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# Unconventional rotational responses of hadronic superfluids in a neutron star caused by strong entrainment and a $\Sigma^-$ hyperon gap

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I show that the usual model of the rotational response of a neutron star, which predicts rotation-induced neutronic vortices and no rotation-induced protonic vortices, does not hold (i) beyond a certain threshold of entrainment interaction strength nor (ii) in case of nonzero  $\Sigma^-$  hyperon gap. I demonstrate that in both these cases the rotational response involves creation of phase windings in electrically charged condensate. Lattices of *bound states of vortices* which are caused these effects can (for a range of parameters) strongly reduce the interaction between rotation-induced vortices with magnetic-field carrying superconducting components.

Microscopic calculations of properties of interior of a neutron star suggest a presence of a mixture of superfluid neutrons, superconducting protons and normal electrons (see e.g. [1] -[5]). Where protons form a type-II superconductor. This model suggests that the magnetic field in the star should induce a large number of vortices in the protonic condensate [1, 4], while rotation should induce neutronic vortex lattice. Because the magnetic field configuration in a young star is very complicated the protonic vortex structure is expected to be a complicated vortex tangle [1, 4]. It is important that in this standard picture for a typical star the average distance between protonic vortices is much larger than the magnetic field penetration length (which is of order of 10-100fm). Such a vortex tangle is indeed a thermodynamically unstable state but estimates suggest that a very large time scale is needed to expel these vortices [1].

The microscopic calculations also indicate that in a neutron star the effective mass of proton is very different from the bare mass [6]. This implies that there is a strong dissipationless drag effect [7]: i.e. superfluid velocity of neutronic condensate drags superfluid density of protonic condensate and vice versa. In particular a neutronic circulation drags along some density of protons resulting in magnetic field generated by neutronic vortex. Therefore a neutronic vortex interacts magnetically with protonic vortices [2].

In the usual model for neutron star, the free energy of the mixture of neutronic and protonic condensates has the form: [2]:

$$F = \frac{1}{2}\rho_{pp}\mathbf{v}_p^2 + \frac{1}{2}\rho_{nn}\mathbf{v}_n^2 + \rho_{pn}\mathbf{v}_p \cdot \mathbf{v}_n + \frac{\mathbf{B}^2}{2} \quad (1)$$

where  $\mathbf{B} = \nabla \times \mathbf{A}$  is magnetic field, while  $\mathbf{v}_n = (1/2m_n)\nabla\phi_n$  and  $\mathbf{v}_p = (1/2m_p)\nabla\phi_p - (e/m_p)\mathbf{A}$  are the superfluid velocities of neutron and proton condensates in units  $\hbar = c = 1$  (generalizations to cases with non-s-wave pairing is straightforward). Here  $m_n \approx m_p = m$  are the bare masses of a neutron and a proton and  $\phi_{n,p}$  are the phases of the corresponding condensates. The third term in (1) represents current-current interaction [7]. Because of it the particle current of one of the condensates

( $\mathbf{w}_{p,n}$ ) is carried by the superfluid velocity of another:

$$\mathbf{w}_p = \rho_{pp}\mathbf{v}_p + \rho_{pn}\mathbf{v}_n; \quad \mathbf{w}_n = \rho_{nn}\mathbf{v}_n + \rho_{pn}\mathbf{v}_p, \quad (2)$$

where  $\rho_{pn} = \rho_{np}$  are the drag coefficients which can be expressed via effective masses  $m_{p,n}^*$  as follows  $\rho_{pn} = \rho_{pp}\frac{m_p^* - m_p}{m_p} = \rho_{nn}\frac{m_n^* - m_n}{m_n}$ . Different microscopic calculations [6] give for  $m_p^*$  the values ranging  $m_p^* \approx 0.3m_p$  to  $m_p^* \approx 0.9m_p$ . Thus the drag strength can be as high as  $\rho_{pn} \approx -0.7\rho_{pp}$ . From the eq.(1) it follows that the electric current induced by protonic circulation or by neutronic drag is given by:

$$\mathbf{J} = e\frac{\mathbf{w}_p}{m} = \frac{e\rho_{pp}}{m^2} \left( \frac{\rho_{pn}}{\rho_{pp}}\nabla\phi_n + \nabla\phi_p - e\mathbf{A} \right). \quad (3)$$

