The Assessment of Dynamic Efficiency of Decision Making Units Using Data Envelopment Analysis

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Dedicated

To the memory of my father,
To my mother who gave me so much,
and
To my wife for her years of love.

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Notation

1. x = Input vector.
2. y = Output vector.
3. $P(x, y)$ = Production Possibility Set.
4. R = Real number, R_+ = Positive real number, R_+^n = n dimensional positive real number.
5. **I**(y) = Input requirement set (the collection of all input vector x that yield at least output vector y).
6. **O**(x) = Output predicable set (All output vector y that can be produced using a given input vector x).
7. i = Indices of input; i = 1, 2,..., m.
8. r = Indices of output; r = 1, 2,..., s.
9. j = Indices of DMUs; j = 1, 2,..., n.
10. j_0 = DMU under assessment.
11. x_{ij} = Amount of input m of DMU j.
12. y_{rj} = Amount of output n of DMU j.

13. $x = (x_1, ..., x_m)$ = General vector of input.

14. $y = (y_1, ..., y_s)$ = General vector of output.

15. $x_j = (x_{1j}, ..., x_{mj})$ = Vector of inputs of DMU j.

16. $y_j = (y_{1j}, ..., y_{sj})$ = Vector of outputs of DMU j.

17. t = Indices of period; t=1,...,T.

18. x^t_{ij} = Amount of input i of DMU j at period t.

19. y^t_{rj} = Amount of output r of DMU j at period t.

20. $x^{1,...,t}_{ij}$ = Path of input i of DMU j over periods 1 to t.

21. $y^{1,...,t}_{ij}$ = Path of output r of DMU j over periods 1 to t.

22. D_i = Input distance function.

23. D_o = Output distance function.

24. M_i = Input-oriented Malmquist productivity index.

25. M_o = Output-oriented Malmquist productivity index.

26. ΔEFF = Efficiency change.

27. ΔTECH = Technical change.

Abbreviations

1. CRS = Constant Returns to Scale.
2. DEA = Data Envelopment Analysis.
3. DFA = Deterministic Frontier Analysis.
4. DMU = Decision Making Units.
5. DRS = Decreasing Returns to Scale.
6. FDH = Free Disposal Hull.
7. IRS = Increasing Returns to Scale.
8. NDRS = Non Decreasing Returns to Scale.
9. NIRS = Non Increasing Returns to Scale.
10. PPS = Production Possibility Set.
11. SD = Strong Disposability (in input and output).
12. SFA = Stochastic Frontier Analysis.
13. VRS = Variable Returns to Scale.
14. WD = Weak Disposability (in input and output).

Synopsis

The concept of a “production function” as means to measuring efficiency began in 1928 with the seminal paper by Cobb and Douglas (1928). However, until the 1950s, production functions were largely used as a tool for studying the functional distribution of income between capital and labour. Farrell’s argument (1957) provides an intellectual basis for redirecting attention from the production function specifically to the deviation from that function as a measure of efficiency. He developed a method so that we can measure efficiency in terms of distance to the “best DMU“ on the frontier isoquant.

Charnes, Cooper and Rhodes (1978) generalised Farrell’s concept to multiple - input multiple - output situations and reformulated it using mathematical programming and thus derived an efficiency measurement known as Data Envelopment Analysis (DEA). Therefore DEA is a linear programming based method for comparing Decision Making Units (DMUs) such as schools, hospitals, etc. In the method originally proposed by Charnes, Cooper and Rhodes (1978) the efficiency of a DMU is defined as a ratio of the weighted sum of outputs to the weighted sum of inputs. Thus in the original DEA approach the notion of time dimension has been ignored.

This thesis proposes a DEA based method for assessing the comparative efficiencies of DMUs operating production processes where input - output levels are inter - temporally dependent. One cause of inter - temporal dependence between input and output levels is stock input which influences output levels over many production periods. Such DMUs cannot be assessed by traditional or ‘static’ DEA. The method developed in the study overcomes the problem of inter - temporal input - output dependence by using input - output ‘paths’ mapped out by operating DMUs over time as the basis of assessing them.

The aim of this thesis is, therefore, firstly, to **address that traditional or “static” DEA fails to capture the efficiency of DMUs with inter - temporal input - output dependence**. Secondly the thesis **develops an approach for measuring efficiency under inter - temporal input - output dependence** by defining an inter - temporal Production Possibility Set (PPS). The method developed uses path of input - output levels associated with DMUs rather than input - output DMUs observed at one point in time as static DEA does. Using this PPS, an assessment framework is developed which parallels that of static DEA.

The thesis **develops mathematical programming models which use input - output** paths to **measure efficiency**, **identify peers** and **target** of performance of DMUs.

The approach is **illustrated using simulated and real data**.

CHAPTER 1: Introduction to efficiency measurement and Data Envelopment Analysis

1.1 Introduction

"Decision Making Units" (DMUs) are units of organisations such as bank branches, universities and hospitals, which typically perform the same function (e.g. bank service in the case of bank branches). A DMU usually uses a set of inputs (resources) to secure a set of outputs (products). The process of transforming inputs into outputs is usually called "production technology".

A description of production technology, in econometrics theory, is expressed by a "production function" which describes the maximum amount of one output that can be produced for given levels of production of the remaining outputs and for given level of input usage ("profit function"). Alternatively, it describes the minimum amount of one input required for the production of given outputs with given amount of all other inputs ("cost function"). Thus a production function specifies a maximum (profit function) or a minimum value (cost function) which can be achieved under the constraints imposed by technology. That is, it describes a boundary, or a "frontier".

For a variety of reasons, interest frequently centres on the distance an observed DMU operates from a frontier, since such a distance provides a measure of the efficiency of the DMU under observation.

This concept of a "production function" as a means to measuring efficiency began in 1928 with the seminal paper by Cobb and Douglas (1928). However, until the 1950s, production functions were largely used as a tool for studying the functional distribution of income between capital and labour. Farrell's argument (1957) provides an intellectual basis for redirecting attention from the production function specifically to the deviation from that function as a measure of efficiency. He developed a method so that we can measure efficiency in terms of distance to the "best DMU" on the frontier isoquant.

This chapter presents a survey of the methods that have been employed for efficiency measurement since Farrell's seminal work. The chapter unfolds as follows.

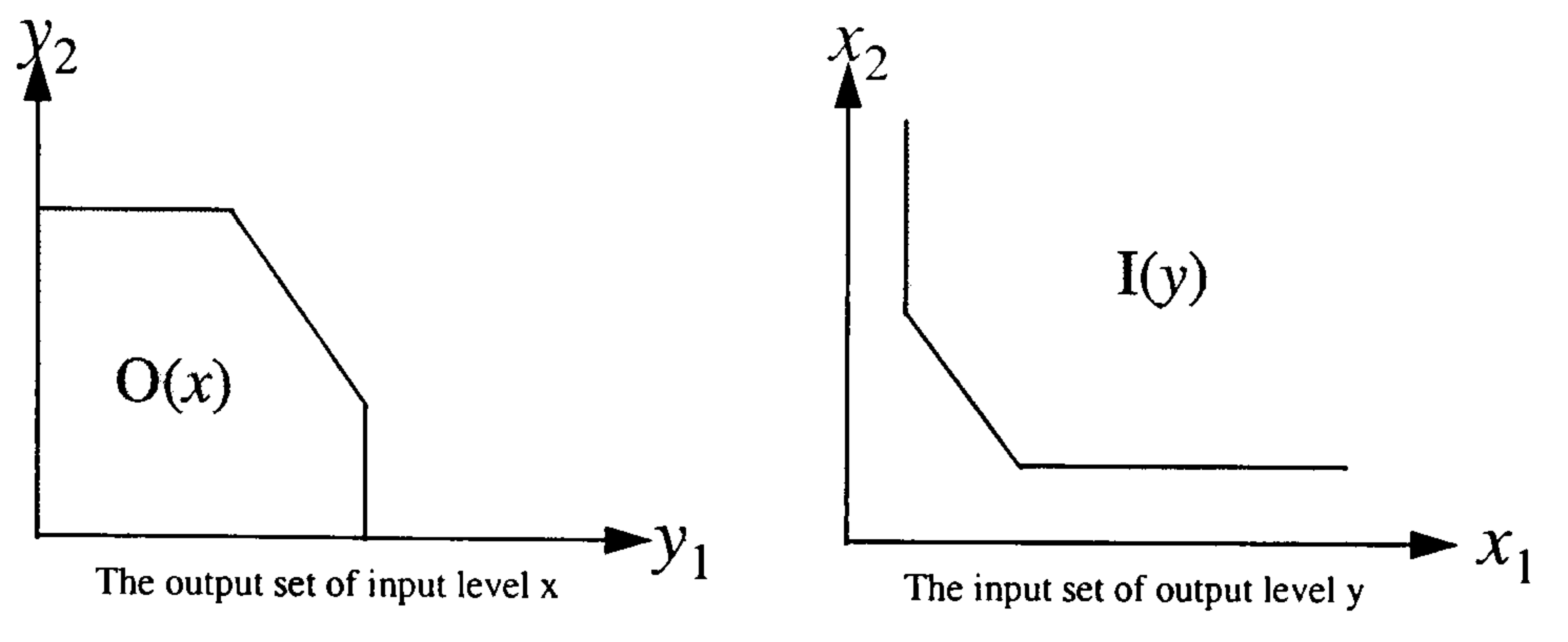
Section (1.2) lays out some of the definitions and terms used later including production technology and "Production Possibility set". Section (1.3) discusses the concept of efficiency measurement, "technical efficiency", "allocative efficiency" and "overall efficiency", in economics with some studies of "parametric frontier analysis". Section (1.4) discusses some studies of non - parametric frontier analysis originally developed by Charnes, Cooper and Rhodes (1978). These represent the concept of Data Envelopment Analysis (DEA) with mathematical details and graphical illustrations. Section (1.5) concludes.

1.2 Production technology

A production technology transforming inputs $x \in R_+^n$ into outputs $y \in R_+^m$ can be represented by input - output correspondences P such that P is the collection of all feasible input - output vectors, i.e.

$$P = \{(x, y) \in R_+^{n+m}, x \text{ can produce } y\} \qquad (1.1).$$

Figure 1-1. The input and output set



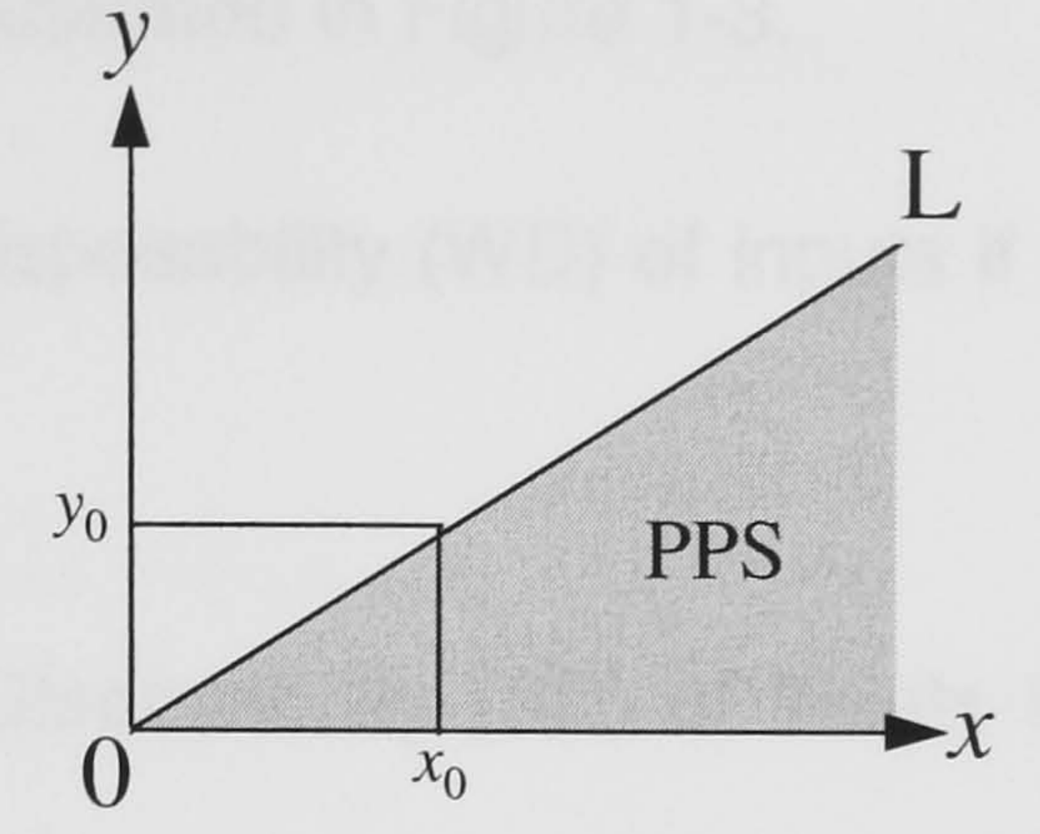
P is usually called the Production Possibility Set (PPS). The set **O**(x) is called the output set, and it denotes the collection of all output vectors $y \in R_+^m$ that are obtainable from the input vector $x \in R_+^n$. The input set **I**(y) denotes the collection of all input vectors $x \in R_+^n$ that yield at least output vector $y \in R_+^m$. The input set and output set are illustrated in Figure 1-1.

The input - output correspondences can be obtained from the PPS as

$$\mathbf{I}(y) = \{x \in R_+^n,\ (x, y) \in P\} \quad \text{and} \quad \mathbf{O}(x) = \{y \in R_+^m,\ (x, y) \in P\}.$$

The relationship between the input set, output set and PPS is shown in Figure 1-2. The PPS of the technology is the area bounded by the x-axis and line L. The output set corresponding to x_0 is $\mathbf{O}(x_0)=[0,\ y_0]$ and the input set corresponding to y_0 is $\mathbf{I}(y_0)=[x_0,\ +\infty)$.

Figure 1-2. The relationship between the input set, output set and PPS



The input set, output set and PPS have one feature in common. They provide a representation of technology in terms of input quantities and corresponding output quantities. In addition two conditions, returns to scale and disposability, are very important to determine the shape of the PPS. These conditions can apply as follows (see for example Färe, Grosskopf and Lovell (1994) p.33-44 and Banker *et al.* (1984) p.1081 and Färe, Grosskopf and Lovell (1985)):

A technology:

I. Exhibits Constant Returns to Scale (CRS) iff $\lambda P = P \ \forall \ \lambda > 0$, where $\lambda P = \{ (\lambda x, \lambda y) \ ; \ \forall \ (x, y) \in P \}$.

II. Exhibits Non Increasing Returns to Scale (NIRS) iff $\lambda P \subseteq P \ ; \ \forall 0 < \lambda \leq 1$ or equivalently if $P \subseteq \lambda P \ ; \ \forall \lambda \geq 1$.

III. Exhibits Non Decreasing Returns to Scale (NDRS) iff $\lambda P \subseteq P \ ; \ \forall \lambda \geq 1$ or equivalently if $P \subseteq \lambda P \ ; \ \forall 0 < \lambda \leq 1$.

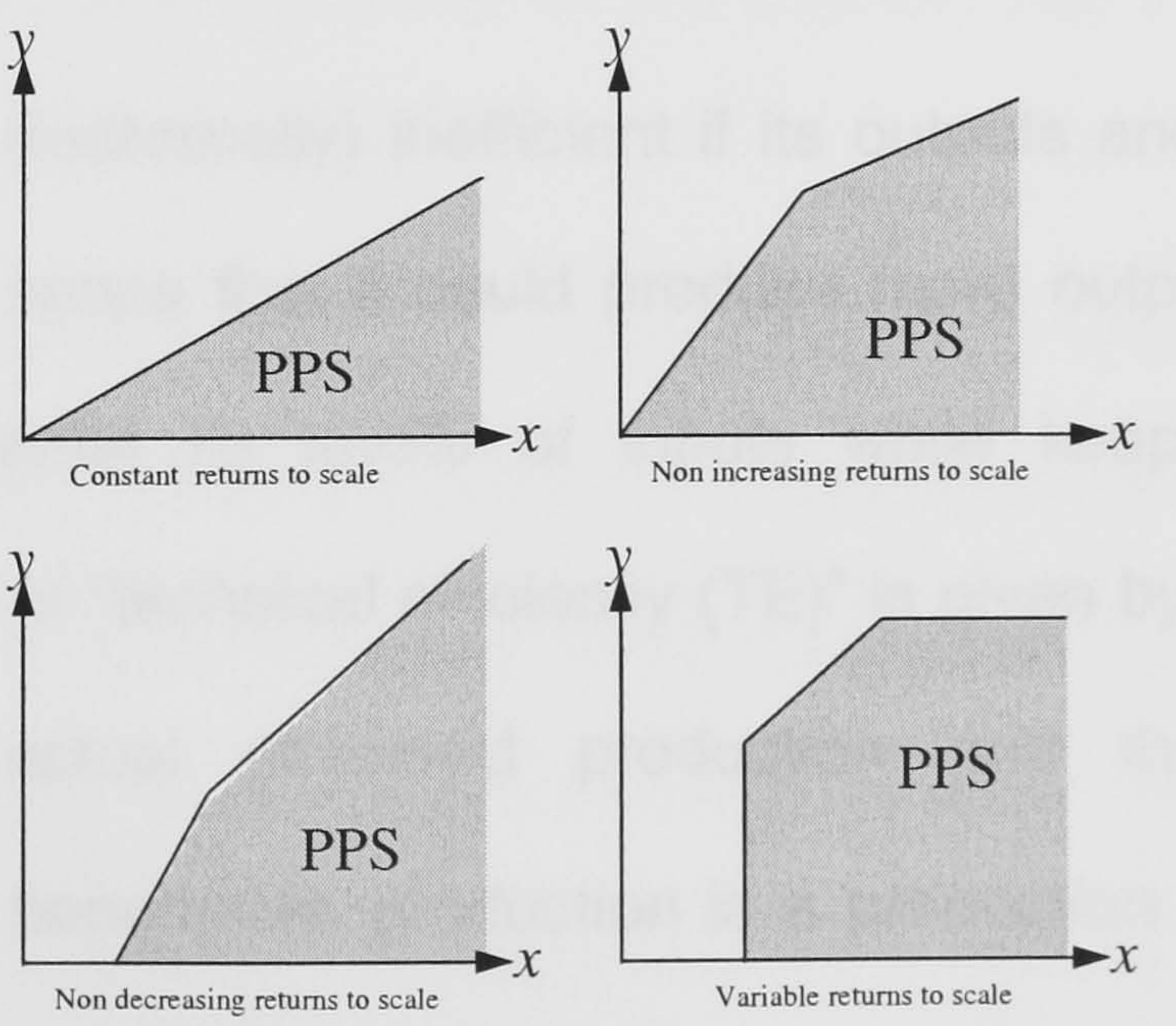
IV. Exhibits Variable Returns to Scales (VRS) iff none of the above returns to scale hold. The returns to scale behaviours of production technology are illustrated in Figure 1-3.

V. Exhibits Weak Disposability (WD) of inputs if $(x, y) \in P$ then $(\lambda x, y) \in P$; $\forall \lambda \geq 1$.

VI. Exhibits Strong Disposability (SD) of inputs if $(x, y) \in P$ and $x' \geq x$ then $(x', y) \in P$.

VII. The output disposability can be defined in a similar way. A PPS is said to exhibit Weak Disposability of output if $(x, y) \in P$ and $(x, \lambda^{-1} y) \in P$; $\forall \lambda \geq 1$ and it exhibits Strong Disposability of output if $(x, y) \in P$ and $y' \leq y$ then $(x, y') \in P$.

Figure 1-3. The returns to scale of production technology



It will be seen later how these assumptions are used to construct a PPS in various approaches to efficiency measurement.

Next the methods of measurement of efficiency are explained.

1.3 Measurement of efficiency

1.3.1 The concept

The seminal article by Farrell (1957) introduced the concept of the "best practice frontier" which outlines the technical limits of what a Decision Making Unit (DMU) can achieve. This best practice frontier, also called the production or the efficiency frontier, specifies for a DMU the maximum quantities of outputs it can produce given any level of inputs and, for any levels of outputs, the minimum quantities of inputs needed for producing the outputs. Using this frontier the concept and the index of technical inefficiency can be defined.

A DMU is said to be (technically) inefficient if its outputs and inputs are below the frontier, in the sense that it could produce more outputs with the available inputs or decrease its levels of inputs while keeping outputs unchanged. The measure of "technical efficiency (TE)" is given by the relative distance between the actual observed production and the "nearest" benchmark production (a benchmark production is a production lying on the frontier).

Figure 1-4. Production frontier for one-input one-output technology

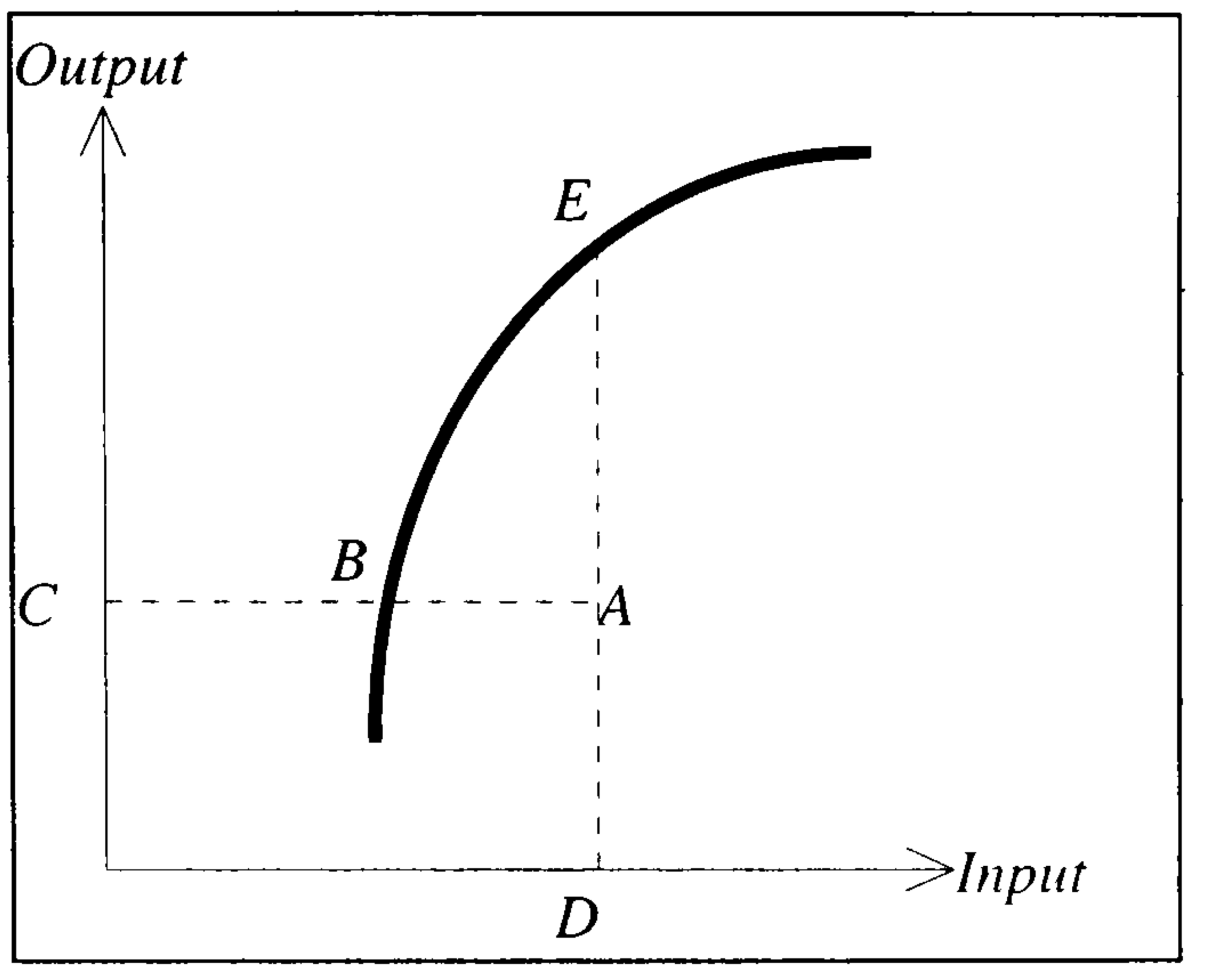


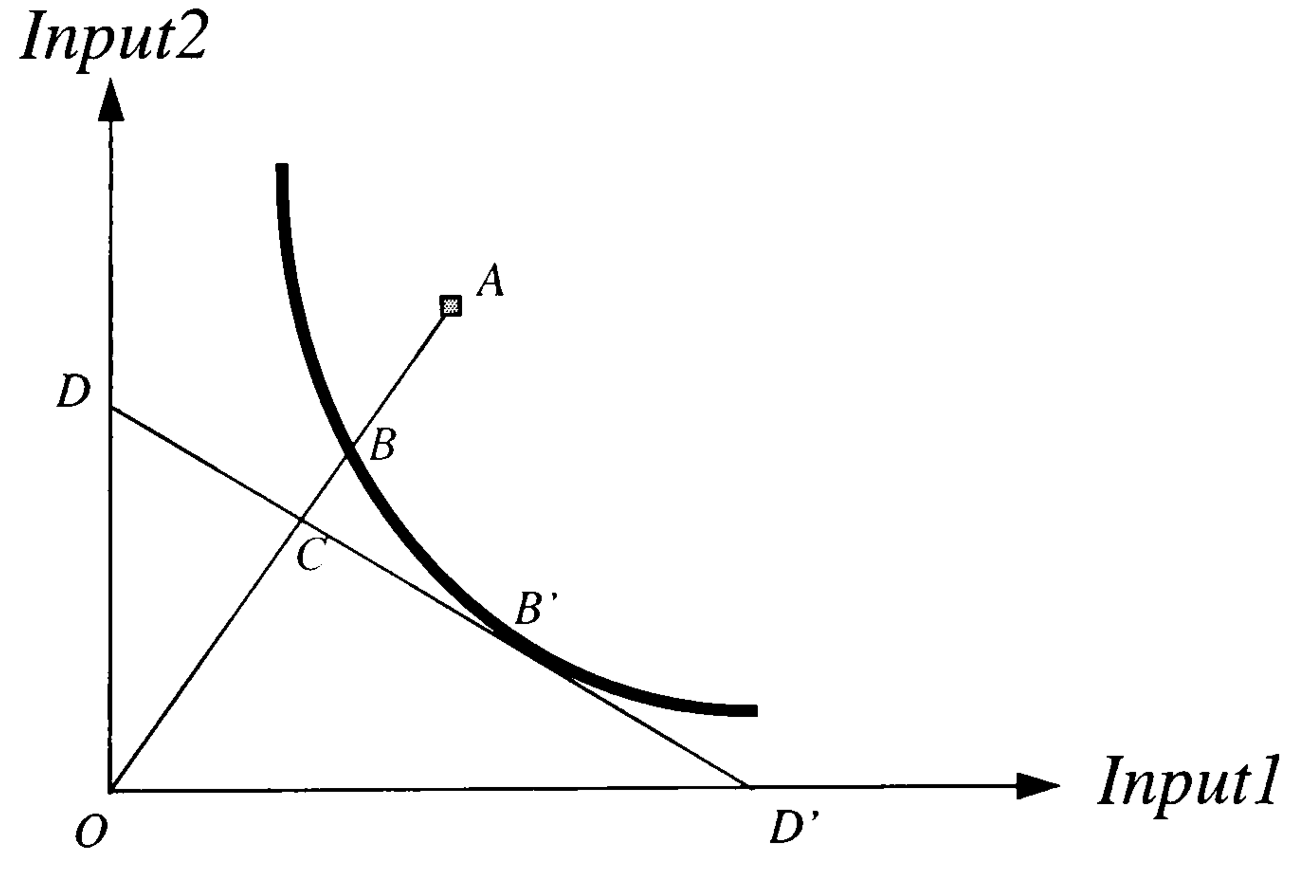
Figure 1-4 represents such a production frontier for the one - input one - output case. If a DMU yields an input - output vector equal to A it is inefficient and its degree of technical efficiency can be given either by an input based indicator $\frac{BC}{AC}$ or by an output based indicator $\frac{DA}{DE}$. Either indicator would be equal to 1 when the actual production lies on the frontier. The DMUs on the frontier are called "*Pareto efficient DMUs*".

Farrell (1957) also proposed another measure, "allocative efficiency (AE)", which reflects the ability of a DMU to use inputs in optimal proportions, given their respective prices and the production technology. This is illustrated in Figure 1-5 for the case of two inputs for one unit output. If the input price ratio is represented by the slope of the isocost line DD', the allocative efficiency of the DMU operating at A is defined to be the ratio $\frac{OC}{OB}$ since the distance BC represents the reduction in production costs that would occur if production were to occur at the allocatively (and technically) efficient point B', instead of at the technically efficient, but allocatively inefficient point B.

The "overall efficiency (OE)", then, is defined to be the ratio $\frac{OC}{OA}$ where the distance AC can also be interpreted in terms of cost reduction. Note that the product of the technical and allocative efficiency provides the measure of overall efficiency.

$$\text{TE} \times \text{AE} = \frac{OB}{OA} \times \frac{OC}{OB} = \frac{OC}{OA} = \text{OE}.$$

Figure 1-5. Production frontier for two inputs and for one unit output

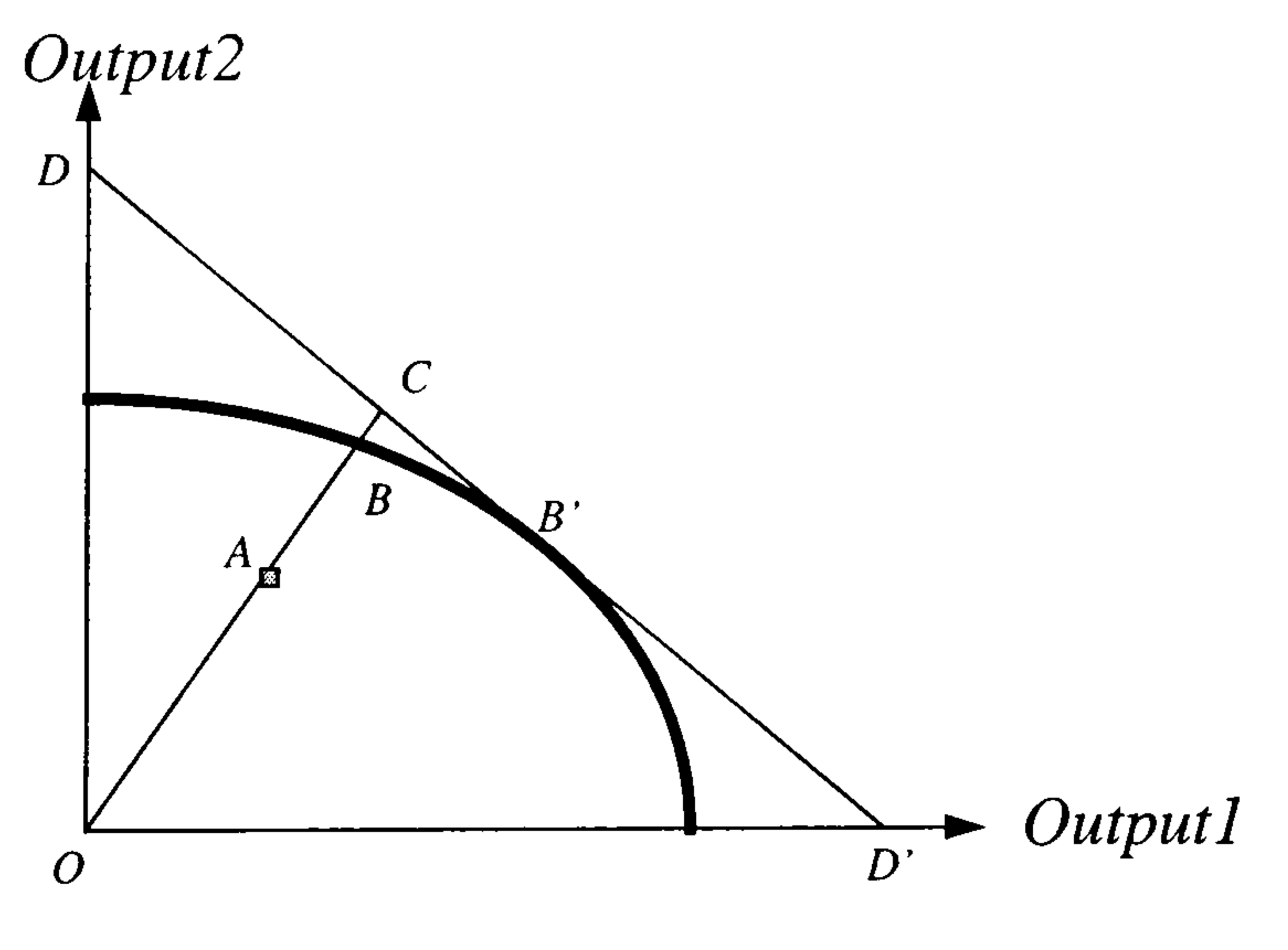


This means allocative (and overall) efficiency is input - oriented. One can illustrate output - oriented measure of allocative (and overall) efficiency by considering the case where production involves two outputs normalised by a single input.

