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Heavy Baryon Chiral Perturbation Theory
with Light Deltas[*]

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Abstract

We demonstrate how heavy mass methods, previously applied to chiral perturbation theory calculations involving the interactions of nucleons and pions, can be generalized to include interactions with the $\Delta(1232)$ in a systematic formalism which we call the "small scale expansion".

Contents

1   Introduction

The subject of chiral perturbation theory has become an important one in contemporary physics, as it represents a procedure by which to make rigorous contact between experimental measurements and the QCD Lagrangian which is thought to generate all hadronic interactions.[1] The way in which this is done is to make use of the underlying chiral symmetry of QCD in order to construct effective Lagrangians in terms of hadrons which retain this symmetry. Of course, in the real world chiral symmetry is broken by quark mass effects but these are assumed to be small and therefore treatable perturbatively. This program was begun in the 1960's but stalled when it was not recognized how to deal with the infinite number of such effective Lagrangians which can be written down.[2] In addition such theories are not renormalizable and this also served as a detriment to further development. Renewed interest, however, was generated in 1979 when a seminal paper by Weinberg demonstrated how to solve both problems.[3] The issue of non-renormalizability was shown to be a red herring. True, a full renormalization of such effective field theories would involve introduction of an infinite number of possible counterterm Lagrangians in order to cancel loop divergences. However, Weinberg showed that provided that one stays at energy,momentum low compared to the chiral scale[4]—$E, p < \Lambda_\chi \sim 1$ GeV—then a consistent renormalization scheme is possible and only a finite number of possible structures and their associated counterterms must be dealt with. These counterterm contributions not only remove loop divergences but also include finite pieces whose size can be determined empirically. Such terms encode the contributions from higher energy sectors of the theory, whose form need not be given explicitly. In addition a consistent power counting scheme can be developed. The point here is that the lowest order effective chiral Lagrangian is well-known to be of order p^2, meaning that it involves either two powers of energy-momentum or of the pseudoscalar mass, while it is easy to see that a one loop diagram generates terms of order p^4. Similarly it is straightforward to characterize any such contributions by its associated power of "momentum" and the program of chiral perturbation theory becomes feasible.

The program was actually carried out by Gasser and Leutwyler who developed a successful formalism to one loop—$\mathcal{O}(p^4)$—in the sector of Goldstone boson (π, K, η) interactions.[5] Their papers stimulated considerable work in this area, and consistent two-loop calculations are now being performed.[6]

This work has been reviewed in a number of places and it is not necessary to repeat it here.[7, 8]

A second arena of activity in this field has been in the low energy properties of the light baryons. In this case, however, things are not so straightforward. The problem is that, in addition to the "small" dimensionful parameters of energy-momentum and pseudoscalar mass, there exists also a "large" dimensionful number—the baryon mass M_B—which is comparable in size to the chiral scale itself, thus rendering the idea of a consistent perturbative treatment doubtful. Indeed Gasser, Sainio and Svarc [9] calculated $\pi - N$ scattering in a relativistic framework and showed that no consistent power counting in analogy to the meson-sector exists. Nevertheless several calculations in the relativistic framework were performed in the 1980s, *e.g.* [10, 11, 12]. However, by generalizing new developments in the field of heavy quark effective theory (HQET) [13], Jenkins and Manohar showed how "heavy baryon" methods could be used to eliminate the large mass term by going to the non-relativistic limit. [14] Subsequently Bernard et al. showed that a consistent power counting formulation of heavy baryon chiral perturbation theory (HBChPT) was possible [15] and did extensive calculations which fully developed the power of such methods for the low energy $\pi - N$ sector. A detailed review of this work is given in ref. [16]. More recently, important contributions to the renormalization of the theory were made by Ecker [17] and by Ecker and Moijzis [18, 19] for the case of SU(2) and Meißner and Müller [20] for the case of SU(3). Furthermore, for an overview of the rapidly evolving field of effective chiral lagrangians for the nucleon-nucleon interaction and applications to few body systems we refer to [21, 22].

Despite the excellent work done by these groups, a number of problems remain. One is that the convergence properties of the perturbative series may require inclusion of an unfortunately large number of terms, especially in the SU(3) version of HBChPT. This is not yet clear, however, and is the subject of current study. [23] A second difficulty is the way in which the contribution of resonant baryon states is handled. In the systematic works mentioned above [15, 16, 17, 18, 19, 20] it is assumed that such states are very heavy compared to the nucleon. In this case they can be integrated out and replaced by a finite piece of a counterterm contribution[1]. However, while this

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[1]There are plenty of "chiral calculations" in the literature which employ effective chiral lagrangians with explicit resonance degrees of freedom, for example see [21, 24] and

may be a reasonable scheme for heavier resonances such as the Roper and higher states, it is of questionable validity for the case of the $\Delta(1232)$, which lies a mere 300 MeV above the nucleon ground state and which couples very strongly to the $\pi - N$ sector. In fact, because of this strong coupling $\Delta(1232)$ contributions begin, in general, quite soon above threshold in those channels wherein such effects are possible. This suggests that instead of including the effects of the resonance by simple counterterms, it would be useful to include $\Delta(1232)$ as an explicit degree of freedom in the effective lagrangian. This has also been advocated by Jenkins and Manohar [25] and their collaborators [24].

Over the past few years [26] we have developed a consistent chiral power counting scheme–the so called "small scale expansion"–which builds upon the systematic HBChPT formalism of refs.[15, 16, 17, 18, 19] and in addition allows for explicit nucleon and delta degrees of freedom to be treated simultaneously in an SU(2) effective chiral lagrangian. Whereas in HBChPT one expands in powers of p analogous to the meson sector, in the "small scale expansion" one sets up a *phenomenological* expansion in the small scale ϵ denoting a soft momentum or the quark-mass or the nucleon-delta mass splitting $\Delta = M_\Delta - M_N$. Δ is a new dimensionful parameter in the theory which stays *finite* in the chiral limit. Strictly speaking the "small scale expansion" therefore has to be regarded as a *phenomenological extension* of pure (HB)ChPT. Given these caveats it should be evident that the addition of spin-3/2 resonances as explicit degrees of freedom to spin 1/2 HBChPT does not lead to one unique lagrangian as in HBChPT, but is dependent on the expansion scheme one is employing. Possible other expansion schemes that come to mind are the $SU(6)$ limit, where nucleon and delta states are mass-degenerate to leading order and the "heavy resonance" limit, where Δ would count as a dimensionful parameter of order p^0. For interesting and extensive work regarding the spin 3/2 resonances in the large N_c limit we refer the reader to ref.[27] In this paper we focus exclusively on the "small scale expansion".

In the next section we present the successful "heavy mass" expansion method developed in refs. [15, 28] for the simple pedagogical example of non-

references therein. In this work we do not comment specifically on these calculations but are only concerned with a *systematic extension* of the highly developed $SU(2)$ HBChPT πN formalism of refs.[15, 16, 17, 18, 19] to $\pi - N - \Delta$ interactions.

relativistic spinor electrodynamics and review spin 1/2 HBChPT. In the following section 3, we then show how this procedure can be generalized to deal with spin 3/2. Section 4 gives the main results of our formal development—giving a consistent chiral perturbative scheme for the joint interactions of pions, nucleons, and deltas to all orders. In section 5, we review the construction of counterterm contributions in the "small scale expansion" and give the pertinent lagrangians to $\mathcal{O}(\epsilon^2)$. We conclude our discussion at this point. First applications of this formalism can be found in [26, 29, 30, 31, 32].

2   1/M Expansion for Spin 1/2 Systems

2.1   Non-relivistic Spin 1/2 Electrodynamics

We begin with a brief review of heavy mass techniques for spin 1/2 systems. [15, 28] First, for simplicity, we assume that only electromagnetic interactions are included. Representing the nucleon as the two-component isospinor

$$\psi_N = \left(\begin{array}{c} \psi_p \\ \psi_n \end{array} \right) \tag{1}$$

with mass parameter M_0, the relativistic spin 1/2 Lagrangian has the familiar form

$$\mathcal{L}_N = \bar{\psi}_N \tfrac{1}{2}\Lambda^\gamma \psi_N \ . \tag{2}$$

For a simple model of spinor electrodynamics where the nucleons do not posses any anomalous magnetic moments one finds

$$\tfrac{1}{2}\Lambda^\gamma = i \not{D} - M_0 \ , \tag{3}$$

with the usual Dirac operator

$$D_\mu = \partial_\mu + i\frac{e}{2}(1 + \tau_3)A_\mu \tag{4}$$

being the covariant derivative. Here e represents the proton charge and is a positive quantity. In general, the four-momentum p_μ can be written as

$$p_\mu = M_0 v_\mu + k_\mu \tag{5}$$

where v_μ is a four-velocity satisfying $v^2 = 1$ and k_μ is a soft momentum satisfying $k_\mu << M_0, \Lambda_\chi$ for all $\mu = 0, 1, 2, 3$. By use of the operators

$$P_\pm = \frac{1}{2}(1 \pm \not{v}) \tag{6}$$

we define the "large"–N–and "small"–h–components of the nucleon field ψ_N via the relations

$$
\begin{aligned}
N(x) &\equiv \exp(iM_0 v \cdot x)P_+\psi_N(x) \ , \\
h(x) &\equiv \exp(iM_0 v \cdot x)P_-\psi_N(x) \ ,
\end{aligned}
\tag{7}
$$

where we have included also the factor $\exp(iM_0 v \cdot x)$ in order to eliminate the mass dependence in the time development factor.

The nucleon Lagrangian then assumes the form

$$\mathcal{L}_N = \bar{N}\mathcal{A}_N N + \bar{h}\mathcal{B}_N N + \bar{N}\tilde{\mathcal{B}}_N h - \bar{h}\mathcal{C}_N h \ , \tag{8}$$

where

$$
\begin{aligned}
\mathcal{A}_N &= P_+(\tfrac{1}{2}\Lambda^\gamma + M_0\not{v})P_+ \ , \\
\mathcal{B}_N &= P_-(\tfrac{1}{2}\Lambda^\gamma + M_0\not{v})P_+ \ , \\
\tilde{\mathcal{B}}_N &\equiv \gamma_0 \mathcal{B}_N^\dagger \gamma_0 = \mathcal{B}_N \ , \\
\mathcal{C}_N &= -P_-(\tfrac{1}{2}\Lambda^\gamma + M_0\not{v})P_- \ .
\end{aligned}
\tag{9}
$$

Using the projection operator identities

$$P_\pm P_\mp = 0 \ , \quad P_\pm P_\pm = P_\pm \ , \quad P_\pm \not{D} P_\pm = \pm v \cdot D P_\pm \ , \quad P_\pm \not{D} P_\mp = \not{D}^\perp \ , \tag{10}$$

one obtains

$$
\begin{aligned}
\mathcal{A}_N &= iv \cdot D \ , \\
\mathcal{B}_N &= i\not{D}^\perp = \tilde{\mathcal{B}}_N \ , \\
\mathcal{C}_N &= 2M_0 + iv \cdot D \ ,
\end{aligned}
\tag{11}
$$

where

$$\not{D}^\perp = \not{D} - \not{v} v \cdot D \tag{12}$$

is the transverse component of $\not{D}$. From Eq.(11) one can easily see that the original relativistic field ψ_N has been decomposed into a "quasi-massless"

("light") field N and a "heavy" field h with a mass parameter of twice the nucleon mass. Quantizing via path-integral methods, the functional integral

$$W[\text{sources}] = \text{const.} \int [dN][d\bar{N}][dh][d\bar{h}] \exp i \int d^4x \, (\mathcal{L}_N + \text{sourceterms}) \tag{13}$$

can be diagonalized via the field-redefinition

$$h' \equiv h - \mathcal{C}_N^{-1} \mathcal{B}_N N \, , \tag{14}$$

i.e.,

$$
\begin{aligned}
W[\text{sources}] &= \text{const.} \int [dN][d\bar{N}][dh'][d\bar{h}'] \exp i \int d^4x \\
&\quad \times \left(\bar{N}(\mathcal{A}_N + \tilde{\mathcal{B}}_N \mathcal{C}_N^{-1} \mathcal{B}_N)N - \bar{h}' \mathcal{C}_N h' + \text{source terms} \right) \\
&= \text{const.} \int [dN][d\bar{N}] det\,(\mathcal{C}_N) \exp i \int d^4x \\
&\quad \times \left(\bar{N}(\mathcal{A}_N + \tilde{\mathcal{B}}_N \mathcal{C}_N^{-1} \mathcal{B}_N)N + \text{source terms} \right) \, .
\end{aligned}
\tag{15}
$$

The determinant can be shown to yield a constant [28], leaving an effective lagrangian written only in terms of the light components–N

$$\mathcal{L}_{\text{eff}} = \bar{N}(\mathcal{A}_N + \tilde{\mathcal{B}}_N \mathcal{C}_N^{-1} \mathcal{B}_N)N \, . \tag{16}$$

For our simple example of spin $1/2$ electrodynamics the inverse operator $\mathcal{C}_N^{-1}$ can be expressed as a series

$$\mathcal{C}_N^{-1} = \frac{1}{2M_0} \sum_{n=0}^{\infty} \left(\frac{-iv \cdot D}{2M_0} \right)^n \, , \tag{17}$$

yielding the desired form for the effective action in terms of an expansion in powers of $1/M_0$. The large nucleon mass M_0 has been moved into interaction vertices, thus providing a convenient theory with only "light" degrees of freedom.

At lowest order we have simply

$$\mathcal{L}_{\text{eff}}^{(1)} = \bar{N}\mathcal{A}_N N = \bar{N}(iv \cdot \partial - \frac{e}{2}(1 + \tau_3)v \cdot A)N \, , \tag{18}$$

resulting in the leading order (free) nucleon propagator

$$D_N(k) = \frac{i}{v \cdot k + i\epsilon} \; , \tag{19}$$

where k is the soft momentum defined in Eq.(5).