When there is a  $2\pi$  phase winding in  $\phi_p$  (i.e. for an integral around the vortex core  $\oint\nabla\phi_p = 2\pi$ ), but no phase windings in  $\phi_n$  (we denote this vortex as (1,0)) the circulation of electric charge is

$$\mathbf{J} = \frac{e\rho_{pp}}{m^2} (\nabla\phi_p - e\mathbf{A}) \quad (4)$$

In the constant density approximation the magnetic field obeys the London equation [8]:  $\mathbf{B} + \lambda^2\text{curl curl}\mathbf{B} = \Phi_0\delta(\mathbf{r})$  where  $\Phi_0$  is the magnetic flux quantum and  $\lambda = m/e\sqrt{\rho_{pp}}$  is the magnetic field penetration length. Imposing a  $2\pi$  winding in  $\phi_p$  in eq. (1) one obtains a vortex with finite energy per unit length:

$$E \approx \int d^2\mathbf{r} \left( \frac{1}{2}\frac{\mathbf{J}^2}{e^2\rho_{pp}} + \frac{1}{2}(\nabla \times \mathbf{A})^2 \right) \approx \left( \frac{\Phi_0}{\lambda} \right)^2 \log \frac{\lambda}{a}, \quad (5)$$

where  $a$  is the cutoff length associated with vortex core.

In the case when a  $2\pi$  winding is imposed only on the neutronic condensates [lets denote it (0,1)], the drag effect produces some protonic current:

$$\mathbf{J} = \frac{e\rho_{pp}}{m^2} \left( \frac{\rho_{pn}}{\rho_{pp}}\nabla\phi_n - e\mathbf{A} \right) = e\rho_{pn}m^2 \left( \nabla\phi_n - e\frac{\rho_{pp}}{\rho_{pn}}\mathbf{A} \right).$$

Indeed this drag current in turn produces magnetic flux which is not quantized but is a linear function of the drag

strength:  $\Phi_d = \oint_{\sigma} d\mathbf{A} = \frac{\rho_{pn}}{\rho_{pp}} \frac{2\pi}{e}$  (here the integration is done over the contour  $\sigma$  located where  $J \approx 0$ ). The main contribution to the energy of (0,1) vortex is associated with the kinetic energy of the superflow of neutrons. This contribution is logarithmically divergent as a function of the system size  $R$ :

$$E_{sf} = \int \frac{1}{2m^2} \rho_{nn} (\nabla\phi_n)^2 \approx \pi \frac{\rho_{nn}}{m^2} \log\left(\frac{R}{a}\right). \quad (6)$$

The secondary contribution is associated with the drag-induced kinetic energy of the protonic supercurrent and the magnetic energy. Because electrically charged currents are localized at the length scale  $\lambda$  (in contrast to neutronic superflow), this contribution is much smaller:

$$E_{ch} \approx \int d^2\mathbf{r} \frac{1}{2} \left( \frac{\mathbf{J}^2}{e^2 \rho_{pp}} + (\nabla \times \mathbf{A})^2 \right) \approx \left( \frac{\Phi_d}{\lambda} \right)^2 \log \frac{\lambda}{a}.$$

If a protonic fluxtube intersects with a neutronic vortex which carries co-directed flux, the magnetic energy and kinetic energy of supercurrents rise. The energy of the intersection can be estimated as:

$$E_{ch}^i \approx \left[ \left( \frac{\Phi_d + \Phi_0}{\lambda} \right)^2 - \frac{\Phi_0^2}{\lambda^2} - \frac{\Phi_d^2}{\lambda^2} \right] \log \frac{\lambda}{a} \approx \frac{2\Phi_d \Phi_0}{\lambda^2} \log \frac{\lambda}{a}$$

According to [3, 4]  $E_{ch}^i \approx 5MeV$ . Because protonic vortex configuration is complicated and because protonic vortices are much more numerous than neutronic vortices [1], there should be strong pinning between the protonic and neutronic vortex matter. Much interest in the physics of vortex interaction was sparked by the recent calculations in [3] which suggest that the picture of strongly pinned protonic and neutronic vortices may be highly inconsistent with the slow precession observed in a few isolated pulsars. Although most of pulsars do not have such precession, this inconsistency casted doubts on the validity of the usual neutron star model. According to [3], the precession would be possible only if the drag strength was orders of magnitude lower than values obtained in microscopic calculations. Following the work [3] some alternative scenarios of magnetic response of a neutron star interior were proposed. In [12], based on the calculations of  $\Sigma^-$  hyperon gap [13], it was proposed that one has a mixture of two oppositely charged condensates which gives further alternatives to type-I/type-II dichotomy [14] (for other aspects of the theory of two-component charged mixture see [15, 16, 17]). The alternative conjecture put forward was that protons form type-I superconductor [3, 9, 10], suggesting that the magnetic field would penetrate star interior via macroscopic normal domains. However it was pointed out in [12] that the energy difference of a neutronic vortex located inside superconducting region versus inside a large type-I domain is quite large, which again leads to a strong interaction between neutronic vortices and flux-carrying