This case is depicted in Figure 1-6 where the point A corresponds to an inefficient DMU which lies below the border of the PPS. Assume DD' is the isorevenue line, thus the (output - oriented) allocative efficiency is defined by $\frac{OB}{OC}$ which has a revenue increasing interpretation (similar to the cost reducing interpretation of allocative efficiency in the input - oriented case). Furthermore, the (output - oriented) overall efficiency is defined as the product of the technical and allocative efficiency measures.

$$\mathrm{OE} = \mathrm{TE} \times \mathrm{AE} = \frac{OA}{OB} \times \frac{OB}{OC} = \frac{OA}{OC}$$

Figure 1-6. Production frontier for two outputs and for one unit of input



1.3.2 The parametric method versus the non - parametric method

There are two empirical methodologies for the measurement of efficiency: "parametric" and "non - parametric".

One distinguishes between these two main alternatives by whether or not the frontier can be specified as a function with constant parameters.

Parametric approach

In the parametric approach of efficiency measurement the production technology is modelled with a single - output production frontier;

$$y_j = f(x_j, \beta) \qquad (1.2)$$

where y denotes output, x denotes a vector of inputs, β is a vector of parameters that is the object of estimation and j denotes the DMU. In reality the functional form of the production model to be estimated may be linear in the logs of output and independent variables such that

$$Y_j = \alpha + \beta X_j \qquad (1.3)$$

where Y_j is the log of the single output of DMU j, X_j is a vector of the logs of its input levels and (α, β) is a vector of unknown parameters.

Technical inefficiency is assumed to enter the production model additively in logarithms (1.3) (or multiplicative in production technology (1.2)) in the form

$$Y_j = \alpha + \beta X_j + \varepsilon_j \quad \text{or} \quad y_j = f(x_j, \beta) \times \varepsilon'_j \qquad (1.4)$$

where ε_j (= log ε'_j) is the indicator of the technical efficiency.

In this approach the aim is the specification and estimation of ε as an efficiency rate. For this in the context of econometric literature there are two distinct models for estimating (1.3) or (1.2) using observed input - output correspondences.

First, "Deterministic Frontier Analysis" (DFA) which measures the technical efficiency relative to a deterministic parametric frontier (see for example Aigner and Chu (1968)). There are a few applications of deterministic production frontiers including Steveneson (1980) and Aguilar (1988). Further discussions appear in Deprins and Simar (1983) and Lovell (1993).

Secondly, "Stochastic Frontier Analysis" (SFA) which measures the technical efficiency relative to a stochastic parametric frontier (see for example Aigner, Lovell and Schmidt (1977)). This stochastic parametric frontier approach assumes ε_j in (1.4) is a composed error term;

$$\varepsilon_j = \nu_j - \mu_j \quad (1.5)$$

where ν_j , is a symmetric normal term capturing randomness outside of the control of the DMU and μ_j (≥ 0) is a one-sided component capturing inefficiency. Further discussion appears in Meeusen *et al.* (1977).

Non - parametric approach

An alternative method of efficiency measurement is "Data Envelopment Analysis" (DEA). This is a non - parametric technique in the sense that no functional form is assumed for the frontier. It measures efficiency relative to a deterministic frontier using linear programming techniques to "envelop" observed input - output vectors as tightly as possible (Charnes, Cooper and Rhodes (1978)). One main advantage of DEA is that it allows several inputs and several outputs to be considered at the same time. In this case, efficiency is measured in terms of inputs or outputs along a ray from the origin.

Historical background of non - parametric efficiency measurement

Koopmans (1951) provided a formal definition of technical efficiency:

A DMU is technically efficient

$\Rightarrow$ if an increase in any output requires a reduction in at least one other output or an increase in at least one input, and or

$\Rightarrow$ if a reduction in any input requires an increase in at least one other input or a reduction in at least one output.

Thus an inefficient DMU could produce the same outputs with less of at least one input, or could use the same inputs to produce more of at least one output.

Farrell (1957) introduced a measure of technical efficiency. To relate the Farrell measure to Koopmans' definition, Shephard (1953, 1970) introduced the "input distance function". Assuming P represents the set of correspondences of input - output as in (1.1). For each $y \in R_+^n$ we may define an "isoquant set"

$$\text{Isoq}(y) = \{x \mid (x, y) \in P \;\&\; (\lambda x, y) \notin P \;;\; \forall\, 0 \leq \lambda < 1\}$$

and an "efficient set"

$$\text{Eff}(y) = \{\, x \mid (x, y) \in P \;\&\; (x', y) \notin P \;;\; \forall\, x' <\neq x\}$$

where $x' <\neq x$ means each element of x is greater than or equal to the corresponding element of x' and x' is different from x.

It is obvious Eff(y) $\subseteq$ Isoq(y). The difference between these two sets will be illustrated later by an example.

Shephard's (1970) input distance function can then be defined as

$$D_i(x, y) = \max\{\, \lambda \mid \frac{x}{\lambda} \in \text{Isoq}(y) \,\}.$$

Clearly $D_i(x, y) \geq 1$ and Isoq(y) = $\{\, x \mid D_i(x, y) = 1 \,\}$ (Shephard (1970)). The "Farrell input - oriented" measure of technical efficiency can now be given as

$$F_i(x, y) = \min\{\, \varphi \mid (\varphi\, x, y) \in P \,\}$$

and it is obvious that $F_i(x, y) \leq 1$, $F_i(x, y) = (D_i(x, y))^{-1}$ and Isoq(y) = $\{\, x \mid F_i(x, y) = 1 \,\}$.

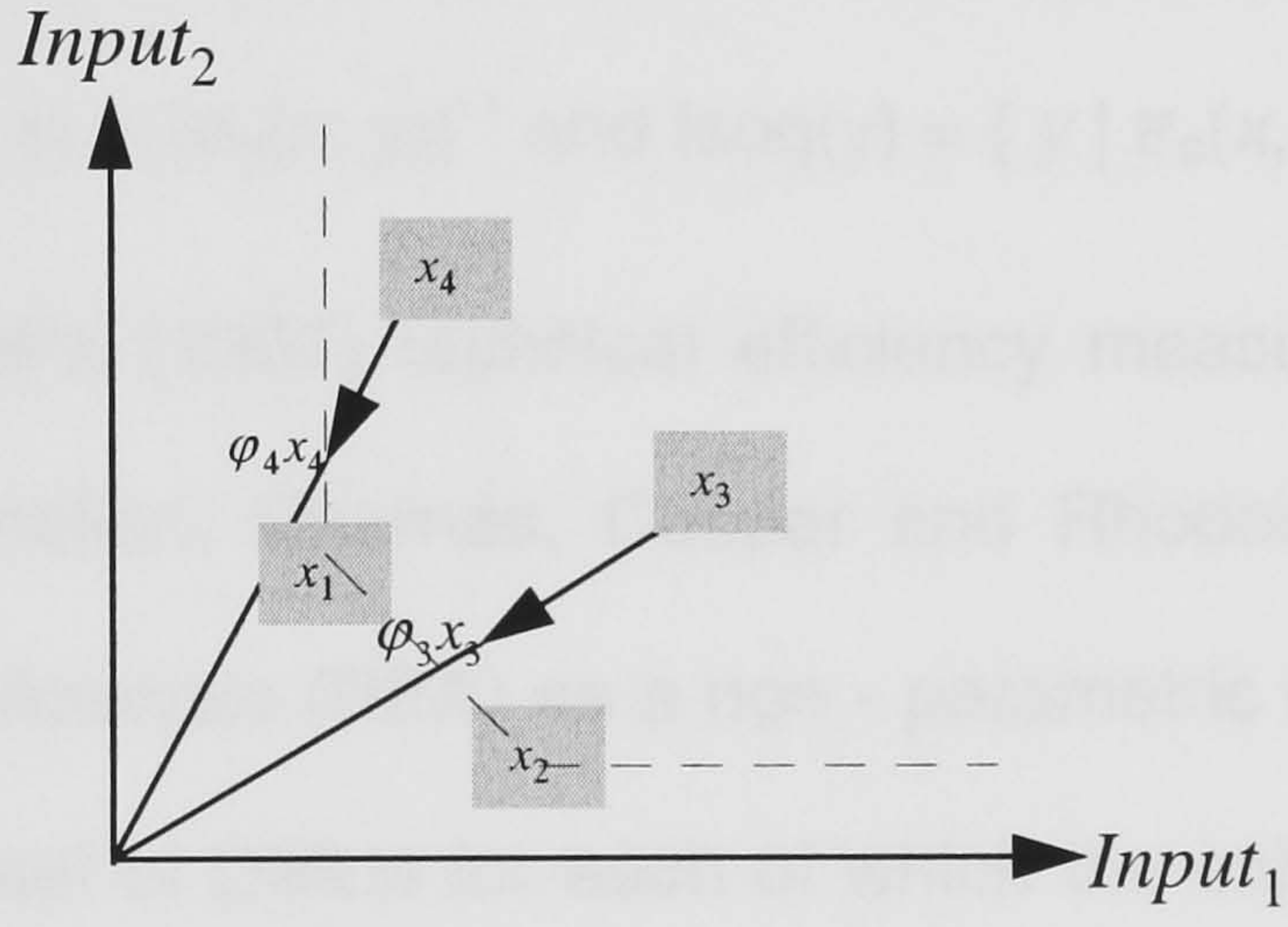
The input set of output y and the input technical efficiency measure are illustrated in Figure 1-7.

It can thus be seen that input vectors $v_3 x_3$ and $v_4 x_4$ can not be contracted radially and still remain capable of producing output vector y. Consequently $F_i(x_1, y) = F_i(x_2, y) = 1$ but $F_i(x_3, y) < 1$ and $F_i(x_4, y) < 1$ (thus $F_i(\varphi_3 x_3, y) = 1$ and

$F_i(\varphi_4 x_4, y)=1$). Also the difference between the efficient set and the isoquant set of output y can be seen in this example as $\varphi_3 x_3 \in \mathrm{Eff}(y)$ but $\varphi_4 x_4 \notin \mathrm{Eff}(y)$ while both $\varphi_3 x_3$ and $\varphi_4 x_4 \in \mathrm{Isoq}(y)$.

Figure 1-7. The input technical efficiency measure

(I(y) = Set of vectors to the right and above of broken line)



Since technical efficiency measurement is sometimes used to investigate output augmentation it is useful to replicate the above definitions in the output orientation. For each $x \in R_+^m$ we could define an isoquant and efficiency sets as follows

$$\mathrm{Isoq}(x) = \{\, y \mid (x, y) \in P \;\&\; (x, \lambda y) \notin P \;;\; \forall\, \lambda > 1 \}$$

$$\mathrm{Eff}(x) = \{\, y \mid (x, y) \in P \;\&\; (x, y') \notin P\ \forall \;;\; y <\neq y' \}$$

with the property that $\mathrm{Eff}(x) \subseteq \mathrm{Isoq}(x)$.

Thus Shephard's (1970) output distance function

$$\mathbf{D}_o(x, y) = \min \{ \lambda \mid \frac{y}{\lambda} \in \mathrm{Isoq}(x) \}$$

provides another measure of efficiency which is of course $\mathbf{D}_o(x, y) \leq 1$. We also have $\mathrm{Isoq}(x) = \{ y \mid \mathbf{D}_o(x, y) = 1 \}$. The "Farrell's output - oriented" measure of technical efficiency can now be defined as

$$\mathrm{F}_o(x, y) = \max \{ \theta \mid (x, \theta y) \in \mathrm{P} \}.$$

Thus we have $\mathrm{F}_o(x, y) = (\mathbf{D}_o(x, y))^{-1}$ and $\mathrm{Isoq}(y) = \{ y \mid \mathrm{F}_o(x, y) = 1 \}$.

Following Farrell's (1957) technical efficiency measure and Shephard's (1970) distance function, Charnes, Cooper and Rhodes (1978) developed Data Envelopment Analysis (DEA) as a non - parametric method of efficiency measurement of a set of DMUs for each of which the only data available are the levels of their multiple inputs and outputs. This approach has been shown to be a significant generalisation of the Farrell method of efficiency measurement and also equivalent to the concepts of "Pareto efficiency".

The method developed in this thesis is an extension to DEA. Therefore the rest of this chapter will discuss the basic and recent developments in DEA.

1.4 Data Envelopment Analysis (DEA)

1.4.1 Basic DEA

Charnes, Cooper and Rhodes (1978) generalised Farrell's measure to multiple - input multiple - output situations and operationalised it using mathematical programming. This method for efficiency measurement became known as "Data Envelopment Analysis" (DEA). Assume a set of observed DMUs, {DMU j; j=1,...,n}, is associated with m inputs, $\{x_{ij}$; i=1,...,m}, and s outputs, $\{y_{rj}$; r=1,...s}. In the method originally proposed by Charnes, Cooper and Rhodes (1978) the efficiency of the j^{th} DMU is defined as follows.

$$\mathrm{Eff} = \frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{rj}}$$

where

y_{rj} = the amount of the r^{th} output from DMU j,

u_r = the weight given to the r^{th} output,

x_{ij} = the amount of the i^{th} input used by DMU j,

v_i = the weight given to the i^{th} input.

The efficiency is then defined as a ratio of the weighted sum of the outputs to the weighted sum of the inputs. Then to measure the efficiency of DMU j_0 Model 1-1 is used.

Model 1-1. DEA ratio model

$$\text{Eff} = \underset{u_r,\, v_i}{\text{Max}} \frac{\sum_r u_r y_{rj_0}}{\sum_i v_i x_{ij_0}}$$

$$s.t.$$

$$\frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \le 1 \qquad ;\forall j$$

$$u_r,\, v_i \ge 0 \qquad ;\forall r, \forall i$$

This fractional model can be easily transformed to a linear programming model (Charnes and Cooper (1962)) as in presented in Model 1-2 and Model 1-3 respectively for input and output orientation case.

Model 1-2. DEA weights model, input-oriented

$$\text{Eff} = \underset{u_r,\, v_i}{\text{Max}} \sum_r u_r y_{rj_0}$$

$$s.t.$$

$$\sum_r u_r y_{rj} - \sum_i v_i x_{ij} \le 0 \qquad ;\forall j$$

$$\sum_i v_i x_{ij_0} = 1$$

$$u_r,\, v_i \ge 0 \qquad ;\forall r, \forall i.$$

Model 1-3. DEA weights model, output-oriented

$$\text{Eff} = \underset{u_r,\, v_i}{\text{Min}} \sum_i v_i x_{ij_0}$$

$$s.t.$$

$$\sum_r u_r y_{rj} - \sum_i v_i x_{ij} \le 0 \qquad ;\forall j$$

$$\sum_r u_r y_{rj_0} = 1$$

$$u_r,\, v_i \ge 0 \qquad ;\forall r, \forall i.$$

The Model 1-2 and Model 1-3 have duals, which measure efficiency with reference to production possibility sets. An axiomatic and self – contained development of such models is presented in Banker, Charnes and Cooper (1984). Let us have the observed DMUs $\{(x_j, y_j)\ j=1,...,n\}$ as defined above. Banker *et al.* (1984) postulated the production possibility set P has the following five properties:

Postulate 1. Non empty. $(x_j, y_j) \in P$ ($\forall$ j=1,...,n) then P is non empty.

Postulate 2. Constant Returns to Scale (CRS). If $(x_j, y_j) \in P$ then for any non-negative scalar $\alpha \geq 0$ $(\alpha x_j, \alpha y_j) \in P$.

Postulate 3. Strong Disposability.

a) If $(x_j, y_j) \in P$ and $x_{j1} \geq x_j$ then $(x_{j1}, y_j) \in P$ (Input Disposability).

b) If $(x_j, y_j) \in P$ and $y_{j1} \leq y_j$ then $(x_j, y_{j1}) \in P$ (Output Disposability).

Postulate 4. Convexity. P is a closed and convex set.

Postulate 5. Minimum extrapolation. P is the intersection of all production sets satisfying postulates 1 to 4 and which contains all the observed DMUs.

If P satisfies the above postulates then P can be expressed as

$$P = \{(x_{j0}, y_{j0}) \text{ s.t. } \textstyle\sum_j \lambda_j x_j \leq x_{j0} \text{ and } \sum_j \lambda_j y_j \geq y_{j0}\ ,\ \lambda_j \geq 0;\ j=1, ..., n\}.$$

The vector $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \in R+^n$ enables us to shrink or expand individual observed DMU for the purpose of constructing an unobserved but feasible DMU.

Combining this PPS with the definition of Farrell's technical efficiency and Shephard's distance function and reformulating it as a linear programming model the following DEA model is obtained for assessing the efficiency of DMU j_0.

Model 1-4. Output oriented - CRS envelopment model

$$
\begin{array}{lll}
\underset{\lambda, h, S_i^-, S_r^+}{\text{Max}} & h + \varepsilon(\sum_r S_r^+ + \sum_i S_i^-) & \\
\text{s.t.} & & \\
& \sum \lambda_j x_{ij} = x_{ij_0} - S_i^- & \forall i \\
& \sum \lambda_j y_{rj} = h y_{rj_0} + S_r^+ & \forall r \\
& S_i^-, S_r^+ \geq 0 & \forall i, \forall r \\
& \lambda_j \geq 0 & \forall j \\
& \varepsilon > 0. &
\end{array}
$$

The Model 1-4 defines the relative efficiency of a DMU in terms of output maximisation. An input minimisation model will be presented later. It is the dual to the weight Model 1-3 except that we also introduced a new element, ε, a positive non-Archimedean (Charnes and Cooper (1984)). Its use ensures that all u_r and $v_i > 0$, so all inputs and outputs are to be accorded some positive value. These values need not to be specified but can be dealt with by computational processes (See for example Ali (1990) and Ali and Seiford (1993)). It is noted that h is maximised first, after which the sum of the slacks in Model 1-4 is maximised. The model then identifies the non-zero slacks, if they exist at an optimal solution, and assurance is provided that no DMUs will not be mistakenly characterised as efficient. This is because an optimal solution could be obtained showing h^*=1 and slacks at zero while alternate solutions exist which associate non-zero slacks with h^*=1, where h^* is the optimum value of h (see Ali *et al.* (1991) and Ali (1992)). In this way, the non-Archimedean element $\varepsilon>0$ is given a computational form without any need to specify it explicitly. (Most DEA computer codes accomplish this in two stages. Stage 1: obtains a value of max h^* with slacks all multiplied by zero rather than $\varepsilon>0$ in the objective function. This h^* is then fixed in Model 1-4 so that cannot be altered in a second stage, which is then directed to maximising the sum of slacks (see Arnold *et al.* (1996) Thanassoulis and Emrouznejad (1996)).

Hence, DMU j_0 is said to be Pareto efficient iff h^*=1 and the optimal values of S_i^{-*} & S_r^{+*} are zero for all i & r (Cooper *et al.* (1999)). This means that no other DMU or combination of DMUs exist which can produce at least the same amount of output as DMU j_0, with less for some resources and / or no more for any other resources.

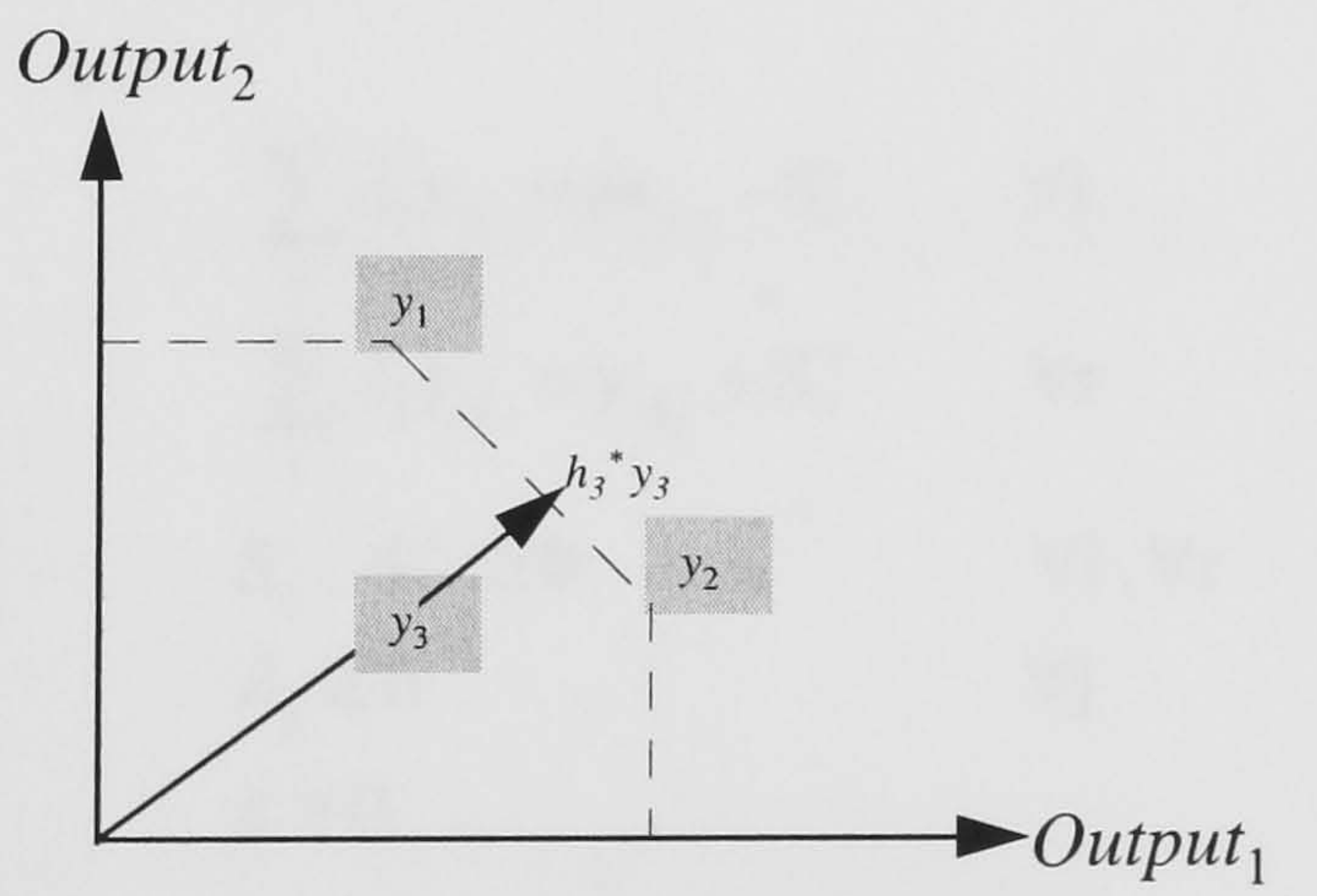
In Model 1-4, S_i and S_r represent slack variables. Thus a slack in an input i, i.e. $S_i^{-*} > 0$, represents an additional inefficient use of input i. A slack in an output r, i.e. $S_r^{+*} > 0$, represents an additional inefficiency in the production of output r.

The DEA Model 1-4 is known as CRS - output - oriented model because it expands the output of DMU j_0 within the CRS - PPS. It should be solved n times once for each DMU being evaluated to generate n optimal sets of values of (h^*, λ^*).

For DMU j_0, DEA efficiency will be the 1/ h^*_{j0}. Therefore:

- If radial expansion is possible Model 1-4 will yield $h^*_{j0} > 1$,
- If radial expansion is not possible Model 1-4 will yield $h^*_{j0} = 1$.

Figure 1-8. The CRS - output - oriented model (Output set of input x)



The positive elements of the optimal values in λ identify the set of dominating DMUs located on the constructed production frontier, against which DMU j_0 is evaluated. The DMUs of this set are called "*peers*" to DMU j_0

(Boussofiane, Dyson and Thanassoulis (1991)). The CRS - output - oriented model is illustrated in Figure 1-8.

Output vector y_3 can be expanded radially and still there is no need to increase its input level x. Consequently h^* in Model 1-4 would be over 1 and its efficiency, $\frac{1}{h^*}$<1. However output vector $h_3^* y_3$ can not be expanded radially using the same amount of input level. Thus $h_3^* y_3$ belongs to the efficient output set and Eff($h_3^* y_3$) = 1. Since $h_3^* y_3$ lies on the line $y_1 y_2$ then y_3 is evaluated against y_1 and y_2 and therefore these DMUs are peers for y_3.

The input oriented model of DEA can be defined in a similar way. The CRS - input model which is dual to Model 1-2 is as follows.

Model 1-5. Input oriented - CRS model

$$\underset{\lambda, h, \mathrm{s}_i^-, \mathrm{s}_r^+}{\text{Min}} \quad \phi - \varepsilon\left(\sum_r \mathrm{S}_r^+ + \sum_i \mathrm{S}_i^-\right)$$

$$\begin{aligned}
&\text{s.t.} \\
&\sum_j \lambda_j \mathrm{x}_{ij} = \phi \mathrm{x}_{ij_0} - \mathrm{S}_i^- && \forall \mathrm{i} \\
&\sum_j \lambda_j \mathrm{y}_{rj} = \mathrm{y}_{rj_0} + \mathrm{S}_r^+ && \forall \mathrm{r} \\
&\mathrm{S}_i^-, \mathrm{S}_r^+ \geq 0 && \forall \mathrm{i}, \forall \mathrm{r} \\
&\lambda_j \geq 0 && \forall \mathrm{j} \\
&\varepsilon > 0. \ .
\end{aligned}$$

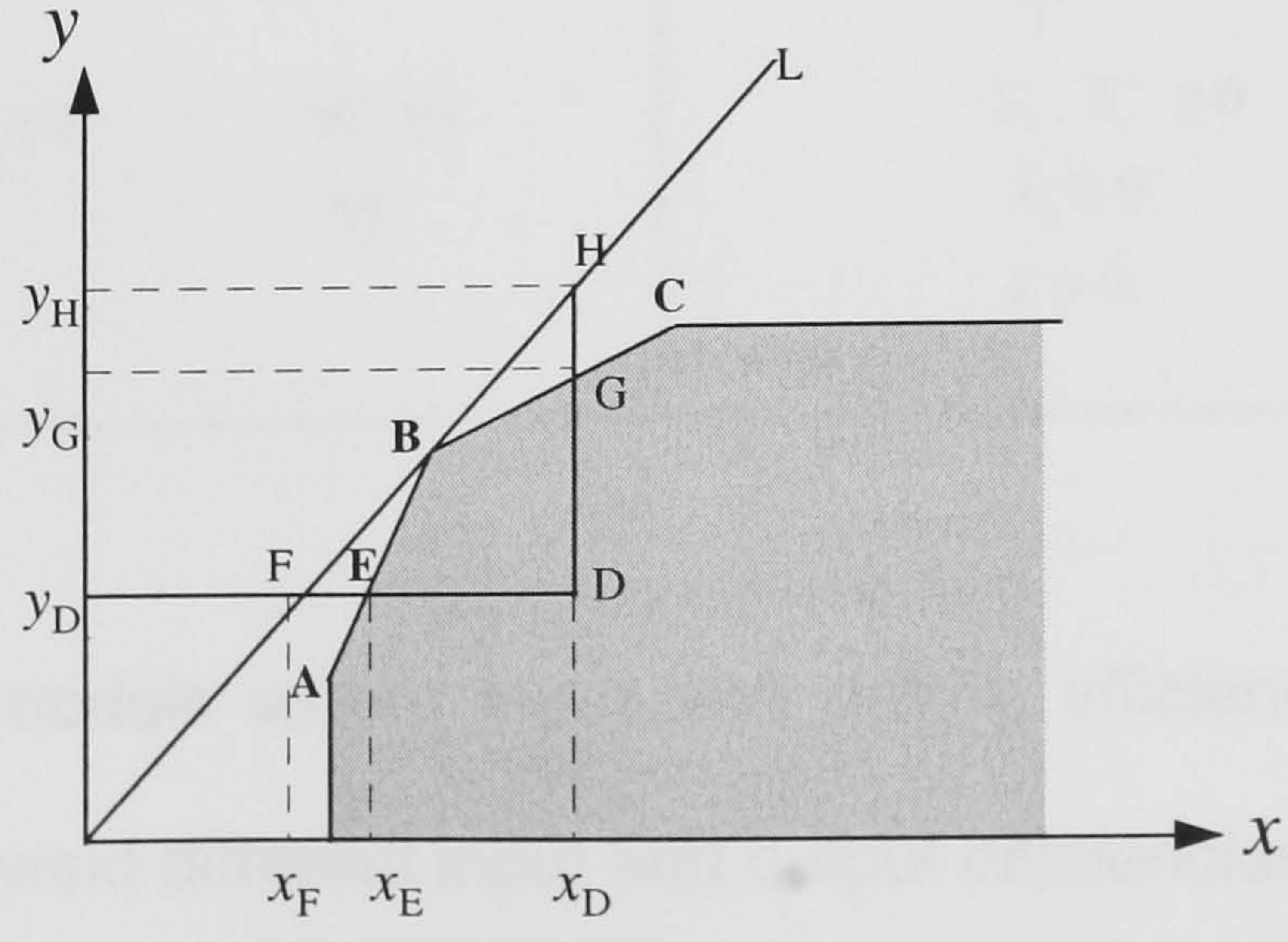
Notations are as in Model 1-4.

Assume that ϕ^* is the optimum value of ϕ. DMU j_0 is said to be Pareto efficient iff $\phi^* = 1$ and the optimal value of S_i^+ and S_r^- are zero ($\forall$ i, r). The efficiency rate of DMUj_0 is ϕ^*.

1.4.2 VRS model (Variable Returns to Scale)

This model was developed by Banker, Charnes and Cooper (1984) and is frequently referred to as the VRS DEA model. The difference between VRS and CRS efficiencies can be illustrated by using Figure 1-9. The figure depicts the production possibility set for the input - output mix (x, y). The line L is the boundary of the PPS for CRS while ABC is the boundary of the PPS for VRS. DMU D with input - output of (x_D, y_D) is inefficient. A measure of (input) inefficiency can be obtained if it is compared to DMU E for VRS and DMU F for CRS. Both E and F have the same output level as D.

Figure 1-9. CRS and VRS efficiency



The fraction $\frac{x_E}{x_D}$ is the VRS-(input) efficiency rate and the fraction $\frac{x_F}{x_D}$ is the CRS-(input) efficiency rate of DMU D. In an analogous manner it can be seen that the fraction $\frac{y_D}{y_G}$ is VRS-(output) efficiency rate and the fraction $\frac{y_D}{y_H}$ is CRS-(output) efficiency rate of DMU D.

Banker, Charnes and Cooper (1984) have extended the original CRS DEA model to assess the VRS efficiency by adding a convexity constraint to it. Specifically their VRS input and output orientation models are as follows.

Model 1-6. Input oriented - VRS model	**Model 1-7. Output oriented - VRS model**
$\min_{\lambda,\phi,S_i^-,S_r^+} \phi-\varepsilon(\sum_r S_r^+ + \sum_i S_i^-)$ s.t. $\sum_j \lambda_j x_{ij} = \phi x_{ij_0} - S_i^- \quad \forall i$ $\sum_j \lambda_j y_{rj} = y_{rj_0} + S_r^+ \quad \forall r$ $\sum_j \lambda_j = 1$ $S_i^-,\ S_r^+ \ge 0 \quad \forall i, \forall r$ $\lambda_j \ge 0 \quad \forall j$ $\varepsilon > 0.$	$\max_{\lambda,\theta,S_i^-,S_r^+} \theta+\varepsilon(\sum_r S_r^+ + \sum_i S_i^-)$ s.t. $\sum_j \lambda_j x_{ij} = x_{ij_0} - S_i^- \quad \forall i$ $\sum_j \lambda_j y_{rj} = \theta y_{rj_0} - S_r^+ \quad \forall r$ $\sum_j \lambda_j = 1$ $S_i^-,\ S_r^+ \ge 0 \quad \forall i, \forall r$ $\lambda_j \ge 0 \quad \forall j$ $\varepsilon > 0.$

Unlike CRS models where input and output efficiency are equal VRS models generally yield different input and output efficiencies.

Non Increasing Returns to Scale (NIRS) and Non Decreasing Returns to Scale (NDRS) are modelled by changing the constrain $\sum_j \lambda_j = 1$ to $\sum_j \lambda_j \geq 1$ and $\sum_j \lambda_j \leq 1$ respectively in Model 1-6 for input and in Model 1-7 for output efficiency.

1.4.3 Other DEA models

Apart from basic DEA models discussed in the previous section researchers have developed further DEA models. Table 1-1 lists some well known DEA models developed since 1978.

Table 1-1. Some well-known DEA models.

Model	References
CRS (Input-oriented, Output - oriented, Ratio model)	Charnes, Cooper, Rhodes (1978)
VRS (Input-oriented, Output - oriented ratio)	Banker, Charnes and Cooper (1984)
Variant Multiplicative	Charnes, Cooper, Seiford and Stutz (1982)
MPSS (Most Productivity Scale Size)	Banker (1984)
Additive	Charnes, Cooper, Golany, Seiford and Stutz (1985)
Invariant Multiplicative	Charnes, Cooper, Seiford and Stutz (1983)
Non - discretionary inputs - outputs	Banker and Morey (1986a)
Categorical Inputs - outputs	Banker and Morey (1986b)
Incorporating Judgement (A prior Knowledge)	Dyson and Thanassoulis (1988)
Preferred targets model	Thanassoulis and Dyson (1992)

Besides developing DEA in theory, practitioners in a number of fields have quickly recognised that DEA is a useful methodology for measuring productive efficiency. Some of the well known applications where DEA is frequently applied are:

- Agricultural and farm industries
- Bank and financial institutions
- Education, schools, colleges and universities
- Health services
- Police and military services
- Transport, airline industry and railroads
- Water industry

Further applications can be found in the Extensive Bibliography of DEA compiled by Emrouznejad and Thanassoulis (1996a, 1996b, 1997) and recent bibliography published by Seiford (1997).

