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Chiral Perturbation Theory and Nucleon Polarizabilities*

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Abstract

Compton scattering offers in principle an intriguing new window on nucleon structure. Existing experiments and future programs are discussed and the state of theoretical understanding of such measurements is explored.

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1 Introduction

One of the attractive features about low energy Compton scattering from hadronic systems is that one can make contact with the meaning of such measurements within the context of classical physics. This has the not insignificant consequence that you can explain to your friends outside particle/nuclear physics what you are doing and why it is of interest! The basic idea here is that of *polarizability*—*i.e.* the deformation induced in a system in the presence of a quasistatic electric/magnetic field.[1] Thus in the presence of an electric field $\vec{E}_0$ a system of charges will deform and an electric dipole moment $\vec{p}$ will result. The electric polarizability α_E is simply the constant of proportionality between the applied field and the induced dipole moment

$$\vec{p} = 4\pi\alpha_E \vec{E}_0 \tag{1}$$

Similarly in the presence of a magnetizing field $\vec{H}_0$ a magnetic dipole moment $\vec{m}$ is generated, with the proportionality characterized by the magnetic polarizability β_M

$$\vec{m} = 4\pi\beta_M \vec{H}_0 \tag{2}$$

Obviously then the polarizabilities are fundamental properties of the hadronic system and probe its underlying structure.

In thinking of how to measure such properties for an elementary particle it is useful to think initially of a simple atomic system such as a hydrogen atom. Then for each such atom there is generated an energy shift

$$\delta U = -\frac{1}{2}4\pi\alpha_E E_0^2 - \frac{1}{2}4\pi\beta_M H_0^2 \tag{3}$$

due to the interaction of the dipole with the fields.[1] Imagining a box filled with a gas characterized by N atoms per unit volume, the energy per unit volume of the system of fields plus atoms will be given by

$$\begin{aligned} u &= \frac{1}{2}(E_0^2 + H_0^2) - \frac{N}{2}(4\pi\alpha_E E_0^2 + 4\pi\beta_M H_0^2) \\ &\equiv \frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2 \end{aligned} \tag{4}$$

where

$$E = E_0(1 - N4\pi\alpha_E), \qquad H = H_0(1 - N4\pi\beta_M) \tag{5}$$

are the effective fields in the gas and

$$\epsilon = 1 + N4\pi\alpha_E, \qquad \mu = 1 + N4\pi\beta_M \tag{6}$$

are the dielectric constant, magnetic permeability respectively. Using the expression

$$n = \sqrt{\epsilon\mu} = 1 + N2\pi(\alpha_E + \beta_M) \tag{7}$$

which relates the index of refraction n to the dielectric constant and magnetic permeability we see that measurement of n for our hypothetical gas would provide a sensitive probe for the sum of electric and magnetic polarizabilities of its individual constituents.

In our case, however, we wish to detect the polarizabilities of an elementary particle—in particular a neutron or proton—and such an index of refraction experiment is not feasible. Nevertheless a means by which to perform such a measurement is suggested by an alternative way by which to express the index of refraction—in terms of the forward Compton scattering amplitude $f_k(0)$[2]

$$n = 1 + N\frac{2\pi}{\omega^2} f_k(0) \tag{8}$$

The connection with the polarizability can be made by use of quantum mechanics. At lowest order for a charged particle one has the Hamiltonian

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 \tag{9}$$

which leads to the well-known Thomson amplitude

$$\mathrm{Amp}_{\mathrm{Comp}} = -\frac{e^2}{m}\hat{\epsilon} \cdot \hat{\epsilon}'^* \tag{10}$$

for Compton scattering. Adding on components of the Hamiltonian corresponding to the polarizabilities—Eq. 3—one finds the modified Compton amplitude

$$\mathrm{Amp}_{\mathrm{Comp}} = \hat{\epsilon} \cdot \hat{\epsilon}'^*(-\frac{e^2}{m} + 4\pi\alpha_E\omega\omega') + \vec{k} \times \hat{\epsilon} \cdot \vec{k}' \times \hat{\epsilon}'^* 4\pi\beta_M \tag{11}$$

