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Abstract

The Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}|$ can be extracted from the rate for the semi-leptonic decay $B \to \pi l^- \bar{\nu}_l$, with little theoretical uncertainty, provided the hadronic form factor for the $B \to \pi$ transition can be measured from some other B decay. In here, we suggest using the decay $B \to \pi J/\psi$. This is a color suppressed decay, and it cannot be properly described within the usual factorization approximation; we use instead a simple and very general phenomenological model for the bdJ/ψ vertex. In order to relate the hadronic form factors in the $B \to \pi J/\psi$ and $B \to \pi l^- \bar{\nu}_l$ decays, we use form factor relations that hold for heavy-to-light transitions at large recoil.

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The main difficulty in extracting the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$, from exclusive semi-leptonic decays such as $\bar{B}_d^0 \to \pi^+ l^- \bar{\nu}_l$, is the theoretical uncertainty associated with the form factor $f_1(q^2)$, in the differential decay rate

$$\frac{d}{dq^2}\Gamma(\bar{B}_d^0 \to \pi^+ l^- \bar{\nu}_l) = \frac{G_F^2}{24\pi^3}|V_{ub}|^2 |\vec{p}_\pi|^3 |f_1(q^2)|^2 \tag{1}$$

($q \equiv p_B - p_\pi$, and $|\vec{p}_\pi|$ is the three-momentum of the pion in the B meson rest-frame). One way to overcome this problem is to compare the semi-leptonic decay to some other B decay, which involves the same hadronic transition, but is proportional to a different CKM matrix element [1]. In here, we suggest using the tree level hadronic decay $B^- \to \pi^- J/\psi$. This decay has been observed recently, at both CLEO [2] and CDF [3]; the present value of the branching ratio is $B(B^- \to \pi^- J/\psi) = (4.4 \pm 2.4) \times 10^{-5}$ [4]. The semi-leptonic decay $\bar{B}_d^0 \to \pi^+ l^- \bar{\nu}_l$ has been seen at CLEO [5], with a preliminary branching ratio $B(\bar{B}_d^0 \to \pi^+ l^- \bar{\nu}_l) \simeq 1.4 \times 10^{-4}$. More data will be necessary, in order to determine the differential decay rate in eq. 1.

The decays $B^- \to \pi^- J/\psi$ or $\rho^- J/\psi$, as their Cabibbo-allowed analogues $B^- \to K^- J/\psi$ or $K^{*-} J/\psi$, and all other B decays to charmonium states, are color suppressed tree level decays. They are notorious for the failure of the common factorization procedure [6] to predict decay rates or polarization ratios [7]. We will use instead the phenomenological model proposed in ref. [8] to describe these decays; it allows for corrections to the factorization result to be included in a simple and economical way. According to that model, and assuming no significant spectator effects, the B decays to J/ψ stem from an effective bqJ/ψ vertex ($q = s$ or d, for the Cabibbo-allowed and suppressed decays, respectively)

$$\begin{aligned} \Lambda^\mu_{bqJ/\psi} &= -\frac{G_F}{\sqrt{2}}\, V_{cb} V_{cq}^* \,(C_2 + \frac{1}{3}C_1)\,[g_0\, q^\mu \, \not{q}(1-\gamma_5) \\ &\quad + g_1\,(m^2_{J/\psi} g^{\mu\nu} - q^\mu q^\nu)\gamma_\nu(1-\gamma_5) + g_2\, m_b i\sigma^{\mu\nu} q_\nu (1+\gamma_5)], \end{aligned} \tag{2}$$

where $C_{1,2}$ are the Wilson coefficients in the weak Hamiltonian, and $q = p_{J/\psi}$. This is the most general expression for the vertex, when, as we do in here, the mass of the light quark q is neglected. In the factorization approximation,

$g_1 = g_0 = f_{J/\psi}/m_{J/\psi}$ and $g_2 = 0$; however, the coefficients $g_{0,1,2}$ deviate from these values due to both perturbative and non-perturbative gluon exchanges. In particular, such QCD effects will generate the Lorentz structure associated with g_2; that is absent from the factorization result, but it is essential to fit the polarization data [8]. In here, the coefficients g_1 and g_2 are to be determined empirically, at $q^2 = m^2_{J/\psi}$, from the data for the B meson decays into J/ψ. The term proportional to the form factor g_0 does not contribute to the decay amplitudes, and so g_0 will be left undetermined.

