capacitance, $\phi = 2\pi\Phi/\Phi_0$ with Φ_0 the flux quantum, $p_\phi = -i\hbar\partial/\partial\Phi$, and $\phi_x = 2\pi\Phi_x/\Phi_0$, where Φ_x is the externally applied flux. Then $U_0 = \Phi_0^2/4\pi^2 L$ and $g = 2\pi L I_c/\Phi_0$, for a ring inductance L and junction critical current I_c. In MQC or qubit designs $\phi_x \ll 1$ and ϕ tunnels between the two lowest wells, centred at $\phi = \phi_\pm = \pm\phi_m$. One also assumes $k_B T < \hbar\Omega_0/2\pi$, where Ω_0 is the Josephson plasma frequency of small oscillations in these wells. The system then truncates [1,2,7] to a two-level Hamiltonian $H_0 = \xi_o(t)\tau_z + \Delta\tau_x$, where $\vec\tau$ is a Pauli vector, $\xi_o(t) \sim U_0\phi_m\phi_x(t)$ is a bias which can be varied in time, and current designs have $|\Delta/\hbar|$ as high as $\sim 1\ GHz$.

Consider now a set of N two-level systems $\{\vec\sigma_k\} \equiv \{\vec s_k, \vec I_k\}$, representing paramagnetic ($\{\vec s_k\}$) and nuclear ($\{\vec I_k\}$) spins [13], at positions $\{\mathbf{r}_k\}$. Those in the SQUID couple to the conduction electron spin density $\vec s_{\rm cond}(\mathbf{r})$ via

$$H_{sp} = \sum_k J_k\vec\sigma_k\cdot\vec s_{\rm cond}(\mathbf{r}_k) + \gamma_k\vec\sigma_k\cdot\vec B(\mathbf{r}_k) , \qquad (2)$$

Spins in the substrate couple to the flux via the 2nd term. The $\{\gamma_k\} \equiv g_k\mu_k$ are spin magnetic moments, and the $\{J_k\}$ represent electronic exchange for the $\{\vec s_k\}$, and hyperfine coupling for the $\{\vec I_k\}$. All $\{\vec s_k\}$ are assumed paramagnetic [14], with Kondo energies $k_B T_K \sim (JE_F)^{1/2}e^{-1/JN(0)} \ll k_B T$, where $N(0)$ is the Fermi-level density of states. The field $\vec B(\mathbf{r}) = \vec B_x + \vec B_\phi(\mathbf{r}) + \sum_k \vec b_k(\mathbf{r})$, where $\vec B_x$ is the external field, $\vec B_\phi(\mathbf{r})$ comes from the SQUID supercurrent, and

$$\vec b_k(\mathbf{r}) = \gamma_k\left[\frac{8\pi}{3}\vec\sigma_k\delta(\mathbf{r}-\mathbf{r}_k) + \frac{\vec\sigma_k}{|\mathbf{r}-\mathbf{r}_k|^3} - \frac{3(\mathbf{r}-\mathbf{r}_k)\cdot\vec\sigma_k}{|\mathbf{r}-\mathbf{r}_k|^5}(\mathbf{r}-\mathbf{r}_k)\right] , \qquad (3)$$

is the dipolar field from $\vec\sigma_k$ if $|\mathbf{r}-\mathbf{r}_k| \ll \lambda_s$, the superconducting penetration depth [15].

¿From $H = H_o + H_{sp}$ we now derive a low-energy Hamiltonian, using the usual instanton method [1] wherein tunneling of ϕ occurs in imaginary time τ. A transition at $\tau = 0$ gives a variation $\phi(\tau) = \phi_m f(\tau)$, with $f(\tau) \sim \pm\tan^{-1}(e^{\Omega_0\tau})$. Consider now the local fields $\vec\omega_k = \gamma_k\vec B(\mathbf{r}_k)$ acting on the $\{\vec\sigma_k\}$, which during the instanton evolve like:

$$\vec\omega_k(\tau) = \omega_k^\perp\vec m_k + \omega_k^\parallel\vec l_k(\tau) , \qquad (4)$$

(cf. Fig. 1), where $|\vec{m}_k| = 1$ and $\vec{l}_k(\tau)$ evolves from $\vec{l}_k^{\pm}$ to $-\vec{l}_k^{\pm}$ during the instanton; $|\vec{l}_k^{\pm}| = 1$ at the end-points. Then, defining $\vec{B}_\phi^{\pm}(\mathbf{r}_k) = [\vec{B}_{\phi=+\phi_m}(\mathbf{r}_k) \pm \vec{B}_{\phi=-\phi_m}(\mathbf{r}_k)]$,

$$\omega_k^{\perp}\vec{m}_k = \gamma_k\{\vec{B}_x + \sum_j \vec{b}_j(\mathbf{r}_k) + \frac{1}{2}\vec{B}_\phi^{+}(\mathbf{r}_k)\} \quad (5)$$

$$\omega_k^{\parallel} = \gamma_k \frac{1}{2}|\vec{B}_\phi^{-}(\mathbf{r}_k)| \equiv \frac{1}{2}\gamma_k \delta B_k \quad (6)$$

