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# Virtual Compton Scattering off the Nucleon in Chiral Perturbation Theory

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## Abstract

We investigate the spin-independent part of the virtual Compton scattering (VCS) amplitude off the nucleon within the framework of chiral perturbation theory. We perform a consistent calculation to third order in external momenta according to Weinberg's power counting. With this calculation we can determine the second- and fourth-order structure-dependent coefficients of the general low-energy expansion of the spin-averaged VCS amplitude based on gauge invariance, crossing symmetry and the discrete symmetries. We discuss the kinematical regime to which our calculation can be applied and compare our expansion with the multipole expansion by Guichon, Liu and Thomas. We establish the connection of our calculation with the generalized polarizabilities of the nucleon where it is possible.

11.30.Rd, 12.39.Fe, 13.60.Fz, 14.20.Fh

## I. INTRODUCTION

Recently, there has been greatly increased activity in the field of virtual Compton scattering (VCS) off the nucleon both on the experimental [1–5] as well as on the theoretical [6–11] side. At the c.w. electron accelerator MAMI (Mainz) the A1 collaboration has taken initial data [2], and proposals for similar experiments have been developed at CEBAF (Newport News) [3,4] or are being prepared at MIT-Bates [5]. Compared with ordinary (real) Compton scattering, VCS off the nucleon offers a much greater variety of experimental possibilities, because the response of the hadronic system to the electromagnetic probe can be investigated by independently varying the energy and the momentum of the initial state photon and thus allows for probing nucleon structure in both transverse and longitudinal modes over a wide kinematic range, covering both the perturbative and nonperturbative regimes. In the former [1], one can describe the VCS process in terms of a perturbation series using the interaction vertices of QCD with quarks and gluons as explicit degrees of freedom. Studies of this perturbative regime of QCD require high resolution. Using an electromagnetic probe, this means that one has to perform an experiment wherein the momentum transfer to the hadronic system mediated by a virtual photon is large, so that we are in the kinematic range of deep inelastic electron scattering. On the other hand, a long history of experimental facts suggests that the effective degrees of freedom of a hadronic system at low energies are not quarks and gluons but baryons and mesons or - in our special case - nucleons and pions. If one performs an electron scattering experiment which does not involve large momentum transfer to the hadronic system, one probes the confined phase of strongly interacting matter and does not resolve its underlying quark structure—our paper will deal with this nonperturbative phase of a hadronic system. (Another interesting line of work is probe the regime *between* the perturbative and nonperturbative phases. It is not clear at which momentum transfer such a “phase transition” will take place, and this will no doubt be addressed by future experiments. )

Despite much effort, even in the nonperturbative regime there remain many questions to be investigated in order to complete our picture of the nucleon and other hadrons. Important quantities which probe the compositeness of a system are its electromagnetic polarizabilities, which characterize the response of the system to an external electric or magnetic field [12]. In real Compton scattering, the electric and magnetic polarizabilities of the nucleon —  $\alpha_0$  and  $\beta_0$  — appear as the first model-dependent coefficients beyond the low-energy theorem (LET) of Low [13] and Gell-Mann and Goldberger [14], and have been measured both for the proton and the neutron [15]. These polarizabilities have also been calculated within various models (for an overview see, *e.g.*, [16,17]). The LET of real Compton scattering has recently been extended to include virtual photons in [6,8], and the generalizations of the polarizabilities were defined in [6], where also a first prediction for these “generalized polarizabilities” was obtained within the framework of a non-relativistic quark model. These results were refined in [9] to include recoil corrections. In [7] an effective Lagrangian approach including nucleon and two-pion resonance mechanisms was used to determine the generalized electric and magnetic polarizability as a function of the initial photon momentum —  $\alpha(|\vec{q}|)$  and  $\beta(|\vec{q}|)$ . A very recent field theoretical calculation [10] investigates the spin-independent polarizabilities within the framework of the linear sigma model. We will herein utilize the technique of heavy baryon chiral perturbation theory and compare our results with these

earlier calculations.

It is well-known that the nonperturbative region can in general be successfully described in terms of the interaction of baryon and pseudoscalar meson degrees of freedom. In the last decade a new approach, chiral perturbation theory [18,19], was developed which describes the low-energy regime of QCD in terms of these effective degrees of freedom (pions and nucleons) while simultaneously requiring the symmetries of the underlying gauge theory and has yielded remarkable results. It was first applied to the sector of pseudoscalar mesons (see, *e.g.*, [20] for a pedagogical introduction) and then extended to case of pion-nucleon interactions [21]. The most recent version, which is known as the “heavy baryon formulation” of chiral perturbation theory [22–24], uses techniques, which are well-known from heavy quark calculations. It allows a consistent power counting scheme to be developed, which was not possible with the former relativistic formulation. Real Compton scattering and the polarizabilities of the nucleon were among the first calculations to be performed in the nucleon sector of chiral perturbation theory [23,25–27], and the experimental verification of those predictions provides an important test of its validity.

In this paper we extend these predictions to the VCS case and point out the strengths as well as the limitations of our approach. After a discussion in section II of the kinematics and the amplitude structure for VCS, we will in section III briefly touch upon the basic ingredients of heavy baryon chiral perturbation theory and then proceed to calculate the tree and loop diagrams to third order in the chiral expansion with respect to external momenta —  $O(p^3)$ . In section IV we analyze the low-energy structure of the VCS amplitude, which follows from general principles like gauge invariance, crossing symmetry and the discrete symmetries [8,11], and predict the structure-dependent constants for the *spin-independent*<sup>1</sup> part of the VCS amplitude, which are beyond the predictive power of such a general approach. Finally, in a concluding section V we discuss the connection of those structure constants to the generalized polarizabilities defined in [6] and relate the two approaches.

## II. KINEMATICS AND CHIRAL EXPANSION

We begin our discussion of VCS off the nucleon by specifying our notation for the process

$$\gamma^*(\varepsilon^\mu, q^\mu) + N(p_i^\mu) \rightarrow \gamma(\varepsilon'^{\mu}, q'^\mu) + N(p_f^\mu). \quad (1)$$

Here the nucleon four-momenta in the initial and final state are denoted by  $p_i^\mu = (E_i, \vec{p}_i)$  and  $p_f^\mu = (E_f, \vec{p}_f)$ , respectively. Since we will not in this paper discuss the component of the VCS amplitude which depends on nucleon spin, we have omitted indices for the nucleon spin states. The initial (final) state photon is characterized by its four-momentum  $q^\mu = (\omega, \vec{q})$  ( $q'^\mu = (\omega', \vec{q}')$ ) and polarization vector  $\varepsilon^\mu = (\varepsilon^0, \vec{\varepsilon})$  ( $\varepsilon'^\mu = (\varepsilon'^0, \vec{\varepsilon}')$ ). Whereas the final state photon is assumed to be real,

$$q'^2 = \omega'^2 - \vec{q}'^2 = 0, \quad (2)$$

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<sup>1</sup>The analogous spin-dependent calculation will be described in a subsequent publication.

the initial state photon is taken to be space-like, *i.e.*

$$q^2 = -Q^2 < 0. \quad (3)$$

Since our discussion refers to an electron scattering experiment, wherein the virtual photon is the exchanged particle between the electron and the hadronic current, we can write the polarization vector of the virtual photon as

$$\varepsilon_\mu = e \bar{u}_{e'} \gamma_\mu u_e / q^2, \quad (4)$$

where  $u_e$  and  $\bar{u}_{e'}$  are the initial and final state electron Dirac spinors,  $\gamma_\mu$  is a Dirac matrix (see, *e.g.*, [28]) and  $e = |e| \approx \sqrt{4\pi/137} > 0$ . Finally, we define the Lorentz invariant momentum transfer

$$t = (q - q')^2, \quad (5)$$

which will be useful in the following discussion. We evaluate the VCS amplitude in the c.m. system,

$$\begin{aligned} \vec{p}_i &= -\vec{q}, \\ \vec{p}_f &= -\vec{q}', \end{aligned} \quad (6)$$

and for our calculation we select a special frame of reference,

$$\begin{aligned} \hat{q} &= (0, 0, 1), \\ \hat{q}' &= (\sin \theta, 0, \cos \theta), \end{aligned} \quad (7)$$

wherein we define an orthonormal set of basis vectors:

$$\hat{e}_z = \hat{q}, \quad (8a)$$

$$\hat{e}_y = \hat{q} \times \hat{q}' / \sin \theta, \quad (8b)$$

$$\hat{e}_x = \hat{e}_y \times \hat{e}_z. \quad (8c)$$

The complete VCS amplitude can in general be expressed in terms of three independent kinematical quantities, *e.g.*,  $\omega'$ ,  $|\vec{q}'|$  and  $\theta$  [8]. Note that electron scattering kinematics implies  $\omega' < |\vec{q}'|$  which follows from energy conservation and  $Q^2 > 0$ . It turns out that our choice of variables facilitates the chiral expansion of the amplitudes, allowing chiral power counting [18] to be applied straightforwardly. The point here is that the chiral expansion is an expansion in terms of external momenta  $p$ , where in VCS on the nucleon the scale is set by the nucleon mass  $M$  and  $4\pi F$ . The pion decay constant has the value  $F = 92.4 \text{ MeV}$ . Now it is a crucial point to define what is meant by the term “external momentum”. In the following analysis we will take the view that *each component* of the four-momenta of the photons and of the three-momenta of the nucleons (see Eq. (6)) has to be “small” compared to  $M$ .<sup>2</sup> As a consequence we will count the terms  $\omega'/M$  and  $|\vec{q}'|/M$  to be of the same chiral