The next higher order is then given by the first $1/M$ correction in Eq.(17). One finds

$$\mathcal{L}_{\text{eff}}^{(2)} = \bar{N}\tilde{\mathcal{B}}_N^{(1)}(\mathcal{C}_N^{-1})^{(0)}\mathcal{B}_N^{(1)}N = \bar{N}\frac{(i\,\slashed{D}^\perp)^2}{2M_0}N \; . \tag{20}$$

Defining the Pauli-Lubanski spin vector

$$S_\mu = \frac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu \tag{21}$$

which obeys the following (d-dimensional) relations

$$S \cdot v = 0, \quad \{S_\mu, S_\nu\} = \frac{1}{2}(v_\mu v_\nu - g_{\mu\nu}), \quad S^2 = \frac{(1-d)}{4},$$
$$[S_\mu, S_\nu] = i\epsilon_{\mu\nu\alpha\beta}v^\alpha S^\beta \; , \quad (d=4) \tag{22}$$

Eq.(20) can then be written in the form

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{1}{2M_0}\bar{N}\left\{(v \cdot D)^2 - D^2 + [S_\mu, S_\nu][D^\mu, D^\nu]\right\}N \; . \tag{23}$$

We recognize the spin-independent term then as a correction to the leading order propagator structure Eq.(19) and its spin-dependent partner as the Dirac component of the magnetic moment. If the spin $1/2$ nucleons are to posses an additional *anomalous* magnetic moment, the latter piece must be augmented by a counterterm of the same form. This is discussed in sections 4, 5. However, the spin-independent piece in Eq.(23) must not be modified by any counterterm contributions. This is associated with "reparametrization invariance" which necessitates that although there exists a freedom under the way in which the momentum p_μ is decomposed into its four-velocity and soft-momentum components, the square of the four-momentum must remain invariant. [33] That is to say, if one observer uses $p_\mu = M_0 v_\mu + k_\mu$ while another defines $p_\mu = M_0 v'_\mu + k'_\mu$ it is required that

$$2M_0 v \cdot k + k^2 = 2M_0 v' \cdot k' + k'^2 \; . \tag{24}$$

Since the leading piece of the effective action in momentum-space involves $v \cdot k$ we know that it must be accompanied by a next order term $\frac{k^2}{2M_0}$ and that the coefficient of this term must be unity for the identity Eq.(24) to hold. A different way to convince oneself that the spin-independent terms in Eq.(23) are protected from extra counterterm contributions for a wide class of theories has to do with the 2-photon (seagull) piece of $\mathcal{L}_{\text{eff}}^{(2)}$ which is quadratic in eA_μ. This structure must generate the familiar Thomson scattering amplitude for a nucleon, whose form is required by rigorous low energy theorems to be [34]

$$\text{Amp}_{\gamma\gamma NN} = \frac{e^2}{M_0} \epsilon \cdot \epsilon' \bar{N} \frac{1}{2}(1 + \tau_3) N \tag{25}$$

Higher order lagrangians $\mathcal{L}_N^{(n)}$, $n \geq 3$ in our example of non-relativistic spin 1/2 electrodynamics can be obtained via Eq.(17) and are suppressed by powers of $1/M_0^{(n-1)}$. The infinite series recovers the full relativistic theory Eq.(3).

Having become familiar with the formalism of (non-relativistic) heavy mass expansions in the simple example of spin 1/2 electrodynamics, it is now straightforward to move on to the example of HBChPT.

2.2 1/M Expansion in HBChPT

Interactions with pions and with general external axial/vector fields $\mathbf{v}_\mu, v_\mu^{(s)}, \mathbf{a}_\mu$ can be included in a chiral-invariant fashion via the operator [9]

$$\tfrac{1}{2}\Lambda^{(1)} = i \not{D} - M_0 + \frac{g_A}{2}\not{u}\gamma_5 \tag{26}$$

where

$$
\begin{aligned}
U &= u^2 = \exp(\frac{i}{F_\pi}\vec{\tau} \cdot \vec{\phi}) \\
D_\mu N &= (\partial_\mu + \Gamma_\mu - iv_\mu^{(s)})N \\
\Gamma_\mu &= \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}u^\dagger(\mathbf{v}_\mu + \mathbf{a}_\mu)u - \frac{i}{2}u(\mathbf{v}_\mu - \mathbf{a}_\mu)u^\dagger \equiv \tau^i\, \Gamma_\mu^i \\
u_\mu &= iu^\dagger\nabla_\mu U u^\dagger \equiv \tau^i\, w_\mu^i \\
\nabla_\mu U &= \partial_\mu U - i(\mathbf{v}_\mu + \mathbf{a}_\mu)U + iU(\mathbf{v}_\mu - \mathbf{a}_\mu).
\end{aligned} \tag{27}
$$

The effective action can be generated as before but now with

$$
\begin{aligned}
\mathcal{A}_N^{(1)} &= iv \cdot D + g_A S \cdot u \\
\mathcal{B}_N^{(1)} &= i \, \slashed{D}^{\perp} - \frac{1}{2} g_A (v \cdot u) \gamma_5 \\
\mathcal{C}_N^{(0)} &= 2M_0 \\
\mathcal{C}_N^{(1)} &= iv \cdot D + g_A S \cdot u
\end{aligned}
\tag{28}
$$

To lowest—$\mathcal{O}(p)$—order there exists a linear coupling to the isovector axial field $\vec{a}_\mu$

$$
\mathrm{Amp}_{NNa} = g_A \bar{N} S^\mu \vec{a}_\mu \cdot \frac{\vec{\tau}}{2} N
\tag{29}
$$

and we recognize $g_A = 1.26$ as the conventional nucleon axial vector coupling constant.

At next—$\mathcal{O}(p^2)$—order we find

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff}}^{(2)} &= \tilde{\mathcal{B}}_N^{(1)} (\mathcal{C}_N^{-1})^{(0)} \mathcal{B}_N^{(1)} \\
&= \frac{1}{2M_0} \bar{N} \Big[(v \cdot D)^2 - D^2 + [S_\mu, S_\nu][D^\mu, D^\nu] \\
&\qquad - ig_A (S \cdot D v \cdot u + v \cdot u S \cdot D) - \frac{1}{4} g_A^2 (v \cdot u)^2 \Big] N
\end{aligned}
\tag{30}
$$

Here the meaning of the first three terms was given above for the case of electromagnetic coupling. However, the D_μ terms now denote *chiral* covariant derivatives containing new pion and axial vector source couplings in addition to the (minimal) photon coupling discussed in the previous section. The fourth piece gives the Kroll-Ruderman term in charged pion photoproduction[35] while the last term starts out as a two-pion coupling to the nucleon with the coupling strength fixed by g_A.

3 1/M Expansion for Spin 3/2 Systems

Before launching into the parallel discussion of the heavy baryon treatment of spin 3/2 systems it is useful to first note a technical but important point about the characterization of such systems.

3.1 Point-invariance and Spin 3/2

We begin with the standard relativistic form of the Lagrangian for a spin 3/2 field Ψ_μ and (bare) mass parameter M_Δ. Throughout this work spin 3/2 fields are represented via a Rarita-Schwinger spinor [36], for an overview of this formalism see Appendix A. We have

$$\mathcal{L}_{3/2} = \bar{\Psi}^\alpha {}^{\frac{3}{2}}\Lambda(A)_{\alpha\beta}\Psi^\beta \tag{31}$$

where

$$\begin{aligned}
{}^{\frac{3}{2}}\Lambda(A)_{\alpha\beta} &= -\Big[(i\slashed{\partial} - M)g_{\alpha\beta} + iA(\gamma_\alpha\partial_\beta + \gamma_\beta\partial_\alpha) \\
&\quad + \frac{i}{2}(3A^2 + 2A + 1)\gamma_\alpha\slashed{\partial}\gamma_\beta + M_\Delta(3A^2 + 3A + 1)\gamma_\alpha\gamma_\beta\Big]
\end{aligned} \tag{32}$$

contains a free and unphysical ("gauge") parameter A ($A \neq -\frac{1}{2}$). The origin of this parameter-dependence lies in the feature that such relativistic spin 3/2 systems must be invariant under the so called "point - transformation" [37, 38]

$$\begin{aligned}
\Psi_\mu(x) &\rightarrow \Psi_\mu(x) + a\gamma_\mu\gamma_\nu\Psi^\nu(x) \\
A &\rightarrow \frac{A - 2a}{1 + 4a}
\end{aligned} \tag{33}$$

which simply says that an arbitrary admixture a ($a \neq -1/4$) of "spurious" or "off-shell" spin 1/2 components $\gamma_\nu\Psi^\nu(x)$ which are *always* present in the relativistic spin 3/2 field $\Psi_\mu(x)$ can be compensated by a corresponding change in the parameter A to leave the Lagrangian Eq.(31) invariant. For $A = -1/3$ one recovers the original lagrangian of Rarita and Schwinger [36], whereas in the more recent literature one tends to use $A = -1$, see *e.g.* [39]. Physical quantities are of course guaranteed to be independent of the choice of the A parameter by virtue of the "KOS-theorem" [40].

For our purposes it is convenient to employ a form for the relativistic spin 3/2 Lagrangian that was written down by Pascalutsa [41]. The advantage of this formulation, when compared with the more familiar one used in ref. [39] for example, lies in the feature that it allows the absorption of any dependence on the unphysical parameter A into a matrix $O_{\mu\nu}^A$, resulting in the form

$$\mathcal{L}_{3/2} = \bar{\Psi}^\alpha O_{\alpha\mu}^A {}^{\frac{3}{2}}\Lambda^{\mu\nu}O_{\nu\beta}^A\Psi^\beta \tag{34}$$

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where

$$O^A_{\alpha\mu} = g_{\alpha\mu} + \frac{1}{2}A\gamma_\alpha\gamma_\mu \tag{35}$$

contains all the A-dependence leaving

$$\tfrac{3}{2}\Lambda^{free}_{\alpha\beta} = -[i(\not{\partial} - M_\Delta)g_{\alpha\beta} - \frac{1}{4}\gamma_\alpha\gamma_\lambda(i\not{\partial} - M_\Delta)\gamma^\lambda\gamma_\beta] \tag{36}$$

as the A-independent leading-order free spin $3/2$ lagrangian with the redefined spin $3/2$ field

$$\psi_\mu(x) = O^A_{\mu\nu}\Psi^\nu(x) . \tag{37}$$

ψ_μ is guaranteed to satisfy all point transformation requirements by construction. Our (non-relativistic) lagrangians written in terms of the ψ_μ fields are therefore independent of A and from now on we will only work with the redefined fields ψ_μ.

3.2 Non-relativistic Spin 3/2 Electrodynamics

Having addressed the requirement of point-invariance, it is useful to begin the development of our formalism, as in the case of the nucleon, by first dealing with the inclusion only of minimal electromagnetic coupling. In order to do so, we must also take account of isotopic spin considerations.

3.2.1 Isospin 3/2

Since the Δ carries $I = 3/2$ it is convenient to use a (Rarita-Schwinger-like) isospurion notation, wherein an isospin $3/2$ state is described by an isospinor which carries also a 3-vector index and which is subject to the constraint

$$\tau^i \psi^i_\mu(x) = 0 , \quad i = 1, 2, 3 , \tag{38}$$

where τ^i is a 2-component Pauli matrix in isospin space. For details on this isospin formalism, the reader is referred to Appendix B. Thus the Δ field is described by the symbol $\psi^i_\mu(x)$, which is both a Lorentz vector and spinor as well as an isotopic vector and spinor. In this case then the relativistic Lagrangian describing the interaction of the delta with the minimally coupled[2]

---

[2]We are not discussing the so-called Rarita-Schwinger gauges [36] here as they would imply constraints on the possible choices for the spin $1/2$ admixture A. We will set up the formalism so that any choice of A except for $(A \neq -1/2)$ can be accommodated.

electromagnetic field is given by

$$\mathcal{L}_{S=3/2,I=3/2} = \bar{\psi}_i^\mu \, {}^{\frac{3}{2}}\Lambda_{\mu\nu}^{(0)ij} \, \psi_j^\nu \tag{39}$$

where the operator

$${}^{\frac{3}{2}}\Lambda_{\mu\nu}^{(0)ij} = -[(i \, \slashed{D}^{ij} - M_\Delta \delta^{ij})g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma^\lambda(i \, \slashed{D}^{ij} - M_\Delta \delta^{ij})\gamma^\lambda\gamma_\nu] \tag{40}$$

besides being 4x4 Dirac matrix and second rank Lorentz-tensor, is also a 2x2 isotopic matrix as well as a second rank tensor in isotopic spin space. Here and below Lorentz indices will always be denoted by Greek symbols while isotopic spin indices will be designated by Roman letters. The minimally coupled covariant derivative is given by

$$D_\mu^{ij} = \partial_\mu\delta^{ij} + ie[\frac{1}{2}(1+\tau_3)\delta^{ij} - i\epsilon^{ij3}]A_\mu \tag{41}$$

which is also a 2x2 matrix as well as a second-rank tensor in isotopic spin space. The interested reader can easily verify that the operator

$$I_3^{ij} = \frac{1}{2}\tau_3\delta^{ij} - i\epsilon^{ij3} \tag{42}$$

when acting on the state Ψ^j given in Eq.(129) merely multiplies each component of the delta by its appropriate value of I_3—e.g.

$$I_3^{3i}\Psi^i = -\sqrt{\frac{2}{3}}\left[\begin{array}{c} \frac{1}{2}\Delta^+ \\ -\frac{1}{2}\Delta^0 \end{array} \right] \tag{43}$$

Having set up our Lagrangian we can proceed to the development of the appropriate heavy baryon formalism.

3.2.2 Transition to Heavy Baryon Fields

In this section we discuss the transformation of the relativistic theory of spin 3/2 particles into the corresponding heavy baryon form. The calculation is analogous to that given for the nucleon sector and described in section 2. However, it is also considerably more complex, since the relativistic Lagrangian formulation of spin 3/2 fields Eq.(39) contains on-shell and off-shell components, spin 3/2 and two independent spin 1/2 degrees of freedom, large

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and small Dirac spinor pieces, leading order and $1/M_\Delta$ contributions, etc., all combined in a very compact notation. Our goal, as stated in the introduction, is to disentangle all these contributions and to develop a systematic $1/M_\Delta$ expansion in terms of a Lagrangian involving the "large" component spin 3/2-isospin 3/2 fields as the only explicit degree of freedom, *without* losing any of the physics contained in Eq.(39).

