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Electromagnetic properties of nucleons and hyperons in a Lorentz covariant quark model

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Abstract. We calculate magnetic moments of nucleons and hyperons and $N \to \Delta\gamma$ transition characteristics using a manifestly Lorentz covariant chiral quark approach for the study of baryons as bound states of constituent quarks dressed by a cloud of pseudoscalar mesons.

PACS. 12.39.Fe Chiral Lagrangians, 12.39.Ki Relativistic quark model, 13.40.Gp Electromagnetic form factors, 14.20.Dh Protons and neutrons, 14.20.Jn Hyperons

1 Introduction

The study of the magnetic moments of light baryons and of the $N \to \Delta\gamma$ transition represents an old and important problem in hadron physics. Many theoretical approaches – lattice QCD, QCD sum rules, Chiral Perturbation Theory (ChPT), various quark and soliton methods, techniques based on the solution of Bethe-Salpeter and Faddeev field equations, etc. – have been applied in order to calculate these quantities.

It should be stressed that analysis of the $N \to \Delta\gamma$ transition is of particular interest because it allows one to probe the structure of both the nucleon and $\Delta(1232)$-isobar and can help to shed light on their possible deformation. This reaction represents a crucial test for the various theoretical approaches. For example, naive quark models based on SU(6) symmetry, which model the nucleon and its first resonance as a spherically symmetric 3q-configurations, fail to correctly describe the electric G_{E2} and Coulomb G_{C2} quadrupole form factors, which vanish in such models in contradistinction with experiment. A comprehensive analysis of the $N \to \Delta\gamma$ transition has been performed, e.g. in Refs. [1].

There are a number of interesting problems which we address in the present paper:

i) if one believes that both valence and sea-quark effects are important in the description of the electromagnetic properties of light baryons, then how large is the contribution of the meson-cloud;
ii) what is the physics required to correctly predict the $M1$ amplitude for the $N \to \Delta$ transition, which is considerably underestimated in constituent quark models;
iii) what input is needed in order to explain the experimental data for $E2/M1$ and $C2/M1$.

To possibly answer the above questions we use a Lorentz covariant chiral quark model recently developed in Ref. [2,3]. The approach is based on a non-linear chirally symmetric Lagrangian, which involves constituent quarks and the chiral (pseudoscalar meson) fields as the effective degrees of freedom. In a first step, this Lagrangian can be used to perform a dressing of the constituent quarks by a cloud of light pseudoscalar mesons and other heavy states using the calculational technique of infrared dimensional regularization (IDR) of loop diagrams. Then within a proper chiral expansion, we calculate the dressed transition operators which are relevant for the interaction of the quarks with external fields in the presence of a virtual meson cloud. In a following step, these dressed operators are used to calculate baryon matrix elements. Note, that a simpler and more phenomenological quark model which was based on the similar ideas of the dressing of the constituent quarks by a meson cloud has been developed in Refs. [4].

We proceed as follows. In Sec. II, we discuss basic notions of our approach. We derive a chiral Lagrangian motivated by baryon ChPT [5], and formulate it in terms of quark and mesonic degrees of freedom. Next, we use this Lagrangian to perform a dressing of the operators of constituent quarks by a cloud of light pseudoscalar mesons and by other heavy states. Then we discuss the calculation of matrix elements of dressed quark operators between baryons states using a specific constituent quark model [6]-[8]. In Sec. III, we apply our approach to the study of magnetic moments of light baryons and to the properties of the $N \to \Delta\gamma$ transition. In Sec. IV we present a summary of our results.

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2 Approach

The chiral quark Lagrangian $\mathcal{L}_{qU}$ (up to order p^4), which dynamically generates the dressing of the constituent quarks by mesonic degrees of freedom, consists of two primary pieces $\mathcal{L}_q$ and $\mathcal{L}_U$:

$$\mathcal{L}_{qU} = \mathcal{L}_q + \mathcal{L}_U\,,\;\; \mathcal{L}_q \,=\, \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \mathcal{L}_q^{(4)} + \cdots\,,$$
$$\mathcal{L}_U = \mathcal{L}_U^{(2)} + \cdots\,. \tag{1}$$

The superscript (i) attached to $\mathcal{L}_{q(U)}^{(i)}$ denotes the low energy dimension of the Lagrangian. The detailed form of the chiral Lagrangian can be found in Ref. [2,3]. Here for transparency we display only leading terms

$$\mathcal{L}_U^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle\,, \mathcal{L}_q^{(1)} = \bar q[i\not{D} - m + \frac{1}{2}g\not{u}\gamma^5]q\,, \tag{2}$$

where q is the quark field, u_μ is the vielbein chiral field, the couplings m and g denote the quark mass and axial charge in the chiral limit. The other notations are specified in [2,3].