In the forward direction then one has

$$\mathrm{Amp}_{\mathrm{Comp}}(\theta = 0) = \hat{\epsilon} \cdot \hat{\epsilon}' \left[-\frac{e^2}{m} + 4\pi(\alpha_E + \beta_M)\omega^2 \right] = 4\pi f_k(0) \tag{12}$$

Then for a *neutral* system, we have from Eq. 8

$$n = 1 + N2\pi(\alpha_E + \beta_M) \tag{13}$$

in agreement with Eq. 7. However, we now have a procedure—Compton scattering—which enables the general extraction of the polarizabilities of an elementary system. Indeed, calculating the cross section we find in general

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left\{\frac{1}{2}(1+\cos^2\theta)+\frac{m}{\alpha}\omega\omega'[\frac{1}{2}(\alpha_E+\beta_M)(1+\cos\theta)^2\right.\\ &+ \left.\frac{1}{2}(\alpha_E-\beta_M)(1-\cos\theta)^2]+\mathcal{O}(\omega^4)\right\} \qquad (14)\end{aligned}$$

so that by measurement of the angular distribution one can extract α_E, β_M experimentally. This program has been carried out for the proton at SAL and MAMI, yielding (here and below all numerical values for polarizabilities will be quoted in the units 10^{-4} fm^3)[3]

$$\alpha_E^p = 12.1 \pm 0.8 \pm 0.5, \qquad \beta_M^p = 2.1 \mp 0.8 \mp 0.5 \qquad (15)$$

In the case of the neutron experiments involving the deuteron are presently underway at both SAL and MAMI, but the best existing number comes from the analysis of a transmission experiment involving neutron scattering on Pb. The idea here is that the existence of a charged particle polarizes the neutron, which then acts back on the charged particle, generating a $1/r^4$ interaction. This leads to a term linear in k in a transmission cross section which can be extracted via careful measurment of its energy dependence. The quoted numbers which arise thereby are[4]

$$\alpha_E^n = 12.6 \pm 1.5 \pm 2.0, \qquad \beta_M = 3.2 \mp 1.5 \mp 2.0 \qquad (16)$$

although the quoted uncertainties are almost certainly too low.[5]

An important contraint in these measurements (and the reason that errors in the case of the magnetic polarizability are accompanied by $\mp$) arises from unitarity and causality—*i.e.* the feature that in the forward direction the Compton scattering amplitude can be represented in terms of a disperson relation involving the total photoabsorption cross section. Using a single subtraction, as indicated from Regge arguments, we have

$$\begin{aligned}\mathrm{Re}f_1(\omega) &= -\frac{e^2}{m}+\frac{\omega^2}{2\pi}\int_0^\infty\frac{d\omega'\sigma_{\rm tot}(\omega')}{\omega'^2-\omega^2}\\ i.e. \qquad & \alpha_E+\beta_M = \frac{1}{2\pi^2}\int_0^\infty\frac{d\omega'}{\omega'^2}\sigma_{\rm tot}(\omega') \qquad (17)\end{aligned}$$

If we include target and beam polarization, things become more interesting. Writing, again in the forward direction,

$$\mathrm{Amp}_{\rm Comp} = 4\pi\left[f_1(\omega)\hat{\epsilon}\cdot\hat{\epsilon}^{'*}+i\omega f_2(\omega)\vec{\sigma}\cdot\hat{\epsilon}^{'*}\times\hat{\epsilon}\right] \qquad (18)$$

then the corresponding dispersion relation for $f_2(\omega)$ is expected to be *unsubtracted*! There exists also in this case a low energy theorem, first given by Gell-Mann, Goldberger and Low in terms of the anomalous magnetic moment of the target.[6] Thus we write

$$f_2(\omega) = -\frac{e^2\kappa^2}{2m^2} + \gamma\omega^2 + \mathcal{O}(\omega^4) \tag{19}$$