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<sup>2</sup>As a practical matter, since delta degrees of freedom are not included directly, the region of applicability is more appropriately  $\omega', |\vec{q}'| < m_\pi$ .

order, whereas a term like  $|\vec{q}|^2/M^2$  is suppressed by one chiral order with respect to the two preceding terms. This has important consequences: Let us, *e.g.*, consider the difference of the initial and final state photon energies, which occurs frequently in the subsequent heavy baryon calculation. From energy conservation one obtains

$$\omega - \omega' = \frac{\omega'^2}{2M} - \frac{|\vec{q}|^2}{2M} + O\left(\frac{r^4}{M^3}\right), \quad (9)$$

where  $r$  stands for either  $\omega'$  or  $|\vec{q}|$ . From Eq. (9) we infer that we must count the difference of the initial and final state photon energies as being one order higher than both energies themselves. Now imagine that we have evaluated a diagram which is of the chiral order  $O(p^n)$  and that part of this amplitude is proportional to  $\omega - \omega'$ . We find that this component of the amplitude can then be neglected because it contributes at the same order as a genuine  $O(p^{n+1})$  diagram. Specifically we will find below that the loop diagrams we wish to calculate are already  $O(p^3)$  and contain many pieces which are proportional to  $\omega - \omega'$ . Such contributions vanish from the beginning in the case of real Compton scattering, and we can also neglect such terms in our  $O(p^3)$  VCS calculation, because they would contribute at the same order as a  $O(p^4)$  contribution, which is not addressed in this paper. Finally, it is useful to present the expansion of  $q^2$  and  $t$  in terms of the three independent kinematical quantities,

$$q^2 = \omega'^2 - |\vec{q}|^2 + O\left(\frac{r^3}{M}\right), \quad (10a)$$

$$t = -\omega'^2 - |\vec{q}|^2 + 2\omega' |\vec{q}| \cos\theta + O\left(\frac{r^3}{M}\right), \quad (10b)$$

which we will also use below.

### III. CALCULATION IN HEAVY BARYON CHIRAL PERTURBATION THEORY

In this section we will extend the  $O(p^3)$  chiral heavy baryon calculation for real Compton scattering [23,27] to the case where  $q^2 < 0$ . In the case of real Compton scattering in the forward direction, a  $O(p^4)$  calculation has been performed in [25,26], which yields only small corrections to the  $O(p^3)$  result. Guided by this observation, we expect our  $O(p^3)$  calculation to provide a reasonable estimate in the kinematical region we are considering. The invariant amplitude for VCS can be written in the form [28]

$$\mathcal{M}_{VCS} = -ie^2 \varepsilon_\mu M^\mu = -ie^2 \varepsilon_\mu \varepsilon'_\nu{}^* M^{\mu\nu}. \quad (11)$$

We will utilize the Coulomb gauge,

$$\varepsilon'^\mu = (0, \vec{\varepsilon}') , \quad \vec{\varepsilon}' \cdot \vec{q}' = 0 \quad (12)$$

for the real photon. We decompose the space components of the virtual photon polarization vector  $\varepsilon^\mu$  into a purely transverse and a purely longitudinal part in our frame of reference,

$$\vec{\varepsilon} = \vec{\varepsilon}_T + \vec{\varepsilon}_L, \quad (13)$$

$$\vec{\varepsilon}_T = \varepsilon_x \hat{e}_x + \varepsilon_y \hat{e}_y, \quad (14)$$

$$\vec{\varepsilon}_L = \varepsilon_z \hat{e}_z. \quad (15)$$

Technically we calculate  $\mathcal{M}_{VCS}$  with an initial photon polarization vector

$$a^\mu = (0, \vec{a}), \quad \vec{a} = \vec{\varepsilon}_T + \frac{q^2}{\omega^2} \vec{\varepsilon} \cdot \hat{q} \hat{q} = \vec{\varepsilon}_T + \frac{q^2}{\omega^2} \varepsilon_z \hat{q} \quad (16)$$

with  $q \cdot a \neq 0$  [29], which simplifies the calculation by substantially reducing the number of Feynman diagrams which must be considered. The invariant amplitude can also be decomposed into a transverse and a longitudinal part. Using current conservation,

$$q_\mu M^\mu = 0, \quad (17)$$

Eq. (11) can be written as

$$\mathcal{M}_{VCS} = ie^2 \left( \vec{\varepsilon}_T \cdot \vec{M}_T + \frac{q^2}{\omega^2} \varepsilon_z M_z \right). \quad (18)$$

The transverse part of the invariant amplitude consists of eight independent structures. We will use a notation similar to that given in [27],

$$\begin{aligned} \vec{\varepsilon}_T \cdot \vec{M}_T &= \vec{\varepsilon}'^* \cdot \vec{\varepsilon}_T A_1 + \vec{\varepsilon}'^* \cdot \hat{q} \vec{\varepsilon}_T \cdot \hat{q}' A_2 \\ &+ i \vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \vec{\varepsilon}_T) A_3 + i \vec{\sigma} \cdot (\hat{q}' \times \hat{q}) \vec{\varepsilon}'^* \cdot \vec{\varepsilon}_T A_4 \\ &+ i \vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \hat{q}) \vec{\varepsilon}_T \cdot \hat{q}' A_5 + i \vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \hat{q}') \vec{\varepsilon}_T \cdot \hat{q}' A_6 \\ &- i \vec{\sigma} \cdot (\vec{\varepsilon}_T \times \hat{q}') \vec{\varepsilon}'^* \cdot \hat{q} A_7 - i \vec{\sigma} \cdot (\vec{\varepsilon}_T \times \hat{q}) \vec{\varepsilon}'^* \cdot \hat{q} A_8, \end{aligned} \quad (19)$$

where  $\sigma_i$  ( $i \in \{x, y, z\}$ ) are the Pauli spin matrices. (Note that in the special case of real Compton scattering time reversal invariance imposes two additional constraints on the amplitudes, as a result of which we find  $A_5 = A_7$  and  $A_6 = A_8$ .) Only two of these amplitudes —  $A_1$  and  $A_2$  — are independent of nucleon spin. For the longitudinal component one finds four independent structures,

$$\begin{aligned} M_z &= \vec{\varepsilon}'^* \cdot \hat{q} A_9 + i \vec{\sigma} \cdot (\hat{q}' \times \hat{q}) \vec{\varepsilon}'^* \cdot \hat{q} A_{10} \\ &+ i \vec{\sigma} \cdot \vec{\varepsilon}'^* \times \hat{q} A_{11} + i \vec{\sigma} \cdot \vec{\varepsilon}'^* \times \hat{q}' A_{12}, \end{aligned} \quad (20)$$

and in this case a single amplitude —  $A_9$  — is spin-independent. In this paper, we restrict our consideration to this spin-independent component of the matrix element,

$$\begin{aligned} \mathcal{M}_{VCS}^{non-spin} &= \mathcal{M}_{VCS} - \mathcal{M}_{VCS}^{spin} \\ &= ie^2 \left[ \vec{\varepsilon}'^* \cdot \vec{\varepsilon}_T A_1 + \vec{\varepsilon}'^* \cdot \hat{q} \vec{\varepsilon}_T \cdot \hat{q}' A_2 + \vec{\varepsilon}'^* \cdot \hat{q} \frac{q^2}{\omega^2} \varepsilon_z A_9 \right], \end{aligned} \quad (21)$$

which can also be obtained from the full amplitude  $\mathcal{M}_{VCS}$  as

$$\mathcal{M}_{VCS}^{non-spin} = \frac{1}{2} \text{tr}(\mathcal{M}_{VCS}). \quad (22)$$

We calculate  $\mathcal{M}_{VCS}^{non-spin}$  using the standard chiral perturbation theory Lagrangian in the heavy baryon formulation to  $O(p^3)$  in the nucleon sector [23,24],

