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Compton Scattering and the Spin Structure of the Nucleon at Low Energies

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We analyze polarized Compton scattering which provides information on the spin-structure of the nucleon. For scattering processes with photon energies up to 100 MeV the spin-structure dependence can be encoded into four independent parameters—the so-called spin-polarizabilities γ_i , $i = 1\dots 4$ of the nucleon, which we calculate within the framework of the “small scale expansion” in SU(2) baryon chiral perturbation theory. Specific application is made to “forward” and “backward” spin-polarizabilities.

I. INTRODUCTION

The subject of Compton scattering from the nucleon has been an active one of late, with much activity on both experimental and theoretical fronts. Historically, Compton scattering off a (point) spin 1/2 target with an anomalous magnetic moment κ has been calculated by Powell [1] and agrees reasonably well with experimental cross sections up to photon energies of 50MeV for unpolarized scattering. If one increases the energy of the incoming photon beam the simple Powell model for a point nucleon fails, as one is picking up sensitivity to the internal structure of the nucleon. One can account for this nucleon structure-dependent effect in *unpolarized* Compton scattering by introducing *two* free parameters—commonly denoted *electric* (α_E) and *magnetic* (β_M) polarizabilities. of the nucleon. It is known that this approach works very well up to photon energies of 100 MeV or so.

Experimentally these polarizabilities for both neutron and proton have been recently measured¹

$$\alpha_E^{(p)} = (12.1 \pm 0.8 \pm 0.5) \times 10^{-4} \text{ fm}^3 \quad [3] \quad (1)$$

$$\beta_M^{(p)} = (2.1 \mp 0.8 \mp 0.5) \times 10^{-4} \text{ fm}^3 \quad [3] \quad (2)$$

$$\alpha_E^{(n)} = (12.6 \pm 1.5 \pm 2.0) \times 10^{-4} \text{ fm}^3 \quad [4] \quad (3)$$

$$\beta_M^{(n)} = (3.2 \mp 1.5 \mp 2.0) \times 10^{-4} \text{ fm}^3 \quad [4] \quad (4)$$

and have been confronted with various theoretical estimates. For example, using SU(2) “heavy baryon” ChPT to $\mathcal{O}(p^4)$ a calculation by Bernard, Kaiser, Schmidt and Meißner yielded [5]

$$\alpha_E^{(p)} = (10.5 \pm 2.0) \times 10^{-4} \text{ fm}^3 \quad (5)$$

$$\beta_M^{(p)} = (3.5 \pm 3.6) \times 10^{-4} \text{ fm}^3 \quad (6)$$

$$\alpha_E^{(n)} = (13.4 \pm 1.5) \times 10^{-4} \text{ fm}^3 \quad (7)$$

$$\beta_M^{(n)} = (7.8 \pm 3.6) \times 10^{-4} \text{ fm}^3 \quad (8)$$

in reasonable agreement with the experimental results of Eqs.(1-4). However, there exist significant theoretical error bars in Eqs.(5-8), associated with uncertainties in the estimation of various counterterm contributions via a “resonance saturation” hypothesis. While the concept of “resonance saturation” is very well established for the $\mathcal{O}(p^4)$ counterterms in the *meson* lagrangian [6], its analogue in the *baryon* sector is far more complex due to the rich structure both in the baryon resonance spectrum and in the variety of couplings between baryons and meson resonances [7]. In fact, the largest uncertainty in

¹The quoted neutron numbers were obtained from an analysis of neutron transmission experiments on a Pb target. However, a recent paper quotes a quite different number. [2]

Eqs.(5-8) is due to the lowest lying nucleon resonance— $\Delta(1232)$. In an attempt to understand this contribution more fully Hemmert, Holstein and Kambor have developed a systematic “small scale expansion” [8] within the SU(2) heavy-mass formulation of baryon ChPT, which allows treatment of both $\Delta(1232)$ and the nucleon as explicit degrees of freedom² rather than simply as a static contribution to counterterms. In this approach, one sets up the calculation as an expansion in the (small) quantity ϵ , which collectively denotes non-relativistic momenta p , the pion mass m_π or the nucleon-delta mass splitting $\Delta = M_\Delta - M_N$.

Nucleon Compton scattering has been calculated recently within this “small scale expansion” framework to $\mathcal{O}(\epsilon^3)$ in ref. [11]. However, this calculation suffered from the lack of accurate information on the strength of the $\pi N\Delta$ and $\gamma N\Delta$ couplings in the “small scale expansion” formalism. Instead, estimates originating from relativistic Born analyses were used for the couplings. This inadequacy can now be overcome. Utilizing a new determination [12] of these couplings within the framework of the “small scale expansion” with the results of ref. [11] one finds

$$\alpha_E^{(p)} = \alpha_E^{(n)} = [12.2(\text{N}\pi - \text{loop}) + 0(\Delta - \text{pole}) + 4.2(\Delta\pi - \text{loop})] \times 10^{-4} \text{ fm}^3, \quad (9)$$

$$\beta_M^{(p)} = \beta_M^{(n)} = [1.2(\text{N}\pi - \text{loop}) + 7.2(\Delta - \text{pole}) + 0.7(\Delta\pi - \text{loop})] \times 10^{-4} \text{ fm}^3. \quad (10)$$

The results are still larger than the currently known experimental information but now much closer to the domain of uncertainty of the chiral $\mathcal{O}(p^4)$ calculation. Obviously $\mathcal{O}(\epsilon^4)$ calculations of α_E, β_M are called for in the “small scale expansion” to study the convergence of the perturbation series³ and are underway.

Our goal in this note, however, is to extend the discussion on *unpolarized* Compton scattering given so far to the more general case of *polarized* Compton scattering. In analogy to the unpolarized case one can parameterize the spin-dependent nucleon structure beyond the anomalous magnetic moment in terms of unknown parameters

²The idea of treating the spin 3/2 baryon resonances as explicit degrees of freedom in “heavy baryon” ChPT has been advocated by Jenkins and Manohar [9]. For estimates of the spin-independent polarizabilities using this SU(3) approach see ref. [10] and references therein.

³The satisfactory agreement between the $\mathcal{O}(p^4)$ calculation in “standard ChPT” [5] and the latest experimental results Eqs.(1-4) depends upon a cancellation between counterterms dominated by $\Delta(1232)$ pole terms and higher order $\pi - N$ continuum contributions. A similar mechanism can presumably be implemented in the “small scale expansion” approach if one pushes the calculation beyond $\mathcal{O}(\epsilon^3)$. A less plausible but alternative mechanism for the reduction of the large delta pole contribution in β_M has been proposed in ref. [10].