where γ is the "spin polarizability," whose relation to the classical properties of the nucleon is a bit more obscure than in the case of its unpolarized analogs, but which can be related in a handwaving fashion to the Faraday effect. Defining $\sigma_\pm(\omega)$ as the photoabsorption cross sections with parallel, antiparallel spin and target helicities, the corresponding dispersion relation yields[7]

$$\begin{aligned} \frac{\pi e^2\kappa^2}{2m^2} &= \int_0^\infty \frac{d\omega}{\omega}(\sigma_+(\omega) - \sigma_-(\omega)) \\ \gamma &= \frac{1}{4\pi^2}\int_0^\infty \frac{d\omega}{\omega^3}(\sigma_+(\omega) - \sigma_-(\omega)) \end{aligned} \tag{20}$$

Here the first expression is the Drell-Hearn-Gerasimov sum rule, while the second provides a dispersive probe of the spin polarizability.

A number of challenges on the experimental front remain

i) more precise determination of the neutron polarizability, either by repeating the ORNL measurement or via $d(\gamma,\gamma)$ studies.

ii) accumulating experimental data utilizing polarization in order to check the DHG and spin polarizability sum rules.[1]

iii) extending the existing measurements in the regime of virtual Compton scattering—$N(e,e'\gamma)N$—in order to provide a probe of the local polarizability structure.

[1] It should be noted in this regard that a possible problem already exists in that if one looks at the isovector component of the DHG sum rule one finds

$$\frac{\pi e^2\kappa_n\kappa_p}{m^2} = +15\mu b \tag{21}$$

vs.

$$-\int_0^\infty \frac{d\omega}{\omega}(\sigma_-(\omega) - \sigma_+(\omega)) = -39\mu b \tag{22}$$

where the dispersive input has been provided by a multipole analysis of existing single pion photoproduction data and model-dependent assumptions about the multipion production.

iv) for later use it should be noted that use of the single pion photoproduction multipole analysis yields a predicted spin polarizability[8]

$$\gamma \approx -1 \times 10^{-4}\mathrm{fm}^4 \tag{23}$$

This then is as far as one can go by means of essentially model independent analysis. In the next section we address the question of how well existing theoretical pictures of the nucleon can confront present and future measurements.

2 Theoretical Approaches

Of course, in addition to having a basic grasp of the underlying physics it is important to attempt a theoretical understanding of the nucleon system and its relation to Compton physics. In this regard, a first approach which one might employ is that of a simple constituent quark model. The idea here is that one can use well-known sum rules for the electric and magnetic polarizabilities[9]

$$\begin{aligned}
\alpha_E &= \frac{\alpha}{3m} < r_p^2 > + 2\alpha \sum_{n\neq 0} \frac{|<n|\sum_i e_i z_i|0>|^2}{E_n - E_0} \\
\beta_M &= -\frac{\alpha}{2m} < (\sum_i e_i \vec{r}_i)^2 > - \frac{\alpha}{6} < \sum_i e_i^2 \frac{r_i^2}{m_i} > \\
&+ 2\alpha \sum_{n\neq 0} \frac{|<n|\sum_i e_i \frac{\sigma_{iz}}{2m_i}|0>|^2}{E_n - E_0}
\end{aligned} \tag{24}$$

Then in a simple harmonic oscillator model of the nucleon there are only two parameters—the quark mass and the oscillator frequency. The former is determined in terms of the nucleon mass—$m = M/3$, while the latter is fixed by the proton charge radius—$\omega = 3/M < r_p^2 >$. There exist a number of problems with this approach

i) The basic scale of the polarizability is too large—$\alpha_E = 2\alpha/M\omega^2 \approx 35$.[9]

ii) The proton electric polarizability is predicted to be significantly larger than that of the neutron[10]

$$\alpha_E^p - \alpha_E^n \simeq \frac{\alpha}{3M} < r_p^2 > \approx 3.8 \tag{25}$$

in contradiction to the central values given in Eqs. 15,16

iii) There exists a large contribution to the magnetic polarizability from the $\Delta(1232)$ intermediate state—$\beta_M^\Delta \approx 12$—which must somehow be cancelled by an equally large diamagnetic term.[11]

At least some of these problems are cured by use of a cloudy bag model with its intrinsic pion cloud and this suggests that perhaps a better approach might be the use of chiral perturbative techniques right from the start, which we next review.

