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TR HEMMERT

BR Holstein
holstein@physics.umass.edu

NC MUKHOPADHYAY

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# NN, N$\Delta$ Couplings and the Quark Model*

Thomas R. Hemmert[a], Barry R. Holstein[a] and Nimai C. Mukhopadhyay[b]
[a]Department of Physics and Astronomy
University of Massachusetts
Amherst, MA 01003
[b]Physics Department
Rensselaer Polytechnic Institute
Troy, NY 12180 - 3590

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**Abstract**

We examine mass-corrected SU(6) symmetry predictions in the quark model relating vector, axial-vector and strong NN and N$\Delta$ coupling, and demonstrate that the experimental N$\Delta$ value is significantly higher than predicted in each case. Nevertheless the Goldberger-Treiman relation is satisfied in both sectors. Possible origins of the discrepancy of the quark model predictions with experiments are discussed.

## 1 Introduction

Particle physicists generally think of the nucleon and the $\Delta(1232)$ baryon as being closely related partners, *i.e.* the internal quark dynamics is assumed to be identical with the mass difference arising from the effects of color-hyperfine interactions. Indeed the simple constituent quark wavefunctions

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display an explicit SU(6) symmetry whereby the nucleon and the delta share a 56-dimensional representation.[1] Within such a model one can calculate the vector and axial form factors $f_i$ and $g_i$ as well as the strong coupling constant $g_{\pi NN}$ for the pion-nucleon system at low momentum transfer. Likewise one can evaluate the corresponding nucleon-delta vector and axial vector form factors $c_i$ and $d_i$, as well as the strong coupling constant $g_{\pi N\Delta}$, at low $q^2$. The overall scale depends, however, on *a priori* unknown quark wave functions of the constituent quarks. By combining the results of the nucleon and delta calculations, one can eliminate this wavefunction dependence and obtain definite predictions relating corresponding NN and N$\Delta$ quantities. As we shall demonstrate, the experimental N$\Delta$ amplitudes are found to be systematically larger than predicted in each case, and the origin of this effect is unclear. Nevertheless, in both NN and N$\Delta$ sectors, we find that the Goldberger-Treiman [2] relation, required by chiral invariance, is valid.

In the succeeding sections, we analyze in turn the vector and axial form factors of the NN and N$\Delta$ systems, as well as the strong $\pi NN$ and $\pi N\Delta$ coupling constants. After examining the validity of the NN and $N\Delta$ Goldberger-Treiman relations, we close our paper with a summary and some speculations concerning these quark model discrepancies.

# 2 Quark Model Calculations

## 2.1 Vector Form Factors

We begin our discussion with the charge-changing polar vector transition between neutron and proton, for which the most general matrix element can be written from spin-parity considerations, in terms of three structure functions $f_i(q^2)$,

$$\begin{aligned} J_\mu^V(\mathbf{p},\mathbf{p}') &= \langle p(\mathbf{p}')|J_\mu^V|n(\mathbf{p})\rangle \\ &= \bar{u}(\mathbf{p}')\left\{f_1(q^2)\gamma_\mu + \frac{if_2(q^2)}{2M}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{2M}q_\mu\right\}u(\mathbf{p}). \qquad (1) \end{aligned}$$

Here $u, \bar{u}$ are plane-wave nucleon spinors, $M$ is the nucleon mass and $q_\mu = p'_\mu - p_\mu$ is the four-momentum transfer. Our goal is to determine the size of these three form factors at low $q^2$ using the constituent quark model. This task is made easier by use of the conserved vector current (CVC) hypothesis, which requires[3]

$$f_1(0) = 1, \quad \text{and} \quad f_3(q^2) = 0, \tag{2}$$

and is consistent with quark model considerations. In order to confront momentum space expressions such as Eq. 1 with coordinate space quark wavefunction calculations, we shall use the wavepacket (WP) formalism, which utilizes the function $\varphi(\mathbf{p})$ defined via[4]

$$|B(\mathbf{x})\rangle = \int d^3p\varphi(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}}|B(\mathbf{p})\rangle \tag{3}$$

with normalization condition[1]

$$\begin{aligned}\int d^3x\ \langle\bar{B}(\mathbf{x})|B(\mathbf{x})\rangle &= \int d^3p\ 2M\ (2\pi)^3|\varphi(\mathbf{p})|^2 \\ &= 1\end{aligned} \tag{4}$$

Note that the hadron bag state $|B(\mathbf{x})\rangle$ is centered about point $\mathbf{x}$ in position space. Center of mass motion of the bag due to the internal quark dynamics will not be taken into account.

Suppose that one is interested in the time component of the momentum space matrix element. *i.e.*

$$\begin{aligned}A_0^{WP} &= \int d^3xd^3p'd^3p\varphi^*(\mathbf{p}')\varphi(\mathbf{p})J_0^V(\mathbf{p},\mathbf{p}')e^{iq\cdot x} \\ &= f_1(0)\end{aligned} \tag{5}$$

In the quark model we can also calculate this transition moment, yielding

$$A_0^{QM} =_{QM} \langle p\uparrow | \int d^3x\ \bar{\psi}_u(\mathbf{x})\gamma_0\psi_d(\mathbf{x})|n\uparrow\rangle_{QM}, \tag{6}$$

where $|N\rangle_{QM}$ represent the quark model state vectors of the nucleon while $\psi_q(\mathbf{x})$ denote quark field operators. As we consider only S-wave quark states throughout, we can represent the field operators as

$$\psi_q(x) = \sum_{spin}[\phi_{0,s}(\mathbf{x})e^{-i\omega_0 t}b_q(s) + \phi^\dagger_{0,s'}(\mathbf{x})e^{i\omega_0 t}b^\dagger_q(s')], \tag{7}$$

