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ES Na

BR Holstein
holstein@physics.umass.edu

Isospin Mixing and Model Dependence*

Euy Soo Na and Barry R. Holstein

Department of Physics and Astronomy
University of Massachusetts
Amherst, MA 01003

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Abstract

We show that recent calculations of $\Delta I = \frac{3}{2}$ effects in nonleptonic hyperon decay induced by $m_d - m_u \neq 0$ are subject to significant model dependence.

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1 Introduction

The isospin breaking caused by the u, d quark mass difference is well-known and significant. Indeed the fact the $m_n > m_p$ and the stability of the proton are a result of this non-degeneracy. Another consequence is that mass and isospin eigenstates are not the same—*e.g.* the physical Λ^0 and π^0 are admixtures of the pure $I = 0, 1$ states Λ_8, Σ_3 and π_8, π_3 respectively. Since such impurities are small—$\mathcal{O}(10^{-2})$—we may write[1]

$$\begin{aligned} \Lambda^0 &\approx \Lambda_8 + \theta_b \Sigma_3 \\ \pi^0 &\approx \pi_3 + \theta_m \pi_8 \end{aligned} \tag{1}$$

where the mixing angle is given in terms of quark mass differences as

$$\theta_m = -\theta_b = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}} \tag{2}$$

where $\hat{m} = \frac{1}{2}(m_u + m_d)$. The size of the quark mass difference is not completely pinned down, but recent work involving mesonic mass differences and $\eta \to 3\pi$ has indicated a value[2]

$$\frac{m_d - m_u}{m_s - \hat{m}} \approx 0.036 \tag{3}$$

which corresponds to a mixing angle

$$\theta_m = -\theta_b \approx 0.016 \tag{4}$$

Perhaps the theoretically cleanest indication of this mixing phenomenon occurs in the semileptonic $K_{\ell 3}$ decays wherein the ratio of reduced matrix elements for the decays

$$K^+ \to \pi^0 e^+ \nu_e \quad \text{and} \quad K_L^0 \to \pi^- e^+ \nu_e \tag{5}$$

are found experimentally to be in the ratio [3]

$$\left(\frac{f_+^{K^+\pi^0}(0)}{f_+^{K_L^0\pi^-}(0)} \right)^{\text{exp}} = 1.029 \pm 0.010 \tag{6}$$

Comparison with the theoretical estimate which arises from mixing

$$\left(\frac{f_+^{K^+\pi^0}(0)}{f_+^{K_L^0\pi^-}(0)}\right)^{\rm theory} = 1+\sqrt{3}\theta_m \tag{7}$$

yields a value

$$\theta_m = 0.017 \pm 0.005 \tag{8}$$

quite consistent with Eq. 4 and bears clear witness to the fact that $\pi^+\pi^0$ are *not* exact isotopic partners.

A particularly interesting and important consequence of this mixing occurs in the arena of nonleptonic weak decays, wherein the enhancement of $\Delta I = \frac{1}{2}$ transitions by a factor of twenty or so over their $\Delta I = \frac{3}{2}$ counterparts has long been an item of study.[4] The reason that particle mixing effects are particularly important in this venue is clear—a $\Delta I = \frac{1}{2}$ transition coupled with mixing of the order of several percent is of the *same order* as bona fide $\Delta I = \frac{3}{2}$ amplitudes. Such mixing contributions must then be subtracted from experimental $\Delta I = \frac{1}{2}$ rule violating amplitudes before confrontation with theoretical $\Delta I = \frac{3}{2}$ calculations is made, and such corrections are generally *significant*. In the case of $K \to 2\pi$, for example, we have

$$\begin{aligned} A(K^+\to\pi^+\pi^0) &\simeq \theta_m A(K^+\to\pi^+\pi_8) \\ A(K^0\to\pi^0\pi^0) &\simeq A(K^0\to\pi_3\pi_3)+2\theta_m A(K^0\to\pi_3\pi_8) \end{aligned} \tag{9}$$