Our first step is to identify the spin 3/2 and (two) spin 1/2 degrees of freedom. Using the (appropriately modified) spin projection operators from the relativistic theory—*cf.* Eq.(125)—we introduce a complete set of orthonormal spin projection operators[3]

$$
\begin{aligned}
_{(33)}P^{3/2}_{\mu\nu} &= g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3}\left(\slashed{v}\gamma_\mu v_\nu + v_\mu\gamma_\nu\slashed{v}\right) \\
_{(11)}P^{1/2}_{\mu\nu} &= \frac{1}{3}\gamma_\mu\gamma_\nu - v_\mu v_\nu + \frac{1}{3}\left(\slashed{v}\gamma_\mu v_\nu + v_\mu\gamma_\nu\slashed{v}\right) \\
_{(22)}P^{1/2}_{\mu\nu} &= v_\mu v_\nu \\
_{(12)}P^{1/2}_{\mu\nu} &= \frac{1}{\sqrt{3}}\left(v_\mu v_\nu - \slashed{v}v_\nu\gamma_\mu\right) \\
_{(21)}P^{1/2}_{\mu\nu} &= \frac{1}{\sqrt{3}}\left(\slashed{v}v_\mu\gamma_\nu - v_\mu v_\nu\right)
\end{aligned}
\tag{44}
$$

obeying

$$
\begin{aligned}
_{(33)}P^{3/2}_{\mu\nu} + {}_{(11)}P^{1/2}_{\mu\nu} \ + \ {}_{(22)}P^{1/2}_{\mu\nu} &= g_{\mu\nu} \\
_{(ij)}P^I_{\mu\nu}\,{}_{(kl)}P^{J,\nu}{}_\delta &= \delta^{IJ}\,\delta_{jk}\,{}_{(il)}P^J_{\mu\delta}
\end{aligned}
\tag{45}
$$

With the use of these projectors we split up ψ^i_μ into three independent degrees of freedom

$$
\begin{aligned}
^{(1)}\psi^i_\mu(x) &= {}_{(11)}P^{1/2}_{\mu\nu}\psi^\nu_j(x) \\
^{(2)}\psi^i_\mu(x) &= {}_{(22)}P^{1/2}_{\mu\nu}\psi^\nu_j(x) \\
^{(3)}\psi^i_\mu(x) &= {}_{(33)}P^{3/2}_{\mu\nu}\psi^\nu_j(x)
\end{aligned}
\tag{46}
$$

---

[3]In the theory of heavy quarks one also has to deal with the problem of spin 3/2 baryons. For a discussion of the use of projection operators in HQET we refer to the article by Falk [42].

in terms of which the operator $\frac{3}{2}\Lambda_{\mu\nu}^{(0)}$, which defines the relativistic Lagrangian, becomes a 3x3 matrix, with

$$
\begin{aligned}
\tfrac{3}{2}_{(33)}\Lambda^{(0)\mu\nu} &= -[(i\,\slashed{D} - M_\Delta)g^{\mu\nu}] \\
\tfrac{3}{2}_{(31)}\Lambda^{(0)\mu\nu} &= \tfrac{3}{2}_{(13)}\Lambda^{(0)\mu\nu} = -[i\,\slashed{D}g^{\mu\nu}] \\
\tfrac{3}{2}_{(23)}\Lambda^{(0)\mu\nu} &= \tfrac{3}{2}_{(32)}\Lambda^{(0)\nu\mu} = 0 \\
\tfrac{3}{2}_{(11)}\Lambda^{(0)\mu\nu} &= -\left[(i\,\slashed{D} - M_\Delta)g^{\mu\nu} + \frac{i}{2}\gamma^\mu\,\slashed{D}\gamma^\nu + 3M_\Delta g^{\mu\nu}\right] \\
\tfrac{3}{2}_{(12)}\Lambda^{(0)\mu\nu} &= -\left[\frac{i}{2}\gamma^\mu\,\slashed{D}\gamma^\nu + \sqrt{3}M_\Delta\,_{(12)}P_{1/2}^{\mu\nu}\right] \\
\tfrac{3}{2}_{(21)}\Lambda^{(0)\mu\nu} &= -\left[\frac{i}{2}\gamma^\mu\,\slashed{D}\gamma^\nu + \sqrt{3}M_\Delta\,_{(21)}P_{1/2}^{\mu\nu}\right] \\
\tfrac{3}{2}_{(22)}\Lambda^{(0)\mu\nu} &= -\left[(i\,\slashed{D} - M_\Delta)g^{\mu\nu} + \frac{i}{2}\gamma^\mu\,\slashed{D}\gamma^\nu + M_\Delta g^{\mu\nu}\right]
\end{aligned} \tag{47}
$$

The leading order contribution here is $\tfrac{3}{2}_{(33)}\Lambda^{(0)\mu\nu}$, as this is the only piece which involves only spin 3/2 degrees of freedom. However, it remains to decompose this field into large and small components and to eliminate the M_Δ dependence in the time development factor as in the analogous nucleon calculation. Thus we define "large" and "small" spin 3/2 fields via

$$
\begin{aligned}
T_\mu^i(x) &= P_+\,^{(3)}\psi_\mu^i(x)\exp(iM_\Delta v\cdot x) \\
^{(3-)}h_\mu^i(x) &= P_-\,^{(3)}\psi_\mu^i(x)\exp(iM_\Delta v\cdot x)
\end{aligned} \tag{48}
$$

which satisfy the constraints

$$
\begin{aligned}
v_\mu T_i^\mu &= \gamma_\mu T_i^\mu = 0 \\
v_\mu\,^{(3-)}h_i^\mu &= \gamma_\mu\,^{(3-)}h_i^\mu = 0
\end{aligned} \tag{49}
$$

Eq.(49) gives the heavy baryon analogues to the subsidiary condition Eq.(115) for the relativistic Rarita-Schwinger spinor of Appendix A. However, we note that $\partial_\mu T_i^\mu = 0$ is in general not true in the heavy baryon formalism, unlike in the relativistic case Eq.(116)

Here the fields $T_i^\mu(x)$ are the (field-redefined) SU(2) isospin analogues of the decuplet fields used in ref.[25], whereas the $^{(3-)}h_i^\mu$ degrees of freedom are not considered there. In addition, if we are not content with just the leading

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order heavy baryon spin 3/2 Lagrangian, then we must also introduce heavy baryon fields for the (off-shell) spin 1/2 components via

$$^{(\alpha\pm)}h^i_\mu(x) = P_\pm \ ^{(\alpha)}\psi^i_\mu(x) \exp(iM_\Delta v \cdot x), \qquad (50)$$

with $\alpha = 1, 2$ labeling the independent spin-1/2 sectors with which we are dealing.

The six resulting degrees of freedom for a spin 3/2 heavy baryon formalism are therefore $T^\mu_i(x)$ and

$$G^i_\mu = \begin{pmatrix} ^{(3-)}h^i_\mu \\ ^{(1+)}h^i_\mu \\ ^{(2-)}h^i_\mu \\ ^{(1-)}h^i_\mu \\ ^{(2+)}h^i_\mu \end{pmatrix}, \qquad (51)$$

where the small component spin 3/2 field and the spin 1/2 contributions have for notational convenience been combined in the column vector $G'^\mu_i(x)$. Suppressing all four-vector and isospin indices the heavy baryon Lagrangian corresponding to Eq.(39) then can be written in the simple form

$$\mathcal{L}_\Delta = \bar{T}\mathcal{A}_\Delta T + \bar{G}\mathcal{B}_\Delta T + \bar{T}\gamma_0\mathcal{B}^\dagger_\Delta\gamma_0 G - \bar{G}\mathcal{C}_\Delta G, \qquad (52)$$

where $\mathcal{A}_\Delta, \mathcal{B}_\Delta, \mathcal{C}_\Delta$ are all matrices containing the (electromagnetic) covariant derivative in our example of (non-relativistic) spin 3/2 electrodynamics and whose explicit forms will be given in the next section. However, while Eq.(52) looks very much like its nucleonic analogue Eq.(8), it must be kept in mind that there are also important differences. Specifically, in terms of our six-component decomposition into large and small spin 3/2 and 1/2 fields $\mathcal{A}_\Delta$ is a number, $\mathcal{B}_\Delta$ is a five-component column vector, while $\mathcal{C}_\Delta$ is a 5x5 matrix.

3.2.3 The Interaction with Photons

We can now proceed to determine the desired effective Lagrangian in terms of the "large" spin 3/2 field T^μ by integrating out the G^μ degrees of freedom just as done for the analogous nucleonic system—shifting G^μ fields via

$$G \to G' + \mathcal{C}^{-1}_\Delta \ \mathcal{B}^{(1)}_\Delta \ T \qquad (53)$$

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the heavy baryon Lagrangian is diagonalized. We then integrate out the unwanted G' fields, yielding

$$\mathcal{L}_\Delta = \bar{T}\left[\mathcal{A}_\Delta + \gamma_0 \mathcal{B}_\Delta^\dagger \gamma_0 \mathcal{C}_\Delta^{-1} \mathcal{B}_\Delta\right] T \tag{54}$$

which is the result we seek. Given that we are only interested in the next-to-leading order corrections to the $\mathcal{O}(\epsilon)$ lagrangians in this paper the process of "integrating out" the unwanted fields is trivial. It just amounts to dropping the (decoupled) G' terms in the lagrangian. However, in general this procedure can be more involved. To carry this procedure to $\mathcal{O}(\epsilon^3)$ one carefully has to keep all the source dependencies to obtain the proper wavefunction renormalization (e.g. [19]). Furthermore, the resulting determinant in general need not be equal to unity anymore as in standard HBChPT because it contains particle and anti-particle excitations of the spin $1/2$ (off-shell) components of the relativistic delta theory. A detailed study of these and other effects is in preparation [43] but the $\mathcal{O}(\epsilon^2)$ results given in this paper should remain unaffected.

Specifically, the leading order (non-relativistic) $\gamma\Delta\Delta$ lagrangian is simply given by the term

$$\mathcal{L}_{\Delta\Delta\gamma}^{(1)} = \bar{T}_i^\mu(x)\, \mathcal{A}_{\Delta,\mu\nu}^{ij}\, T_j^\nu(x) \tag{55}$$

where

$$\mathcal{A}_{\Delta,\mu\nu}^{ij} = -i\, v \cdot D^{ij}\, g_{\mu\nu} . \tag{56}$$

The (free) leading order Δ propagator in the spin $3/2$ isospin $3/2$ subspace is then

$$D_F(p)_{\mu\nu}^{ij} = \frac{-i}{v \cdot k + i\epsilon}\, P_{\mu\nu}^{3/2}\, \xi_{3/2}^{ij} , \tag{57}$$

with the d-dimensional non-relativistic spin $3/2$ projector

$$\begin{aligned}
P_{\mu\nu}^{3/2} &\equiv\ P_v^+\ _{(33)}P_{\mu\nu}^{3/2}\, P_v^+ \\
&=\ g_{\mu\nu} - v_\mu v_\nu + \frac{4}{d-1} S_\mu S_\nu
\end{aligned} \tag{58}$$

and the isospin $3/2$ projector $\xi_{3/2}^{ij}$ defined in Appendix B. Note that we have split up the Δ momentum as

$$p_\mu = M_\Delta v_\mu + k_\mu , \tag{59}$$

where k_μ refers to off-shell momentum in the propagator Eq.(57).

	3-	1+	2-	1-	2+
3-	M	.	.	.	.
1+	.	M	M	.	.
2-	.	M	M	.	.
1-	.	.	.	M	M
2+	.	.	.	M	M

Table 1: M_Δ dependent terms in the matrix $\mathcal{C}_\Delta$

The first higher order—$1/M_\Delta$—correction then has the form

$$\mathcal{L}^{(2)}_{\Delta\Delta\gamma} = \bar{T}\tilde{\mathcal{B}}_\Delta (\mathcal{C}^{-1}_\Delta)^{(0)} \mathcal{B}_\Delta T \tag{60}$$

which looks very much like the corresponding nucleon expression Eq.(18) except that $\mathcal{B}, \mathcal{C}$ are vector, matrix quantities as emphasized above. The column vector $\mathcal{B}$ is straightforwardly found to be

$$\mathcal{B}^{ij}_{\Delta,\mu\nu} = - \begin{pmatrix} P_- \left[{}_{(33)} P^{3/2}_{\mu\nu} i\, \slashed{D}^{ij}_\perp \right] P_+ \\[2ex] 0 \\[2ex] 0 \\[2ex] P_- \left[{}_{(11)} P^{1/2}_{\mu\nu} i\, \slashed{D}^{ij}_\perp \right] P_+ \\[2ex] 0 \end{pmatrix} \tag{61}$$

However, determining the inverse of the 5x5 matrix $\mathcal{C}_\Delta$ presents more of a challenge. Following our procedure in the case of the nucleon, we shall use a representation in terms of a power series in $(1/M_\Delta)^n$. Thus in order to find the leading piece $(C^{-1}_\Delta)^{(0)}$ we need only take into account the terms in matrix $\mathcal{C}_\Delta$ that depend explicitly on M_Δ. These mass dependent terms in $\mathcal{C}_\Delta$ are schematically displayed in Table 1, with "." denoting kinetic energy contributions. One recognizes a convenient block-diagonal structure in the mass terms, which facilitates the construction of the inverse matrix. The explicit construction for $\mathcal{C}^{-1}_\Delta$ at $\mathcal{O}(1/M_\Delta)$ is given in Appendix C, refuting claims of its non-existence [44] by explicit construction.