$\gamma_i, i = 1..4$, which are called the *spin-polarizabilities* of the nucleon. [13]. In section 2 we give a definition for spin-polarizabilities in terms of a low-energy expansion of the Compton amplitude and in section 3 we present the ChPT predictions for the γ_i to $\mathcal{O}(\epsilon^3)$ in the “small scale expansion” scheme of ref. [8]. In particular, we have calculated all contributions to the spin-dependent structure of the nucleon which arise from $\pi - N$ continuum states, $\Delta(1232)$ pole graphs, $\pi - \Delta$ continuum states and $\pi^0\gamma\gamma$ anomaly effects to $\mathcal{O}(\epsilon^3)$. We analyze the underlying physics behind each γ_i and conclude with a discussion of the case of the so called *forward* and *backward* spin-polarizabilities of the nucleon, for which some experimental information is available from multipole analyses and fits to the *unpolarized* cross sections [14,15].

II. SPIN DEPENDENT COMPTON SCATTERING

Assuming invariance under parity, charge conjugation and time reversal symmetry the general amplitude for Compton scattering can be written in terms of six structure dependent functions $A_i(\omega, \theta)$, $i = 1..6$, with $\omega = \omega'$ denoting the photon energy in the c.m. frame and θ being the c.m. scattering angle:

$$\begin{aligned}
T = & A_1(\omega, \theta) \vec{\epsilon}^{*'} \cdot \vec{\epsilon} + A_2(\omega, \theta) \vec{\epsilon}^{*'} \cdot \hat{k} \vec{\epsilon} \cdot \hat{k}' \\
& + iA_3(\omega, \theta) \vec{\sigma} \cdot (\vec{\epsilon}^{*'} \times \vec{\epsilon}) + iA_4(\omega, \theta) \vec{\sigma} \cdot (\hat{k}' \times \hat{k}) \vec{\epsilon}^{*'} \cdot \vec{\epsilon} \\
& + iA_5(\omega, \theta) \vec{\sigma} \cdot [(\vec{\epsilon}^{*'} \times \hat{k}) \vec{\epsilon} \cdot \hat{k}' - (\vec{\epsilon} \times \hat{k}') \vec{\epsilon}^{*'} \cdot \hat{k}] \\
& + iA_6(\omega, \theta) \vec{\sigma} \cdot [(\vec{\epsilon}^{*'} \times \hat{k}') \hat{\epsilon} \cdot \hat{k}' - (\vec{\epsilon} \times \hat{k}) \vec{\epsilon}^{*'} \cdot \hat{k}] \quad (11)
\end{aligned}$$

Here $\vec{\epsilon}, \hat{k}$ ($\vec{\epsilon}', \hat{k}'$) are the polarization vector, direction of the incident (final) photon while $\vec{\sigma}$ represents the (spin) polarization vector of the nucleon.

Following general conventions we separate the pion-pole (“anomalous”) contributions (c.f. Fig.2a) from the remaining (“regular”) terms. We write

$$\begin{aligned}
A_i(\omega, \theta) = & A_i(\omega, \theta)^{\pi^0\text{-pole}} + A_i(\omega, \theta)^{\text{regular}} \quad (12) \\
& i = 1..6
\end{aligned}$$

The anomalous contributions to $\mathcal{O}(\epsilon^3)$ are given in Appendix A for completeness. In the following we concentrate on the regular parts of the amplitude.

One now performs a low-energy expansion⁴ of the six independent (regular) structure functions $A_i(\omega, \theta)^{\text{regular}}$ in powers of the photon energy ω . We note that the A_i are real functions for the case $\omega < m_\pi$, with m_π being the mass of the pion. For the case of a proton target of mass M_N with anomalous magnetic moment $\kappa^{(p)}$ one finds

$$\begin{aligned}
A_1(\omega, \theta)_{c.m.}^{\text{reg.}} = & -\frac{e^2}{M_N} - \frac{e^2}{4M_N^3} (1 - \cos \theta) \omega^2 \\
& + 4\pi \left(\alpha_E^{(p)} + \cos \theta \beta_M^{(p)} \right) \omega^2 \\
& + \frac{4\pi}{M_N} \left(\alpha_E^{(p)} + \beta_M^{(p)} \right) (1 + \cos \theta) \omega^3 \\
& + \mathcal{O}(\omega^4) \quad (13)
\end{aligned}$$

$$\begin{aligned}
A_2(\omega, \theta)_{c.m.}^{\text{reg.}} = & \frac{e^2}{M_N^2} \omega - 4\pi \beta_M^{(p)} \omega^2 \\
& - \frac{4\pi}{M_N} \left(\alpha_E^{(p)} + \beta_M^{(p)} \right) \omega^3 + \mathcal{O}(\omega^4) \quad (14)
\end{aligned}$$

$$\begin{aligned}
A_3(\omega, \theta)_{c.m.}^{\text{reg.}} = & \left[1 + 2\kappa^{(p)} - (1 + \kappa^{(p)})^2 \cos \theta \right] \frac{e^2}{2M_N^2} \omega \\
& + 4\pi \left[\gamma_1^{(p)} - (\gamma_2^{(p)} + 2\gamma_4^{(p)}) \cos \theta \right] \omega^3 \\
& - \frac{(2\kappa^{(p)} + 1)e^2}{8M_N^4} \cos \theta \omega^3 + \mathcal{O}(\omega^4) \quad (15)
\end{aligned}$$

$$A_4(\omega, \theta)_{c.m.}^{\text{reg.}} = -\frac{(1 + \kappa^{(p)})^2 e^2}{2M_N^2} \omega + 4\pi \gamma_2^{(p)} \omega^3 + \mathcal{O}(\omega^4) \quad (16)$$

$$A_5(\omega, \theta)_{c.m.}^{\text{reg.}} = \frac{(1 + \kappa^{(p)})^2 e^2}{2M_N^2} \omega + 4\pi \gamma_4^{(p)} \omega^3 + \mathcal{O}(\omega^4) \quad (17)$$

$$A_6(\omega, \theta)_{c.m.}^{\text{reg.}} = -\frac{(1 + \kappa^{(p)}) e^2}{2M_N^2} \omega + 4\pi \gamma_3^{(p)} \omega^3 + \mathcal{O}(\omega^4) \quad (18)$$

Note that for each structure function the leading order terms in the ω expansion are given by *model-independent* Born contributions for scattering from a spin 1/2 point particle (with an allowed anomalous magnetic moment) and are fixed by the low energy theorems (LET) of current algebra. For example, in the case of forward scattering $\vec{k} = \vec{k}'$ with the transversality condition $\vec{\epsilon} \cdot \vec{k} = \vec{\epsilon}^{*'} \cdot \vec{k} = 0$ only two of the six structure functions survive

$$A_1(\omega, 0)_{c.m.} = 4\pi f_1(\omega); \quad \rightarrow f_1(0)_{\text{LET}} = -\frac{e^2 Z^2}{4\pi M_N} \quad (19)$$

$$A_3(\omega, 0)_{c.m.} = 4\pi \omega f_2(\omega); \quad \rightarrow f_2(0)_{\text{LET}} = -\frac{e^2 \kappa^2}{8\pi M_N^2} \quad (20)$$

yielding the familiar Thomson result [17] with $Z = 1$ for a proton and the target spin-dependent LET found many years ago by Gell-Mann, Goldberger and Low [18].