1.5 Conclusion

DEA is a non - parametric approach of frontier analysis for assessing the technical efficiency of DMUs, such as bank branches, hospitals, schools, etc. In the DEA models the relative efficiency is calculated by measuring the distance between the observed and efficient input - output levels of DMUs.

The introductory chapter has outlined the efficiency measurement methods, particularly DEA as a non - parametric approach. DEA has been widely used for comparing of the efficiency of DMUs. However the DEA models discussed in this chapter do not take into account the time dimension. The next chapter will discuss the use of DEA over time and the motivation for this thesis.

CHAPTER 2: Using DEA on panel data and the motivation for dynamic efficiency

2.1 Introduction

The DEA models that have been presented so far in this thesis do not take into consideration the time dimension. With panel data, one has input - output observations for each DMU and at each time period (such as Year, Month and so on). The DEA models developed by Charnes *et al.* (1978) or their extensions (Charnes *et al.* (1995)) can be used to assess DMUs cross - sectionaly within each period of time. However the drawback of a cross - sectional analysis is that it provides only a snapshot of a process which evolves through time. Consequently cross - sectional analysis provides only a partial, and possibly a misleading, evaluation of performance. For this reason

DEA has more recently been used with panel data in various approaches. DEA was first applied to panel data by Charnes, Clark *et al.* (1985), Färe (1986), and in a much wider range by Färe *et al.* (1992, 1995a 1995b and 1997). The advantage of panel data is that it offers the opportunity of obtaining a longer term evaluation of the performance of DMUs.

Perhaps the "window analysis" approach by Charnes, Clark *et al.* (1985) is of considerable importance as a pioneering attempt to deal with the problem of time and it has given some valuable insights into the issues involved.

Much more recently, researchers (Tulkens *et al.* (1995)) have dealt with other forms of DEA assessment over time by studying technical progress and technical regress using non - parametric models. For example Sengupta (1995) presents models for dealing with limited inter - temporal dependence of inputs - outputs while Färe *et al.* (1992 and 1997) developed an index, "Malmquist index", for measuring productivity change over time.

The aims of this chapter are to overview cross - sectional analysis by DEA and develop the motivation for this thesis. Section (2.2) describes "window analysis" as a first approach in the DEA literature dealing with time series data. Section (2.3) discusses aggregate efficiency. Cross - sectional analysis of DEA for both "contemporaneous" and " sequential" technology will be presented in section (2.4). Sections (2.5) and (2.6) discuss the "diachronic performance" measurement and "network model" which were developed recently by Färe *et al.* (1996 and 1997). A brief review of dynamic DEA developed by Sengupta (1995) is given in section (2.7). Section (2.8) concludes.

2.2 Window analysis

A method for detecting trends over time in efficiency scores is provided by the window analysis methodology of Charnes, Clark *et al.* (1985). In this approach the set of T periods is divided into a sequence of overlapping sub - periods of equal length. Each DMU is seen as a different DMU in each period. The methodology defines a sequence of windows consisting of periods { 1, ..., τ } for the first window, periods { 2, ..., τ +1 } for the second window and so on through periods { T-(τ +1), ..., T } for the last window. The DEA problem is solved $n\times\tau$ times in each window where n is the number of observed DMUs. The efficiency rate of each DMU can be tracked through the sequence of overlapping sub - periods. For example Table 2-1 can be constructed as a result of the DEA assessments carried out in an assessment of n DMUs. The three figures in each row correspond to the efficiency rating for each DMU in the window relating to the row. For example the efficiencies of DMU 1 taken as a separate DMU in years 1, 2 and 3 in the first window are 93.5%, 89.3% and 91.8% respectively.

Table 2-1. Window analysis of n DMUs in 10 periods, with the length of 3

Year →	1	2	3	4	5	6	7	8	9	10
DMU 1										
Window 1	93.5	89.3	91.8							
Window 2		78.9	94.3	84.8						
Window 3			90.4	89.6	93.8					
:				.	.	.				
Window 8								97.4	83.9	89.6
DMU 2										
Window 1	88.5	91.8	90.8							
:		.	.	.						
:			.	.	.					
:										
DMU n										
Window 1	92.5	87.3	87.1							
Window 2		79.9	91.3	82.4						
Window 3			89.2	83.9	90.1					
:				.	.	.				
Window 8								92.3	89.1	90.3

The figures in each column give a view of the efficiency of a DMU during a year. The efficiency values reflect the relative performance of the DMU in a given year as the comparator set of DMU is progressively changed. The figures across each row indicate how the efficiency of the DMU changes with time within a given window. The length of the window is a matter of judgement by the analyst. Windows might cover periods of time over which operating conditions are similar or where seasonal effects on performance are similar. However the window analysis provides no evidence on the nature of any technical change.

2.3 Aggregate technology

Assume a production technology over T periods (t = 1, ..., T). To obtain the efficiency of DMUs one possibility is to construct a single PPS made from the summation of inputs and the summation of outputs for the entire life of DMUs. This is called aggregate technology and its PPS can be defined as

$$P^{\Sigma} = \{ (X_j, Y_j)) \mid X_j \text{ can produce } Y_j;$$

$$\text{where} \quad X_j = \Sigma_t x_j(t) \text{ and } Y_j = \Sigma_t y_j(t) \; ; \; \forall j \}.$$

With reference to this PPS the efficiency of each aggregated DMU can be obtained from standard DEA models (e.g. Model 1-2 and Model 1-3) where input - output levels are aggregated over the whole life of DMUs. This efficiency ratio is called aggregate efficiency and it does not provide any evidence on the efficiency of DMUs in a specific period of time. To obtain efficiency of DMUs in each period we have to employ cross - sectional analysis as described below.

2.4 Cross - sectional analysis

One way to compare DMUs over time is to define a PPS in each time period. The PPS of each period can be expressed by

$$P^{t} = \{ (x(t), y(t)) \mid x(t) \text{ can produce } y(t) \text{ under certain conditions of technology at period } t \}.$$

Tulkens *et al.* (1995) named this PPS "contemporaneous technology". So in this technology a sequence of T PPS's is constructed one for each period. DMUs can then be assessed within each period - specific PPS using standard DEA models (e.g. Model 1-2 and Model 1-3).

Tulkens *et al.* (1995) also introduced another technology by defining a PPS at each point in time t using the observations from the beginning up until t. This technology is called "sequential technology" and can be denoted as

$$P^{1,\ldots t} = \{ (x(s), y(s)) \mid x(s) \text{ can produce } y(s);\ s=1,2\ldots,t \}$$

Thus again in this technology a sequence of T PPS's is constructed one for each period. DMUs in this technology are assessed using standard DEA Model 1-2 and Model 1-3 and the PPS's for each s=1 to s=t, t being combined into a single PPS.

It is noted that the PPS sets in contemporaneous technology are not nested while PPS sets in sequential technology are nested. i.e.

$$P^{1,\ldots t} \subseteq P^{1,\ldots t+1}.$$

It can be readily deduced that the sequential PPS in the last period of time contains all DMUs observed in each contemporaneous PPS from t=1 to t=T.

Thus feasible DMUs in contemporaneous PPS at a specific time are feasible in sequential PPS at that time too. i.e.

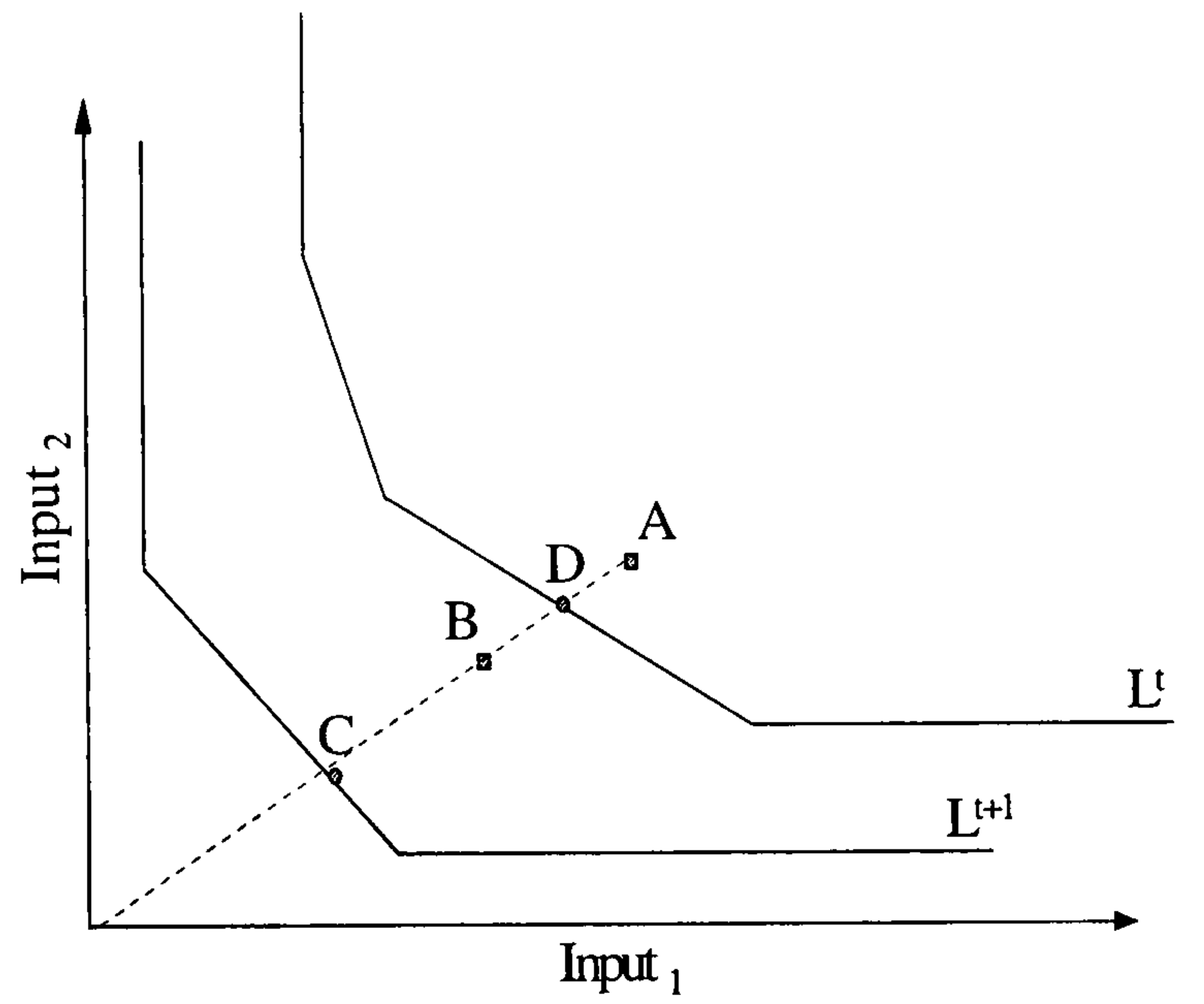
$$P^{t} \subseteq P^{1,\ldots,t} \subseteq P^{1,\ldots,t+1} \subseteq P^{1,\ldots,T}.$$

It follows that the efficiency rate of a DMU in sequential technology at a specific time t is not higher than its efficiency rate in contemporaneous technology at that time.

The “cross - sectional performance” of a DMU relates to time period *t* and it is assessed relative to the best observed practice in that time period based on contemporaneous (or sequential) PPS. Cross - sectional efficiency offers a snap - shot of the performance of a unit in the time period concerned. It fails to identify the progress or regress over time either of the efficient boundary itself or of a given operating unit. This point is illustrated in Figure 2-1.

Figure 2-1 shows the efficient boundary L^t and L^{t+1} for producing a unit of output in periods *t* and *t*+1 respectively. Two inputs are used in the production process. Now assume a production unit operates at A in time period *t* and at B in time period *t*+1. Clearly the unit is more productive in time period *t*+1 in that it secures a unit of output using much lower input levels than in period t.

Figure 2-1. Cross - sectional efficiency does not reflect diachronic productivity changes.



Nevertheless, the cross - sectional efficiency of the unit in time period *t* is $\frac{OD}{OA}$ and in time period *t*+1 is $\frac{OC}{OB}$. Since $\frac{OC}{OB} < \frac{OD}{OA}$ the cross - sectional efficiencies of the unit convey the incorrect impression that its performance deteriorates over time.

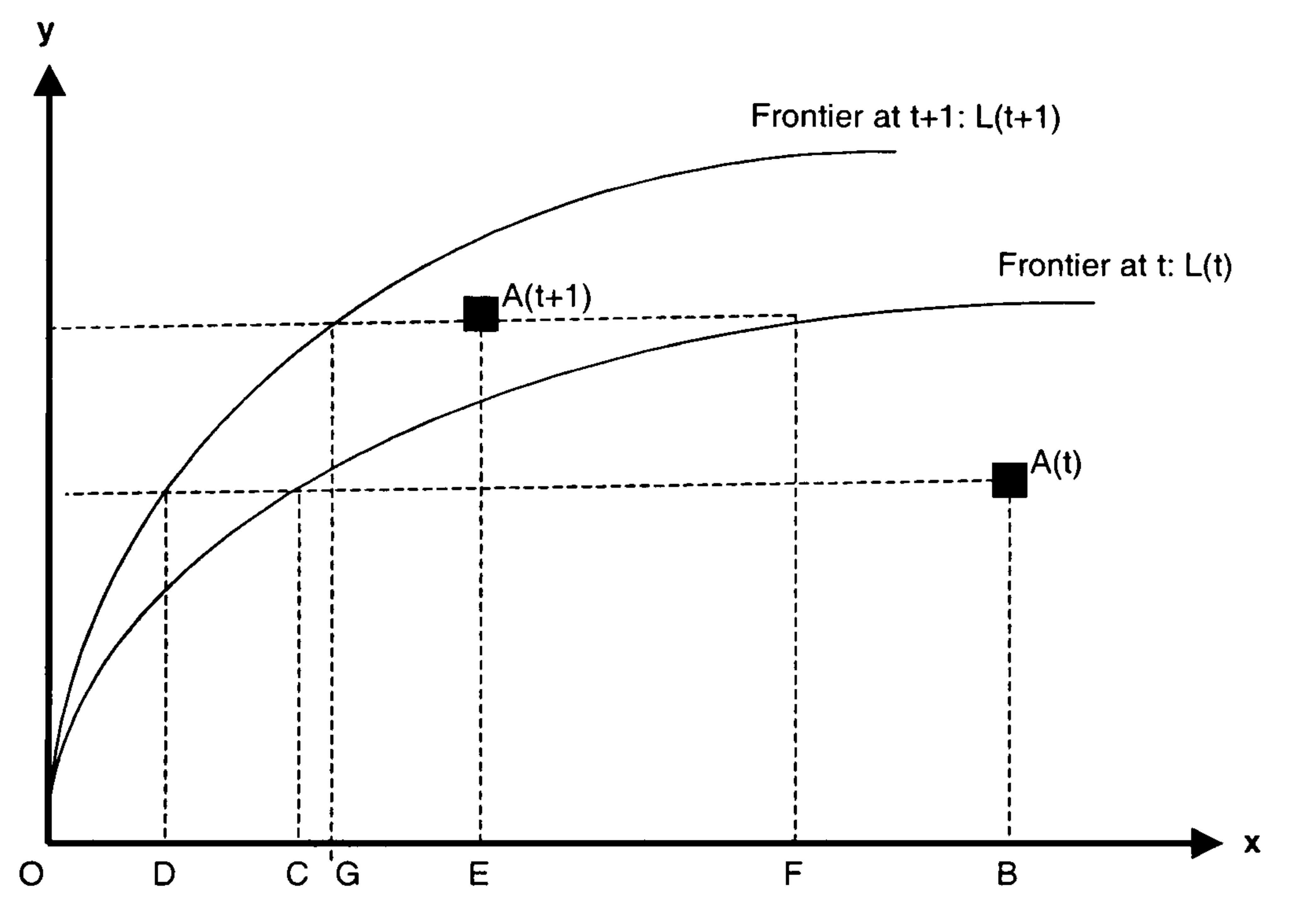
2.5 Diachronic performance measurement

The problem is addressed using *diachronic* performance measurement. One approach frequently used for measuring productivity change over time is that developed by Färe *et al.* (1992 and 1997) using a

"Malmquist index" (See Malmquist (1953)). Färe *et al.* (1992) decompose the total productivity change of a unit into that attributable to the 'shift' in the efficient boundary between period *t* and *t+1* and that attributable to the 'catch - up' of the unit's efficiency. The catch - up factor reflects the change in the cross - sectional efficiency of an operating unit as we move from time period *t* to time period *t*+1. The boundary shift term reflects the movement in the efficient boundary from time period *t* to time period *t*+1 in terms of how much more (less) input is needed to secure a given level of output, under efficient operation. For more details of the approach see Färe *et al.* (1992 and 1995a).

The concept of Malmquist productivity index can be illustrated by Figure 2-2 following Färe *et al.* (1992), Berg *et al.* (1992), Price and Weyman-Jones (1996). In this Figure, a production frontier is representing the efficient level of output y than can be produced from a given level of input x. We only represent a single-input single-output case but it can be extended to multi-input multi-output in the framework of defining DEA models. The assumption made is that the frontiers can shift over time. The frontiers thus obtained in the current, t, and future, t+1, time periods are labelled accordingly. When inefficiency is assumed to exist, the relative movement of any given operational unit over time will therefore depend on both its position relative to the corresponding frontier (efficiency change) and the position of the frontier itself (technical change). If inefficiency is ignored, then the productivity growth over time will be unable to distinguish between improvements that derive from an operational unit "catching up" to its own frontier, or those that result from the frontier itself shifting up over time.

Figure 2-2. Malmquist productivity index and its decomposition



Now assume A(t) represents an input output bundle for some given operational unit in period t. Thus an input-based measure of efficiency can be deduced by the horizontal distance ratio OC/OB. That is, inputs can be reduced in order to make production technically efficient with respect to the frontier in period t. By comparison and with respect to the same frontier, in period t, an input based measure for operational unit A(t+1) can be defined with the ratio of OF/OE. Since the frontier has shifted, OF/OE exceeds unity, even though A(t+1) is technically inefficient when compared to the period t+1 frontier.

With using Malmquist input-oriented productivity index, it is possible to decompose this total productivity change between the two periods into technical change and efficiency change. Note that, some researchers use the input oriented measures of Malmquist index (see for example Berg, *et al.* (1992) and Funkuyama (1995)) but many others use the output orientation of the Malmquist index. We define the Malmquist index as in input based measure. This is also in line with our dynamic extension to the Malmquist index in Chapter 7.

The input based Malmquist productivity index could be formulated as:

$$M_i^{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \left[\frac{D_i^t(x^{t+1}, y^{t+1})}{D_i^t(x^t, y^t)} \times \frac{D_i^{t+1}(x^{t+1}, y^{t+1})}{D_i^{t+1}(x^t, y^t)} \right]^{1/2} .$$

Where D_i is the input distance function and $M_i^{t+1}(x^t, y^t, x^{t+1}, y^{t+1})$ is the productivity of the most recent production unit, i.e. A(t+1), using period t+1 technology relative to the earlier production unit, i.e. A(t), with respect to t technology. A value greater than unity will indicate positive total factor productivity growth between the two periods. Following Färe *et al.* (1995a) an equivalent way of writing this index is:

$$M_i^{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_i^{t+1}(x^{t+1}, y^{t+1})}{D_i^t(x^t, y^t)} \left[\frac{D_i^t(x^{t+1}, y^{t+1})}{D_i^{t+1}(x^{t+1}, y^{t+1})} \times \frac{D_i^t(x^t, y^1)}{D_i^{t+1}(x^t, y^t)} \right]^{1/2}$$

or

M=ΔTECH × ΔEFF

where

$$\Delta EFF = \frac{D_i^{t+1}(x^{t+1}, y^{t+1})}{D_i^t(x^t, y^t)}$$

$$\Delta TECH = \left[\frac{D_i^t(x^{t+1}, y^{t+1})}{D_i^{t+1}(x^{t+1}, y^{t+1})} \times \frac{D_i^t(x^t, y^1)}{D_i^{t+1}(x^t, y^t)} \right]^{1/2}$$

In this view M, the Malmquist total factor productivity index, is the product of a measure of technical progress, ΔTECH, as measured by shifts in a frontier at period t+1 and period t (average geometrically) and a change in efficiency, ΔEFF, over the same period.

In order to calculate these indexes it is necessary to solve several sets of linear programming problems as presented in Model 2-1. Assume there are n DMUs and that each DMU consumes varying amounts of m different inputs to produce s outputs in each period t. The j^{th} DMU, in period t, is therefore represented by the vectors (x_j^t, y_j^t). The purpose is to construct a non-parametric envelopment frontier over the data points such that all observed DMUs lie on or below the production frontier. The calculation exploits the fact that the input distance functions (D_i) used to construct the Malmquist index are the reciprocals of the Farrell (1957) input oriented technical efficiency measure (see Chapter 1). The first two linear programs are where the technology and observation to be evaluated are from the same period, and the solution value is less than or equal to unity. The second two linear programs occur where the reference technology is constructed from data in one period, whereas the observation to be evaluated is from another period. Assuming constant returns to scale the following four linear programs are used to calculate the Malmquist index and its components.

Model 2-1. Linear programming models for calculation of the Malmquist index and its components.

$$
\begin{array}{ll}
[D_i^t(x_t, y_t)]^{-1} = Min\ \phi & \\
\text{s.t.} & \\
\sum_j \lambda_j x_{ij}^t \leq \phi x_{ij0}^t & \forall i \\
\text{s.t.} & \\
\sum_j \lambda_j y_{rj}^t \geq y_{rj0}^t & \forall r \\
\lambda_j \geq 0 & \forall j
\end{array}
$$

$$
\begin{array}{ll}
[D_i^{t+1}(x_{t+1}, y_{t+1})]^{-1} = Min\ \phi & \\
\text{s.t.} & \\
\sum_j \lambda_j x_{ij}^{t+1} \leq \phi x_{ij0}^{t+1} & \forall i \\
\text{s.t.} & \\
\sum_j \lambda_j y_{rj}^{t+1} \geq y_{rj0}^{t+1} & \forall r \\
\lambda_j \geq 0 & \forall j
\end{array}
$$

$$
\begin{array}{ll}
[D_i^{t+1}(x_t, y_t)]^{-1} = Min\ \phi & \\
\text{s.t.} & \\
\sum_j \lambda_j x_{ij}^{t+1} \leq \phi x_{ij0}^t & \forall i \\
\text{s.t.} & \\
\sum_j \lambda_j y_{rj}^{t+1} \geq y_{rj0}^t & \forall r \\
\lambda_j \geq 0 & \forall j
\end{array}
$$

$$
\begin{array}{ll}
[D_i^t(x_{t+1}, y_{t+1})]^{-1} = Min\ \phi & \\
\text{s.t.} & \\
\sum_j \lambda_j x_{ij}^t \leq \phi x_{ij0}^{t+1} & \forall i \\
\text{s.t.} & \\
\sum_j \lambda_j y_{rj}^t \geq y_{rj0}^{t+1} & \forall r \\
\lambda_j \geq 0 & \forall j
\end{array}
$$

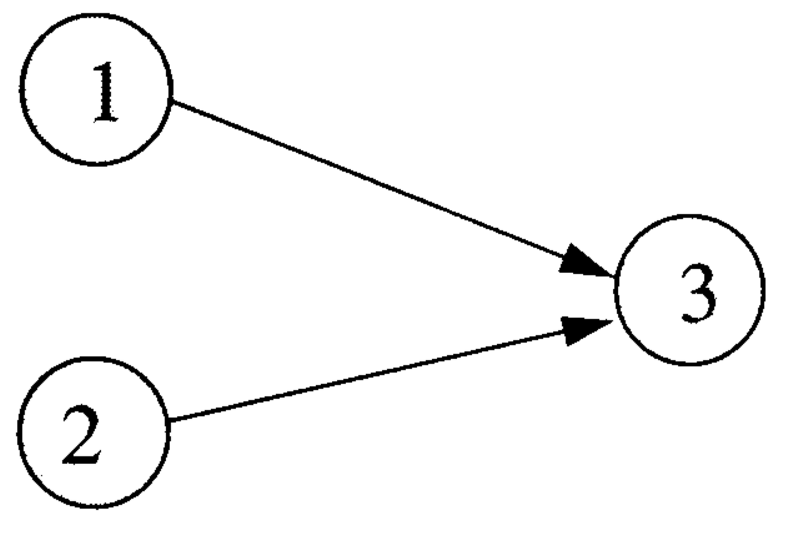
By solving these linear programming models it is possible to provide four efficiency and productivity indexes for each observed DMU. Regarding change in efficiency, technical efficiency increases (decreases) if and only if the optimum ΔEFF is greater (less) than one and ΔEFF can be obtained by solving the first two linear programming models. An interpretation of the technological change is that technical progress (regress) has occurred if ΔTECH is greater (less) than one.

Färe *et al.* (1997) recognised that this diachronic performance measure also ignores the issue of assessing comparative efficiency of DMUs with inter - temporal input - output dependence. They then addressed the problem using a "network model" (Färe *et al.* (1997)). In Chapter 7 we will extend the productivity Malmquist index in line with the efficiency model that will be introduced in this thesis.

2.6 Network model of DEA

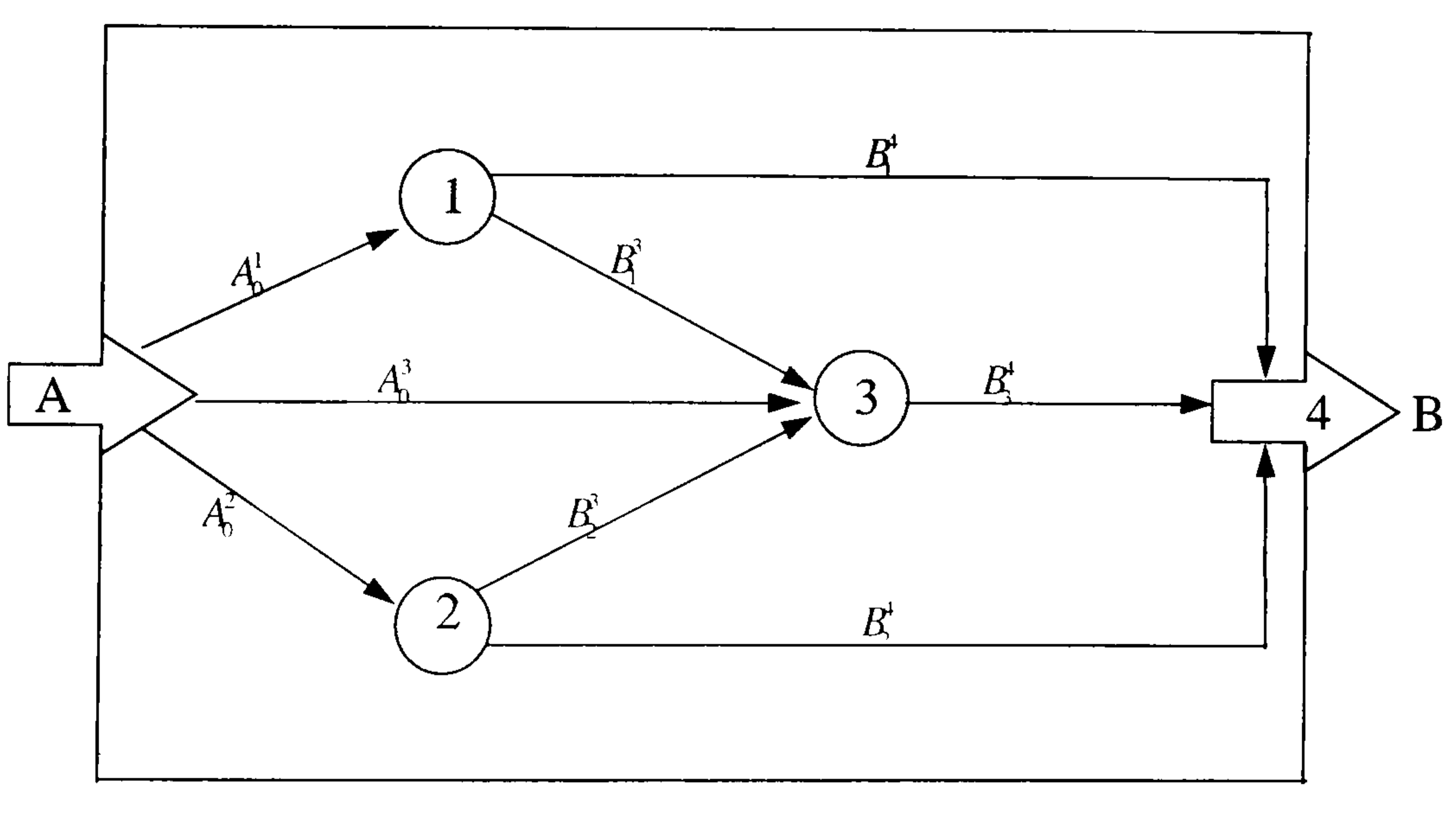
Färe *et al.* (1997) address "intermediate inputs" by introducing a network model. In a multi stage process an intermediate input is a product produced by technology in one stage and used as input in another stage. Therefore the key issue in their model is to divide a technology into sub - technologies so that the sub - technologies are connected in a network to form the overall frontier or reference technology. Looking at the production as a series of sub - technologies allows us to explicitly model intermediate inputs or products. For simplification let us restrict a network to include only the technologies which are modelled in a directed network as shown in Figure 2-3.

Figure 2-3. Sub - technologies



Let us add a process for distributing exogenous inputs and a process for collecting final outputs. The extended network model is illustrated in Figure 2-4.

Figure 2-4. The network technology



Assuming that total available exogenous input is denoted by A and A^i_0 ; i=1,2,3 denotes the amount of the vector of exogenous input used in sub -

technology i, then $A \geq \sum_i A^i_0$. Further assume that B^j_i denotes the amount of output from sub - process i that is delivered to sub - process j. Thus for the network depicted in Figure 2-4 the total output produced at say sub - process 1 is $B_1^3 + B_1^4$, where B_1^3 is the vector of intermediate products produced by sub - process 1 and used as input in sub - process 3. B_1^4 denotes the amount of output from process 1 that becomes final output.

Regarding the collection node (4), given that each sub - technology produced distinct products, the final output vector $B \in R+^m$ consists of $B_1^4 \in R+^{m1}$, $B_2^4 \in R+^{m2}$ and $B_3^4 \in R+^{m3}$ where m = m1 + m2 + m3 and $B = (B_1^4, B_2^4, B_3^4)$. To formalise the network technology, we assume that there are k=1,...,K observations of $(B_1^3, B_1^4)^k$, $(B_3^4)^k$, $(B_2^3, B_2^4)^k$, $(A^1_0)^k$, $(A^2_0)^k$, $(A^3_0)^k$ and A^k. Färe presented a piece - wise linear technology associated with this network model in terms of output sets (see for example Färe *et al.* (1997) p.22).

With reference to this PPS the efficiency of each DMU can be obtained from standard DEA models (e.g. Model 1-2 and Model 1-3) where inputs - outputs of DMU k are defined as follows.

Inputs

- $(A^1{}_0)^k$
- $(A^2{}_0)^k$
- $(A^3{}_0)^k$
- $(B_1{}^3)^k$
- $(B_2{}^3)^k$

Outputs

- $(B_1{}^4)^k$
- $(B_2{}^4)^k$
- $(B_3{}^4)^k$

This model takes into account the intermediate output but it does not deal with capital input which offers output over a number of periods. Sengupta (1995) has addressed the problem of capital input in various DEA models.

2.7 Dynamic efficiency , a different aspect

Sengupta (1995, 1996) has extended some DEA models for dynamic and stochastic purposes. He formulated various dynamic models which clarify the economic concepts of allocative efficiency and technical change. The bases of these models are that the production and cost frontier are viewed dynamically over time. Technological change and adjustment of inputs over time are some of the major sources of dynamic efficiency in production units. His extension of static to dynamic DEA models is mainly on allocative efficiency, or price efficiency, rather than technical efficiency. Recall that two types of efficiency measures are usually distinguished in production economics. One is *technical* efficiency, which measures the success in producing maximum outputs from a given set of inputs. The other is the *allocative* efficiency, which measures a DMU's success in choosing an optimal set of inputs under a given set of input prices. This measure is sometimes also called *price* efficiency. The advantage of technical efficiency is that we do not require prices for inputs.

Sengupta in various dynamic models used allocative efficiency to determine the optimum levels of inputs, whereas the technical efficiency model treats the observed inputs and outputs as given, and tests if each DMU achieves its maximum possible levels of output for given levels of inputs. In some cases Sengupta dealt with capital input as it has output effects spread over several periods ahead and developed a cost minimisation model in the framework of DEA. Here we present a formulation of one of his models but for a comprehensive discussion see Sengupta (1995).