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)}, \quad (23)$$

with

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}_v (i v \cdot D + g_A S \cdot u) N_v, \quad (24a)$$

$$\begin{aligned} \mathcal{L}_{\pi N}^{(2)} = & -\frac{1}{2M} \bar{N}_v \left\{ D \cdot D - (v \cdot D)^2 \right. \\ & \left. - \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} v^\rho S^\sigma \left[ f_+^{\mu\nu} (1 + 4c_6) + 2v^{(s),\mu\nu} (1 + 2c_7) \right] \right\} N_v, \end{aligned} \quad (24b)$$

$$\begin{aligned} \mathcal{L}_{\pi N}^{(3)} = & \frac{1}{2M^2} \bar{N}_v \left\{ \left[ f_+^{\mu\nu} \left( c_6 + \frac{1}{8} \right) + v^{(s),\mu\nu} \left( c_7 + \frac{1}{4} \right) \right] \right. \\ & \left. \times \varepsilon_{\mu\nu\rho\sigma} S^\sigma i D^\rho + \text{h.c.} \right\} N_v, \end{aligned} \quad (24c)$$

where  $\varepsilon_{0123} = 1$ . We have only kept those terms which contribute to a  $O(p^3)$  VCS calculation. In particular terms linear in the photon fields, which vanish in our gauge, have been omitted. Moreover, we note that  $\mathcal{L}_{\pi N}^{(3)}$  contributes only to the spin-dependent piece of the VCS amplitude and is irrelevant for the spin-independent part. The velocity-dependent field  $N_v$  is projected out from the nucleon Dirac spinor  $\Psi_N$ ,

$$N_v = \exp[iMv \cdot x] P_v^+ \Psi_N, \quad (25)$$

where the projection operator is given by

$$P_v^+ = \frac{1}{2} (1 + \not{v}). \quad (26)$$

where  $v^\mu$  is a velocity vector satisfying  $v^2 = 1$ . The covariant derivative is defined as

$$D_\mu = \partial_\mu + \Gamma_\mu - i v_\mu^{(s)}, \quad (27)$$

where

$$\Gamma_\mu = \frac{1}{2} \left\{ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger \right\}, \quad (28a)$$

$$u = U^{\frac{1}{2}}, \quad (28b)$$

$$U = \exp(i \vec{\tau} \cdot \vec{\pi} / F). \quad (28c)$$

In Eq. (28c)  $\vec{\tau}$  are the conventional Pauli isospin matrices, while  $\vec{\pi}$  represents the interpolating pion field. The field strength tensors are defined as

$$v_{\mu\nu}^{(s)} = \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)}, \quad (29a)$$

$$f_+^{\mu\nu} = u F_L^{\mu\nu} u^\dagger + u^\dagger F_R^{\mu\nu} u, \quad (29b)$$

$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i [r^\mu, r^\nu], \quad (29c)$$

$$F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i [l^\mu, l^\nu], \quad (29d)$$



and to the order considered the corresponding coefficients are related to the anomalous magnetic moments,

$$c_6 = \frac{\mu_V}{4} - \frac{1}{4} = \frac{1}{4}(\mu_p - \mu_n - 1) , \quad (30a)$$

$$c_7 = \frac{\mu_S}{2} - \frac{1}{2} = \frac{1}{2}(\mu_p + \mu_n - 1) . \quad (30b)$$

In our application the right- and left-handed currents,  $r_\mu$  and  $l_\mu$ , are the isovector part of the electromagnetic current and  $v_\mu^{(s)}$  is the isoscalar piece,

$$\begin{aligned} r_\mu &= -\frac{e}{2}A_\mu\tau_3 , \\ l_\mu &= -\frac{e}{2}A_\mu\tau_3 , \\ v_\mu^{(s)} &= -\frac{e}{2}A_\mu . \end{aligned} \quad (31)$$

The Pauli-Lubanski spin vector is given as

$$S^\mu = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu \quad (32)$$

with  $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ . Finally, we define the quantity

$$u_\mu = i \left\{ u^\dagger (\partial_\mu - ir_\mu) u - u (\partial_\mu - il_\mu) u^\dagger \right\} . \quad (33)$$

In the heavy baryon calculation we can write the initial and final nucleon momenta as

$$\begin{aligned} p_i^\mu &= Mv^\mu + t_i^\mu , \\ p_f^\mu &= Mv^\mu + t_f^\mu , \end{aligned} \quad (34)$$

where  $t_i^\mu$  and  $t_f^\mu$  denote off-shell momenta. We choose  $v^\mu = (1, 0, 0, 0)$  such that  $v \cdot a = 0$ , where the polarization vector  $a^\mu$  is given in Eq. (16).

In the pion sector we can restrict ourselves to the  $O(p^2)$  Lagrangian,

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4} \text{tr} [(\nabla_\mu U)^\dagger \nabla^\mu U] , \quad (35)$$

where

$$\nabla_\mu U = \partial_\mu U + \frac{1}{2}ieA_\mu[\tau_3, U] . \quad (36)$$

represents the covariant derivative. Here we have omitted the usual mass term, because we only need to consider pion-photon vertices generated by  $\mathcal{L}_{\pi\pi}^{(2)}$ . We also observe that power counting arguments, in principle, require the inclusion of a tree diagram with a  $\pi^0$  in the  $t$ -channel in our calculation (Fig. 1 (d)). For this diagram we need the lowest order Wess-Zumino Lagrangian [30,31], which has odd intrinsic parity and is  $O(p^4)$ . However, this diagram does not contribute to the spin-independent part of the VCS amplitude,  $\mathcal{M}_{VCS}^{non-spin}$ ,

but only to its spin-dependent counterpart,  $\mathcal{M}_{VCS}^{spin}$ . For this reason we will not discuss it in the following. Finally, we wish to emphasize that we do not require any additional diagrams compared to the calculation for real Compton scattering [23]. The complete set of diagrams we have to calculate is given in Figs. 1 ((a)  $s$ -channel, (b)  $u$ -channel, (c) contact diagram) and 2 (loop diagrams). In the following we will treat the tree and loop parts of the amplitudes separately,

$$A_i = A_i^{tree} + A_i^{loop}. \quad (37)$$

### A. Tree Diagrams

We expect the  $s$ - and  $u$ -channel Born diagrams (Fig. 1 (a,b)) and the contact diagram (Fig. 1 (c)) to generate the structure-independent part of the amplitude which was predicted for real Compton scattering by Low [13] and Gell-Mann and Goldberger [14]. In [23] it has been shown that the heavy baryon approach reproduces this LET, which must be the case for any consistent calculation based on gauge invariance, Lorentz invariance and crossing symmetry. For the case of VCS we obtain

$$A_1^{tree} = -\frac{1}{2M} (1 + \tau_3), \quad (38a)$$

$$A_2^{tree} = \frac{1}{2M^2} (1 + \tau_3) |\vec{q}|, \quad (38b)$$

$$A_9^{tree} = -\frac{1}{2M} (1 + \tau_3) + \frac{1}{2M^2} (1 + \tau_3) |\vec{q}| \cos \theta + \frac{1}{4M^2} (1 + \tau_3) \frac{|\vec{q}|^2}{\omega'}. \quad (38c)$$

Recently the LET has been extended to the case of virtual Compton scattering on the nucleon [6,8]. We find that our results at  $O(p^3)$  exactly agree with the predictions, which behave as  $1/M$  and  $1/M^2$  in [8]. However, the terms of order  $1/M^3$  which have also been derived in [8] as well as the  $q^2$  corrections to the zeroth order nucleon form factors, which are proportional to  $\frac{1}{M} \frac{1}{(4\pi F)^2}$ , [23] are beyond the predictive power of our  $O(p^3)$  chiral calculation.<sup>3</sup> We note that we do not generate any contributions to the spin-independent part of the VCS amplitude in the case of VCS on the neutron, because the photon only couples to the nucleon charge at tree-level in the spin-independent sector, but not to its magnetic moment.

### B. Loop Diagrams

In contrast to the three diagrams in Figs. 1 (a) – (c) discussed above we expect the loop diagrams in Fig. 2 (and the  $t$ -channel diagram from Fig. 1 (d)) to contribute to the

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<sup>3</sup>Such effects will be considered in a subsequent  $O(p^4)$  publication.

“structure-dependent,” and thus model-dependent terms beyond the LET. Qualitatively, this can be understood as follows: In the sense of Low’s method [32] the  $s$ - and  $u$ -channel diagrams of Figs. 1 (a) and (b) generate the most singular terms, whereas the contact interaction of Fig. 1 (c) is required by gauge invariance. On the other hand, the one-particle irreducible loop diagrams of Fig. 2 yield regular contributions to the VCS amplitude, which are of higher order in the external momenta. These contributions are not predicted by gauge invariance alone. Nevertheless, we will see that an extended low-energy expansion will enable us to derive *some* constraints for a calculation of the structure-dependent amplitude (see section IV).

In the calculation of the loop contribution to the amplitudes,  $A_i^{loop}$ , it turns out, as already pointed out above, that the number of loop diagrams is reduced to 9 by the choice of the gauge and of the velocity  $v$ . By power counting arguments, the only possibility of generating  $O(p^3)$  loop diagrams consists in using interactions from  $\mathcal{L}_{\pi N}^{(1)}$  and  $\mathcal{L}_{\pi\pi}^{(2)}$ . As a consequence the photon-nucleon interaction vanishes at lowest order,  $\mathcal{L}_{\gamma NN}^{(1)} = 0$ , and we need not consider any loop diagrams which contain this kind of interaction. Using the interactions of Eqs. (24a), (24b) and (35), we obtain the invariant amplitudes for the 9 diagrams in Fig. 2,  $\mathcal{M}_{VCS}^{(1)}$  to  $\mathcal{M}_{VCS}^{(9)}$ . The exact results are listed in the appendix . For some diagrams we cannot carry out the integrations over one or even two Feynman parameters analytically and could, in principle, proceed to evaluate these integrals numerically. However, since we want to establish a connection with the general low-energy expansion of the structure-dependent part of the VCS amplitude, we expand the expressions in Eq. (A2) in terms of the two external momenta,  $\omega'$  and  $|\vec{q}|$ , using Eqs. (9), (10a) and (10b). This expansion will scale with the mass of the particle propagating in the loop. Consequently, we will obtain a power series in  $r/m_\pi$  where  $r$  stands for  $\omega'$  or  $|\vec{q}|$ .<sup>4</sup> We could in principle extend the  $r/m_\pi$  expansion to an arbitrary given order within the framework of a  $O(p^3)$  calculation. In this context we want to point out that it is crucial not to confuse the expansion in  $r/m_\pi$  with the chiral expansion in  $r/M$ . The result of the chiral  $O(p^3)$  calculation, expanded to the order  $r^4$  then reads

$$\begin{aligned}
A_1^{loop} &= \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi} \left[ \frac{5}{96} \omega'^2 + \frac{1}{192} \omega' |\vec{q}| \cos \theta \right. \\
&\quad + \frac{17}{1920} \frac{1}{m_\pi^2} \omega'^4 + \frac{19}{1920} \frac{1}{m_\pi^2} \omega'^3 |\vec{q}| \cos \theta \\
&\quad - \frac{1}{384} \frac{1}{m_\pi^2} \omega'^2 |\vec{q}|^2 - \frac{1}{320} \frac{1}{m_\pi^2} \omega'^2 |\vec{q}|^2 \cos^2 \theta \\
&\quad \left. + \frac{1}{960} \frac{1}{m_\pi^2} \omega' |\vec{q}|^3 \cos \theta \right], \tag{39a} \\
A_2^{loop} &= \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi} \left[ -\frac{1}{192} \omega' |\vec{q}| - \frac{1}{384} \frac{1}{m_\pi^2} \omega'^3 |\vec{q}| \right.
\end{aligned}$$

