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General multi-product, multi-pollutant market pollution permit model: a variational inequality approach

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Abstract

In this paper, we develop a new model in marketable pollution permits that consists of perfectly competitive, multi-product, multi-pollutant firms. The formulation and qualitative analysis of the model, as well as the computational approach, are based on the theory of variational inequalities.

1. Introduction

The problem of environmental pollution results partly from the absence of prices of certain scarce environmental goods, such as clean air and water. Because there is no cost attached to the discharge of pollutants, and there is no incentive for the reduction of discharges, industrial firms emit an excessive amount of pollution that, in many cases, is more than can be absorbed by the environment.

One economic approach to the pollution-reduction problem is to set up a market for pollution permits. In this approach, at least in theory, the pollution-emitters have a choice between reducing emissions by employing some abatement tech-

nology, or purchasing permits from other emitters that hold excess permits. The permits in such a market would be completely transferrable, so that the participating emitters could trade the permits, depending on their cost of control. As Tietenberg (1980) notes, this reallocation of permits can lead to substantial reductions in pollution, under competitive conditions, while ensuring that environmental quality standards are met.

An alternative approach to the pollution-reduction problem is to place an appropriate price on the polluting firms. It turns out that the corrective price takes the form of a Pigouvian tax (Pigou, 1920), which, if set equal to the marginal external cost, will induce the polluting firm to internalize the full social costs of its contribution to the pollution damage (Baumol and Oates, 1988). Yet another approach, i.e. that of unit subsidies, is equivalent to the tax approach, in that it can establish the same incentive for abatement as would a tax of an identical magnitude. However, these two approaches have different implications for the profitability of production in the polluting industry and the long-run entry-exit decisions (Cropper and Oates, 1992).

In this paper, we develop a general market model in pollution permits, in which individual firms will minimize costs, while ensuring that the externally set environmental quality standards are met. The individual firms produce multiple products and emit multiple pollutants. We assume that the firms are perfectly competitive in their output markets, as well as in the permit markets. Because we are analyzing firm behavior at large, each source of pollution takes the price of its output and the pollutant-specific permit as given, because it is assumed that each source in a region is small relative to the entire economy. The model deals explicitly with spatial differentiation, through the use of a diffusion matrix that maps emissions from sources to receptor points that are dispersed in space. This is especially important because studies show that, for certain pollutants, if spatial differentiation is not built into the system, then the cost-savings from employing an economic-incentive-based approach will be lost (Mendelsohn, 1986).

The marketable pollution permit approach is selected for analysis in this paper, for a variety of reasons. The primary reason is that the regulatory body then has direct control over the quantity of emissions in the region, because it has the authority to issue pollution permits equal to the efficient quantity of pollution. The permit approach also enjoys the advantage of familiarity, because it is a modified form of the current regulatory approach. The strongest motive for marketable pollution permits, however, is that it provides an incentive for polluters to adopt new abatement and control technologies to reduce the level of emissions, because the excess number can be sold in the form of marketable assets.

The methodology that we utilize for the formulation, qualitative analysis and computation of the equilibrium pattern in markets for pollution control is the theory of variational inequalities. The theory of variational inequalities has already been utilized to study a plethora of equilibrium problems in economics and operations research (Nagurney, 1993). In particular, it has been used recently to study equilibrium problems with policy interventions in the setting of spatial economic markets (Nagurney and Zhao, 1991; Nagurney et al., 1995, 1996) and in financial markets (Dong, 1995; Nagurney and Dong, 1995). The framework devel-

oped in this paper yields the profit-maximized quantities of the multiple products; the equilibrium quantities of the various emissions; the equilibrium allocation of pollution licenses; and the prices of the licenses. The use of variational inequality theory in environmental economics has yet to be fully explored. Here, we aim to make a contribution in this direction.

The paper is organized as follows. In Section 2, we consider the behavior of the individual firm and develop the optimization problem faced by the firm. Subsequently, we present the economic conditions that govern the market model and then derive the variational inequality formulation of the equilibrium conditions. We also establish that the equilibrium pattern does satisfy environmental quality standards imposed by the government, provided that the sum of licenses for each pollutant for a receptor point is equal to the imposed standard for that point. Furthermore, the equilibrium allocation of licenses is independent of the initial allocation of licenses, provided that the sums are satisfied. These are important features from an operational standpoint. In addition, we provide some qualitative properties of the equilibrium pattern.

In Section 3, an algorithm is proposed for the computation of equilibrium, accompanied by the conditions for convergence. The algorithm — a modified projection method — is then applied to compute solutions to several numerical examples in Section 4. Finally, we summarize our results and present the conclusions in Section 5.

2. The model

In this section, we develop a multi-product, multi-pollutant market model in pollution permits that yields the profit-maximizing quantities of firms' products, the efficient quantities of emissions, and the equilibrium allocation of pollution licenses, in addition to the prices of the licenses.

We consider m sources of industrial pollution, which are firms that are fixed in location, with a typical source denoted by i. There are n receptor points, with a typical receptor point denoted by j. We assume that, in general, the firms and the receptor point are spatially separated. Let there be r different pollutants emitted by the sources, with a typical pollutant denoted by t. Each source produces a vector of emissions denoted by e_i, where $e_i = (e_i^1, \ldots, e_i^t, \ldots, e_i^r)$ and where the component e_i^t denotes the amount of pollutant t emitted by source i. We further group the vectors $\{e_1, \ldots, e_m\}$ into the vector $e \in R_+^{mr}$.

We assume, as given, an $m \times n \times r$ diffusion matrix $\mathbf{H}$, where the component h_{ij}^t denotes the contribution that one unit of emission by source i makes to the average pollutant concentration of type t at receptor point j. This idea comes from Montgomery (1972), who stated that the emission vector e_i can be mapped into concentrations by the diffusion matrix $\mathbf{H}$ so that the resultant emissions will not exceed the externally set standard.

Let a permit denote a license, the possession of which will allow a source to emit a specific pollutant at some specific receptor point. Hence, each polluter will have to hold a portfolio of licenses to cover all the relevant monitored receptor points.