On the other hand, the higher order terms in ω are model-dependent quantities and the comparison here between theoretical predictions and experimentally measured values provides an often sensitive test of the validity of the theoretical picture of the nucleon being employed.

Here, *e.g.*, the electric and magnetic polarizabilities α_E and β_M discussed above enter the amplitude at $\mathcal{O}(\omega^2)$ and measure the system’s deformation in quasi-static electric (\vec{E}) and magnetizing (\vec{H}) external fields [19]

$$\vec{d} = 4\pi \alpha_E \vec{E}, \quad \vec{m} = 4\pi \beta_M \vec{H}, \quad (21)$$

with \vec{d} , (\vec{m}) denoting the induced electric (magnetic) dipole moment.

⁴A Taylor expansion in the energy for the anomalous parts of the amplitude is problematic due to the rapid variation of the pion-pole contributions with energy. See Appendix A for details.

For the case of a microscopic target the situation gets more complicated due to the possibility of target spin as an additional degree of freedom that responds to external electric and magnetic fields. For example, one can construct an induced spin-dependent dipole \vec{p}_s via $\vec{p}_s = \gamma_3 \vec{\nabla}(\vec{S} \cdot \vec{B})$, where \vec{S} denotes the vector of the target spin and γ_3 corresponds to a *spin-dependent polarizability* of the target. Ragusa has analyzed⁵ the case of a spin 1/2 target [13] and found that there exist four independent quantities γ_i , $i = 1, \dots, 4$ which enter at $\mathcal{O}(\omega^3)$ and probe the spin-structure dependent pieces of the Compton amplitude. These “spin-polarizabilities” are perhaps less familiar than their spin-independent counterparts α_E , β_M but they offer equally sensitive probes of nucleon structure.

Actually one combination of these terms *is* well-known. If we consider forward and spin-dependent scattering then one identifies the forward spin-polarizability γ_0 —

$$\gamma_0 = \gamma_1 - \gamma_2 - 2\gamma_4. \quad (22)$$

The reason for the importance of this term is that if, based on Regge arguments, one makes the assumption that the forward spin-flip amplitude obeys an unsubtracted dispersion relation one finds the result

$$f_2(\omega) = \frac{1}{4\pi^2} \int_W^\infty \frac{s ds}{s^2 - \omega^2} [\sigma_-(s) - \sigma_+(s)], \quad (23)$$

where σ_\pm are the photo-absorption cross sections for parallel and anti-parallel alignments of the photon and target helicities and $W = m_\pi + m_\pi^2/(2M_N)$ denotes the threshold energy for an associated (neutral) pion in the intermediate state. At $\omega = 0$ this becomes

$$\frac{\pi e^2 \kappa^2}{2M_N^2} = - \int_W^\infty \frac{ds}{s} [\sigma_-(s) - \sigma_+(s)] \quad (24)$$

which is the well-known Drell-Hearn-Gerasimov (DHG) sum rule and has received a good deal of recent attention [21].

Differentiating Eq.(23) with respect to ω^2 one finds a related sum rule for γ_0

$$\gamma_0 = \frac{1}{4\pi^2} \int_W^\infty \frac{ds}{s^3} [\sigma_-(s) - \sigma_+(s)], \quad (25)$$

⁵We have carried out the ω -expansion of the structure functions A_i in the c.m. frame, whereas Ragusa’s analysis was performed in the Breit-frame. In the Breit frame the structure-dependent parts of the amplitudes A_1 and A_2 are even functions of ω . When transforming from the Breit frame to the c.m. frame we generate additional terms which are odd in ω . However, one remains with a total of two spin-independent and four spin-dependent quantities to $\mathcal{O}(\omega^3)$. For a detailed discussion we refer to [20].

which was found originally by Gell-Mann, Goldberger and Thirring (GGT) [22]. The recent interest in the GGT sum rule Eq.(25) has been triggered by the fact that it relies on the same data sets required to test the DHG sum rule thus providing a very sensitive check with even faster high energy convergence due to the increased power in the energy denominator.

At present neither the forward spin-flip amplitude nor the spin-dependent cross sections have yet been measured, but preparations are underway in several laboratories around the world [23]. However, a model-dependent analysis of the dispersive integral Eq.(25) has been performed, relying on multipole analyses of single-pion photoproduction data, yielding⁶ [14]

$$\gamma_0^{\text{disp.}} \approx \begin{cases} -1.34 \times 10^{-4} \text{ fm}^4 & p \\ -0.38 \times 10^{-4} \text{ fm}^4 & n \end{cases} \quad (26)$$

We shall return to this result below.

Finally, we want to note the recent determination by the LEGS group of the corresponding spin-polarizability which one obtains in the *backward* direction

$$\gamma_\pi = \gamma_1 + \gamma_2 + 2\gamma_4. \quad (27)$$

They obtain [15]

$$\delta_p \equiv -\gamma_\pi^{\text{exp}} = (27.7 \pm 2.3 + 2.8 / - 2.4) \times 10^{-4} \text{ fm}^4 \quad (28)$$

from a dispersion-based global fit to low energy Compton scattering data, yielding a dramatic difference in magnitude between the forward and the backward spinpolarizabilities. Now there exist some uncertainties in this determination because higher order (*i.e.* ω^4) structure dependent terms beyond α_E , β_M in the low-energy expansion of $A_1(\omega, \theta)^{\text{reg.}}$, $A_2(\omega, \theta)^{\text{reg.}}$ in Eqs.(13, 14) enter in the unpolarized cross section at the same order as the spin-polarizabilities and are very poorly known at this point. Nevertheless, we want to point out that the significant difference in magnitude between $\gamma_0^{(p)}$ and $\gamma_\pi^{(p)}$ can be easily explained once one connects these two different linear combinations of the four spin-polarizabilities with the underlying physics using ChPT.