Any bare quark operator (both one- and two-body) can be dressed by a cloud of pseudoscalar mesons and heavy states in a straightforward manner by use of the effective chirally-invariant Lagrangian $\mathcal{L}_{qU}$. To calculate the electromagnetic transitions between baryons we project the dressed electromagnetic quark operator $J_{\mu,\,\rm em}^{\rm dress}$ between the corresponding baryon states. The master formula is:

$$\langle B(p')|\, J_{\mu,\,\rm em}^{\rm dress}(q)\,|B(p)\rangle \,=\, (2\pi)^4\,\delta^4(p'-p-q)$$
$$\times\; \bar u_B(p')\biggl\{\gamma_\mu\,F_1^B(q^2) \,+\, \frac{i}{2\,m_B}\,\sigma_{\mu\nu}q^\nu\,F_2^B(q^2)\biggr\}u_B(p)$$
$$= (2\pi)^4\,\delta^4(p'-p-q)[M_\mu^V(q^2) + M_\mu^T(q^2)] \tag{3}$$

where

$$M_\mu^V(q^2) = \sum_{q=u,d,s} f_D^q(q^2)\,\langle B(p')|\, j_{\mu,q}^{\rm bare}(0)\,|B(p)\rangle \tag{4}$$
$$M_\mu^T(q^2) = \sum_{q=u,d,s} i\,\frac{q^\nu}{2\,m_q}\,f_P^q(q^2)\,\langle B(p')|\, j_{\mu\nu,q}^{\rm bare}(0)\,|B(p)\rangle$$

$B(p)$ and $u_B(p)$ are the baryon state and spinor, m_B is the baryon mass. The explicit forms of $f_D^q(q^2)$ and $f_P^q(q^2)$ are given in Ref. [2]. Here we focus on the diagonal $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions (the extension to the nondiagonal transitions and transitions involving higher spin states like the $\Delta(1232)$ isobar is straightforward). $F_1^B(q^2)$ and $F_2^B(q^2)$ are the Dirac and Pauli baryon form factors. In the master equation (3) we express the matrix elements of the dressed quark operator in terms of the matrix elements of the bare operators. In our application we deal with the bare quark operators for vector $j_{\mu,q}^{\rm bare}(0)$ and tensor $j_{\mu\nu,q}^{\rm bare}(0)$ currents defined as

$$j_{\mu,q}^{\rm bare}(0) \,=\, \bar q(0)\,\gamma_\mu\,q(0)\,,\;\; j_{\mu\nu,q}^{\rm bare}(0) \,=\, \bar q(0)\,\sigma_{\mu\nu}\,q(0)\,. \tag{5}$$

Equations (3)-(5) contain our main result: we perform a model-independent factorization of the effects of hadronization and confinement contained in the matrix elements of the bare quark operators $j_{\mu,q}^{\rm bare}(0)$ and $j_{\mu\nu,q}^{\rm bare}(0)$ and the effects dictated by chiral symmetry (or chiral dynamics) which are encoded in the relativistic form factors $f_D^q(q^2)$ and $f_P^q(q^2)$. Due to this factorization the calculation of $f_D^q(q^2)$ and $f_P^q(q^2)$, on one side, and the matrix elements of $j_{\mu,q}^{\rm bare}(0)$ and $j_{\mu\nu,q}^{\rm bare}(0)$, on the other side, can be done independently. In particular, in a first step we derived a model-independent formalism based on the ChPT Lagrangian, which is formulated in terms of constituent quark degrees of freedom, for the calculation of $f_D^q(q^2)$ and $f_P^q(q^2)$. Note, that the separate bare and the meson cloud contributions to the baryon form factors are defined as

$$M_\mu^{\rm bare}(q^2) = M_\mu^{V;0}(q^2)\,, \tag{6}$$
$$M_\mu^{\rm cloud}(q^2) = M_\mu^V(q^2) - M_\mu^{V;0}(q^2) + M_\mu^T(q^2)$$

where

$$M_\mu^{V;0}(q^2) = \sum_{q=u,d,s} f_D^q(0)\,\langle B(p')|\, j_{\mu,q}^{\rm bare}(0)\,|B(p)\rangle \tag{7}$$

and $f_D^q(0) = e_q$ is the quark charge.