Let l_{ij}^t denote the number of licenses for pollutant t at point j held by source i, and let us group licenses for each firm i into a vector $l_i \in R_+^{nr}$. We then further group the vectors $\{l_1, \ldots, l_m\}$ into the vector $l \in R_+^{mnr}$. We assume throughout that some initial allocation of licenses, i.e. $l_{ij}^{t(0)}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$; $t = 1, \ldots, r$, has been made by the regulatory government authority.

Furthermore, let p_j^t denote the price of the license for pollutant t that affects receptor point j, and let us group the prices into the vector $p \in R_+^{nr}$. Also, assume that the market in pollution licenses is perfectly competitive, i.e. each source of pollution takes the price of the license to pollute at a specific point as given and cannot affect the price itself, because each source is small relative to the entire economy.

We also assume that the sources are perfectly competitive in their input and output markets. Let there be o different outputs produced by the sources, with a typical output denoted by k. Each of the sources produces a vector of products denoted by q_i, where $q_i = (q_{i1}, \ldots, q_{ik}, \ldots, q_{io})$ and where q_{ik} is the quantity of product k produced by source i. We group the vectors $\{q_1, \ldots, q_m\}$ into the vector $q \in R_+^{mo}$. Also, let π_k denote the price of the product which is assumed to be taken by a typical firm.

We further assume that the firms exhibit profit-maximizing behavior. To describe mathematically the behavior of the firms in the market, we first discuss the cost function faced by a typical firm and, subsequently, we define a market equilibrium relative to an initial allocation of licenses. Next, we derive the variational inequality formulation of the governing equilibrium conditions, the solution to which yields the efficient quantities of product, emissions and licenses along with the license prices.

2.1. The cost function

To maximize profit, a competitive firm needs to minimize the cost of producing its optimal level of output. The cost function measures the minimum cost of producing some level of output, given fixed factor prices. Following the treatment by Henderson and Quandt (1980), we derive the cost function from the information on the production functions, the cost expression and the expansion path function.

A production function is an equation or a schedule that shows the maximum amount of output that can be produced from any specified set of inputs, given the existing technology. In our case, the production function for the product q_{ik} depends on the employment of firm i's resources to the production of q_i vector. Assume that a firm employs v types of input, with a typical input denoted by s. Let x_{is} denote the input that firm i employs in its production activity and let Ψ_i^k denote the transformation function of inputs into outputs. Hence, it follows that

$$q_{ik} = \Psi_i^k(x_{i1}, \ldots, x_{iv}) \qquad k = 1, \ldots, o. \tag{1}$$

For each pollutant t, the emissions function, in turn, depends on the employment of firm i's resources to the abatement activity for the vector e_i and on the

vector q_i. We include the vector q_i because the emissions of a firm are directly related to the production activity. Let Υ_i^t denote the transformation function of the inputs and the output into the emissions. Therefore, we may write

$$e_i^t = \Upsilon_i^t(x_{i1}, \ldots, x_{iv}, q_i), \qquad t = 1, \ldots, r. \tag{2}$$

Let the total cost of production G_i be given by

$$G_i = \sum_{s=1}^{v} w_{is} x_{is} + b \tag{3}$$

where w_{is} denotes the price of input x_{is}, and b denotes the cost of any fixed inputs.

If the production function is strictly quasi-concave, then every point of tangency between an isoquant and an isocost line is the solution of both a constrained-maximum problem and a constrained-minimum problem. The locus of all the tangency points gives us the expansion path for the firm. A rational, profit-maximizing firm will select only input combinations that lie on its expansion path.

Formally, this path is an implicit function of the inputs that can be expressed as

$$g(x_{i1}, \ldots, x_{iv}) = 0. \tag{4}$$

Assume that the systems of equations given by Eqs. (1)–(4) can be reduced to a single equation in which cost is stated as an explicit function of the level of output of products and emissions, and the input prices plus the fixed cost of inputs, i.e.

$$G_i = \sum_{s=1}^{v} \Phi_i(e_i, q_i, w_{is}) + b. \tag{5}$$

This cost function Φ_i is non-decreasing, homogeneous of degree one, and concave with respect to the input prices w_{is}. We can assume that the input prices are invariant, so that the cost function depends on the emissions and output level, plus the fixed cost of inputs. Furthermore, this fixed cost of fixed inputs b must be paid by the firm i, regardless of whether or not it engages in production. Hence, the cost function takes the general form

$$G_i = G_i(e_i, q_i). \tag{6}$$

2.2. A firm's optimization problem

We now construct a constrained optimization problem in which a typical firm has to take into account the joint cost of production and emissions abatement, as well as the cost of purchasing pollution licenses.

Because the price of the product q_{ik} is π_k, each firm i acquires a revenue of

$$\sum_{k=1}^{o} \pi_k q_{ik}. \tag{7}$$

In addition, the value of a firm's initial endowment of licenses can be expressed as

$$\sum_{j=1}^{n} \sum_{t=1}^{r} p_j^{t*} l_{ij}^{t0}$$

where p_j^{t*} denotes the given price of a license to pollute a specific pollutant t at receptor point j, which, under the assumption of perfect competition in the license markets, is assumed as given.

Also, the net cost of purchasing licenses for a specific pollutant t at all the receptor points is given by

$$\sum_{j=1}^{n} p_j^{t*} \left(l_{ij}^{t} - l_{ij}^{t0} \right).$$

Consequently, the total net cost of purchasing licenses to cover all the emitted pollutants is given by

$$\sum_{t=1}^{r} \sum_{j=1}^{n} p_j^{t*} \left(l_{ij}^{t} - l_{ij}^{t0} \right). \tag{8}$$

We assume that each firm in the market is profit maximizing, so that it can be characterized by a utility function that measures its profit or net revenue. Hence, the utility function u_i that faces each such firm i; $i = 1, \ldots, m$, can be expressed as the difference between the total revenue acquired by a firm and the total cost incurred by the firm. We have

$$u_i = u_i(q_i, e_i, l_i) = \sum_{k=1}^{o} \pi_k q_{ik} - G_i(e_i, q_i) - \sum_{j=1}^{n} \sum_{t=1}^{r} p_j^{t*} \left(l_{ij}^{t} - l_{ij}^{t0} \right). \tag{9}$$

A firm's optimization problem can be then expressed as

$$\max u_i(q_i, e_i, l_i) \tag{10}$$

subject to

$$h_{ij}^{t} e_i^{t} \leq l_{ij}^{t}, \qquad j = 1, \ldots, n, t = 1, \ldots, r \tag{11}$$

and the non-negativity constraints

$$q_{ik} \geq 0, \qquad e_i^{t} \geq 0, \qquad k = 1, \ldots, o, t = 1, \ldots, r. \tag{12}$$

Eq. (11) states that each firm is allowed to have an average rate of emission for pollutant t that produces no more pollution at any point than the amount which the firm is licensed to cause at that point.