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<sup>4</sup>This expansion does not increase the chiral order, because  $r$  and  $m_\pi$  are both treated as “small” parameters.

$$+ \frac{1}{320} \frac{1}{m_\pi^2} \omega'^2 |\vec{q}|^2 \cos \theta - \frac{1}{960} \frac{1}{m_\pi^2} \omega' |\vec{q}|^3 \Big], \quad (39b)$$

$$A_9^{loop} = \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi} \left[ \frac{5}{96} \omega'^2 + \frac{17}{1920} \frac{1}{m_\pi^2} \omega'^4 \right. \\ \left. + \frac{7}{960} \frac{1}{m_\pi^2} \omega'^3 |\vec{q}| \cos \theta - \frac{7}{960} \frac{1}{m_\pi^2} \omega'^2 |\vec{q}|^2 \right]. \quad (39c)$$

In order to obtain  $A_9^{loop}$ , we made use of Eqs. (8a) and (16).

#### IV. LOW-ENERGY EXPANSION, STRUCTURE COEFFICIENTS AND POLARIZABILITIES

In the previous section we saw that the  $O(p^3)$  heavy baryon result reproduces the terms required by Low's method in the tree-level amplitudes. This section will deal with a general parametrization of the structure- or model-dependent part of  $\mathcal{M}_{VCS}^{non-spin}$ . In [11] a general low-energy parametrization  $O$  of the structure-dependent amplitude for virtual Compton scattering on a spin-zero target, *e.g.* a pion, has been worked out. The corresponding expression for the nucleon is more complicated, as in Dirac space it involves a general  $4 \times 4$ -matrix. However, we can apply the parametrization of [11] to the *spin-independent* part of the VCS amplitude off the nucleon. Indeed the form of the scattering amplitude for the spin-independent part of the reaction (1) is the same as for the process [33]

$$\gamma^*(\varepsilon^\mu, q^\mu) + \pi(p_i^\mu) \rightarrow \gamma(\varepsilon'^\mu, q'^\mu) + \pi(p_f^\mu). \quad (40)$$

Before discussing the structure-dependent terms, one has to specify which convention has been used for splitting the total amplitude  $\mathcal{M}_{VCS}$  into a Born contribution and a structure-dependent part. This issue is addressed in [8,11] in quite some detail. We follow the convention of [6,8] where the Born contribution is calculated using Dirac and Pauli form factors,  $F_1$  and  $F_2$ . In fact, to the order considered here, the tree-level diagrams of Fig. 1 (a) to (c) generate the same contributions to  $A_1$ ,  $A_2$  and  $A_9$  as a covariant Born term calculation involving  $F_1$  and  $F_2$  [8].

The most general spin-independent contribution to the structure-dependent VCS amplitude can then be written as

$$O = \varepsilon_\mu O^{\mu\nu} \varepsilon_\nu'^* = O^{(1)} + O^{(2)} + O^{(3)} + O^{(4)} + O(k^5), \quad (41)$$

where the terms of increasing orders of  $k$  ( $k$  stands for  $q$  or  $q'$ ) are the following:

$$O^{(1)} = 0, \quad (42a)$$

$$O^{(2)} = g_0 [\varepsilon \cdot q' \varepsilon'^* \cdot q - q \cdot q' \varepsilon \cdot \varepsilon'^*] \\ + \tilde{c}_1 [(q + q') \cdot (p_i + p_f) (\varepsilon \cdot (p_i + p_f) \varepsilon'^* \cdot q + q' \cdot \varepsilon (p_i + p_f) \cdot \varepsilon'^*) \\ - 2q \cdot q' \varepsilon \cdot (p_i + p_f) \varepsilon'^* \cdot (p_i + p_f) - 2q \cdot (p_i + p_f) q' \cdot (p_i + p_f) \varepsilon \cdot \varepsilon'^*], \quad (42b)$$

$$O^{(3)} = 0, \quad (42c)$$

$$O^{(4)} = [g_{2a} q \cdot q' + g_{2b} q^2 + 4g_{2c} q \cdot (p_i + p_f) q' \cdot (p_i + p_f)] [\varepsilon \cdot q' \varepsilon'^* \cdot q - q \cdot q' \varepsilon \cdot \varepsilon'^*]$$

$$\begin{aligned}
& + \left[ \tilde{c}_{3a} q \cdot q' + \tilde{c}_{3b} q^2 + 4\tilde{c}_{3c} q \cdot (p_i + p_f) q' \cdot (p_i + p_f) \right] \\
& \times \left[ (q + q') \cdot (p_i + p_f) (\varepsilon \cdot (p_i + p_f) \varepsilon'^* \cdot q + \varepsilon \cdot q' \varepsilon'^* \cdot (p_i + p_f)) \right. \\
& \quad \left. - 2q \cdot q' \varepsilon \cdot (p_i + p_f) \varepsilon'^* \cdot (p_i + p_f) - 2q \cdot (p_i + p_f) q' \cdot (p_i + p_f) \varepsilon \cdot \varepsilon'^* \right] \\
& + c_3 \left[ 2q^2 (q \cdot (p_i + p_f) q' \cdot (p_i + p_f) \varepsilon \cdot \varepsilon'^* - q \cdot q' \varepsilon \cdot (p_i + p_f) \varepsilon'^* \cdot (p_i + p_f)) \right. \\
& \quad \left. + (q + q') \cdot (p_i + p_f) q^2 (\varepsilon \cdot (p_i + p_f) \varepsilon'^* \cdot q - \varepsilon \cdot q' \varepsilon'^* \cdot (p_i + p_f)) \right. \\
& \quad \left. + 2(q + q') \cdot (p_i + p_f) q \cdot q' \varepsilon \cdot q \varepsilon'^* \cdot (p_i + p_f) \right. \\
& \quad \left. - 4q \cdot (p_i + p_f) q' \cdot (p_i + p_f) \varepsilon \cdot q \varepsilon'^* \cdot q \right]. \tag{42d}
\end{aligned}$$

Here we have used the same notation for the unknown structure coefficients as in [11]. We then match our chiral calculation of the  $O(p^3)$  spin-independent amplitude with the low-energy expansion, Eq. (41), demanding

$$-ie^2 O = \mathcal{M}_{VCS}^{loop, non-spin}. \tag{43}$$

A consistent matching procedure requires that we make use of identical approximations for the kinematical quantities (see Eqs. (9), (10a) and (10b)) in both the chiral loop calculation and the low-energy expansion, Eq. (41). To be specific, we have neglected corrections of the type  $\omega'/M$  and  $|\vec{q}|/M$  (and higher). After this procedure we are able to determine the unknown structure coefficients in Eqs. (42b) and (42d) from our chiral calculation. Using Eq. (41) we obtain the following parametrization of the spin-independent amplitudes:

$$\begin{aligned}
A_1^{structure} &= \omega'^2 \left[ -g_0 - 8M^2 \tilde{c}_1 \right] + \omega' |\vec{q}| \cos \theta g_0 \\
& \quad + \omega'^4 \left[ -g_{2a} - g_{2b} - 16M^2 g_{2c} + 8M^2 c_3 - 8M^2 \tilde{c}_{3a} - 8M^2 \tilde{c}_{3b} - 128M^4 \tilde{c}_{3c} \right] \\
& \quad + \omega'^3 |\vec{q}| \cos \theta \left[ 2g_{2a} + g_{2b} + 16M^2 g_{2c} + 8M^2 \tilde{c}_{3a} \right] \\
& \quad + \omega'^2 |\vec{q}|^2 \left[ g_{2b} - 8M^2 c_3 + 8M^2 \tilde{c}_{3b} \right] + \omega'^2 |\vec{q}|^2 \cos^2 \theta \left[ -g_{2a} \right] \\
& \quad + \omega' |\vec{q}|^3 \cos \theta \left[ -g_{2b} \right], \tag{44a}
\end{aligned}$$

$$\begin{aligned}
A_2^{structure} &= \omega' |\vec{q}| \left[ -g_0 \right] \\
& \quad + \omega'^3 |\vec{q}| \left[ -g_{2a} - g_{2b} - 16M^2 g_{2c} \right] + \omega'^2 |\vec{q}|^2 \cos \theta g_{2a} + \omega' |\vec{q}|^3 g_{2b}, \tag{44b}
\end{aligned}$$

$$\begin{aligned}
A_9^{structure} &= \omega'^2 \left[ -g_0 - 8M^2 \tilde{c}_1 \right] \\
& \quad + \omega'^4 \left[ -g_{2a} - g_{2b} - 16M^2 g_{2c} + 8M^2 c_3 - 8M^2 \tilde{c}_{3a} - 8M^2 \tilde{c}_{3b} - 128M^4 \tilde{c}_{3c} \right] \\
& \quad + \omega'^3 |\vec{q}| \cos \theta \left[ g_{2a} + 8M^2 \tilde{c}_{3a} \right] + \omega'^2 |\vec{q}|^2 \left[ g_{2b} + 8M^2 c_3 + 8M^2 \tilde{c}_{3b} \right]. \tag{44c}
\end{aligned}$$