The lowest order chiral Lagrangian coupling pions and nucleons can be written as

$$\mathcal{L} = \bar{\Psi}(i \not{D} - M + \frac{1}{2} g_A \not{u} \gamma_5)\Psi \tag{26}$$

where (to lowest chiral order) g_A is the nucleon axial decay constant and M is the nucleon mass. The coupling to pions is provided by

$$U = u^2 = \exp(\frac{i}{F_\pi}\vec{\tau}\cdot\vec{\phi}_\pi) \tag{27}$$

while the vector field u_μ is given by

$$u_\mu = iu^\dagger \nabla_\mu U u^\dagger \tag{28}$$

The covariant derivative $D_\mu\Psi = \partial_\mu\Psi + \Gamma_\mu\Psi$ is given by the connection

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, u] - \frac{i}{2}u^\dagger(v_\mu + a_\mu)u - \frac{i}{2}u(v_\mu - a_\mu)u^\dagger \tag{29}$$

Even at this level this simple form has a number of phenomenological successes:

i) The Goldberger-Treiman relation—$Mg_A = F_\pi g_{\pi NN}$—is valid to the accuracy of a few percent.[12]

ii) In muon capture the prediction

$$\frac{g_P}{g_A} = \frac{2Mm_\mu}{m_\pi^2 + 0.9m_\mu^2} = 7.0 \tag{30}$$

is verified (although the recent TRIUMF radiative muon capture data seems to be at odds with this prediction.)[13]

iii) The Kroll-Ruderman theorem

$$E_{0+}^{\pi^\pm N} = \pm\frac{\sqrt{2}eg_A}{8\pi F_\pi} \tag{31}$$

has recently been verified in threshold charged pion photoproduction.[14]

One clearly wants to go to loop level in order to enforce unitarity and to provide a stringent test of these ideas. Loop diagrams introduce divergences, but these can, of course, be absorbed by empirical counterterm contributions, just as in the mesonic sector. However, there is one complication which arises for baryons. If one simply uses Eq. 26 and calculates the associated loop diagrams using relativistic perturbation theory, then one finds that a given loop diagram contributes to many different orders in the mass/momentum expansion (generically denotes by p)—one does not have consistent power counting. In order to remedy this problem, a Foldy-Wouthuysen transformation is performed,[15] so that one ends up with an expansion both in p/Λ_χ, where Λ_χ is the chiral scale, as well as in p/M, where M is the baryon mass. Of course, the classic procedure of Foldy and Wouthuysen is performed in the Hamiltonian picture, which it is more convenient for our purposes to utilize a Lagrangian framework, as we now show.

The procedure of Foldy and Wouthuysen is one where coupling between the large and small components of the baryon wavefunction is eliminated.[15] Thus writing, for example,

$$\Psi(x,t) \equiv e^{-iMt} \begin{pmatrix} N(x,t) \\ H(x,t) \end{pmatrix} \tag{32}$$

We can write the Lagrangian of the system in terms of upper, lower components N, H as

$$\mathcal{L} = \bar{N}AN + \bar{H}BN + \bar{N}\gamma_0 B^\dagger \gamma_0 H + \bar{H}CH \tag{33}$$

This form can be diagonalized via the definition

$$H' = H + C^{-1}BN \tag{34}$$

which yields

$$\mathcal{L} = \bar{N}(A - \gamma_0 B^\dagger \gamma_0 C^{-1} B)N + \bar{H}' C H' \tag{35}$$