The time varying $\omega_k^{\parallel}\vec{l}_k(\tau)$ causes transitions of $\vec{\sigma}_k$; writing $|\sigma_k^{+}>= T_k^{\pm}|\sigma_k^{-}>$, which defines a transition matrix $T_k^{\pm}$ for the passage $\vec{l}_k^{-} \to \vec{l}_k^{+}$, we have in general

$$T_k^{\pm} = \exp\left\{-\frac{1}{\hbar}\int_{-}^{+} d\tau \omega_k^{\parallel}\vec{l}_k(\tau)\cdot\vec{\sigma}_k\right\} \equiv e^{-i\varphi_k + \vec{\alpha}_k\cdot\vec{\sigma}_k} , \quad (7)$$

operating on $\vec{\sigma}_k$, where φ_k, α_k are typically complex [9]. In the long interval (over times $\sim \hbar/\Delta$) between instantons, $\vec{\sigma}_k$ sits in a static field $\vec{\omega}_k^{\pm} \equiv \vec{\omega}_k(\vec{l}_k = \vec{l}_k^{\pm})$; but during instantons, $T_k^{\pm}$ operates. This gives immediately the diagonal and non-diagonal terms in an effective Hamiltonian of "Central Spin" form [9,10], valid at energy scales $\ll \hbar\Omega_o$:

$$H_{\text{eff}} = \left[\Delta\hat{\tau}_+ e^{-i\sum_k \vec{\alpha_k}\cdot\vec{\sigma}_k} + H.c.\right] + \sum_k \left(\omega_k^{\perp}\vec{\sigma}_k\cdot\vec{m}_k + \hat{\tau}_z\omega_k^{\parallel}\vec{\sigma}_k\cdot\vec{l}_k\right) + \xi_o(t)\hat{\tau}_z , \quad (8)$$

where the φ_k are absorbed into the physical Δ. Eigenstates $|\sigma\rangle$ of $\hat{\tau}_z$, with $\sigma = \pm 1$ for $\tau_z = \uparrow, \downarrow$, are converted to a pair of 2^N-fold multiplets of coupled SQUID/spin bath states, with linewidth $E_0^2 = \sum_k(\omega_k^{\parallel})^2$ and normalised distributions $W_\sigma(\xi) = (2/\pi E_0^2)^{1/2}e^{-2(\xi-\sigma\xi_o)^2/E_o^2}$, in energy ξ. The parameter E_o will be crucial to decoherence- note it only depends on the *change* δB_k in field in (6), and is much easier to determine than the total field $\vec{B}(\mathbf{r}_k)$.

The above derivation ignores the slow spin diffusion and spin-lattice relaxation in the spin bath (which cause the vectors $\vec{l}_k$,$\vec{m}_k$ to become dynamic variables). This occurs on timescales $\sim \mu s$ or longer, ie., $\gg \hbar/\Delta$, and is not relevant to decoherence in the present problem.

2. *Example*: The parameters $\omega_k^{\perp}$, $\omega_k^{\parallel}$ are determined from the fields $\vec{B}_\phi^{\pm}(\mathbf{r}_k)$ (ie., knowing the supercurrent distributions corresponding to $\phi = \pm\phi_m$), once we know $\{\gamma_k\}$ and $\{\mathbf{r}_k\}$ for all relevant nuclear and paramagnetic spins. We therefore assume homogeneous concentrations x_r, x_J, and x_s of paramagnetic spins in the ring, junction, and substrate respectively, for the geometry in Fig. 2, as well as a single nuclear species, with one nucleus per unit cell. Simple magnetostatics gives the results in Table I for $\omega_k^{\perp}$ and $\omega_k^{\parallel}$. The vector $\vec{\alpha}_k$ depends on the detailed path followed by $\vec{l}_k(\tau)$ during tunneling. Since $\vec{l}_k(\tau)$ changes very quickly (on a timescale Ω_o^{-1}), from time-dependent perturbation theory $|\vec{\alpha}_k| \sim \omega_k^{\parallel}/\hbar\Omega_o$, and the mean number of bath spins flipping per transition is $\lambda \sim \frac{1}{2}\sum_k |\vec{\alpha}_k|^2 \sim E_o^2/\hbar^2\Omega_0^2$ (with an associated decoherence rate $\lambda\Delta/\hbar \sim \Delta E_o^2/\hbar^3\Omega_o^2$). Thus $\lambda \ll 1$ unless E_o is unreasonably large.