Demanding the validity of the matching relation Eq. (43) we obtain a system of linear equations for the structure coefficients, yielding

$$g_0 = \frac{1}{192} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi}, \tag{45a}$$

$$\tilde{c}_1 = -\frac{11}{192} \frac{1}{8M^2} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi}, \tag{45b}$$

$$g_{2a} = \frac{1}{320} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi^3}, \tag{45c}$$

$$g_{2b} = -\frac{1}{960} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi^3} \quad (45d)$$

$$g_{2c} = \frac{1}{1920} \frac{1}{16M^2} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi^3}, \quad (45e)$$

$$c_3 = -\frac{3}{1280} \frac{1}{8M^2} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi^3}, \quad (45f)$$

$$\tilde{c}_{3a} = \frac{1}{240} \frac{1}{8M^2} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi^3}, \quad (45g)$$

$$\tilde{c}_{3b} = -\frac{1}{256} \frac{1}{8M^2} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi^3}, \quad (45h)$$

$$\tilde{c}_{3c} = -\frac{9}{640} \frac{1}{128M^4} \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi^3}. \quad (45i)$$

Moreover, we wish to stress that it is actually a *prediction* of the low-energy expansion, Eq. (44a), that the expression for  $A_1$  *cannot* involve the structures  $|\vec{q}|^2$  and  $|\vec{q}|^4$ . Similarly the expansion of the amplitude  $A_9$ , Eq. (44c), excludes structures of the type  $\omega' |\vec{q}| \cos \theta$ ,  $\omega'^2 |\vec{q}|^2 \cos^2 \theta$ ,  $\omega' |\vec{q}|^3 \cos \theta$  and  $|\vec{q}|^4$ . When we compare these predictions with the result of our loop calculation, we find that each of these constraints is satisfied. We also note that our  $O(p^3)$  calculation gives a zero result for structures with an odd power of  $r$  ( $r \in \{\omega', |\vec{q}|\}$ ), which is required by the general expression in [11] obtained by imposing gauge invariance, crossing symmetry and the discrete symmetries.

It is useful at this point to interpret the structure coefficients which we have obtained. The coefficients which originate from  $O^{(2)}$  —  $g_0$  and  $\tilde{c}_1$  — are closely related to the electric and magnetic polarizabilities of the nucleon in real Compton scattering,  $\alpha_0$  and  $\beta_0$ . This can be seen by considering the limit of real Compton scattering in the c.m. system, *i.e.*  $|\vec{q}| = \omega = \omega'$ . If we apply the definitions for the polarizabilities, (see, *e.g.*, [27])

$$\begin{aligned} \alpha_0 + \beta_0 &= \frac{e^2}{8\pi} \frac{\partial^2}{\partial \omega^2} A_1(\omega = 0, \theta = 0), \\ \beta_0 &= -\frac{e^2}{4\pi} \left( \frac{A_2}{\omega' |\vec{q}|} \right) (\omega = 0, \theta = 0), \end{aligned} \quad (46)$$

to Eqs. (39a) and (39b), we determine

$$\begin{aligned} \alpha_0 &= \frac{5e^2 g_A^2}{384\pi^2 m_\pi F^2}, \\ \beta_0 &= \frac{e^2 g_A^2}{768\pi^2 m_\pi F^2}, \end{aligned} \quad (47)$$

which coincide with the results found in [23]. We can then express the structure constants  $g_0$  and  $\tilde{c}_1$  in terms of  $\alpha_0$  and  $\beta_0$ ,

$$g_0 = \frac{4\pi}{e^2} \beta_0, \quad (48a)$$

$$\tilde{c}_1 = -\frac{\pi}{2e^2 M^2} (\alpha_0 + \beta_0). \quad (48b)$$

An alternative way by which to derive this relation without explicitly taking the real Compton limit is to compare our results for  $A_1$ ,  $A_2$  and  $A_9$  with the low-energy expansion of the VCS amplitude of [8]. We find that this method yields identical results for the polarizabilities  $\alpha_0$  and  $\beta_0$  once the transformation to our convention is made.

## V. COMPARISON WITH OTHER CALCULATIONS

We now investigate the connection of our results with the multipole expansion of [6], where possible. We begin by recalling the primary features of such an expansion. Guichon, Liu and Thomas parametrize the structure-dependent part of the VCS amplitude in terms of essentially the same kinematical quantities as do we, namely,  $\omega'$ ,  $|\vec{q}|$ ,  $\theta$ . However, they expand the structure-dependent part of the amplitude in terms of  $\omega'$ , *keeping only the first non-vanishing, linear term*. (They do not consider  $\omega'^2$  terms because they are of higher order in their expansion.) The multipole expansion then generates various combinations of  $\cos\theta$  and powers of  $|\vec{q}|$  and, furthermore, suggests the definition of generalized polarizabilities characterized by angular momentum quantum numbers of the respective partial waves. The crucial difference in comparison with the low-energy expansion introduced above consists in the feature that the starting point of the Guichon *et al.* analysis is the  $\omega'$  expansion around  $\omega' = 0$ , wherein  $|\vec{q}|$  can be chosen arbitrarily, (*i.e.* not necessarily small), once  $\omega'$  has a small value. One can see the difference between the two expansion schemes most easily by looking at the quantity  $\omega$ . Rewriting Eq. (9), we obtain

$$\omega = \omega' + \frac{\omega'^2}{2M} - \frac{|\vec{q}|^2}{2M} + O\left(\frac{r^4}{M^3}\right). \quad (49)$$

On the other hand, Guichon, Liu and Thomas use

$$\omega|_{\omega'=0} = -\frac{|\vec{q}|^2}{2M} + O\left(\frac{|\vec{q}|^4}{M^3}\right). \quad (50)$$

From Eqs. (49) and (50) we conclude that we will not, in general, be able to express the results of our calculation for the structure coefficients in terms of the generalized polarizabilities. In our approximation scheme we take the quantities  $\omega$  and  $\omega'$  to be equal and consider terms up to  $\omega'^4$  whereas [6] only keeps one power of  $\omega'$  and starts the expansion of  $\omega$  with a term which due to its  $1/M$  suppression would be higher order in our scheme. Using Eq. (50) the expansions of the model-dependent parts of the spin-independent amplitudes in [6] read

$$\begin{aligned} A_1^{non-Born} &= -\sqrt{\frac{3}{8}}\omega' |\vec{q}| \cos\theta P^{(11,11)0}(|\vec{q}|) \\ &\quad - \sqrt{\frac{3}{2}}\omega'\omega|_{\omega'=0} P^{(01,01)0}(|\vec{q}|) \\ &\quad - \frac{3}{2}\omega' |\vec{q}|^2 \hat{P}^{(01,1)0}(|\vec{q}|) + O(\omega'^2), \end{aligned} \quad (51a)$$

$$A_2^{non-Born} = \sqrt{\frac{3}{8}}\omega' |\vec{q}| P^{(11,11)0}(|\vec{q}|) + O(\omega'^2), \quad (51b)$$

$$A_9^{non-Born} = -\sqrt{\frac{3}{2}}\omega'\omega|_{\omega'=0} P^{(01,01)0}(|\vec{q}|) + O(\omega'^2), \quad (51c)$$

where we had to modify the definition of  $A_9^{non-Born}$  in [6], because therein the Lorentz gauge,  $\varepsilon \cdot q = 0$ , is used, while we work in the gauge defined in Eq. (16). Note that the generalized polarizabilities  $P^{(11,11)0}$ ,  $P^{(01,01)0}$  and  $\hat{P}^{(01,1)0}$  are functions of  $|\vec{q}|$  only.