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Putting all these results together we arrive finally at the first non-relativistic correction to the leading order minimally coupled $\gamma\Delta\Delta$ Lagrangian Eq.(55)

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\Delta\Delta\gamma} &= -\frac{1}{2M_\Delta}\bar{T}^\mu_i(x)[(v\cdot D^{ik}v\cdot D^{kj} - D^{ik}_\alpha D^{kj}_\beta g^{\alpha\beta})g_{\mu\nu} \\
&+ [S^\alpha, S^\beta](D^{ik}_\alpha D^{kj}_\beta - D^{ik}_\beta D^{kj}_\alpha)g_{\mu\nu}]T^\nu_j
\end{aligned}
\tag{62}
$$

The physics here is identical to the case of the nucleon—we recognize immediately the kinetic energy term and the interaction of the Dirac moment with the electromagnetic field. Likewise, from the former we can read off the seagull term which generates the Thomson scattering amplitude

$$
\mathrm{Amp}_{\gamma\gamma TT} = -\frac{e^2}{M_\Delta}\epsilon\cdot\epsilon'\bar{T}^i_\mu(\delta_{ij}\frac{1}{2}(3+\tau_3) - i\epsilon_{3ij}(1+\tau_3) - \delta_{i3}\delta_{j3})g_{\mu\nu}T^j_\nu
\tag{63}
$$

whose form is required by the low energy theorems.

3.3 1/M Expansion for Chiral Spin 3/2 Lagrangians

In the previous subsection we carried out the calculation in some detail since when interactions with pions have to be included the form becomes considerably more complex and it is useful to have become familiar with the formalism in the simpler minimally-coupled case. For the leading order relativistic chiral spin 3/2 lagrangian we employ the matrix

$$
\begin{aligned}
{}^{\frac{3}{2}}\Lambda^{\pi ij}_{\mu\nu} &= -[(i\,\slashed{D}^{ij} - M_\Delta\delta^{ij})g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma^\lambda(i\,\slashed{D}^{ij} - M_\Delta\delta^{ij})\gamma_\lambda\gamma_\nu \\
&+ \frac{g_1}{2}g_{\mu\nu}\,\slashed{u}^{ij}\gamma_5 + \frac{g_2}{2}(\gamma_\mu u^{ij}_\nu + u^{ij}_\mu\gamma_\nu)\gamma_5 \\
&+ \frac{g_3}{2}\gamma_\mu\,\slashed{u}^{ij}\gamma_5\gamma_\nu]
\end{aligned}
\tag{64}
$$

where

$$
u^{ij}_\mu = \xi^{ik}_{3/2}\,u_\mu\,\xi^{kj}_{3/2}
\tag{65}
$$

is used to generate the pion couplings[4] . Note that D^{ij}_μ is now the chiral covariant derivative which includes coupling to the pions as well as to external

---

[4]In the definition of u^{ij}_μ one can have additional terms proportional to $i\epsilon^{ijk}w^k_\mu$. However, one can absorb their contributions into redefinitions of the coupling constants g_i, $i = 1, 2, 3$ with the help of the isospin algebra given in Appendix B.

vector,axial fields $\mathbf{v}_\mu, v_\mu^{(s)}, \mathbf{a}_\mu$ via

$$D_\mu^{ij} = \partial_\mu \delta^{ij} + C_\mu^{ij} \tag{66}$$

where

$$C_\mu^{ij} = \delta^{ij} \left(\Gamma_\mu - iv_\mu^{(s)} \right) - 2i\epsilon^{ijk}\Gamma_\mu^k \tag{67}$$

We also have extended the lagrangian of the previous section by appending three interaction terms with coupling constants g_1, g_2, g_3, which represent the most general, chiral- and Lorentz-invariant way to couple a pion to a spin 3/2 particle. Note that g_2, g_3 only contribute if one,both of the spin 3/2 fields is,are off-mass-shell. In the following it will become clear that off-shell parameters such as g_2, g_3 do not pose a problem for the "small scale expansion" but can be treated on the same footing as any other coupling constant when performing the transition from the fully relativistic lagrangians to the non-relativistic ones. Although little if anything is known at this point about the off-shell couplings g_2, g_3, one could in principle utilize the SU(6) quark model in order to estimate the size of the physical pion-delta coupling constant g_1, yielding

$$g_1 = \frac{9}{5}g_A . \tag{68}$$

However, we will treat g_1 on the same footing as any other low energy constant in chiral perturbation theory and trust that in subsequent work g_1, g_2, g_3 can be extracted from fits to experimental data.

It is then straightforward to use projection operators to find the forms for the quantities $\mathcal{A}_\Delta, \mathcal{B}_\Delta, \mathcal{C}_\Delta$

$$\mathcal{A}_{\Delta,\mu\nu}^{(1),ij} = -[iv \cdot D^{ij} + g_1 S \cdot u^{ij}]g_{\mu\nu} \tag{69}$$

$$\mathcal{B}_{\Delta,\mu\nu}^{(1)ij} = - \begin{pmatrix} P_-[_{(33)}P_{\mu\nu}^{3/2}i \slashed{D}^{\perp ij} - \frac{1}{2}g_{1(33)}P_{\mu\nu}^{3/2}v \cdot u^{ij}\gamma_5]P_+ \\ P_+[(\frac{2}{3}g_1 + g_2)S_\mu u_\nu^{ij}]P_+ \\ P_-[-\frac{1}{2}g_2 v_\mu u_\nu^{ij}\gamma_5]P_+ \\ P_-[_{(11)}P_{\mu\nu}^{1/2}i \slashed{D}^{\perp ij}]P_+ \\ 0 \end{pmatrix} \tag{70}$$

In the case of the matrix $\mathcal{C}_\Delta^{-1}$ we require only the leading $\mathcal{O}(\epsilon^0)$ form already given in Appendix C since we are working only to $\mathcal{O}(\epsilon^2)$.

To leading order we have then

$$\mathcal{L}_{\Delta\Delta}^{(1)} = \bar{T}_i^\mu(x) \, \mathcal{A}_{\Delta,\mu\nu}^{(1)ij} \, T_j^\nu(x) \tag{71}$$

with $\mathcal{A}_{\Delta,\mu\nu}^{(1)ij}$ given in Eq.(69). The form of this leading order lagrangian should not come as a surprise. For the special case of $v^\mu = (1,0,0,0)$ it corresponds to the non-relativistic delta isobar model from the 1970s [45]. For a general v^μ it reproduces the lagrangian of Jenkins and Manohar [25]. All mass dependence is gone[5], as desired. The first term contains the kinetic energy of the spin 3/2 particle and provides, among other things, a minimally coupled interaction with the electromagnetic field, whereas the second term starts out as a one pion vertex with derivative coupling to the spin 3/2 fields.

At next—$\mathcal{O}(\epsilon^2)$—order we find the $1/M_\Delta$ Lagrangian for spin 3/2 particles as

$$
\begin{aligned}
\mathcal{L}_\Delta^{(2)} &= \tilde{\mathcal{B}}_\Delta^{(1)} (\mathcal{C}_\Delta^{-1})^{(0)} \mathcal{B}_\Delta^{(1)} \\
&= \frac{1}{2M_\Delta} \, \bar{T}_i^\mu(x) \left\{ \left[D_\alpha^{ik} D_\beta^{kj} \, g^{\alpha\beta} - v \cdot D^{ik} v \cdot D^{kj} \right] \, g_{\mu\nu} \right. \\
&\qquad + g_1 \, i \left(S \cdot D^{ik} v \cdot u^{kj} + v \cdot u^{ik} S \cdot D^{kj} \right) \, g_{\mu\nu} \\
&\qquad - \left[S^\alpha, S^\beta \right] \left(D_\alpha^{ik} D_\beta^{kj} - D_\beta^{ik} D_{kj}^\alpha \right) \, g_{\mu\nu} \\
&\qquad + \frac{g_1^2}{4} v \cdot u^{ik} v \cdot u_{kj} \, g_{\mu\nu} \\
&\qquad \left. - u_\mu^{ik} u_\nu^{kj} \, \frac{g_1^2 + 4 g_1 g_2 + 3 g_2^2}{3} \right\} T_j^\nu(x) \, . \tag{72}
\end{aligned}
$$

We can now discuss the physics contained in this new $1/M_\Delta$ Lagrangian for spin 3/2 particles, only mentioning the most prominent features. As before, the first term in Eq.(72) contains the kinetic energy and the two photon vertex which accounts for Thomson scattering. The second term corresponds to the Kroll-Ruderman term in the nucleon sector, i.e. it contains a one photon one pion vertex. Note that in our construction of the $\mathcal{O}(\epsilon^2)$ counterterms in section 5 we do not expect to find a term that has the

---

[5]Note that for the case of a coupled spin 1/2 - spin 3/2 system in general one is left with a residual mass dependence in the non-relativistic lagrangians. This will be discussed in section 4.

same structure as either one of these two expressions—neither vertex gets renormalized at $\mathcal{O}(\epsilon^2)$ due to reparameterization invariance [33], just as in the nucleon system. The third expression gives a one photon vertex at $\mathcal{O}(1/M_\Delta)$, which corresponds to a photon coupled to the Dirac moment of a spin $3/2$ particle. This vertex will get renormalized by counterterms corresponding to anomalous magnetic moments of spin $3/2$ particles. Finally, the fourth and fifth term, among other contributions, generate two-pion vertices at lowest order. Note that the fifth term does not have an analogue in the nucleon system and depends on the unknown $\mathcal{O}(\epsilon)$ off-shell coupling constant g_2 of Eq.(64).

At this point we have achieved our goal of constructing the next to leading order spin $3/2$ Lagrangian for a pure spin $3/2$ system. In principle one can generalize our procedure to obtain even higher order corrections. However, in most practical applications of this formalism one has to deal with the somewhat more complex situation of having both spin $3/2$ ($\Delta(1232)$) and spin $1/2$ degrees of freedom (nucleons) present simultaneously. We now shift the discussion to this more general case then, noting that we will reutilize many of the structures that we defined in this section.

4 Simultaneous 1/M Expansion for Coupled Delta and Nucleon Systems

4.1 Nucleon-Delta Transition Lagrangians

For the non-relativistic reduction of the $\pi N \Delta$ system we first require the relativistic lagrangians which couple N and Δ degrees of freedom and satisfy invariance under "point-transformations" as discussed in section 3.1. We note that the general relativistic $\mathcal{O}(\epsilon^n)$ $N\Delta$ transition lagrangian can be written as

$$\mathcal{L}^{(n)}_{\pi N \Delta}[A] \;\; = \;\; \bar{\psi}_N \, O^{(n)}_\mu \, \Theta^{\mu\nu}_A \Psi_\nu + h.c. \;, \tag{73}$$

where $O^{(n)}_\mu$ is a general chiral transition matrix to order ϵ^n and $\Theta^A_{\mu\nu}$ is the most general (Dirac-) tensor that guarantees "point-transformation" invariance (Eq.(33)) for the relativistic spin $3/2$ Rarita-Schwinger field Ψ_μ. Nath,

Etemadi and Kimel [38] have determined this most general tensor to be

$$\Theta^A_{\mu\nu} = g_{\mu\nu} + \left[z_i + \frac{1}{2}\left(1 + 4z_i\right)A\right]\gamma_\mu\gamma_\nu \, , \tag{74}$$

where z_i denotes a free parameter which governs the coupling of the (off-shell) spin $1/2$ components to a given matrix $O^{(n)}_{\mu,\,i}$. Guided by the note by Pascalutsa [41], we again observe that the A-dependence factors out and write

$$\begin{aligned}
\Theta^A_{\mu\nu} &= \left(g_{\mu\alpha} + z_i\gamma_\mu\gamma_\alpha\right)\left(g^\alpha_\nu + \frac{A}{2}\gamma^\alpha\gamma_\nu\right) \\
&= \Theta_{\mu\alpha}(z_i)\, O^{\alpha\,A}_\nu \, ,
\end{aligned} \tag{75}$$

with $O^A_{\alpha\beta}$ given by Eq.(35). Therefore we can subsume all dependencies on the (unphysical) parameter A via the same field redefinition Eq.(37) already introduced in our discussion of the pure spin $3/2$ sector in section 3.1 and find

$$\mathcal{L}^{(n)}_{\pi N\Delta} = \bar{\psi}_N\, O^{(n)}_{\mu,\,i}\, \Theta^{\mu\nu}(z_i)\, \psi_\nu + h.c. \, , \tag{76}$$

where ψ_ν denotes the field-redefined spin $3/2$ field of Eq.(37).

For the leading—$\mathcal{O}(\epsilon)$–order relativistic $N\Delta$ transition lagrangian we therefore write[6]

$$\mathcal{L}^{(1)}_{\pi N\Delta} = g_{\pi N\Delta}\left\{\bar{\psi}^\mu_i\, \Theta_{\mu\alpha}(z_0)\, w^\alpha_i\, \psi_N + \bar{\psi}_N\, w^{\alpha\dagger}_i\, \Theta_{\alpha\mu}(z_0)\, \psi^\mu_i\right\} \tag{77}$$

with

$$\Theta_{\mu\nu}(z_0) = g_{\mu\nu} + z_0\gamma_\mu\gamma_\nu \, . \tag{78}$$

We therefore note that in the leading order *relativistic* $N\Delta$ lagrangian one finds two independent $\pi N\Delta$ couplings—the so-called "on-shell" coupling constant $g_{\pi N\Delta}$ and the so-called "off-shell parameter" z_0. While a determination of $g_{\pi N\Delta}$ within the "small scale expansion" is given in the next section, there is no agreement in the literature about the magnitude and status of z_0. Many

---

[6]We have made the assumption that the leading order $N\Delta$ lagrangian can be written down without terms that involve the nucleon or delta equations of motion. See section 5.3 for a more detailed discussion of this issue.

years ago Peccei [46] argued for $z_0 = -1/4$, while Nath, Etemadi and Kimel [38] determined $z_0 = +1/2$ from requirements of local causality. More recently the RPI-group [39] has argued to leave all "off-shell parameters" as free parameters to be fitted from physical observables. In this work we will adopt the later approach and treat z_0 as a free parameter to be determined in future applications of this theory, as our formalism does not *require* a specific choice for it.