We let λ_{ij}^{t} denote the Lagrange multiplier associated with the jt th constraint of Eq. (11), and we group these multipliers into the vector $\lambda_i \in R_+^{nr}$. We note that λ_{ij}^{t}

may be interpreted as the marginal cost of pollution abatement for pollutant t associated with firm i and receptor point j; we now term this Lagrange multiplier as the marginal abatement cost.

The utility function for each perfectly competitive firm is concave with respect to licenses. Assuming that the utility function $u_i(q_i, e_i, l_i)$ is continuously differentiable and concave with respect to e_i and q_i, the necessary and sufficient conditions for an optimal product-specific output and pollutant-specific emissions, license and marginal abatement cost pattern $(q_i^*, e_i^*, l_i^*, \lambda_i^*)$ is that this pattern is non-negative and satisfies the inequality

$$
\sum_{k=1}^{o} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial q_{ik}} - \pi_k \right] (q_{ik} - q_{ik}^*) + \sum_{t=1}^{r} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^{t*} h_{ij}^t \right]
$$

$$
\times (e_i^t - e_i^{t*}) + \sum_{j=1}^{n} \sum_{t=1}^{r} \left(p_j^{t*} - \lambda_{ij}^{t*} \right) \left(l_{ij}^t - l_{ij}^{t*} \right)
$$

$$
+ \sum_{j=1}^{n} \sum_{t=1}^{r} \left(l_{ij}^{t*} - h_{ij}^t e_i^{t*} \right) \left(\lambda_{ij}^t - \lambda_{ij}^{t*} \right) \geq 0
$$

$$
\forall q_{ik} \geq 0,\ e_i^t \geq 0,\ l_{ij}^t \geq 0,\ \lambda_{ij}^t \geq 0;\ j = 1, \ldots, n,\ k = 1, \ldots, o,\ t = 1, \ldots, r.
$$
(13)

Note that, in equilibrium, a similar inequality needs to hold for each of the other perfectly competitive firms.

We now give a further interpretation to Eq. (13) in terms of equilibrium conditions. Our goal here is to provide an economic interpretation to the optimality condition of Eq. (13). In particular, the first term in Eq. (13) describes the following equilibrium condition. For each product k, $k = 1, \ldots, o$, we have

$$
\left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial q_{ik}} - \pi_k \right]
\begin{cases}
= 0, & \text{of } q_{ik}^* > 0 \\
\geq 0, & \text{if } q_{ik}^* = 0.
\end{cases}
$$
(14)

This condition states that, when a positive quantity of output is produced by a firm, the principle of price equals marginal cost holds. However, if the marginal cost of production is greater than the price charged for the output, then it does not benefit the firm to produce and, consequently, $q_{ik}^* = 0$.

The second term in Eq. (13) describes the following equilibrium condition. For each pollutant t, $t = 1, \ldots, r$, we have

$$
\left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^{t*} h_{ij}^t \right]
\begin{cases}
= 0, & \text{if } e_i^{t*} > 0 \\
\geq 0, & \text{if } e_i^{t*} = 0.
\end{cases}
$$
(15)

Recall that λ_{ij}^t is the Lagrange multiplier attached to the corresponding emissions constraint, and can be viewed as the marginal abatement cost to be borne by firm i. The term $\lambda_{ij}^t h_{ij}^t$ is the shadow price times the ambient concentration for pollutant t at receptor point j, and can be interpreted as the shadow

value to firm i of emissions constraint on ambient concentrations of pollutant t at receptor point j. Also $\partial G_i(e_i^*, q_i^*)/\partial e_i^t$ is the marginal cost of reducing emissions borne by firm i. Hence, the typical firm will only emit if this marginal cost equals minus the marginal abatement cost. However, if the marginal cost of reducing emissions is greater than minus the marginal abatement cost by the firm to reduce emissions, then it will be infeasible for the firm to emit any pollutants, hence, e_i^{t*} will be zero.

Likewise, the third term in Eq. (13) describes the following equilibrium condition. For each receptor point j, $j = 1, \ldots, n$, and for each pollutant t, $t = 1, \ldots, r$, we have

$$\left(p_j^{t*} - \lambda_{ij}^{t*} \right) \begin{cases} = 0, & \text{if } l_{ij}^{t*} > 0 \\ \geq 0, & \text{if } l_{ij}^{t*} = 0. \end{cases} \tag{16}$$

In other words, the final distribution of licenses will be positive only when the marginal abatement cost of pollution borne (λ_{ij}^{t*}) by source i for pollutant t that affects receptor point j is equal to the price of the specific pollutant license associated with the receptor point j, i.e. p_j^{t*}. However, when the actual price of the license is greater than the marginal abatement cost, the final distribution of that license is zero.

Finally, the fourth term in Eq. (13) describes the following equilibrium condition. For each receptor point j, $j = 1, \ldots, n$, and for each pollutant t, $t = 1, \ldots, r$, we have

$$\left(h_{ij}^t e_i^{t*} - l_{ij}^{t*} \right) \begin{cases} = 0, & \text{if } \lambda_{ij}^{t*} > 0 \\ \geq 0, & \text{if } \lambda_{ij}^{t*} = 0. \end{cases} \tag{17}$$

This system has the following interpretation. When the emissions constraint is binding, the shadow price λ_{ij}^{t*} associated with the constraint reflects this fact, by taking on a positive value. However, when the constraint is non-binding, the shadow price associated with the constraint is driven to zero.