[1]For Dirac spinors we use the convention $u(\mathbf{p},s) = \sqrt{E+m}\begin{pmatrix}\chi_s \\ \frac{\sigma\cdot p}{E+m}\chi_s\end{pmatrix}$ [4]

where

$$\phi_{0,s}(\mathbf{x}) = \begin{pmatrix} iu(\mathbf{x})\chi_s \\ l(\mathbf{x})\sigma \cdot \hat{\mathbf{x}}\chi_s \end{pmatrix}. \tag{8}$$

Here $u(\mathbf{x}), l(\mathbf{x})$ correspond to upper, lower components of the quark wavefunctions respectively and are normalized via

$$\int d^3x \phi^\dagger(\mathbf{x})\phi(\mathbf{x}) = \int d^3x (u^2(\mathbf{x}) + l^2(\mathbf{x})) = 1. \tag{9}$$

Using these expressions we can evaluate Eq. 6 yielding

$$\begin{aligned} A_0^{QM} &= \int d^3x \, (u_u(\mathbf{x})u_d(\mathbf{x}) + l_u(\mathbf{x})l_d(\mathbf{x})) \\ &= 1, \end{aligned} \tag{10}$$

in the limit of SU(2) symmetry. Lastly, equating quark model and wavepacket expressions we find

$$f_1(0) = 1, \tag{11}$$

as required by CVC.

In order to evaluate the weak magnetic form factor—$f_2(0)$—we take a first moment of the matrix element, yielding

$$\begin{aligned} A_1^{WP} &= \int d^3x d^3p' d^3p \varphi^*(\mathbf{p}')\varphi(\mathbf{p}) \frac{1}{2}\epsilon_{ij3} x^i J_V^j(\mathbf{p}, \mathbf{p}') e^{iq\cdot x} \\ &= \frac{f_1(0) + f_2(0)}{2M}, \end{aligned} \tag{12}$$

where $\epsilon_{ijk}$ is the completely antisymmetric Levi-Civita tensor. Again by CVC we require[3]

$$f_1(0) + f_2(0) = 1 + \kappa_p - \kappa_n = 4.7, \tag{13}$$

where $\kappa_i$ is the anomalous magnetic moment of the nucleon $i$.

On the other hand, in the constituent quark model,

$$\begin{aligned} A_1^{QM} &= {}_{QM}\langle p \uparrow | \int d^3x \, \frac{1}{2}\epsilon_{ij3} x^i \left[\bar{\psi}_u(x)\gamma^j \psi_d(x)\right] |n \uparrow\rangle_{QM} \\ &= \frac{5}{9}\int d^3x \, |\vec{x}|[u_u(\mathbf{x})l_d(\mathbf{x}) + u_d(\mathbf{x})l_u(\mathbf{x})]. \end{aligned} \tag{14}$$

Equating wavepacket and quark model results we have the prediction[2]

$$f_2^{QM} = \frac{10M}{9}\int d^3x\; |\vec{x}|[u_u(\mathbf{x})l_d(\mathbf{x}) + u_d(\mathbf{x})l_u(\mathbf{x})] - 1. \tag{18}$$

Next we turn to the corresponding N-$\Delta$ polar vector transition, for which the most general matrix element can be written, using spin-parity considerations, in terms of four form factors $c_i(q^2)$ as

$$\begin{aligned} J^V_{\mu\,\Delta^{++}p}(\mathbf{p},\mathbf{p}') &= \langle \Delta^{++}(\mathbf{p}')|J^V_{\mu\,\Delta p}|p(\mathbf{p})\rangle \\ &= \bar{\Delta}^{\nu\,++}(\mathbf{p}')\Big[\Big(\frac{c_1(q^2)}{2M}\gamma^\lambda + \frac{c_2(q^2)}{4M^2}q^\lambda + \frac{c_3(q^2)}{4M^2}p^\lambda\Big) \\ &\quad \times(q_\lambda g_{\mu\nu} - q_\nu g_{\lambda\mu}) + c_4(q^2)g_{\mu\nu}\Big]\gamma_5 u(\mathbf{p}). \end{aligned} \tag{19}$$

Here the $\Delta^{++}$ is represented in terms of a Rarita-Schwinger spinor [8] and the momentum transfer is defined as $q_\mu = p'_\mu - p_\mu$. Using CVC we require $c_4(q^2) = 0$. In order to extend the wavepacket formalism to delta states, represented as free particle Rarita-Schwinger spinors, we define a wavepacket function $\rho(\mathbf{p})$ via

$$|\Delta_\mu(\mathbf{x})\rangle = \int d^3p\; \rho(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}}|\Delta_\mu(\mathbf{p})\rangle, \tag{20}$$

normalised as

$$\begin{aligned} \int d^3x\; \langle\bar{\Delta}^\mu(\mathbf{x})|\Delta_\mu(\mathbf{x})\rangle &= \int d^3p\; 2M_\Delta\; (2\pi)^3|\rho(\mathbf{p})|^2 \\ &= 1. \end{aligned} \tag{21}$$

[2] For example, using MIT bag model wavefunctions[5] one finds

$$f_2(0) = 2.5, \tag{15}$$

which, when center of mass corrections are included, is boosted to[6]

$$f_2(0) = 3.2, \tag{16}$$

and is in reasonable agreement with the CVC and experimental value[7]

$$f_2^{CVC}(0) = \kappa_p - \kappa_n = 3.7. \tag{17}$$

However, most of our considerations below will be independent of specific choices for quark wavefunctions.