Let us now compare Eq. (39a) with Eq. (51a). First of all, we note that in virtual Compton scattering the term containing  $\alpha_0$  is proportional to  $\omega\omega'$  before we have made any kinematical approximations. This can be seen from Eqs. (42b) and (48b). When we insert Eq. (49) for  $\omega$  the leading term will be proportional to  $\omega'^2$ , whereas applying Eq. (50) the leading term will be proportional to  $\omega'\omega|_{\omega'=0}$ . The important observation here is that in both expansion schemes these terms have the same coefficient which is proportional to  $\alpha_0$ . We conclude that our result for the  $\omega'^2$  term in Eq. (39a) serves as a prediction for  $\alpha_0$  in [6],

$$P^{(01,01)0}(|\vec{q}|=0) = -\frac{4\pi}{e^2}\sqrt{\frac{2}{3}}\alpha_0 = \sqrt{\frac{2}{3}}(g_0 + 8M^2\tilde{c}_1). \quad (52)$$

With an analogous chain of arguments we find that the term  $\omega'|\vec{q}|\cos\theta$  in Eq. (51a) can be directly compared with the corresponding term in Eq. (39a), yielding

$$P^{(11,11)0}(|\vec{q}|=0) = -\frac{4\pi}{e^2}\sqrt{\frac{8}{3}}\beta_0 = -\sqrt{\frac{8}{3}}g_0. \quad (53)$$

We see that the result for the magnetic polarizability  $\beta_0$  obtained from Eqs. (39a) and (51a) is consistent with the fact that both Eq. (39b) and Eq. (51b) can also be parametrized in terms of this polarizability as a pre-factor of the  $\omega'|\vec{q}|$  term. As regards Eq. (51c) we observe that the parametrization in terms of  $P^{(01,01)0}$  is consistent with our result for the pre-factor of the  $\omega'|\vec{q}|$  term in Eq. (39c). Now what about  $\hat{P}^{(01,1)0}(|\vec{q}|=0)$  in Eq. (51a)? In order to determine this polarizability from our calculation, one would have to calculate the coefficient of the  $\omega'|\vec{q}|^2$  term in Eq. (39a). Due to the approximations of Eqs. (9), (10a) and (10b), a  $O(p^3)$  heavy baryon calculation does not generate a term of this type. So we cannot determine this polarizability in our calculation. We expect the leading term contributing to  $\hat{P}^{(01,1)0}(|\vec{q}|=0)$  to be at least  $1/M$  suppressed. Hence its evaluation requires a consistent  $O(p^4)$  calculation, which is in progress. This is borne out by the linear sigma model calculation [10] which finds that the coefficient of  $\omega'|\vec{q}|^2$  is indeed  $1/M$  suppressed. Furthermore, in [10] it was shown numerically that  $\hat{P}^{(01,1)0}$  is not independent of the generalized polarizabilities  $\alpha$  and  $\beta$ .

Let us turn to the remaining seven structure constants,  $g_{2a}$ ,  $g_{2b}$ ,  $g_{2c}$ ,  $c_3$ ,  $\tilde{c}_{3a}$ ,  $\tilde{c}_{3b}$ ,  $\tilde{c}_{3c}$ , which parametrize the  $r^4$  terms in Eq. (44). First we observe in Eqs. (39a) and (51a) that, strictly speaking, we will only be able to compare terms which contain  $\omega'$  (and under certain circumstances also  $\omega'^2$ ) but not the terms with  $\omega'^3$  and  $\omega'^4$ . These are higher-order corrections in Eq. (51a) but not in Eq. (39a), where  $\omega'$  and  $|\vec{q}|$  are counted as being of the same order. This suggests that the multipole expansion of Eq. (51) and the chiral expansion of Eq. (39) have different domains of application. We will discuss the appropriate kinematical region for each of the two expansions later. We conclude that the terms  $\omega'^3, \omega'^4$  belong to the model-dependent piece of the VCS amplitude, but do not fit into the expansion scheme in



terms of the generalized polarizabilities of Eq. (51). Let us now discuss the  $\omega'^2 |\vec{q}|^2 \cos^2 \theta$  term in Eq. (39a). We cannot expect to find an analogous term in Eq. (51a), because the quadratic dependence on  $\cos \theta$  indicates a term of higher multipolarity which has not been considered in the special application of Eq. (51a), where the angular momentum of the final state photon is fixed at  $L' = 1$ . As regards the  $\omega'^2 |\vec{q}|^2$  terms in Eqs. (39a) and (39c) we find that the factors  $\omega'^2$  originate from the product  $\omega\omega'$  before we apply Eq. (9). This can be seen from Eqs. (42d), (44a) and (44c). As a result we might be tempted to interpret the coefficient of the  $\omega'^2 |\vec{q}|^2$  terms as  $\frac{d}{d|\vec{q}|^2}\alpha(|\vec{q}|=0)$ . However, we find different solutions from matching the transverse and the longitudinal results of Eqs. (39a) and (39c) with Eqs. (51a) and (51c). Inspecting Eqs. (44a) and (44c) we observe that the difference between the transverse and the longitudinal result can be explained by the fact that the structure constant  $c_3$  enters the terms with a different sign. If we recall that the transverse amplitude  $A_1^{loop}$  contains contributions from higher multipoles than  $L' = 1$ , we come to the conclusion that the  $\omega'^2 |\vec{q}|^2$  term can also get contributions from such multipoles, in particular from the constant part of the Legendre polynomial for  $L' = 2$ . For this reason we cannot determine  $\frac{d}{d|\vec{q}|^2}\alpha(|\vec{q}|=0)$  from this amplitude. In the longitudinal amplitude  $A_9^{loop}$ , however, we find no contributions from higher multipoles. For this reason we use the longitudinal amplitude for the determination of  $\frac{d}{d|\vec{q}|^2}\alpha(|\vec{q}|=0)$  and, using Eqs. (39c), (44c) and (51c), arrive at

$$\begin{aligned} \frac{d}{d|\vec{q}|^2}\alpha(|\vec{q}|=0) &= \frac{e^2}{4\pi} (g_{2b} + 8M^2c_3 + 8M^2\tilde{c}_{3b}) \\ &= -\frac{7e^2g_A^2}{3840\pi^2m_\pi F^2}. \end{aligned} \quad (54)$$

The situation is more obvious for  $\frac{d}{d|\vec{q}|^2}\beta(|\vec{q}|=0)$ . If we compare the term  $\omega' |\vec{q}|^3 \cos \theta$  in Eq. (39a) with Eqs. (44a) and (51a) we find

$$\frac{d}{d|\vec{q}|^2}\beta(|\vec{q}|=0) = -\frac{e^2}{4\pi}g_{2b} = \frac{e^2g_A^2}{3840\pi^2m_\pi F^2}. \quad (55)$$

From Eq. (51b) we can read off that we could expect the same coefficient for the  $\omega |\vec{q}|^3$  term in Eqs. (39b) and (44b), and we find that our results are consistent with Eq. (51b). This is supported by the fact that we have no indication of higher multipoles entering  $A_2^{loop}$  in our calculation which could obscure the  $L' = 1$  results.

Summing up, we have established a connection between the multipole expansion [6] and the low-energy coefficients [11]  $g_0$ ,  $\tilde{c}_1$ ,  $g_{2b}$  and the linear combination  $c_3 + \tilde{c}_{3b}$ , which is reflected in the following expansion of the electric and magnetic polarizabilities of the nucleon,

$$\begin{aligned} \alpha(|\vec{q}|) &= \alpha_0 \left( 1 - \frac{7}{50} \frac{|\vec{q}|^2}{m_\pi^2} + O\left(\frac{|\vec{q}|^4}{m_\pi^4}\right) \right) \\ &= \frac{e^2}{4\pi} \left( -[g_0 + 8M^2\tilde{c}_1] \right. \\ &\quad \left. + |\vec{q}|^2 (g_{2b} + 8M^2c_3 + 8M^2\tilde{c}_{3b}) + O\left(\frac{|\vec{q}|^4}{m_\pi^4}\right) \right), \end{aligned} \quad (56a)$$

$$\begin{aligned}
\beta(|\vec{q}|) &= \beta_0 \left( 1 + \frac{1}{5} \frac{|\vec{q}|^2}{m_\pi^2} + O\left(\frac{|\vec{q}|^4}{m_\pi^4}\right) \right) \\
&= \frac{e^2}{4\pi} \left( g_0 - |\vec{q}|^2 g_{2b} + O\left(\frac{|\vec{q}|^4}{m_\pi^4}\right) \right).
\end{aligned} \tag{56b}$$

We wish to point out Eqs. (56a) and (56b) exactly agree with the corresponding terms in the relativistic field-theoretical calculation within the linear sigma model [10]. In comparison with the results of [6,7,9] our value for the electric polarizability of the proton,  $\alpha_0 = 12.8 \times 10^{-4} \text{ fm}^3$  is much larger and our value for the magnetic polarizability of the proton,  $\beta_0 = 1.3 \times 10^{-4} \text{ fm}^3$  is smaller than in those calculations. (We have used the numerical values  $m_\pi = 135 \text{ MeV}$ ,  $F = 92.4 \text{ MeV}$  and  $g_A = 1.26$ .) The slope of the corresponding generalized electric polarizability,  $\frac{d}{d|\vec{q}|^2} \alpha(|\vec{q}|=0)$ , is found to be considerably larger than in the effective Lagrangian [7] and the constituent quark model [6,9] calculations. The slope of the generalized magnetic polarizability,  $\frac{d}{d|\vec{q}|^2} \beta(|\vec{q}|=0)$ , even has a different sign compared with other calculations. For the neutron we find the same analytical expressions for these quantities as for the proton.

We cannot give an interpretation of the structure coefficients  $g_{2a}$ ,  $g_{2c}$ ,  $\tilde{c}_{3a}$ ,  $\tilde{c}_{3c}$  and the missing linear combination  $c_3 - \tilde{c}_{3b}$  in terms of generalized polarizabilities. However, all structure coefficients in Eq. (42d), not only the subset which we could interpret in terms of generalized polarizabilities, have to be considered if one performs an experiment in a kinematical region where  $\omega'$  is comparable with  $|\vec{q}|$ . Furthermore, we find that, in addition to the convergence radius of the chiral Lagrangian, which is given by  $r/M$ , the inelastic threshold of single pion production sets a primary limit to our calculation. The low-energy parametrization in terms of structure coefficients is only valid in a regime below the pion threshold, where  $\omega$  and  $|\vec{q}|$  do not approach  $m_\pi$ . This is clearly a different kinematical regime than the one of the multipole expansion [6], which is predominantly suited for arbitrary  $|\vec{q}|$ , which are much larger than the very small  $\omega'$ . (Of course, in principle our calculation is exact for  $\omega' < m_\pi$  and  $|\vec{q}| < m_\pi$  if we keep the finite integrals in Eq. (A2) instead of expanding the integrands, as done above.)