We now describe the system of equalities and inequalities that govern the quantities and prices of the licenses in the region at equilibrium. Mathematically, the economic system conditions that govern market clearance in pollution permits are as follows. For each receptor point j, $j = 1, \ldots, n$, and for each pollutant t, $t = 1, \ldots, r$, we have

$$\sum_{i=1}^{m} \left(l_{ij}^{t0} - l_{ij}^{t*} \right) \begin{cases} = 0, & \text{if } p_j^{t*} > 0 \\ \geq 0, & \text{if } p_j^{t*} = 0. \end{cases} \tag{18}$$

This system states that, if the price of a license for pollutant t at a point j is positive, then the market for licenses at that point must clear; if there is an excess supply of licenses for a particular pollutant t at a receptor point, then the price of a license at that point must be zero.

We now define an equilibrium by combining the optimality conditions for each firm and the market clearing conditions for the pollution permits.

Definition (an equilibrium). A vector $(q^*, e^*, l^*, \lambda^*, p^*) \in R_+^{mo+mr+2mnr+nr}$ is an equilibrium point of the market model for pollution permits developed above if and only if it satisfies Eq. (13) for all firms i, and the system of equalities and inequalities of Eq. (18) for all receptor points $j = 1, \ldots, n$, and for all pollutants $t = 1, \ldots, r$.

2.3. Variational inequality formulation

We now derive the variational inequality formulation of the equilibrium conditions of Eq. (13) and (18) that govern the above multi-product, multi-pollutant market model. The variational inequality problem is a unified framework within which all the above inequalities and equalities can be expressed as a single inequality. As mentioned in the Introduction, this framework has already been used to formulate a plethora of equilibrium problems that arise in economics and operations research (Nagurney, 1993). However, its potential in environmental economics has yet to be fully explored.

Theorem 1 *(variational inequality formulation).* *A vector of firm production outputs, emissions, licenses and associated marginal abatement costs, along with the vector of permit prices, i.e.*

$$(q^*, e^*, l^*, \lambda^*, p^*) \in R^{mo+mr+2mnr+nr}$$

is an equilibrium point if and only if it satisfies the variational inequality problem, i.e.

$$\sum_{i=1}^{m} \sum_{k=1}^{o} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial q_{ik}} - \pi_k \right] (q_{ik} - q_{ik}^*)$$

$$+ \sum_{i=1}^{m} \sum_{t=1}^{r} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^{t*} h_{ij}^t \right] (e_i^t - e_i^{t*})$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left(p_j^{t*} - \lambda_{ij}^{t*} \right) \left(l_{ij}^t - l_{ij}^{t*} \right)$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left(l_{ij}^{t*} - h_{ij}^t e_i^{t*} \right) \left(\lambda_{ij}^t - \lambda_{ij}^{t*} \right)$$

$$+ \sum_{j=1}^{n} \sum_{t=1}^{r} \sum_{i=1}^{m} \left(l_{ij}^{t0} - l_{ij}^{t*} \right) \left(p_j^t - p_j^{t*} \right) \geq 0,$$

$$\forall (q, e, l, \lambda, p) \in R_+^{mo+mr+2mnr+nr} \tag{19}$$

Proof. See Appendix A.

We now put the variational inequality of Eq. (19) into standard form (Nagurney, 1993). Define the column vector $(X) \equiv (q, e, l, \lambda, p) \in R_+^{mo+mr+2mnr+nr}$ and $F(X)$

as the row vector that consists of the row vectors $(G(X), E(X), L(X), \Lambda(X), P(X))$, where $G(X)$ is the mo-dimensional vector with component ik given by $\partial G_i / \partial q_{ik} - \pi_k$; $E(X)$ is the mr-dimensional vector with component it given by

$$\frac{\partial G_i}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^t h_{ij}^t$$

$L(X)$ is the mnr-dimensional vector with component ijt given by $p_j^t - \lambda_{ij}^t$; $\Lambda(X)$ is the mnr-dimensional vector with component ijt given by $l_{ij}^t - h_{ij}^t e_i^t$, and $P(X)$ is the nr-dimensional vector with jtth component by

$$\sum_{i=1}^{m} \left(l_{ij}^{t0} - l_{ij}^t \right).$$

The variational inequality of Eq. (19) can now be expressed as follows. Determine $X^* \in K$, such that

$$F(X^*)(X - X^*) \geq 0, \qquad \forall X \in K \tag{20}$$

where $K \equiv \{(X) \equiv (q, e, l, \lambda, p) \in R_+^{mo + mr + 2mnr + nr}\}$.

We now establish in the following corollary that the equilibrium pattern is independent of the initial license allocation, provided that the sum of the licenses for each receptor point for each pollutant is fixed. We then discuss what the value of that sum should be, given the environmental quality standards.

Corollary 1. *If $l_{ij}^{t0} \geq 0$ for all $i = 1, \ldots, m$, $j = 1, \ldots, n$ and $t = 1, \ldots, r$ and $\sum_{i=1}^{m} l_{ij}^{t0} = Q_j^t$ for $j = 1, \ldots, n$ and $t = 1, \ldots, r$, with each Q_j^t fixed and positive, then the equilibrium pattern $(q^*, e^*, l^*, \lambda^*, p^*)$ is independent of $\{l_{ij}^{t0}\}$.*

Proof. The first four terms in the variational inequality of Eq. (19) are independent of $\{l_{ij}^{t0}\}$, whereas the last term only depends on the sum $\sum_{i=1}^{m} l_{ij}^{t0}$ for $j = 1, \ldots, n$ and $t = 1, \ldots, r$. $\quad \square$

Hence, any initial allocation of licenses that maintains the desired sum of the licenses for each receptor point and pollutant will not affect the equilibrium pattern.

We now present a theorem which provides a manner of determining the appropriate sums of the initial licenses for each receptor point and pollutant. It is an important result for operationalizing this approach.