Comparing Eq. 4 and Eq. 21 we make the following identification between the two wavepacket functions for our calculations of $N-\Delta$ transition moments:

$$\varphi(\mathbf{p})\sqrt{2M} = \rho(\mathbf{p})\sqrt{2M_\Delta} \tag{22}$$

Unlike in the case of $N-N$ transitions (Eq. 5) the time component of Eq. 19 vanishes in both wavepacket and quark model calculations, as expected. Note that in our use of the wavepacket formalism the three-momenta of initial and final particle are forced to be the same. [3] In the case of nucleon-delta transitions this implies $q^2 = q_0^2 \neq 0$ due to their differing mass. For the first non-vanishing moment we find in the wavepacket formalism

$$\begin{aligned} B_1^{WP} &= \int d^3x d^3p' d^3p \; \rho^*(\mathbf{p}')\varphi(\mathbf{p}) \; \frac{1}{2}\epsilon_{ij3}x^i J^{j\,V}_{\Delta^{++}p}(\mathbf{p},\mathbf{p}')e^{iq\cdot x} \\ &= \sqrt{\frac{1}{6}}\frac{M+M_\Delta}{2M_\Delta}\Big[\frac{c_1(q_0^2)}{2M}\left(1+\frac{M_\Delta+M}{2M}\right)+\frac{c_2(q_0^2)}{2M}\frac{q_0^2}{4M^2} \\ &\quad +\frac{c_3(q_0^2)}{2M}\left(\frac{M_\Delta^2-M^2-q_0^2}{8M^2}\right)\Big] \end{aligned} \tag{24}$$

with $q_0 = M_\Delta - M$ and

$$\begin{aligned} B_1^{QM} &= {}_{QM}\langle\Delta^{++}\uparrow|\int d^3x\; \frac{1}{2}\;\epsilon_{ij3}x^i\left[\bar{\psi}_u(x)\gamma^j\psi_d(x)\right]|p\uparrow\rangle_{QM} \\ &= \frac{4}{3}\sqrt{\frac{1}{6}}\int d^3x|\vec{x}|[u_d(\mathbf{x})l_u(\mathbf{x})+u_u(\mathbf{x})l_d(\mathbf{x})] \end{aligned} \tag{25}$$

[3]If we want to account explicitly for the four-momentum $k_\mu$ transferred to the initial hadron bag by a photon or a $W^\pm$ the situation gets much more complicated. For a generic matrix element $M$ and a generic form factor $F(q^2)$ we have

$$\begin{aligned} M^{WP'} &= \int d^3x d^3p d^3p' \rho^*(\mathbf{p}')\varphi(\mathbf{p})F((p'-p)^2)e^{-i(\mathbf{p}'-\mathbf{p}-\mathbf{k})\cdot\mathbf{x}} \\ &= (2\pi)^3\int d^3p\;\rho^*(\mathbf{p}+\mathbf{k})\;\varphi(\mathbf{p})\;F((E'-E)^2-\mathbf{k}^2) \end{aligned} \tag{23}$$

Note that in this case we cannot make use of Eq. 22 because the two wavepacket functions $\varphi(\mathbf{p})$ and $\rho(\mathbf{p})$ are not evaluated at the same momentum. In order to solve this problem we would have to model one of these functions explicitly. In our calculations we therefore set $\mathbf{k}=0$ and approximate the static bag form factor $F(q^2)\to F((E'-E)^2)\approx F((M'-M)^2)$

One way to avoid this extra model dependence is to account for recoil of the final state hadron bag. This was done for some of our calculations [31] and the effects were found to be of the same order as the static bag's center of mass corrections. We therefore feel justified to neglect the factor $\exp(i\mathbf{k}\cdot\mathbf{x})$ in our calculations.

in the quark model.

Equating these expressions we have the prediction

$$c_1(q_0^2)\left(1+\frac{M_\Delta+M}{2M}\right)+c_2(q_0^2)\frac{(M_\Delta-M)^2}{4M^2}+c_3(q_0^2)\frac{M_\Delta-M}{4M}$$
$$=\frac{2M_\Delta}{M+M_\Delta}\times\frac{8M}{3}\int d^3x|\vec{x}|[u_d(\mathbf{x})l_u(\mathbf{x})+u_u(\mathbf{x})l_d(\mathbf{x})]. \qquad (26)$$

However, rather than use specific quark wavefunctions, we can employ Eq. 18 to write the equivalent form

$$c_1(q_0^2)\left(1+\frac{M_\Delta+M}{2M}\right)+c_2(q_0^2)\frac{(M_\Delta-M)^2}{4M^2}+c_3(q_0^2)\frac{M_\Delta-M}{4M}$$
$$=\frac{2M_\Delta}{M+M_\Delta}\times\frac{12}{5}\left(1+f_2(0).\right) \qquad (27)$$

We note that in the limit $M_\Delta = M$ this becomes the familiar SU(6) prediction

$$c_1(0)=\frac{6}{5}(1+f_2(0)). \qquad (28)$$

We can proceed to calculate a second moment of Eq. 19:

$$\begin{aligned} B_2^{WP} &= \int d^3x d^3p' d^3p\ \rho^*(\mathbf{p}')\varphi(\mathbf{p})\ [3x_3^2-\vec{x}^2]J^{0\,V}_{\Delta^{++}p}(\mathbf{p},\mathbf{p}')e^{iq\cdot x} \\ &= \sqrt{\frac{3}{2}}\frac{(M+M_\Delta)}{2M_\Delta}\frac{1}{M^2}\left[c_1(q_0^2)+\frac{M_\Delta-M}{2M}c_2(q_0^2)+\frac{1}{2}c_3(q_0^2)\right]. \qquad (29)\end{aligned}$$

However, in the constituent quark model, this moment vanishes as our quark wavefunctions (Eq. 8) are purely S-wave.

$$\begin{aligned} B_2^{QM} &= {}_{QM}\langle\Delta^{++}\uparrow|\int d^3x\ [3x_3^2-\vec{x}^2]\bar{\psi}_u(x)\gamma^0\psi_d(x)|p\uparrow\rangle_{QM} \\ &= 0. \qquad (30)\end{aligned}$$

Experimentally this restriction turns out to be well justified because the vector part of the nucleon-delta transition is dominated by the M1 amplitude [9]. Combining the results of Eq. 29, Eq. 30 and Eq. 27 we find the prediction

$$c_1(q_0^2)=\frac{2M_\Delta}{M+M_\Delta}\times\frac{6}{5}\left(1+f_2(0)\right). \qquad (31)$$

