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# Low Energy Tests of Chiral Symmetry<sup>1</sup>

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## Abstract

The present status of low energy tests of chiral invariance via chiral perturbation theory is reviewed, both in the meson and baryon sectors, and future prospects are discussed.

## 1 Introduction

When I was a student back in the 1960's, the goal of particle and nuclear physicists was to seek the fundamental laws of nature, and one of the requirements of such a "fundamental" law was that it be renormalizable. Well now it's 1996 and we've learned a few things in these three decades. One is that nonrenormalizable effective field theories can be just as if not more useful than their renormalizable siblings in certain situations. One of these is the case of QCD, where we have what we feel is a correct model of nature. However, it is written in terms of the "wrong" degrees of freedom (quarks and gluons rather than hadrons) and is impossible to solve because of its strong coupling and inherent nonlinearity. Much more useful in the arena of low energy physics is an effective Lagrangian, which is written in terms of experimental degrees of freedom—mesons and baryons—and which encodes the symmetries of the underlying QCD interaction—specifically for our purposes chiral symmetry, which exists in the limit in which the quark mass can be taken as vanishing. This is a program which was begun in the 60's with the effective two-derivative Lagrangian[1]

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4} \text{Tr} D_\mu U D^\mu U^\dagger + \frac{F_\pi^2}{4} \text{Tr} 2B_0 m (U + U^\dagger) \quad (1)$$

which describes the interaction of the Goldstone fields  $\phi_i, i = 1..8$ . Here  $F_\pi = 92.4$  MeV is the pion decay constant,  $m$  is the quark mass matrix,  $B_0$  is a phenomenological constant and

$$U = \exp\left(\frac{i}{F_\pi} \sum_{j=1}^8 \lambda_j \phi_j\right) \quad (2)$$

is a nonlinear function of the fields which transforms as  $LUR^\dagger$  under chiral rotations. When used at tree level this interaction is rather successful in predicting low energy

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Coefficient	Value	Origin
$L_1$	$0.65 \pm 0.28$	$\pi\pi$ scattering
$L_2$	$1.89 \pm 0.26$	and
$L_3$	$-3.06 \pm 0.02$	$K_{\ell 4}$ decay
$L_5$	$2.3 \pm 0.2$	$F_K/F_\pi$
$L_9$	$7.1 \pm 0.3$	$\langle r_\pi^2 \rangle$
$L_{10}$	$-5.6 \pm 0.3$	$\pi \rightarrow e\nu\gamma$

Table 1: Gasser-Leutwyler coefficients  $L_i$  and the means of determination.

interactions.[1] For example, expanding to fourth order in the fields one finds the well-known Weinberg predictions for  $\pi\pi$  scattering lengths[2]

$$a_0^0 = \frac{7m_\pi}{32\pi F_\pi^2} \quad a_0^2 = -\frac{m_\pi}{16\pi F_\pi^2} \quad (3)$$

which are confirmed by experiment. However, in order to go further and include loop effects one must include additional four-derivative pieces into the effective Lagrangian, with ten phenomenological constants  $L_i, i = 1\dots 10$  which can be determined experimentally as shown by Gasser and Leutwyler,[3] yielding values for these parameters as given in Table 1.

## 2 Mesons

As mentioned above, the one loop chiral expansion has been carried out in the case of Goldstone boson interactions by many investigators. As emphasized by Weinberg,[4] this is basically an expansion in energy-momentum with a scale parameter  $\Lambda_\chi \sim 1$  GeV, so that one is entitled to quit at one loop provided that energies are small compared to this scale. It is this for this reason that this is called chiral *perturbation* theory ( $\chi$ pt). Although ten seems at first like a large number of parameters,  $\chi$ pt is very predictive and the extent to which these predictions are valid is at some level a probe of the validity of QCD itself. This subject has been extensively reviewed in many places[5] and there is in general very good agreement between predicted and measured quantities as shown in Table 2 The one area here where there is a possible problem has to do with the required relationship between the charged pion polarizability  $\alpha_E^\pi$  and the axial structure function  $h_A$  in radiative pion decay[8]

$$\alpha_E^\pi = \frac{\alpha h_A}{\sqrt{2}F_\pi m_\pi} \quad (4)$$

The chirally required value for the polarizability— $2.8 \times 10^{-4} \text{ fm}^3$ —is at variance with the value found at Serpukov via radiative pion scattering[6] but not with that found