The "heavy" components H' can now be integrated out, yielding an effective Lagrangian in terms of the "light" components N

$$\begin{aligned} W &= \int [dN][d\bar{N}][dH'][d\bar{H}'] \exp i \int d^4x \mathcal{L}(N, H') \\ &= \text{const.} \times \int [dN][d\bar{N}] \exp i \int d^4x \mathcal{L}_{eff}(N) \end{aligned} \tag{36}$$

Using this form an extensive series of calculations involving low energy nucleon electromagnetic interactions have been carried out by the group of Bernard, Kaiser and Meissner (BKM).[16] In particular, in the case of nucleon Compton scattering at one loop—$\mathcal{O}(p^3)$—order they find results

$$\alpha_E^p = \alpha_E^n = 10\beta_M^p = 10\beta_M^n = \frac{5e^2 g_A^2}{384\pi^2 F_\pi^2 m_\pi} = 12.2 \tag{37}$$

which are in remarkable agreement with experiment! As a reality check, it should be noted, however, that in the case of the spin polarizability the one loop prediction

$$\gamma = \frac{e^2 g_A^2}{96\pi^3 F_\pi^2 m_\pi^2} = 4.5 \times 10^{-4}\mathrm{fm}^4 \tag{38}$$

is opposite in sign to the value Eq. 23 given by the sum rule. Also, BKM together with Schmidt have extended their calculation to $\mathcal{O}(p^4)$, yielding[17]

$$\begin{aligned} \alpha_E^p &= 10.5 \pm 2.0 \qquad \alpha_E^n = 13.4 \pm 1.5 \\ \beta_M^p &= 3.5 \pm 3.6 \qquad \beta_M^n = 7.8 \pm 3.6 \end{aligned} \tag{39}$$

The error bars here are associated with the feature that BKM estimated the contribution of appropriate resonances, such as the $\Delta(1232)$ by integrating them out and including only their contribution to counterterms. Also there exist various uncertainties associated with higher order loop contributions.

However, while treating resonant contributions in this way is satisfactory in general, it is not so clear that this is appropriate for the $\Delta(1232)$ due to its strong coupling and its proximity to the nucleon. Thus I, together with Thomas Hemmert and Joachim Kambor, have developed a scheme whereby a chiral expansion can be performed consistently with the Δ included as a specific degree of freedom.[18] The difficulty here is that the Rarita-Schwinger representation of the spin 3/2 field has too many degrees of freedom—in addition to the desired spin 3/2 piece there exist *two* independent spin 1/2 sectors. Also, for each sector there is a large and a small component. We handled this problem by using projection operators to identify each piece. Thus we were able to represent the

Rarita-Schwinger field as a six component "spinor"

$$\Psi^\mu = e^{-iMt}\begin{pmatrix} \Delta^\mu_{\frac{3}{2}} \\ H^\mu_{\frac{3}{2}} \\ \ell^{1\mu}_{\frac{1}{2}} \\ h^{1\mu}_{\frac{1}{2}} \\ \ell^{2\mu}_{\frac{1}{2}} \\ h^{2\mu}_{\frac{1}{2}} \end{pmatrix} \tag{40}$$

Then one writes the spin 3/2 Lagrangian in terms of these six fields and integrates out all but the desired $\Delta^\mu_{\frac{3}{2}}$ component. There are a few technically challenging features such as taking the inverse of a 5×5 matrix, but basically this is just a repeat of what was done in the case of the Dirac—spin 1/2—field. What results is a generalized chiral expansion in terms of "small" quantities p, m_π and $\Delta \equiv m_\Delta - m_N$, which we denote generically by ϵ. There exist in general two kinds of additional contributions to the usual heavy baryon results. One is from the diagrams wherein the $\Delta(1232)$ appears as a simple pole, while the other is where the Δ contributes as part of a loop term. Using couplings determined empirically we find in this way at $\mathcal{O}(\epsilon^3)$[19]

$$\begin{aligned} \alpha_E &= 12.2(\mathrm{N-pole}) + 0(\Delta - \mathrm{pole}) + 4.2(\Delta - \mathrm{loop}) \\ \beta_M &= 1.2(\mathrm{N-loop}) + 7.2(\Delta - \mathrm{pole}) + 0.7(\Delta - \mathrm{loop}) \\ \gamma &= 4.6(\mathrm{N-loop}) - 2.4(\Delta - \mathrm{pole}) - 0.2(\Delta - \mathrm{loop}) \end{aligned} \tag{41}$$