The aim of the model presented here is to allow a DMU to compute the time path of optimal input usage over a period of time. Assume there are n DMUs and that each DMU consumes varying amount of m different inputs to produce s outputs in each period t. The input and output of j^{th} DMU, for period t, are therefore x_{ij}^t, i=1, ..., m and y_{rj}^t, r=1, ..., s. Let q_i be the price attached to input i.

Hence Model 2-2 can capture the minimum price for DMU_j.

Model 2-2. A DEA price efficiency model

$$
\begin{aligned}
&\min_{x,\lambda} \quad \sum_i q_i x_i \\
&s.t. \\
&\sum_j \lambda_j x_{ij} \le x_i \qquad \forall \mathrm{i}, \\
&\sum_j \lambda_j y_{rj} \ge y_{rj0} \qquad \forall \mathrm{r}, \\
&\sum_j \lambda_j = 1, \\
&\mathrm{x_i} \ge 0 \quad \forall \mathrm{i} \ , \lambda_j \ge 0 \quad \forall \mathrm{j}
\end{aligned}
$$

In this model q_i is the input price attached to input i, and x_i is the i^{th} input optimally dedicated by DMU j along with the weights λ. Let λ^* be the optimal solution of the above LP model. The minimal cost of unit j is given by $c_j^*=\Sigma_i q_i x^*_i$ where the observed cost of the same unit is $c_j=\Sigma_i q_i x_i$ where x^*_i is the optimal solution of the LP Model 2-2.

Hence the overall efficiency of the DMU_j would be defined as

$OE_j = \Sigma_i q_i x^*_i / \Sigma_i q_i x_i$.

A dynamic extension of this, as developed by Sengupta and presented in Model 2-3, is a model where the DMU_j uses an objective function to choose the sequence of decision variables $x_i(t)$ over a planning horizon. The objective of this model is to minimise the expected present value of the total cost subject to the constraints in Model 2-2 but for each period.

Model 2-3. A dynamic DEA price efficiency model

$$
\begin{aligned}
&\underset{x(t),\lambda(t)}{Min} \sum_t \rho(t)c(t) \\
&s.t. \\
&\sum_j \lambda_j(t)x_{ij}(t) \le x_i(t) \qquad \forall i, \\
&\sum_j \lambda_j(t)y_{rj}(t) \ge y_{rj0}(t) \qquad \forall r, \\
&\sum_j \lambda_j(t) = 1 \qquad \forall r, \\
&x_i(t) \ge 0 \qquad \forall i, \\
&\lambda_j(t) \ge 0 \qquad \forall j.
\end{aligned}
$$

Where $c(t)=\Sigma_i q_i(t)x_i(t)$ is the total cost in period t and ρ is a known discount factor.

According to Sengupta this model can be improved if we could make a distinction between the current and capital inputs and then minimise a discounted stream of costs for both current and capital inputs in the DEA framework.

Therefore he developed a type of *dynamic* formulation for when capital inputs are treated differently from the current inputs. Assume x_i (i=1, ..., m-1) are current inputs and z is a single capital input. If $q_m(t)$ is the input price of capital input, then $q_m(t)z(t)$ can be treated as the investment in durable goods in the process. If we assume continuous discounting at an instantaneous rate r, the total equivalent cost of the production unit is

$$c(t) = \Sigma_i q_i(t)\, x_i(t) + rq_m(t)z(t).$$

Minimising this cost function subject to the DEA constraints in Model 2-3 we are able to measure the overall efficiency of DMU. If x* is the optimal input, then the overall inefficiency of DMU_j in the use of capital input is given by

$$OE_j(z)=rq_mz^*/rq_mz = z^*/z.$$

In the dynamic case with the introduction of a planning horizon the objective will be choosing current and capital inputs so as to minimise the total cost over the horizon $0<t<T$.

This is presented in Model 2-4 which is a typical cost minimisation DEA model that is developed by Sengupta.

Based on this model if the observed path of capital expansion equals the optimal path for every t then the model would exhibit dynamic efficiency; otherwise any divergence of the two paths would generate inefficiency over time.

Model 2-4. A dynamic DEA price efficiency model treating capital and current inputs differently

$$\int_{t=0}^{T} e^{-rt} c(t)dt$$

$$s.t.$$

$$\sum_j \lambda_j(t)x_{ij}(t) \le x_i(t) \qquad \forall \text{i},$$

$$\sum_j \lambda_j(t)z_j(t) \le z(t)$$

$$\sum_j \lambda_j(t)y_{rj}(t) \ge y_{rj0}(t) \qquad \forall \text{r},$$

$$\sum_j \lambda_j(t) = 1 \qquad \forall \text{r},$$

$$\text{x}_\text{i}(t) \ge 0 \qquad \forall \text{i},$$

$$\lambda_j(t) \ge 0 \qquad \forall \text{j}.$$

Where $c(t) = \Sigma_i q_i(t)\, x_i(t) + rq_m(t)z(t)$ is the total cost in period t and r is a known discount factor.

Sengupta has also developed a series of dynamic efficiency models using optimum control theory (Sengupta 1995). As mentioned earlier in almost of his models he expanded DEA using the concept of cost minimisation. He therefore either attaches prices to inputs and develops DEA models treating capital and current input differently, or he extends the concept of allocative efficiency to a dynamic model.

However not only the input prices are unlikely to be known or relevant for certain contexts, there are certain basic objections to keeping prices constant in a dynamic model. Firstly, the efficiency measures will be biased if the observed input prices fluctuate widely over time and inputs are adjusted to the past or to the expected future prices which differ from the current ones. Secondly, the price or allocative efficiency measure is very sensitive to error of measurement in estimating factor prices. These objections are much less valid when developing a dynamic model for technical efficiency, since we do not have prices in the model. Therefore the models developed in this thesis are different from those of Sengupta as we do not use input prices.

Our approach could be seen more close to the network technology as developed by Färe *et al.* (1997) in the sense that the network technology is also useful for when we have intermediate input/ output. However network technology is more useful for when in a multi - stage production process an output in the middle of the process can be turned as input after that. We are aiming to introduce a longer assessment DEA model defining a unit as a path over several periods; in particular treating current and capital input differently.

2.8 Conclusion and Motivation for dynamic efficiency

In this chapter methods for assessing relative efficiency of DMUs over time were reviewed. The drawback of these methods is that they provide only a snapshot of a process which evolves through time. Consequently the approaches provide only partial and possibly misleading evaluation of performance for production processes with inter - temporal input - output which is the area to be addressed in this thesis.

The issue of assessing comparative efficiency of DMUs where output levels over a given period of time depend at least in part on prior resources has been so far largely ignored in the literature. The approach developed in this thesis considers general forms of inter - temporal input - output dependence and in the general multi - period production process but particularly it is useful for when we have capital stock.

CHAPTER 3: How static efficiency measures can fail to capture true performance

3.1 Introduction

This chapter demonstrates how static efficiency can fail to capture the true performance of DMUs whose operations have not ceased at the time of assessment and where output levels over a given period of time depend at least in part on resource levels in prior periods. A typical application area of this kind is that where DMUs secure their outputs using resources which include capital stock. Such stock, which may occasionally be upgraded, affects output levels over a continuous time interval which may span several assessment periods. In such cases traditional or 'static' approaches to assessing performance break down because they implicitly assume that there is “correspondence” between “coincident” input - output levels. The distinction between correspondence and coincidence of input - output levels is as follows:

⇒ “Coincident input - output” levels are those observed during the same time period;

⇒ “Corresponding input - output“ levels are those where the output levels are caused exclusively by the input levels.

Where correspondence of coincident input - output levels does not hold we have “inter - temporal input - output dependence”. This chapter contains a taxonomy of inter - temporal input - output dependencies and a discussion of their causes. The chapter unfolds as follows.

Section (3.2) discusses the classification of production processes. Section (3.3) highlights some causes of inter - temporal input - output dependencies. Section (3.4) provides an example of a production process with inter - temporal input - output and its treatment by static DEA. Conclusions are drawn in section (3.5).

3.2 A classification of production processes

Depending on the duration of the life of operating DMUs and on the nature of any inter - temporal dependence of input - output levels three types of production process can be discerned:

- Single period;
- Multi - period without inter - temporal input - output dependence;
- Multi - period with inter - temporal input - output dependence.

3.2.1 Single period production processes

In such production processes clearly the issue of inter - temporal dependence of input - output levels does not arise. Thus there is correspondence between the coincident input and output levels of each DMU and efficiency can be assessed by the DEA models developed by Charnes *et al.* (1978) as discussed in earlier chapters or their extensions as described in Charnes *et al.* (1995).

3.2.2 Multi - period production processes without inter - temporal input - output dependence

These processes do have contemporaneous correspondence of input - output levels. Thus the DEA models developed by Charnes *et al.* (1978) or their extensions Charnes *et al.* (1995) can be used to assess the DMUs concerned. However, DMUs are now in existence over several time - periods and issues arise as to their performance *over time* rather than just at each specific point in time. In essence, in multi - period production processes performance can be assessed in two contexts: *cross - sectionally* and *diachronically*. Models for such assessments were outlined in Chapter 2.

3.2.3 Multi - period production processes with inter - temporal input - output dependence

This is the case examined in this thesis. The DMUs operate over a continuing sequence of time periods and we do not have correspondence of coincident input - output levels. A clear example of inter - temporal impact of input is advertising. While advertising is normally treated as a single period business expense its impact can cover many periods. Dhalla (1976) states that "management must view advertising as a capital investment with sales revenue generated like a stream over time". White *et al.* (1996) state that "advertising expenditures should be analysed as a long - term investment in an invisible asset by utilising capital budgeting". Therefore advertising behaves much more like an inter - temporal input rather than a single - period expense as it produces a multi - period "future income stream". We refer to production processes with such dependencies as “inter - temporal production processes”.

3.3 Causes of inter - temporal input - output dependencies

Some of the main causes of inter - temporal input - output dependence are those of "capital stock", "lagged output", and "capital output". These causes are elaborated below.

3.3.1 Capital Stock

Capital stock, such as robots in car plants, enhance productivity. The productive life of capital stock spans in general many time periods such as years or quarters typically used for recording coincident input - output data. Inter - temporal dependence of input - output levels is caused by changes in the level of capital stock, such as those due to capital investments. Asset acquisition does not generally lead to an instantaneous rise in productivity and may indeed initially lead to its drop. This is because of the 'adjustment' and 'disruption' processes generally associated with asset acquisition. The adjustment process is typically referred to as the 'learning curve' as DMUs need to learn how to use new assets acquired. Asset acquisition can also entail disruption due to the need to integrate the new with existing assets. The duration and timing of the adjustment and disruption effects will generally differ from DMU to DMU depending on their asset acquisition activities.

Sengupta (1993, 1994 and 1995) highlights some reasons why static assessments fail to measure efficiency of DMUs with capital input. They include :

- The actual process is in fact a progressive process, in the sense that it is accumulating real capital, having more real equipment at the end of a period under consideration than it had at the beginning. We can not analyse it in a static framework. In a static framework we must replace the changing stock of capital by constant stock of capital, which is not realistic (Sengupta (1994)).

- Capital inputs have a multi period dimension, since they generate outputs over many periods, yet the standard DEA applications are based exclusively on one period's input. This biases efficiency comparisons against the capital-intensive processes (Sengupta (1995)).

- The decision making units which are compared in terms of relative efficiency, may take more than one period to adjust to capital input changes and this inter - temporal adaptivity is ignored by the standard DEA application (Sengupta (1995)).

3.3.2 Lagged Output

In some production situations output can lag input in a way which makes it difficult to establish correspondence between input and output. One case in point is that of promotion of sales. Consider, for example, sales teams promoting financial products such as personal insurance, pension plans etc.

Over some given assessment period a team may use the bulk of its time to make a wide range of introductory contacts with potential clients, hold explanatory workshops on the products for sale etc. Actual recorded sales may be low during such a period. However, the team may have been successful in building up goodwill among potential clients which will manifest itself in increased sales over future periods. Thus, in essence, there is a lag between sales effort and actual sales. Such a lag may span several assessment periods which makes it difficult to establish correspondence between input (time devoted to promoting sales) and output (sales achieved) within a given assessment period.

3.3.3 Capital Output

In certain production contexts it is possible for intermediate or capital output to be created which is not directly measurable but can enhance productivity in subsequent periods. An example of intermediate or capital output is that of research. Typically research output is measured by the number of research papers or reports published, research grants obtained and so on (the important but difficult issue of the quality of the research output is ignored here). A research team may generate intermediate output in the form of research ideas and provisional research results which are incomplete for publication. Such intermediate output is in effect 'work - in progress' and cannot be captured by the usual research output measures. Yet it may have important implications for a team's productivity in subsequent periods.

Capital output whose generation and / or impact spans several assessment periods distorts the correspondence of input - output levels within any given assessment period.

Next a simple example of inter - temporal production process is provided to show how the static DEA framework may provide incorrect results.

3.4 An Example of inter- temporal production and its treatment by static DEA

An inter - temporal production function

The inter - temporal effects are easily demonstrated by considering a simple DMUs with two inputs, capital stock (Z) and period - specific input (x), and a single output (y). A period - specific input is an input that is used up in one period and has no further impacts on output. Assume that for DMUs the technology is expressed by a production function as follows.

$$y^t = \begin{cases} 2.4Z^{t-1} + 0.2x^t, & 0 \le Z^{t-1} \le \frac{1}{3}x^t \\ 1.2Z^{t-1} + 0.6x^t, & \frac{1}{3}x^t \le Z^{t-1} \le \frac{3}{4}x^t \\ 0.27Z^{t-1} + 1.3x^t, & Z^{t-1} \ge \frac{3}{4}x^t \end{cases} \tag{3.1}$$

Where x^t is the period - specific input, Z^t is capital stock of the starts of period t and y^t is output at period t. Z^0 is the level of initial capital stock at t=0. The technology is such that any amount of capital stock in period t-1 will impact output in period t. For example as can be seen in (3.1) the level of output in period t depends on the ratio of capital stock in the previous period t-1 to the period - specific input in period t; $\frac{Z^{t-1}}{x^t}$.

- If this ratio is lower than $\frac{1}{3}$ then stock of capital at t-1 makes a substantial contribution to output produced in period t, while
- If this ratio is greater than $\frac{3}{4}$ then period - specific input at t makes a substantial contribution to output produced in period t (x and Z are measured in the same units).

Figure 3-1. The impact of capital stock in period t-1 on output in period t for one unit of period - specific input associated with technology (3.1)

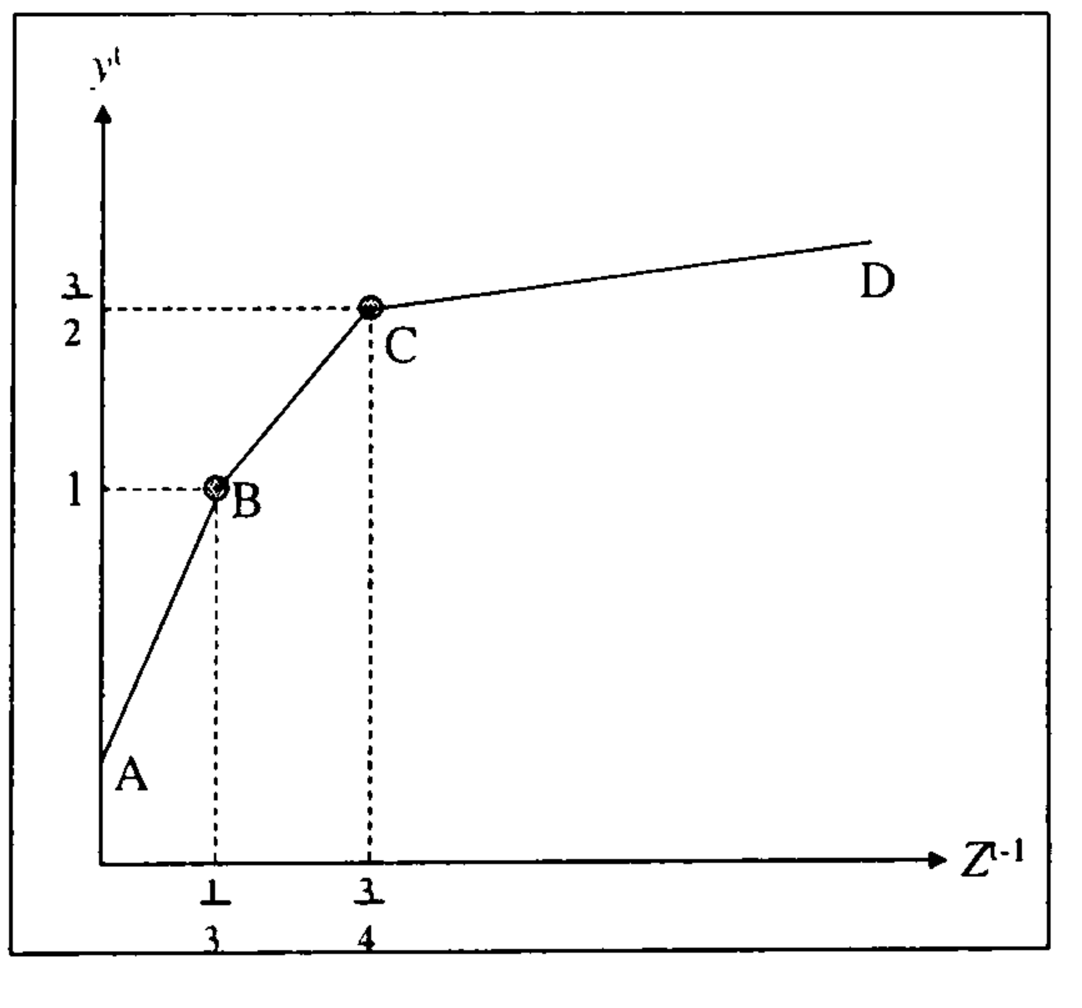


Figure 3-1 shows how the capital stock in period t-1 impacts on the output at t, assuming the period - specific input is constant at the level of 1.

Static DEA assessment

Now consider 4 DMUs associated with the above technology which have the input - output levels shown in Table 3-1.

Table 3-1. Observed DMUs associated with the inter - temporal technology in (3.1)

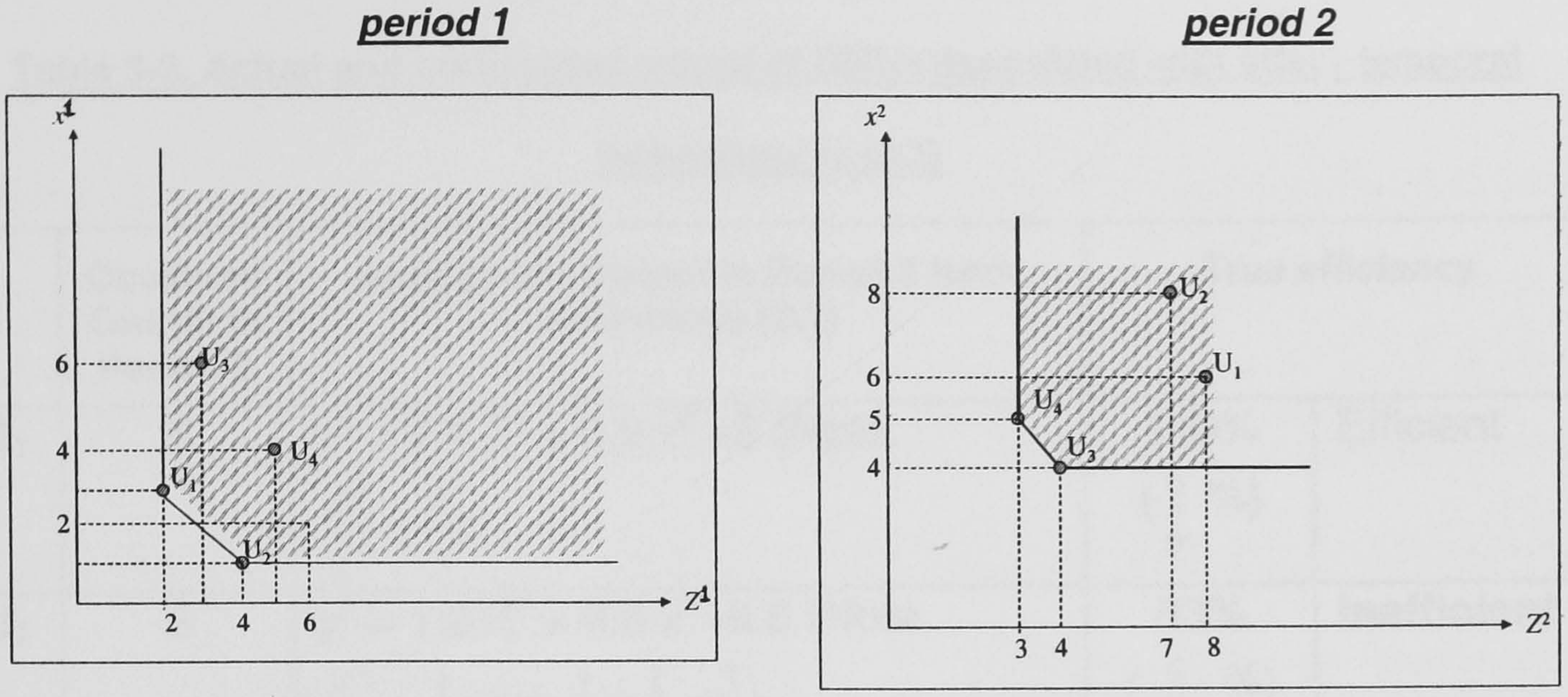
	Inputs in period 1 per six unit of output		**Inputs in period 2 per six units of output**	
	z^1	x^1	z^2	x^2
U_1	2	3	8	6
U_2	4	1	7	8
U_3	3	6	4	4
U_4	5	4	3	5

The results of static DEA efficiency Model 1-4 are illustrated in Figure 3-2. In static DEA, a model with two inputs, period - specific and capital stock, and single output is solved. It indicates that:

- In the first period U_1 and U_2 are efficient DMUs while U_3 and U_4 are inefficient DMUs.

- In the second period U_3 and U_4 are efficient DMUs while U_1 and U_2 are inefficient DMUs.

Figure 3-2. Static efficiency model (contemporaneous technology)



True performance

The static approach ignores the inter - temporal impact of the previous stock of capital which causes the output to rise during future periods. In particular looking at technology (3.1) it is known that in the second period, U_1 and U_3 are efficient DMUs and U_2 and U_4 are inefficient. The reasons are summarised in Table 3-2.

Table 3-2 shows the observed output and anticipated output from the technology (3.1) in period 2. This indicates that

- U_1 and U_3 are truly efficient while
- U_2 and U_4 are truly inefficient.

These results differ from those of static efficiency shown in Figure 3-2.

Table 3-2. Actual and anticipated output of DMUs associated with inter - temporal technology in (3.1)

	Observed Output in Period 2	Anticipated Output in Period 2 from technology (3.1)	True efficiency	
U_1	6	$y^2 = 2.4\ Z^1 + 0.2\ x^2 = 6$ (Note. $\frac{Z^1}{x^2} = \frac{1}{3}$)	100% ($\frac{6}{6}$ %)	Efficient
U_2	6	$y^2 = 1.2\ Z^1 + 0.6\ x^2 = 9.6$ (Note. $\frac{Z^1}{x^2} = \frac{1}{2}$ with $\frac{1}{3} \le \frac{1}{2} \le \frac{3}{4}$)	63% ($\frac{6}{9.6}$ %)	**Inefficient**
U_3	6	$y^2 = 0.27\ Z^1 + 1.3\ x^2 = 6$ (Note. $\frac{Z^1}{x^2} = \frac{3}{4}$)	100% ($\frac{6}{6}$ %)	Efficient
U_4	6	$y^2 = 0.27\ Z^1 + 1.3\ x^2 = 7.85$ (Note. $\frac{Z^1}{x^2} = 1$ with $\frac{3}{4} \le 1$)	76% ($\frac{6}{7.85}$ %)	**Inefficient**

This simple example demonstrates how the static DEA approach may provide incorrect results when DMUs are operating under inter - temporal input - output dependence.

3.5 Conclusion

A classification of production process and three particular causes of inter - temporal input - output dependence including “capital input”, “lagged output” and “capital output” were described in this Chapter.

In multi - period processes with inter - temporal input - output dependencies some input (such as capital stock) may impact future output levels. In such cases standard DEA which is a static approach for assessing the relative efficiency of DMUs fails because it implicitly assumes that no inter - temporal impact of input - output levels exists.

A simple example was used to illustrate how the DEA static efficiency model does not reflect inter - temporal efficiency of production technology. Further example of clarifying the difference between dynamic and static efficiency models in capturing inter - temporal input - output will be given in Chapter 5 where a new measure of inter - temporal input - output dependence is introduced.

The next chapter addresses how to build a new PPS for cases where inter - temporal input - output is taken into account when measuring efficiency.

CHAPTER 4: Defining a Production Possibility Set over input - output paths

4.1 Introduction

In the previous chapter a simple example was presented illustrating that the static efficiency obtained from ordinary DEA models does not reflect true performance under inter - temporal production technology. In such technologies DMU efficiency depends upon their input - output levels over time. In the “input - orientation” static DEA framework inefficiency is assessed by measuring how far a DMU's observed input levels are from a "best practice" set of input levels on the frontier. Given the DMU’s output levels there is a similar definition of “output - oriented” inefficiency (see for example Färe (1988)). We follow this framework and compare DMUs by measuring how far a DMU's input vector is from a best practice set of input levels over time, given the DMU's output levels. The key methodological problem is that the true technically best frontier is unknown and must be estimated from observed input - output correspondences. The difference between alternative methodologies in DEA largely reflects alternative maintained assumptions in estimating the frontier. Unlike the static DEA approaches which assess inefficiency at each period we introduce a "dynamic efficiency" model which assess inefficiency through the DMU's life taking into account inter - temporal dependence of input - output levels.

We introduce for this purpose the concept of a "DMU - path" and use it to define a technology of production which reflects inter - temporal input - output dependence. Then the necessary and sufficient conditions for a dynamic path to be input - efficient will be presented. This concept of input - efficiency will be discussed in Chapter 5.

This chapter defines and illustrates a PPS using paths of input - output coincidences over time. The chapter unfolds as follows.

Section (4.2) introduces the concept of a path capturing inter - temporal input - output dependence. Section (4.3) extends the standard PPS to define a PPS using paths of DMU input - output levels. An example will be given to illustrative the new PPS in section (4.4). Section (4.5) is an extension to section (4.3) to redefine the PPS capturing initial and terminal stock of capital input. Conclusions are drawn in section (4.6).

4.2 *Capturing inter - temporal input - output correspondence using input - output paths*

Correspondence of coincident input - output levels is at the heart of the definition of the "Production Possibility Set" (PPS) used to assess the comparative performance of DMUs in static DEA. See for example (Banker *et al.* (1984) p.1081) and (Tulkens and Vanden Eeckaut (1995) p.475) for the definition of the PPS in static DEA and "Free Disposal Hull" (FDH) approaches to measuring efficiency respectively. Input - output correspondence is fundamental to performance measurement since what must be measured is useful output secured against the resources (inputs) used for its procurement. This fundamental requirement of input - output correspondence does not alter in the presence of inter - temporal input - output dependence.

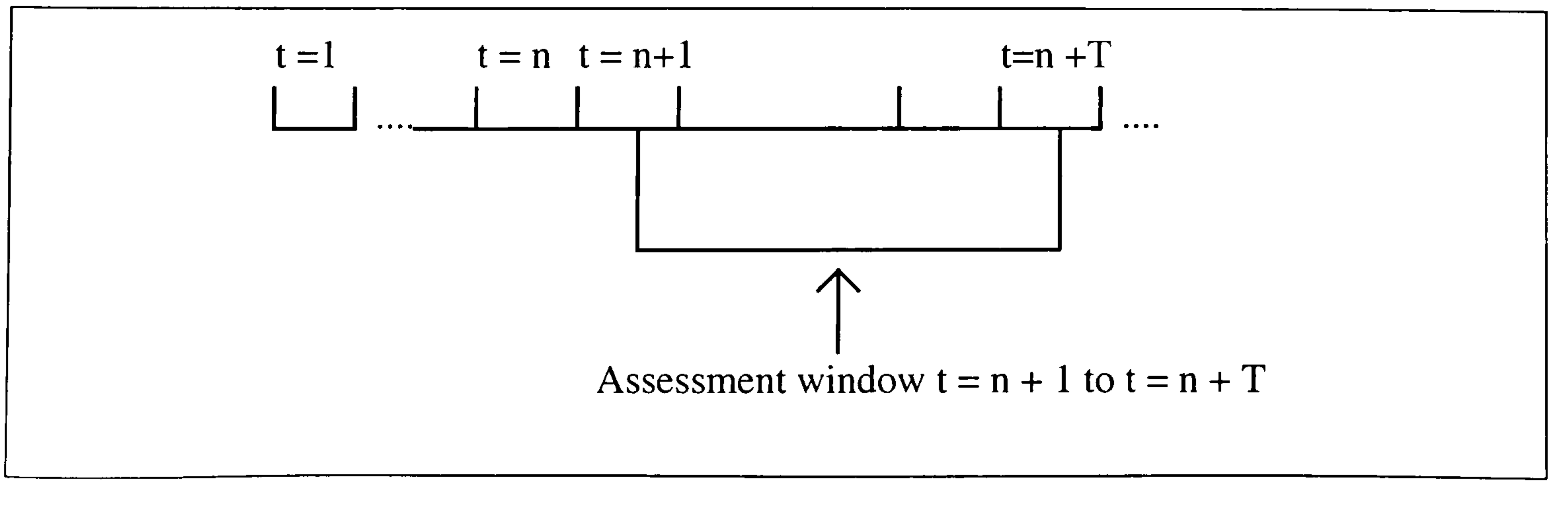
Thus, we need a method of capturing inter - temporal input - output correspondence which in many situations is more appropriately expressed dynamically. When investment or prices change, DMUs do not respond immediately, nor do they delay their response. Rather, they spread their response over a period of time. Of course the nature of such responses would vary from DMU to DMU, a major differentiating factor being the durability of the DMU of interest.

The concept of assessment path and assessment window

Inter - temporal input - output correspondence can be captured through the use of "paths" of coincident input - output levels as follows. Consider a DMU j which came into existence n + T time periods ago, it has been in existence up to the current i.e. the $(n+T)^{th}$ period and it is expected to continue in existence after the end of the current period. Let us further assume that input - output coincidences (x_j^t, y_j^t) are observed, where x_j^t = $(x_{1j}^t, x_{2j}^t, \ldots x_{mj}^t)$ are the input levels and y_j^t = $(y_{1j}^t, y_{2j}^t, \ldots y_{sj}^t)$ are the output levels observed in time period t at DMU j. Finally let the final T periods ending up at the current i.e. $(n+T)^{th}$ period be referred to as the "assessment window".

Therefore the sequence (x_j^t, y_j^t) t = n+1 ... n+T can be defined as the "assessment path" of DMU j and denoted $(x_j^{1,2,\ldots,T}, y_j^{1,2,\ldots,T})$. The concept of the assessment path of DMU j is illustrated graphically in Figure 4-1.

Figure 4-1. The assessment path of DMU j is the sequence of its input - output levels from t=n+1 to t=n+T.



In the case of a stock input (e.g. capital) the levels within the path reflect its variation over time such as might be caused by occasional investment activity. The shorter the periods into which the assessment window is subdivided the more accurate the reflection of the varying levels of the underlying continuous variable of stock input. In the case of a flow input (e.g. recurrent operating expenditure) the levels within the path reflect the resource used up during each period in the procurement of outputs.

Let us now consider an assessment window covering the entire life of the DMU. The assessment path can be said to capture the input - output correspondence represented by the DMU. This is because all inputs used by the DMU are reflected in the assessment path as are the corresponding outputs procured, irrespective of the time lag between inputs and corresponding outputs.

The concept of a path covering the entire life of a DMU is useful for seeing how input - output correspondence under inter - temporal effects can be captured in a path. However, a path covering the entire life of a DMU is not very practical. In most situations the DMUs are expected to continue in existence long into the future and what management usually wants is to measure performance over a 'sensible' length of time leading up to the present. In view of this we need to restrict our attention to assessment paths which cover a part of the life of a DMU. The path covering the last T periods of a DMU's life, e.g. from $t = n+1$ to $t = n+T$ in Figure 4-1, is the type of assessment path which can be constructed in practice. In the remainder of this chapter we will focus on paths of this type.

Is the length of the assessment path important?

It is evident that the length of an assessment window is a matter of judgement by the analyst. It should reflect the input - output correspondence mapped out by a DMU over the assessment window. This is because most lagged and capital output effects are likely to relate to inputs within the window while any adjustment period will represent a short proportion of the time covered by the window. Therefore the length of the assessment window to be used is a matter of judgement formed in the light of output lag, adjustment periods and capital output effects likely to apply to the situation modelled. How many assessment periods are used within the assessment window is also an issue which needs to be addressed and to which we shall return after presenting the assessment method to be used.

