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## Comment on “Hausdorff Dimension of Critical Fluctuations in Abelian Gauge Theories”

In their Letter [1], Hove, Mo, and Sudbø derive a simple connection between the anomalous scaling dimension,  $\eta$ , of the U(1) universality class order parameter,  $\phi(\mathbf{x})$ , and the Hausdorff dimension,  $D_H$ , of critical loops:

$$\eta + D_H = 2. \quad (1)$$

In the loop representation, the correlator  $G(\mathbf{r}) = \langle \phi(\mathbf{r})\phi^*(0) \rangle \propto r^{-(d-2+\eta)}$  describes the distribution of the end-points in open loops. For definiteness, one may think of the high-temperature-expansion loops for the lattice  $|\phi|^4$ -model.

The analysis of Ref. [1] might seem absolutely compelling, being just a translation of the hyperscaling hypothesis into the loop language: *At the critical point there should be about one loop of diameter  $r$  per volume element  $r^d$*  [2]. Nevertheless, given the result  $\eta = 0.0380(4)$  of Ref. [3], the relation (1) is in strong contradiction with the value  $D_H = 1.7655(20)$  which we obtained for the 3D  $|\phi|^4$ -model with suppressed leading corrections to scaling [3] (and also—with a bit less accuracy—for the standard bond-current model [4], and its special version with excluded loop overlaps and self-crossings). The simulations were done with the Worm algorithm [5].

The hidden flaw in the treatment of Ref. [1] is as follows. When introducing the self-similar expression

$$P(\mathbf{r}; N) \propto N^{-\rho} F(r/N^\Delta), \quad \Delta = 1/D_H \quad (2)$$

for the probability to find the ends of an open loop of length  $N$  being distance  $\mathbf{r}$  away from each other, which is then used to establish the connection between the open and closed loops, the authors take for granted that  $F(0)$  is finite. While looking innocent, this is an *arbitrary* assumption, since the self-similar form (2) is valid only for  $r \gg a$ , where  $a$  is a microscopic cutoff (e.g., the lattice period). Strictly speaking, a closed loop of length  $N$  corresponds to  $F(a/N^\Delta)$  rather than to  $F(0)$ , and one has to work with the generic asymptotic form

$$F(x) \propto x^\theta \quad \text{at} \quad x \ll 1, \quad (3)$$

with some exponent  $\theta$ . With Eq. (3), the hyperscaling argument yields  $\rho = (d - \theta)/D_H$ , and from  $G(r) \propto \int dN P(\mathbf{r}; N)$  one then obtains

$$\eta + D_H = 2 - \theta. \quad (4)$$

Using high-precision data for  $\eta$  and  $D_H$  mentioned above, we find  $\theta = 0.1965(20)$ .

It is instructive to explicitly verify Eq. (3) by simulating  $P(r; N)$ . In Fig. 1 we present results of such a simulation for the  $|\phi|^4$ -model. We plot the value of  $P(r, N)N^{d\Delta}$  as a function of  $r$  for three different values of  $N$ . In view of the self-similarity of  $P(r, N)$ , the qualitative difference between the cases of  $\theta \neq 0$  and  $\theta = 0$  is readily seen. In the former case, curves for different values of  $N$  should merge for  $r/N^\Delta \ll 1$ —and they do in Fig. 1. In the latter case, as  $r \rightarrow 0$  one should see a fan of curves with essentially different slopes and a common origin at  $r = 0$ .

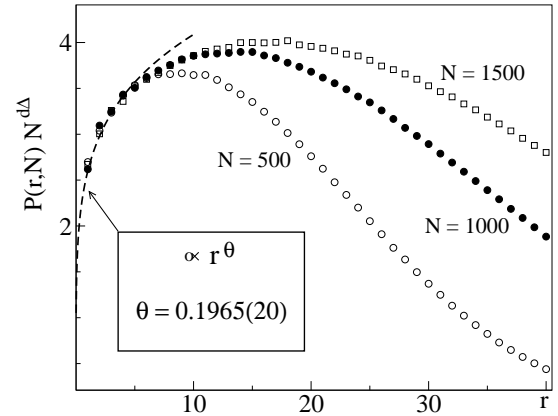


FIG. 1: Distribution of open loops over radii for three different values of  $N$ . The Worm algorithm simulation [5] was done for the loop representation (high-temperature expansion) of the 3D lattice  $|\phi|^4$ -model with  $L = 192^3$  sites at the special critical point with suppressed leading corrections to scaling [3].

One important implication of Eq. (4) in the absence of additional relation between  $D_H$ ,  $\eta$  and  $\theta$ , is that the anomalous scaling dimension *can not* be deduced from simulations of closed loops which determine  $D_H$  only.

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