In order to have everything written in terms of experimentally accessible quantities the only thing left to do is the scaling down of the form factor $c_1(q^2)$ from the time-like point $q_0^2 = (M_\Delta - M)^2 = 0.086\mathrm{GeV}^2$ to the photon-point $q^2 = 0$. For the $q^2$ behavior we use the empirical parametrization[10]

$$c_1(q^2) = \frac{1}{\sqrt{1 - \frac{q^2}{1.43\mathrm{GeV}^2}}} \times \frac{c_1(0)}{(1 - \frac{q^2}{0.71\mathrm{GeV}^2})^2}, \tag{32}$$

which, when extrapolated to the time-like region, yields

$$\begin{aligned} c_1(0)^{th.} &= \frac{1}{1.34}\,\frac{2M_\Delta}{M + M_\Delta} \times \frac{6}{5}\,(1 + f_2(0)) \\ &\approx 4.8. \end{aligned} \tag{33}$$

In order to see how well these predictions work, we note that $c_1(0), c_3(0)$ can be determined by use of CVC and the analogous electromagnetic transition $\gamma p \to \Delta^+$ for which the most general gauge-invariant matrix element has the form

$$\begin{aligned} M_{\gamma p \Delta^+} = -e\sqrt{\frac{2}{3}}\Delta^+_\mu(\mathbf{p_\Delta})\Big[&\frac{h_1(0)}{2M}\Big((M_\Delta + M)\epsilon^\mu + \gamma\cdot\epsilon\; p_N^\mu\Big) \\ &+\frac{h_3(0)}{4M^2}\Big(\frac{1}{2}(M_\Delta^2 - M^2)\epsilon^\mu + p_N\cdot\epsilon\; p_N^\mu\Big)\Big]\gamma_5 u(\mathbf{p_N}) \end{aligned} \tag{34}$$

where $\epsilon^\mu$ denotes the polarization four-vector of the (real) photon. From photoproduction experiments in the $\Delta$ region one extracts[9]

$$h_1^{exp.}(0) = 5.10 \pm 0.55, \quad h_3^{exp.}(0) = -5.41 \pm 0.85. \tag{35}$$

Then, using the CVC constraint

$$c_i(0) = \left[\sqrt{\frac{2}{3}}h_i(0)\right]\cdot\sqrt{3}, \tag{36}$$

one finds

$$c_1(0)^{exp.} = 7.21 \pm 0.78, \tag{37}$$

$$c_3(0)^{exp.} = -\;7.65 \pm 1.20. \tag{38}$$

We note that the result for $c_1(0)$ is more than 30% larger than its quark model mass-corrected SU(6) prediction Eq. 33. This result is rather surprising given the relatively simple physics involved and traditional gluonic hyperfine effects which explain the the $N, \Delta$ splitting are unable to account for this excess M1 strength in the N-$\Delta$ region [11], although we shall return to this point later.

## 2.2 The Axial Form Factors

Moving to the axial vector current, we define the general axial matrix element between neutron and proton in terms of three structure functions $g_i(q^2)$:

$$\begin{aligned} J^A_\mu(\mathbf{p},\mathbf{p}') &= \langle p(\mathbf{p}')|A_\mu|n(\mathbf{p})\rangle \\ &= \bar{u}(\mathbf{p}')\left[g_1(q^2)\gamma_\mu + \frac{ig_2(q^2)}{2M}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{2M}q_\mu\right]\gamma_5 u(\mathbf{p}). \end{aligned} \tag{39}$$

In the SU(2) limit, $g_2(q^2) = 0$ from the G-invariance considerations[14]. Also $g_3(q^2)$ contains the pion pole, and is, strictly speaking, outside the simple constituent quark model [15]. Finally, in the case of $g_1(q^2)$ we find

$$\begin{aligned} C_0^{WP} &= \int d^3x d^3p d^3p' \varphi^*(\mathbf{p}')\varphi \mathbf{p}) J^3_A(\mathbf{p},\mathbf{p}')e^{iq\cdot x} \\ &= g_1(0), \end{aligned} \tag{40}$$

in the wavepacket approach, and

$$\begin{aligned} C_0^{QM} &= {}_{QM}\langle p\uparrow|\int d^3x\ \bar{\psi}_u(x)\gamma^3\gamma_5\psi_d(x)|n\uparrow\rangle_{QM} \\ &= \frac{5}{3}\left[1-\frac{4}{3}\int d^3x\ l_u(\mathbf{x})l_d(\mathbf{x})\right], \end{aligned} \tag{41}$$

in the constituent quark model. Equating these expressions, we find

$$g_1(0) = \frac{5}{3}\left[1-\frac{4}{3}\int d^3x\ l_u(\mathbf{x})l_d(\mathbf{x})\right]. \tag{42}$$

If we set $l_i(\mathbf{x}) = 0$, we recover the well-known SU(6) result $g_1(0) = \frac{5}{3}$, in the nonrelativistic quark model[16]. It is the inclusion of the lower component of the relativistic wavefunctions which brings this value down to the experimental number $g_1^{exp.}(0) = 1.262 \pm 0.004$[17]. Thus we require[4]

$$\int d^3x\ l_u(\mathbf{x})l_d(\mathbf{x}) \approx 0.4. \tag{43}$$

[4]We note that this result is approximately obtained by using the MIT bag wavefunctions[6]. However, we do not wish to specify any specific model.

For the axial N-$\Delta$ transition, the matrix element can be written from spin-parity arguments in terms of four form factors $d_i(q^2)$

$$\begin{aligned} J^A_{\mu\Delta N}(\mathbf{p},\mathbf{p}') &= \langle\Delta^{++}(\mathbf{p}')|A^\Delta_\mu|p(\mathbf{p})\rangle \\ &= \bar{\Delta}^{++\nu}(\mathbf{p}')\Big[d_1(q^2)g_{\mu\nu} + \frac{d_2(q^2)}{M^2}P^\alpha\left(q_\alpha g_{\mu\nu} - q_\nu g_{\alpha\mu}\right) \\ &\qquad -\frac{d_3(q^2)}{M^2}p_\nu q_\mu + i\frac{d_4(q^2)}{M^2}\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta\gamma_5\Big]u(\mathbf{p}), \end{aligned} \tag{44}$$

where $P_\mu = p'_\mu + p_\mu$ and $q_\mu = p'_\mu - p_\mu$. (In Appendix A, we give the connection between the form factors $d_i(q^2)$ and the $C^A_i(q^2)$ often used in previous works[18].)