4.3 Defining a Dynamic PPS

Let us consider n DMUs (DMU_j j=1,..., n) each have an input - path $x_j^{1,\ldots T}$ and an output - path $y_j^{1,\ldots,T}$ where $x_j^{1,\ldots T} = (x_{1j}^{1,\ldots T}, \ldots, x_{mj}^{1,\ldots T})$ and $y_j^{1,\ldots,T} = (y_{1j}^{1,\ldots,T}, \ldots, y_{sj}^{1,\ldots,T})$. Thus input and output - paths can be vectors of input - paths and output - paths in the case of multi - input multi - output DMUs. Thus the entire life of a DMU can be divided to k overlapping windows $W_1 = [1,\ldots,\tau]$,

W_2=[2,...,τ+1], ..., W_k=[T-τ+1,...,T]. A “Dynamic PPS” with reference to each window can be defined. For the sake of simplification let us focus on window W_1 which covers time periods t=1,...,T. A dynamic PPS P can be expressed as a set of input - paths and output - paths ($x^{1,...T}$, $y^{1,...,T}$) such that;

$P = \{ (x^{1,...T}, y^{1,...,T}) \mid \text{input - path } x^{1,...T} \text{ can produce output - path } y^{1,...,T}\}$.

Following the construction of the PPS in DEA, (e.g. Banker *et al.* (1984) p. 1081), it is assumed that P has the following properties:

i. P is non-empty

All observed paths $\{(x_j^{1,2,...,T}, y_j^{1,2,...,T}) , j=1,2,...n\} \in P$.

ii. Strong disposability of input

If $(x_j^{1,2,...,T}, y_j^{1,2,...,T}) \in P$ and $x^{1,2,...,T} \geq x_j^{1,2,...,T}$, then $(x^{1,2,...,T}, y_j^{1,2,...,T}) \in P$ where $x^{1,2,...,T} \geq x_j^{1,2,...,T}$ means $x^t \geq x_j^t$ for t=1,2,...,T, and $x^t \geq x_j^t$ means that at least one element of x^t is greater than the corresponding element of x_j^t.

iii. Strong disposability of output

If $(x_j^{1,2,...,T}, y_j^{1,2,...,T}) \in P$ and $y^{1,2,...,T} \leq y_j^{1,2,...,T}$, then $(x_j^{1,2,...,T}, y^{1,2,...,T}) \in P$.

iv. No output can be produced without some input (‘No free lunch’)

$(x_j^{1,2,...,T}, 0) \in P$; but if $y_j^{1,2,...,T} \geq 0$ then $(0, y_j^{1,2,...,T}) \notin P$.

v. Constant Returns to Scale

If $(x_j^{1,2,\ldots,T}, y_j^{1,2,\ldots,T}) \in P$ then for each positive real value $\lambda>0$ we have $(\lambda x_j^{1,2,\ldots,T}, \lambda y_j^{1,2,\ldots,T}) \in P$.

vi. Minimum extrapolation

P is the closed and convex set satisfying i-v.

A dynamic PPS P which satisfies the above postulates can be constructed from the observed assessment paths $(x_j^{1,2,\ldots,T}, y_j^{1,2,\ldots,T})$, $j = 1 \ldots n$ as follows:

$$P = \{(x^{1,2,\ldots,T}, y^{1,2,\ldots,T}) \mid x^{1,2,\ldots,T} \geq \sum_j \lambda_j x_j^{1,2,\ldots,T};$$

$$y^{1,2,\ldots,T} \leq \sum_j \lambda_j y_j^{1,2,\ldots,T};\ \lambda_j \in R+,\ j=1,\ldots,n\} \qquad (4.1).$$

The next section illustrates P as defined in (4.1) using a graphical example.

4.4 Illustration of the dynamic PPS

To clarify the difference between the dynamic PPS presented in this thesis and static DEA PPS consider an inter - temporal technology which consists of two observed DMUs as presented in Table 4-1. The DMUs use a single input to secure a standard unit of output.

Table 4-1. Input levels per standard output

	Period 1	Period 2
	x^1	x^2
U_1	40	20
U_2	10	60

Table 4-1 shows that DMU U_1 starts with a large amount of input in period one and uses less in period two while DMU U_2 starts with a small amount of input in period one and rapidly increases it in period two.

Static contemporaneous technology (see Tulkens *et al.* (1995)) defines two PPS's, one for each period. The PPS in period one is

$$P^1 = \{ (x^1, y^1) \mid x^1 \text{ can produce } y^1 \text{ in the first period} \}.$$

Therefore the input requirement set to secure output of y^1_0 in period one is

$$P^1 (y^1_0) = \{ x^1 \mid x^1 \geq 10 \}.$$

The PPS in period two is

$$P^2 = \{ (x^2, y^2) \mid x^2 \text{ can produce } y^2 \text{ in the second period} \}.$$

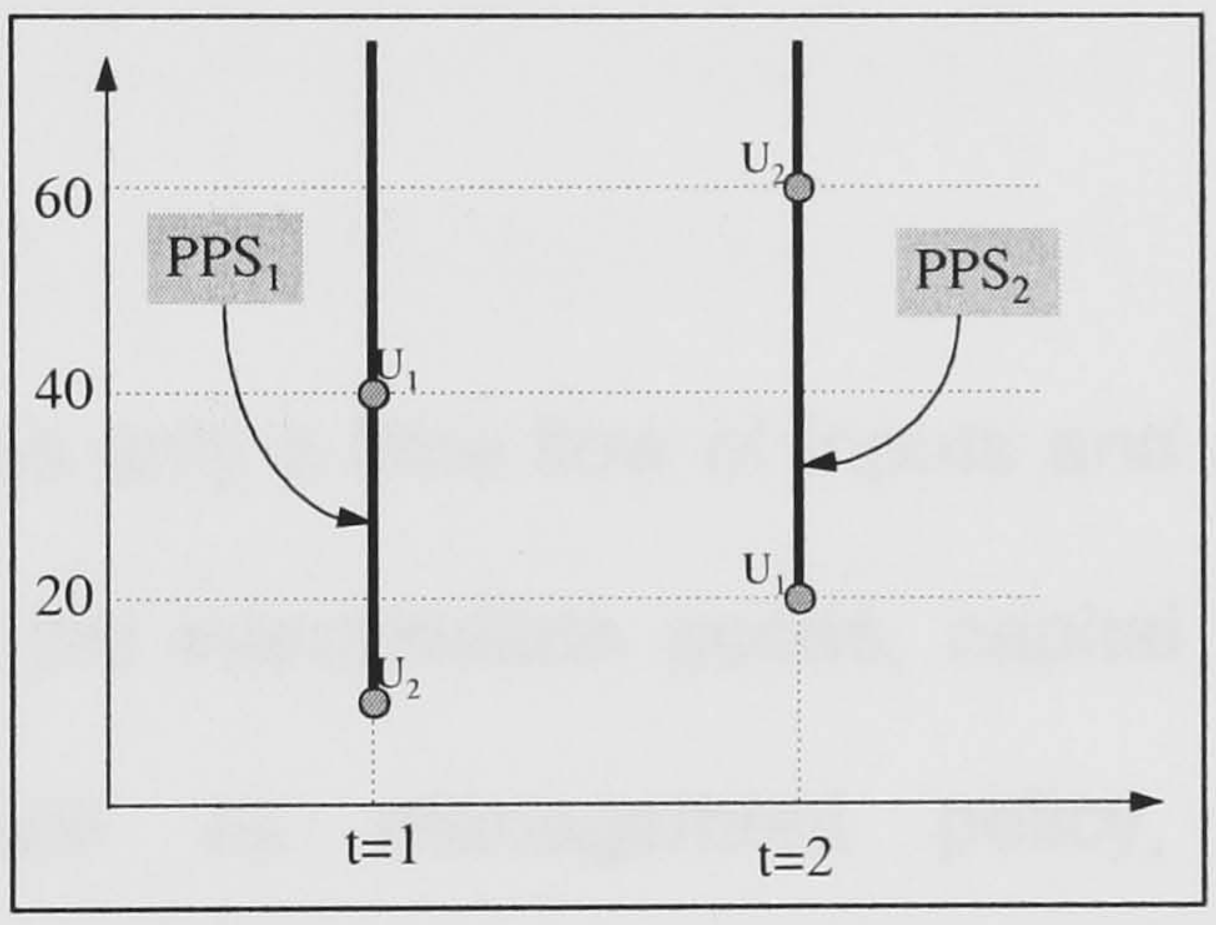
Therefore the input requirement set to secure output of y^2_0 in period two is

$$P^2 (y^2_0) = \{ x^2 \mid x^2 \geq 20 \}.$$

These indicate that, for instance, an input path associated with the input value of 10 in the first period and an input value of 20 in the second period is a feasible path. This definition of contemporaneous technology expresses that the PPS contains all DMUs above of (10, 20) as illustrated in Figure 4-2. This PPS is

$$\mathbf{PPS(y=1) = \{(x^1, x^2) \mid x^1 \geq 10, x^2 \geq 20\} = \{(10\alpha^1, 20\alpha^2) \mid \alpha^1 \ \&\ \alpha^2 \geq 1 \}.}$$

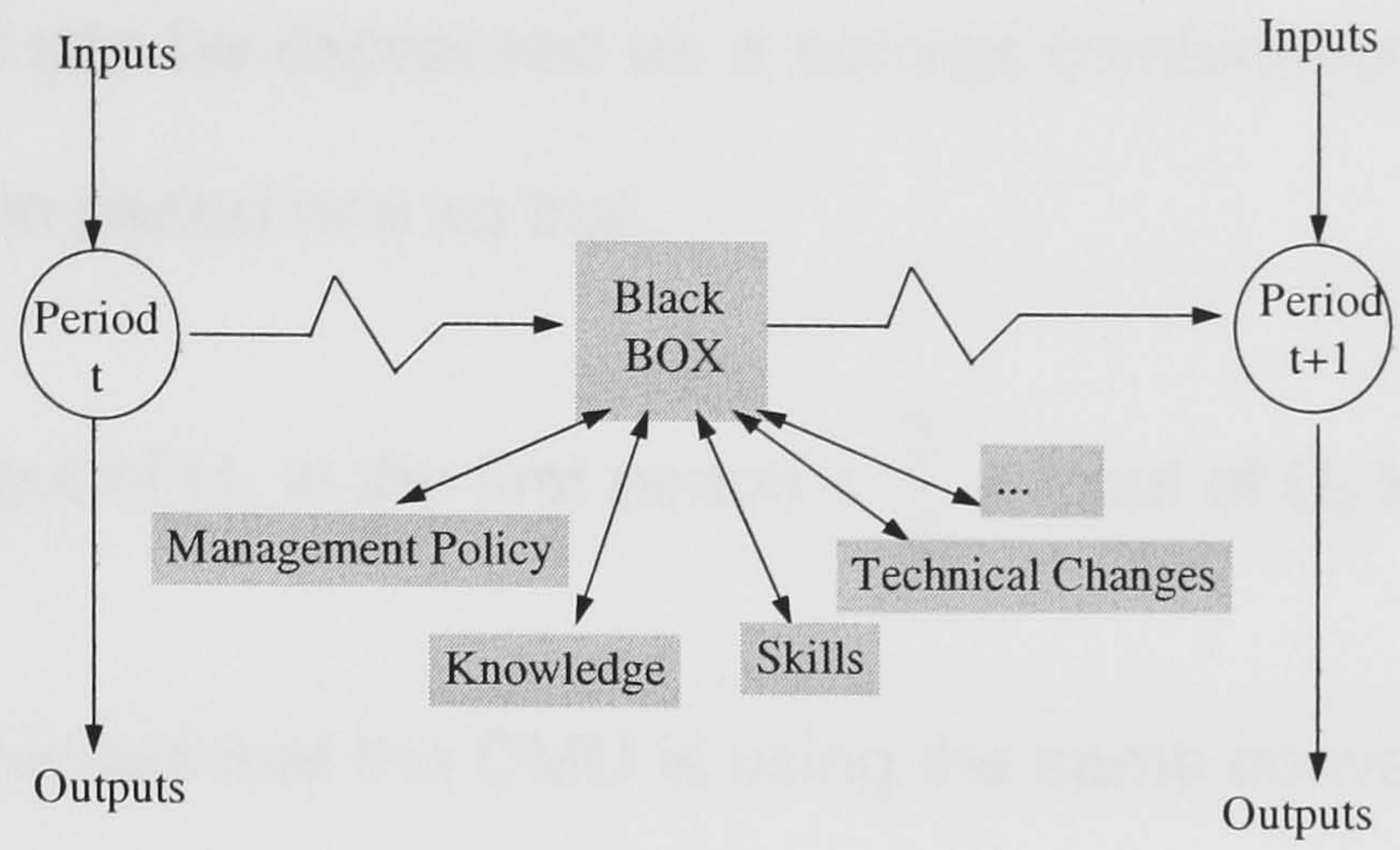
Figure 4-2. Static PPS's for each one of two periods of time



However an input path (10, 20) may not be feasible in dynamic PPS as is now explained.

In a dynamic process there is a "black box" converting inputs to outputs as the DMUs move from one period to the next (Figure 4-3).

Figure 4-3. Dynamic Process



In a dynamic process only a time flow of inputs and outputs is observed, without ever observing the intermediate goods, capital equipment or some invisible capability such as management policy, skills, knowledge, technological change, etc. which may have been produced and utilised inside the black box. This black box will lead managers to change the input quantities from period t to period t+1.

For instance consider the above example of only two observed DMUs with a single input (40, 20) and (10, 60) per unit of output respectively. If the same management policy as in the two observed DMUs is adopted, and assuming the same invisible capability such as skills, knowledge, technological change then if in the first period a DMU uses a low level of input, it is expected to use a large amount of input in the second period (DMU

U_2). Similarly, if in the first period a DMU uses a large level of input, it is expected to use a small amount of input in the second period (DMU U_1).

Now assume a DMU starts with input level of 20 per unit of output in the first period. This can be expressed as a convex combination of the inputs of DMU U_1 and U_2 in period one so that:

$$20 = \frac{1}{3} \times \text{input of } U_1 \text{ in the first period} + \frac{2}{3} \times \text{input of } U_2 \text{ in the first period.}$$

It is then expected that the DMU is using the same convex combination of input of DMU U_1 and U_2 in period two. Thus for this it needs to use at least 47.77 units of input in the second period, that is

$$\frac{1}{3} \times \text{input of } U_1 \text{ in second period} + \frac{2}{3} \times \text{input of } U_2 \text{ in second period} = 47.77\,.$$

So an input path of (20, 47.77) can be assumed a feasible path by reason of convexity of the PPS over time. Note that an input path of (20, 40) indicated by the static PPS is not feasible in the dynamic PPS because starting with an input of 20 in the first period we need an input of at least 47.77 in the second period to secure one unit of output.

On the other hand, if the plan in the second period is to use an input level of 40 then this DMU should start with an input level of 25 at least in the first period. This is because

$$40 = \frac{1}{2} \times \text{input of } U_1 \text{ in second period} + \frac{1}{2} \times \text{input of } U_2 \text{ in second period.}$$

This suggests that the DMU needs input level of 25 units in the first period, that is

$$\frac{1}{2} \times \text{input of } U_1 \text{ in first period} + \frac{1}{2} \times \text{input of } U_2 \text{ in first period} = 25.$$

Thus a path of (10, 20) which is feasible and in fact efficient in contemporaneous static DEA technology is not feasible in the dynamic PPS.

Stated in another way, the issue is whether the input available in one period allows managers an unrestricted choice of production process in the period after;

- If such a unrestricted choice is possible, the process will not be dynamic because the production process in the next period will not be built on the past process (hence the DMU in the second period can be seen as a new DMU in the analysis).

- If such an unrestricted choice is not possible the process is dynamic. In this case all feasible DMUs constructed from observed DMUs should admit the policy of observed DMUs in each black box for moving from one period to the next.

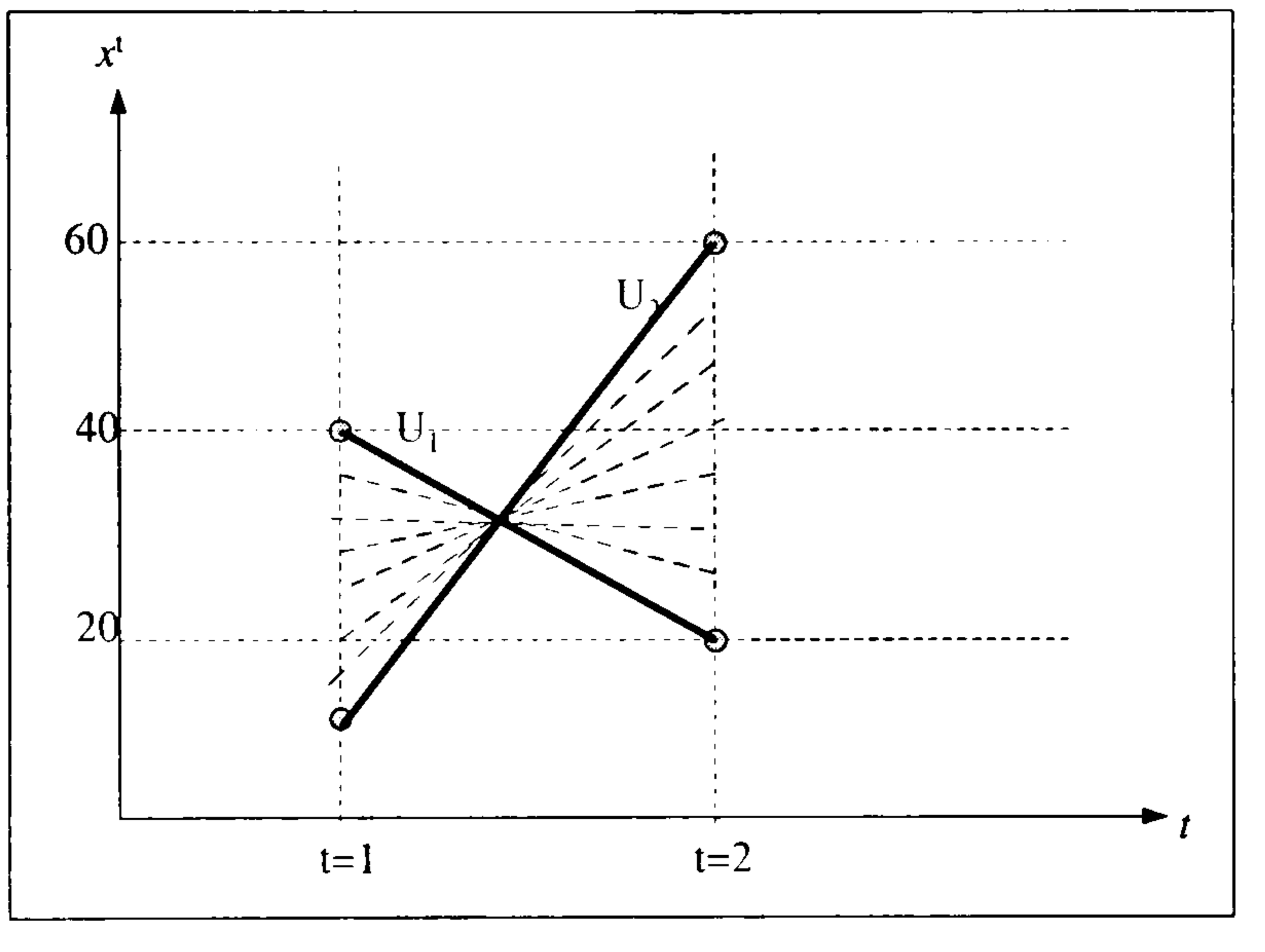
Under the dynamic PPS, using the two observed DMUs in Table 4-1 it can be seen that a feasible set of input levels in periods 1 and 2 is

$$\{ ((10\alpha_1 + 40\alpha_2), (60\alpha_1,+ 20\alpha_2)) \; ; \text{s.t. } \alpha_1 + \alpha_2 = 1, \alpha_1, \alpha_2 \geq 0 \}$$

which is a convex combination of the path of U_1 and U_2. These convex combination paths are assumed feasible and are illustrated Figure 4-4. It is clear that the choice of input in period two is conditional on the level of input used in period one.

Figure 4-4. A set of convex combination of two observed paths over two periods of time

$$A = \{(x^1, x^2) \mid x^1 = (10\alpha_1 + 40\alpha_2), x^2 = (60\alpha_1 + 20\alpha_2), \alpha_1 + \alpha_2 =1 , \alpha_1, \alpha_2 \geq 0 \}$$



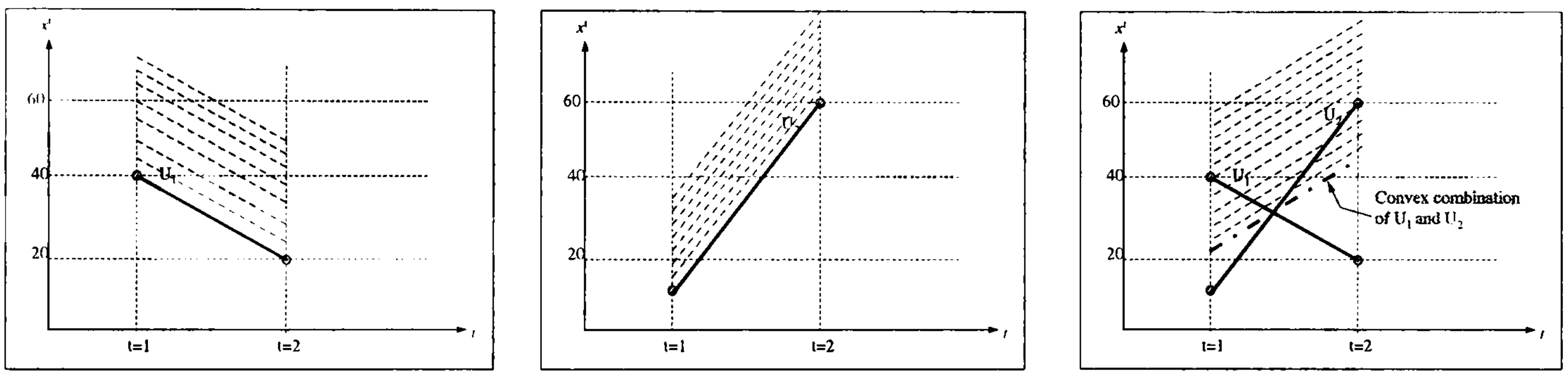
The dynamic PPS will be constructed by adding strong disposability to the set A. This is illustrated in Figure 4-5 where;

- B is a set of paths constructed by strong disposability to the path of U_1,
- C is a set of paths constructed by strong disposability to the path of U_2 and

- D is a set of paths constructed by strong disposability to a path of a convex combination of U_1 & U_2 which itself is an element of A in Figure 4-4.

Figure 4-5. Sets of paths constructed from strong disposability of two observed paths over two periods

$\mathbf{B = \{(x^1, x^2) \mid x^1 \geq 40, x^2 \geq 20\}}$	$\mathbf{C = \{(x^1, x^2) \mid x^1 \geq 10, x^2 \geq 60\}}$	$\mathbf{D = \{(x^1, x^2) \mid x^1 \geq (10\lambda_1 + 40\lambda_2), x^2 \geq (60\lambda_1 + 20\lambda_2), \lambda_1 + \lambda_2 = 1, \lambda_1, \lambda_2 \geq 0\}}$



Therefore the dynamic PPS is the smallest convex closed set which contains A, B, C and D.

Mathematically, if the feasible input - path (x^1, x^2) is denoted by $x^{1,2}$ the full PPS is expressed as follows.

The path $x^{1,2} \in$ PPS iff

$$\mathbf{x^1} \geq (10\lambda_1 + 40\lambda_2),\ \mathbf{x^2} \geq (60\lambda_1 + 20\lambda_2),$$

$$\lambda_1 + \lambda_2 = 1, \lambda_1, \lambda_2 \geq 0.$$

Thus on the contemporaneous static DEA technology there is no relationship between the convex combination of input - output levels in one period and the convex combination of input - output levels in another period. (Note that in contemporaneous technology the PPS is defined as:

$$PPS = \{(x^1, x^2) \mid \mathbf{x^1} \geq (10\lambda_1 + 40\lambda_2), \mathbf{x^2} \geq (60\lambda_3 + 20\lambda_4),$$

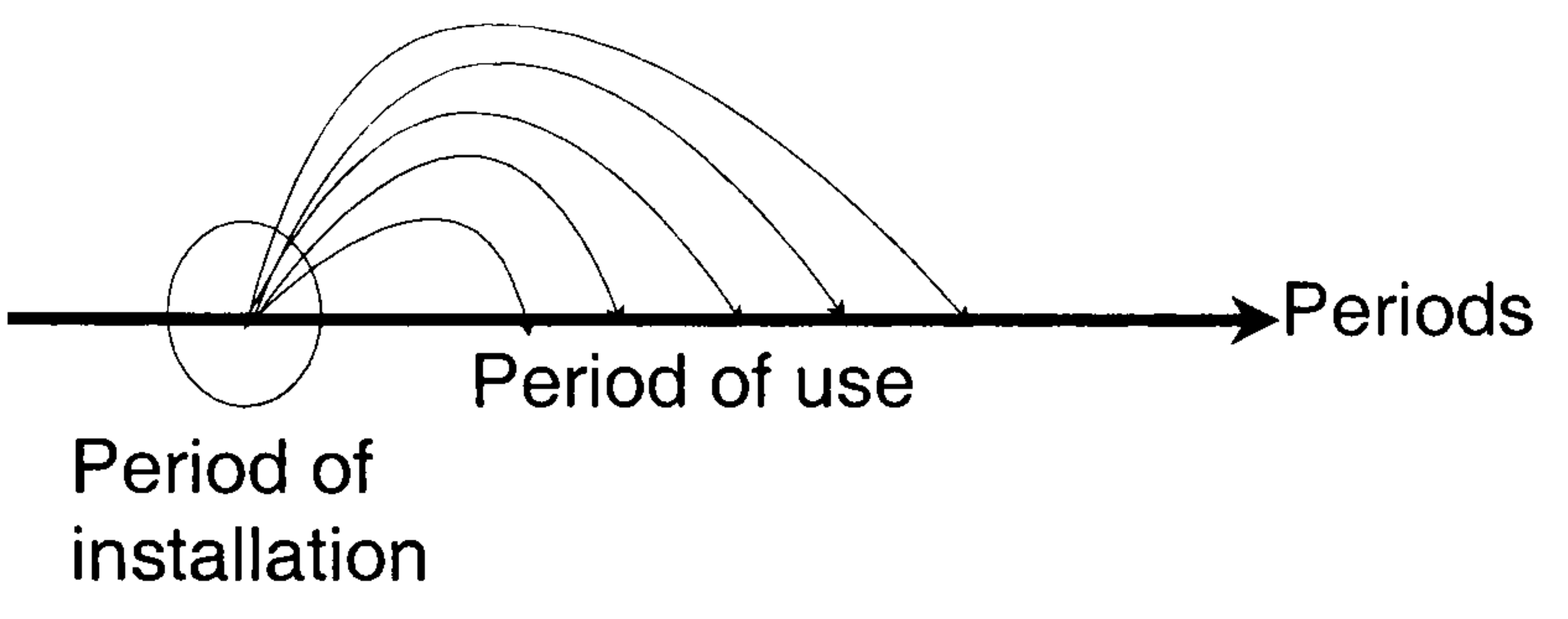
$$\lambda_1 + \lambda_2 = 1, \lambda_3 + \lambda_4 = 1, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0\}).$$

4.5 Capturing initial and terminal stock of capital within the PPS

There is, however, a further aspect which is important from a capital theory viewpoint. As noted earlier, capital is viewed here as stock. Once, a capital input is implemented, it produces a flow of outputs in future periods (see Figure 4-6).

It is clear from Figure 4-6 that there is lagged production of output from changes in capital taking place at some point in time. So long as lagged outputs are within the assessment window used we are not concerned about their timing. However, where lagged output due to changes in capital stock made within the assessment window, falls outside of it, and also where output within the assessment window is the result of changes in capital prior to the assessment window, then lagged output of this kind needs to be reflected in the dynamic assessment. Thus one distinction between static and dynamic PPS is that the definition reflects initial and terminal conditions of capital stock. Whereas, the static PPS does not require these two additional conditions.

Figure 4-6. The flow of output from capital



Now focus upon DMU - paths from the point of view of terminal stock in each assessment path. To clarify this issue, consider two feasible paths P and P', both of a finite duration and length t=1, ..., τ. Assume that they start with the same level of capital stock in the first period and they provide identical output streams $y^{1,\dots,\tau} = y'^{1,\dots,\tau}$ but that the terminal capital stocks differ, with K^{τ} $>\neq$ K'^{τ}. Thus path P provides more terminal capital stock than P' which can contribute to future outputs. Clearly, then, this capability of path P should be reflected in its assessment. However;

- If period τ is literally the end of the life of the DMU - path, then terminal capital stock of path P can not be used to produce output in future and can be ignored;

- On the other hand if DMU - path P survives after period τ then having more of terminal capital stock than DMU P' will enable higher future output at DMU - path P.

Let us assume that stock of capital input at period τ can be used for producing output in future.

To take into account this assumption in the PPS, terminal capital stock must be treated as another output.

Similar discussion can be made for initial capital stock in the assessment window. If, in time horizon t=1,2,...,τ, the initial capital stocks of DMUs at t=0 are not identical the PPS should take into account the difference between those DMUs which start with a large and those which start with a small quantity of capital stock at the beginning of the process under consideration. Initial capital stock should be reflected in the PPS as another input, as it can be converted to output within the assessment period.

Restating the dynamic PPS to reflect initial and terminal stock input

Let us consider a window of periods $t = \tau,\ \tau + 1,\ ...,\ \tau + T$. Assume that the set of inputs, I={1, ..., m}, can be divided into two sub - sets of period - specific inputs and capital - inputs, respectively I_1 and I_2 such that

$$I_1 \ \& \ I_2 \subseteq I,\ I_1 \cup I_2 = I \text{ and } I_1 \cap I_2 = \varnothing .$$

Then the set of inputs is:

period - specific input paths: $x^{\tau,\ \tau+1,\ \ldots,\ \tau+T}$;

changes in stock input paths: $z^{\tau,\ \tau+1,\ \dots,\ \tau+T}$;

Initial - stock inputs: $Z^{\tau-1}$.

The set of outputs is :

output - paths: $y^{\tau,\ \tau+1,\ \dots,\ \tau+T}$;

terminal - stock inputs as outputs: $Z^{\tau+T}$

For example in case of capital the changes in stock inputs will be reflected by investment.

This raises the issue of how to estimate the level of initial and terminal stock of inputs. The details of how to estimate such values are not directly addressed in this thesis. However one possibility is to reflect stock input by means of converting it to a capital value which takes into account the age and productive capabilities of the stock. Depreciation is of use as a means of reflecting in monetary terms the age of stock of capital.

The PPS within the assessment window $\tau,\dots,\tau+T$ can be now stated as follows:

$$P = \{(x^{\tau,\dots,\tau+T}, z^{\tau,\dots,\tau+T}, y^{\tau,\dots,\tau+T})|$$

$$x_i^t \geq \sum_j \lambda_j x_{ij}^t; \qquad \forall\ t=\tau,\dots,\tau+T\ \&\ i \in I_1$$

$$z_i^t \geq \sum_j \lambda_j z_{ij}^t; \qquad \forall\ t=\tau,\dots,\tau+T\ \&\ i \in I_2$$

$$y^t \leq \sum_j \lambda_j y_j^t; \qquad \forall\ t=\tau,\dots,\tau+T$$

$$Z_i^{\tau-1} \geq \sum_j \lambda_j Z_{ij}^{\tau-1}; \qquad \forall\ i \in I_2$$

$$Z_i^{\tau+T} \leq \sum_j \lambda_j Z_{ij}^{\tau+T}; \qquad \forall\ i \in I_2$$

$$\lambda_j \in R+ \qquad \forall\ j\ \} \qquad (4.2)$$

Note that if it is assumed that there is only one period, then this PPS will collapse to the static PPS which was discussed in Chapter one.

4.6 Conclusion

The relative efficiency of a DMU is calculated from the distance of its input levels to those of efficient DMUs (or linear combination of efficient DMUs). The inter - temporal input - output dependence is at the heart of the definition of the PPS used to assess dynamic efficiency. This chapter has introduced the concept of DMU paths. Then it has defined a dynamic PPS of DMU input – output levels over time. An example was given to illustrate dynamic PPS and to reveal its difference from static PPS.

In the PPS developed one important issue is to capture initial and terminal stock of input. Therefore extra constraints were included in the definition of the PPS to take into account the initial level of stock and capability of enhancing product from the DMU's terminal stock of input.

The next chapter uses the PPS as defined in this chapter to measure the relative efficiency of the assessment path of a DMU.

CHAPTER 5: Measuring the comparative efficiency of an assessment path

5.1 Introduction

So far in this thesis it has been shown that static DEA assessment fails to capture true performance of DMUs with inter - temporal input - output dependence. Thus for these DMUs a dynamic PPS was defined in Chapter 4. This chapter introduces and illustrates a measure for comparative efficiency of an assessment path. The dynamic efficiency measure of DMU - paths will be introduced in two phases. In the first phase an efficiency model is introduced to illustrate the basic idea of comparing assessment paths. An example will be given to illustrate the difference between dynamic and static efficiency. In the second phase a more general case of dynamic efficiency of DMU - paths will be introduced. This model will be based upon the PPS in (4.2) so that it can capture initial and terminal stock of capital input. The chapter unfolds as follows.