The dependence on sample size is the most striking result in Table I and Fig. 2, the contributions of all spins to E_o increasing rapidly as R *decreases*- the increase in the number of spins with R is more than offset by the weaker fields. The contribution from the substrate can easily be reduced by making it very thin and surrounding it with ^4He, and one may also reduce E_o by reducing d,h,x, and ϕ_m [16].

3. *SQUID dynamics*: A proper calculation of the SQUID dynamics requires adding to (8) a coupling to an oscillator bath, representing electrons, photons, and phonons. In the absence of a spin bath, the dominant electronic contribution gives a decoherence rate [7,8] $\Gamma_\phi^e = 16\pi\phi_m^2 k_B T/Re^2 = \tilde{\alpha}_e kT/\hbar$ where $\tilde{\alpha}_s = (16\pi\phi_m^2\hbar\Omega_o/E_c)Q^{-1}$, $Q(T)$ is the SQUID Q-factor, and E_c the junction charging energy. In a simple RCSJ model with shunt resistance R_s and junction resistance R_j one has $R^{-1} = R_s^{-1} + R_j^{-1}$ (and $R_j(T) \sim R_o(1+e^{\Delta_{BCS}/T})/2$, where Δ_{BCS} is the superconducting gap and R_o the normal junction resistance), so that Q and $\tilde{\alpha}_s$ are T-independent at low T.

Let us now calculate the SQUID dynamics for the Hamiltonian (8), and thence the spin bath contribution to τ_ϕ^{-1}; we then compare this to the electronic contribution. Assuming $\lambda \ll 1$ (see above) we drop the $\{\alpha_k\}$ from (8). Since $\omega_k^{\parallel}, \omega_k^{\perp} \ll \Delta$, we can treat these couplings perturbatively; and a quick check of the numbers in Table I shows $\omega_k^{\parallel} \ll \omega_k^{\perp}$ almost always. Since in general $\vec{m}$ and $\vec{l}$ are neither parallel nor perpendicular, we choose $\vec{m}$ as the spin quantization axis $\hat{\vec{z}}$. The component of $\vec{l}$ parallel to $\vec{m}$ is dealt with by redefining $\xi \to \xi + \sum_k \omega_k^{\parallel}\cos(\widehat{\vec{l}\vec{m}})\sigma_k^z$, which now depends on the spin bath state. Because $\vec{l}_x$ has a transverse component, whenever the SQUID state changes the coupling term $\sum_k \omega_k^{\parallel}\vec{\sigma}_k^x\ \tau_z$ forces environmental spins to precess in a new local magnetic field, which can be viewed quantum mechanically as SQUID-induced transitions between the environmental states [9]. To quantify this effect we calculate the time correlation function $P_{\uparrow\uparrow}(t)$, the probability [1] that $\tau_z = 1$ at time t (ie., $\phi = +\phi_m$) if it was 1 at $t = 0$, after integrating out the spin bath. An instanton expansion gives (assuming $\xi \ll |\Delta|$):

$$P_{\uparrow\uparrow} = \sum_{nm} |i\Delta|^{2(n+m)} \prod_{a=1}^{2n} dt_a \prod_{b=1}^{2m} dt'_b F\left[\tau_z(t), \tau_z(t')\right] \quad (9)$$

summed over "outgoing" and "return" paths $\tau_z(t), \tau'_z(t)$. The influence functional F [19] given from (8), assuming

$|\vec{\alpha}_k| = 0$, $\omega_k^\perp \ll k_B T$, and $\omega_k^\parallel \ll \omega_k^\perp$, is

$$\ln F = -\sum_k \frac{(\omega_k^\parallel)^2}{8\hbar^2} | \int_0^t ds e^{\frac{i}{\hbar}\omega_k^\perp s} \big[\tau_z(s) - \tau_z'(s)\big]|^2 \quad (10)$$

(assuming an initial thermal environmental state). We distinguish 2 regimes:

(a) Strong decoherence regime ($E_0 \gg \Delta$): Here F is negligible if instanton/anti-instanton pairs are separated by times $> \hbar/E_o$, and we find immediately that $F \sim \exp[-E_o^2(t_i - t_{i+1})^2/2\hbar^2]$, implying $P_{\uparrow\uparrow}(t) \sim (1/2)[1 + e^{-t/\tau_R}]$, with the rate

$$\tau_R^{-1} = \frac{2\Delta^2}{\hbar^2} \int_0^\infty dt e^{-E_o^2 t^2/2\hbar^2} = \sqrt{2\pi}\frac{\Delta^2}{\hbar E_o} , \quad (11)$$

ie., completely incoherent quantum relaxation.