4.2   Leading Order Lagrangians

The full relativistic Lagrangian that we must consider for a system of nucleon, Δ and pion degrees of freedom consists of four terms:

$$\mathcal{L} = \mathcal{L}_{\pi N} + \mathcal{L}_{\pi \Delta} + \mathcal{L}_{\pi \Delta N} + \mathcal{L}_{\pi \pi} \tag{79}$$

The piece of the Lagrangian solely involving pions is given by the standard expressions of ref. [5] and is not affected by the heavy baryon transformations to $\mathcal{O}(\epsilon^2)$. For the other three components of the Lagrangian we follow the same philosophy as in section 2. We start from the leading order relativistic Lagrangians in order to construct the leading order heavy baryon Lagrangians. The relativistic πN, $\pi \Delta$ and $\pi N \Delta$ Lagrangians have been given above in Eqs.(26,64) and Eq.(77).

Given our detailed discussion in section 2, one can now read off the leading heavy baryon Lagrangians by replacing the relativistic fields with their corresponding "large" heavy baryon fields

$$\psi_N(x) \quad \rightarrow \quad P_v^+ \, N(x) \, \exp(-iM_0 \, v \cdot x) \tag{80}$$

$$\psi_\mu^i(x) \quad \rightarrow \quad P_v^+ \, T_j^v(x) \, \exp(-iM_0 \, v \cdot x) \tag{81}$$

For nucleons, the transition from relativistic $\psi(x)$ to heavy baryon $N(x)$ was reviewed in section 2. The prescription given here to obtain heavy baryon delta degrees of freedom $T^\mu(x)$ is identical to the definition in Eq.(48), except that we now make the choice

$$\exp(-iM_0 v \cdot x) \tag{82}$$

for the exponential time dependence in order to recover the standard form of $\mathcal{L}_{\pi N}^{(1)}$ as given in ref. [15]. We find then, to lowest order

$$\mathcal{L}_{\pi N}^{(1)} \quad = \quad \bar{N} \left[i \, v \cdot D \, + \, g_A \, S \cdot u \right] N$$

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$$\mathcal{L}_{\pi\Delta}^{(1)} = -\bar{T}_i^\mu \left[i \, v \cdot D^{ij} - \delta^{ij}\Delta_0 + g_1 \, S \cdot u^{ij} \right] g_{\mu\nu} \, T_j^\nu$$

$$\mathcal{L}_{\pi N\Delta}^{(1)} = g_{\pi N\Delta} \left\{ \bar{T}_i^\mu \, g_{\mu\alpha} \, w_i^\alpha \, N \; + \; \bar{N} \, w_i^{\alpha\dagger} \, g_{\alpha\mu} \, T_i^\mu \right\} \tag{83}$$

with $\Delta_0 = M_\Delta - M_0$ being the (bare) nucleon-delta mass splitting. Aside from this parameter, there are no other changes in the leading order delta Lagrangian $\mathcal{L}_{\pi\Delta}^{(1)}$ when compared with Eq.(71) of section 3, as expected. In contrast to the underlying relativistic lagrangian Eq.(77) the leading order non-relativistic $\pi N\Delta$ Lagrangian $\mathcal{L}_{\pi N\Delta}^{(1)}$ contains only the on-shell coupling structure proportional to $g_{\pi N\Delta}$. This has also been noted by Lucio and Napsuciale [47].

The non-relativistic constituent quark model in the SU(6) limit suggests

$$g_{\pi N\Delta}^{QM} = g_A \frac{6}{5\sqrt{2}} \approx 1.07 \; . \tag{84}$$

However, in contrast to NN and $\Delta\Delta$ transitions it is known that for the case of the nucleon-delta transition the quark model estimates are not very reliable [48]. We regard it therefore as mandatory that all $N\Delta$-transition couplings be fit from experiment. This said, we have obtained [43] a consistent determination[7] of $g_{\pi N\Delta}$ from a fit to the decay width of the delta resonance to *leading order* in the "small scale expansion", which *is* consistent with the SU(6) quark model

$$g_{\pi N\Delta}^{\mathrm{HHK}} = 1.05 \pm 0.02 \; . \tag{85}$$

Details of this determination will be given in a future communication as we are just concerned with the general structure of the lagrangians in this paper.

We now move on to consider the $\mathcal{O}(\epsilon^2)$—$1/M$—corrections to these leading order lagrangians.

4.3 1/M Corrected Lagrangians for Nucleon-Delta Interactions

The leading order relativistic Lagrangians of section 4.2 contain all the information necessary in order to construct the first $1/M$ corrections.

---

[7]Note that the leading "small scale expansion" value for $g_{\pi N\Delta}$ given here is different from the one obtained in the phenomenological (relativistic) analyses of pion photoproduction in the $\Delta(1232)$ region (*e.g.* [39]), which is mainly due to the non-relativistic kinematics of the heavy mass method.

At this point it is useful to clarify what is meant by "$1/M$" in a system which contains *two* independent mass scales, the (bare) nucleon mass M_0 and the (bare) delta mass M_Δ. First, we rewrite all mass dependent expressions into the set M_0 and $\Delta = M_\Delta - M_0$. In order to establish a systematic chiral power counting for our Lagrangians, we then take the limit $\Delta << M_0$ and expand all quantities as a power series in $1/(2M_0)$, with appearances of the "small" scale Δ therefore restricted to numerators[8]. This suppression by twice the (bare) mass of the nucleon is what we mean by the phrase $1/M$. Let us note here, that the choice M_0, Δ as the two independent mass scales is not unique, but provides us with the most convenient way to match onto the already existing heavy baryon formalism for pure nucleon degrees of freedom, as for example laid out in ref.[16].

For notational simplicity we will again suppress all four-vector and isospin indices. Aside from the "large" spin 3/2 component $T^\mu(x)$, we now also have to consider the other five delta degrees of freedom already discussed in section 3.2. As before, we denote these collectively $G^\mu(x)$, but now with the mass parameter being M_0. Furthermore, using the "large" and "small" heavy nucleon fields N, h defined in section 2, we have a complete set of degrees of freedom with which to rewrite the relativistic Lagrangians in term of heavy baryon fields—

$$
\begin{aligned}
\mathcal{L}_{\pi N} &= \bar{N}\mathbf{A}_N N + \bar{h}\,\mathbf{B}_N N + \bar{N}\,\gamma_0 \mathbf{B}_N^\dagger \gamma_0\,h - \bar{h}\,\mathbf{C}_N\,h \\
\mathcal{L}_{\pi\Delta} &= \bar{T}\mathbf{A}_\Delta T + \bar{G}\,\mathbf{B}_\Delta T + \bar{T}\,\gamma_0\mathbf{B}_\Delta^\dagger\gamma_0\,G - \bar{G}\,\mathbf{C}_\Delta\,G \\
\mathcal{L}_{\pi\Delta N} &= \bar{T}\mathbf{A}_{\Delta N} N + \bar{G}\,\mathbf{B}_{\Delta N} N + \bar{T}\,\gamma_0\mathbf{D}_{N\Delta}^\dagger\gamma_0\,h + \bar{G}\,\gamma_0\mathbf{C}_{N\Delta}^\dagger\gamma_0\,h \\
&+ \bar{N}\,\gamma_0\mathbf{A}_{\Delta N}^\dagger\gamma_0\,T + \bar{N}\,\gamma_0\mathbf{B}_{\Delta N}^\dagger\gamma_0\,G + \bar{h}\,\mathbf{D}_{N\Delta}T + \bar{h}\,\mathbf{C}_{N\Delta}\,G \quad (86)
\end{aligned}
$$

If one is only interested in the $\mathcal{O}(\epsilon^2)$ $1/M$ corrections, several useful simplifications occur in these forms:

i) all matrices $\mathbf{A}_X, \mathbf{B}_X, \mathbf{C}_X, \mathbf{D}_X$ are only needed to $\mathcal{O}(\epsilon)$. The structures depending on $\mathbf{A}_X^{(1)}$ matrices are then given by the $\mathcal{O}(\epsilon)$ Lagrangians of Eq.(83).

ii) the matrix $\mathbf{D}_{N\Delta}^{(1)}$ is found to be identically zero.

---

[8]This is certainly correct for all terms appearing in our Lagrangians. However, the "small" scale Δ can enter into the denominator of amplitudes through the spin 3/2 propagator or spin 3/2 kinematic coefficients. For an example, see our discussion on neutral pion photoproduction in ref.[26]

iii) all terms in Eq.(86) that couple $G^\mu(x)$ and $h(x)$ degrees of freedom can be discarded in this section as they only start contributing at $\mathcal{O}(\epsilon^4)$.

In order to obtain the Lagrangians at $\mathcal{O}(\epsilon^2)$, one can therefore simply use the variable shift of ref. [15] for the "small" nucleon component

$$h \to h' + \mathbf{C}_N^{-1} \, \mathbf{B}_N^{(1)} \, N \tag{87}$$

and then integrate out the $h'(x)$ degrees of freedom as in section 2. Similarly, we introduce an analogous shift for the $G^\mu(x)$ fields

$$G \to G' + \mathbf{C}_\Delta^{-1} \, \mathbf{B}_\Delta^{(1)} \, T + \mathbf{C}_\Delta^{-1} \, \mathbf{B}_{\Delta N}^{(1)} \, N, \tag{88}$$

in order to write our Lagrangians exclusively in terms of the "large" fields $T^\mu(x)$ and $N(x)$. Note that the variable change in Eq.(88) is more complex than the corresponding one in section 3 due to the addition of spin $1/2$ fields. Finally, integrating out the $G'(x)$ degrees of freedom[9] leaves us with the following formal expressions for the $\mathcal{O}(\epsilon^2)$ $1/M$ Lagrangians:

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(2)} &= \bar{N}\left[\gamma_0 \mathbf{B}_N^{(1)\dagger}\gamma_0 \mathbf{C}_N^{-1}\mathbf{B}_N^{(1)} + \gamma_0\mathbf{B}_{\Delta N}^{(1)\dagger}\gamma_0\mathbf{C}_\Delta^{-1}\mathbf{B}_{\Delta N}^{(1)}\right] N \\
\mathcal{L}_{\pi\Delta}^{(2)} &= \bar{T}\gamma_0\mathbf{B}_\Delta^{(1)\dagger}\gamma_0\mathbf{C}_\Delta^{-1}\mathbf{B}_\Delta^{(1)}T \\
\mathcal{L}_{\pi\Delta N}^{(2)} &= \bar{T}\gamma_0\mathbf{B}_\Delta^{(1)\dagger}\gamma_0\mathbf{C}_\Delta^{-1}\mathbf{B}_{\Delta N}^{(1)}N + \bar{N}\gamma_0\mathbf{B}_{\Delta N}^{(1)\dagger}\gamma_0\mathbf{C}_\Delta^{-1}\mathbf{B}_\Delta^{(1)}T
\end{aligned}
\tag{89}
$$

We begin our discussion with the $\mathcal{O}(\epsilon^2)$ $1/M$ Lagrangian for nucleons $\mathcal{L}_{\pi N}^{(2)}$. Aside from the well known piece $\gamma_0\mathbf{B}_N^{(1)\dagger}\gamma_0\mathbf{C}_N^{-1}\mathbf{B}_N^{(1)}$ of Eq.(30) in section 2.2, we encounter the new contributions $\gamma_0\mathbf{B}_{\Delta N}^{(1)\dagger}\gamma_0\mathbf{C}_\Delta^{-1}\mathbf{B}_{\Delta N}^{(1)}$ which are proportional to $g_{\pi N\Delta}^2$. As one can very clearly see from the matrix structure, these terms arise from intermediate $G^\mu(x)$ states that were integrated out, resulting in new $1/M$ suppressed two-pion vertices in the nucleon Lagrangian. Starting from the relativistic nucleon-delta transition Lagrangian $\mathcal{L}_{\pi N\Delta}$ and translating into the heavy baryon formalism as outlined in section 3 for the case of the pure spin $3/2$ Lagrangian, we find the explicit representation for the matrix $\mathbf{B}_{\Delta N}^{(1)}$

---

[9]Any possible effects by the associated determinant can again only start showing up at $\mathcal{O}(\epsilon^3)$ and are therefore not considered here.

$$
\mathbf{B}^{(1)i}_{\Delta N, \mu} \;\; = \;\;
\begin{pmatrix}
0 \\[1.2em]
g_{\pi N\Delta} \left(1 + 3z_0\right) P_{+\;(11)} P^{1/2}_{\mu\nu} \, P_{+} \, w^{i\nu} \\[1.2em]
g_{\pi N\Delta} \, \sqrt{3} \, z_0 \, P_{-\;(21)} P^{1/2}_{\mu\nu} \, P_{+} \, w^{i\nu} \\[1.2em]
g_{\pi N\Delta} \, \sqrt{3} \, z_0 \, P_{-\;(12)} P^{1/2}_{\mu\nu} \, P_{+} \, w^{i\nu} \\[1.2em]
g_{\pi N\Delta} \left(1 + z_0\right) P_{+\;(22)} P^{1/2}_{\mu\nu} \, P_{+} \, w^{i\nu}
\end{pmatrix}
\tag{90}
$$

Furthermore, we need $\mathbf{C}^{-1}_{\Delta}$ at $\mathcal{O}(1/M)$ which is given in Appendix C. After some algebra one then arrives at

$$
\mathcal{L}^{(2)}_{\pi N} \;\; = \;\; \bar{N} \, \gamma_0 \mathbf{B}^{(1)\dagger}_{N} \gamma_0 \, \mathbf{C}^{-1}_{N} \, \mathbf{B}^{(1)}_{N} \, N - \frac{1}{2M_0} \, \bar{N} \left[g^{(2a)}_{\pi\pi NN} \, w^{i\dagger} \cdot S \, \xi^{ij}_{3/2} \, S \cdot w^j \right.
$$
$$
\left. + g^{(2b)}_{\pi\pi NN} \, w^{i\dagger} \cdot v \, \xi^{ij}_{3/2} \, v \cdot w^j \, \right] N
\tag{91}
$$