III. CHPT RESULTS

The technical details of $\mathcal{O}(\epsilon^3)$ calculations in Compton scattering with explicit nucleon and $\Delta(1232)$ degrees of freedom are discussed in ref. [11]. Here we give only the results, in terms of coupling constants g_A , b_1 , $g_{\pi N \Delta}$ defined via the effective Lagrangians

⁶For claims that this multipole analysis is inconsistent with the DHG sum rule see ref. [14].

$$\mathcal{L}_N^{(1)} = \bar{N} (iv \cdot D + g_A S \cdot u) N, \quad (29)$$

$$\mathcal{L}_{N\Delta}^{(1)} = g_{\pi N\Delta} \bar{T}_i^\mu w_\mu^i N + \text{h.c.}, \quad (30)$$

$$\mathcal{L}_{N\Delta}^{(2)} = \frac{ib_1}{M_N} \bar{T}_i^\mu S^\nu f_{+\mu\nu}^i N + \dots, \quad (31)$$

$$\mathcal{L}_\Delta^{(1)} = -\bar{T}_\mu^i \left(iv \cdot D^{ij} - \Delta \xi_{3/2}^{ij} + \dots \right) g^{\mu\nu} T_\nu^j, \quad (32)$$

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F_\pi^2}{4} \text{Tr} \left[(\nabla_\mu U)^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger \right], \quad (33)$$

where $i, j = 1, 2, 3$ are isospin indices and $\xi_{3/2}^{ij} = \frac{2}{3} \delta^{ij} - \frac{i}{3} \epsilon^{ijk} \tau^k$ is an isospin 3/2 projection operator with Pauli-matrix τ^k . Here U denotes a nonlinear representation of the pion field with pion decay constant F_π and N represents an (isodoublet) nucleon field of mass M_N . The delta degrees of freedom are encoded in the fields T_λ^i and are described in terms of a Rarita-Schwinger representation both in spin and isospin space. The mass-parameter Δ can be chosen to correspond to the physical mass-difference between nucleon and delta states and S_μ denotes the Pauli-Lubanski spin-vector

$$S^\mu = \frac{i}{2} \gamma_5 \sigma^{\mu\nu} v_\nu, \quad (34)$$

with heavy baryon four-velocity v_μ .

The chiral field-tensors in Eqs.(29-33) are related to pion π^i and photon fields A_μ via

$$D_\mu = \partial_\mu - i \frac{e}{2} (1 + \tau_3) A_\mu + \dots, \quad (35)$$

$$D_\mu^{ij} = \partial_\mu \delta^{ij} - i \frac{e}{2} (1 + \tau^3) A_\mu \delta^{ij} + e \epsilon^{i3j} A_\mu + \dots, \quad (36)$$

$$u_\mu = -\frac{1}{F_\pi} \tau^i \partial_\mu \pi^i + \frac{e}{F_\pi} A_\mu \epsilon^{i3j} \pi^i \tau^j + \dots, \quad (37)$$

$$w_\mu^i = -\frac{1}{F_\pi} \partial_\mu \pi^i - \frac{e}{F_\pi} A_\mu \epsilon^{i3j} \pi^j + \dots, \quad (38)$$

$$f_{+\mu\nu}^i = e \delta^{i3} (\partial_\mu A_\nu - \partial_\nu A_\mu) + \dots, \quad (39)$$

$$\nabla_\mu U = \partial_\mu U - i \frac{e}{2} A_\mu [\tau_3, U] + \dots, \quad (40)$$

$$\chi = m_\pi^2 + \dots, \quad (41)$$

where m_π is the mass of the pion in the limit of exact $SU(2)$ isospin symmetry.

Furthermore, we need the anomalous $\pi^0 \gamma \gamma$ -vertex provided by the Wess-Zumino-Witten Lagrangian [24]

$$\mathcal{L}_{\pi^0 \gamma \gamma}^{(WZW)} = -\frac{e^2}{32\pi^2 F_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \pi^0, \quad (42)$$

where $\epsilon_{0123} = 1$ and $F_{\mu\nu}$ corresponds to the electromagnetic field tensor.

Having defined the relevant lagrangians and coupling constants, we now present the results of our $\mathcal{O}(\epsilon^3)$ calculation for the four isoscalar spin-polarizabilities $\gamma_i^{(s)}$. All *regular* isovector spinpolarizabilities $\gamma_i^{(v)}$ are identically zero at $\mathcal{O}(\epsilon^3)$. Choosing the heavy baryon velocity-vector $v^\mu = (1, 0, 0, 0)$ and working in the Coulomb gauge

$v \cdot \epsilon = v \cdot \epsilon' = 0$ we have calculated the contributions from $\pi - N$ continuum states (Fig.3), $\Delta(1232)$ pole graphs (Figs.2b+c) and $\pi - \Delta$ continuum states (Fig.4). We note that the nucleon Born contributions up to $\mathcal{O}(\epsilon^3)$ (Fig.1) do not contribute to the polarizabilities as defined in Eqs.(13-18) but provide the model-independent LETs of current algebra discussed in section 2. The (anomalous) contributions from neutral pion exchange (Fig.2a) are also excluded here and can be found in Appendix A.

For the $\pi - N$ continuum contributions (Fig.3) to $\mathcal{O}(\epsilon^3)$ in the ‘‘small scale expansion’’ we find

$$\gamma_1^{(s) N\pi} = + \frac{e^2 g_A^2}{96\pi^3 F_\pi^2 m_\pi^2}; \quad (43)$$

$$\gamma_2^{(s) N\pi} = + \frac{e^2 g_A^2}{192\pi^3 F_\pi^2 m_\pi^2}; \quad (44)$$

$$\gamma_3^{(s) N\pi} = + \frac{e^2 g_A^2}{384\pi^3 F_\pi^2 m_\pi^2}; \quad (45)$$

$$\gamma_4^{(s) N\pi} = - \frac{e^2 g_A^2}{384\pi^3 F_\pi^2 m_\pi^2}; \quad (46)$$

We note that our $\mathcal{O}(\epsilon^3)$ results for the $\pi - N$ continuum agree with the $\mathcal{O}(p^3)$ HBChPT calculation of Bernard, Kaiser and Meißner [16]. Finite shifts to their NN couplings due to the presence of explicit delta degrees of freedom will only occur in higher order lagrangians and do not affect the πNN coupling g_A to the order we are calculating. For a detailed discussion of this issue we refer to ref. [12].