The lowest order effective chiral Lagrangian describing this process is

$$\mathcal{L}_w = c_1 {\rm tr}\left(\lambda_6 D_\mu U D^\mu U^\dagger\right) \tag{10}$$

where

$$U = \exp\left(\frac{i}{F_\pi}\sum_j \lambda_j\phi_j\right) \tag{11}$$

is the usual chiral structure, with $F_\pi = 92.4$ MeV being the pion decay constant.[5] Then we find

$$A(K^+\to\pi^+\pi_8) = -\sqrt{2}A(K^0\to\pi_3\pi_8) = \sqrt{\frac{2}{3}}A(K^0\to\pi_3\pi_3) \tag{12}$$

If we then *define* the empirical $\Delta I = \frac{3}{2}$ amplitude via

$$ {}^{\frac{3}{2}}d_K^{\rm exp} = \frac{3A(K^+ \to \pi^+\pi^0)}{2A(K^0 \to \pi^0\pi^0) + A(K^0 \to \pi^+\pi^-)} \approx 0.069 \tag{13} $$

then the mixing contribution to ${}^{\frac{3}{2}}d_K$ is found to be

$$ {}^{\frac{3}{2}}d_K^{\rm mix} \simeq \sqrt{\frac{2}{3}}\theta_m \simeq 0.013 \tag{14} $$

leaving the isospin "pure" piece

$$ {}^{\frac{3}{2}}d_K^{\rm pure} = {}^{\frac{3}{2}}d_K^{\rm exp} - {}^{\frac{3}{2}}d_K^{\rm mix} \approx 0.056 \tag{15} $$

This analysis is fairly straightforward and is essentially model-independent, depending only on the underlying chiral symmetry of QCD. On the other hand, things are not so simple in the corresponding hyperon decay analysis, to which we now turn.

2 Nonleptonic Hyperon Decay

In the case of nonleptonic hyperon decay, things are more complex. Indeed there exist both S-wave (parity-violating) and P-wave (parity-conserving) amplitudes A and B respectively defined via

$$ {\rm Amp}(P \to P'\pi) = \bar{u}_{P'}(A + B\gamma_5)u_P \tag{16} $$

Also, there exist *seven* different channels with $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ components in each. We define $\Delta I = \frac{3}{2}$ parameters via

$$ \begin{aligned} c_\Lambda^{\frac{3}{2}} &= -\sqrt{\frac{1}{3}}\left[{\rm Amp}(\Lambda^0 \to p\pi^-) + \sqrt{2}{\rm Amp}(\Lambda^0 \to n\pi^0)\right] \\ c_\Sigma^{\frac{3}{2}} &= {\rm Amp}(\Sigma^+ \to n\pi^+) - {\rm Amp}(\Sigma^- \to n\pi^-) - \sqrt{2}{\rm Amp}(\Sigma^+ \to p\pi^0) \\ c_\Xi^{\frac{3}{2}} &= -\frac{2}{3}\left[{\rm Amp}(\Xi^- \to \Lambda^0\pi^-) + \sqrt{2}{\rm Amp}(\Xi^0 \to \Lambda^0\pi^0)\right] \end{aligned} \tag{17} $$

for A, B amplitudes respectively. The experimental values for each quantity are given in Table 1,[4] where all quoted numbers are in units of 10^{-7}. Since corresponding $\Delta I = \frac{1}{2}$ quantities are of order 5-15 ($\times 10^{-7}$) the $\Delta I = \frac{3}{2}$ suppression is clear.

In order to estimate the mixing contributions to these parameters, one needs a realistic model for nonleptonic hyperon decay and this is where the problem lies. Indeed in the standard picture S-wave amplitudes are given by the PCAC-commutator contributions[6]

$$\begin{aligned} < \pi^a P'|\mathcal{H}_w^{PV}|P > &= -\frac{i}{F_\pi} < P'|[F_a^5, \mathcal{H}_w^{PV}]|P > \\ &= -\frac{i}{F_\pi} < P'|[F_a, \mathcal{H}_w^{PC}]|P > \end{aligned} \tag{18}$$

while P-waves are represented by baryon pole terms

$$\begin{aligned} < \pi^a P'|\mathcal{H}_w^{PC}|P > &= \sum_{P''} < \pi^a P'|P'' > \frac{i}{m_P - m_{P''}} < P''|\mathcal{H}_w^{PC}|P > \\ &+ \sum_{P''} < P'|\mathcal{H}_w^{PC}|P'' > \frac{i}{m_{P'} - m_{P''}} < \pi^a P''|P > \end{aligned} \tag{19}$$