Section (5.2) defines "dynamic efficiency in a window" of a DMU - path, taken as the unit of assessment. Then it introduces a measure of dynamic efficiency for comparing DMU - paths. Section (5.3) provides an illustrative assessment of dynamic efficiency with hypothetical data, based on an inter - temporal production process and it outlines the difference between static and dynamic efficiency where initial and terminal stock of capital are available. Sections (5.4) shows how to capture initial and terminal stock of capital in a dynamic efficiency model. Conclusions are drawn in section (5.5).

5.2 An inter - temporal DEA model

Definition of a dynamic efficient path in a window

We begin by extending the definition of Pareto efficiency to assessment paths. Drawing from Charnes *et al.* (1978) p. 433, Solow (1970), Abel *et al.* (1989) and Burmeister (1980) a Pareto efficient path can be defined as follows.

An observed path $P^{1,\ldots,\tau}$ is called a "*Pareto efficient path*" over the time horizon $t=1,\ldots,\tau$ if no alternative feasible path exists over the same time horizon along which

a) Less input can be used in some time period while producing at least the same output - path or

b) More output is produced in some period while using no more than the same input - path.

We shall refer to an assessment path, which is Pareto efficient in the foregoing sense as a "dynamic efficient path in window t=1,..., τ". We call dynamic efficient path in window t=1,..., τ because the efficiency is only in window t=1,..., τ. The obvious distinction is that dynamic efficiency requires consideration of how the DMU performs through the period from t=1 to t=τ. In contrast, static efficiency requires the consideration of how the DMU performs at each period and ignores the inter - temporal impact through the assessment window.

Dynamic efficiency measure, a comparison of DMU - paths

With reference to the PPS presented in section (4.3) the following linear programming model can be used to determine whether the assessment path ($x_{j0}^{1,2,\dots,\tau}$, $y_{j0}^{1,2,\dots,\tau}$) of DMU j_0 is dynamically efficient within window t=1,...,τ.

Model 5-1. Dynamic efficiency within window t=1,...,τ.

$$\text{Min } \alpha_0 = \frac{\sum_{t=1}^{\tau} \alpha^t}{\tau} - \varepsilon \left(\sum_{t=1}^{\tau} \sum_{i=1}^{m} S_i^{t-} + \sum_{t=1}^{\tau} \sum_{r=1}^{s} S_r^{t+} \right)$$

$$\text{s.t.} \sum_{j}^{N} \lambda_j x_{ij}^t = \alpha^t x_{ij_0}^t - S_i^{t-} \qquad ; i = 1...m, t = 1...\tau$$

$$\sum_{j}^{N} \lambda_j y_{rj}^t = y_{rj_0}^t + S_r^{t+} \qquad ; r = 1...s, t = 1...\tau$$

$$\lambda_j \geq 0; \forall j$$

$$S_i^{t-}, S_r^{t+} \geq 0 \ \forall\, i, r \text{ and t.}$$

Where x_{ij}^t is the level of input i and y_{rj}^t is the level of output r observed in period t at DMU j.

An optimal solution to Model 5-1 specifies a production point ($x_i^t, i = 1...m, y_r^t, r = 1...s$, t = 1... τ) within the PPS of the assessment window, where

$$x_i^t = \sum_{j}^{N} \lambda_j^* x_{ij}^t = \alpha_t^* x_{ij_0}^t - S_i^{t-*} \quad i = 1...m, t = 1...\tau$$
$$y_r^t = \sum_{j}^{N} \lambda_j^* y_{ij}^t = y_{ij_0}^t + S_r^{t+*} \quad r = 1...s, t = 1...\tau. \tag{5.1}$$

The superscript * denotes the optimal value of the corresponding variable in Model 5-1.

Note that when τ = 1 Model 5-1 collapses to Model 5.9 in Charnes *et al.* (1978) (p. 433) used to measure efficiency in the single production period context.

The assessment path specified in expression (5.1) is "dynamically efficient in window t=1,...,τ" in line with the earlier definition. By virtue of Model 5-1. it is the case that there exists no path within the PPS which offers a reduction in one of the input levels x_i^t for any i or t without either a consequent rise in some other input level or a reduction in the level of at least one of the outputs in some time period t. By implication, when at the optimal solution to Model 5-1;

$$\alpha_t^* = 1 \ \forall t, \ S_i^{t-*} = 0 \ \forall \ i, t \ \text{ and } S_r^{t+*} = 0 \ \forall \ r, t$$

the assessment path of DMU j_0 is "dynamically efficient in window t=1,...,τ". In such a case;

$$\alpha_o = 1.$$

Where the assessment path of DMU j_0 is not dynamically efficient, the value of α_o can be seen as a measure of its "dynamic (input) efficiency". Specifically, α_o measures the average proportion to which the observed input levels of DMU j_0 can be contracted without detriment to any one of its output levels in any time period while maintaining its input mix in each period of the assessment window. Each component α_t^* measures the extent to which the input levels in period t can be lowered radially under efficient production. The cross - sectionally radial measure of efficiency used is consistent with the basic notion in DEA of not imposing a prior value system over input - output levels within each time period.

The difference between static and dynamic efficiency

It can be seen that the inter - temporal Model 5-1 with T periods (m inputs and s outputs) is analogous to a one - period static DEA model with $m \times T$ inputs and $s \times T$ outputs except that α^t varies with t, unlike static DEA where we would have $\alpha^t = \alpha \; \forall t$.

Thus one way to view Model 5-1 is as one which sub - divides the assessment window into shorter periods in order to reflect changes in the levels of inputs and outputs which have inter - temporal dependence.

Model 5-1 then gives flexibility to the unit being assessed as to which time period it chooses to reduce the use of resources in order to gain the maximum efficiency rating over the assessment window. In this way the model gives explicit recognition to the fact that different units can operate with different resource profiles over time and still being efficient.

Subdivision of periods within an assessment window

The measure of efficiency yielded by Model 5-1 will not alter if the periods of the assessment window are aggregated or subdivided, provided the input - output levels in the new periods are obtainable by simply scaling the input - output levels of the original periods. This can be readily seen by noting that such scaling of input - output levels merely generates redundant constraints within Model 5-1 in going from the original to the new periods within the assessment window.

In the more general case, however, where the subdivision or aggregation of the original periods does not preserve the ratios of the original input - output levels the efficiency measure α_o will be assessment period subdivision variant. This is as should be since the aim is to assess DMUs by charting their resource use and output creation over time during the assessment window.

Thus an important question is which assessment window subdivision yields the more reliable efficiency measure. The answer is the sub-division is a subject of judgement by the analyst and it may differ in different applications. Say, for example, in assessing universities windows of 3 or 4 years may prove more accurate in reflecting correspondence between input - output levels of DMUs. This is because the cycle of study in universities is about 3 years. Obviously a balance has to be struck between reflecting accurately the input - output path of each DMU and the number of assessment periods used.

Referent assessment paths in dynamic efficiency

The assessment paths corresponding to positive λ values at the optimal solution to Model 5-1 will be referred to as the "referent assessment paths" or "efficient peer paths" of DMU j_0. The dynamic efficiency rating of the assessment path of DMU j_0 is with reference to these input - output paths.

Alternative measures of dynamic efficiency

With the approach developed in this chapter it is possible to identify whether or not a DMU - path is Pareto efficient. However, how far is a DMU - path from its peer(s) on the frontier is another question. Just as in static DEA, here too there is no unique measure of a DMU's distance from the best practice frontier. In Chapter 8 some alternative measures of dynamic efficiency will be discussed.

The next section illustrates the assessment of the dynamic efficiencies of a set of hypothetical DMUs and contrasts the results obtained, with those that would be obtained in a 'static' DEA framework.

5.3 An illustrative assessment of the dynamic efficiencies of hypothetical DMUs

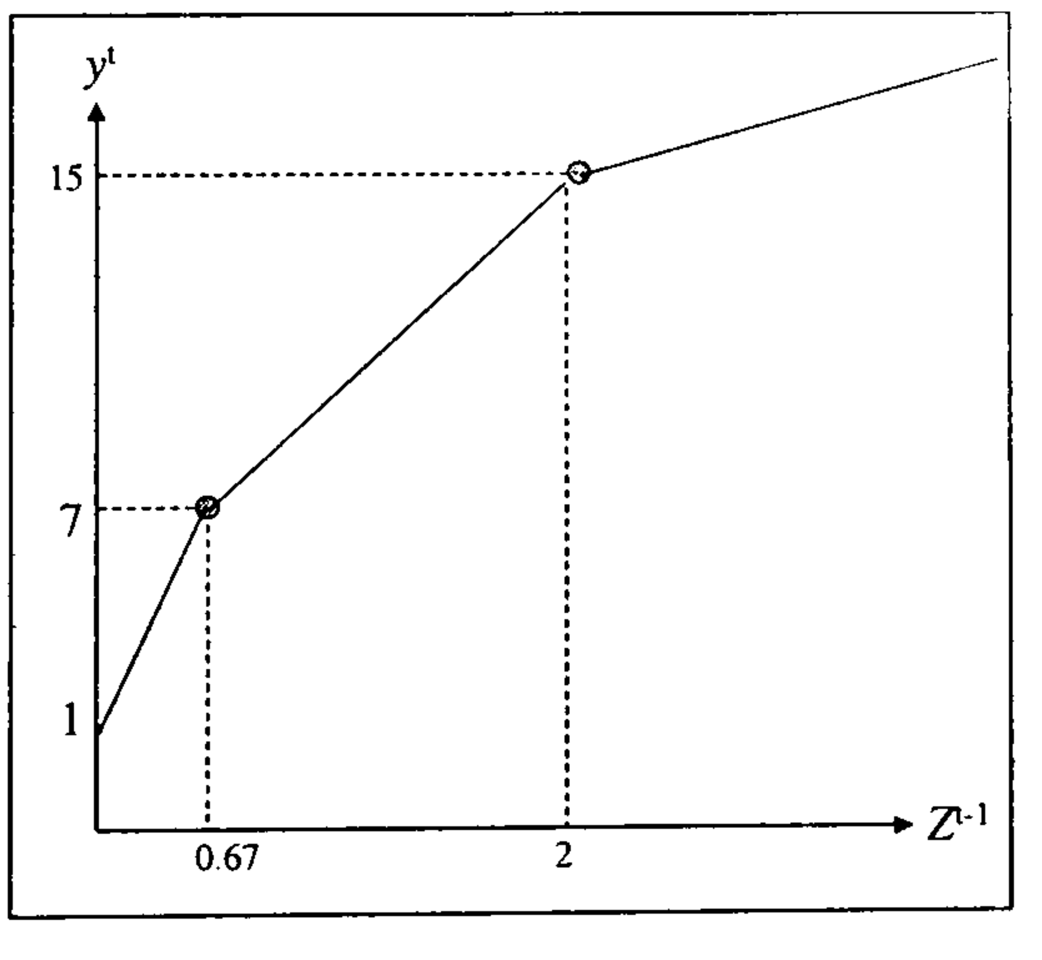
In order to compare the static and dynamic DEA approaches a set of 10 DMUs associated with a specific production function is chosen. We will use a technology in which there is inter - temporal dependence where stock input of a period affects output in the subsequent period but not beyond that.

Using hypothetical input levels, the output levels of DMUs were generated in line with the inter - temporal technology (5.2) (see also section (3.4)). The index t indicates the time period.

An inter - temporal production technology

$$y^t = \begin{cases} 9Z^{t-1} + x^t, & 0 \le Z^{t-1} \le 0.67x^t \\ 6Z^{t-1} + 3x^t, & 0.67x^t \le Z^{t-1} \le 2x^t \\ 3Z^{t-1} + 9x^t, & 2x^t \le Z^{t-1} \end{cases} \tag{5.2}$$

Figure 5-1. The impact of stock input at t-1 on output at t, for $x^t = 1$



Expression (5.2) shows a continuous production function where output at each period depends on flow input, x, and stock input, Z. Stock input is measured at the start of each period. It is the fraction of stock input at t-1 to the period - specific input at t, i.e. $\frac{Z^{t-1}}{x^t}$ which impacts the contribution of Z to output produced at t. The impact of stock input at t-1 on output at t, for $x^t = 1$, is illustrated in Figure 5-1. The figure clearly shows the output increase when the capital stock increases.

Let us assume that there are 10 observed DMUs U_1 ... U_{10} over four periods which operate under the production technology in expression (5.2). Their data appear in Table 5-2 generated using the arbitrary inputs in Table 5-1.

Table 5-1. Inputs of 10 hypothetical DMUs in 4 periods.

	Initial stock	Period 1		Period 2		period 3		period 4	
	z^0	x^1	z^1	x^2	z^2	x^3	z^3	x^4	z^4
U1	50	40	100	40	20	40	20	40	20
U2	50	40	20	40	100	40	20	40	20
U3	50	40	20	40	20	40	100	40	20
U4	50	40	20	40	20	40	20	40	100
U5	50	40	40	40	40	40	40	40	40
U6	50	80	10	90	40	80	50	120	60
U7	50	80	10	120	40	90	50	80	60
U8	50	30	40	10	100	10	180	20	60
U9	50	140	10	180	10	130	20	190	20
U10	50	10	20	20	20	20	10	10	10

The output paths in Table 5-2 as generated have 100% efficiency.

Table 5-2. Output of 10 DMUs in 4 periods of time under expression (5.2).

	Output path ($y^{1,2,3,4}$)	
	(t1, t2, t3, t4)	Total
U1	(420, 810, 870, 930)	3030
U2	(420, 540, 870, 930)	2760
U3	(420, 540, 630, 930)	2520
U4	(420, 540, 630, 690)	2280
U5	(420, 630, 750, 870)	2670
U6	(530, 630, 840, 1260)	3260
U7	(530, 660, 870, 1140)	3200
U8	(390, 360, 660, 1290)	2700
U9	(590, 720, 760, 1000)	3070
U10	(240, 390, 450, 390)	1470

The DMUs in Table 5-2 have been assessed using Model 5-2 and windows of two periods. The model is solved with windows of two periods because the production technology has two-period interdependence of input - output levels. Model 5-2 is an instance of Model 5-1.

Model 5-2. Dynamic efficiency of hypothetical data in periods τ-1, τ.

$$
\begin{aligned}
&\text{For } \tau = 2, 3, 4: \\
&\text{Min } \alpha_0 = \frac{\alpha^{\tau-1}+\alpha^{\tau}}{2} - \varepsilon\Big(\sum_{t=\tau-1}^{\tau} S_x^{t-} + S_z^{t-} + S_y^{t+}\Big) \\
&\text{s.t. } \sum_{j=1}^{10} \lambda_j x_j^t = \alpha^t x_{j_0}^t - S_x^{t-} \qquad ; t = \tau-1, \tau \\
&\sum_{j=1}^{10} \lambda_j z_j^t = \alpha^t z_{j_0}^t - S_z^{t-} \qquad ; t = \tau-1, \tau \\
&\sum_{j=1}^{10} \lambda_j y_j^t = y_{j_0}^t + S_y^{t+} \qquad ; t = \tau-1, \tau \\
&\lambda_j \geq 0;\ \forall j \\
&S_x^{t-},\ S_z^{t-},\ S_y^{t+} \geq 0\ \ \forall\, t.
\end{aligned}
$$

The static DEA efficiencies were also calculated under contemporaneous and aggregate technology using in Model 5-3 and Model 5-4 respectively.

Model 5-3. Static DEA efficiency in period t.

$$
\begin{aligned}
&\text{For } t = 1, 2, 3, 4 \\
&\text{Min } \phi^t - \varepsilon(S_x^{t-} + S_Z^{t-} + S_y^{t+}) \\
&\text{s.t. } \sum_{j=1}^{10} \lambda_j x_j^t = \phi^t x_{j_0}^t - S_x^{t-} \\
&\sum_{j=1}^{10} \lambda_j Z_j^t = \phi^t Z_{j_0}^t - S_Z^{t-} \\
&\sum_{j=1}^{10} \lambda_j y_j^t = y_{j_0}^t + S_y^{t+} \\
&\lambda_j \geq 0; \forall j \;\&\; S_x^{t-}, S_Z^{t-}, S_y^{t+} \geq 0\ \ \forall\, t.
\end{aligned}
$$

Model 5-4. Aggregate DEA efficiency of hypothetical data.

$$
\begin{aligned}
&\text{Min } \phi - \varepsilon(S_X^- + S_Z^- + S_Y^+) \\
&\text{s.t. } \sum_{j=1}^{10} \lambda_j X_j = \phi X_{j_0} - S_X^- \\
&\sum_{j=1}^{10} \lambda_j Z_j = \phi Z_{j_0} - S_Z^- \\
&\sum_{j=1}^{10} \lambda_j Y_j = Y_{j_0} + S_Y^+ \\
&\lambda_j \geq 0; \forall j \;\&\; S_X^-, S_Z^-, S_Y^+ \geq 0. \\
&\text{where} \\
&X_j = \sum_{t=1}^{4} x_j^t,\ Z_j = \sum_{t=0}^{4} z_j^t,\ Y_j = \sum_{t=1}^{4} y_j^t\ \ \forall j.
\end{aligned}
$$

We assume, in Model 5-3, that DMUs are using x and Z as inputs to produce single output y in the assessment period. In each time, Z is he total capital stock up to and including the last period under assessment. In Model 5-4 we use the aggregate levels of input and output over the horizon t_1 to t_4. In this model X and Y are, respectively, the total level of the current input and the total level of the output over periods t_1 to t_4. Z is the total level of capital invested within time horizon t_0 to t_4 which includes initial capital investment as well as all the invested capital over periods t_1 to t_4. The dynamic, static and aggregate efficiency results are summarised in Table 5-3.

Table 5-3. Comparison of static , aggregate and dynamic efficiency results of the 10 DMUs described in Table 5-1 and Table 5-2.

	Contemporaneous technology t1, t2, t3, t4				Aggregate technology	Dynamic efficiency		
						(t1 & t2)	(t2 & t3)	(t3 & t4)
U1	0.67	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U2	1.00	0.67	1.00	1.00	0.91	1.00	0.97	1.00
U3	1.00	1.00	0.72	1.00	0.83	1.00	0.72	1.00
U4	1.00	1.00	1.00	0.74	0.75	1.00	0.72	0.66
U5	0.89	0.93	0.93	0.94	0.88	0.83	0.80	0.77
U6	1.00	0.91	0.91	1.00	0.91	1.00	0.48	0.56
U7	1.00	0.88	0.92	1.00	0.89	0.99	0.47	0.53
U8	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U10	1.00	0.95	1.00	1.00	1.00	1.00	1.00	1.00

Two main questions arise here. First, why is there such a big difference between the dynamic efficiency and static efficiency scores? Second, why could the dynamic efficiency Model 1-1 not capture the true performance in some cases? We answer these using some examples.

For instance, consider unit U_4 that is the least efficient unit in the aggregate model (efficiency = 75%) and it is the least efficient unit in the fourth period of the static contemporaneous model (efficiency=74%). A study of capital investment of this unit shows that almost of the investment made by U_4 is in period t_4. However none of the static models could capture the impact of this investment within the assessment periods. Probably if the production process continues this unit will become much more efficient in the next period since its production process suggests a high level of output in periods following periods of investment.

A comparison of capital investment by U1, U2, U3 and U4 and their static efficiency scores are very informative with regard to their investment plan. All these units use the same level of current inputs in each period. The total invested capital by these units is the same but their investment sequencing is different. Major investment of unit U_1 is in the first period of the production process. Therefore this unit becomes inefficient, and in fact the least efficient unit in period t_1 according to the static contemporaneous technology. We have the same results for other three units, U_2 is the least efficient unit with high level of capital stock in period 2. So is U_3 in period 3 and U_4 in period 4. In all static contemporaneous technology a high level of capital investment means lower efficiency, e.g. U_1 in Period t_1, U_2 in Period t_2,

U_3 in Period t_3 and U_4 in Period t_4. However the aggregate efficiency model distinguishes between earlier investment and late investment. As it is expected earlier investment would have more benefit to the unit than late investment. This can be readily seen from a comparison of the aggregate efficiency results of these four units. The aggregate efficiency scores are ordered exactly as the investments, i.e. U_1 is more efficient that U_2, U_2 is more efficient that U_3 and U_3 is more efficient that U_4.

The dynamic efficiency Model 5-1 also fails to capture the efficiency of some DMUs under certain circumstances. A study of the data and dynamic efficiency scores shows that this must be investigated according to the level of initial capital stock and / or the level of capital stock remaining at the end of the assessment window. We give three examples here.

First, consider dynamic efficiency in the window covering periods t_2 and t_3. Within this window U_2 and U_3 are using the same level of capital and current input in total. However the dynamic efficiency score of U_3 (=0.72) is much lower than the dynamic efficiency score of U_2 (=0.97). Why has this happened? The invested capital of 100 in period t_2 would return in output format within the assessment window while the invested capital of 100 for U_3 would not. This is why the dynamic efficiency of U_3 is much lower than that of U_2.

As a second example we consider U_3 and U_4 in dynamic efficiency window made up of periods t_3 and t_4. U_4 shows less efficient than U_3 while both units use the same level of capital and current inputs in total. Again the late investment of capital input in U_4 could not be captured by dynamic efficiency Model 5-1 as this would increase the output beyond the window under assessment.

As a third example but different from the previous two we consider U_1 and U_4 under dynamic efficiency within the window made up of periods t_2 and t_3. Interestingly both units have exactly the same level of capital and current inputs in both periods under assessment but U_1 becomes dynamically efficient while U_4 is inefficient with very low efficiency score of 0.72. Why has the dynamic efficiency Model 5-1 assigned such very different scores to these units with the same levels of current and capital inputs? The answer lies in the continuous nature of the production process. In this particular case a high level of capital invested by U_1 in period t_1, prior to assessment window impacts on the efficiency score obtained within periods t_2 and t_3.

The above examples clarify the weakness of the dynamic efficiency Model 5-1 in capturing the efficiency scores properly when a DMU has a huge amount of capital invested prior to the assessment window and / or when the unit accumulates in some assessment window a large amount of capital stock, probably for further production in future periods.

The example clearly illustrates how snap - shot static efficiencies can fail to capture true performance when there is inter - temporal dependence of input - output levels. The dynamic efficiency model captures better the performance of DMUs in such cases. However this dynamic efficiency model could not capture the impact of any stock input at the end of the assessment window, nor could it capture the difference in stock at the start of the assessment window had there been any.

This is the main reason that in the next section we are aiming to capture the role of end level of capital stock as well as the role of initial investment by introducing further constraints to the model.

5.4 Capturing initial and terminal - stock in the dynamic efficiency model

Assume that we have a set of DMU - paths over the periods $t=\tau, \ldots, \tau+T$. Assume further that the set of inputs can be divided into two sub - sets, one of period - specific inputs and the second of capital inputs. Let us denote these two sub - sets I_1 and I_2 such that;

$$I_1 \text{ and } I_2 \subseteq \{1, \ldots, m\}, \; I_1 \cup I_2 = \{1, \ldots, m\} \text{ and } I_1 \cap I_2 = \varnothing.$$

I_1 is the set of period - specific and I_2 is the set of capital inputs.

In the previous chapter it was argued that if $\tau+T$ is literally the end of the DMU, then we can ignore terminal capital input. However if DMUs survive after period $\tau+T$ then Model 5-1 must be reformulated to capture both initial and terminal stock or capital inputs.

Model 5-5. Dynamic efficiency within window $t = \tau,\ \tau+1,\ ...,\ \tau+T$ to take into account the initial and terminal stock of capital.

$$\text{Min } \alpha = \frac{\sum_{t=\tau}^{\tau+T} \alpha^t}{T} - \varepsilon\Big(\sum_{t=\tau}^{\tau+T}\sum_{i\in I_1} S_i^{t-} + \sum_{t=\tau}^{\tau+T}\sum_{i\in I_2} \delta_i^{t-} + \sum_{t=\tau}^{\tau+T}\sum_{r=1}^{s} S_r^{t+} + \sum_{i\in I_2}\gamma_i^- + \sum_{i\in I_2}\gamma_i^+\Big)$$

s.t.

$$\text{C1:} \quad \sum_{j=1}^{N}\lambda_j x_{ij}^t = \alpha^t x_{ij_0}^t - S_i^{t-} \quad ; i\in I_1,\ t=\tau,...,\tau+T$$

$$\text{C2:} \quad \sum_{j=1}^{N}\lambda_j z_{ij}^t = \alpha^t z_{ij_0}^t - \delta_i^{t-} \quad ; i\in I_2,\ t=\tau,...,\tau+T$$

$$\text{C3:} \quad \sum_{j=1}^{N}\lambda_j y_{rj}^t = y_{rj_0}^t + S_r^{t+} \quad ;\ r=1,...,s,\ \ t=\tau,...,\tau+T$$

$$\text{C4:} \quad \sum_{j}^{N}\lambda_j Z_{ij}^{\tau+T} = Z_{ij_0}^{\tau+T} + \gamma_i^+ \quad ;\ i\in I_2$$

$$\text{C5:} \quad \sum_{j=1}^{N}\lambda_j Z_{ij}^{\tau-1} = Z_{ij_0}^{\tau-1} - \gamma_i^- \quad ;\ i\in I_2$$

$$\lambda_j \ge 0; \forall j,\ S_i^{t-}\ge 0,\ \delta_i^{t-}\ge 0\ (\forall t, \forall i\in I_1),\ S_r^{t+}\ge 0(\forall r,\forall t),\ \gamma_i^+\ge 0,\ \gamma_i^-\ge 0(\forall i\in I_2)$$

where;

$I_1 \subset \{1,...,m\}$ are flow inputs,

$I_2 \subset \{1,...,m\}$ are those inputs that their end - stock will be converted, directley or indirectley, into more output some type at some future period.

$Z_{ij}^{\tau-1}$ is the initial - stock of capital of type i for DMU j; $i\in I_2$,

$Z_{ij}^{\tau+T}$ is the end - stock capital of type i for DMU j; $i\in I_2$.

With reference to the PPS (4.2), Model 5-1 can be reformulated to Model 5-5 to take into account both initial and terminal stock inputs using constraints sets **C1**- **C5** as follows:

$\Rightarrow$ **C1** are **period - specific** input constraints,

$\Rightarrow$ **C2** are **stock - change** input constraints,

⇒ **C3** are **output** constraints and,

⇒ **C4** are **end - stock** constraints,

⇒ **C5** are **initial - stock** constraints.

Model 5-5 modifies Model 5-1 essentially by adding constraint sets C4 and C5. Set C4 treats terminal capital stock as an output and that is why constraint sets C3 and C4 are essentially the same. Constraint C5 treats initial stock of capital as an exogenously fixed input. Thus the model measures the extent to which inputs, both flow and stock, can be reduced further, given the initial and terminal stock input of the unit and given its output levels during the assessment window.

5.5 Conclusion

In this chapter an extension was made to the definition of Pareto efficiency from the static case where Pareto efficiency is defined with the reference to DMUs to one where Pareto efficiency is defined with reference to paths of DMUs. We have also defined a measure of dynamic efficiency. The measure was used in our model is radial in each period. Alternative measures of dynamic efficiency will be introduced in Chapter 8.

The dynamic DEA models developed in this chapter can capture initial and terminal stock of capital input where terminal stock input impacts future output.

Hence Model 5-1 was introduced as a first step for measuring the dynamic efficiency. If there is no capital input in the production process, or we ignore the role of initial investment and the role of end capital stock, this model can be used for measuring the dynamic efficiency of DMUs.

An example was used to illustrate how snap - shot static efficiencies can fail to capture true performance when there is inter - temporal dependence of input - output levels. The dynamic efficiency model captures better the performance of DMUs in such cases. However example shows that Model 5-1 is fail to capture true performance in some cases with high level of initial capital and / or with high level of end stock capital. Therefore a new model was introduced, Model 5-5, which is the base of our analysis for the rest of this thesis.

The next chapter generalises the comparison of the two methods by looking at a larger number of DMUs with more complex comparative performance relationships.

CHAPTER 6: A simulation study comparing static and dynamic efficiency measures

6.1 Introduction

This chapter compares more comprehensively static and dynamic DEA efficiency. The chapter uses simulation data drawn from two different scenarios. The scenarios differ in that one holds the data constant and varies the technology and the other holds the technology constant and varies the data. In this manner any bias in the results which is technology or data specific can be identified.

In each scenario there are 10 runs. Each run has 100 DMUs each one observed over 15 periods of time. Hence

⇒ In **scenario (I)** the input - output path (data) of DMUs is kept constant and we vary the technology which describes the inter - temporal input - output dependence.

⇒ In **scenario (II)** the technology is kept constant and we vary the data set.

The chapter unfolds as follows.

Section (6.2) lays out scenario (I). In this section the method of generating the data set is discussed and both static and dynamic efficiency are compared against true efficiency. Section (6.3) lays out scenario (II). In this scenario an inter - temporal Cobb- Douglas production function is employed to generate a large data set under varying input levels. We then compare static and dynamic efficiency. Section (6.4) compares static and dynamic DEA models across the two scenarios. Conclusions are drawn in section (6.5).

6.2 Scenario I: Constant input data and varying technology

Assume a technology with two inputs, flow x and stock change z, and a single output y. The values of the input variables, x and z, are generated randomly and independently from uniform distributions with range [1, 100] and with means and standard deviations varying over time as in Table 6-1.

Table 6-1. Mean and stdv. of input variables

Period	Flow input (x)		Change in stock input (z)	
	Mean	Stdv.	Mean	Stdv.
t1	52	28	57	26
t2	48	29	53	26
t3	51	30	56	27
t4	52	29	57	26
t5	48	29	53	26
t6	50	27	55	24
t7	51	31	56	28
t8	47	28	52	25
t9	51	28	55	26
t10	53	28	57	26
t11	55	29	59	26
t12	55	28	59	25
t13	43	28	49	25
t14	50	26	54	24
t15	53	31	58	28

Therefore in this assessment we have three input variables:

⇒ Flow input, x, that is the input used up in each given period,

⇒ Stock input, Z, accumulated over many periods and

⇒ Change in stock input, z, that is the difference of stock input from one period to the next.

The actual values of flow input, stock change and stock input are shown in Tables A1 - A3 (Appendix A) respectively.

To generate the output levels, the data in Tables A1 - A3 (Appendix A) are used with the following function (see Burmeister (1980)) which is the inter - temporal production function (see also Chapter 3).

$$y = f(x,z,t) = \begin{cases} \alpha_1 Z^{t-1} + \beta_1 x^t \ ; & 0 \le \dfrac{Z^{t-1}}{x^t} \le c_1 \\ \alpha_2 Z^{t-1} + \beta_2 x^t \ ; & c_1 \le \dfrac{Z^{t-1}}{x^t} \le c_2 \\ \alpha_3 Z^{t-1} + \beta_3 x^t \ ; & c_2 \le \dfrac{Z^{t-1}}{x^t} \end{cases} \qquad (6.1)$$

Where y represents the maximum amount of a single output that can be produced from flow input x^t and the level of stock input at the end of period t-1, Z^{t-1}. Then 10 technologies of this kind with different parameters are considered. These technologies are labelled TEC1 - TEC10 as in Table 6-2.

Table 6-2. Parameters in different technologies of type (6.1)

Technology	α_1	β_1	α_2	β_2	α_3	β_3	c_1	c_2
TEC1	9	1	6	3	3	9	0.67	2
TEC2	12	1	6	5.02	3	11	0.67	2
TEC3	9	1.5	6	3	4	10	0.5	3.5
TEC4	5	9	7	8	9	1	0.5	3.5
TEC5	2	8	2.75	3	3	2.5	0.67	2
TEC6	3	6	3	6	1	10	0.67	2
TEC7	8	3	5	4.5	6	1	0.5	3.5
TEC8	6	8	3	9.5	2	13	0.5	3.5
TEC9	3	12	6	10	10	2	0.67	2
TEC10	7	5	2	4	4	0.5	0.5	3.5

For example technology TEC1 is as follows;

$$y(t) = f(x,z,t) = \begin{cases} 9Z^{t-1} + x^t \ ; & 0 \le \dfrac{Z^{t-1}}{x^t} \le 0.67 \\ 6Z^{t-1} + 3x^t \ ; & 0.67 \le \dfrac{Z^{t-1}}{x^t} \le 2 \\ 3Z^{t-1} + 9x^t \ ; & 2 \le \dfrac{Z^{t-1}}{x^t} \end{cases} \qquad (6.2)$$

The parameters in each TEC have been selected so as to maintain the continuity of the production function. As in Banker, Chang and Cooper (1996) the value $0 \le e^t_j \le 1$ is used to represent the efficiency associated with observation j at period t, so that

$$\hat{y}^t_j = y^t_j \times e^t_j \ ; \ j=1,2,...,n \qquad (6.3).$$

$\hat{y}^t_j$ is the efficient output level in line with the underlying technology in (6.1).

Therefore for each observation we have

$$\hat{y}^t_j \le y^t_j \ .$$

which accords with the characterisation of y^t_j as always being the maximal amount obtained from utilised values of x_j and Z_j. True efficiency figures in e^t_j will provide the benchmark against which the performance of static and dynamic models can be judged.

The efficiencies e_j^t used are such that for 10% of DMUs output levels are exactly as the technology would predict and the DMUs are efficient over all periods (i.e. $e_j^t = 1$, t=1,..., 15). Overall 25% of the DMUs are efficient in each period. However 60% of these efficient DMUs differ in general from one period to the next.

Mean and standard deviation of efficiency rate of DMUs in each period are as in Table 6-3. The efficiencies generated are listed in Table A4 (Appendix A).