Theorem 2. An equilibrium vector achieves environmental quality standards as represented by the vector $(Q_1, \ldots, Q_n)$, where $Q_j = (Q_j^1, \ldots, Q_j^r)$, provided that $\sum_{i=1}^{m} l_{ij}^{t0} = Q_j^t$ for $j = 1, \ldots, n$ and $t = 1, \ldots, r$.

Proof. From Eq. (11), we have that, for each firm i, $i = 1, \ldots, m$, and for each pollutant t, $t = 1, \ldots, r$, that

$$h_{ij}^t e_i^{t*} \leq l_{ij}^*, \qquad \forall j = 1, \ldots, n, \quad \forall t = 1, \ldots, r$$

Moreover, it follows from the equilibrium conditions of Eq. (18) and from the assumption on the initial license allocations that

$$\sum_{i=1}^{m} h_{ij}^{t} e_i^{t*} \leq \sum_{i=1}^{m} l_{ij}^{t*} \leq \sum_{i=1}^{m} l_{ij}^{t0} = Q_j^{t}, \qquad \forall j = 1, \ldots, n, \quad \forall t = 1, \ldots, r \qquad \square$$

2.4. Qualitative properties

Here, we investigate certain qualitative properties of the equilibrium. In particular, we establish properties of the function $F(X)$ that are needed for convergence of the algorithm in Section 3.

Lemma 1. If the utility function of Eq. (9) is concave for each firm i, then $F(X)$ is monotonic.

Proof. See Appendix A.

Lemma 2. The function $F(X)$ is Lipschitz continuous, i.e. there exists a positive constant L, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \qquad \forall X^1, X^2 \in K \tag{21}$$

under the assumption that the utility functions have bounded second-order derivatives.

Proof. This follows from the same arguments as the proof of Lemma 3 in Nagurney (1993).

3. The algorithm

Here, we present an algorithm for the solution of the variational inequality of Eq. (20) — equivalently, Eq. (19) — that governs the multi-product, multi-pollutant market equilibrium model for pollution permits. The algorithm resolves the variational inequality problem into very simple subproblems, each of which can be solved explicitly and in closed form.

The algorithm that we propose for the computation of the equilibrium pattern is the modified projection method of Korpelevich (1976). The algorithm is guaranteed to converge, provided that F satisfies only the monotonicity condition and the Lipschitz continuity condition, assuming that a solution exists.

The statement of the modified projection method is as follows.

Step 1: *Initialization.* Set $X^0 \in K$. Let $\beta = 1$ and let α be a scalar such that $0 < \alpha < 1/L$, where L is the Lipschitz continuity constant (cf. Eq. (21)).

Step 2: *Computation.* Compute $\bar{X}^\beta$ by solving the variational inequality subproblem

$$\left[\bar{X}^\beta + \alpha F(X^{\beta-1})^\mathrm{T} - X^{\beta-1}\right]^\mathrm{T}(X - \bar{X}^\beta) \geq 0, \qquad \text{for all } X \in K \tag{22}$$

Step 3: *Adaptation. Compute* X^β *by solving the variational inequality subproblem*

$$\left[X^\beta + \alpha F(\bar{X}^\beta)^\mathrm{T} - X^{\beta-1}\right]^\mathrm{T}(X - X^\beta) \geq 0, \qquad \text{for all } X \in K \tag{23}$$

Step 4: Convergence verification. *If* $max|X_l^\beta - X_l^{\beta-1}| \leq \epsilon$ *for all l, with* $\epsilon > 0$ (*a prespecified tolerance*), *then stop; else, set* $\beta = \beta + 1$, *and go to Step* 2.

We now discuss the modified projection method more fully. We first recall the definition of the projection of x on the closed convex set K with respect to the Euclidean norm, we denote this by $P_K x$ as

$$y = P_K x = \arg \min_{z \in K}\|x - z\| \tag{24}$$

In particular, we note that (cf. Theorem 1.2, Nagurney, 1993) $\bar{X}^\beta$ generated by the modified projection method as the solution to the variational inequality subproblem of Eq. (22) is actually the projection of $X^{\beta-1} - \alpha F(X^{\beta-1})^\mathrm{T}$ on the closed convex set K, where K here is simply the non-negative orthant. In other words, we have

$$\bar{X}^\beta = P_K\left[X^{\beta-1} - \alpha F(X^{\beta-1})^\mathrm{T}\right] \tag{25}$$

Similarly, X^β generated by the solution to the variational inequality subproblem of Eq. (23) is the projection of $X^{\beta-1} - \alpha F(\bar{X}^\beta)^\mathrm{T}$ on the non-negative orthant, i.e.

$$X^\beta = P_K\left[X^{\beta-1} - \alpha F(\bar{X}^\beta)^\mathrm{T}\right] \tag{26}$$

Because the feasible set here is of the box type, the above projections immediately decompose across the coordinates of the feasible set. In fact, the solution of each variable encountered in Eqs. (22) and (23) amounts to projecting on to R_+ separately.

Consequently, we can provide closed-form expressions for the solution of the problems of Eqs. (22) and (23). In particular, we have that Eq. (22) can be solved as follows. For all firms i, $i = 1, \ldots, m$, products k, $k = 1, \ldots o$, and pollutants t, $t = 1, \ldots, r$ set

$$\bar{q}_{ik} = \max\left\{0, \alpha\left[\frac{\partial G_i(e_i^{\beta-1}, q_i^{\beta-1})}{\partial q_{ik}} + \pi_k\right] + q_{ik}^{\beta-1}\right\} \tag{27}$$

and

$$\bar{e}_t^{t\beta} = \max\left\{0, \alpha\left[\frac{\partial G_i(e_i^{\beta-1}, q_i^{\beta-1})}{\partial e_i^t} - \sum_{j=1}^n \lambda_{tj}^{t\beta-1}h_{ij}\right] + e_t^{t\beta-1}\right\} \tag{28}$$