As long as final state interactions do not play a significant role [9], it follows from eq. 2 that the amplitude for the $B^- \to \pi^- J/\psi$ decay is given by

$$\begin{aligned} A(B^- \to \pi^- J/\psi) &= -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{cd} (C_2 + \frac{1}{3} C_1) 2 m_B |\vec{p}_\pi| m_{J/\psi} \\ &\quad \times \left[g_1 f_1(m^2_{J/\psi}) + g_2 m_b s(m^2_{J/\psi}) \right] \end{aligned} \tag{3}$$

($|\vec{p}_\pi|$ is the three-momentum of the pion in the B meson rest-frame); $f_1(q^2)$ is the same form factor as in the semi-leptonic decay $\bar{B}^0_d \to \pi^+ l^- \bar{\nu}_l$, and $s(q^2)$ is the form factor associated with the Lorentz structure of the g_2 term in eq. 2. In the $m_d \to 0$ limit that we consider in here, these form factors satisfy the relation

$$f_1(q^2) = -(m_B - E_\pi + |\vec{p}_\pi|) s(q^2) \ , \tag{4}$$

that was derived in ref. [10] (but see also the earlier work by Stech, in ref. [11]), from the constituent quark picture for the hadronic transition. This, and other form factor relations for heavy-to-light transitions, follow in the static limit for the heavy b quark and the ultra-relativistic limit for the recoiling light quark; they are independent of the exact form of the wave-functions of the mesons. They hold best at large recoil momenta, as is the case for the $B \to \pi$ transition at $q^2 = m^2_{J/\psi}$ [10]. The $B^- \to \pi^- J/\psi$ decay rate is then

$$\begin{aligned} \Gamma(B^- \to \pi^- J/\psi) &= \frac{G_F^2}{4\pi} |V_{cb}|^2 \sin^2\theta_c (C_2 + \frac{1}{3} C_1)^2 |\vec{p}_\pi|^3 m^2_{J/\psi} \\ &\quad \times \left| f_1(m^2_{J/\psi}) g_1 \left(1 - \frac{g_2}{g_1} \right) \right|^2 \end{aligned} \tag{5}$$

(for simplicity, we ignore the small pion mass, but this is not necessary).

This expression can now be used to eliminate from eq. 1 the dependence on the hadronic matrix element f_1. We obtain

$$\frac{1}{\Gamma(B^- \to \pi^- J/\psi)} \left[\frac{d}{dz}\Gamma(\bar{B}_d^0 \to \pi^+ l^- \bar{\nu}_l)\right]_{z=\frac{m_{J/\psi}^2}{m_B^2}}$$

$$= \frac{1}{6\pi^2}\left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{1}{\sin^2\theta_c}\frac{m_B^2}{m_{J/\psi}^2}\left|(C_2 + \frac{1}{3}C_1)g_1\left(1 - \frac{g_2}{g_1}\right)\right|^{-2} , \tag{6}$$

where $z \equiv q^2/m_B^2$; this is our main result. The remaining task is to extract from the data for the B decays to J/ψ the value of the parameters $g_{1,2}$. This was done in ref. [8]; however, the derivation in there relied on a specific model for the q^2 dependence of the hadronic form factors. Once again, we can use the heavy-to-light form factor relations of refs. [10] and [11], and avoid the model dependence in the evaluation of $g_{1,2}$.

The ratio g_2/g_1 can be extracted from the polarization in the decay $B \to K^* J/\psi$. Using both the effective bsJ/ψ vertex and the heavy-to-light form factor relations, we obtain for the $B \to K^* J/\psi$ helicity amplitudes

$$\frac{A_+}{A_0} = 0 \tag{7}$$

$$\frac{A_-}{A_0} = -\frac{2m_{J/\psi}}{m_B - E_{K^*} + |\vec{p}_{K^*}|}\left(1 - \frac{g_2}{g_1}\frac{m_B(m_B - E_{K^*} + |\vec{p}_{K^*}|)}{m_{J/\psi}^2}\right) \times \left(1 - \frac{g_2}{g_1}\frac{m_B}{m_B - E_{K^*} + |\vec{p}_{K^*}|}\right)^{-1} , \tag{8}$$

where E_{K^*} and $|\vec{p}_{K^*}|$ are the energy and three-momentum of the K^* in the B meson rest-frame. The polarization ratio

$$\frac{\Gamma_L}{\Gamma} \equiv \frac{|A_0|^2}{|A_0|^2 + |A_-|^2 + |A_+|^2} = \frac{1}{1 + |A_-/A_0|^2} \tag{9}$$

can then be used to determine g_2/g_1. With $\Gamma_L/\Gamma = 0.78 \pm 0.07$ [12], we obtain $g_2/g_1 = 0.24 \pm 0.03$ (the two-fold ambiguity in the solution is resolved using the value found in ref. [8]).