Obviously in the case of the electric or magnetic polarizabilities the $\Delta(1232)$ contributions are large and destroy agreement with experiment, while in the case of the spin polarizability the corrections are significant and in the right direction but are not large enough to bring about agreement with the sum rule value. In any case it is clear that $\mathcal{O}(\epsilon^4)$ calculations are abolutely necessary, and these are underway.

Before leaving this section it should be noted that a recent analysis by the LEGS group of the world set of proton Compton scattering data has produced a number for the *backward* spin-polarizability γ_π[20]

$$\gamma_\pi = (-28.0 \pm 2.8 \pm 2.5) \times 10^{-4}\mathrm{fm}^4 \tag{42}$$

This quantity is simply the 180°-scattering analog of the usual spin polarizability. A direct measurement, of course, requires a polarized beam *and* target. However, the angular distribution of the unpolarized cross section is also sensitive to γ_π and that is how it was extracted. On the theoretical side the backward spin-polarizability is dominated by the anomaly contribution from the pion pole term, which alone yields a predicted effect

$$\gamma_\pi^{anomaly} = -44 \times 10^{-4}\mathrm{fm}^4 \tag{43}$$

which is an order of magnitude larger than the size of its forward scattering analog—γ—to which the pion pole diagram does not contribute. However, in addition to the pole term there are additional contributions from the usual N, Δ loop and pole diagrams, which tend to make the predicted value for γ_π somewhat smaller in magnitude but in basic agreement with the measured number[21]

$$\gamma_\pi^{\mathrm{theo}} = [-44(\mathrm{anomaly})+4.6(N-\mathrm{loop})+2.4(\Delta-\mathrm{pole})-0.2(\Delta-\mathrm{loop})]\times 10^{-4}\mathrm{fm}^4 \tag{44}$$

However, in this case not only additional theoretical work extending these results to $\mathcal{O}(\epsilon^4)$ will be required, but also direct experimental measurement using polarized beam and target in order to have real confidence in the measured number. Such experiments can be expected at MAMI, LEGS, as well as at the free electron laser backscattering facility now under development at Duke.

3 Virtual Compton Scattering

A new frontier in this area is represented by the subject of virtual Compton scattering (VCS). There are two different ways in which this can be manifested. One is to have a real incident photon but for the final photon to fragment into a Dalitz pair.[22] This corresponds to positive q^2 and will not be discussed here. Rather we concentrate on the case that the initial photon is produced in an electron scattering process, with $q^2 < 0$, but scatters from a target to a real final state photon. This sort of process leads to probes of nucleon structure via so called generalized polarizabilities, as we show below, and has generated approved experimental programs at MAMI, CEBAF and BATES. A significant theoretical interest has also developed in VCS, with many papers already having appeared.[23]

On the experimental side VCS offers a significant advantage over its usual Compton counterpart in that event rates possible with virtual photons are much enhanced. However, there is at the same time an associated cost in that the desired process is hidden in general behind a huge background due to Bethe-Heitler scattering, wherein the source of the final photon is simply bremsstrahlung from either the initial or final state electron. On the experimental side this means that typical measurements, which take place in "parallel kinematics" (*i.e.* zero angle ϕ between the lepton scattering plane and the hadronic scattering plane) must extract the interesting signal from a huge but calculable background flux. The finite size of the detectors at MAMI, which extend out to $\phi = \pm 22^\circ$, ameliorates some of this effect, but it remains a significant problem—the calculated generalized polarizability "signal" is only a very small component in a large Bethe-Heitler "background."[24] On the other hand, using the new and "portable" OOPS detectors, the BATES experiment will be able to employ perpendicular kinematics—$\phi = 90^\circ$—which puts the signal and background on a nearly even footing.[25] In any case, as theorists we can easily isolate the Bethe-Heitler from the VCS signal—