For all firms i, $i = 1, \ldots, m$, all receptor points j, $j = 1, \ldots, n$, and all pollutants t, $t = 1, \ldots, r$, set

$$\bar{l}_{ij}^{t\beta} = \max\left\{0, \alpha\left(-p_j^{t\beta-1} + \lambda_{ij}^{t\beta-1}\right) + l_{ij}^{t\beta-1}\right\} \tag{29}$$

and

$$\bar{\lambda}_{ij}^{t\beta} = \max\left\{0, \alpha\left(-l_{ij}^{t\beta-1} + h_{ij}e_t^{t\beta-1}\right) + \lambda_{ij}^{t\beta-1}\right\} \tag{30}$$

Finally, for all receptor points j, $j = 1, \ldots, n$, and all pollutants t, $t = 1, \ldots, r$, set

$$\bar{p}_j^{t\beta} = \max\left\{0, \alpha\left(-\sum_{i=1}^{m} l_{ij}^{t0} + \sum_{i=1}^{m} l_{ij}^{t\beta-1}\right) + p_j^{t\beta-1}\right\} \tag{31}$$

The variational inequality subproblem of Eq. (23) can be solved explicitly in closed form in a similar manner.

Convergence is given in the following:

Theorem 3. The modified projection method described above converges to the solution of the variational inequality of Eq. (13) under the assumptions that the utility functions have bounded second-order derivatives and are concave.

Proof. It follows from Lemmas 1 and 2 that the function $F(X)$ is both monotonic and Lipschitz continuous, under the stated assumptions. Hence, as established in Theorem 2 in Korpelevich (1976), the modified projection method is guaranteed to converge under these conditions. □

4. Numerical examples

Here, we present numerical examples that illustrate the model presented in Section 2, along with the performance of the algorithm presented in Section 3. We consider two examples, both of which are quadratic in form. The cost function used in these examples is in terms of input prices, outputs, emissions and other technological parameters. The algorithm was coded in FORTRAN 77. The system used was the IBM SP2. The CPU times below are reported exclusive of input — output and initialization times.

Example 1.

This example consists of five firms and 10 receptor points. Each firm produces two outputs and emits two pollutants. Each firm in the permit market faces a joint production and emission-abatement cost of the form

$$G_i(e_i, q_i) = (w_1 \times w_2)^{1/2} \left[\sum_{t=1}^{2} a1_{it} e_i^t \right.$$

$$\left. \times 1 + \frac{1}{2} \sum_{t=1}^{2} a2_{it}(e_i^t)^2 + \sum_{k=1}^{2} b1_{ik} q_{ik} + \frac{1}{2} \sum_{k=1}^{2} b2_{ik}(q_{ik})^2 + f_i \right] \quad (32)$$

where w_1 and w_2 represent the prices of the two inputs, and $a1_{it}$, $a2_{it}$, $b1_{ik}$, $b2_{ik}$ and f_i are the technological parameters. The values for these parameters are given in Table 1.

The initial allocation of the licenses for pollutants t, $t = 1, 2$, i.e. the l_{ij}^{t0} terms were set as follows: $l_{ij}^{t0} = 0$, if $i \leq j$, for $t = 1, 2$; otherwise, $l_{ij}^{t0} = 5t$. The diffusion matrix H terms, i.e. the h_{ij}^t terms were set as follows: $h_{ij}^1 = 0.0001\frac{i}{j}$, and $h_{ij}^2 = 0.0005\frac{i}{j}$ for pairs i, j.

In this example, we assume that the typical firm produces a higher quantity of product 1 and emits a higher amount of pollutant 1. Hence, we initialized the algorithm with $q_{i1}^0 = 40$ and $q_{i2}^0 = 20$ for all firms i. Also, the prices charged for these products are set at $\pi_1 = 30$ and $\pi_2 = 20$. Furthermore, because the two pollutants emitted would be dissimilar, the initial values for the emissions are set at $e_i^{1^0} = 10$ and $e_i^{2^0} = 5$ for all the firms i. All the remaining initial variables are set equal to 1.

We set $\alpha = 0.01$ and used $\epsilon = 0.0001$ for the convergence tolerance.

The algorithm converged in 99 iterations and 0.05 CPU seconds, and yielded the equilibrium output vector for product 1 as

$$q_1^* = (13.33, 6.92, 14.16, 10.77, 6.33)$$

It gave the equilibrium output vector for product 2 as

$$q_2^* = (5.60, 9.21, 8.18, 4.36, 7.16)$$

It gave the equilibrium emission vector for pollutant 1 as

$$e^{1*} = (4.00, 2.51, 3.33, 5.05, 4.17)$$

Table 1
Example 1 — production and emission cost paramters

Firm i	w_1	w_2	f_i	$a1_{i1}$	$a1_{i2}$	$a2_{i1}$	$a2_{i2}$	$b1_{i1}$	$b1_{i2}$	$b2_{i1}$	$b2_{12}$
1	5	10	3.2	−2.0	−1.5	0.5	0.8	2.0	3.2	2.1	3.0
2	5	10	1.1	−1.0	−2.0	0.4	0.6	3.0	2.5	3.9	1.9
3	5	10	8.0	−2.0	−1.0	0.6	0.7	3.1	2.0	1.9	2.2
4	5	10	3.9	−1.5	−3.0	0.3	0.8	2.0	3.0	2.6	3.9
5	5	10	6.0	−2.5	−1.0	0.6	0.5	3.5	2.1	4.0	2.5

It gave the equilibrium emission vector for pollutant 2 as

$$e^{2*} = (1.84, 3.33, 1.43, 3.75, 2.00)$$

and it gave the license price vector for both pollutants $t = 1, 2$ as

$$p'^* = (0.00, 0.00, 0.00, 0.00, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00).$$

We also computed the maximum error for these terms, where the error is the value of the corresponding function term in the variational inequality problem of Eq. (19). Specifically, when q_{ik}^* is positive in the equilibrium condition of Eq. (14), it follows that

$$\sum_{k=1}^{0} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial q_{ik}} \right]$$

should be equal to π_k, i.e. the price for the product k. If this equality does not hold, then q_{ik}^* is zero. Similarly, the equilibrium conditions of Eqs. (15)–(18) are required to be satisfied. The maximum error for the expressions in the variables of the variational inequality problem is 0.0001. We do not report all the license values and abatement costs, because there are too many values.