Reaction	Quantity	Theory	Expt.
$\gamma \rightarrow \pi^+\pi^-$	$\langle r_\pi^2 \rangle$ (fm <sup>2</sup> )	0.44 (input)	$0.44 \pm 0.02$
$\gamma \rightarrow K^+K^-$	$\langle r_K^2 \rangle$ (fm <sup>2</sup> )	0.44	$0.34 \pm 0.05$
$\pi^+ \rightarrow e^+\nu_e\gamma$	$h_V(m_\pi^{-1})$	0.027	$0.029 \pm 0.017$
	$h_A/h_V$	0.46 (input)	$0.46 \pm 0.08$
$K^+ \rightarrow e^+\nu_e\gamma$	$(h_A + h_V)(m_\pi^{-1})$	0.038	$0.043 \pm 0.003$
$\pi^+ \rightarrow e^+\nu_e e^+e^-$	$r_A/h_V$	2.6	$2.6 \pm 0.6$
$\gamma\pi^+ \rightarrow \gamma\pi^+$	$(\alpha_E + \beta_M)(10^{-4})$ fm <sup>3</sup>	0	$1.4 \pm 3.1$
	$\alpha_E(10^{-4})$ fm <sup>3</sup>	2.8	$6.8 \pm 1.4$ [6]
			$2.2 \pm 1.1$ [7]
$K \rightarrow \pi e^+\nu_e$	$\xi = f_-(0)/f_+(0)$	-0.13	$-0.20 \pm 0.08$
	$\lambda_+$ (fm <sup>2</sup> )	0.067	$0.065 \pm 0.005$
	$\lambda_0$ (fm <sup>2</sup> )	0.040	$0.050 \pm 0.012$

Table 2: Comparison between chiral predictions and experimental values for parameters in the Goldstone sector.

at SLAC in  $\gamma\gamma \rightarrow \pi^+\pi^-$ . [7] This is clearly an area which deserves further study and work in this regard is presently underway at DAΦNE, Fermilab and Mainz.

### 3 Baryons

In the case of pion-nucleon interactions, chiral perturbative calculations can also be performed. However, things are much less clean for reasons which will become clear. One begins as before with the simplest  $\pi N$  Lagrangian having chiral symmetry

$$\mathcal{L}_{\pi N} = \bar{N}(i \not{D} - m_N + \frac{g_A}{2} \not{\psi} \gamma_5)N \quad (5)$$

where  $D_\mu$  is a covariant derivative,  $g_A$  is a coupling constant to be determined, and  $u_\mu = iu^\dagger \nabla_\mu U u^\dagger$  with  $u^2 = U$ . Expanding to lowest order we find

$$\mathcal{L}_{\pi N} = \bar{N}(i \not{\partial} - m_N - ig_A \vec{\tau} \cdot \vec{A} \gamma_5 - \frac{g_A}{F_\pi} \vec{\tau} \cdot \nabla \vec{\phi} + \dots)N \quad (6)$$

so that  $g_A$  is to be identified with the axial coupling in neutron beta decay. Also we see that chiral symmetry requires the Goldberger-Treiman relation between  $g_A$  and the  $\pi NN$  coupling constant [9]

$$F_\pi g_{\pi NN} = m_N g_A \quad (7)$$

Using the best present values we have

$$1201 \text{ MeV} = 92.4 \text{ MeV} \times 13.0 \quad \text{vs.} \quad 939 \text{ MeV} \times 1.26 = 1183 \text{ MeV} \quad (8)$$

$\pi N \rightarrow \pi N$	Scattering lengths, $\sigma_{\pi N}$
$\pi N \rightarrow \pi\pi N$	LET's, $\pi\pi$ Scattering lengths
$\gamma N \rightarrow \gamma N$	Polarizabilities, DHG sum rule
$\gamma N \rightarrow \pi N$	LET's
$\gamma^* N \rightarrow \pi N$	LET's, $g_A(q^2)$
$\gamma N \rightarrow \pi\pi N$	Chiral loop effects in $\pi^0\pi^0$