Equating the wavepacket result

$$\begin{aligned} D_0^{WP} &= \int d^3x d^3p d^3p' \rho^*(\mathbf{p}')\varphi(\mathbf{p})\; J^{3A}_{\Delta N}(\mathbf{p},\mathbf{p}')e^{iq\cdot x} \\ &= \sqrt{\frac{2}{3}}\left(d_1(q_0^2) + d_2(q_0^2)\frac{M_\Delta^2 - M^2}{M^2}\right), \end{aligned} \tag{45}$$

where $q_0 = M_\Delta - M$, with that given in the corresponding quark model calculation

$$\begin{aligned} D_0^{QM} &= {}_{QM}\langle\Delta^{++}\uparrow|\int d^3x\;\bar{\psi}_u(x)\gamma^3\gamma_5\psi_d(x)|p\uparrow\rangle_{QM} \\ &= 2\sqrt{\frac{2}{3}}\left[1 - \frac{4}{3}\int d^3x\; l_u(\mathbf{x})l_d(\mathbf{x})\right], \end{aligned} \tag{46}$$

we find

$$d_1(q_0^2) + d_2(q_0^2)\frac{M_\Delta^2 - M^2}{M^2} = 2\left[1 - \frac{4}{3}\int d^3x l_u(\mathbf{x})l_d(\mathbf{x})\right]. \tag{47}$$

As in the case of the vector formfactors, we want to scale the $d_i(q^2)$ from $q_0^2 = (M_\Delta - M)^2 = 0.086\text{GeV}^2$ down to the photon-point $q^2 = 0$. For the $q^2$ dependence we use the empirical parametrisation[23]

$$d_i(q^2) = d_i(0)\frac{1 + 1.21\frac{q^2}{2\text{GeV}^2 - q^2}}{(1 - \frac{q^2}{M_A^2})^2}, \tag{48}$$

where $M_A$ ranges from 1.14 to 1.28 GeV, and extrapolate it to the timelike region:

$$d_{1,2}(q_0^2) \approx d_{1,2}(0) \times (1.17 \pm 0.03). \tag{49}$$

Comparing with the corresponding expression for the neutron-proton transition (Eq. 42) we can now write Eq. 47 entirely in terms of experimental quantities as

$$d_1(0) + d_2(0)\, \frac{M_\Delta^2 - M^2}{M^2} = \frac{d_{1,2}(0)}{d_{1,2}(q_0^2)} \times \frac{6}{5}\, g_1(0), \tag{50}$$

which can be tested in charged current neutrino-nucleon scattering. In the degenerate limit $M_\Delta = M$, this becomes the well-known SU(6) relation $d_1(0) = \frac{6}{5}\, g_1(0)$.

Before confronting this prediction with experiment, we examine additional relations which arise in the quark model. As in the nucleon case $d_3(q^2)$ contains the pion pole and is outside the simple constituent quark formalism. In the case of $d_4(q^2)$, our use of the S-wave wavefunctions yields $d_4(q^2) = 0$, which is consistent with other calculations (*cf.* appendix A). Finally, calculating a first moment of the axial current, and omitting $d_3(q^2)$ contributions, we find

$$\begin{aligned} D_1^{WP} &= -i \int d^3x d^3p d^3p'\; x_3 \rho^*(\mathbf{p}')\varphi(\mathbf{p})\; J^{0A}_{\Delta N}(\mathbf{p},\mathbf{p}') e^{iq\cdot x} \\ &= \sqrt{\frac{2}{3}} \left( \frac{d_1(q_0^2)}{2M_\Delta} + d_2(q_0^2) \frac{M + M_\Delta}{M^2} \right). \end{aligned} \tag{51}$$

The corresponding quark model calculation of this moment gives

$$\begin{aligned} D_1^{QM} &= -i\; {}_{QM}\langle \Delta^{++} \uparrow | \int d^3x\; x_3\; \bar{\psi}_u(x)\gamma_0\gamma_5\psi_d(x) | p \uparrow\rangle_{QM} \\ &= \frac{2}{3}\sqrt{\frac{2}{3}} \int d^3x\; x\, [u_u(\mathbf{x}) l_d(\mathbf{x}) - u_d(\mathbf{x}) l_u(\mathbf{x})] \\ &\approx 0, \end{aligned} \tag{52}$$

which is a consequence of SU(2) symmetry[19] and brings about an additional relation between $d_1(0)$ and $d_2(0)$:

$$d_1(0) + 2\, d_2(0)\, \frac{M_\Delta(M + M_\Delta)}{M^2} = 0. \tag{53}$$

Using this result Eq. 50 can be written in the simpler form

$$d_1(0)^{th.} = \frac{1}{1.17} \times \frac{6}{5}\, g_1(0)\, \frac{2M_\Delta}{M_\Delta + M}, \tag{54}$$

$$d_2(0)^{th.} = -\frac{1}{1.17} \times \frac{6}{5}\, g_1(0)\, \frac{M^2}{(M + M_\Delta)^2}. \tag{55}$$

We now examine the experimental determination of the axial N-Δ transition form factors. In previous papers it was assumed that $d_1(0)$ is the only major contributing form factor to the cross section $\nu p \to \Delta^{++}\mu^-$, at low $q^2$. For example, Barish *et al.* find[20]

$$d_1^{exp.}(0) = 2.0 \pm 0.4. \tag{56}$$

We show their low $q^2$ data points in Figure 1. Note the large uncertainty in this result but also that it is consistent with the old model calculation by Adler[21], as newly parametrized by Schreiner *et al.*[22].