Example 2

The second example also consists of five firms, 10 receptor points, two products and two pollutants. However, in this example, we include an interaction term in the joint production and emission abatement cost. Specifically, this cost function has the form

$$G_i(e_i, q_i) = (w_1 \times w_2)^{1/2} \left[\sum_{t=1}^{2} c1_{it} e_i^t + \sum_{t=1}^{2} c2_{it} (e_i^t)^2 \right.$$

$$\left. + \sum_{k=1}^{2} d1_{ik} q_{ik} + \sum_{k=1}^{2} d2_{ik} (q_{ik})^2 \right]$$

$$+ (w_1 \times w_2)^{1/2} \left[\sum_{t=1}^{2} \sum_{k=1}^{2} h1_{it} h2_{ik} (e_i^t q_{ik}) \right] \quad (33)$$

where w_1 and w_2 are the prices of the two inputs, and $c1_{it}$, $c2_{it}$, $d1_{ik}$, $d2_{ik}$, $h1_{i1}$, $h1_{i2}$, $h2_{i1}$ and $h2_{i2}$ are the technological parameters. The values for these parameters are given in Table 2.

The joint production and emission abatement cost is more general in this example, because it includes an additional term that would handle the interaction between the output produced and pollutants emitted.

To analyze a different scenario in this example, we consider firm i as initially producing a greater quantity of product 2 and emitting a greater amount of

Table 2
Example 2 — production and emission cost parameters

Firm i	w_1	w_2	$c1_{i1}$	$c1_{i2}$	$c2_{i1}$	$c2_{i2}$	$d1_{i1}$	$d1_{i2}$	$d2_{i1}$	$d2_{i2}$	$h1_{i1}$	$h1_{i2}$	$h2_{i1}$	$h2_{i2}$
1	5	10	-1.0	-2.5	0.2	0.25	2.0	1.5	2.2	1.6	0.1	0.2	0.1	0.2
2	5	10	-2.1	-1.6	0.8	0.35	4.0	2.0	3.8	2.1	0.1	0.3	0.2	0.2
3	5	10	-2.0	-3.0	0.3	0.25	2.5	1.5	2.6	1.2	0.2	0.2	0.1	0.1
4	5	10	-1.0	-2.0	0.2	0.50	2.1	3.0	2.0	3.5	0.1	0.3	0.3	0.1
5	5	10	-2.4	-1.5	0.8	0.15	3.0	1.5	3.5	1.7	0.2	0.2	0.1	0.1

pollutant 2. Hence, the initial values for the two products are set as $q_{i1}^0 = 20$ and $q_{i2}^0 = 40$. Also, the prices charged for the products are set at $\pi_1 = 25$ and $\pi_2 = 40$. Furthermore, the emissions are initialized as $e_i^{1^0} = 5$ and $e_i^{2^0} = 10$. The rest of the data used are identical to these of Example 1.

The algorithm converged in 325 iterations and 0.17 seconds of CPU times, and yielded the equilibrium output vector for product 1 as

$$q_1^* = (5.21, 2.76, 4.30, 5.70, 3.13)$$

It gave the equilibrium output vector for product 2 as

$$q_2^* = (11.96, 9.01, 15.98, 5.27, 11.29)$$

It gave the equilibrium emission vector for pollutant 1 as

$$e^{1*} = (1.65, 1.09, 1.98, 1.68, 1.14)$$

It gave the equilibrium emission vector for pollutant 2 as

$$e^{2*} = (4.31, 1.61, 5.60, 1.67, 4.56)$$

and it gave the license price vector for both pollutants $t = 1, 2$ as

$$p^{t*} = (0.00, 0.00, 0.00, 0.00, 3.00, 3.00, 3.00, 3.00, 3.00, 3.00).$$

Again, we do not report the license values and abatement costs, because there are too many of them. Moreover, each iteration is remarkably simply and computationally very efficient, because closed-form expressions are used.

5. Conclusions

In this paper, we have utilized the theory of variational inequalities for the formulation, analysis and computation of equilibrium solutions to a market model in pollution permits that consists of multi-product, multi-pollutant firms. The emissions are spatially differentiated.

In particular, we first derived the optimality conditions that govern each perfectly competitive firm, and then obtained the economic conditions that govern the

licenses to pollute and the license prices. We also gave an economic interpretation to these conditions. All the systems of equalities and inequalities were then shown to satisfy a single inequality, i.e. a finite-dimensional variational inequality problem.

We then turned to the qualitative analysis of the model and established certain properties; in particular, the monotonicity and Lipschitz continuity of the function that enters the variational inequality model, under reasonable conditions on the cost functions that face each firm. The same conditions are needed to establish the convergence of the proposed algorithm, i.e. the modified projection method. In the context of our problem, this algorithm resolves what we expect to be a large-scale problem into simple subproblems, each of which can be solved simultaneously and in closed form.

We subsequently applied the algorithm to numerical examples to illustrate its performance. Further research can entail incorporating other behaviors into the model, such as oligopolistic behavior, sensitivity analysis, as well as evaluation of alternative policy scenarios. Finally, to aid further in the operationalization of this framework, the importance of empirical work cannot be underestimated.