$$\mathrm{Amp}^{\mathrm{tot}}_{\mathrm{VCS}} = \mathrm{Amp}^{\mathrm{Bethe-Heitler}} + ie^2(\vec{\epsilon}_T \cdot \vec{M}_T + \frac{q^2}{\omega^2}\epsilon_z M_z) \qquad (45)$$

where we have separated the VCS component into longitudinal and transverse components and have used the shorthand

$$\epsilon_\mu = \bar{u}_{e'}\gamma_\mu u_e/q^2 \qquad (46)$$

It is then straightforward to identify twelve—four longitudinal and eight transverse—independent structure functions,

$$\begin{aligned}
\vec{\epsilon}_T \cdot \vec{M}_T &= \vec{\epsilon}\,' \cdot \vec{\epsilon}_T A_1 + \vec{\epsilon}\,' \cdot \hat{q}\vec{\epsilon}_T \cdot \hat{q}' A_2 \\
&+ i\vec{\sigma} \cdot \vec{\epsilon}\,' \times \vec{\epsilon}_T A_3 + i\vec{\sigma} \cdot \hat{q}' \times \hat{q}\vec{\epsilon}\,' \cdot \vec{\epsilon}_T A_4 \\
&+ i\vec{\sigma} \cdot \vec{\epsilon}\,' \times \hat{q}\vec{\epsilon}_T \cdot \hat{q}' A_5 - i\vec{\sigma} \cdot \vec{\epsilon}\,' \times \hat{q}'\vec{\epsilon}_T \cdot \hat{q}' A_6 \\
&- i\vec{\sigma} \cdot \vec{\epsilon}_T \times \hat{q}'\vec{\epsilon}\,' \cdot \hat{q} A_7 - i\vec{\sigma} \cdot \vec{\epsilon}_T \times \hat{q}\vec{\epsilon}\,' \cdot \hat{q} A_8 \\
M_z &= \vec{\epsilon}\,' \cdot \hat{q} A_9 + i\vec{\sigma} \cdot \hat{q}' \times \hat{q}\vec{\epsilon}\,' \cdot \hat{q} A_{10} \\
&+ i\vec{\sigma} \cdot \vec{\epsilon}\,' \times \hat{q} A_{11} + i\vec{\sigma} \cdot \vec{\epsilon}\,' \times \hat{q}' A_{12}
\end{aligned} \qquad (47)$$

For each structure function one expands

$$A_i = A_i^{\mathrm{Born}} + \text{generalized polarizabilities} \qquad (48)$$

where here A_i^{Born} signifies the nucleon pole diagrams with on-shell form factors, while the generalized polarizabilities have been defined by Guichon et al. in the low-energy approximation of including terms only up to linear order in the real photon energy ω'.[26]

Multipoles	S	Inter. St.	$(\rho' L', \rho L)S$
$L1 \times L1$	0,1	$\frac{1}{2}^-, \frac{3}{2}^-$	$P^{(01,01)S}$
$L1 \times E1$	0,1	$\frac{1}{2}^-, \frac{3}{2}^-$	$\hat{P}^{(01,1)S}$
$M1 \times M1$	0,1	$\frac{1}{2}^+, \frac{3}{2}^+$	$P^{(11,11)S}$
$L2 \times M2$	1	$\frac{3}{2}^-$	$P^{(01,12)1}$
$M1 \times L2$	1	$\frac{3}{2}^+$	$P^{(11,02)1}$
$M1 \times L0$	1	$\frac{1}{2}^+$	$P^{(11,00)1}$
$M1 \times E2$	1	$\frac{1}{2}^+$	$\hat{P}^{(11,2)1}$

Table 1: Generalized polarizabilities as defined by Guichon et al.[26]