In Figure 2 we show the results of a more recent experiment by Kitagaki et al.[23] using a neutrino beam of $< E_\nu > \approx 1.6$ GeV and a deuterium target. They measured the ratio of two exclusive neutrino cross sections and found

$$R \equiv \frac{\frac{d\sigma}{dq^2}(\nu d \to \mu^- \Delta^{++} n)|_{q^2 \approx 0}}{\frac{d\sigma}{dq^2}(\nu d \to \mu^- pp)|_{q^2 \approx 0}} = 0.50 \pm 0.05, \tag{57}$$

for the point of lowest $q^2 = 0.1$ GeV$^2$. Stimulated by this result, we note that the corresponding cross sections can be expressed, at $q^2 = 0$, as

$$\begin{aligned}
\frac{d\sigma}{dq^2}(\nu n \to \mu^- p)|_{q^2=0} &= \cos^2\theta_C \frac{G_F^2}{2\pi} \left( f_1^2(0) + g_1^2(0) \right), \\
\frac{d\sigma}{dq^2}(\nu p \to \mu^- \Delta^{++})|_{q^2=0} &= \cos^2\theta_C \frac{G_F^2}{6\pi}\, \frac{(M_\Delta + M)^2}{M_\Delta^2 + M^2}\, \frac{2ME_\nu + (M^2 - M_\Delta^2)}{2ME_\nu} \\
&\quad \cdot \frac{M^2}{M_\Delta^2} \left[ d_1(0) + d_2(0) \frac{M_\Delta^2 - M^2}{M^2} \right]^2, \qquad (58)
\end{aligned}$$

where $G_F$ denotes the Fermi constant and $\cos\theta_C$ represents $V_{ud}$ in the KM-matrix. (Note that we have here neglected all terms depending on the mass of the muon.) Extrapolating the experimental result of Eq. 57 to $q^2 = 0$, we find

$$
\begin{aligned}
d_1(0) + d_2(0)\frac{M_\Delta^2 - M^2}{M^2} &= \sqrt{R}\,\sqrt{6}\,\sqrt{f_1^2(0) + g_1^2(0)}\,\frac{\sqrt{M_\Delta^2 + M^2}}{M_\Delta + M} \\
&\quad \times \frac{M_\Delta}{M}\,\sqrt{\frac{ME_\nu}{2ME_\nu + (M^2 - M_\Delta^2)}}. \qquad (59)
\end{aligned}
$$

The two main uncertainties in this result come from the spread in the neutrino beam energy and from the experimental error bar in Eq. 57. Depending on whether we use the mean neutrino beam energy $< E_\nu >= 1.6$ GeV or the peak in the neutrino beam energy distribution $E_\nu^{\mathrm{pk}} = 1.2$ GeV, one gets

$$
\begin{aligned}
d_1(0) + d_2(0)\frac{M_\Delta^2 - M^2}{M^2} &= 2.08 \pm 0.10 (E_\nu = 1.6 \text{ GeV}), \\
&= 2.18 \pm 0.11 (E_\nu = 1.2 \text{ GeV}). \qquad (60)
\end{aligned}
$$

Note that the error bars include only those of Eq. 57. One can see that the uncertainty coming from the neutrino beam energy is of the same magnitude. We, therefore, conclude

$$
d_1(0) + d_2(0)\frac{M_\Delta^2 - M^2}{M^2} = 2.1 \pm 0.2. \qquad (61)
$$

Use of Eq. 53 enables us to determine $d_1(0)$ and $d_2(0)$ independently:

$$
\begin{aligned}
d_1^{exp.}(0) &= 2.1\,\frac{2M_\Delta}{M_\Delta + M} \\
&= 2.4 \pm 0.25, \\
\\
d_2^{exp.}(0) &= -2.1\,\frac{M^2}{(M + M_\Delta)^2} \\
&= -0.4 \pm 0.05. \qquad (62)
\end{aligned}
$$

Comparing these experimental results with the predictions of the quark model (Eq. 54, Eq. 55)

$$
d_1^{QM}(0) = 1.5 \pm 0.05 \quad \text{and} \quad d_2^{QM}(0) = -0.25 \pm 0.005, \qquad (63)
$$

we come to the conclusion that once more the quark model significantly underestimates the strength of the axial transition form factors—for the dominant form factor $d_1(0)$, we find a deviation of more than 35% ! (Note that the result of Kitagaki *et al.* (Eq. 62) is consistent with the previous determination by Barish *et al.* (Eq. 56), though the central values are somewhat different.)

Having now examined the axial vector amplitudes for NN and NΔ transitions, we move on to the analogous strong pion couplings.

## 2.3 Strong Coupling Constants

In the case of the strong couplings, we begin by defining effective Lagrangians for the $\pi NN$ and $\pi N\Delta$ interaction

$$\begin{aligned}
\mathcal{L}_{\pi NN} &= -ig_{\pi NN}\ \bar{u}(\mathbf{p}')\gamma_5\tau^i u(\mathbf{p})\pi^i, \\
\mathcal{L}_{\pi N\Delta} &= \frac{g_{\pi N\Delta}}{2M}\bar{\Delta}^i_\mu(\mathbf{p}')g^{\mu\nu}u(\mathbf{p})\partial_\nu\pi^i + H.c..
\end{aligned} \tag{64}$$

Since the pion is a pseudoscalar, these both represent P-wave amplitudes, which can be projected out via

$$M^{WP}_{P-wave} = \int d^3x d^3p' d^3p\ \varphi^*(\mathbf{p}')\varphi(\mathbf{p})\ x_3\ M_{B'B\pi}(\mathbf{p},\mathbf{p}')\ e^{iq\cdot x}. \tag{65}$$