(b) Weak decoherence regime ($E_o \ll \Delta$): This problem is easily solved by going to the basis of eigenstates of $\hat{\tau}_x$. Their associated spin multiplets are widely separated in energy, so real transitions between them are impossible, and a perturbation expansion in $\omega_k^\parallel/\Delta$ gives

$$H_{\rm eff} \sim \left[\Delta + \sum_{kk'} \frac{\omega_k^\parallel \omega_{k'}^\parallel}{2\Delta} \hat{\sigma}_k^x \hat{\sigma}_{k'}^x \right] \hat{\tau}_x + \sum_k \omega_k^\perp \hat{\sigma}_k^z \quad (12)$$

plus terms $\sim O((\omega_k^\parallel)^4/\Delta^3)$. Assuming $E_o \ll k_B T$ (so all states of a multiplet have equal thermal weight), we find that

$$\tau_\phi^{-1} \sim \sum_{kk'} \omega_k^\parallel \omega_{k'}^\parallel / 8\Delta \ \approx \ E_o^2/8\hbar\Delta \quad (13)$$

ie., roughly $\Delta\tau_\phi/2\pi\hbar \sim (\Delta/E_o)^2$ oscillations survive before phase decoherence sets in. We emphasize that *no energy relaxation from* $|1\rangle$ *to* $|0\rangle$ *is involved here*- this requires coupling to electronic excitations [1,2,7,8].

Comparing now electronic and spin bath decoherence rates, we see a crossover from electronic-dominated to spin bath-dominated decoherence around a temperature $T_c \sim (2\pi)^{1/2}\Delta^2/k_B\tilde{\alpha}_s E_o$ (strong decoherence regime), or $T_c \sim E_o^2/8k_B\tilde{\alpha}_s\Delta$ (weak decoherence regime). In the former case coherence will never be seen at any T; in the latter case the decoherence time will saturate at the value in (13), below T_c. In both cases the low T decoherence is controlled by the spin bath. We emphasize, on the other hand, that the spin bath will be almost invisible in MQT experiments- it causes litle dissipation, and adds to the tunneling exponent a factor $\delta S \sim \pi E_o/\hbar\Omega_o$, so typically $\delta S \ll 1$ (and is T-independent!).

4. *Connection to Experiments*: The concentration, type, and location of the paramagnetic impurities in the sample will usually be very uncertain. Luckily, (11) and (12) show we only need to know E_0 to parametrise the spin bath effects. In both regimes it can be determined by dynamic "fast passage" resonant tunneling experiments, from which one can extract $W(\epsilon) = W_+(\epsilon) + W_-(\epsilon)$ by inverting the data [20]. In the weak decoherence regime we can also look directly at the lineshape via microwave absorption between $|0\rangle$ and $|1\rangle$ manifolds.

What this means is that (i) we may characterise the spin bath, extracting E_o, via well-established experimental techniques, and then (ii) test the theory herein by comparing the predictions for $P_{\uparrow\uparrow}(t)$ in (11) and (13) with experiment. Note, incidentally, that just as in the nanomagnetic case [9–11], wide T-independent resonant peaks in the sweep experiments are circumstantial evidence for a spin bath-mediated mechanism (oscillator bath-mediated relaxation rates are T-dependent and typically *increase* as one moves further from resonance). Such peaks (of width $\sim 0.4\ K$) were seen in recent experiments [20], indicating a value $E_o \sim 0.2\ K$ for this particular SQUID. If one is to see MQC, or to make superconducting qubits, E_o must be reduced by at least 10^2. Ways to do this were indicated by our example- one wants very pure rings (including even isotopic purification if possible) with large R, small h, small junctions, thin substrates, and small ϕ_m [16].

In this paper we have shown how the low-T decoherence time in a SQUID must saturate at a value controlled by coupling to the spin bath. We thank the Institute for Theoretical Physics in Santa Barbara, where some of this work was done, and grant INTAS-2124 from the European community.

Figures

Fig. 1 (a) Evolution in imaginary time τ of the field $\vec{\omega}_k$ acting on $\vec{\sigma}_k$, during tunneling of ϕ from $-\phi_m$ to $+\phi_m$. The field begins at $\vec{\omega}_k^-$ and end at $\vec{\omega}_k^+$. (b) A typical "path" (in the path integral sense) for $\tau_z(t)$, as the SQUID tunnels between $|-\phi_m>$ and $|\phi_m>$. The transition occurs in a "bounce time" $\sim \Omega_0^{-1}$; the time between transitions $\sim 2\pi\hbar/\Delta$.