Table 3: Examples of nucleon reactions which have been examined via  $\chi$ pt.

and the agreement to better than two percent strongs confirms the validity of chiral invariance in the nucleon sector. A second probe in this arena arises from the feature that the nucleon matrix element of the axial current also includes a pion pole contribution, leading to a prediction that in the muon capture process one requires[10]

$$r_P = \frac{g_P(q^2 = -0.9m_\mu^2)}{g_A(q^2 = -0.9m_\mu^2)} = \frac{2m_N m_\mu}{m_\pi^2 + 0.9m_\mu^2} = 7.0 \quad (9)$$

which can be experimentally checked. Present results are

$$r_P = \begin{cases} 7.4 \pm 2.0[11] & \mu^- \text{ capture } ^3\text{He} \\ 6.5 \pm 2.4[12] & \mu^- \text{ capture H} \\ 10 \pm 1[13] & \text{radiative } \mu^- \text{ capture H} \end{cases} \quad (10)$$

Obviously the discrepancy in the case of the radiative capture needs to be further explored.

In order to go further one requires loop corrections, just as in the mesonic case. A problem arises here that for the nucleons one has an additional dimensionful parameter— $m_N$ —which is the same size as the chiral scale, which makes the entire renormalization procedure doubtful. This problem can be gotten around, however, by using so-called heavy baryon methods, which are equivalent to the use of a Foldy-Wouthuysen transformation and which make a consistent power counting scheme possible. Of course, renormalization introduces new low energy constants (six, *e.g.* at  $\mathcal{O}(p^2)$ ) but nevertheless this program has been carried out, predominantly by the group of Bernard, Kaiser, and Meissner,[14] and applications are reported in many systems: One area of particular interest here is that of pion photoproduction. In this case for charged production the feature that the pion derivative in Eq. 6 is covariant leads to the Kroll-Ruderman predictions for the  $E_{0+}$  (electric dipole) multipole at threshold

$$E_{0+}^{\text{th}} = \begin{cases} \sqrt{2}D(1 - \frac{3}{2}\mu + \mathcal{O}(\mu^2)) & \pi^+ n \\ -\sqrt{2}D(1 - \frac{1}{2}\mu + \mathcal{O}(\mu^2)) & \pi^- p \end{cases} \quad (11)$$

where  $D = eg_{\pi NN}/8\pi m_N = 23.9 (\times 10^{-3}/m_\pi)$  and  $\mu \equiv m_\pi/m_N$ . In this case theory and experiment are in good agreement, as shown below. However, the experimental

	theory	expt.
$E_{0+}^{\text{th}}(\pi^+n)$	26.3	$27.9 \pm 0.5[15]$ $28.8 \pm 0.7[16]$
$E_{0+}^{\text{th}}(\pi^-p)$	-31.3	$-31.4 \pm 1.3[15]$ $-31.2 \pm 1.2[17]$

Table 4: Threshold values of  $E_{0+}$  for charged pion photoproduction ( $\times 10^{-3}/m_\pi$ ).

	theory	expt.
$E_{0+}(\pi^0p)(\times 10^{-3}/m_\pi)$	-1.2	$-1.31 \pm 0.08[20]$ $-1.3 \pm 0.5 \pm 0.6[21]$
$P_1/ \vec{q} (\pi^0p)(\times \text{GeV}^{-2})$	0.480	$0.47 \pm 0.01[20]$ $0.41 \pm 0.03[21]$

Table 5: Threshold parameters for neutral pion photoproduction.

numbers quoted are from old emulsion measurements and results from the recent SAL experiment are eagerly awaited.

In the case of neutral pion photoproduction, things are more interesting. In this case the venerable Low Energy Theorems (LET) for the threshold  $E_{0+}$  multipole

$$E_{0+}^{\text{th}} = \begin{cases} -D(\mu - \frac{1}{2}\mu^2(3 + \kappa_p) + \mathcal{O}(\mu^3)) = -2.3 & \pi^0p \\ -D(\frac{1}{2}\mu^2\kappa_n + \mathcal{O}(\mu^3)) = +0.5 & \pi^0n \end{cases} \quad (12)$$

were shown in 1991 to be incorrect due inappropriate assumptions concerning analyticity.[18] New loop contributions at  $\mathcal{O}(\mu^2)$