In conclusion, we have investigated the spin-averaged amplitude of virtual Compton scattering within the framework of heavy baryon chiral perturbation theory to  $O(p^3)$ . We have determined the generalized spin-independent electromagnetic polarizabilities of the nucleon [6]. In order to obtain analytic expressions, we have restricted our calculation to  $\alpha_0$  and  $\beta_0$  of real Compton scattering and the slopes of the generalized polarizabilities with respect to  $|\vec{q}|^2$ . We have also performed an alternative low-energy expansion of the VCS amplitude [11] which is not restricted to first order in the energy of the outgoing photon as is the multipole expansion. We made a prediction, based on chiral symmetry, for the 9 *a priori* unknown structure coefficients characterizing the spin-independent part of the VCS amplitude up to  $O(r^4)$ . All predictions of the loop calculation are the same for the proton and the neutron at  $O(p^3)$ , but will differ in a  $O(p^4)$  evaluation.

## APPENDIX A: LOOP AMPLITUDES

Using standard techniques we obtain the following invariant amplitudes for the Feynman diagrams in Fig. 2 with the Lagrangian interactions, Eqs. (24a), (24b) and (35):

$$\mathcal{M}_{VCS}^{(1)} = -\frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v \varepsilon'^* \cdot a \frac{1}{v \cdot (l + t_i + q) + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} N_v, \quad (\text{A1a})$$

$$\mathcal{M}_{VCS}^{(2)} = -\frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v \varepsilon'^* \cdot a \frac{1}{v \cdot (l + t_i - q') + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} N_v, \quad (\text{A1b})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(3)} &= \frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v a \cdot l \varepsilon'^* \cdot (2l - q') \\ &\times \frac{1}{v \cdot (l + t_i + q - q') + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} \frac{1}{(l - q')^2 - m_\pi^2 + i0^+} N_v, \end{aligned} \quad (\text{A1c})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(4)} &= \frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v \varepsilon'^* \cdot l a \cdot (2l + q) \\ &\times \frac{1}{v \cdot (l + t_i + q - q') + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} \frac{1}{(l + q)^2 - m_\pi^2 + i0^+} N_v, \end{aligned} \quad (\text{A1d})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(5)} &= \frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v \varepsilon'^* \cdot l a \cdot (2l - q) \\ &\times \frac{1}{v \cdot (l + t_i) + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} \frac{1}{(l - q)^2 - m_\pi^2 + i0^+} N_v, \end{aligned} \quad (\text{A1e})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(6)} &= \frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v a \cdot l \varepsilon'^* \cdot (2l + q') \\ &\times \frac{1}{v \cdot (l + t_i) + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} \frac{1}{(l + q')^2 - m_\pi^2 + i0^+} N_v, \end{aligned} \quad (\text{A1f})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(7)} &= \frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v (v \cdot (l + q' - q) v \cdot l - l \cdot (l + q' - q)) \\ &\times \varepsilon'^* \cdot (2(l - q) + q') a \cdot (2l - q) \\ &\times \frac{1}{v \cdot (l + t_i) + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} \frac{1}{(l - q)^2 - m_\pi^2 + i0^+} \frac{1}{(l + q' - q)^2 - m_\pi^2 + i0^+} N_v, \end{aligned} \quad (\text{A1g})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(8)} &= \frac{e^2 g_A^2}{2F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v (v \cdot (l + q' - q) v \cdot l - l \cdot (l + q' - q)) \\ &\times a \cdot (2(l + q') - q) \varepsilon'^* \cdot (2l + q') \\ &\times \frac{1}{v \cdot (l + t_i) + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} \frac{1}{(l + q')^2 - m_\pi^2 + i0^+} \frac{1}{(l + q' - q)^2 - m_\pi^2 + i0^+} N_v, \end{aligned} \quad (\text{A1h})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(9)} &= -\frac{e^2 g_A^2}{F^2} \int \frac{d^d l}{(2\pi)^d} \bar{N}_v (v \cdot (l + q' - q) v \cdot l - l \cdot (l + q' - q)) \\ &\times \varepsilon'^* \cdot a \frac{1}{v \cdot (l + t_i) + i0^+} \frac{1}{l^2 - m_\pi^2 + i0^+} \frac{1}{(l + q' - q)^2 - m_\pi^2 + i0^+} N_v. \end{aligned} \quad (\text{A1i})$$

We have only retained the spin-independent parts of the  $d$ -dimensional integrals. We note that we do not find any dependence on the nucleon isospin, which means that the loop contributions for proton and neutron are identical. Carrying out the integration over the loop momentum  $l$  we find that the sum of all amplitudes is finite. The individual amplitudes can be written as

$$\mathcal{M}_{VCS}^{(1)} = -\frac{i}{2} \frac{e^2 g_A^2}{F^2} a \cdot \varepsilon'^* J_0(\omega, m_\pi^2), \quad (\text{A2a})$$

$$\mathcal{M}_{VCS}^{(2)} = -\frac{i}{2} \frac{e^2 g_A^2}{F^2} a \cdot \varepsilon'^* J_0(-\omega', m_\pi^2), \quad (\text{A2b})$$

$$\mathcal{M}_{VCS}^{(3)} = \frac{ie^2 g_A^2}{F^2} a \cdot \varepsilon'^* \int_0^1 dx J_2'(-\omega'(1-x) + \omega, m_\pi^2), \quad (\text{A2c})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(4)} = & \frac{ie^2 g_A^2}{F^2} \left[ a \cdot \varepsilon'^* \int_0^1 dx J_2'(-\omega' + \omega(1-x), m_\pi^2 - q^2 x(1-x)) \right. \\ & \left. + \varepsilon'^* \cdot qa \cdot q \int_0^1 dx \frac{x}{2} (2x-1) J_0'(-\omega' + \omega(1-x), m_\pi^2 - q^2 x(1-x)) \right], \quad (\text{A2d}) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(5)} = & \frac{ie^2 g_A^2}{F^2} \left[ a \cdot \varepsilon'^* \int_0^1 dx J_2'(\omega x, m_\pi^2 - q^2 x(1-x)) \right. \\ & \left. + \varepsilon'^* \cdot qa \cdot q \int_0^1 dx \frac{x}{2} (2x-1) J_0'(\omega x, m_\pi^2 - q^2 x(1-x)) \right], \quad (\text{A2e}) \end{aligned}$$

$$\mathcal{M}_{VCS}^{(6)} = \frac{ie^2 g_A^2}{F^2} a \cdot \varepsilon'^* \int_0^1 dx J_2'(-\omega' x, m_\pi^2), \quad (\text{A2f})$$

$$\begin{aligned} \mathcal{M}_{VCS}^{(7)} = & 2 \frac{ie^2 g_A^2}{F^2} \int_0^1 dx \int_0^1 dy \\ & \times \left[ a \cdot \varepsilon'^* \left[ -5(1-y) \right. \right. \\ & \times J_6''(\omega(y+x(1-y)) - \omega'y, m_\pi^2 - t(1-x)y(1-y) - q^2 x(1-x)(1-y)^2) \\ & + \left( \frac{1}{2} (t - q^2) (1-y)^2 (y+x(1-y) + y(1-x)) \right. \\ & + q^2 (1-x)(1-y)^2 (y+x(1-y)) - \omega'^2 y(1-y)^2 \\ & \left. \left. - \omega^2 (1-x)(1-y)^2 (y+x(1-y)) + \omega\omega' (1-y)^2 (y+x(1-y) + y(1-x)) \right) \right. \\ & \left. \times J_2''(\omega(y+x(1-y)) - \omega'y, m_\pi^2 - t(1-x)y(1-y) - q^2 x(1-x)(1-y)^2) \right] \\ & + a \cdot q' \varepsilon'^* \cdot q \left[ ((1-x)(1-y)^2(1-6y) + y(1-y)(x+y(1-x))) \right. \\ & \times J_2''(\omega(y+x(1-y)) - \omega'y, m_\pi^2 - t(1-x)y(1-y) - q^2 x(1-x)(1-y)^2) \\ & + \left( -\omega'^2 (1-x)y^2(1-y)^3 - \omega^2 (1-x)^2 y(1-y)^3 (y+x(1-y)) \right. \\ & + \omega\omega' (1-x)y(1-y)^3 (y+x(1-y) + y(1-x)) \\ & + \frac{1}{2} (t - q^2) (1-x)y(1-y)^3 (y+x(1-y) + y(1-x)) \\ & \left. \left. + q^2 (1-x)^2 y(1-y)^3 (y+x(1-y)) \right) \right] \end{aligned}$$