The calculation of the matrix elements of the bare quark operators $j_{\mu,q}^{\rm bare}$ and $j_{\mu\nu,q}^{\rm bare}$ can then be relegated to quark models based on specific assumptions about hadronization and confinement with taking into account of certain constraints dictated by Lorentz and gauge invariance and chiral symmetry [2,3]. Here we consistently employ the relativistic three-quark model (RQM) [6]-[8] to compute such matrix elements. The RQM was previously successfully applied for the study of properties of baryons containing light and heavy quarks [7]-[8]. The main advantages of this approach are: Lorentz and gauge invariance, a small number of parameters, and modelling of effects of strong interactions at large (~ 1 fm) distances. A preliminary analysis of the electromagnetic properties of nucleons has been performed in Ref. [6] where the effects of valence quarks have been consistently taken into account. Here we extend this analysis to the case of hyperons as well as to the $N \to \Delta\gamma$ transitions and we include meson-cloud effects. The basic idea of RQM is to model the coupling of baryons to their valence quarks using the three-quark currents [9] which are also extensively used in QCD sum rules [10]. For the octet baryon states one can write two possible currents (vector J_B^V and tensor J_B^T), while for the decuplet states only one current – J_D. In particular, for the proton and Δ^+-isobar the currents look as:

$$J_p^V = \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 d^{a_1}u^{a_2}C\gamma_\mu u^{a_3}$$
$$J_p^T = \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 d^{a_1}u^{a_2}C\sigma_{\mu\nu} u^{a_3} \tag{8}$$
$$J_{\Delta^+}^\mu = \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(d^{a_1}u^{a_2}C\gamma^\mu u^{a_3} + 2u^{a_1}u^{a_2}C\gamma^\mu d^{a_3})\,,$$

where a_i are the color indices and C is the charge conjugation matrix.

3 Physical applications

In this section we consider the application of our technique to the problem of magnetic moments of light baryons and the static characteristics of the $N \to \Delta\gamma$ transition. We calculate the contributions of both valence and sea-quarks to these quantities using the approach discussed above. In particular, we present results for magnetic moments of light baryons (Table 1) and properties of $N \to \Delta\gamma$ transition (Table 2): magnetic, electric and Coulombic form factors G_{M1}, G_{E2} and G_{C2}, helicity amplitudes $A_{3/2}$ and $A_{1/2}$ at zero recoil, ratios of multipoles EMR $= E2/M1 = -G_{E2}/G_{M1}$ and CMR $= C2/M1 = -G_{C2}/G_{M1}$, transition dipole $\mu_{N\Delta}$ and quadrupole $Q_{N\Delta}$ moment, $\Delta^+ \to p + \gamma$ decay width. In Tables 1 and 2 we show the contributions both of the valence quarks ($3q$) and of the meson cloud and compare the total results with data [11].

As stressed above, for the octet states there exist two possible choices for the three-quark current: vector and tensor. A preliminary analysis (see also Ref. [6,3]) showed that these two types of currents give practically the same (or at least very similar) results in the case of the static properties of light baryons, *e.g.*, magnetic moments. This result is easily understood because the vector and tensor currents of the baryon octet become degenerate in the nonrelativistic limit. Also, the magnetic moments of light baryons are dominated by the nonrelativistic contributions, with relativistic corrections being of higher order and small. This explains why the simple nonrelativistic quark approaches work so well in the description of the magnetic moments of light baryons. Therefore, in order to distinguish between the two types of currents of the baryon octet we need to examine quantities which are dominated by relativistic effects. Two such quantities are the well known ratios $E2/M1$ and $C2/M1$ of the multipole amplitudes characterizing the $N \to \Delta\gamma$ transition. Here we find that the sole use of vector and tensor currents gives *opposite* results for the signs of these ratios. In particular, the use of the pure vector current for the proton gives reasonable results for $E2/M1$ and $C2/M1$ both with a *correct* (negative) sign, while the use of the pure tensor current yields ratios with *wrong* (positive) sign. Therefore, the study of the ratios $E2/M1$ and $C2/M1$ allows one to select the appropriate current for the description of the bound-state structure of the baryon octet (nucleons and hyperons). It is interesting to note that in the QCD sum rule method [10] dealing with current quarks the vector current structure is also preferred. This choice originally gave an explanation of the nucleon mass, while the use of the tensor current yields a suppression of the nucleon mass due to the "bad" chiral properties of this type of the three-quark current. We would like to stress, however, that this preference of the vector current for the description of the baryon octet in our approach and in QCD sum rules is apparently just coincidental because here we are dealing with constituent quarks instead of current quarks. For the EMR and CMR ratios we present our predictions at zero recoil ($Q^2 = 0$) and at the finite value $Q^2 = 0.06$ GeV^2 (recently the A1 Collaboration at Mainz [12] measured these quantities at this kinematic point). Our predictions are in good agreement with the experimental data of the LEGS Collaboration at Brookhaven [13] and of the GDH, A1 and A2 Collaborations at Mainz [12,14].