$$\Delta E_{0+}^{\text{th}} = -D\mu^2\left(\frac{m_N}{4F_\pi}\right)^2 \quad (13)$$

are large and destroyed the agreement which appeared to exist between the original LET predictions and preliminary  $\pi^0p$  experiments at both Mainz and at Saclay. However, there has been a good deal of recent activity. On the theoretical side, Bernard et al. have performed an analysis at  $\mathcal{O}(p^4)$ . [18] In doing so they require the values of two counterterms. Estimating these via  $\Delta, \rho, \omega$  dominance, they predict a value for  $E_{0+}$  in good agreement with new experiments performed at both MAMI and SAL. This may be somewhat accidental, as the convergence of the series appears slow— $E_{0+} = C(1 - 1.26 + 0.59 + \dots)$ . However, it has been pointed out that the P-wave calculations do not suffer from this slow convergence, yielding what should be a very solid prediction

$$\frac{1}{|\vec{q}|}P_1 \equiv (M_{1+} - M_{1-} + 3E_{1+})$$

	theory	expt.
$\alpha_E^p$	10.5	$11.6 \pm 0.6 \pm 0.6$ [25]
$\beta_M^p$	3.5	$2.6 \mp 0.6 \mp 0.6$ [25]
$\alpha_E^n$	13.4	$12.6 \pm 1.5 \pm 2.0$ [26]
$\beta_M^n$	7.8	$3.2 \mp 1.5 \mp 2.0$ [26]

Table 6: Experimental values of electric and magnetic polarizabilities compared to chiral predictions at  $\mathcal{O}(p^4)$ . (All  $\times 10^{-4} \text{ fm}^3$ )

$$= \frac{D}{M_N} \left[ 1 + \kappa_p - \mu \left( 1 + \frac{1}{2} \kappa_p - \frac{g_{\pi NN}^2 (10 - 3\pi)}{48\pi} \right) + \dots \right] = 0.480 \text{ GeV}^{-2} \quad (14)$$

There may be a remaining problem in the size of the  $E_{1+}$  multipole,[21] but this involves a significant theoretical cancellation.

A second arena of activity is that of Compton scattering. In this case recent precise measurements of both the proton and neutron polarizabilities have been performed yielding values as shown below. As can be observed, the electric polarizability of the neutron is comparable to but slightly larger than its proton counterpart. This is interesting since a valence quark model cannot produce such a result, yielding instead a prediction[22]

$$\alpha_E^p - \alpha_E^n = \frac{\alpha}{3M_N} (\langle r_p^2 \rangle - \langle r_n^2 \rangle) \approx 4.6 (\times 10^{-4} \text{ fm}^3) \quad (15)$$

On the other hand a chiral expansion of the polarizability does not have this problem, starting off from equal values for the proton and neutron[23]

$$\alpha_E^p = \alpha_E^n = 10\beta_M^p = 10\beta_M^n = \frac{e^2 g_A^2}{192\pi^3 F_\pi^2 M_N} \left( \frac{5\pi}{2\mu} + \dots \right) = 12.4 (\times 10^{-4} \text{ fm}^3) \quad (16)$$

A full  $\mathcal{O}(p^4)$  calculation of both electric and magnetic polarizabilities, with counterterms evaluated via resonance dominance yields very satisfactory results as shown Table 6, although again the convergence of the series is in doubt.[24] Similarly a good deal of work has been done in the case of polarized Compton scattering, both experimentally and theoretically, but we do not have the space to discuss this here.

## 4 Conclusions

Obviously in a short report such as this the discussion above can provide only a brief introduction to the multitude of work which is presently underway. Additional areas of activity include