Fig. 2 A model DC SQUID ring, of height h, radius R, on a substrate of volume R^3. The junction is a weak link of length d and radius r. We also show, for this example, the various contributions to E_o as a function of R, assuming $h = 1\ \mu m$, $r = 50\ nm$, $d = 200\ nm$, and an insulating substrate with a concentration $x_S = 10^{-5}$ of paramagnetic impurities. We suppose ring and junction are made from Al (with $\gamma_N^{Al} = 11.094 MHz/T$, and $I = 5/2$), with concentrations $x_J, x_r = 10^{-6}$ paramagnetic impurities. For definiteness we assume all paramgnetic impurities have spin $s = 1$, $\gamma_s = 2\mu_B = 14 GHz/T$.

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[13] For the simple Zeeman coupling in (2) the only effect of having spin $> 1/2$ is to multiply E_0 by $2I$ (nuclear spins) or $2s$ (paramagnetic spins).
[14] If $k_B T_K \sim \Delta_{BCS}$, the BCS energy gap, then we must also include mid-gap states in the calculations - see Ref. [12]. We ignore this possibility here.
[15] We have dropped Suhl-Nakamura contributions to the nuclear fields here (cf H. Suhl, Phys. Rev. **109**, 606 (1958); T. Nakamura, Prog. Th. Phys. **20**, 542 (1958); although they change $\omega_k^{\perp}$ (increasing it) they do not affect the crucial parameter $\omega_k^{\parallel}$, which governs E_o.
[16] Designs with $\phi_m \ll 1$ (eg., [17]) have reduced decoherence, since $E_o \propto \phi_m$. One can even have "quiet" designs [18], with $\phi_m = 0$, ie., a *zero* diagonal coupling (the terms $\propto \hat{\tau}_z$ in Eq.(8)). We make two cautionary remarks here. First, although $\omega_k^{\parallel}$, E_o, and hence decoherence are greatly reduced, α_k must be recalculated in such designs, and can change much less- one may not then drop α_k from (8), since spin flips will now be the major source of decoherence. Second, such designs will probably not show what is usually meant by MQC [1,2] if the overlap between states $|\uparrow\rangle, |\downarrow\rangle$ is no longer small (although they may still function as qubits).