Table 1. Magnetic moments of light baryons.

	Bare (3q)	Meson cloud	Total	Data [11]
μ_p	2.614	0.179	2.793	2.793
μ_n	-1.634	-0.279	-1.913	-1.913
μ_Λ	-0.579	-0.034	-0.613	-0.613 ± 0.004
μ_{Σ^+}	2.423	0.148	2.571	2.458 ± 0.010
μ_{Σ^-}	-0.960	-0.223	-1.183	-1.160 ± 0.025
μ_{Ξ^0}	-1.303	-0.082	-1.385	-1.250 ± 0.014
μ_{Ξ^-}	-0.567	0.012	-0.555	-0.651 ± 0.003
$\lvert\mu_{\Sigma^0\Lambda}\rvert$	1.372	0.245	1.617	1.61 ± 0.08
$\mu_{N\Delta}$	2.984	0.354	3.338	3.642 ± 0.019

Table 2. Results for the N $\to \Delta\gamma$ transition. Notations: EMR and CMR in % (subscripts 0 and 0.06 mean the values of $Q^2 = 0$ and 0.06 GeV^2), A_i in 10^{-3} GeV$^{-1/2}$, $Q_{N\Delta}$ in fm^2, $\Gamma_{\Delta\to N\gamma}$ in MeV.

	Bare (3q)	Meson cloud	Total	Data [11]
EMR_0	-3.41	0.31	-3.10	-2.5 ± 0.5
$\mathrm{EMR}_{0.06}$	-3.34	0.33	-3.01	-2.28 ± 0.29
CMR_0	-3.95	0.26	-3.69	
$\mathrm{CMR}_{0.06}$	-5.13	0.35	-4.78	-4.81 ± 0.27
$A_{1/2}$	-110.0	-14.3	-124.3	-135 ± 6
$A_{3/2}$	-219.4	-25.3	-244.7	-250 ± 8
G_{E2}	0.125	0.002	0.127	0.137 ± 0.012
G_{M1}	3.655	0.434	4.089	4.460 ± 0.023
G_{C2}	0.144	0.007	0.151	
$Q_{N\Delta}$	-0.098	-0.001	-0.099	-0.108 ± 0.009
$\mu_{N\Delta}$	2.984	0.354	3.338	3.642 ± 0.019
$\Gamma_{\Delta\to N\gamma}$	0.49	0.12	0.61	0.58 - 0.67

4 Summary

In this paper we have calculated the magnetic moments of light baryons as well as the $N \to \Delta\gamma$ transition properties using a manifestly Lorentz covariant chiral quark approach to the study of baryons as bound states of constituent quarks dressed by a cloud of pseudoscalar mesons. Our main results are:

- The contribution of the meson cloud to the static properties of light baryons is up to 20%, which is consistent with the perturbative nature of their contribution and, together with the relativistic corrections, helps to explain how the 30% shortfall in the SU(6) prediction is ameliorated;

- We get a reasonable description for the dipole magnetic moment $\mu_{N\Delta}$ due to the enhancement of the valence quark contribution [3];

- The multipole ratios EMR and CMR are sensitive to the choice of the proton current: vector J_p^V or tensor J_p^T. The use of a pure vector current J_p^V gives a reasonable description of the data. The pure tensor current J_p^T gives results for EMR and CMR with the wrong (positive) sign. However, a small admixture of the tensor current is possible, and forthcoming experiments can give a strong restriction on the mixing parameter of such currents [3];

- We presented a detailed analysis of the light baryon observables all of which are in good agreement with experimental data.

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