domains. Let me stress that importantly the magnetic response of a type-I superconductor is highly nonuniversal. Generically this magnetic response involves formation of multi-domains. The precise shape of the domains depends on a number of factors and is magnetic-history dependent [18]. Importantly a boundary between a normal metal and a superconductor in magnetic field implies that there is a magnetic field and supercurrents within the range of  $\lambda$  at this boundary. Thus even if protons form a type-I superconductor, neutronic vortices will be strongly interacting with the boundaries of normal domains which typically are plentiful in a type-I superconductor [18]. The strength of the resulting pinning of neutronic vortices will depend on the domain's structure (which is usually very complicated). Besides that the mechanism responsible for formation of large normal domains in a type-I superconductor is associated with the dominance of attractive core-core interaction between type-I vortices. However this force has the range of the coherence length. For a dilute vortex system with low mobility (as expected to be the case in a neutron star) the coalescence tendency of type-I vortices (and therefore the tendency to form large normal domains) would therefore be extremely small. So in general, the interaction strength between neutronic vortex lattice and flux-carrying structures in type-I case should not be expected to be dramatically lower than the interaction between neutronic vortices and protonic vortices in a type-II case.

Therefore it is an interesting question if there exists a mechanism which, in principle, could lead to any significant coupling reduction between neutronic and protonic vortices. Here I propose two scenarios which lead to such an effect. First let me observe that beyond a certain threshold of the drag strength, the usual neutronic vortices are *not thermodynamically stable*. That is, the vortex lattices form because such a state minimizes the free energy in a rotating system. The free energy of a (0,1) vortex in a system rotating with the angular velocity  $\Omega$  is [8]

$$F_r^{(0,1)} \approx \pi \frac{\rho_{nn}}{m^2} \log\left(\frac{R}{a}\right) + \left(\frac{\Phi_d}{\lambda}\right)^2 \log \frac{\lambda}{a} - \mathbf{M}\Omega, \quad (7)$$

Where  $\mathbf{M}$  is the vortex momentum [8]:

$$M = \int d^3r \left[ \frac{r}{m} \rho_{pn} |(\nabla\phi_n - e \frac{\rho_{pp}}{\rho_{pn}} \mathbf{A})| + \frac{r}{m} \rho_{nn} |\nabla\phi_n| \right] \quad (8)$$

In eq. (8) the first contribution comes from protonic circulation, while the second term corresponds to the neutronic circulation. The supercurrent of protons is concentrated around the vortex within the range of penetration length (of order of 10 or 100fm in the standard picture), while the characteristic size of the star is of order of 10 km. Therefore the protonic contribution  $\int d^3r (r/m) \rho_{pn} |(\nabla\phi_n - e(\rho_{pp}/\rho_{pn})\mathbf{A})|$  in the vortex momentum (the last term in Eq. (7)) is much smaller than

the neutronic contribution (which is not exponentially localized). Note also that this small contribution has the role of the *energy penalty* because  $\rho_{pn} < 0$  and thus protons are circulating in the direction opposite to the neutronic circulation. Let us now rewrite the equation (1) as

$$F = \int d\mathbf{r} \frac{1}{2} \left[ \left( \rho_{nn} - \frac{\rho_{pn}^2}{\rho_{pp}} \right) (\nabla \phi_n)^2 + \rho_{pp} \left( \nabla \phi_p + \frac{\rho_{pn}}{\rho_{pp}} \nabla \phi_n - e\mathbf{A} \right)^2 + (\nabla \times \mathbf{A})^2 \right] \quad (9)$$