The weak parity-conserving baryon-baryon amplitudes are characterized via SU(3) F, D couplings as

$$< P_j|\mathcal{H}_w^{PC}|P_i >= \bar{u}_j(-if_{6ij}F + d_{6ij}D)u_i \tag{20}$$

The strong mesonic coupligns are represented in terms of the generalized Goldberger-Treiman relation as[7]

$$g_A^{ijk} = \frac{2F_\pi}{m_j + m_k} g^{ijk} \tag{21}$$

with the pseudoscalar couplings g^{ijk} given in terms of SU(3) f, d couplings as

$$g^{ijk} = -2(-if_{ijk}f + d_{ijk}d)g \tag{22}$$

with $g^2/4\pi \approx 13$. Then, for example, we have

$$\begin{aligned} A(\Sigma^+ \to p\pi^0) &= \frac{1}{F_\pi}(D - F) \\ A(\Lambda^0 \to p\pi^-) &= \frac{1}{\sqrt{3}F_\pi}(D + 3F) \\ &\quad etc. \end{aligned} \tag{23}$$

for S-wave amplitudes and

$$\begin{aligned}
B(\Sigma^+ \to p\pi^0) &= 2g(m_N + m_\Sigma)(\frac{(f+d)(F-D)}{2m_N(m_\Sigma - m_N)} \\
&- \frac{2f(F-D)}{2m_\Sigma(m_\Sigma - m_N)}) \\
B(\Lambda^0 \to p\pi^-) &= \frac{2}{\sqrt{3}}g(m_N + m_\Lambda)(\frac{(f+d)(3F+D)}{2m_N(m_\Lambda - m_N)} \\
&- \frac{2d(F-D)}{(m_\Sigma + m_\Lambda)(m_\Sigma - m_N)}) \\
&\mathit{etc.} \qquad (24)
\end{aligned}$$

for P-waves. In the case of the strong couplings the values[8]

$$f + d = 1, \quad \frac{d}{f} = 1.8 \qquad (25)$$

are generally accepted. However, there is no consensus for the weak parameters F, D. If one employs the values $D/F = -0.42$ and $F/2F_\pi = 1.13 \times 10^{-7}$ which provide a good fit to S-wave terms A then a poor fit is given for P-waves as shown as "model 1" in Table 1. On the other hand, using $D/F = -0.85$ and $F/2F_\pi = 1.83 \times 10^{-7}$ yields a good P-wave representation but a poor S-wave fit—*cf.* "model 2" in Table 1.[8] This problem has been known for a long time, and a definitive and widely accepted solution has yet to be found. One intriguing possibility was put forth by LeYaouanc et al., who point out that a reasonable fit to *both* S- and P-wave amplitudes can be provided (*cf.* "model 3" in Table 2) by appending intermediate state contributions from SU(6) $70, 1^-$ states to usual S-wave commutator terms.[10] Such contributions, of course, vanish in the soft pion limit if SU(3) invariance obtains, but in the real world such contributions can be sizable and when estimated using a simple constituent quark model seem to be able to provide a satisfactory resolution to the S/P dilemma. Of course, this suggestion is not unique and other possibilities have been proposed. However, our purpose in this note is not to provide a solution to the problem of hyperon decay but rather to study the model dependence of the mixing estimates.