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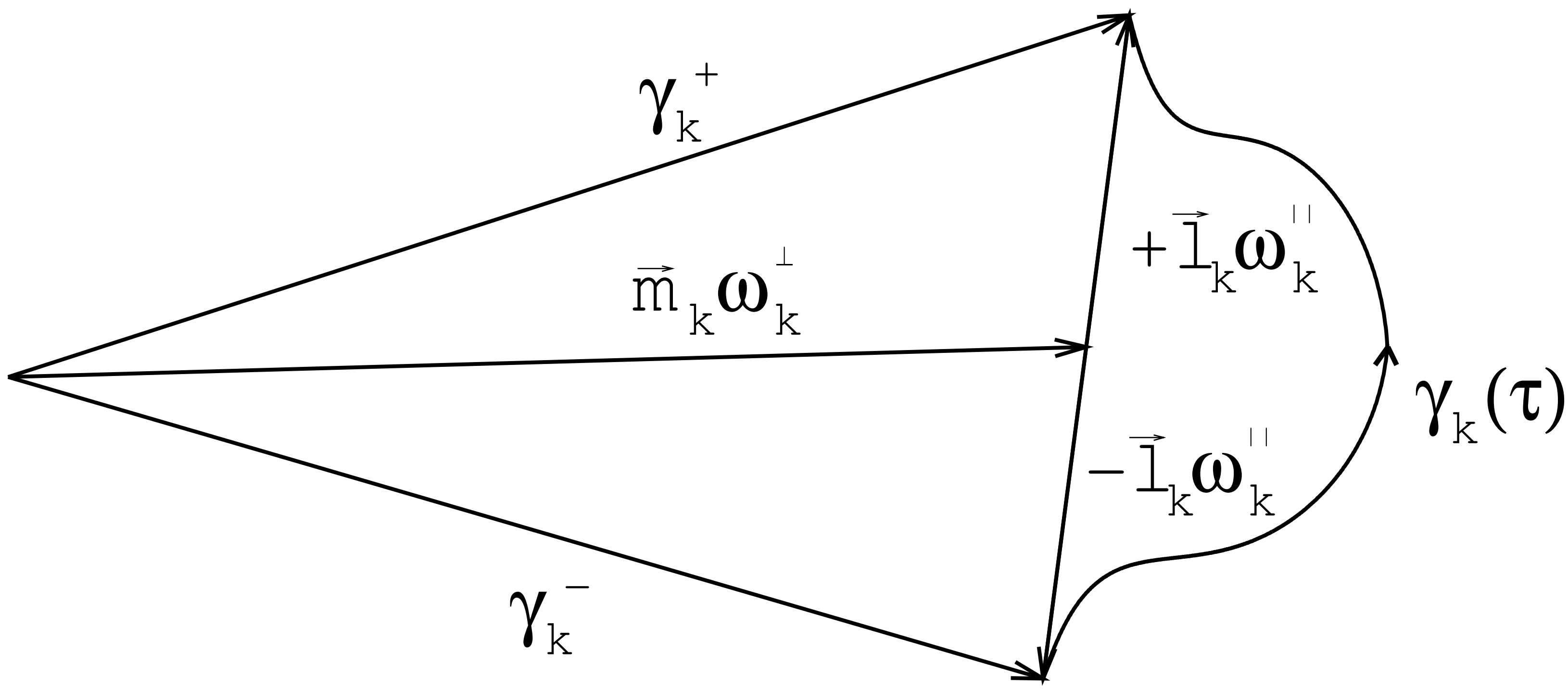
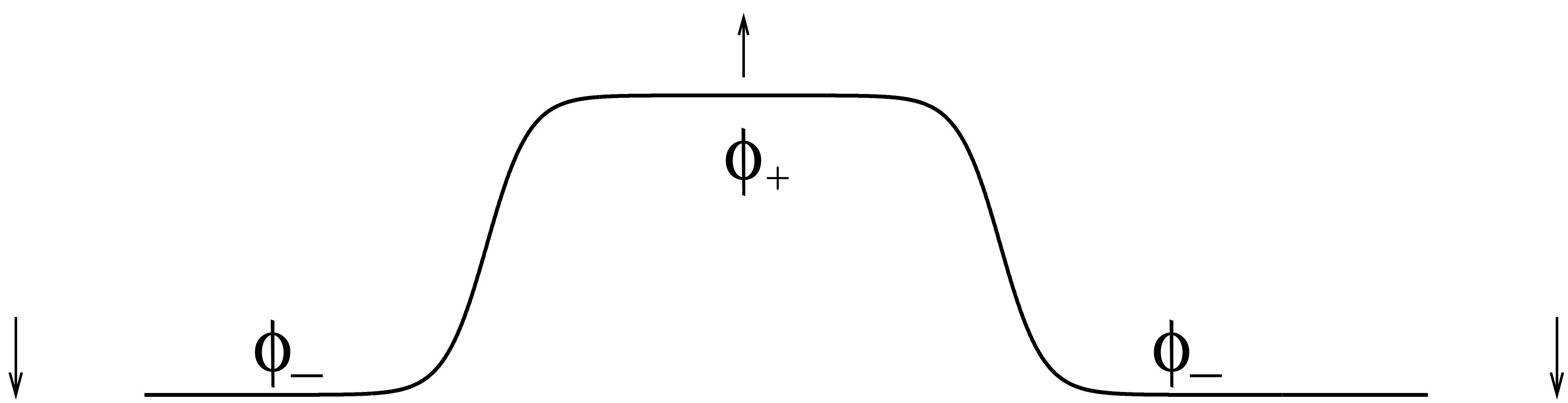
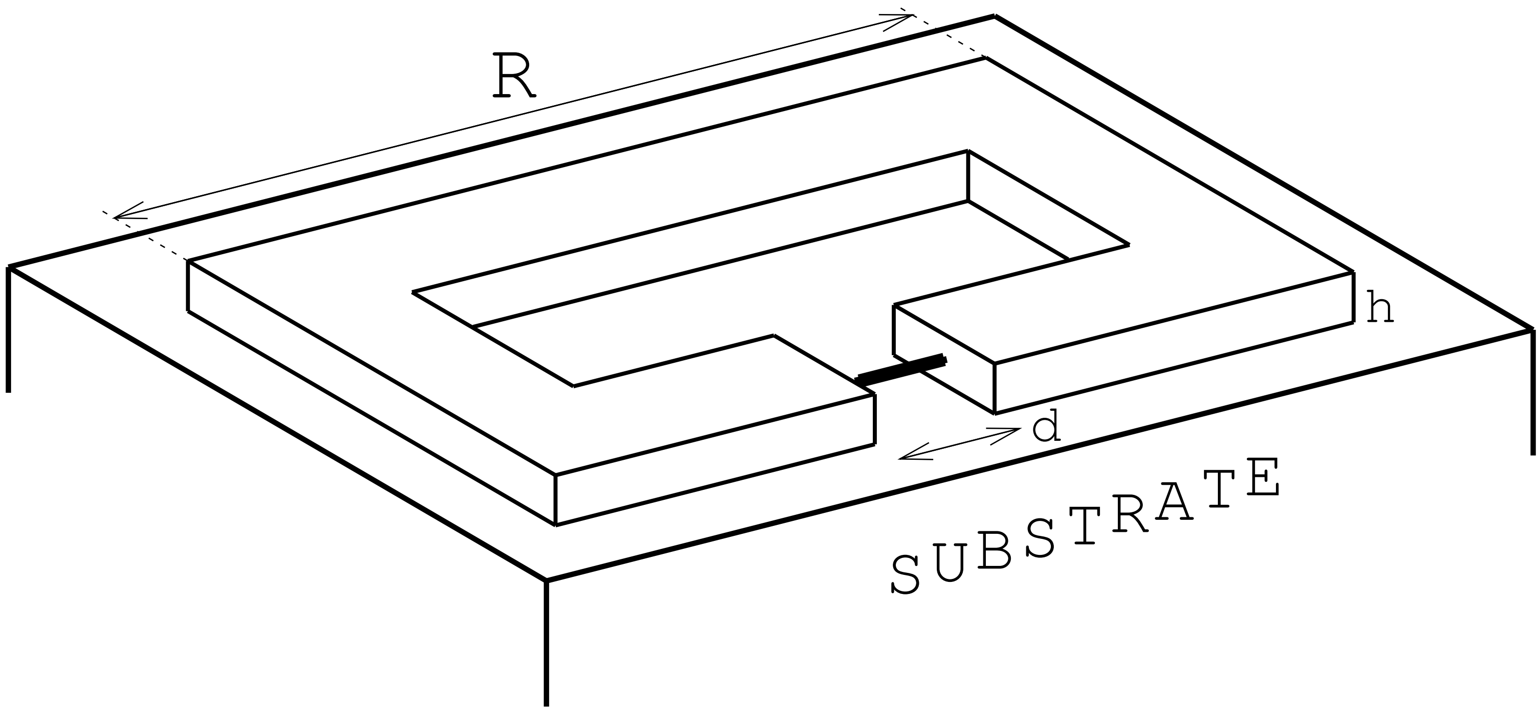
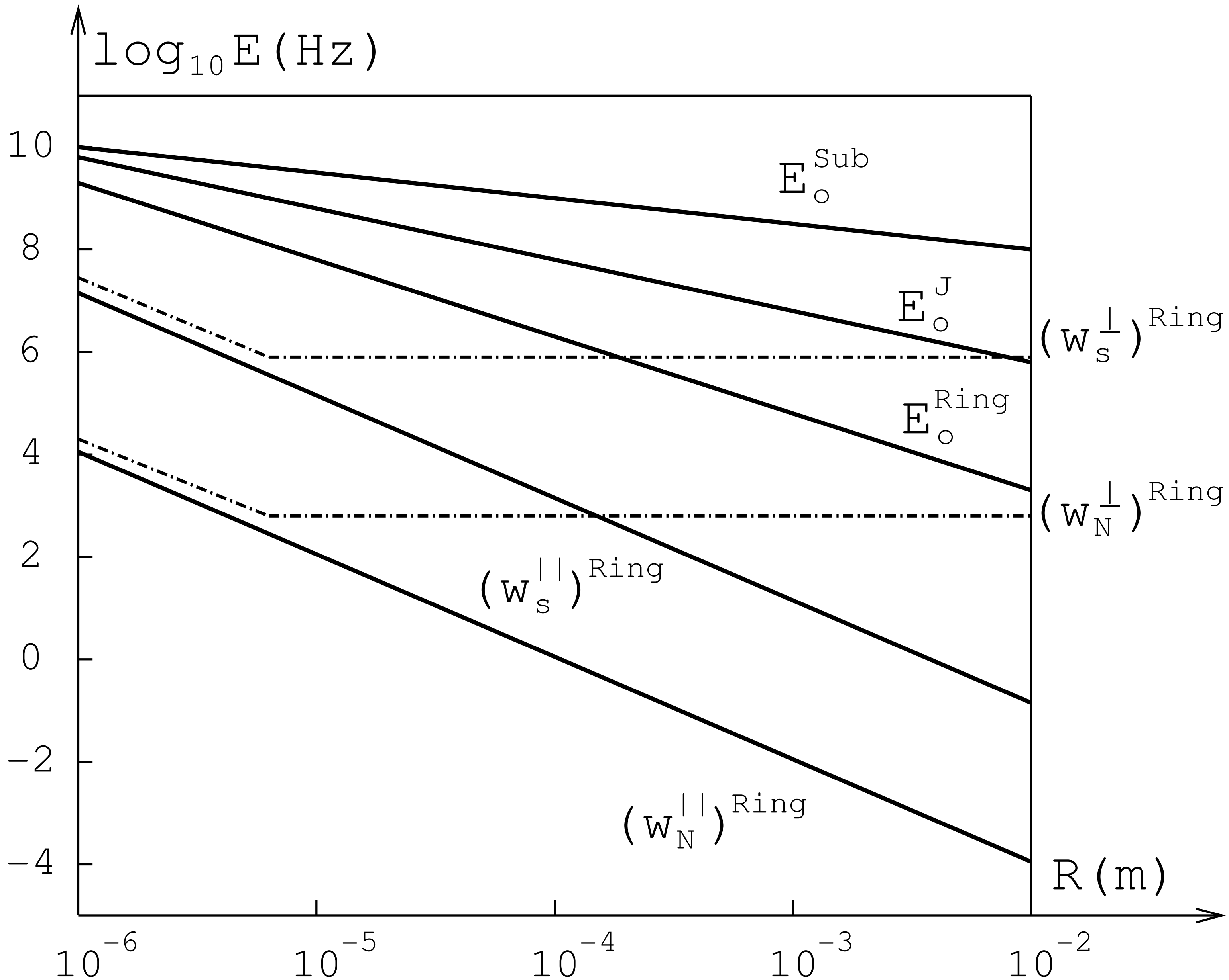
	Ring	Junction	Substrate
N_N	$4Rh\lambda_L n_N$	$\pi r^2 d n_N$	$\sim R^3 n_N$
N_s	$x_J N_N$	$x_J N_N$	$x_s N_N$
δB_k	Φ_0/R^2	$\Phi_0/(Rr)$	Φ_0/R^2
$\omega_k^\parallel$	$\gamma_k \delta B_k/2$	$\gamma_k \delta B_k/2$	$\gamma_k \delta B_k/2$
E_0^N	$I\gamma_N \Phi_0(\lambda_L h n_N)^{1/2}/R^{3/2}$	$I\gamma_N \Phi_0(\pi d n_N)^{1/2}/2R$	$I\gamma_N \Phi_0 n_N^{1/2}/2R^{1/2}$
E_0^s	$s\gamma_s \Phi_0(x_J \lambda_L h n_N)^{1/2}/R^{3/2}$	$s\gamma_s \Phi_0(x_J \pi d n_N)^{1/2}/2R$	$s\gamma_s \Phi_0(x_s n_N)^{1/2}/2R^{1/2}$
$\omega_N^\perp$		$\gamma_N[\Phi_0/R^2 + (\mu_o/4\pi)(\gamma_N + \gamma_s x_{J,s})C_N n_N]$	
$\omega_s^\perp$		$\gamma_s[\Phi_0/R^2 + (\mu_o/4\pi)(\gamma_N + \gamma_s x_{J,s})C_N n_N]$	
$\lvert\vec{\alpha}_k\rvert$	$\gamma_k \Phi_o/2R^2\hbar\Omega_o$	$\gamma_k \Phi_o/2Rr\hbar\Omega_o$	$\gamma_k \Phi_o/2R^2\hbar\Omega_o$