$$\begin{aligned}
& \times J_0'' \left( \omega(y+x(1-y)) - \omega'y, m_\pi^2 - t(1-x)y(1-y) - q^2x(1-x)(1-y)^2 \right) \Big] \\
& + a \cdot q \varepsilon'^* \cdot q \left[ (1-y) \left( y+x(1-y) - \frac{1}{2} \right) (6(1-x)(1-y) - (y+x(1-y))) \right. \\
& \times J_2'' \left( \omega(y+x(1-y)) - \omega'y, m_\pi^2 - t(1-x)y(1-y) - q^2x(1-x)(1-y)^2 \right) \\
& + \left( \omega'^2(1-x)y(1-y)^3 \left( y+x(1-y) - \frac{1}{2} \right) \right. \\
& + \omega^2(1-x)^2(1-y)^3(y+x(1-y)) \left( y+x(1-y) - \frac{1}{2} \right) \\
& - \omega\omega'(1-x)(1-y)^3(y+x(1-y) + y(1-x)) \left( y+x(1-y) - \frac{1}{2} \right) \\
& - \frac{1}{2}(t-q^2)(1-x)(1-y)^3(y+x(1-y) + y(1-x)) \left( y+x(1-y) - \frac{1}{2} \right) \\
& \left. - q^2(1-x)^2(1-y)^3(y+x(1-y)) \left( y+x(1-y) - \frac{1}{2} \right) \right) \\
& \left. \times J_0'' \left( \omega(y+x(1-y)) - \omega'y, m_\pi^2 - t(1-x)y(1-y) - q^2x(1-x)(1-y)^2 \right) \right] \Big] , \quad (\text{A2g})
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{VCS}^{(8)} &= 2 \frac{ie^2 g_A^2}{F^2} \int_0^1 dx \int_0^1 dy \\
& \times \left[ a \cdot \varepsilon'^* \left[ -5(1-y) \right. \right. \\
& \times J_6'' \left( \omega y - \omega'(y+x(1-y)), m_\pi^2 - t(1-x)y(1-y) - q^2xy(1-y) \right) \\
& + \left( -\omega^2 y(1-y)^2 - \omega'^2(1-x)(1-y)^2(y+x(1-y)) \right. \\
& + \omega'\omega(1-y)^2(y+x(1-y) + y(1-x)) + q^2 y(1-y)^2 \\
& \left. \left. - \frac{1}{2}(q^2-t)(1-y)^2(y+x(1-y) + y(1-x)) \right) \right. \\
& \times J_2'' \left( \omega y - \omega'(y+x(1-y)), m_\pi^2 - t(1-x)y(1-y) - q^2xy(1-y) \right) \Big] \\
& + a \cdot q' \varepsilon'^* \cdot q \left[ \left( (1-x)(1-y)^2(1-6y) + y(1-y)(y+x(1-y)) \right) \right. \\
& \times J_2'' \left( \omega y - \omega'(y+x(1-y)), m_\pi^2 - t(1-x)y(1-y) - q^2xy(1-y) \right) \\
& + \left( -\omega^2(1-x)y^2(1-y)^3 \right. \\
& - \omega'^2(1-x)^2 y(1-y)^3(y+x(1-y)) \\
& + \omega\omega'(1-x)(1-y)^3 y(y+x(1-y) + y(1-x)) \\
& + q^2(1-x)y^2(1-y)^3 \\
& \left. + \frac{1}{2}(t-q^2)(1-x)y(1-y)^3(y+x(1-y) + y(1-x)) \right) \\
& \left. \times J_0'' \left( \omega y - \omega'(y+x(1-y)), m_\pi^2 - t(1-x)y(1-y) - q^2xy(1-y) \right) \right] \Big]
\end{aligned}$$

$$\begin{aligned}
& + a \cdot q \varepsilon'^* \cdot q \left[ (1-y)(1-7y) \left( y - \frac{1}{2} \right) \right. \\
& \times J_2'' \left( \omega y - \omega' (y + x(1-y)), m_\pi^2 - t(1-x)y(1-y) - q^2 xy(1-y) \right) \\
& + \left( -\omega^2 y^2 (1-y)^2 \left( y - \frac{1}{2} \right) \right. \\
& - \omega'^2 (1-x)y(1-y)^2 (y + x(1-y)) \left( y - \frac{1}{2} \right) \\
& + \omega \omega' y (1-y)^2 (y + x(1-y) + y(1-x)) \left( y - \frac{1}{2} \right) \\
& + q^2 y^2 (1-y)^2 \left( y - \frac{1}{2} \right) \\
& \left. - \frac{1}{2} (q^2 - t) y (1-y)^2 \left( y - \frac{1}{2} \right) (y + x(1-y) + y(1-x)) \right) \\
& \left. \times J_0'' \left( \omega y - \omega' (y + x(1-y)), m_\pi^2 - t(1-x)y(1-y) - q^2 xy(1-y) \right) \right] \quad (A2h)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{VCS}^{(9)} &= \frac{ie^2 g_A^2}{F^2} a \cdot \varepsilon'^* \int_0^1 dx \left[ (d-1) J_2' \left( -(\omega' - \omega)x, m_\pi^2 - tx(1-x) \right) \right. \\
& \left. + \left( (\omega' - \omega)^2 x(1-x) - tx(1-x) \right) J_0' \left( -(\omega' - \omega)x, m_\pi^2 - tx(1-x) \right) \right]. \quad (A2i)
\end{aligned}$$

Here we have used the definitions

$$J_0(\omega, m_\pi^2) = -4L\omega + \frac{\omega}{8\pi^2} \left( 1 - 2\ln \frac{m_\pi}{\mu} \right) - \frac{1}{4\pi^2} \sqrt{m_\pi^2 - \omega^2} \arccos \frac{-\omega}{m_\pi} + \mathcal{O}(d-4), \quad (A3a)$$

$$J_2(\omega, m_\pi^2) = \frac{1}{d-1} \left[ (m_\pi^2 - \omega^2) J_0(\omega, m_\pi^2) - \omega \Delta_\pi \right], \quad (A3b)$$

$$\begin{aligned}
J_6(\omega, m_\pi^2) &= \frac{1}{d^2 + 6d + 5} \left( (d+5)m_\pi^2 - 6\omega^2 \right) J_2(\omega, m_\pi^2) \\
& + \frac{1}{d^2 + 6d + 5} \omega^2 (\omega^2 - m_\pi^2) J_0(\omega, m_\pi^2) \\
& + \frac{1}{d^2 + 6d + 5} \left( \omega^3 - \omega m_\pi^2 \left( 1 + \frac{5}{d} \right) \right) \Delta_\pi. \quad (A3c)
\end{aligned}$$

In Eqs. (A3a), (A3b) and (A3c) we have used the same conventions as [27],

$$\begin{aligned}
\Delta_\pi &= 2m_\pi^2 \left( L + \frac{1}{16\pi^2} \ln \frac{m_\pi}{\mu} \right) + \mathcal{O}(d-4), \\
L &= \frac{\mu^{d-4}}{16\pi^2} \left[ \frac{1}{d-4} + \frac{1}{2} (\gamma_E - 1 - \ln 4\pi) \right], \quad (A4)
\end{aligned}$$

where we introduced the Euler-Mascharoni constant,  $\gamma_E = 0.557215$ , and the scale  $\mu$  in the dimensional regularization scheme we use for the evaluation of the integrals. With  $J_i'$  and  $J_i''$  we denote the first and second partial derivative with respect to  $m_\pi^2$ ,

$$J_i'(\omega, m_\pi^2) = \frac{\partial}{\partial(m_\pi^2)} J_i(\omega, m_\pi^2), \quad (A5a)$$

$$J_i''(\omega, m_\pi^2) = \frac{\partial^2}{\partial(m_\pi^2)^2} J_i(\omega, m_\pi^2). \quad (A5b)$$

## APPENDIX: ACKNOWLEDGMENTS

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FIGURES

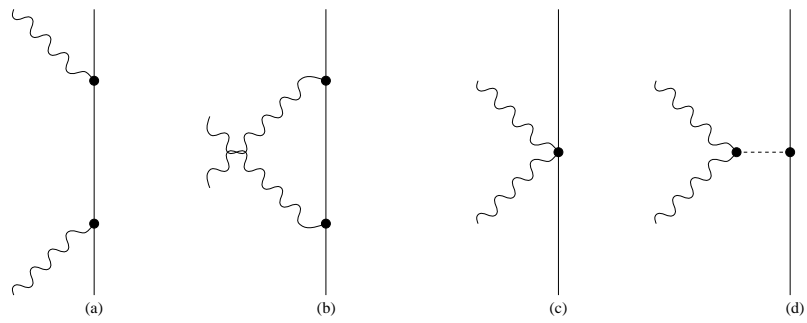


FIG. 1. Tree diagrams in virtual Compton scattering.

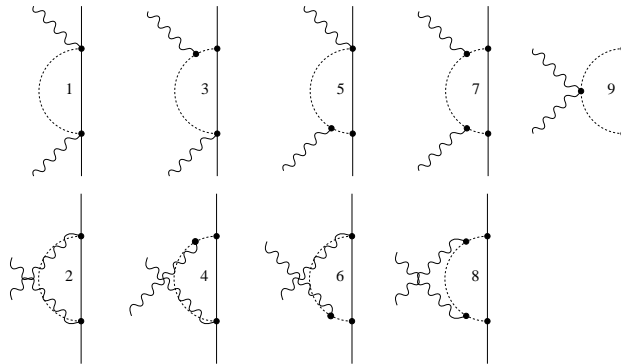


FIG. 2. Loop diagrams in virtual Compton scattering.