Here we separated the energy associated with the electrically neutral superflow (given by the first term). It is associated with the phase gradient decoupled from  $\mathbf{A}$ . The remaining terms describe the kinetic energy of the electrically charged currents (represented by the phase gradients coupled to vector potential  $\mathbf{A}$ ) and the magnetic energy. In a free energy of (0,1) vortex in a rotating frame, there is (i) a free energy penalty coming from the negative-drag-induced momentum of protons, and (ii) energy penalties from the kinetic energy of protonic currents and magnetic field. Let me observe that the simplest vortex with phase winding only in the neutronic phase is not thermodynamically stable when  $|\rho_{pn}|/\rho_{pp} > 1/2$ . Thermodynamic stability is achieved when the free energy is minimized with respect to all the degrees of freedom which include phase windings. Previously it was assumed that rotation induces vortices with only neutronic phase winding [2]. However from the eq. (9) we can see that if there is a  $2\pi$  winding in  $\phi_n$ , and  $-\rho_{pn}/\rho_{pp} > 1/2$ , the system can minimize the free energy in a rotating frame by creating an additional  $2\pi$  winding in the protonic phase  $\phi_p$  (around the same core). Let's denote such composite vortices (1,1). Its free energy is

$$F_r^{(1,1)} \approx \pi \frac{\rho_{nn}}{m^2} \log\left(\frac{R}{a}\right) + \left(\frac{\Phi_0(1 + \frac{\rho_{pn}}{\rho_{pp}})}{\lambda}\right)^2 \log\frac{\lambda}{a} - M\Omega \quad (10)$$

Note that microscopic calculations [6] estimate the drag strength in a neutron star can be as high as  $\rho_{pn}/\rho_{pp} \approx -0.7$ . Let me observe that with *increased* drag strength beyond the threshold  $-\rho_{pn}/\rho_{pp} > 1/2$ , the magnetic energy and the energy of protonic currents around a rotation-induced (1,1) vortex *decreases*. Moreover the additional phase winding does not affect the dominant neutronic negative contribution (coming from  $-M\Omega$ ) to the free energy (7). Also because the protonic current of (1,1) vortex

$$\mathbf{J}^{(1,1)} \propto \left( \left[ 1 - \frac{|\rho_{pn}|}{\rho_{pp}} \right] \nabla \phi_n - e\mathbf{A} \right) \quad (11)$$

has a circulation opposite to that of a (0,1) vortex

$$\mathbf{J}^{(0,1)} \propto \left( -\frac{|\rho_{pn}|}{\rho_{pp}} \nabla \phi_n - e\mathbf{A} \right) \quad (12)$$

it carries an opposite momentum. Thus instead of a positive free energy penalty  $-\mathbf{M} \cdot \boldsymbol{\Omega}$  it produces a negative free energy gain in this term. Therefore  $F_r^{(1,1)} < F_r^{(0,1)}$  for  $-\rho_{pn}/\rho_{pp} > 1/2$  and thus the (0,1) neutronic vortices are not thermodynamically stable in this regime. Instead a superfluid supports rotation by creating a lattice of composite vortices (1,1). Observe that in general, for a given superfluid momentum, there is also a possibility to minimize the energy of charged currents for smaller ratios of  $|\rho_{pn}|/\rho_{pp}$  by creating a  $2\pi N$  winding in  $\phi_n$  and compensating the contribution of  $\nabla \phi_n$  in the second term in (9) by a  $2\pi M$  “counter-winding” in  $\phi_p$ . But in the case of a vortex with a multiple winding in  $\phi_n$  the first term in (9) depends quadratically on  $N$  while the last term depends on it linearly. Because in a neutron star, the  $\rho_{nn}$  is very large compared to  $\rho_{pn}$  and  $\rho_{pp}$  the composite vortices with multiple windings in  $\phi_n$ , like e.g. (2,1) should not be thermodynamically stable.

Another physical consequence of the possible formation of composite (1,1) vortices is that for  $-\rho_{pn}/\rho_{pp} > 1/2$  with *increased* drag strength  $|\rho_{pn}|$  there is a *decrease* in the protonic current and magnetic energy contributions to the energy cost of an intersection between the rotation induced vortex and protonic (1,0) flux tube. For nearly parallel vortices it is given by

$$E_{ch} \approx 2\Phi_0^2(1 + \rho_{pn}/\rho_{pp})\lambda^{-2} \log(\lambda/a). \quad (13)$$

Also the (1,1) vortices should have reduced interaction with the boundaries of the normal domains (if protonic condensate is type-I). To estimate more accurately vortex-vortex interaction potential, it is important to observe that the composite (1,1) vortex has a core in protonic condensate due to protonic phase winding. This gives rise to the attractive core-core interaction with the range of coherence length [19] (which originates from minimization of the energy cost of suppression of the protonic order parameters in the cores). Therefore if protons form type-II condensate then for  $\rho_{pn}/\rho_{pp} > 0.5$  the interaction between approximately co-directed (1,1) and (1,0) vortices is further minimized by core-core interaction because both (1,1) and (1,0) vortices have zeros of protonic condensate.