For the corresponding quark model multipole we find

$$M^{QM}_{P-wave} = -iN_\pi \int d^3x\ x_3\ {}_{QM}\langle B'|\bar{\psi}_u(x)\gamma_5\psi_d(x)|B\rangle_{QM}, \tag{66}$$

where $N_\pi$ is an unknown normalization constant associated with the created pion. Equating hadronic and quark model amplitudes, we find

$$\begin{aligned}
g^{QM}_{\pi NN}(0) &= \frac{10}{9\sqrt{2}}M\ N_\pi \int d^3x\ |\vec{x}|\left[u_u(x)l_d(x) + u_d(x)l_u(x)\right], \\
g^{QM}_{\pi N\Delta}(q_0^2) &= \frac{8}{3}\frac{MM_\Delta}{M+M_\Delta}\ N_\pi \int d^3x\ |\vec{x}|\left[u_u(x)l_d(x) + u_d(x)l_u(x)\right],
\end{aligned} \tag{67}$$

with $q_0 = M_\Delta - M$. In the limit $M_\Delta = M$ these relations reduce to the familiar SU(6) prediction

$$\begin{aligned}
\frac{g_{\pi N\Delta}(0)}{g_{\pi NN}(0)} &\approx \frac{g_{\pi N\Delta}(m_\pi^2)}{g_{\pi NN}(m_\pi^2)} \\
&= \frac{6\sqrt{2}}{5} = 1.70.
\end{aligned} \tag{68}$$

Assuming that the $q^2$ behavior of $g_{\pi N\Delta}$ scales like the axial nucleon-delta transition form factor, as given by the associated generalized Goldberger-Treiman relation, we obtain the mass-corrected theoretical prediction

$$\begin{aligned}
\frac{g_{\pi N\Delta}(0)}{g_{\pi NN}(0)} &\approx \frac{g_{\pi N\Delta}(m_\pi^2)}{g_{\pi NN}(m_\pi^2)} \\
&\approx \frac{1}{1.17} \times \frac{6\sqrt{2}}{5} \frac{2M_\Delta}{M + M_\Delta} \\
&\approx 1.6. \qquad (69)
\end{aligned}$$

Using the most recent value for the $\pi NN$ coupling[12]

$$g_{\pi NN}^{exp.} = 13.05 \pm 0.31, \qquad (70)$$

and the value of $g_{\pi N\Delta}$ extracted from a K-matrix analysis of phase shifts[9, 13]

$$g_{\pi N\Delta}^{exp.} = 28.6 \pm 0.3, \qquad (71)$$

we find

$$\frac{g_{\pi N\Delta}^{exp.}}{g_{\pi NN}^{exp.}} = 2.21 \pm 0.08, \qquad (72)$$

which again exceeds by 25% the mass-corrected SU(6) prediction.

We now move on to discuss the Goldberger-Treiman relations which connect the axial formfactors to the strong coupling constants.

# 3 Goldberger-Treiman Relations

Despite our inability to make direct contact between the strong interactions of elementary particles and the QCD Lagrangian which presumably underlies such interactions, it is possible to exploit, at low energies, the (approximate) chiral symmetry of QCD in order to provide rigorous predictive power. In the NN sector, an example of this is the Goldberger-Treiman relation[2]

$$F_\pi g_{\pi NN}(q^2) = M g_1(q^2), \qquad (73)$$

which is derived most simply via the PCAC relation

$$\partial^\mu A_\mu^i = F_\pi m_\pi^2 \phi_\pi^i, \qquad (74)$$

assuming pion pole dominance of the pseudoscalar formfactor $g_3(q^2)$, which is consistent with a recent experiment[24]. Here $F_\pi = 92.4$ MeV is the pion decay constant. Note that the relation is strictly valid only at the *same* value of momentum transfer for both strong and axial couplings. However, in checking its experimental validity one generally uses $g_1(0)$ and $g_{\pi NN}(m_\pi^2)$. We thus expect a slight violation of GT, various methods of taking this effect into account were analysed by Dominguez[25]. Defining

$$\Delta_\pi = 1 - \frac{M g_1(0)}{F_\pi g_{\pi NN}(m_\pi^2)}, \tag{75}$$

we anticipate $\Delta_\pi \approx 0.02$ from diagrams such as those in Figure 3. Experimentally things are not as clear, however, because of the presently uncertain value of $g_{\pi NN}$ at $q^2 = m_\pi^2$. The situation is summarized in Table 1, and we see that things work to better than 5%[26] in any case .

Table 1:

| | $g_{\pi NN}^2/4\pi$ | $\Delta_\pi$ |
|---|---|---|
| $\pi^\pm$ | $13.54 \pm 0.05$ [12] | 0.017 |
| | $13.31 \pm 0.27$ [27] | 0.008 |
| | $14.28 \pm 0.18$ [28] | 0.043 |
| $\pi^0$ | $13.47 \pm 0.11$ [12] | 0.014 |
| | $13.55 \pm 0.13$ [29] | 0.017 |
| | $14.52 \pm 0.40$ [30] | 0.051 |

Similarly one can derive the corresponding Goldberger-Treiman relation in the N$\Delta$ sector[31]. Using PCAC and assuming pion pole dominance of the pseudoscalar form factor $d_3(q^2)$ one finds

$$F_\pi g_{\pi N\Delta}(q^2) = \sqrt{2} M d_1(q^2). \tag{76}$$

Again checking the validity of this result using $g_{\pi N\Delta}(m_\pi^2)$ and $d_1(0)$ one expects a violation of size

$$\begin{aligned} \Delta_\pi^\Delta &= 1 - \frac{\sqrt{2} M d_1(0)}{F_\pi g_{\pi N\Delta}(m_\pi^2)} \\ &\approx 0.02 \end{aligned} \tag{77}$$

where the size of $\Delta_\pi^\Delta$ is estimated using the diagrams shown in Figure 4. Dillig and Brack came to similar results[32]. Given the large uncertainty in $g_{\pi N\Delta}$ we need not worry about this small deviation at this point.