One realizes that the two new structures start out as $\pi\pi NN$ coupling terms, as expected from the leading order $\pi N\Delta$ lagrangian Eq.(77). The new two-pion coupling constants $g^{(2a)}_{\pi\pi NN}, g^{(2b)}_{\pi\pi NN}$ are given in terms of the $\mathcal{O}(\epsilon)$ nucleon-delta coupling constants $g_{\pi N\Delta}, z_0$ and are likely to play a significant role in low-energy pion-nucleon scattering and near-threshold two-pion production amplitudes. We note that these two coupling constants are related to the $\mathcal{O}(\epsilon^2)$ counterterms c_2, c_3, c_4 of ref.[16]. A new analysis of the anatomy of these counterterms is called for. Here we only define the general structures

$$
\begin{aligned}
g^{(2a)}_{\pi\pi NN} &= g^2_{\pi N\Delta} \, \frac{4}{3} \left(1 + 8z_0 + 12z_0^2\right) \\
g^{(2b)}_{\pi\pi NN} &= g^2_{\pi N\Delta} \, \frac{1}{3} \left(5 - 8z_0 - 4z_0^2\right)
\end{aligned}
\tag{92}
$$

We now move on to the $\mathcal{O}(\epsilon^2)$ $1/M$ Lagrangian for deltas $\mathcal{L}^{(2)}_{\pi\Delta}$. Noting that $\mathbf{B}^{(1)}_{\Delta} = \mathcal{B}^{(1)}_{\Delta}$ and employing $\mathbf{C}^{-1}_{\Delta}$ from Appendix C, we determine

$$
\mathcal{L}^{(2)}_{\pi\Delta} = \frac{1}{2M_0} \, \bar{T}^{\mu}_i(x) \quad \Big\{ \quad \Big[D^{ik}_{\alpha} D^{kj}_{\beta} \, g^{\alpha\beta} - v \cdot D^{ik} v \cdot D^{kj} \Big] \, g_{\mu\nu}
$$

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$$
\begin{aligned}
+ \quad & g_1 \, i \left(S \cdot D^{ik} v \cdot u^{kj} + v \cdot u^{ik} S \cdot D^{kj} \right) \, g_{\mu\nu} \\
- \quad & \left[S^\alpha, S^\beta \right] \left(D^{ik}_\alpha D^{kj}_\beta - D^{ik}_\beta D^{kj}_\alpha \right) \, g_{\mu\nu} \\
+ \quad & \frac{g_1^2}{4} v \cdot u^{ik} v \cdot u^{kj} \, g_{\mu\nu} \\
- \quad & g_{\pi\pi\Delta\Delta} \, u^{ik}_\mu u^{kj}_\nu \Big\} \, T^\nu_j(x)
\end{aligned}
\tag{93}
$$

with

$$
g_{\pi\pi\Delta\Delta} = \frac{1}{3} \left(g_1^2 + 4 \, g_1 \, g_2 + 3 \, g_2^2 \right)
\tag{94}
$$

This form is identical to that of Eq.(72) except for the change $1/2M_\Delta \to 1/2M_0$ and the associated physics has already been discussed in section 3.4. The $1/M$ suppressed two-pion delta-delta coupling $g_{\pi\pi\Delta\Delta}$ is in principle determined in terms of the $\mathcal{O}(\epsilon)$ coupling constants g_1, g_2. At present, however, we have no information about the off-shell coupling constant g_2 and therefore treat $g_{\pi\pi\Delta}$ as a low energy constant of $\mathcal{O}(\epsilon^2)$ to be fitted from future experiment. Theoretically speaking, it nevertheless has a status different from the $\mathcal{O}(\epsilon^2)$ counterterms of section 5.

Finally, we turn to the $\mathcal{O}(\epsilon^2)$ $1/M$ nucleon-delta transition Lagrangian $\mathcal{L}_{\pi N\Delta}$. With $\mathbf{B}^{(1)}_\Delta$ and $\mathbf{B}^{(1)}_{\Delta N}$ from above and $(\mathbf{C}^{-1}_\Delta)^{(0)}$ from Appendix C we find

$$
\mathcal{L}^{(2)}_{\pi\Delta N} = \frac{-1}{2M_0} \, \bar{T}^\mu_i(x) \left[h_{\pi\pi N\Delta} \, u^{ij}_\mu \xi^{jk}_{3/2} \, S \cdot w^k + 2 \, g_{\pi N\Delta} \, i D^{ij}_\mu \xi^{jk}_{3/2} \, v \cdot w^k \right] N(x) + h.c.
\tag{95}
$$

with

$$
h_{\pi\pi N\Delta} = \frac{2}{3} \, g_{\pi N\Delta} \left(g_1 + 2 \, g_2 + 4 \, g_1 \, z_0 + 6 \, g_2 \, z_0 \right)
\tag{96}
$$

The first term in this $\mathcal{O}(\epsilon^2)$ nucleon-delta transition Lagrangian starts out as a two-pion vertex. In fact, this is the lowest order vertex of this kind, as the corresponding $\mathcal{O}(\epsilon)$ Lagrangian $\mathcal{L}^{(1)}_{\pi\Delta N}$ Eq.(77) contains only vertices involving odd number of pions. The new coupling constant $h_{\pi\pi N\Delta}$ is of the same type as $g_{\pi\pi\Delta\Delta}$. In principle it is fixed in terms of the $\mathcal{O}(\epsilon)$ coupling constants $g_{\pi N\Delta}, z_0, g_1, g_2$, but again in practice we are ignorant about the size of g_2 and therefore must treat it as a free parameter of $\mathcal{O}(\epsilon^2)$, at least for now. The second term in Eq.(95) is completely determined by the $\mathcal{O}(\epsilon)$ coupling constant $g_{\pi N\Delta}$ and starts out as a $1/M$ suppressed one-pion vertex,

which is non-vanishing even at threshold. As we have shown in ref.[26], this structure contributes to neutral pion photoproduction at order ϵ^3.

This concludes our discussion on the $1/M$ corrections at $\mathcal{O}(\epsilon^2)$. We next introduce a general formalism which allows construction of terms to arbitrary order in the chiral expansion.

4.4 The Small Scale Expansion to All Orders

We begin our all orders discussion from the set of nucleon and delta Lagrangians that have already been translated into the heavy baryon formalism, using the notation introduced above to illustrate our method. As has been done in section 3, we will use path integral methods to integrate out "small" components of the Dirac wavefunctions. The first step is to decouple the $h(x)$ and $G^\mu(x)$ degrees of freedom via the change of variables

$$G \rightarrow G' + \mathbf{C}_\Delta^{-1} \, \gamma_0 \mathbf{C}_{N\Delta}^\dagger \gamma_0 \, h, \tag{97}$$

resulting in the new set of Lagrangians

$$
\begin{aligned}
\mathcal{L}'_{\pi N} &= \bar{N} \, \mathbf{A}_N \, N + \bar{h} \, \tilde{\mathbf{B}}_N \, N + \bar{N} \, \gamma_0 \tilde{\mathbf{B}}_N^\dagger \gamma_0 \, h - \bar{h} \, \tilde{\mathbf{C}}_N \, h \\
\mathcal{L}'_{\pi\Delta} &= \bar{T} \, \mathbf{A}_\Delta \, T + \bar{G}' \, \mathbf{B}_\Delta \, T + \bar{T} \, \gamma_0 \mathbf{B}_\Delta^\dagger \gamma_0 \, G' - \bar{G}' \, \mathbf{C}_\Delta \, G' \\
\mathcal{L}'_{\pi\Delta N} &= \bar{T} \, \mathbf{A}_{\Delta N} \, N + \bar{G}' \, \mathbf{B}_{\Delta N} \, N + \bar{T} \, \gamma_0 \tilde{\mathbf{D}}_{N\Delta}^\dagger \gamma_0 \, h \\
&+ \bar{h}\tilde{\mathbf{D}}_{N\Delta}T + \bar{N}\gamma_0\mathbf{B}_{\boldsymbol{\Delta N}}^\dagger\gamma_0\mathbf{G}' + \bar{\mathbf{N}}\gamma_0\mathbf{A}_{\boldsymbol{\Delta N}}^\dagger\gamma_0\mathbf{T}
\end{aligned} \tag{98}
$$

with

$$
\begin{aligned}
\tilde{\mathbf{B}}_N &= \mathbf{B}_N + \mathbf{C}_{N\Delta} \, C_\Delta^{-1} \, \mathbf{B}_{\Delta N} \\
\tilde{\mathbf{C}}_N &= \mathbf{C}_N - \mathbf{C}_{N\Delta} \, C_\Delta^{-1} \, \mathbf{C}_{N\Delta}^\dagger \\
\tilde{\mathbf{D}}_{N\Delta} &= \mathbf{D}_{N\Delta} + \mathbf{C}_{N\Delta} \, C_\Delta^{-1} \, \mathbf{B}_\Delta
\end{aligned} \tag{99}
$$

The next step involves decoupling the "small" fields $h(x), G^{\mu\prime}(x)$ from the "large" fields $N(x), T^\mu(x)$—we shift the fields according to

$$
\begin{aligned}
G' &\rightarrow G'' + \mathbf{C}_\Delta^{-1} \, \mathbf{B}_\Delta \, T + \mathbf{C}_\Delta^{-1} \, \mathbf{B}_{\Delta N} \, N \\
h &\rightarrow h' + \tilde{\mathbf{C}}_N^{-1} \, \tilde{\mathbf{B}}_N \, N + \tilde{\mathbf{C}}_N^{-1} \, \tilde{\mathbf{D}}_{N\Delta} \, T
\end{aligned} \tag{100}
$$

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and then integrate out the quantities $h'(x), G^{\mu''}(x)$. The resulting Lagrangians, which no longer involve couplings to $h', G^{\mu''}$ read

$$
\begin{aligned}
\tilde{\mathbf{L}}_{\pi N} &= \bar{N}\mathbf{A}_N N + \bar{N}\left[\,\gamma_0\tilde{\mathbf{B}}_N^\dagger\gamma_0\,\tilde{\mathbf{C}}_N^{-1}\,\tilde{\mathbf{B}}_N + \gamma_0\mathbf{B}_{\Delta N}^\dagger\gamma_0\,\mathbf{C}_\Delta^{-1}\,\mathbf{B}_{\Delta N}\right]N \\
\tilde{\mathbf{L}}_{\pi\Delta} &= \bar{T}\mathbf{A}_\Delta T + \bar{T}\left[\,\gamma_0\mathbf{B}_\Delta^\dagger\gamma_0\,\mathbf{C}_\Delta^{-1}\,\mathbf{B}_\Delta + \gamma_0\tilde{\mathbf{D}}_{N\Delta}^\dagger\gamma_0\,\tilde{\mathbf{C}}_N^{-1}\,\tilde{\mathbf{D}}_{N\Delta}\,\right]T \\
\tilde{\mathbf{L}}_{\pi\Delta N} &= \bar{T}\mathbf{A}_{\Delta N}N + \bar{T}\left[\,\gamma_0\tilde{\mathbf{D}}_{N\Delta}^\dagger\gamma_0\,\tilde{\mathbf{C}}_N^{-1}\,\tilde{\mathbf{B}}_N + \gamma_0\mathbf{B}_\Delta^\dagger\gamma_0\,\mathbf{C}_\Delta^{-1}\,\mathbf{B}_{\Delta N}\right]N \\
&+ \bar{N}\,\gamma_0\mathbf{A}_{\Delta N}^\dagger\gamma_0\,T + \bar{N}\left[\,\gamma_0\tilde{\mathbf{B}}_N^\dagger\gamma_0\,\tilde{\mathbf{C}}_N^{-1}\,\tilde{\mathbf{D}}_{N\Delta} + \gamma_0\mathbf{B}_{\Delta N}^\dagger\gamma_0\,\mathbf{C}_\Delta^{-1}\,\mathbf{B}_\Delta\right]T
\end{aligned}
$$
(101)

One can then read off the desired $1/M$ corrections to arbitrary order from the matrix expressions in the square brackets. In particular, one can verify that the $\mathcal{O}(\epsilon^2)$ $1/M$ Lagrangians of section 4.3 are correct and complete.

Finally, we would like to stress that matrices $\mathbf{A}_X$, $\mathbf{B}_X$, and $\mathbf{C}_X$ are derived from the corresponding relativistic lagrangians. The corresponding expressions have been given in previous sections mostly only to leading order. However, when going to higher orders in the expansion, it is necessary to augment such terms by counterterm Lagrangians.

In section 5 we construct the counterterm Lagrangians of $\mathcal{O}(\epsilon^2)$ in the small scale expansion.

5 Counterterms

Before undertaking applications of the formalism developed above there remains one important step. Working in an effective field theory framework, it is mandatory to include all possible local terms allowed by the symmetry requirement. These so called counterterms appear at each order ϵ^n of the low energy expansion, starting at order ϵ^2. Also, when loop contributions are taken into account, various divergences will arise which must be absorbed into the counterterm component of the effective action. For example, in HBChPT the most general structures have been given up to order p^3 in [18] for the case of SU(2). Here we explicitly construct the next-to-leading order heavy baryon NN-, $N\Delta$- and $\Delta\Delta$-counterterm lagrangians, starting from the corresponding relativistic chiral baryon lagrangians. The general methods described here can be generalized for the construction of arbitrary higher order counterterm lagrangians in the three sectors of the theory. However,

we shall quote the results explicitly only to $\mathcal{O}(\epsilon^2)$. Also, because of space limitations we shall be content merely to sketch the derivation of these results. A more complete discussion is available in ref.[49].

The basic procedure which we utilize is to construct the most general form of the relativistic Lagrangian in the sector being considered. We refer to [9, 11] for examples from the spin 1/2 sector. In performing this task, it should be kept in mind that the only information we are using about the strong interactions at low energies are the symmetries which the effective lagrangian and the meson/baryon fields have to obey. In particular, these are symmetries under parity transformations, charge conjugation, hermitean conjugation and overall Lorentz-invariance, as well as invariance under chiral vector and chiral axial-vector transformations. Violation of one of these symmetries is the only justification to omit a possible structure in the counterterm lagrangians to the order we are working. Our approach is to implement these symmetries on the level of the relativistic lagrangians and then to perform the (non-relativistic) 1/M expansion.