There is another scenario leading to appearance of composite vortices and possible electromagnetic coupling reduction in a neutron star. It has been discussed that a star can have another electrically charged condensate associated with  $\Sigma^-$  Cooper pairs [12, 13]. Let me observe that from the theory of two-component charged mixture [16] it follows that the presence of the second charged component alters qualitatively the rotational response of the system (if intercomponent Josephson coupling is negligible [16]). That is, in the free energy functional of such a system, one can form a gauge-invariant combination of the phase gradients  $\propto (\nabla(\phi_p + \phi_\Sigma))$  which is decoupled from the vector potential [15, 16] (here  $\phi_\Sigma$  is the phase of hyperon condensate). It means that if another charged

component is present, then there is an additional superfluid mode (i.e. besides the neutronic) associated with co-flow of oppositely charged neutronic and hyperonic Cooper pairs. Such co-flows transfer mass without charge transfer. From the calculations in [15, 16] it follows the system has two kinds of vortex excitations which carry identical momenta associated with these co-flows. These vortices have phase windings ( $\Delta\theta_p = 2\pi, \Delta\theta_\Sigma = 0$ ) and ( $\Delta\theta_p = 0, \Delta\theta_\Sigma = 2\pi$ ). On the other hand these vortices carry different fractions of magnetic flux quanta:

$$\Phi_i = \pm \frac{|\Psi_i|^2}{m_i} \left[ \frac{|\Psi_p|^2}{m_p} + \frac{|\Psi_\Sigma|^2}{m_\Sigma} \right]^{-1} \Phi_0, \quad (14)$$

where  $|\Psi_{(p,\Sigma)}|^2$  and  $m_{(p,\Sigma)}$  are protonic and hyperonic condensate's number densities and masses. The density  $\nu$  of rotation-induced vortices in a mixture of charged condensates depends on the angular velocity  $\Omega$  as [16]

$$\nu = \frac{(m_p + m_\Sigma)\Omega}{\pi\hbar}. \quad (15)$$

Therefore these vortices should be more numerous than neutronic vortices (observe that this effect takes place no matter how small the superfluid density of the second charged component is). At the same time the phase stiffness of the composite superfluid mode is tiny  $\frac{|\Psi_p|^2}{m_p} \frac{|\Psi_\Sigma|^2}{m_\Sigma} \left[ \frac{|\Psi_p|^2}{m_p} + \frac{|\Psi_\Sigma|^2}{m_\Sigma} \right]^{-1}$  [15, 16]. It is indeed much smaller than the phase stiffness of the neutronic condensate and therefore ordering energies of the rotation-induced protonic/hyperonic vortex lattice are much smaller than those of neutronic vortex lattice. In this system therefore there will be a competition between minimization of vortex lattice ordering energy versus minimization of the kinetic energy of charged currents and magnetic energy. Both the neutronic vortices and the rotation-induced vortices in a charged condensate mixture [15] carry fractional magnetic flux. It is known from the other examples [16, 20] that this kind of competitions results in formation of composite vortex lattices if the ordering energy of one of the vortex sublattices is low (it is indeed the case in the system in question). In particular one neutronic vortex can anchor one or several rotation-induced vortices in the charged condensates mixture. There is certainly a range of parameters where the resulting composite vortices carry very small magnetic flux. This would result in a different scenario of electromagnetic coupling reduction of a rotation-induced vortex lattice and magnetic field carrying structures [21]. Observe also that in a mixture the London law is modified [16].

In conclusion, in the usually assumed picture, a rotation of a neutronal star induces neutronic vortex lattices, and magnetic field produces protonic fluxtubes. It is also assumed in the usual picture that no vortices in the protonic condensate can be induced by rotation since single-component charged condensate reacts to rotation via the

London law. In this work it was shown that a mixture of hadronic superfluids in a neutron star has a qualitatively different rotational response if drag strength exceeds a certain threshold  $\rho_{pn}/\rho_{pp} < -1/2$  (microscopic calculations suggest that  $\rho_{pn}/\rho_{pp}$  can be as large as  $-0.7$ ) or if several charged condensates are present. In both these cases rotation actually does produce phase windings in a charged condensate i.e. protonic (or  $\Sigma^-$ -hyperonic) vortices. Both these scenarios, may lead to a strong reduction of electromagnetic coupling between a rotation-induced vortex lattice and magnetic-flux carrying structures. If the resulting interaction strength falls below the temperature scale, in some region of a neutron star, it can lead to decoupling of superfluid and flux-bearing components.

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