- i)  $\gamma p \rightarrow \pi^0 \pi^0 p$  for which the significant near threshold cross section observed via the TAPS group at Mainz is being confronted with the pion loop corrected amplitude. The loop correction is large but it is too early to tell whether it agrees with the experimental findings.
- ii)  $\gamma^* p \rightarrow \pi^0 p$  for which  $k^2 \simeq -0.1 \text{ GeV}^2$  data from both NIKHEF and MAMI seems to contradict at least some of the chiral perturbative predictions. However, this value of momentum transfer is probably above the range where one can expect agreement, and we await the lower  $k^2$  data to be taken at Mainz.
- iii)  $\pi N \rightarrow \pi \pi N$  for which previous analysis in order to extract the  $\pi\pi$  scattering lengths has utilized the Olsson-Turner parameterization, which is inconsistent with a modern chiral analysis.
- iv)  $K_{\ell 4}$  measurements at DAΦNE and elsewhere should be able to resolve the question concerning the use of standard vs. generalized chiral perturbation theory.
- v)  $\vec{\gamma} \vec{p} \rightarrow \gamma p$  measurements at CEBAF and elsewhere combined with precise resonance photoproduction data should shed light on the validity of the DHG sum rule.
- vi) on the theoretical side it is important to include effects of the  $\Delta(1240)$  as a baryonic degree of freedom and not just as a heavy state which contributes to a counterterm. Work to this end is underway and should appear soon.

Overall then I hope that I have been able to convey the sense that the area of low energy tests of the standard model via chiral perturbation theory is an active and exciting one, with plenty of work remaining to be done on both the theoretical *and* experimental sides.

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## References

- [1] See, *e.g.* S. Gasiorowicz and D.A. Geffen, Rev. Mod. Phys. **41**, 531 (1969).
- [2] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).
- [3] J. Gasser and H. Leutwyler, Ann. Phys. (NY) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).
- [4] S. Weinberg, Physica **A96**, 327 (1979).
- [5] See, *it e.g.* B.R. Holstein, Int. J. Mod. Phys. **A7**, 7873 (1992).



- [6] Yu. M. Antipov et al., Phys Lett. **B121**, 445 (1983).
- [7] D. Babusci et al., Phys. Lett. **B277**, 158 (1992).
- [8] J.F. Donoghue and B.R. Holstein, Phys. Rev. **D48**, 137 (1993).
- [9] M.L. Goldberger and S.B. Treiman, Phys. Rev. **110**, 1478 (1958).
- [10] For a modern treatment see V. Bernard, N. Kaiser, and U.G. Meissner, Phys. Rev. **D50**, 6899 (1994).
- [11] L.B. Auerbach et al., Phys. Rev. **138**, B127 (1965); D.R. Clay et al., Phys. Rev. **140**, B586 (1965).
- [12] G. Bardin et al., Phys. Lett. **B104**, 320 (1981).
- [13] Preliminary TRIUMF result, M. Blecher (private communication).
- [14] For a useful summary, see V. Bernard, N. Kaiser and U.G. Meissner, Int. J. Mod. Phys. **E4**, 193 (1995).
- [15] J.P. Burg, Ann. De Phys. (Paris) **10**, 363 (1965).
- [16] M.J. Adamovitch et al., Sov. J. Nucl. Phys. **2**, 95 (1966).
- [17] E.L. Goldwasser et al., *Proc. XII Int. conf. n High Energy Physics, Dubna, 1964*, ed. Ya.-A.Smorodinsky, Atomizdat, Moscow (1966).
- [18] V. Bernard, J. Gasser, N. Kaiser and U.G. Meissner, Phys. Lett. **B268**, 291 (1991).
- [19] V. Bernard, N. Kaiser, and U.G. Meissner, to be published in Zeit. fur Physik.
- [20] M. Fuchs et al., Phys. Lett. **B368**, 20 (1996).
- [21] J.C. Bergstrom et al., Phys. Rev. **C53**, R1052 (1996).
- [22] B.R. Holstein, Comm. Nucl. Part. Phys. **20**, 301 (1992).
- [23] V. Bernard, N. Kaiser and L.G. Meissner, Phys. Rev. Lett. **67**, 1515 (1991); Nucl. Phys. **B373**, 364 (1992).
- [24] V. Bernard, N. Kaiser, A. Schmidt, and U.G. Meissner, Phys. Lett. **B3119**, 269 (1993).
- [25] F.J. Federspiel et al., Phys. Rev. Lett. **67**, 1511 (1991); E.L. Hallin et al., Phys. Rev. **C48**, 1497 (1993); A. Zeiger et al., Phys. Lett. **B278**, 34 (1992).
- [26] J. Schmiedmyer et al., Phys. Rev. Lett. **66**, 1015 (1991).