Appendix A

A1. Proof of Theorem 1

Assume that $(q^*, e^*, l^*, \lambda^*, p^*) \in R^{mo + mr + 2mnr + nr}$ is an equilibrium. Note that Eq. (13) thus holds for all firms $i = 1, \dots, m$, and that, summing it over all firms, one obtains

$$\sum_{i=1}^{m} \sum_{k=1}^{o} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial q_{ik}} - \pi_k \right] (q_{ik} - q_{ik}^*)$$

$$+ \sum_{i=1}^{m} \sum_{t=1}^{r} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^{t*} h_{ij}^t \right] (e_i^t - e_i^{t*})$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left(p_j^{t*} - \lambda_{ij}^{t*} \right) \left(l_{ij}^t - l_{ij}^{t*} \right)$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left(l_{ij}^{t*} - h_{ij}^t e_i^{t*} \right) \left(\lambda_{ij}^t + \lambda_{ij}^{t*} \right) \geq 0,$$

$$\forall (q, e, l, \lambda) \in R_+^{mo + mr + 2mnr}. \tag{A1}$$

Also, from the system of Eq. (18), we can conclude that the equilibrium must satisfy

$$\sum_{j=1}^{n} \sum_{t=1}^{r} \sum_{i=1}^{m} \left(l_{ij}^{t0} - l_{ij}^{t*} \right) \left(p_j^t - p_j^{t*} \right) \geq 0, \qquad \forall p \in R_+^{nr}. \tag{A2}$$

Indeed, note that, if $p_j^{t*} > 0$, then Eq. (18) gives that

$$\sum_{i=1}^{m} \left(l_{ij}^{t0} - l_{ij}^{t*} \right) = 0$$

and Eq. (A2) must hold. However, if $p_j^{t*} = 0$, then Eq. (18) gives that

$$\sum_{i=1}^{m} \left(l_{ij}^{t0} - l_{ij}^{t*} \right) \geq 0$$

and Eq. (A2) must hold.

Finally, summing Eqs. (A1) and (A2), one obtains the variational inequality of Eq. (19).

We now establish the converse of the proof, i.e. that the solution to Eq. (19) also satisfies Eqs. (13) and (18). Let $(q^*, e^*, l^*, \lambda^*, p^*) \in R^{mo + mr + 2mnr + nr}$ be a solution of Eq. (19). If one lets $q_{ik}^t = q_{ik}^{t*}$ for all i, k; $e_i^t = e_i^{t*}$ for all i; $l_{ij}^t = l_{ij}^{t*}$ for all i, j, t; $\lambda_{ij}^t = \lambda_{ij}^{t*}$, for all i, j, t; and substitutes these values into Eq. (19), one obtains

$$\sum_{j=1}^{n} \sum_{t=1}^{r} \sum_{i=1}^{m} \left(l_{ij}^{t0} - l_{ij}^{t*} \right) \left(p_j^t - p_j^{t*} \right) \geq 0, \qquad \forall p \in R_+^{nr} \tag{A3}$$

which implies the system conditions of Eq. (18).

Similarly, if one lets $p_j^t = p_j^{t*}$ for j, t, and substitutes these values into Eq. (19), then one obtains

$$\sum_{i=1}^{m} \sum_{k=1}^{o} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial q_{ik}} - \pi_k \right] (q_{ik} - q_{ik}^*)$$

$$+ \sum_{i=1}^{m} \sum_{t=1}^{r} \left[\frac{\partial G_i(e_i^*, q_i^*)}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^{t*} h_{ij}^t \right] (e_i^t - e_i^{t*})$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left(p_j^{t*} - \lambda_{ij}^{t*} \right) \left(l_{ij}^t - l_{ij}^{t*} \right)$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left(l_{ij}^{t*} - h_{ij}^t e_i^{t*} \right) \left(\lambda_{ij}^t - \lambda_{ij}^{t*} \right) \geq 0,$$

$$\forall (q, e, l, \lambda) \in R_+^{mo + mr + 2mnr} \tag{A4}$$

which implies that Eq. (13) must hold for the firms. □

A2. Proof of Lemma 1

We will establish that $F(X)$ is monotone, i.e.

$$[F(X^1) - F(X^2)](X^1 - X^2) \geq 0, \qquad \forall X^1, X^2 \in K \tag{A5}$$

In view of the definition of $F(X)$ in the above model, Eq. (25) takes the form

$$\sum_{i=1}^{m} \sum_{k=1}^{o} \left\{ \left[\frac{\partial G_i(e_i^1, q_i^1)}{\partial q_{ik}} - \pi_k \right] - \left[\frac{\partial G_i(e_i^2, q_i^2)}{\partial q_{ik}} - \pi_k \right] \right\} (q_{ik}^1 - q_{ik}^2)$$

$$+ \sum_{i=1}^{m} \sum_{t=1}^{r} \left\{ \left[\frac{\partial G_i(e_i^1, q_i^1)}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^{t1} h_{ij}^t \right] - \left[\frac{\partial G_i(e_i^2, q_i^2)}{\partial e_i^t} + \sum_{j=1}^{n} \lambda_{ij}^{t2} h_{ij}^t \right] \right\} (e_i^{t1} - e_i^{t2})$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left[\left(p_j^{t1} - \lambda_{ij}^{t1} \right) - \left(p_j^{t2} - \lambda_{ij}^{t2} \right) \right] \left(l_{ij}^{t1} - l_{ij}^{t2} \right)$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} \left[\left(l_{ij}^{t1} - h_{ij}^t e_i^{t1} \right) - \left(l_{ij}^{t2} - h_{ij}^t e_i^{t2} \right) \right] \left(\lambda_{ij}^{t1} - \lambda_{ij}^{t2} \right)$$

$$+ \sum_{j=1}^{n} \sum_{t=1}^{r} \sum_{i=1}^{m} \left[\left(l_{ij}^{t0} - l_{ij}^{t1} \right) - \left(l_{ij}^{t0} - l_{ij}^{t2} \right) \right] \left(p_j^{t1} - p_j^{t2} \right). \tag{A6}$$

After combining and simplifying terms, Eq. (A6) reduces to

$$\sum_{i=1}^{m} \sum_{k=1}^{o} \left[\frac{\partial G_i(e_i^1, q_i^1)}{\partial q_{ik}} - \frac{\partial G_i(e_i^2, q_i^2)}{\partial q_{ik}} \right] (q_{ik}^1 - q_{ik}^2)$$

$$+ \sum_{i=1}^{m} \sum_{t=1}^{r} \left[\frac{\partial G_i(e_i^1, q_i^1)}{\partial e_i^t} - \frac{\partial G_i(e_i^2, q_i^2)}{\partial e_i^t} \right] (e_i^{t1} - e_i^{t2}). \tag{A7}$$

However, under the assumption that the utility functions are concave, we know that minus the gradient of the utility function is monotone; hence, the expression in Eq. (A7) must be greater than or equal to zero. □

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