TABLE I. Parameters for the SQUID in Fig. 2, for nuclear (N) and paramagnetic (S) spins in the bulk ring, the junction, and the substrate. N_N and N_s count all spins within a penetration depth λ_L of the surface - in the ring we assume $h > \lambda_L > r$ (if $\lambda_L > h$, then substitute h instead of λ). We assume $B_x \sim \Phi_0/R^2$. The number density n_N of nuclear spins $\{\vec{I}_k\}$ (with $|\vec{I}_k| = I$) is $\sim 1/a_0^3$, where a_0 is the lattice parameter; x_J and x_s are paramagnetic impurity concentrations per site (with $|\vec{s}_k| = s$) in SQUID and substrate. Values for $\omega^\perp$ for ring and substrate spins (left blank in the Table) are the same as for the junction; $C_N \sim 5-10$ is a geometrical factor describing the effective number of nearest neighbour spins. Finally, $E_0^2 = \sum_k (\omega_k^\parallel)^2$, so $E_0^s \sim N_s^{1/2}\omega_s^\parallel$ and $E_0^N \sim N_N^{1/2}\omega_N^\parallel$ (see text).


γ_k^+
$\vec{m}_k \omega_k^\perp$
$+\vec{l}_k \omega_k^{\parallel}$
$\gamma_k(\tau)$
$-\vec{l}_k \omega_k^{\parallel}$
γ_k^-


ϕ_+
ϕ_-
ϕ_-


R
h
d
SUBSTRATE



log10E(Hz)
10
8
6
4
2
0
−2
−4
E_o^Sub
E_o^J
(w_s^⊥)^Ring
E_o^Ring
(w_N^⊥)^Ring
(w_s^||)^Ring
(w_N^||)^Ring
R(m)
10^−6
10^−5
10^−4
10^−3
10^−2