Next we give the $\mathcal{O}(\epsilon^3)$ contribution to the spin-polarizabilities due to $\Delta(1232)$ Born graphs (Figs.2b+c):

$$\gamma_1^{(s) \Delta} = 0; \quad (47)$$

$$\gamma_2^{(s) \Delta} = -\frac{e^2 b_1^2}{4\pi 9M^2 \Delta^2}; \quad (48)$$

$$\gamma_3^{(s) \Delta} = 0; \quad (49)$$

$$\gamma_4^{(s) \Delta} = +\frac{e^2 b_1^2}{4\pi 9M^2 \Delta^2}; \quad (50)$$

while for the $\mathcal{O}(\epsilon^3)$ $\pi - \Delta$ continuum terms (Fig.4) we determine

$$\gamma_1^{(s) \Delta\pi} = \frac{e^2 g_{\pi N\Delta}^2}{4\pi 54\pi^2 F_\pi^2} \left[-\frac{\Delta^2 + 2m_\pi^2}{(\Delta^2 - m_\pi^2)^2} + \frac{3\Delta m_\pi^2 \ln R}{(\Delta^2 - m_\pi^2)^{\frac{5}{2}}} \right]; \quad (51)$$

$$\gamma_2^{(s) \Delta\pi} = \frac{e^2 g_{\pi N\Delta}^2}{4\pi 54\pi^2 F_\pi^2} \left[\frac{1}{\Delta^2 - m_\pi^2} - \frac{\Delta \ln R}{(\Delta^2 - m_\pi^2)^{\frac{3}{2}}} \right]; \quad (52)$$

$$\gamma_3^{(s) \Delta\pi} = \frac{e^2 g_{\pi N\Delta}^2}{4\pi 108\pi^2 F_\pi^2} \left[\frac{1}{\Delta^2 - m_\pi^2} - \frac{\Delta \ln R}{(\Delta^2 - m_\pi^2)^{\frac{3}{2}}} \right]; \quad (53)$$

$$\gamma_4^{(s) \Delta\pi} = \frac{e^2 g_{\pi N\Delta}^2}{4\pi 108\pi^2 F_\pi^2} \left[-\frac{1}{\Delta^2 - m_\pi^2} + \frac{\Delta \ln R}{(\Delta^2 - m_\pi^2)^{\frac{3}{2}}} \right], \quad (54)$$

with

$$R = \frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}. \quad (55)$$

For the parameter set $F_\pi = 92.4$ MeV, $m_\pi = 138$ MeV, $M_N = 938$ MeV, $\Delta = 294$ MeV and the axial coupling $g_A = 1.26$ determined from neutron beta decay we give the $\mathcal{O}(\epsilon^3)$ predictions for the spin-polarizabilities in Table I. The $\pi N\Delta$ ($\gamma N\Delta$) coupling constant $g_{\pi N\Delta}$ (b_1) has been fixed from the experimental $\Delta(1232)$ decay width and found to be⁷ [12]

$$g_{\pi N\Delta}^{(1)} = 1.05 \pm 0.02; \quad b_1^{(2)} = 3.85 \pm 0.15. \quad (56)$$

As Table I clearly shows, $\gamma_1^{(s)}, \gamma_3^{(s)}$ are dominated by the contributions from the $\pi - N$ continuum (Fig.3) in this $\mathcal{O}(\epsilon^3)$ calculation, whereas $\gamma_2^{(s)}, \gamma_4^{(s)}$ receive sizeable corrections due to delta pole graphs (Fig.2b+c). At this order these are the only two of the four spin-polarizabilities which receive contributions from delta pole exchange (see Eqs.(47-50)) via two successive magnetic dipole (M1) $\gamma N\Delta$ transitions. With respect to the spin-polarizabilities one therefore has to perform at least an $\mathcal{O}(\epsilon^4)$ calculation in order to be sensitive to the electric quadrupole (E2) $\gamma N\Delta$ transition moment. Furthermore, we note that the effects of the $\pi - \Delta$ continuum (Fig.4) are found to be much smaller than any of the other analyzed channels, as expected. It is interesting to note that $\gamma_2^{(p,n)}$ displays a strong cancelation at this order between the $N\pi$ -loop diagrams and the $\Delta(1232)$ pole graphs, possibly indicating a similar cancelation mechanism as the one postulated [5,11] for the magnetic polarizability β_M at $\mathcal{O}(\epsilon^4)$!

We now move on to discuss the connection with available experiments pertaining to the spin-polarizabilities.

IV. COMPARISON WITH EXPERIMENT

As mentioned above, so far there exists only limited experimental information on two particular linear combinations of spin-polarizabilities γ_i .

In the forward direction we can compare the $\mathcal{O}(\epsilon^3)$ ChPT predictions with the multipole analysis of ref. [14]. Via Eq.(22) one finds

⁷It is important to note that these values are obtained by consistent use of the ‘‘small scale expansion’’ and are therefore appropriate for use with other calculations at $\mathcal{O}(\epsilon^3)$. In reference [11] considerably larger values $g_{\pi N\Delta} = 1.5 \pm 0.2$, $b_1^2 = 6.3 \pm 1.75$, obtained from a tree level relativistic analysis, were employed. We are now convinced that the correct procedure is the one employed in the present work, so that the Δ effects obtained in ref. [11] should be appropriated rescaled downward by nearly a factor of two.

$$\begin{aligned} \gamma_0^{(s) th.} &= [4.6 (N\pi - \text{loop}) - 2.4 (\Delta - \text{pole}) \\ &\quad - 0.2 (\Delta\pi - \text{loop})] \times 10^{-4} \text{ fm}^4, \\ &= + 2.0 \times 10^{-4} \text{ fm}^4, \end{aligned} \quad (57)$$

where we have used the same parameter set as in the previous section. We also note that all anomalous contributions from Eqs.(A7-A10) in Appendix A cancel out exactly to $\mathcal{O}(\epsilon^3)$ in this particular combination of spin-polarizabilities. This makes the isoscalar combination $\gamma_0^{(s)}$ directly accessible in experiments and thus provides a very sensitive test of the predicted interference between pion-nucleon and delta dynamics. At present, we can only conclude that in the $\mathcal{O}(\epsilon^3)$ calculation one obtains the forward spin-polarizability of the proton with the opposite sign when comparing with the existing multipole analysis Eq.(26). Our delta pole contribution reduces the large positive result of the $\pi - N$ continuum substantially, but at this order we have no clear indication yet whether this trend will continue at higher orders to lead to an overall negative result.