5.1 Building Blocks and Chiral Counting Rules

One begins the process by itemizing the various building blocks from which to form such a Lagrangian. In addition to the already defined structures we employ:

$$
\begin{aligned}
\chi_\pm &= 2B \left[u^\dagger \left(\mathbf{s} + i\tilde{\mathbf{p}} \right) u^\dagger \pm u \left(\mathbf{s} + i\tilde{\mathbf{p}} \right)^\dagger u \right] \equiv \tau^i \, \chi_\pm^i \,, \\
\chi_\pm^{(s)} &= \frac{1}{2} \, Tr \left(\chi_\pm \right) \,, \\
f_{\mu\nu}^\pm &= u^\dagger F_{\mu\nu}^R u \pm u F_{\mu\nu}^L u^\dagger \equiv \tau^i \, f_{\pm\mu\nu}^i \,, \\
F_{\mu\nu}^X &= \partial_\mu F_\nu^X - \partial_\nu F_\mu^X - i \left[F_\mu^X, F_\nu^X \right] ; \quad X = L, R \,, \\
F_\mu^R &= \mathbf{v}_\mu + \mathbf{a}_\mu \,, \quad F_\mu^L = \mathbf{v}_\mu - \mathbf{a}_\mu \,, \\
v_{\mu\nu}^{(s)} &= \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)} \,, \\
w_{\mu\nu}^i &= \frac{1}{2} \, Tr \left(\tau^i \left[D_\mu, u_\nu \right] \right) \,.
\end{aligned}
\tag{102}
$$

Here $\mathbf{s}$ ($\tilde{\mathbf{p}}$) denote an external scalar (pseudoscalar) field, whereas $\mathbf{v}_\mu$ ($\mathbf{a}_\mu$) correspond to an external isovector vector (axial-vector) field. Following

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ref.[18] we have also defined $v_\mu^{(s)}$ denoting an external isoscalar vector field, which we need for processes involving external photons.

Having defined the building blocks of our lagrangians, we note that each has certain properties under chiral power counting. Thus, for example, forms such as

$$M_0, \; M_\Delta, \; \psi_N, \; \psi_\mu^j, \; D_\mu \psi_N, \; D_\mu^{ij} \psi_\nu^j, \; U \tag{103}$$

count as $\mathcal{O}(\epsilon^0)$, while others, e.g.

$$\nabla_\mu U, \; u_\mu, \; w_\mu^i, \; \Delta, \; (i \not{D} - M_0) \psi_N, \; \left(i \not{D}^{ij} - M_\Delta\right) \psi_\mu^j \tag{104}$$

contribute at $\mathcal{O}(\epsilon^1)$, while still others

$$m_q, \; \chi_\pm, \; \chi_\pm^{(s)}, \; \chi_\pm^i, \; f_{\mu\nu}^\pm, \; v_{\mu\nu}^{(s)}, \; f_{\pm\mu\nu}^i, \; [D_\mu, u_\nu], \; w_{\mu\nu}^i \tag{105}$$

count as $\mathcal{O}(\epsilon^2)$. [10] Using these forms then one can easily construct Lorentz-invariant relativistic Lagrangians which begin at any particular order of chiral counting.

5.2 Transformation Rules

The ways by which to enforce parity, charge conjugation invariance and hermiticity in relativistic chiral lagrangians for baryons are well-known and do not need repeating here. For details we refer to [11, 49]. However, less familiar perhaps are the strictures associated with invariance under chiral rotations under which the structures defined above transform according to [5, 9]

$$
\begin{aligned}
U = u^2 &\rightarrow V_R \, U \, V_L^\dagger \\
u &\rightarrow V_R \, u \, h^\dagger = h \, u \, V_L^\dagger \\
\nabla_\mu U &\rightarrow V_R \, \nabla_\mu U \, V_L^\dagger \\
\psi_N &\rightarrow h \, \psi_N \\
X &\rightarrow h \, X \, h^\dagger \quad \text{with} \quad X = D_\mu, \, u_\mu, \, \chi_\pm, \, \chi_\pm^{(s)}, \, v_{\mu\nu}^{(s)}, \, f_{\mu\nu}^\pm \,.
\end{aligned} \tag{106}
$$

---

[10]We work in the framework of standard ChPT and not the Generalized ChPT of J. Stern et al. [50].

Here the operators V_L (V_R) denote chiral rotations among the left- (right-) handed quarks of the underlying QCD lagrangian, whereas $h = h(V_R, V_L, \pi)$ corresponds to the "compensator field" [51, 52] of the nonlinear chiral representation, defined in Eq.(106).

Defining chiral vector V_V and chiral axial V_A transformations via

$$V_V(\beta^i) = V_R + V_L \ , \quad V_A(\alpha^i) = V_R - V_L \ , \tag{107}$$

we can give a representation of the compensator field h for infinitesimal chiral SU(2) transformations:

$$h(\alpha^i, \beta^j, \pi^k) = 1 + i\frac{\tau \cdot \beta}{2} - \frac{i}{2F_0} \ \alpha^i \pi^k \epsilon^{ikl} \ \frac{\tau^l}{2} + \mathcal{O}(\alpha^2, \beta^2, \pi^2) \ . \tag{108}$$

Here the three (infinitesimal) rotation angles α^i correspond to chiral axial rotations. This symmetry sector of the hadronic theory is spontaneously broken at low energies, resulting in the emission and absorption of three associated Goldstone bosons π^k (pions) with associated decay constant F_0, as is evident from Eq.(108).

For the chiral objects with explicit isospin index we use the transformation law

$$Y^i \ \rightarrow \ h^{ij} \ Y^j \ h^\dagger \ , \quad Y^i = w^i_\mu, \ \chi^i_\pm, \ f^i_{\pm\mu\nu}, \ w^i_{\mu\nu} \ , \tag{109}$$

with the chiral response matrix[11]

$$h^{ij} = \left[\delta^{ij} + \left(\frac{i}{2}\delta^{ij}\tau^k - \epsilon^{ijk} \right) \beta^k + \frac{1}{2F_0} \left(i\delta^{ij}\epsilon^{abk}\frac{\tau^a}{2}\pi^b + \pi^i\delta^{jk} - \delta^{ik}\pi^j \right) \alpha^k \right] \ .$$

Finally, the covariant derivative acting on the I=3/2 field and the spin 3/2 isospin 3/2 field itself transform as

$$\begin{aligned} \psi^i_\mu \ &\rightarrow \ h^{ij} \ \psi^j_\mu \\ D^{ij}_\nu\psi^j_\mu \ &\rightarrow \ h^{ia} D^{ab}_\nu\psi^b_\mu \ . \end{aligned} \tag{110}$$

---

[11]Ellis and Tang have given an expression for the spin 3/2 isospin 3/2 compensator field h^{ij} without going back to an infinitesimal representation. [53]

5.3 $\mathcal{O}(\epsilon^2)$ Counterterm Lagrangians

Having established the behavior of building blocks under chiral rotations, we now construct the appropriate relativistic lagrangians. We start from a set of relativistic interaction structures in each sector (NN, ΔN, $\Delta\Delta$) which conserve parity and are Lorentz invariant. In the second step we match the free isospin-indices in all possible combinations. The final step consists of ensuring hermiticity and C-invariance. We note the following simplifications of possible structures:

a) All O(ϵ^2) *relativistic* lagrangians can be written free of the equations of motions (EOM) via field redefinitions. EOM terms can only start appearing in O(ϵ^3) lagrangians.

b) All Rarita-Schwinger fields are accompanied by a "Theta-tensor" Eq. (78) which contains a free parameter governing the coupling to spin $3/2$ off-shell degrees of freedom. Throughout this work we assume that *all* off-shell dependent structures involving relativistic spin $3/2$ fields can be rewritten into the "Theta-tensor" form, using Dirac identities and field redefinitions. All structures that involve the dot-product of a Dirac matrix and a Rarita-Schwinger spinor ($\gamma_\mu \psi^\mu$) are therefore accounted for solely by "Theta-tensors".

c) One can always move a (covariant) derivative onto the remaining fields in the relativistic structure via integration by parts, as the whole lagrangian is only unique up to a total derivative.

d) As a consequence of a) and c) there are no structures $\slashed{D}$, $\sigma_{\mu\nu}D^\nu$ acting on a baryon field in the O(ϵ^2) lagrangians. We have also assumed that the O(ϵ) $N\Delta$ lagrangian Eq.(77) can be written free of $\slashed{D}$ structures (EOM terms). Therefore the relativistic O(ϵ^2) $N\Delta$ lagrangians also do not contain the structure $\bar{\psi}_i^\mu (\slashed{D} w_\mu^i)\psi_N + h.c.$.

e) There are no structures of the type $D_\mu \psi^\mu$ at O(ϵ^2), as this relation is a consequence of the O(ϵ) EOM for the relativistic spin $3/2$ field in combination with assumption b).

f) For the same reasoning as in e) we have also omitted the structure $\bar{\psi}_i^\mu \psi^i D_\mu \psi_N + h.c.$ in the O(ϵ) $N\Delta$ lagrangian Eq.(77). The O(ϵ^2) contribution contained in this term has been accounted for after an integration by parts.

The structures proportional to $\mathcal{A}_N^{(2)}$ which are consistent with invariance under charge- and hermitean conjugation can then be written in the form

[15]

$$\mathcal{L}_N^{(2)} = \bar{N}\Big\{ c_1 2\chi_+^{(s)} + c_2 \left(v \cdot u\right)^2 + c_3 u \cdot u + c_4 \left[S_\mu, S_\nu\right] u^\mu u^\nu \qquad (111)$$

$$+ c_5 \left[\chi_+ - \chi_+^{(s)}\right] - \frac{i}{4M_0} [S^\mu, S^\nu]_- \left[c_6 f_{\mu\nu}^+ + 2c_7 v_{\mu\nu}^{(s)}\right] \Big\} N \ .$$

We note explicitly that Eq.(111) contains only the counterterms of the $O(\epsilon^2)$ nucleon-nucleon lagrangian! In order to obtain the complete $O(\epsilon^2)$ spin $1/2$ lagrangian one still has to add the leading $1/M$ NN-structures which have been calculated in chapter 4.3. We now give a brief discussion of some of the physics contained in the counterterm lagrangian:

The first structure in Eq.(111) contains the isoscalar component of the quark-mass contribution to the mass of the nucleon. The terms proportional to c_2, c_3, c_4 start out as $NN\pi\pi$ vertices, which are new structures beyond the $NN\pi\pi$-vertex (Weinberg term) incorporated in the chiral connection D_μ of the leading order NN-lagrangian. The fifth term represents the isovector component (i.e. $\sim m_u - m_d$) of the quark-mass contribution to the mass of the nucleon and therefore vanishes in the SU(2) isospin symmetry limit. Finally, the structure c_6 (c_7) can be related to the isovector (isoscalar) anomalous magnetic moment of the nucleon.

In the same way one can treat the ΔN and $\Delta\Delta$ sectors of the theory. Once more we only list the results—details can be found in ref.[49].

$$\mathcal{L}_{\Delta N}^{(2)} = \bar{T}_i^\mu \frac{1}{2M_0} \Big[b_1 \, i f_{+\mu\nu}^{\,i} S^\nu + b_2 \, i f_{-\mu\nu}^{\,i} v^\nu + b_3 \, i w_{\mu\nu}^i v^\nu$$

$$+ b_4 \, w_\mu^i S \cdot u + b_5 \, u_\mu S \cdot w^i \Big] N + h.c. \qquad (112)$$

The physics incorporated in these five resulting structures can very easily be interpreted. The first term in the lagrangian provides the M1 isovector[12] $\gamma N\Delta$ transition moment, whereas the second term can be determined from one of the axial-vector nucleon-delta transition form factors at zero four-momentum transfer. The structure proportional b_3 represents a new $\pi N\Delta$

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[12]As expected, there is no structure in Eq.(112) depending on an external isoscalar vector field, as it cannot contribute to the $\Delta I = 1$ transition of the $N\Delta$ system.

coupling, that is independent of the leading coupling $g_{\pi N\Delta}$. The remaining two terms start out as $N\Delta\pi\pi$ couplings, which are forbidden in the leading order lagrangian. Again, in order to obtain the complete $\mathcal{O}(\epsilon^2)$ lagrangian in the small "scale expansion" one has to add the leading $1/M$ $N\Delta$-structures of chapter 4.3 to this lagrangian.

Finally, we list the corresponding $\Delta\Delta$ form

$$
\begin{aligned}
\mathcal{L}_{\Delta\Delta}^{(2)} &= \bar{T}_i^\mu \left[a_1\chi_+^{(s)} + a_2 \left(v \cdot u\right)^2 + a_3\, u \cdot u + a_4 \left[S_\alpha, S_\beta\right] u^\alpha u^\beta \right. \\
&\qquad \left. +a_5 \left(\chi_+ - \chi_+^{(s)}\right)\right] g_{\mu\nu}\, \delta^{ij}\, T_j^\nu\; + i\,\bar{T}_i^\mu \left[a_6\, f_{\mu\nu}^+ + a_7\, v_{\mu\nu}^{(s)}\right] \delta^{ij}\, T_j^\nu\; + \\
&\quad +\bar{T}_i^\mu \left[a_8 \left\{w_\alpha^a, w_\beta^b\right\} \left(\delta^{ia}\delta^{jb} + \delta^{ib}\delta^{ja}\right) g_{\mu\nu} g^{\alpha\beta}\right] T_j^\nu \\
&\quad +\bar{T}_i^\mu \left[a_9 \left\{w_\alpha^a, w_\beta^b\right\} \left(\delta^{ia}\delta^{jb} + \delta^{ib}\delta^{ja}\right) g_{\mu\nu} v^\alpha v^\beta\right] T_j^\nu \\
&\quad +\bar{T}_i^\mu \left[a_{10} \left\{w_\alpha^a, w_\beta^b\right\} \left(\delta^{ia}\delta^{jb} + \delta^{ib}\delta^{ja}\right) g_\mu^\alpha g_\nu^\beta\right] T_j^\nu \\
&\quad +\bar{T}_i^\mu \left[a_{11} \left\{w_\alpha^a, w_\beta^b\right\} \delta^{ij}\delta^{ab} g_\mu^\alpha g_\nu^\beta\right] T_j^\nu\;.
\end{aligned}
\tag{113}
$$

Once more we note that one has to add the corresponding $1/M$ spin $3/2$ lagrangian of chapter 4.2 to Eq.(113) in order to obtain the $O(\epsilon^2)$ delta-delta lagrangian which is both reparameterization invariant and constitutes the proper non-relativistic limit of the relativistic nucleon-delta-pion system. The physics content of the first seven terms is straightforward and is in general identical to the corresponding NN case described above. The remaining 4 structures start out as $\Delta\Delta\pi\pi$ vertices.