Table 6-3. Mean and stdv. of true efficiency (e)

Period	Mean	Stdv.
t1	0.87	0.11
t2	0.88	0.10
t3	0.87	0.11
t4	0.88	0.10
t5	0.88	0.11
t6	0.87	0.11
t7	0.87	0.11
t8	0.86	0.11
t9	0.87	0.11
t10	0.87	0.10
t11	0.87	0.12
t12	0.88	0.11
t13	0.87	0.11
t14	0.88	0.11
t15	0.86	0.11

The 10 technologies in Table 6-2 have been selected to secure a mixed impact of stock and flow input. They can be classified in 3 groups as follows.

⇒ Group 1: Technologies (TEC1 - TEC5, TEC9 and TEC10) have output levels in which the impact of stock and flow input vary depending on the ratio of stock to flow input as in Table 6-2;

⇒ Group 2: Technologies TEC6 and TEC8 have outputs which are highly impacted by flow input. The flow input coefficients are much bigger than those of stock input and both inputs are measured in the same units. Compare, for example in TEC6, the coefficients of current inputs are (3, 3, 1) against the coefficients of capital inputs which are (6, 6, 10). Obviously this technology is highly influenced by capital input than current input. The same is true for TEC8.

⇒ Group 3: Technology TEC7 has output which is highly impacted by stock input. The stock input coefficients are much bigger than those of flow input. Compare the coefficient of capital stock which are (3, 4.5, 1) against the coefficient of current input which are (8, 5, 6). The technology shows it is dominated by current input with a very little impact by capital.

We are intended to show that for technologies in group 1 and 3 static DEA models must perform worse than for technologies in group 2 as static DEA models do not reflect well stock input. Therefore it is expected that for technologies in group 1 and 3 dynamic DEA models should perform better than static DEA models.

6.2.1 Static efficiency scores for simulated data

The static DEA efficiencies were obtained by solving the CRS DEA model in each period and for each DMU. The inputs are two contemporaneous, flow and stock and there is only one output.

The average of static DEA efficiencies across all DMUs and for each time period are presented in the second row of Tables B1a - B10a (Appendix A) respectively for technologies TEC1 - TEC10. For example, in Table B1a (Appendix A) the mean of 0.5798 under t4 in the row labelled "static" is the average obtained over all DMUs in period t4 using the static DEA efficiency model. The averages of absolute deviations of static DEA from true efficiency are shown in the first row of Tables B1b - B10b (Appendix A) for technologies TEC1 - TEC10 respectively.

6.2.2 Dynamic efficiency scores for simulated data

Dynamic efficiencies have been computed using Model 5-5. In this model there are three types of input; flow input, stock input and stock change. As noted in Chapter 5 the length of the assessment window used and its relationship with the lag in inter - temporal effects involved will affect the results in the assessment. As can be seen in expression (6.1) the lag in the technologies used is one period. Thus any assessment window of length of two or more periods will be sufficient. To investigate the impact of length of window used we solve dynamic efficiency model using windows of length from

2 to 15 periods. Dyn-2 will be used to denote the measurement of dynamic efficiency with a window of length of two periods and so on for Dyn-3,..., Dyn-15.

Table 6-4. Mean efficiency for replication in 15 periods for 100 DMUs

	TEC1	TEC2	TEC3	TEC4	TEC5	TEC6	TEC7	TEC8	TEC9	TEC10	Average
True	0.851	0.851	0.851	0.851	0.851	0.851	0.851	0.851	0.851	0.851	0.851
Static	0.618	0.641	0.605	0.625	0.721	0.717	0.615	0.738	0.638	0.674	0.659
Dyn-2	0.765	0.777	0.757	0.767	0.804	0.816	0.754	0.827	0.743	0.78	0.779
Dyn-3	0.843	0.851	0.837	0.827	0.853	0.869	0.829	0.875	0.811	0.842	0.844
Dyn-4	0.887	0.892	0.88	0.86	0.881	0.898	0.87	0.9	0.849	0.875	0.879
Dyn-5	0.93	0.936	0.923	0.908	0.924	0.939	0.915	0.94	0.895	0.92	0.923
Dyn-6	0.954	0.959	0.948	0.968	0.945	0.958	0.94	0.959	0.921	0.943	0.949
Dyn-7	0.972	0.975	0.966	0.976	0.961	0.972	0.959	0.972	0.942	0.96	0.966
Dyn-8	0.983	0.986	0.979	0.969	0.973	0.982	0.972	0.982	0.958	0.972	0.976
Dyn-9	0.991	0.993	0.987	0.967	0.983	0.989	0.983	0.989	0.971	0.982	0.984
Dyn-10	0.995	0.996	0.992	0.977	0.989	0.993	0.989	0.993	0.981	0.989	0.989
Dyn-11	0.997	0.997	0.995	0.985	0.992	0.995	0.992	0.995	0.986	0.992	0.993
Dyn-12	0.998	0.998	0.997	0.997	0.995	0.997	0.995	0.997	0.991	0.995	0.996
Dyn-13	0.999	0.999	0.998	0.998	0.997	0.998	0.997	0.998	0.994	0.997	0.997
Dyn-14	0.999	1	0.998	0.998	0.997	0.998	0.997	0.998	0.995	0.997	0.998
Dyn-15	1	1	0.999	0.999	0.998	0.999	0.998	0.999	0.995	0.998	0.998

The averages of dynamic DEA efficiency models Dyn-2 to Dyn-15 are presented in the row labelled "Dyn-2" to "Dyn-15" in Tables B1a - B10a (Appendix A) respectively for technologies TEC1 - TEC10. The average efficiencies have been computed for each window over all DMUs. For example, in Table B1a the mean of 0.8577 in the row labelled "Dyn-3", under t4, is the average obtained over all DMUs from the dynamic efficiency model associated with data generated using technology TEC1 and for window of length of 3 periods (i.e. Periods of t2, t3 and t4).

The averages of absolute deviations of dynamic DEA efficiencies from true efficiencies are shown in the rows labelled "Dyn-2" to "Dyn-15" in Tables B1b - B10b (Appendix A) for technologies TEC1 - TEC10.

The overall averages of static and dynamic efficiency for all technologies TEC1 - TEC10 in each period are summarised in Table 6-4. In this table, for example, 0.843 under TEC1 in the row labelled "Dyn-3" is the mean dynamic efficiency of all DMUs when assessed in windows of 3 periods (t1, t2 and t3), (t2, t3 and t4), ..., (t13, t14 and t15).

6.2.3 Analysis of the results across all technologies

We will compare dynamic with static DEA efficiency scores by reference to mean efficiencies and mean absolute deviations from true efficiencies for each technology.

The results of static DEA vary in accuracy. For example in technology TEC3 the mean static efficiency is very far from its true mean while in technology TEC8 the mean static efficiency is closer to true mean efficiency among the static DEA results. Figure 6-1 conveys this information pictorially.

Figure 6-1. The difference between static DEA and true mean efficiency

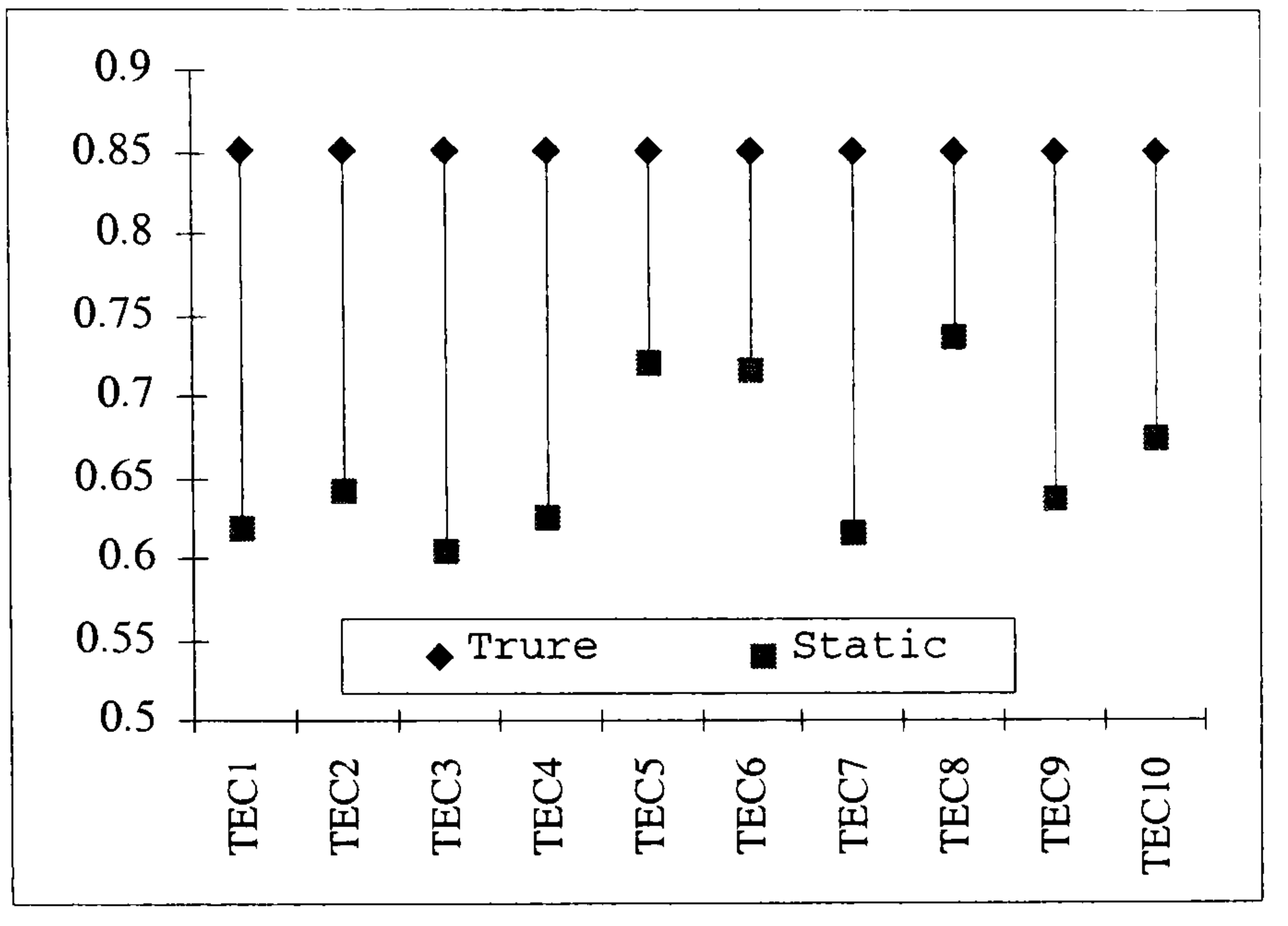


Figure 6-1 shows that the static DEA efficiency performs better in technologies

TEC8, TEC5, TEC6, TEC10, TEC2, TEC9, TEC4, TEC1, TEC7, TEC3 (6.4)

in that order.

This suggests that:

- Having the higher inter - temporal input - output dependence in the technology reduces the accuracy of efficiency estimated in static DEA models. Such technologies were TEC3 in group 1 and TEC7 in group 3. For example in TEC7 output is highly impacted by stock input (see the coefficients of stock and flow inputs of TEC7 in Table 6-2). The difference between static and true efficiency in TEC7 can be readily seen from Figure 6-1 where:

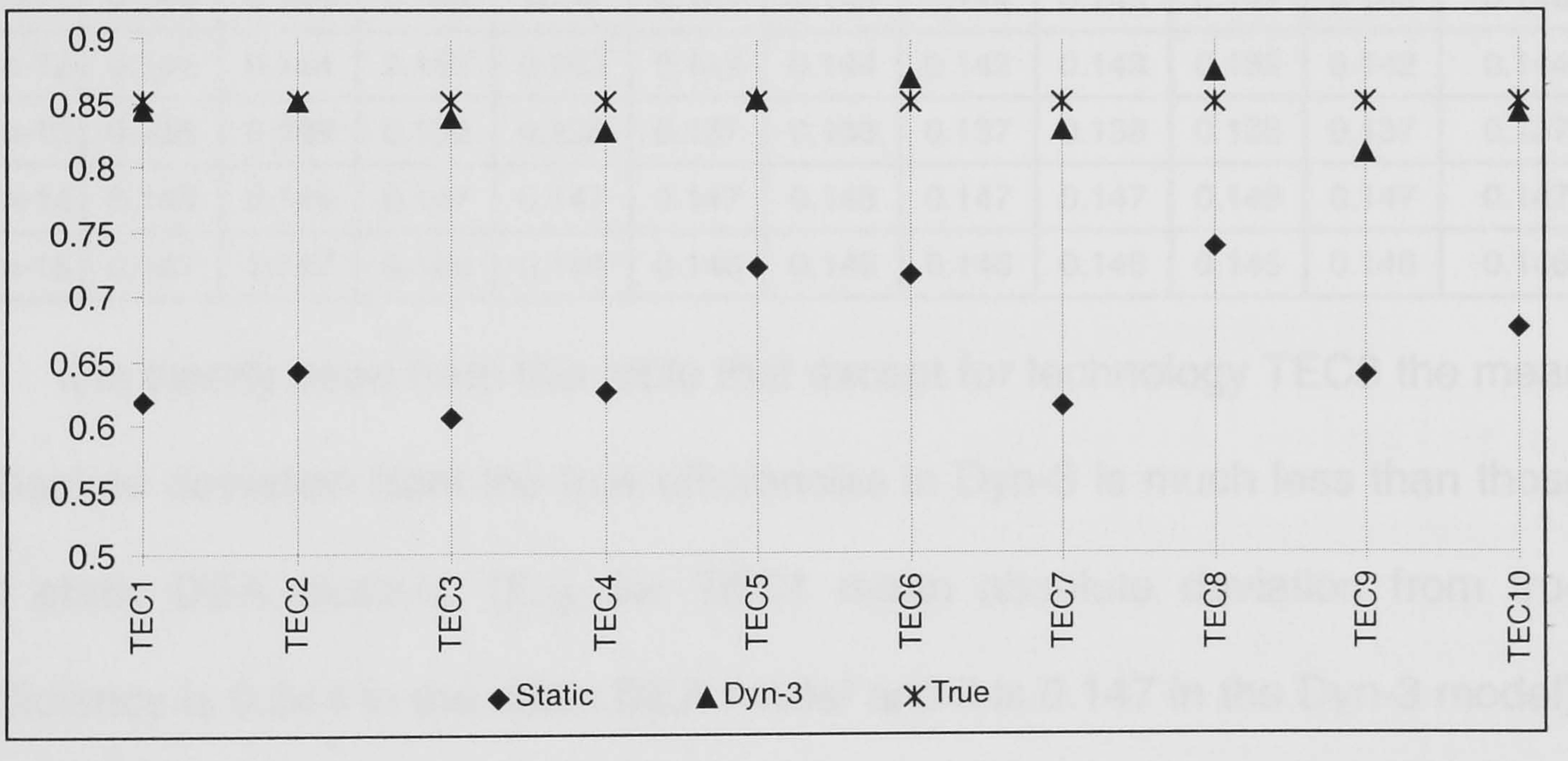
- The overall mean efficiency in static DEA is 0.615 (see column TEC7 in the row labelled "Static" in Table 6-4) while
- The true mean efficiency is 0.851 (see column TEC7 in the row labelled "True" in Table 6-4).

- Having lower inter - temporal input - output dependence improves the accuracy of efficiency in static DEA models. Technologies with lower inter - temporal input - output dependence are TEC6 and TEC8 in group 2. For example output in TEC8 is highly impacted by flow input (see coefficients of stock and flow inputs of TEC8 in Table 6-2). For TEC8:
  - The overall mean efficiency in static DEA is 0.738 (see column TEC8 in the row labelled "Static" in Table 6-4) while
  - The true mean efficiency is 0.851 (see column TEC8 in the row labelled "True" in Table 6-4).

It is evident in Table 6-4 that Dyn-3 captures true performance better than the other dynamic models. For the time being it might be noted that the technologies of scenario (I) have maximum lag between stock input and output of 3 periods.

Figure 6-2 illustrates this for each technology comparing static and Dyn-3 against the true performance. The figure clearly shows that the Dyn-3 efficiency model captures true performance better than the static efficiency model.

Figure 6-2. Average efficiency in simulation (I) for replication in technologies TEC1 through TEC10



Further evidence of the comparative performance of static and dynamic DEA models is provided by the mean absolute deviations between true and computed efficiencies. These results are summarised in Table 6-5 (Dyn-x efficiencies used are as in Table 6-4.)

Table 6-5. Mean absolute deviation from true efficiency

	TEC1	TEC2	TEC3	TEC4	TEC5	TEC6	TEC7	TEC8	TEC9	TEC10	Average
Static	0.244	0.221	0.255	0.255	0.138	0.145	0.244	0.122	0.222	0.185	0.203
Dyn-2	0.175	0.167	0.177	0.157	0.137	0.135	0.172	0.131	0.172	0.149	0.157
Dyn-3	0.147	0.143	0.145	0.135	0.124	0.125	0.14	0.123	0.141	0.129	0.135
Dyn-4	0.128	0.127	0.127	0.137	0.121	0.121	0.125	0.121	0.127	0.122	0.126
Dyn-5	0.128	0.128	0.125	0.145	0.115	0.118	0.12	0.118	0.119	0.116	0.123
Dyn-6	0.13	0.131	0.128	0.143	0.122	0.125	0.124	0.125	0.123	0.122	0.127
Dyn-7	0.132	0.133	0.13	0.11	0.127	0.13	0.127	0.13	0.124	0.127	0.127
Dyn-8	0.139	0.14	0.136	0.126	0.131	0.136	0.132	0.135	0.127	0.131	0.133
Dyn-9	0.142	0.143	0.14	0.15	0.137	0.14	0.137	0.14	0.133	0.137	0.14
Dyn-10	0.141	0.142	0.14	0.16	0.138	0.141	0.139	0.14	0.136	0.138	0.141
Dyn-11	0.146	0.146	0.145	0.16	0.143	0.145	0.144	0.145	0.141	0.143	0.146
Dyn-12	0.144	0.144	0.143	0.153	0.142	0.144	0.142	0.143	0.139	0.142	0.144
Dyn-13	0.138	0.139	0.138	0.138	0.137	0.138	0.137	0.138	0.135	0.137	0.137
Dyn-14	0.148	0.148	0.147	0.147	0.147	0.148	0.147	0.147	0.146	0.147	0.147
Dyn-15	0.147	0.147	0.146	0.146	0.146	0.146	0.146	0.146	0.145	0.146	0.146

It is clearly seen from this table that except for technology TEC8 the mean absolute deviation from the true efficiencies in Dyn-3 is much less than those of static DEA models. (E.g. for TEC1 mean absolute deviation from true efficiency is 0.244 in the static DEA model and it is 0.147 in the Dyn-3 model). One reason why static DEA is performing better in technology TEC8 is that stock input has very little impact on output compared to flow input (see coefficients of flow and stock inputs of TEC8 in Table 6-2).

Clearly the strength of impact of stock input is very important for the accuracy of static efficiency measurement in DEA. The simulation shows that if the impact of stock is very high then the static efficiency fails to capture the true performance of DMUs while the dynamic DEA model captures true performance better.

6.2.4 Analysis of the results on a selected technology

Table 6-6 presents results from the simulation of technology TEC1. The efficiency means obtained from the static DEA and dynamic DEA models over 15 periods and different lengths of window are shown. To assess the impact of the length of the window in dynamic efficiency the model was solved for different lengths of window and the results in Table 6-6 are based on lengths of 2 to 15 periods. For this technology the estimates from dynamic DEA Dyn-3 are better than from other lengths of window.

Table 6-6. Mean efficiency scores in simulation (I), technology TEC1

	T1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.852	0.760	0.599	0.580	0.612	0.572	0.606	0.545	0.562	0.566	0.569	0.612	0.625	0.585	0.631	0.618
Dyn-2		0.852	0.858	0.855	0.757	0.740	0.757	0.725	0.760	0.702	0.728	0.751	0.735	0.746	0.748	0.765
Dyn-3			0.852	0.858	0.899	0.902	0.834	0.825	0.837	0.823	0.851	0.808	0.812	0.830	0.831	0.843
Dyn-4				0.852	0.858	0.899	0.928	0.924	0.884	0.890	0.892	0.880	0.905	0.869	0.866	0.887
Dyn-5					0.941	0.946	0.924	0.927	0.932	0.926	0.945	0.916	0.916	0.924	0.936	0.930
Dyn-6						0.957	0.963	0.952	0.947	0.956	0.958	0.966	0.947	0.943	0.952	0.954
Dyn-7							0.972	0.976	0.969	0.969	0.972	0.978	0.977	0.965	0.967	0.972
Dyn-8								0.983	0.986	0.981	0.982	0.986	0.985	0.986	0.979	0.983
Dyn-9									0.991	0.992	0.990	0.989	0.992	0.993	0.992	0.991
Dyn-10										0.995	0.995	0.995	0.992	0.993	0.996	0.995
Dyn-11											0.997	0.998	0.998	0.994	0.996	0.997
Dyn-12												0.998	0.999	0.999	0.996	0.998
Dyn-13													0.999	0.999	0.999	0.999
Dyn-14														0.999	0.999	0.999
Dyn-15															1.000	1.000

The mean absolute deviations between true and estimated efficiency parallel the performance of the mean values. The mean absolute deviations are shown Table 6-7.

Table 6-7. Mean absolute deviation between true and estimated efficiency in simulation (I), technology TEC1

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.009	0.099	0.266	0.289	0.236	0.281	0.26	0.316	0.291	0.304	0.284	0.234	0.269	0.277	0.243	0.244
Dyn-2		0.137	0.115	0.117	0.167	0.19	0.172	0.192	0.164	0.231	0.215	0.182	0.207	0.175	0.189	0.175
Dyn-3			0.135	0.114	0.118	0.129	0.156	0.144	0.138	0.157	0.158	0.162	0.17	0.157	0.166	0.147
Dyn-4				0.127	0.126	0.131	0.125	0.101	0.119	0.13	0.137	0.14	0.113	0.153	0.131	0.128

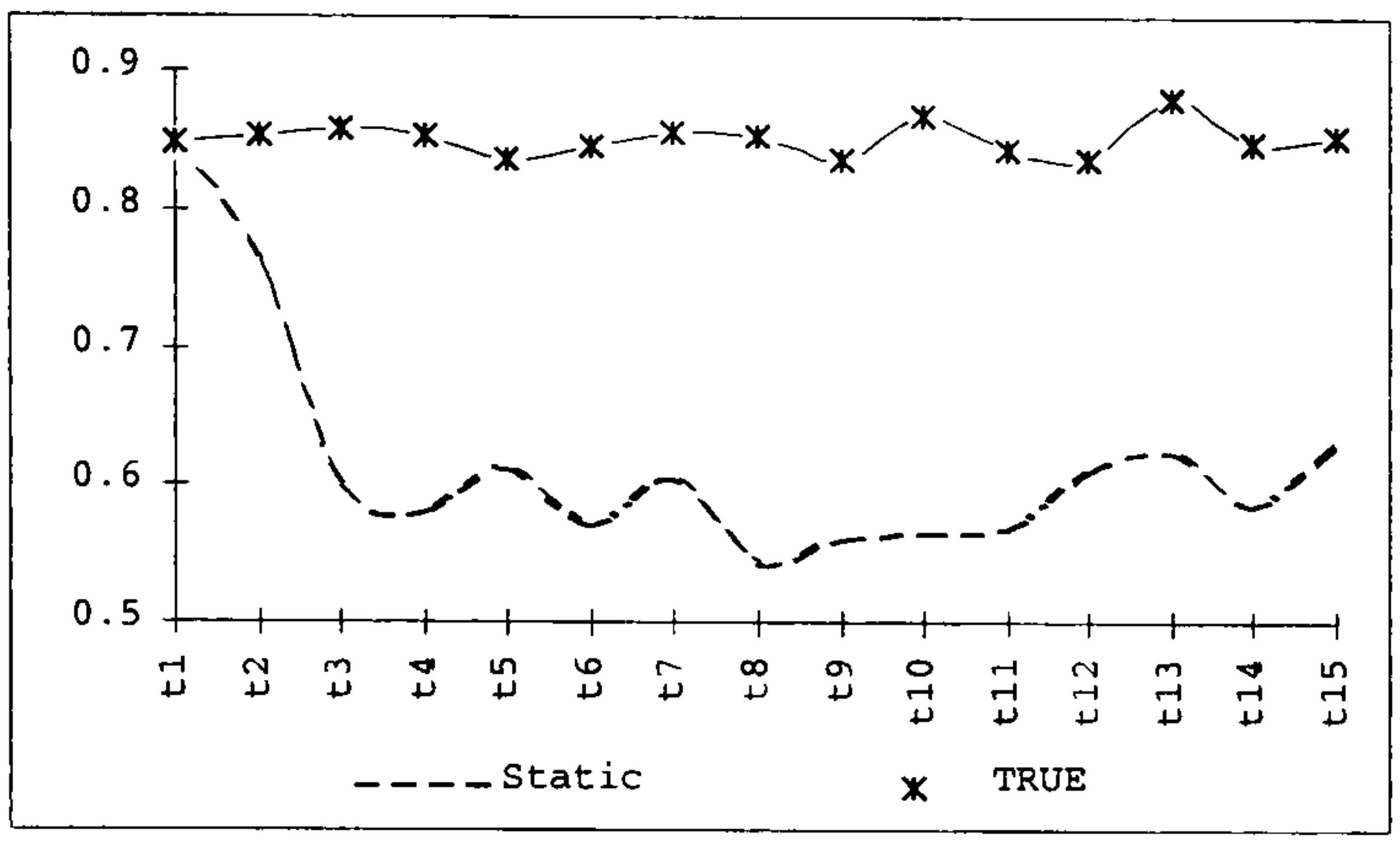
Figure 6-3 shows graphically the results in Table 6-7. Figure 6-3a shows that static efficiency always underestimates true efficiency.

The use of paths in dynamic efficiency reflects any output resulting from earlier stock input. The dynamic efficiencies of window with length 2, 3, and 4 are illustrated in Figure 6-3b, Figure 6-3c and Figure 6-3d respectively.

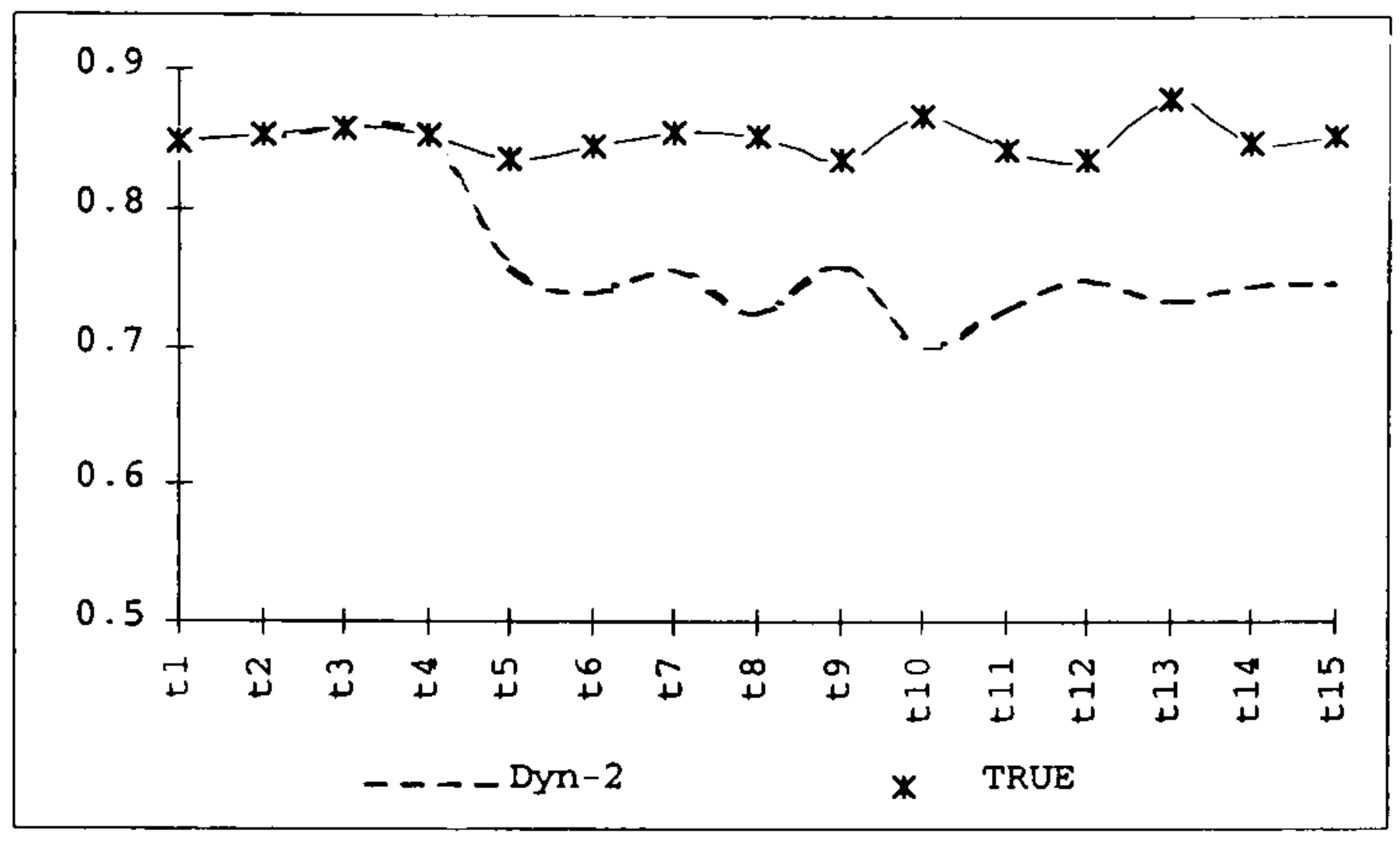
Dynamic efficiency of window with length 3 more or less matches true performance. This is because in this technology DMUs take 3 periods to adjust to stock level changes. That is all stock input changes up to and in period t-2, make up stock input Z^{t-1} which impacts output in period t. Thus the lag between accumulated stock change at the end of t-2 and period t is 3 periods.