We note that the forward spin-polarizabilities have also been calculated some time ago in relativistic one-loop ChPT, yielding [25]

$$\gamma_0^{(p) 1\text{-loop}} = +2.2 \times 10^{-4} \text{ fm}^4, \quad (58)$$

$$\gamma_0^{(n) 1\text{-loop}} = +3.2 \times 10^{-4} \text{ fm}^4. \quad (59)$$

However, we want to remind the reader that relativistic 1-loop ChPT does not possess a systematic chiral power counting [26] and only gives an *indication* of some of the higher order corrections in the $\pi - N$ sector. In ref. [25] it was further argued that the addition of *phenomenological* delta exchange graphs can lead to negative results for the forward spin-polarizabilities in agreement with the multipole analysis Eq.(26). It remains to be seen whether a systematic calculation to $\mathcal{O}(\epsilon^4)$ of all effects of $\Delta(1232)$ contributions will support this finding. On the experimental side a measurement scattering circularly polarized photons off a polarized proton in the forward direction would thus provide an independent check on the GGT sum rule Eq.(25) and the associated multipole analysis Eq.(26).

Aside from this very limited information on the γ_i in the forward direction recently an analysis of the corresponding spin-polarizability in the *backward direction* has been reported [15]. Determining the pertinent linear combination Eq.(27) of spin-polarizabilities accessible in this particular angular direction we find the $\mathcal{O}(\epsilon^3)$ result

$$\begin{aligned} \gamma_\pi^{(s) th.} &= [4.6 (N\pi - \text{loop}) + 2.4 (\Delta - \text{pole}) \\ &\quad - 0.2 (\Delta\pi - \text{loop})] \times 10^{-4} \text{ fm}^4, \\ &= + 6.8 \times 10^{-4} \text{ fm}^4, \end{aligned} \quad (60)$$

in dramatic disagreement to the reported LEGS number Eq.(28). However, in addition to the uncertainty of higher order contributions affecting the experimental extraction procedure noted in the previous section, we

believe the main origin of this discrepancy to arise from *anomalous* contributions. While to $\mathcal{O}(\epsilon^3)$ these contributions cancel exactly in the forward direction, they all interfere constructively and yield a *maximal* contribution in the backward direction. From Appendix A one finds for proton, neutron

$$\gamma_\pi^{(p) \text{ anom.}} = -43.5 \times 10^{-4} \text{ fm}^4 \text{ (anomaly)}, \quad (61)$$

$$\gamma_\pi^{(n) \text{ anom.}} = +43.5 \times 10^{-4} \text{ fm}^4 \text{ (anomaly)}, \quad (62)$$

yielding

$$\begin{aligned} \delta &= - \left[\gamma_\pi^{(s)} + \gamma_\pi^{\text{anom.}} \right] \\ \rightarrow \delta_p^{\text{th.}} &= +36.7 \times 10^{-4} \text{ fm}^4, \\ \rightarrow \delta_n^{\text{th.}} &= -50.3 \times 10^{-4} \text{ fm}^4, \end{aligned} \quad (63)$$

which is roughly consistent with but is slightly larger in absolute value than the reported backward “spin-polarizability” measurement on the proton Eq.(28). Certainly a detailed analysis of all higher order corrections in unpolarized backward scattering has to be performed before one can draw strong conclusions on the discrepancy and ultimately the result Eq.(28) should of course be checked in a polarized experiment. Nevertheless we have convincingly shown that the *regular* contribution of γ_π to $\mathcal{O}(\epsilon^3)$ is of the same order of magnitude as γ_0 and there is no theoretical reason to expect otherwise. Any pion-nucleon and delta physics will be obscured in a backward-measurement unless it is possible to subtract the large anomalous contributions. If this can be done, then it should be possible to see the *constructive* interference between the $\pi - N$ continuum contributions and the delta pole graphs in $\gamma_\pi^{(s)}$, as opposed to the *destructive* interference in $\gamma_0^{(s)}$ in the case of forward scattering! However, this may require a fully polarized experiment.

It is certainly desirable to also perform polarized Compton scattering experiments off polarized proton targets for angles θ different from 0 and 180 degrees. Analyses regarding the experimental feasibility are in progress.⁸ These experiments would allow the determination of additional linear combinations of the four spin-polarizabilities and finally lead to high precision information on the complete set of spin-structure parameters of the nucleon at low energies. Theoretical investigations into favorable energy, angle and spin-orientation ranges are also underway and will be reported in a future communication. However, in any case it is essential to extend the ChPT calculations to $\mathcal{O}(\epsilon^4)$ in order to gain a better understanding of the theoretical uncertainties.

⁸R. Miskimen and M. Pavan, private communication.

V. CONCLUSIONS

We have in this note reviewed the theoretical status of the four nucleon spin-polarizabilities γ_i in the framework of ChPT as obtained in polarized Compton scattering. We have analyzed in detail the contributions of the $\pi - N$ continuum, (anomalous) neutral pion exchange, $\Delta(1232)$ pole graphs and the $\pi - \Delta$ continuum. At this point connection with experimental information can only be made for particular linear combinations of the spin-polarizabilities accessible in forward or backward Compton scattering. Our $\mathcal{O}(\epsilon^3)$ predictions are qualitatively consistent with the new LEGS backward spin-polarizability number. On the theoretical side an $\mathcal{O}(\epsilon^4)$ calculation is clearly called for to determine the convergence of the perturbative series, whereas the upcoming Compton experiments with polarized photons scattering off polarized nucleons should provide important information on sign and magnitude of the individual spin-polarizabilities.

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APPENDIX A: ANOMALOUS CONTRIBUTIONS

The anomalous contributions to the six structure functions arising from the Wess-Zumino-Witten functional [24] read

$$A_1^{\pi^0 \text{-pole}}(\omega, t) = 0, \quad (A1)$$

$$A_2^{\pi^0 \text{-pole}}(\omega, t) = 0, \quad (A2)$$

$$A_3^{\pi^0 \text{-pole}}(\omega, t) = + \frac{e^2 g_A}{8\pi^2 F_\pi^2} \tau_3 \frac{\omega t}{m_\pi^2 - t}, \quad (A3)$$

$$A_4^{\pi^0 \text{-pole}}(\omega, t) = 0, \quad (A4)$$

$$A_5^{\pi^0 \text{-pole}}(\omega, t) = - \frac{e^2 g_A}{8\pi^2 F_\pi^2} \tau_3 \frac{\omega^3}{m_\pi^2 - t}, \quad (A5)$$

$$A_6^{\pi^0 \text{-pole}}(\omega, t) = + \frac{e^2 g_A}{8\pi^2 F_\pi^2} \tau_3 \frac{\omega^3}{m_\pi^2 - t}, \quad (A6)$$

with $t = -2\omega^2(1 - \cos\theta)$ and τ_3 being a Pauli matrix in nucleon-isospin space.