6 Conclusions

The subject of chiral perturbation theory in the baryon sector has by now become highly developed. Indeed predictions have been made and in many cases experimentally confirmed for a variety of processes involving $\pi - \gamma - N$ interactions, as summarized in ref. [16]. In this work, however, effects of the $\Delta(1232)$ are included only in terms of contributions to the various counterterms which contribute to the various reactions. Because of this treatment, any such predictions in channels to which the $\Delta(1232)$ can contribute are necessarily restricted to the very-near threshold region. Indeed the ΔN mass difference is of the same order of magnitude as the "small" parameter m_π and cannot be neglected in studies of non-threshold phenomena. In this context we have developed a procedure by which the $\Delta(1232)$ can be treated

as an explicit degree of freedom in such heavy baryon chiral perturbative studies. The method we have used is a simple generalization of the familiar nucleon technique of integrating out the "heavy" lower component in favor of the "light" upper component, but is more complex due to the presence of additional spin-1/2 components together with the desired spin 3/2 structure in the Rarita-Schwinger formalism. Nevertheless, we have shown how it can be accomplished using heavy baryon projection operators which isolate the various spin/spinor components of the wavefunction. This procedure opens the way then for a rigorous expansion not just in powers of energy-momentum p and pion mass m_π but also simultaneously in the small quantity $\Delta_0 = (M_\Delta - M_0)$—we call this an expansion in ϵ as opposed to the usual expansion in p and m_π which are generically noted by p—and will hopefully allow extension of the near-threshold predictions given in ref. [16] into higher energy domains. Herein we have generated a start to this process by presenting the formalism which makes such a program possible. We have shown how, using projection operators, the heavy spin-3/2 field can be isolated from its Rarita-Schwinger form and have, in a path integral context, constructed the lowest order effective action. We also included coupling to nucleons and showed how the program can be carried out in the general case. Forms for possible counterterm Lagrangians were presented up to $\mathcal{O}(\epsilon^2)$. At this point one can begin to apply the formalism to physical processes and parameters. Studies in this regard are under way.

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A The Rarita-Schwinger Formalism

The free spin 3/2 field of mass M_Δ, represented as a vector-spinor field $\Psi_\mu(x)$, satisfies the equation of motion

$$(i\gamma_\nu\partial^\nu - M_\Delta)\Psi_\mu(x) = 0 \tag{114}$$

with the subsidiary condition

$$\gamma_\mu\Psi^\mu(x) = 0 \tag{115}$$

Given these two equations, one can also show

$$\partial_\mu\Psi^\mu(x) = 0 \tag{116}$$

We now expand the spin 3/2 field into plane wave states of definite spin $s_\Delta = -\frac{3}{2}...+\frac{3}{2}$ and momentum p to give an explicit representation of $\Psi_\mu(x)$ in terms of (anti)-particle creation and annihilation operators $b, b^\dagger$, $(d, d^\dagger)$ respectively

$$\Psi_\mu(x) = \sum_{s_\Delta} \int \frac{d^3p}{J_F} \left(b(\mathbf{p}, s_\Delta)\, u_\mu(\mathbf{p}, s_\Delta)e^{-ip\cdot x} + d^\dagger(\mathbf{p}, s_\Delta)\, v_\mu(\mathbf{p}, s_\Delta)e^{ip\cdot x} \right) \tag{117}$$

where $u_\mu(\mathbf{p}, s_\Delta)$ is called a Rarita-Schwinger spinor. For the energy dependent normalization constant J_F we choose

$$J_F = (2\pi)^3 \frac{E}{M_\Delta} \ . \tag{118}$$

The Rarita-Schwinger spinor for the spin 3/2 field is constructed by coupling a spin 1 vector $e_\mu(\mathbf{p}, \lambda)$ to a spin 1/2 Dirac spinor $u(\mathbf{p}, s)$ via Clebsch-Gordon coefficients and then boosting to a velocity $\mathbf{v} = \mathbf{p}/M_\Delta$

$$u_\mu(\mathbf{p}, s_\Delta) = \sum_{\lambda,s} (1\lambda\frac{1}{2}s|\frac{3}{2}s_\Delta)\, e_\mu(\mathbf{p}, \lambda)\, u(\mathbf{p}, s) \tag{119}$$

where [45]

$$e^\mu(\mathbf{p}, \lambda) = \left(\frac{\hat{\mathbf{e}}_\lambda \cdot \mathbf{p}}{M_\Delta}, \ \hat{\mathbf{e}}_\lambda + \frac{\mathbf{p}(\hat{\mathbf{e}}_\lambda \cdot \mathbf{p})}{M_\Delta(p_0 + M_\Delta)} \right) \tag{120}$$

$$u(\mathbf{p}, s) = \sqrt{\frac{E + M_\Delta}{2M_\Delta}} \left(\begin{array}{c} \chi_s \\ \frac{\sigma\cdot\mathbf{p}}{E+M_\Delta}\chi_s \end{array} \right) \tag{121}$$

40

For the unit vectors $\hat{\mathbf{e}}_\lambda$, $\lambda = 0, \pm 1$ appearing in Eq. 119 we use a spherical representation

$$\hat{\mathbf{e}}_+ = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \qquad \hat{\mathbf{e}}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \hat{\mathbf{e}}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \qquad (122)$$

The anti-particle spinors $v_\mu(\mathbf{p}, s_\Delta)$ can be constructed analogously.

It is important to note that the spin 3/2 field, due to its construction via a direct spin 1 - spin 1/2 coupling, always contains spurious spin 1/2 degrees of freedom. It is therefore useful to introduce a complete set of orthonormal spin projection operators, which enable separation of the spin 3/2 and spin 1/2 components:

$$\left(P^{3/2}\right)_{\mu\nu} + \left(P_{11}^{1/2}\right)_{\mu\nu} \;+\; \left(P_{22}^{1/2}\right)_{\mu\nu} = g_{\mu\nu} \qquad (123)$$

$$\left(P_{ij}^I\right)_{\mu\delta} \left(P_{kl}^J\right)^\delta{}_\nu \;=\; \delta^{IJ} \delta_{jk} \left(P_{il}^J\right)_{\mu\nu} \qquad (124)$$

with

$$\left(P^{3/2}\right)_{\mu\nu} = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}\left(\gamma \cdot p\, \gamma_\mu p_\nu + p_\mu \gamma_\nu \gamma \cdot p\right)$$

$$\left(P_{11}^{1/2}\right)_{\mu\nu} = \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{p_\mu p_\nu}{p^2} + \frac{1}{3p^2}\left(\gamma \cdot p\, \gamma_\mu p_\nu + p_\mu \gamma_\nu \gamma \cdot p\right)$$

$$\left(P_{22}^{1/2}\right)_{\mu\nu} = \frac{p_\mu p_\nu}{p^2}$$

$$\left(P_{12}^{1/2}\right)_{\mu\nu} = \frac{1}{\sqrt{3}p^2}\left(p_\mu p_\nu - \gamma \cdot p\, p_\nu \gamma_\mu\right)$$

$$\left(P_{21}^{1/2}\right)_{\mu\nu} = \frac{1}{\sqrt{3}p^2}\left(\gamma \cdot p\, p_\mu \gamma_\nu - p_\mu p_\nu\right) \qquad (125)$$

Note that there exist *two* spin 1/2 degrees of freedom in addition to the desired spin 3/2 component. Finally, we also give the following useful properties of the spin projection operators:

$$\left[\slashed{p}, \left(P^{3/2}\right)_{\mu\nu}\right]_- = 0 \qquad (126)$$

$$\left\{\slashed{p}, \left(P_{ij}^{1/2}\right)_{\mu\nu}\right\}_+ = 2\,\delta^{ij}\left(P_{ij}^{1/2}\right)_{\mu\nu}\slashed{p} \qquad (127)$$

B   Isospurion Formalism for $\Delta(1232)$

$\Delta(1232)$ is an isospin 3/2 system. The four physical states $\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$ can be described by treating the spin 3/2 field $\Psi_\mu(x)$ as an isospin-doublet and attaching an additional isovector index $i = 1, 2, 3$ to it. The resulting field, $\Psi^i_\mu(x)$, is therefore a vector-spinor field both in spin and in isospin space. Note that the vector-spinor construction in isospin space would allow for six states, we therefore introduce a subsidiary condition, analogously to Eq.(115), to eliminate two degrees of freedom

$$\tau^i \, \Psi^i_\mu(x) = 0 \tag{128}$$

where τ^i represents the three Pauli matrices.

For the three isospin doublets we use the representation

$$\Psi^1_\mu = \frac{1}{\sqrt{2}} \left[\begin{array}{c} \Delta^{++} - \frac{1}{\sqrt{3}}\Delta^0 \\ \frac{1}{\sqrt{3}}\Delta^+ - \Delta^- \end{array} \right]_\mu$$

$$\Psi^2_\mu = \frac{i}{\sqrt{2}} \left[\begin{array}{c} \Delta^{++} + \frac{1}{\sqrt{3}}\Delta^0 \\ \frac{1}{\sqrt{3}}\Delta^+ + \Delta^- \end{array} \right]_\mu$$

$$\Psi^3_\mu = -\sqrt{\frac{2}{3}} \left[\begin{array}{c} \Delta^+ \\ \Delta^0 \end{array} \right]_\mu \tag{129}$$

As in Appendix A, one can construct a complete set of orthonormal isospin projection operators [45], which we use extensively, to separate the isospin 3/2 from the isospin 1/2 components

$$\xi^{3/2}_{ij} + \xi^{1/2}_{ij} = \delta^{ij} \tag{130}$$

$$\xi^I_{ij}\xi^J_{jk} = \delta^{IJ}\xi^J_{ik} \tag{131}$$

with

$$\xi^{3/2}_{ij} = \delta^{ij} - \frac{1}{3}\tau^i\tau^j = \frac{2}{3}\delta^{ij} - \frac{i}{3}\epsilon_{ijk}\tau^k \tag{132}$$

$$\xi^{1/2}_{ij} = \frac{1}{3}\tau^i\tau^j = \frac{1}{3}\delta^{ij} + \frac{i}{3}\epsilon_{ijk}\tau^k \tag{133}$$

	3-	1+	2-	1-	2+
3-	x_{11}^{-1}	0	0	0	0
1+	0	y_{11}^{-1}	y_{12}^{-1}	0	0
2-	0	y_{21}^{-1}	y_{22}^{-1}	0	0
1-	0	0	0	z_{11}^{-1}	z_{12}^{-1}
2+	0	0	0	z_{21}^{-1}	z_{22}^{-1}

Table 2: Block substructure of matrix C_Δ^{-1} at $\mathcal{O}(1/M)$

C   Inverses of Matrices $\mathbf{C_\Delta}$ at $\mathcal{O}(1/M)$

At $\mathcal{O}(1/M)$, matrices $\mathcal{C}_\Delta^{-1}$ (section 2) and $\mathbf{C}_\Delta^{-1}$ (section 3) exhibit a block-diagonal substructure with block matrices $\tilde{X}^{-1}$, $\tilde{Y}^{-1}$ and $\tilde{Z}^{-1}$, as displayed in Table 2. At this order they read

$$\tilde{X}_{\mu\nu}^{-1} = \frac{-1}{2m} \, P_{-\ (33)} P_{\mu\nu}^{3/2} \, P_-$$

$$\tilde{Y}_{\mu\nu}^{-1} = \frac{-1}{6m} \left[\begin{array}{cc} -3P_{+\ (11)} P_{\mu\nu}^{1/2} \, P_+ & -\sqrt{3}P_{+\ (12)} P_{\mu\nu}^{1/2} \, P_- \\ -\sqrt{3}P_{-\ (21)} P_{\mu\nu}^{1/2} \, P_+ & 3 \, P_{-\ (22)} P_{\mu\nu}^{1/2} \, P_- \end{array} \right]$$

$$\tilde{Z}_{\mu\nu}^{-1} = \frac{-1}{6m} \left[\begin{array}{cc} 3 \, P_{-\ (11)} P_{\mu\nu}^{1/2} \, P_- & -3 \, \sqrt{3} \, P_{-\ (12)} P_{\mu\nu}^{1/2} \, P_+ \\ -3 \, \sqrt{3} \, P_{+\ (21)} P_{\mu\nu}^{1/2} \, P_- & 5 \, P_{+\ (22)} P_{\mu\nu}^{1/2} \, P_+ \end{array} \right], \quad (134)$$

with $m = M_\Delta$ for the pure spin $3/2$ case (section 3) and $m = M_0$ for the case of simultaneous nucleon and delta degrees of freedom (section 4).

At this point we want to remind the reader concerning the nature of the $1/M$ expansion in the latter case. We use M_0 and $\Delta = M_\Delta - M_0$ as the two independent mass scales in the theory and work in the limit $\Delta << M_0$ in order to establish a systematic chiral power counting. We have therefore truncated $\mathbf{C}_\Delta^{-1}$ at $\mathcal{O}(1/M)$ in order to arrive at Eq.(134). It happens to be the case that there is no explicit Δ dependence in $\mathbf{C}_\Delta^{-1}$ at leading order. However, this will not be true at $\mathcal{O}(1/M^2)$, the matrices $\mathcal{C}_\Delta^{-1}$ and $\mathbf{C}_\Delta^{-1}$ will then start to look very different.

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