Using the value $d_1(0)$ extracted from neutrino scattering experiments (Eq. 56 and Eq. 62), we can now try to test the validity of the generalised Goldberger-Treiman relation Eq. 77 in Table 2:

Table 2:

| $d_1(0)$ | "predicted" $g_{\pi N\Delta}$ |
|---|---|
| $2.0 \pm 0.40$(Eq. 56) | $29 \pm 6$ |
| $2.4 \pm 0.25$(Eq. 62) | $34.5 \pm 3.6$ |

We observe that our extracted $d_1(0)$ (Eq. 62) indicates a number at the upper end of the present range for $g_{\pi N\Delta} \approx 26 - 33$, but is consistent with presently known information. We conclude that despite the large violations of the mass-corrected SU(6) predictions for $d_1(0)$ and $g_{\pi N\Delta}(0)$ the generalized Goldberger-Treiman relation (Eq. 76), demanded by chiral invariance, remains valid, within errors. Since (broken) chiral symmetry is a property of the fundamental QCD Lagrangian this result is perhaps not unexpected, but is nonetheless reassuring.

# 4 Conclusion

In the previous sections we have examined the corrections which exist between vector, axial and strong couplings between nucleons and their counterparts in the NΔ sector. In the limit of degenerate nucleon and delta masses, these relations are simply the result of SU(6) symmetry. However, use of the constituent quark model provides significant mass-dependent corrections to these predictions. In each case the experimental NΔ coupling was found to be significantly larger than its predicted value, by amounts ranging from 25% to 35%. Despite these violations of the (mass-corrected) SU(6) predictions, the connection between the axial and strong NΔ couplings required by chiral symmetry—the Goldberger-Treiman relation—remains valid in both NN and $N\Delta$ sectors.

A question which has not been satisfactorily answered in previous investigations is the origin of these surprisingly large symmetry violations. Conventional attempts to understand such SU(6) violations via hyperfine gluonic interactions have not been successful [11]. However, the chiral soliton (Skyrmion) model also yields model-independent relations between diagonal and off-diagonal electroweak form factors, and those for the strong couplings[33]. These predictions work much better in experimental tests, suggesting the important role of degrees of freedom beyond those in the constituent quark model. It is interesting to note that the tree-level effective Lagrangian in the chiral soliton model already contains higher order physics derivable from chiral perturbation theory [34].

This brings us to another promising line of thinking, provided by heavy baryon techniques [35, 36], wherein one undertakes a rigorous expansion of transition amplitudes in terms of powers of q/M. Within this approach, one would expect the lowest order parameters to obey the symmetries of the constituent quark model. However, in higher order, these quantities are renormalized by the meson loop corrections

$$\mathcal{Q} \to \mathcal{Q}(1 - \lambda \frac{m_K^2}{16\pi^2 F_K^2} \ln(\frac{m_K^2}{\Lambda_\chi^2})), \tag{78}$$

where $\lambda$ is a constant of $\mathcal{O}(1)$, which depends upon the process being considered and $\Lambda_\chi \sim 1GeV$ is the chiral scale parameter. Since $m_K^2/16\pi^2 F_K^2 \sim 25\%$, such corrections have the potential to lead to symmetry violations of the size found above. However, as yet this is only speculation and further work is needed in order to test this hypothesis, as will be reported in a future communication.

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# A Notation

Many experimental papers on neutrino induced delta production use the notation of Llewellyn-Smith [18] and Schreiner *et al.*[22]. In this notation the N-$\Delta$ vector transition current reads

$$\begin{aligned} J^V_{\mu\Delta^i N}(\mathbf{p},\mathbf{p}') = \sqrt{3}\, \bar{\Delta}^\nu_i(\mathbf{p}') \Big\{ \Big[ \frac{C_3^V(q^2)}{M}\gamma^\lambda + \frac{C_4^V(q^2)}{M^2} p'^\lambda + \frac{C_5^V(q^2)}{M^2} p^\lambda \Big] \\ \times (q_\lambda g_{\mu\nu} - q_\nu g_{\lambda\mu}) + C_6^V(q^2) g_{\mu\nu} \Big\} \gamma_5 u(\mathbf{p}). \end{aligned} \tag{79}$$

$M$ denotes the mass of the nucleon and the four momentum transfer is defined as $q_\mu = p'_\mu - p_\mu$. Comparing with Eq. 19 we find the following connections to our formfactors $c_i(q^2)$:

$$c_1(0) = 2\sqrt{3} C_3^V(0), \tag{80}$$

$$c_2(0) = 4\sqrt{3}C_4^V(0), \tag{81}$$
$$c_3(0) = 4\sqrt{3}(C_4^V(0) + C_5^V(0)), \tag{82}$$
$$c_4(q^2) = \sqrt{3}C_6^V(q^2) = 0. \tag{83}$$

Eq. 83 results from CVC requirements.

For the N-$\Delta$ axial transition current experimentalists tend to use

$$\begin{aligned} J^A_{\mu\Delta^i N}(\mathbf{p},\mathbf{p}') &= \sqrt{3}\,\bar{\Delta}_i^\nu(\mathbf{p}')\Big\{\Big[\frac{C_3^A(q^2)}{M}\gamma^\lambda + \frac{C_4^A(q^2)}{M^2}p'^\lambda\Big](q_\lambda g_{\nu\mu} - q_\nu g_{\lambda\mu}) \\ &\quad + C_5^A(q^2)g_{\mu\nu} + \frac{C_6^A(q^2)}{M^2}q_\mu q_\nu\Big\}u(\mathbf{p}). \end{aligned} \tag{84}$$

This current corresponds to Eq. 44, if we make the following identifications

$$d_4(q^2) = \frac{\sqrt{3}}{2}\frac{M}{M_\Delta}C_3^A(q^2) = 0. \tag{85}$$

In our approach, the vanishing of this form factor arises from restricting ourselves solely to quarks bound in S-wave states. Several other calculations are consistent with our result (see Schreiner [22] for an overview of some) though the reasoning might be different.

Having eliminated $d_4(q^2)$ we find for the other form factors

$$d_1(0) = \sqrt{3}C_5^A(0), \tag{86}$$
$$d_2(0) = \frac{\sqrt{3}}{2}C_4^A(0), \tag{87}$$
$$d_3(0) = \frac{\sqrt{3}}{2}(2C_6^A(0) - C_4^A(0)). \tag{88}$$

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