Due to the small energy-denominators these functions are typically not Taylor expanded in ω but kept in full as anomalous contributions. If one wishes to extract contributions of this six amplitudes to the spin-polarizabilities, one obtains

$$\gamma_1^{(v) \text{ anom}} = -\frac{e^2 g_A}{16\pi^3 F_\pi^2 m_\pi^2} = -21.7 \times 10^{-4} \text{ fm}^4, \quad (\text{A7})$$

$$\gamma_2^{(v) \text{ anom}} = 0, \quad (\text{A8})$$

$$\gamma_3^{(v) \text{ anom}} = +\frac{e^2 g_A}{32\pi^3 F_\pi^2 m_\pi^2} = +10.9 \times 10^{-4} \text{ fm}^4, \quad (\text{A9})$$

$$\gamma_4^{(v) \text{ anom}} = -\frac{e^2 g_A}{32\pi^3 F_\pi^2 m_\pi^2} = -10.9 \times 10^{-4} \text{ fm}^4, \quad (\text{A10})$$

which are all in the isovector channel at this order.

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$\gamma_i^{(s)}$	$N\pi$ -loop	Δ -pole	$\Delta\pi$ -loop	Sum
$\gamma_1^{(s)}$	+4.56	0	-0.21	+4.35
$\gamma_2^{(s)}$	+2.28	-2.40	-0.23	-0.35
$\gamma_3^{(s)}$	+1.14	0	-0.12	+1.02
$\gamma_4^{(s)}$	-1.14	+2.40	+0.12	+1.38

TABLE I. $\mathcal{O}(\epsilon^3)$ predictions for the isoscalar spin-polarizabilities $\gamma_i^{(s)}$. All results are given in the units 10^{-4} fm^4 . All isovector spin-polarizabilities $\gamma_i^{(v)}$ are identically zero to this order.

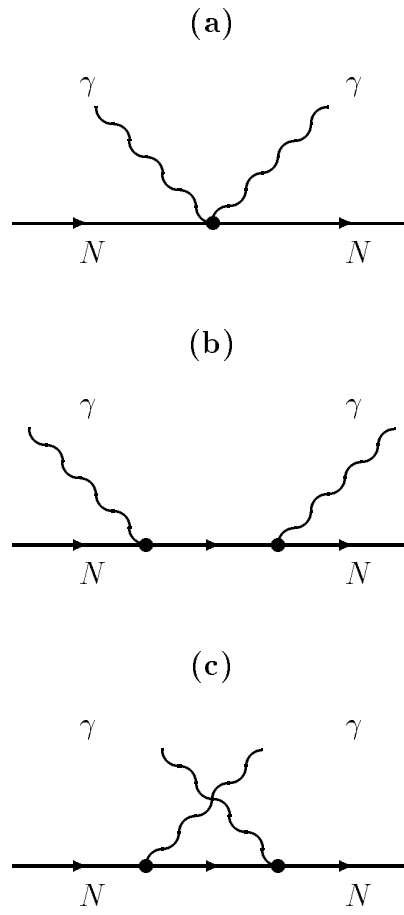


Figure 1: $\mathcal{O}(\epsilon^3)$ Born graph contributions.

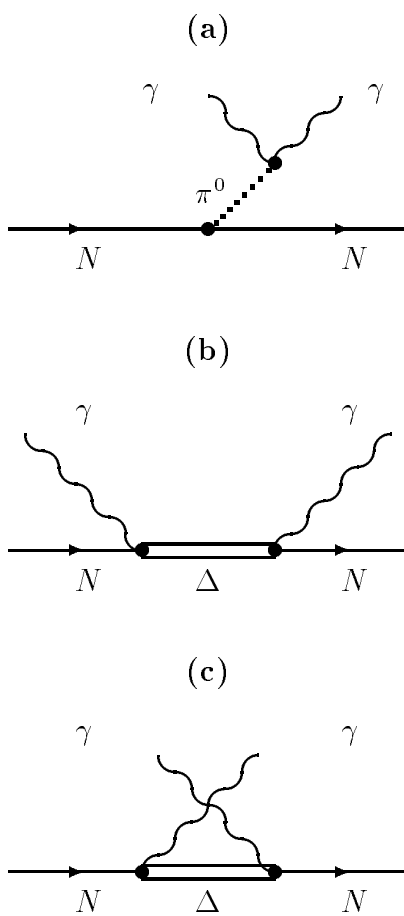


Figure 2: $\mathcal{O}(\epsilon^3)$ structure dependent Born contributions.