We should point out that, in general, the ratio g_2/g_1 may have a nontrivial phase [13] that we ignore, for now. It can be determined from a more detailed study of the angular correlations in the $B \to K^* J/\psi \to (K\pi)(e^+e^-)$ decay. The distribution in the angles θ_K, θ_{e^+} and ϕ (respectively, the polar angles of the K and e^+ momenta with respect to the momenta of the parent particles K^* and J/ψ, and the azimuthal angle between the K^* and J/ψ decay planes) is determined by the quantities [14]

$$\alpha_1 \equiv \frac{Re(A_+A_0^* + A_-A_0^*)}{|A_0|^2 + |A_-|^2 + |A_+|^2} \tag{10}$$

$$\beta_1 \equiv \frac{Im(A_+A_0^* - A_-A_0^*)}{|A_0|^2 + |A_-|^2 + |A_+|^2} \tag{11}$$

$$\alpha_2 \equiv \frac{Re(A_+A_-^*)}{|A_0|^2 + |A_-|^2 + |A_+|^2} \tag{12}$$

$$\beta_2 \equiv \frac{Im(A_+A_-^*)}{|A_0|^2 + |A_-|^2 + |A_+|^2} \ . \tag{13}$$

A measurement of the coefficients α_1 and β_1, which in our model are

$$\alpha_1 = \frac{\Gamma_L}{\Gamma} Re\left(\frac{A_-}{A_0}\right) \tag{14}$$

$$\beta_1 = -\frac{\Gamma_L}{\Gamma} Im\left(\frac{A_-}{A_0}\right) \ , \tag{15}$$

will allow a determination of the amplitude and phase of g_2/g_1. On the other hand, a measurement of the coefficients α_2 and β_2, which vanish in our model, provides an interesting test of the approximation $m_s \to 0$, that was used in both the bsJ/ψ vertex and in the $B \to K^*$ form factors.

As for $|g_1|$, it is determined from the branching ratio for the inclusive decay $B \to J/\psi +$ anything,

$$\begin{aligned} B(B \to J/\psi + \text{anything}) &= [\Gamma(b \to sJ/\psi) + \Gamma(b \to dJ/\psi)]/\Gamma \\ &= \frac{G_F^2}{16\pi}\tau_B |V_{cb}|^2 (C_2 + \frac{1}{3}C_1)^2 m_b^5 \left(1 - \frac{m_{J/\psi}^2}{m_b^2}\right)^2 \frac{m_{J/\psi}^2}{m_b^2} \\ &\quad \times |g_1|^2 \left(|1 - \frac{g_2}{g_1}|^2 + 2\frac{m_{J/\psi}^2}{m_b^2}|1 - \frac{g_2}{g_1}\frac{m_b^2}{m_{J/\psi}^2}|^2\right) \ . \end{aligned} \tag{16}$$

With $B(B \to J/\psi + \text{anything}) = (0.82 \pm 0.08)\%$ [6], the value that was found above for g_2/g_1, and taking $m_b = m_B$ and $\tau_B = 1.6$ psec, we obtain $|V_{cb}(C_2 + C_1/3)g_1| = (1.81 \pm 0.14) \times 10^{-3}$.

Finally, the expression in eq. 6 becomes

$$\frac{1}{\Gamma(B^- \to \pi^- J/\psi)} \left[\frac{d}{dz}\Gamma(\bar{B}_d^0 \to \pi^+ l^- \bar{\nu}_l)\right]_{z=\frac{m_{J/\psi}^2}{m_B^2}}$$
$$= |V_{ub}|^2 \times (0.54 \pm 0.06) \times 10^6 \ , \tag{17}$$

which can then be used to determine $|V_{ub}|$ (the error corresponds to the present experimental uncertainty in our determination of the parameters $g_{1,2}$). Corrections to this result are necessary, if the ratio g_2/g_1 proves to have a significant phase. The residual theoretical uncertainty in our result is that associated with the heavy-to-light form factor relations of refs. [10] and [11], which are valid in the limit of a static heavy b quark and a massless recoiling quark. From the analysis in ref. [10], it is expected that this approximation holds well for the $B \to \pi$ transition, throughout most of the kinematic range and, in particular, for $q^2 = m_{J/\psi}^2$. Corrections to the form factor relations will be larger, in the case of the $B \to \rho$ or $B \to K^*$ transitions (hence our choice of the semileptonic decay $B \to \pi l^- \bar{\nu}_l$, rather than $B \to \rho l^- \bar{\nu}_l$). These transitions play a role in the evaluation of g_2/g_1, and so this should be the dominant source of the theoretical uncertainty in our final result. As we pointed out before, the size of that uncertainty can be probed experimentally, with a detailed angular analysis of the $B \to K^* J/\psi$ or $B \to \rho J/\psi$ decays.

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