As summarized in Table 1, there exist ten such terms—three of which are spin-independent and seven requiring polarization for their measurement. However, it was subsequently demonstrated by Drechsel et al. that a consistent treatment of crossing symmetry and charge conjugation invariance yields four additional constraints—one for S=0 and three for S=1.[27] In the spin-independent case one can then eliminate $\hat{P}^{(01,1)0}$ and write everything in terms of just the two *generalized* polarizabilities

$$\begin{aligned} \alpha_E(\bar{q}) &= -\frac{e^2}{4\pi}\sqrt{\frac{3}{2}}P^{(01,01)0}(\bar{q}) \\ \beta_M(\bar{q}) &= -\frac{e^2}{4\pi}\sqrt{\frac{3}{8}}P^{(11,11)0}(\bar{q}) \end{aligned} \tag{49}$$

which reduce to the usual quantities in the real photon limit $\bar{q} \to 0$. The meaning of these quantities is also clear. When one applies an electric or magnetic field to a charged system the induced electric or magnetic dipole moments are in general functions of position, whose Fourier transform in $\bar{q}$ are just the generalized polarizabilities given above. In the spin-dependent case it is not so clear which generalized spin-polarizbilities to eliminate, but in any case charge conjugation invariance implies that there exist only four independent $\bar{q}$-dependent quantities.

In the first—unpolarized—experiments what will be measured are three independent combinations

$$\begin{aligned} P_{LL}(\bar{q}) &= -2\sqrt{6}MG_E(Q_0^2)P^{(01,01)0}(\bar{q}) \\ P_{TT}(\bar{q}) &= \frac{3}{2}G_M(Q_0^2)\left[2\omega_0 P^{(01,01)1}(\bar{q}) + \sqrt{2}\bar{q}^2(P^{(10,12)1}(\bar{q}) + \sqrt{3}\hat{P}^{(01,1)1}(\bar{q}))\right] \end{aligned}$$

$$
\begin{aligned}
P_{LT}(\bar{q}) &= \sqrt{\frac{3}{2}}\frac{M\bar{q}}{\sqrt{Q_0^2}}G_E(Q_0^2)P^{(11,11)0}(\bar{q}) + \frac{\sqrt{3}\sqrt{Q_0^2}}{2\bar{q}}G_M(Q_0^2) \\
&\times \left[P^{(11,00)1}(\bar{q}) + \frac{\bar{q}^2}{\sqrt{2}}P^{(11,02)1}(\bar{q})\right] \qquad (50)
\end{aligned}
$$

The leading terms here are the LL and LT pieces so that one's initial sensitivity will be to the electric and magnetic generalized polarizabilities given in Eq. 49. A particularly interesting test here will be the measurement of the magnetic polarizability $\beta_M(\bar{q})$, for which loop effects in heavy baryon chiral perturbation theory predict a temporary rise (!) at low $\bar{q}^2$ in contradistinction to quark models which predict a steady decrease due to form factor effects.[28] The news here then is good and bad. On the one hand the technique of virtual Compton scattering offers a new and potentially high resolution probe of nucleon structure. On the other hand the new information is available of significant background due to Bethe-Heitler and nucleon Born diagrams and very high precision experiments will be required in order to harvest this potentially rich crop of data.

4 Conclusions

During the past decade Compton scattering has become an important tool for the probing of hadron structure, and this will no doubt continue into the new millenium. Indeed there remain significant challenges for both experimentalists and theorists in this regard. In the former case, the challenges will be to improve upon existing measurements of both proton and (especially) neutron polarizabilities, as well as to utilize polarized beam and target technology to provide new spin-polarizability measurements. Experiments yielding polarized photoabsorption cross sections should also become available in order to test the various sum rule predictions. Virtual Compton scattering programs at the electron machines will provide a rich lode of new generalized polarizability information. On the theoretical side the challenge will be to understand this rich trove of information. One frontier is to provide chiral calculations at $\mathcal{O}(\epsilon^4)$. A second is to relate this information to dispersive approaches which provide the high mass contributions to sum rules for these quantities. Finally, I personally would like to develop also a *physical* understanding for the meaning of each of these generalized polarizabilities so that we can communicate with colleagues what the excitement of these measurements really means.

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