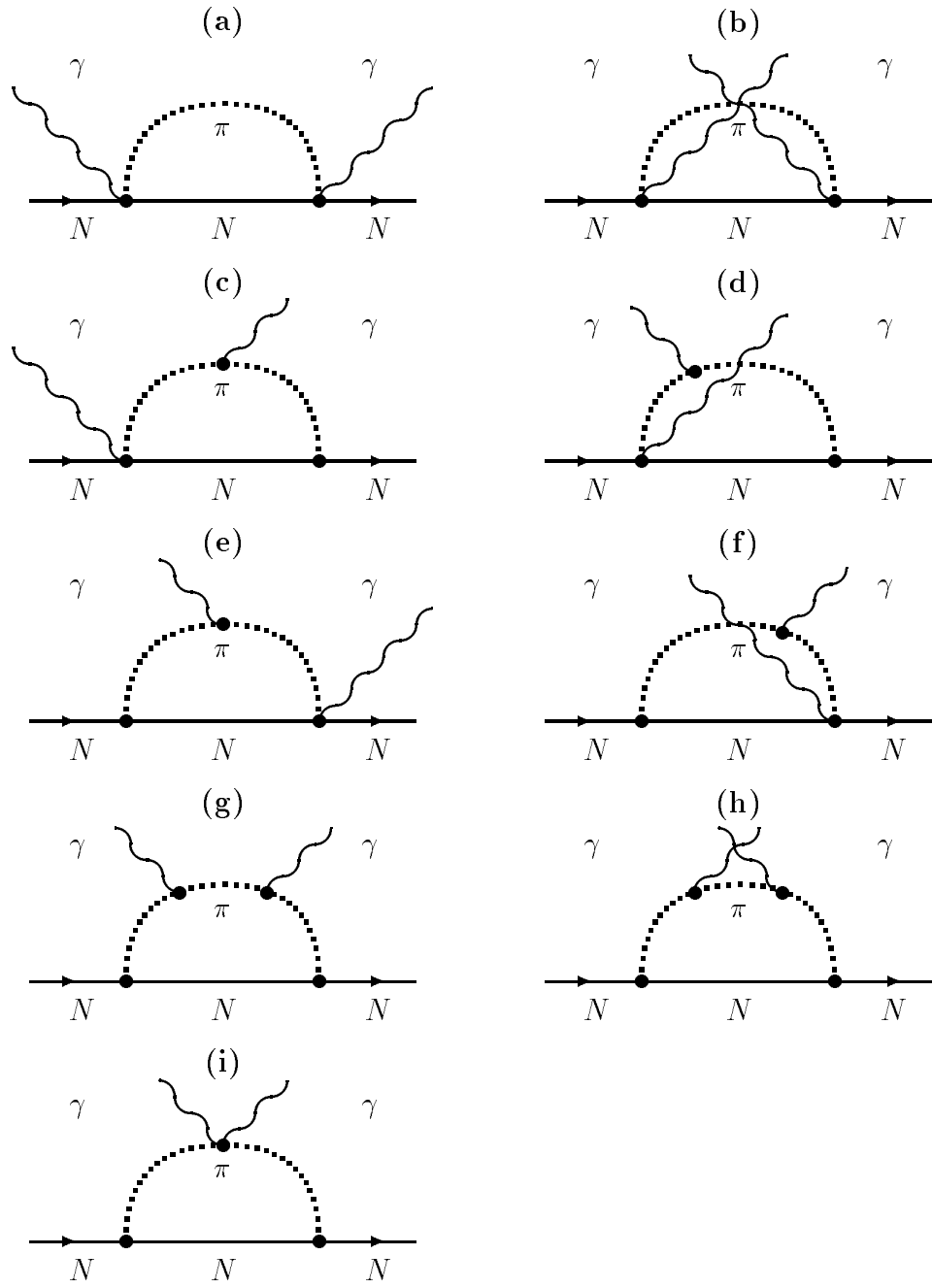


Figure 3: $\mathcal{O}(\epsilon^3)$ $N\pi$ loop diagrams.

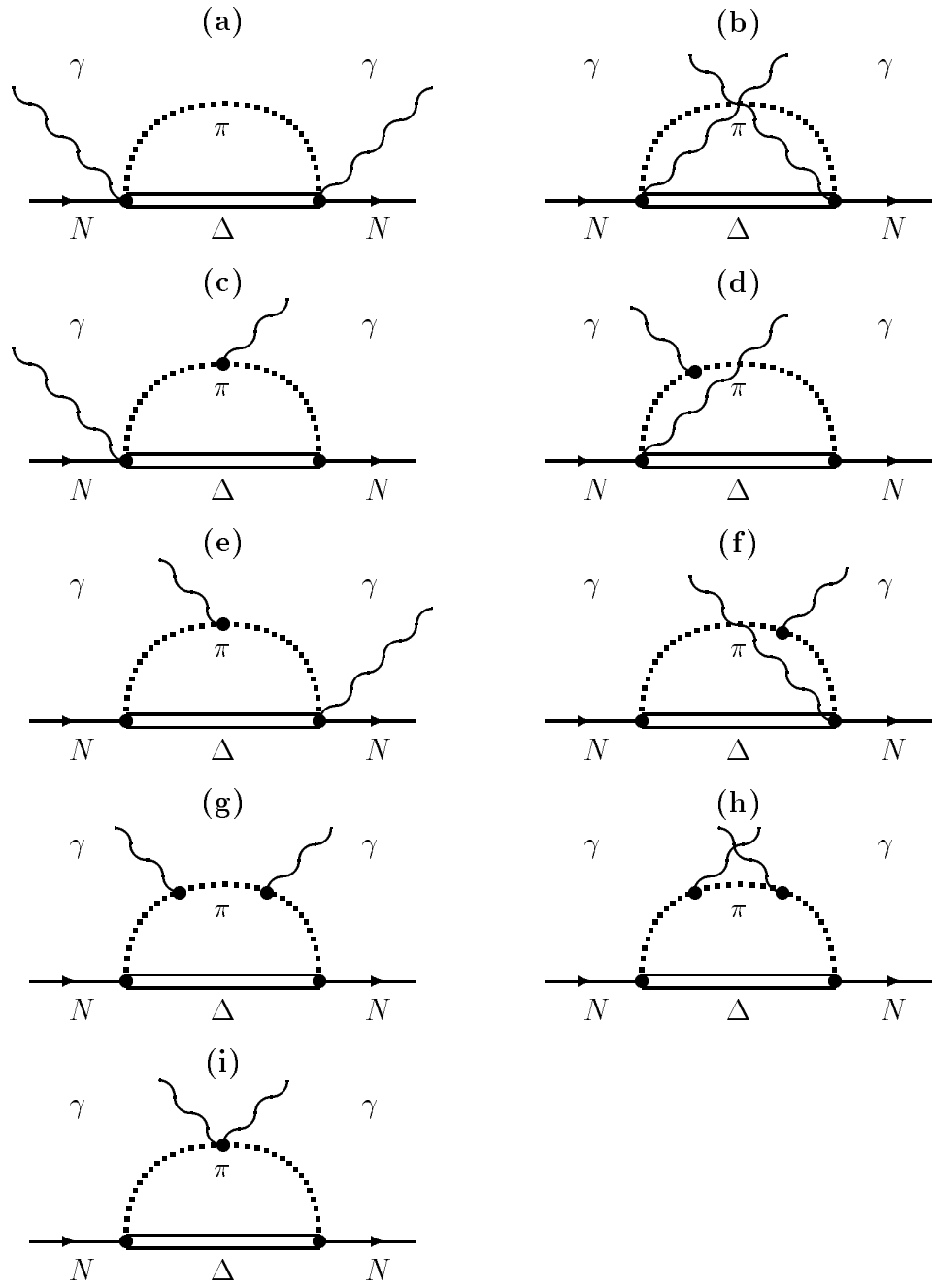


Figure 4: $\mathcal{O}(\epsilon^3)$ $\Delta\pi$ loop diagrams.