In these various pictures of hyperon decay one can easily calculate the size of the mixing contributions to the experimental $\Delta I = \frac{1}{2}$ rule violating

	S-waves				P-waves			
	exp	model 1	model 2	model 3	exp	model 1	model 2	model 3
Λ^0_-	3.25	3.36	4.55	3.21	22.1	30.6	25.9	25.9
Σ^+_0	-3.27	-3.20	-6.78	-3.44	26.6	15.4	32.6	32.6
Σ^-_-	4.27	4.53	9.59	4.87	-1.44	-8.7	-1.1	-1.1
Ξ^-_-	-4.51	-4.45	-8.15	-5.08	16.6	-5.9	17.6	17.6

Table 1: Shown are values for the S-wave hyperon decay amplitudes A for various channels as obtained experimentally and in models. All numbers are to be multiplied by 10^{-7}. Models 1,2,3 are described in the text.

parameters $c_i^{\frac{3}{2}}$. In order to accomplish this program one requires various unphysical weak decay amplitudes, but these are straightforwardly calculated in the various models, yielding the results

$$\begin{aligned}
A(\Sigma^+ \to p\pi_8) &= -\frac{\sqrt{3}}{F_\pi}(D-F) + \frac{1}{F_\pi}(m_\Sigma - m_N)30C \\
A(\Lambda^0 \to n\pi_8) &= \frac{1}{\sqrt{2}F_\pi}(D+3F) + \frac{1}{F_\pi}(m_\Lambda - m_N)3\sqrt{6}C \\
A(\Sigma^0 \to p\pi^-) &= \frac{1}{F_\pi}(-D+F) - \frac{1}{F_\pi}(m_\Sigma - m_N)18\sqrt{3}C \\
A(\Xi^0 \to \Lambda^0\pi_8) &= -\frac{1}{\sqrt{2}F_\pi}(-D+3F) - \frac{1}{F_\pi}(m_\Xi - m_\Lambda)2\sqrt{6}C
\end{aligned} \quad (26)$$

where F, D are the weak decay parameters defined in Eq. 20 and

$$C = \frac{1}{4\sqrt{3}} G \cos\theta_C \sin\theta_C \frac{<\psi^s|\delta(r_1 - r_2)|\psi^s>}{m^2R^2\omega} \quad (27)$$

is a parameter defined by Le Yaouanc et al. which arises from the 1^- intermediate state contributions. From the S-wave fit given in Table 1 one determines $C \simeq 3.9 \times 10^{-9}$ and can then calculate the various contributions to $d_i^{\frac{3}{2}}$, yielding the results shown in Table 2.

Study of the numbers given in this table reveals the point of our note—mixing contributions to $\Delta I = \frac{3}{2}$ weak decay amplitudes are of the same size as the experimental numbers themselves *and* are quite model dependent.

	exp	model 1	model 2	model 3
$A_\Lambda^{\frac{3}{2}}$	0.059	-0.005	-0.113	-0.048
$A_\Xi^{\frac{3}{2}}$	-0.227	-0.051	-0.081	-0.098
$A_\Sigma^{\frac{3}{2}}$	0.485	0.118	0.249	0.317
$B_\Lambda^{\frac{3}{2}}$	0.141	0.500	0.546	0.546
$B_\Xi^{\frac{3}{2}}$	0.530	0.504	0.791	0.791
$B_\Sigma^{\frac{3}{2}}$	6.022	-0.256	-0.542	-0.542

Table 2: Shown are the predicted values of $\Delta I = 3/2$ amplitudes for both parity conserving and violating sectors of the hyperon decays compared to their experimental values. Models 1,2,3 are described in the text.

Indeed, Maltman recently calculated the values given for model 1, obtaining numbers which represent generally $\sim$ 25% corrections for S-waves and $\sim$ 100% corrections for P-waves.[11] We see, however, that results can be very different for models which are equally capable or describing the hyperon decay data. For instance, in the successful model of LaYouanc et al. the corrections in both S- and P-wave channels are found to be $\sim$ 100%, while we see from comparison of models 1 and 2 that even in the basic model the results are very sensitive to the values for the weak F,D coefficients which are chosen. We do not claim here then to reliably calculate the size of the simulated $\Delta I = \frac{3}{2}$ effect—rather to merely note the rather significant model dependence of same. This result has interesting implications for those attempting to calculate bona fide $\Delta I = \frac{3}{2}$ effects in nonleptonic decays when comparison with experiment is attempted, but those are the subject of another paper.

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