Figure 6-3. Mean efficiency results from simulation (I) in technology TEC1

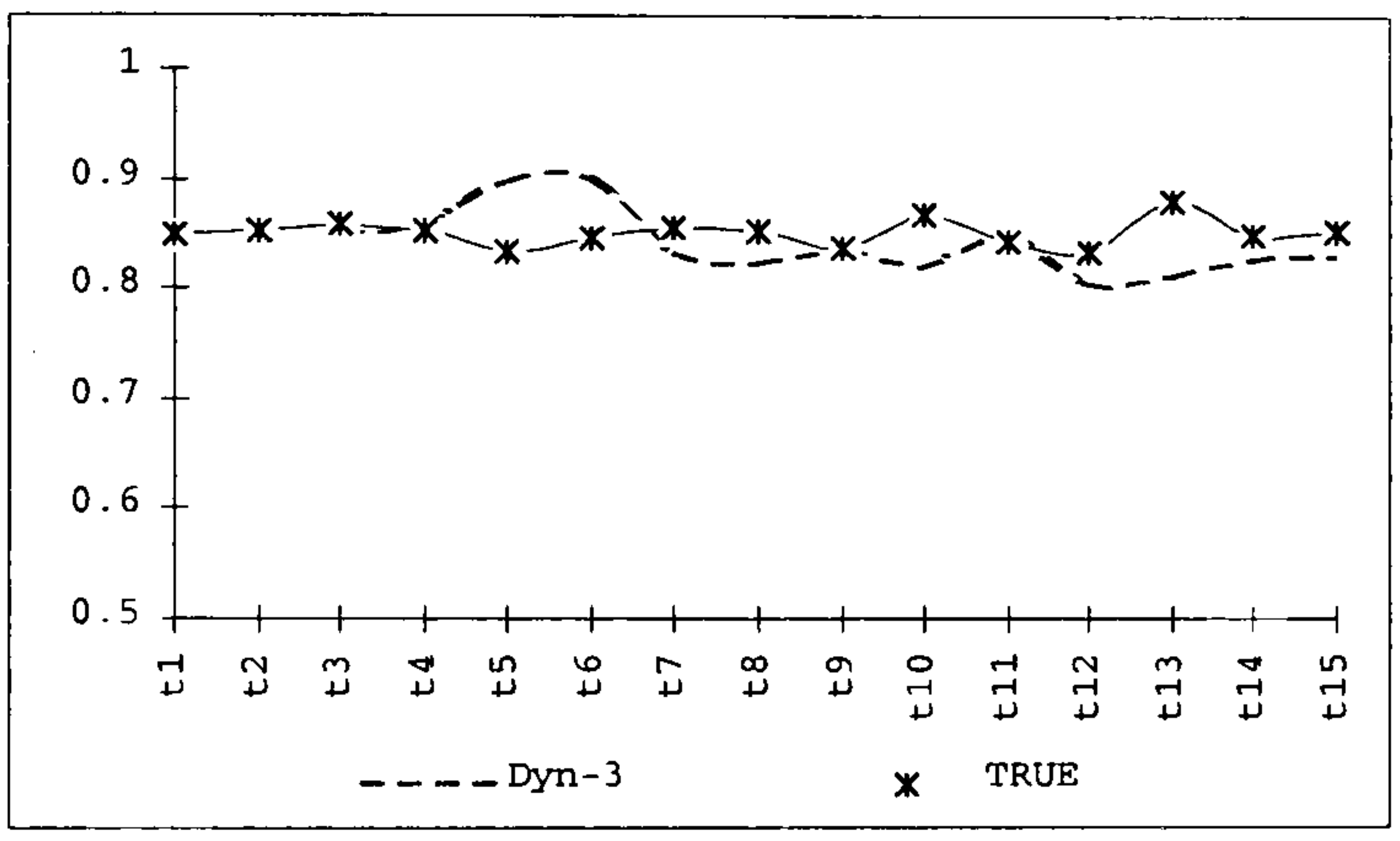
a) Static against True efficiencies



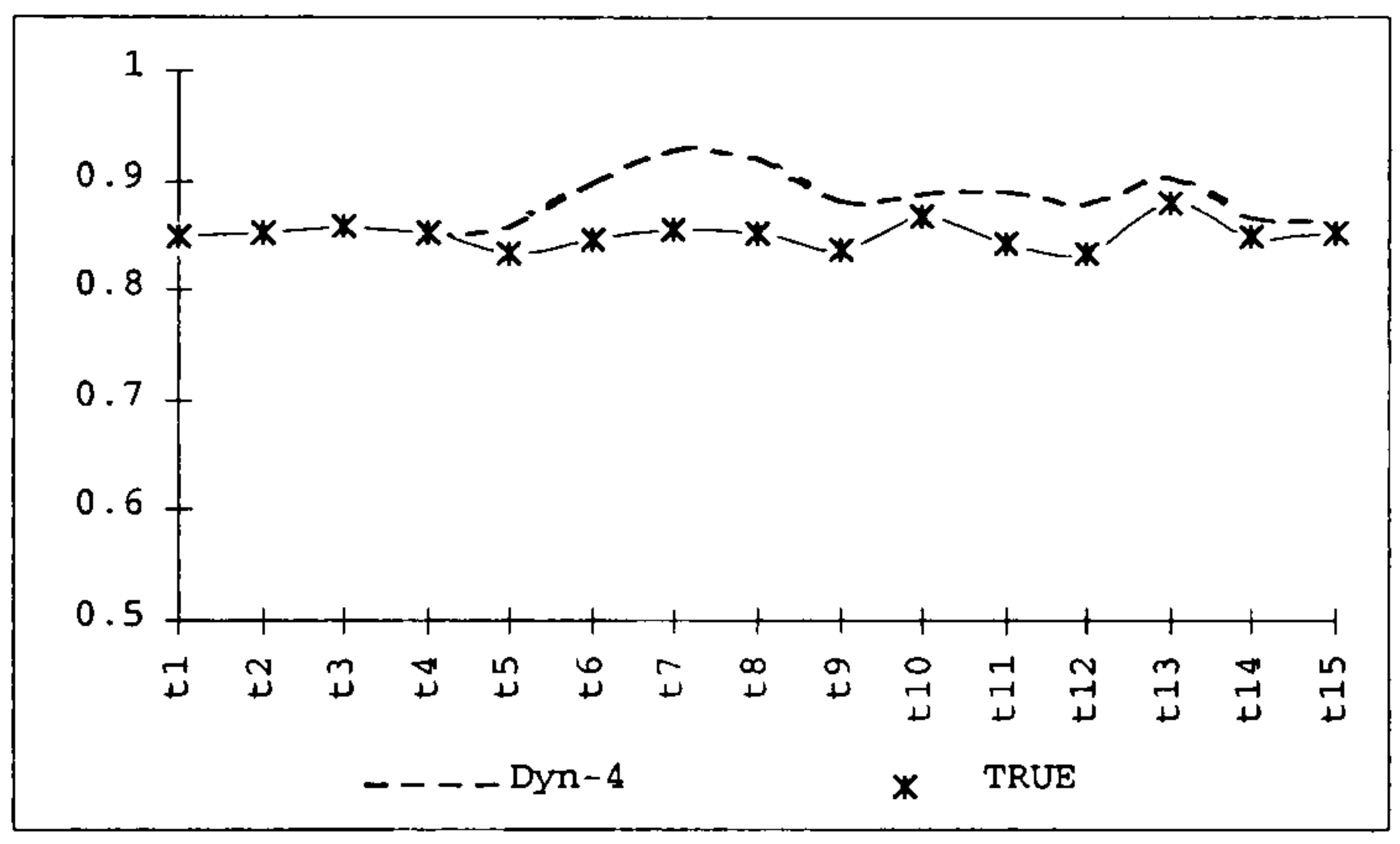
b) Dyn-2 model against True efficiencies



c) Dyn-3 model against True efficiencies



d) Dyn-4 model against True efficiencies

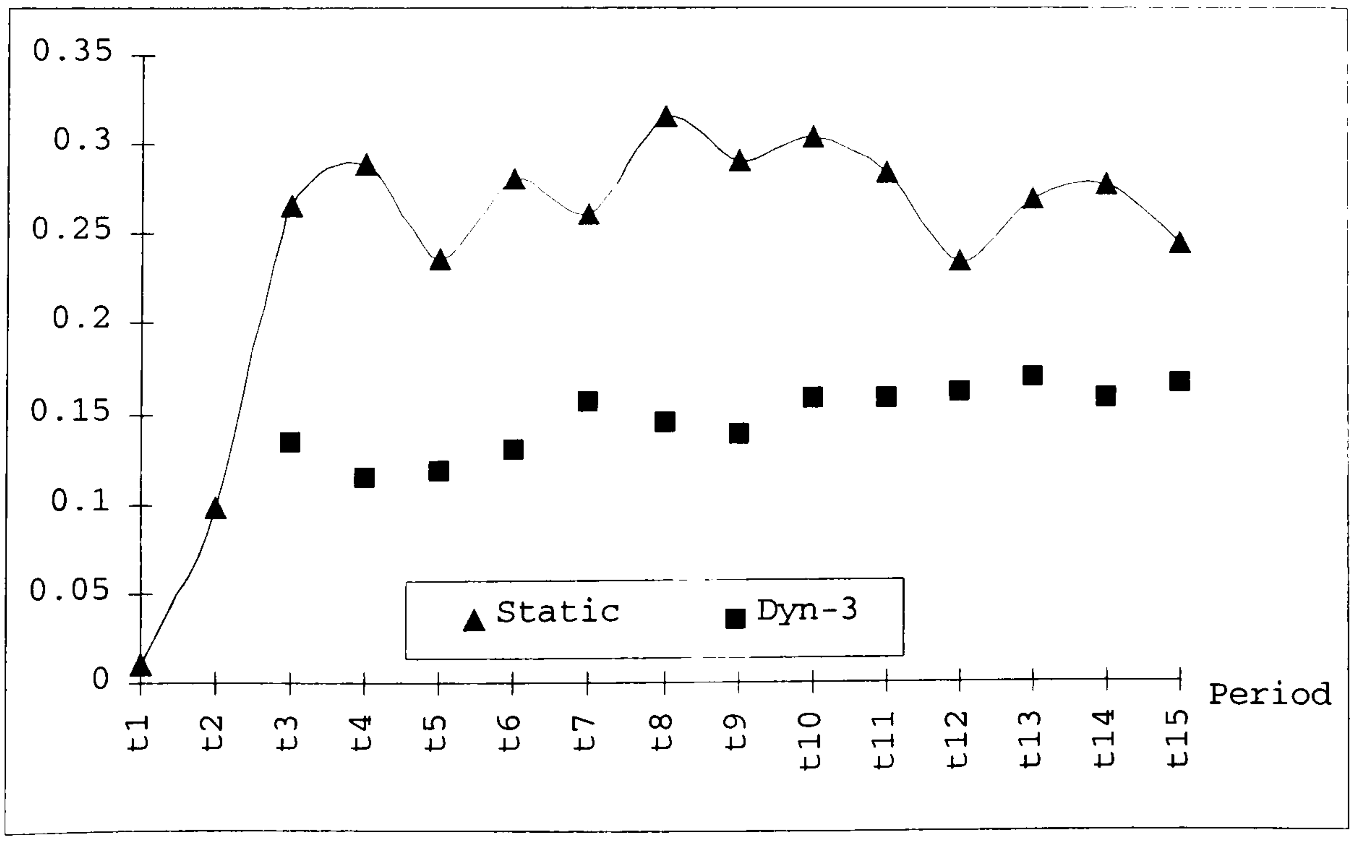


However Figure 6-3d and Table 6-4 show that dynamic efficiency results with length of 4^{+} (more than 4) always overestimate efficiency scores. It is evident in Table 6-4 that dynamic efficiency scores in the larger windows (e.g. Dyn-14 and Dyn-15) approach 1. This is as we expect because when the length of the window increases the number of constraints in Model 5-5 increases which can only increase the optimal value of the objective function being minimised. Put another way, DMUs have more opportunity to appear efficient by having a “high” output level in at least one time period.

The mean absolute deviations from true efficiency shown in Table B1b in Appendix A are plotted in Figure 6-4. The Dyn-3 mean absolute deviations are consistently better than those of the static DEA efficiencies. The same is true for all technologies as can be seen in Tables B1b - B10b (Appendix A).

In conclusion, The results of scenario (I) show that the impact of inter - temporal input - output dependence is very important in efficiency measurement. The simulation shows that those technologies that are highly impacted by stock input are assessed especially inaccurately by static DEA. Dynamic DEA captures better the performance of DMUs. However, the length of window used in dynamic efficiency can impact the accuracy of the results obtained.

Figure 6-4. Mean absolute deviation from true efficiency, scenario (I) in technology TEC1



The next scenario examines the results obtained with single technology using different data sets.

6.3 Scenario II: Constant technology and varying input data

Our aim in this scenario is to investigate the impact of input - output paths profiles on the comparison of static and dynamic DEA efficiency models. For this purpose we use a single technology and vary the paths. We use a function taken from Banker, Chang and Cooper (1996) as our technology. It is a piece wise Cobb-Douglas function with inter - temporal effects and Constant Returns to Scale (see 6.5).

$$y = f(x,Z,t) = \begin{cases} 10(x^t)^{0.2}(Z^{t-1})^{0.8}, & 0 \le \dfrac{x^t}{Z^{t-1}} \le 0.4 \;, \\ 14.42(x^t)^{0.6}(Z^{t-1})^{0.4}, & 0.4 \le \dfrac{x^t}{Z^{t-1}} \le 1 \;, \\ 14.42(x^t)^{0.35}(Z^{t-1})^{0.65}, & 1 \le \dfrac{x^t}{Z^{t-1}} \le 3 \;, \\ 8.33(x^t)^{0.85}(Z^{t-1})^{0.15}, & 3 \le \dfrac{x^t}{Z^{t-1}} \;, \end{cases} \tag{6.5}$$

In (6.5) y represents the maximum amount of a single output that can be produced from flow input x^t and stock input Z^{t-1}. The parameters in this technology were selected to maintain continuity and Constant Returns to Scale. Some 10 different input sets are used.

In scenario (II) the value of the input variables, flow input x (in the range of [10, 100]) and stock change z (in the range of [10, 20]), are generated randomly and independly from uniform distributions.

At any point in time t, x^t and Z^t are regarded as exogenous and y^t is regarded as an endogenous variable since it is determined by the production technology. Once exogenous variables are known the production technology may be used to generate the endogenous variable y. Ten sets of exogenous variables x and Z were generated. The value of the output y for DMU j was then calculated using the following equation:

$$\hat{y}_j^t \text{ (x, Z)} = y_j^t \text{ (x, Z)} \times e_j^t \qquad ; j=1,2,...,n \quad (6.6)$$

where e_j^t is a residual term and it stands for the true efficiency for DMU j in period t.

For generating true efficiencies the same method was used with the same distributions as in scenario (I). The true efficiency is generated in the range of [0.30,1] using the uniform distribution with a mean of 0.75. Table A5 (Appendix A) shows the efficiencies generated. The 10 data sets used are denoted SET1 to SET10.

6.3.1 Analysis of the results

As in scenario (I), the static DEA efficiencies in each SET were obtained by solving the CRS DEA model in each period and for each DMU using two contemporaneous inputs, flow and stock, and one output. The dynamic efficiency scores were obtained from dynamic efficiency Model 5-5 associated with three inputs, flow input, stock input and stock change input.

The mean efficiencies across all DMUs and for each assessment window in scenario (II) are summarised in Tables C1 - C10 (Appendix A) respectively for SET1 - SET10.

The average of static DEA efficiency across all DMUs and for each time period are presented in the second row of Tables C1 – C10 (Appendix A) respectively for SET1 – SET10. For example in Table C2 the mean of 0.719 under t4 in the row labelled "Static" is the average obtained over all DMUs solving the static DEA efficiency model for data at period t4 in SET2. Dynamic efficiency results were computed using Model 5-5 and it was solved for window of length of 2 to 15 periods. The results are summarised in Table C1 – C10 (Appendix A) in the rows labelled "Dyn-2" to "Dyn-15". For example, in Table C2 the mean of 0.712 under t4 and in the row labelled "Dyn-3" is the average efficiency obtained over all DMUs using the dynamic efficiency model with length of 3 periods on the input - output levels in SET2. The overall mean efficiencies obtained for each set of 100 DMUs are summarised in Table 6-8 bellow.

Table 6-8. Mean efficiency of 10 sets SET1-SET10 of 100 DMUs, over 15 periods

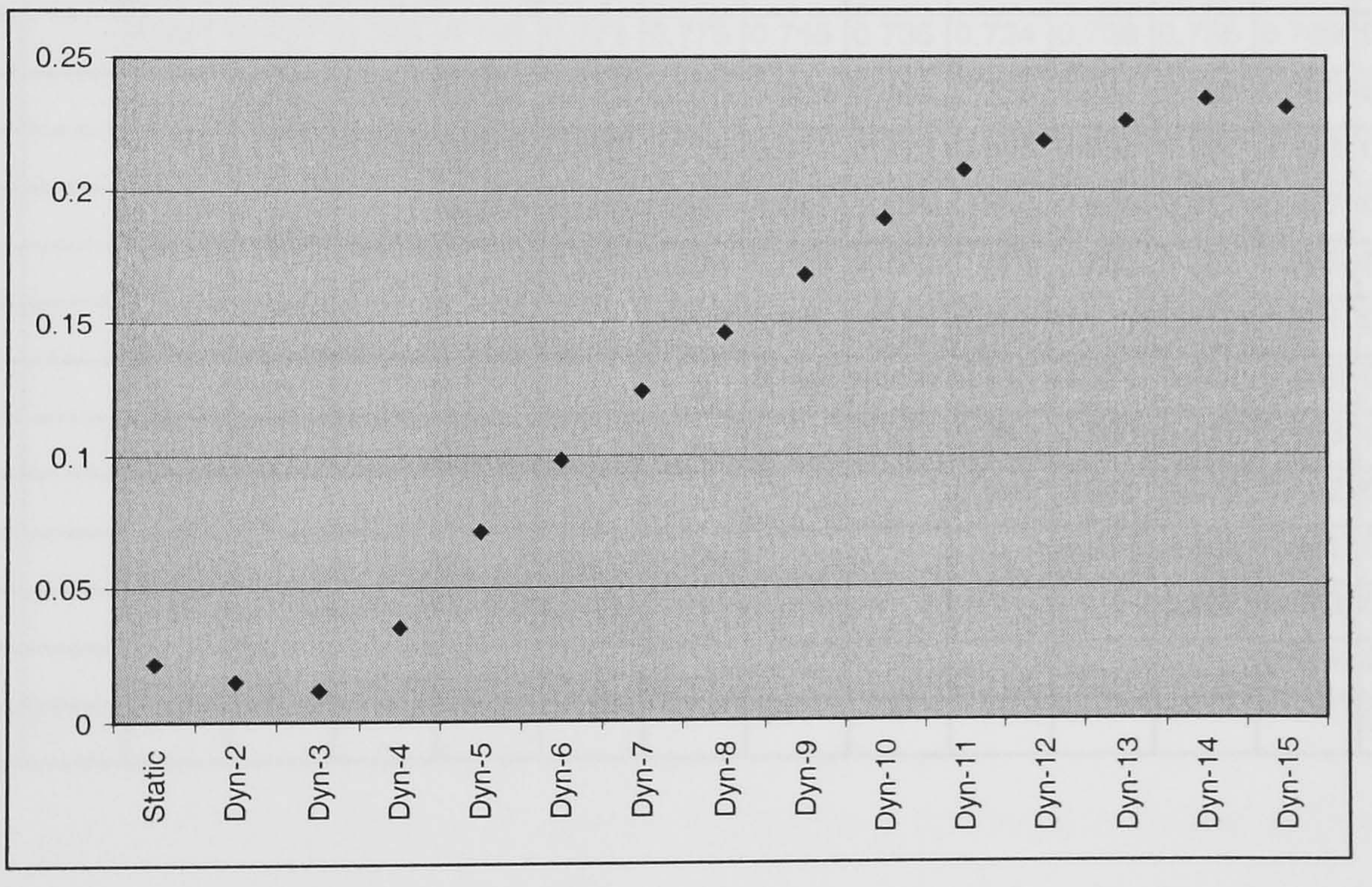
	SET1	SET2	SET3	SET4	SET5	SET6	SET7	SET8	SET9	SET10	Average
TRUE	0.734	0.734	0.734	0.734	0.734	0.734	0.734	0.734	0.734	0.734	0.734
Static	0.713	0.712	0.710	0.715	0.712	0.709	0.714	0.713	0.712	0.714	0.712
Dyn-2	0.722	0.717	0.718	0.720	0.718	0.719	0.721	0.717	0.718	0.720	0.719
Dyn-3	0.746	0.742	0.743	0.749	0.743	0.742	0.749	0.745	0.750	0.750	0.746
Dyn-4	0.774	0.761	0.763	0.774	0.762	0.765	0.774	0.766	0.773	0.777	0.769
Dyn-5	0.811	0.798	0.803	0.808	0.799	0.802	0.809	0.804	0.807	0.810	0.805
Dyn-6	0.837	0.825	0.830	0.840	0.826	0.826	0.838	0.829	0.837	0.834	0.832
Dyn-7	0.865	0.849	0.854	0.858	0.850	0.856	0.863	0.858	0.863	0.864	0.858
Dyn-8	0.887	0.872	0.875	0.887	0.872	0.879	0.886	0.875	0.885	0.884	0.880
Dyn-9	0.906	0.896	0.900	0.902	0.896	0.899	0.905	0.900	0.906	0.909	0.902
Dyn-10	0.929	0.916	0.921	0.925	0.916	0.925	0.926	0.917	0.927	0.928	0.923
Dyn-11	0.941	0.934	0.939	0.948	0.934	0.939	0.945	0.938	0.943	0.947	0.941
Dyn-12	0.955	0.947	0.951	0.951	0.947	0.953	0.954	0.949	0.951	0.957	0.952
Dyn-13	0.965	0.958	0.961	0.961	0.958	0.964	0.962	0.962	0.956	0.956	0.960
Dyn-14	0.968	0.964	0.967	0.968	0.964	0.973	0.968	0.967	0.973	0.969	0.968
Dyn-15	0.965	0.965	0.968	0.959	0.965	0.970	0.968	0.968	0.969	0.958	0.965

In Table 6-8, for example, 0.761 under SET2 in the row labelled "Dyn-4" is the mean dynamic efficiency of all DMUs when assessed in windows of 4 periods (t1, t2, t3, t4), (t2, t3, t4, t5), ..., (t12, t13, t14, t15).

A look at the average efficiencies in Table 6-8 shows that dynamic efficiency with a window of 3 periods again performs better than all other DEA models. It is readily seen from Table 6-8 that the dynamic efficiency with longer windows (above 6 or 7) overestimates the true efficiency while the static DEA model underestimates the true efficiency. This is parallel with what we obtained in scenario (I).

Figure 6-5 shows the mean absolute deviation between true and computed DEA efficiencies for all 10 data sets and 15 periods. In this figure it is clearly shown that the mean absolute deviation is lowest in Dyn-3. This highlights the importance in dynamic DEA of choosing assessment windows with appropriate length.

Figure 6-5. The overall mean of absolute deviation from true efficiency across all DMUs in scenario (II)



6.3.2 Analysis of the impact in a selected SET

If we know the lag between input and output the length of window we use in the dynamic DEA model can be set to capture it.

Take for example SET1. Results of the first run for static efficiency and dynamic efficiency models are summarised in Table 6-9. This run indicates that dynamic efficiency with window of length of 2 and 3 are much closer to the true performance. However it is clear from Table 6-9 that static DEA model and Dyn-2 underestimates the efficiency score while Dyn-4 overestimates the efficiency scores.

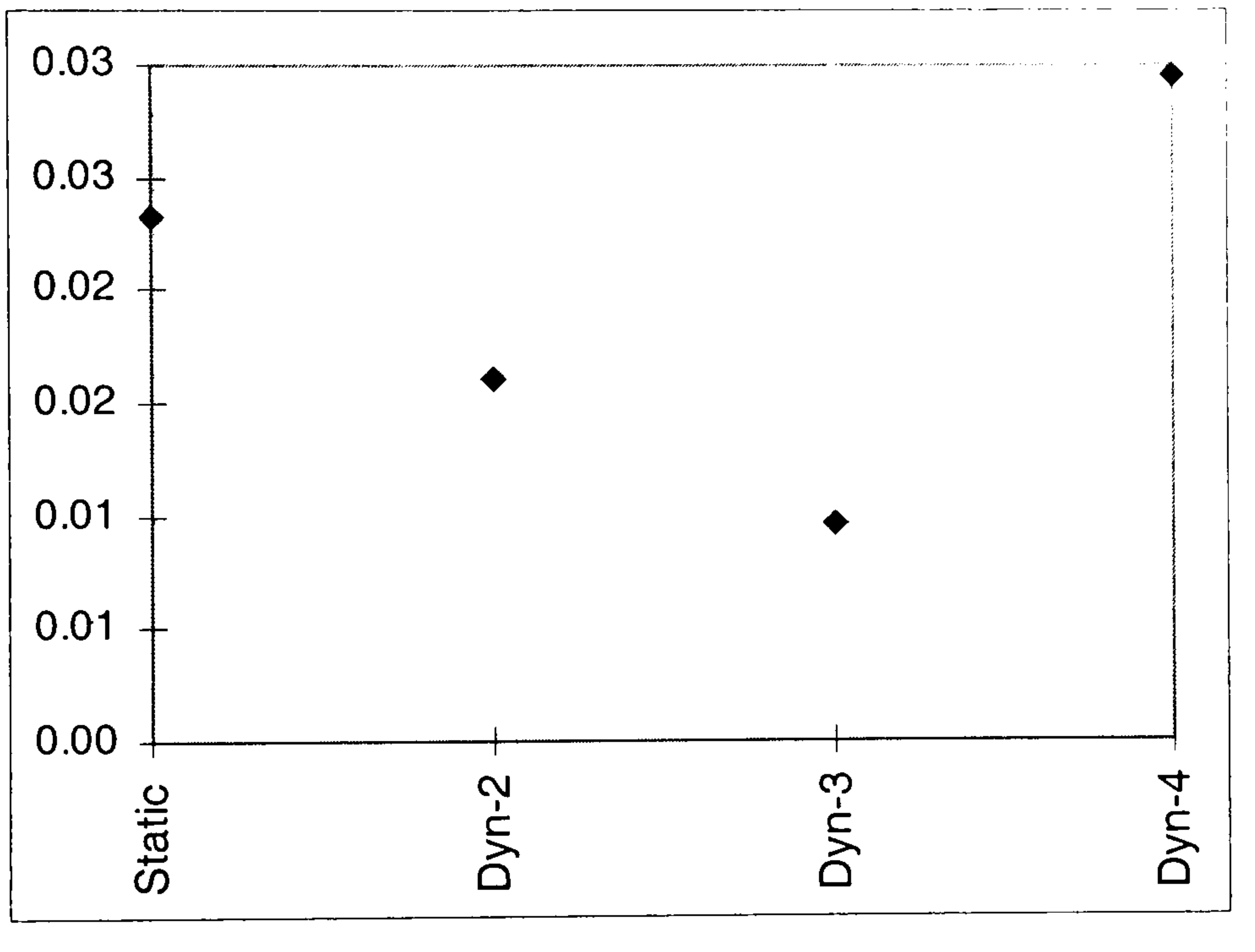
Table 6-9. Average efficiency in scenario (II) for data set SET1

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.737	0.701	0.729	0.737	0.730	0.721	0.710	0.684	0.687	0.718	0.725	0.732	0.665	0.693	0.732	0.713
Dyn-2		0.745	0.730	0.719	0.722	0.742	0.753	0.726	0.701	0.698	0.732	0.738	0.735	0.685	0.675	0.722
Dyn-3			0.741	0.727	0.743	0.735	0.771	0.775	0.755	0.736	0.724	0.738	0.756	0.769	0.724	0.746
Dyn-4				0.741	0.736	0.757	0.779	0.775	0.804	0.809	0.786	0.767	0.763	0.780	0.786	0.774
Dyn-5					0.823	0.826	0.837	0.836	0.811	0.815	0.815	0.805	0.797	0.793	0.765	0.811
Dyn-6						0.853	0.858	0.857	0.841	0.837	0.840	0.838	0.817	0.817	0.817	0.837
Dyn-7							0.882	0.874	0.869	0.876	0.875	0.880	0.857	0.853	0.822	0.865
Dyn-8								0.908	0.886	0.888	0.899	0.912	0.873	0.871	0.859	0.887
Dyn-9									0.903	0.903	0.915	0.922	0.912	0.898	0.886	0.906
Dyn-10										0.927	0.941	0.939	0.918	0.943	0.904	0.929
Dyn-11											0.948	0.945	0.939	0.942	0.931	0.941
Dyn-12												0.959	0.955	0.961	0.946	0.955
Dyn-13													0.967	0.973	0.956	0.965
Dyn-14														0.971	0.966	0.968
Dyn-15															0.965	0.965

All results obtained from 10 different data sets using technology (6.5) are similar with what was obtained in the first run. These results appear in Tables C1 - C10 (Appendix A).

The absolute deviation of average from true efficiencies in the first data set in simulation (II) is illustrated in Figure 6-6. This figure clearly shows that the absolute deviation of average efficiency from true efficiency in Dyn-3 is less than the other dynamic models and static model.

Figure 6-6. Mean absolute deviation from true efficiency in scenario (II) for data set SET1



Therefore, scenario (II) confirms that the dynamic efficiency model captures the true performance better according to evaluate the efficiency of DMUs in 10 data sets generated from a Cobb - Douglas inter - temporal production function.

6.4 *Comparing static and dynamic DEA models across the two scenarios*

Table 6-10 shows the summary of results on DEA efficiencies across the two scenarios. The first two numerical columns contain the results of scenario (I) and the second two columns the results of the scenario (II).

Table 6-10. Summaries of the results in scenario (I) and scenario(II)

	Scenario (I) for technologies TEC1 - TEC10		Scenario (II) for data set SET1 - SET10	
	Overall average efficiency	Overall absolute deviation between true and estimated DEA efficiencies	Overall average efficiency	Overall absolute deviation between true and estimated DEA efficiencies
True	0.851		0.734	
Static	0.659	0.203	0.712	0.021
Dyn-2	0.779	0.157	0.719	0.015
Dyn-3	0.844	0.135	0.746	0.012
Dyn-4	0.879	0.126	0.769	0.035
Dyn-5	0.923	0.123	0.805	0.071
Dyn-6	0.949	0.127	0.832	0.098
Dyn-7	0.966	0.127	0.858	0.124
Dyn-8	0.976	0.133	0.880	0.146
Dyn-9	0.984	0.14	0.902	0.168
Dyn-10	0.989	0.141	0.923	0.189
Dyn-11	0.993	0.146	0.941	0.207
Dyn-12	0.996	0.144	0.952	0.218
Dyn-13	0.997	0.137	0.960	0.227
Dyn-14	0.998	0.147	0.968	0.234
Dyn-15	0.998	0.146	0.965	0.232

The overall average efficiency in scenario (I), which is reported in column 2 of Table 6-10, was obtained from mean efficiency across all DMUs in TEC1 to TEC10. Similarly the overall efficiency in scenario (II), which is reported in column 4 of Table 6-10, was obtained from mean efficiency across all DMUs in SET1 to SET10.

In the same way overall absolute deviation between true and estimated efficiencies was obtained as an average across TEC1 – TEC10 in scenario (I) and across SET1 – SET10 in scenario (II).

In scenario (II) the true average efficiency is 0.734, the mean static DEA efficiency is 0.712. Of the dynamic DEA efficiencies the closest mean to the true mean is offered by Dyn-3 with the average of 0.746. In scenario (I) the true average efficiency is 0.851 while again the closest dynamic DEA efficiencies are those of Dyn-3 with average of 0.844. The static DEA efficiency has mean of 0.659. Thus it is concluded that the static efficiencies are worse than the best dynamic efficiencies, in Dyn-3, for both scenario (II), where we keep the technology constant and scenario (I), where we vary the technology. This is also confirmed by the mean absolute deviations in Table 6-10 under static and Dyn-3.

Several other important conclusions can be drawn from these simulations:

I. In all 20 case studies of TEC1 TEC10 and SET1 – SET10 dynamic DEA has captured true performance better than static DEA. In all cases it was the better estimation of efficiency as well as receiving better average of deviations with true efficiency.

II. All cases were aimed at testing the effect of length of window. As expected, the window of length 3 becomes the better in the efficiency estimation for these special production technologies. However more research is needed on the issue of what window length is better.

III. TEC1 – TEC10 were intended to test the effect of changing the production technology while the data remain constant. The efficiency results showed little sensitivity to the changing of production technology.

SET1 – SET10 were run to check the effect of changing the data under a given production technology. It is found that there was little impact on the accuracy of results.

6.5 Conclusion

In this chapter we compared static and dynamic DEA efficiency models when DMUs operate under inter - temporal input - output dependence. For this purpose simulated data has been used.

Two scenarios were considered. In scenario (I) static and dynamic DEA models were compared under different production technologies keeping input - output paths constant. In scenario (II) the approaches were compared under changing input - output paths, keeping technology the same. In each scenario 10 runs of 100 DMUs over 15 periods were examined.

In all cases at least one dynamic DEA model performed better than the static DEA model. The window performing best under dynamic DEA was that which matched most closely the lag of inter - temporal effects. However, further investigation is needed of the impact of the length of window on dynamic efficiency. At this stage it can be suggested that the selection of length of window will depend on the nature of operations of the DMUs and the lag of inter – temporal effects.

An analysis of the dynamic efficiency obtained in both approaches across all windows indicates that the length of window in dynamic efficiency is important and it should be selected in line with the inter - temporal technology which mainly depends on the process of transferring capital input to output. The results show that the degree of accuracy of static DEA is also very dependent on the technology. For example we found that static DEA captures the efficiency of TEC8 in scenario (I) better than it captures the efficiency score of the other technologies. Certainly, in TEC3 static efficiency is far closer to true efficiency. Why does static DEA better in some technologies and fails to capture the true performance in others? The answer must be sought in the degree of inter - temporal effects in the technology operated by the DMUs. Between technologies TEC8 and TEC3, we note that TEC3 is very dependent on the current input while TEC8 is influenced by capital input. (Compare the current and capital coefficients of TEC3 and TEC8 in Table 6-2). What can be generalised from this is that if DMUs are operating under a technology that is highly influenced by capital input we explicitly ignore the role of future production in static DEA and the static model fails to assess the efficiency of such DMUs. In other words:

- High inter - temporal dependence of input output would reduce the accuracy of efficiency obtained in static DEA.
- Lower inter - temporal dependence would improve the accuracy of efficiency scores obtained in static DEA.

This generalisation can be clearly seen in the next chapter where we use real data for measuring the dynamic efficiency of industrialised countries. We will see that DMUs with high level of capital investment become less efficient in static DEA while they are showing more efficient under dynamic DEA.

CHAPTER 7: An Assessment of the Efficiency and Productivity of Industrialised Countries Using Dynamic DEA Models

7.1 Introduction

Analysis of production efficiency of industrialised countries, which is directly interested in the question of whether certain countries perform better than others in producing more output with the same or less inputs, is an example of the importance of estimating production relationships. In order to estimate production relationships we need to develop appropriate measures for the two major inputs into production activity, namely labour and capital. A physical asset once installed is capable of contributing several years of outputs for the production unit that uses it. This implies that we must take into account investments made in the previous years in order to produce a measure of the efficiency and productivity for any given year.

In this chapter we use dynamic efficiency and compare our results with previous work on the analysis of efficiency and productivity of OECD (Organisation for Economic Cooperation and Development) countries. Our sample constructed from 17 countries consist of: AUSTRALIA, AUSTRIA, BELGIUM, CANADA, DENMARK, FINLAND, FRANCE, GERMANY, GREECE, IRELAND, ITALY, JAPAN, NORWAY, SPAIN, SWEDEN, UK, USA.

We shall use the data from Färe, Grosskopf, Norris and Zhang (1994) (hereafter FGNZ) who calculated the efficiency and productivity of OECD countries. In a separate study, Taskin and Zaim (1997) show the importance of efficiency gains as a source of labour productivity convergence in high and low income countries including those in the OECD. Both studies capture the role of capital stock and they assume that production in each period is carried out by using capital and labour.

However, studies of the kind used by FGNZ are 'static', using one period of time (e.g. one year) at a time which captures only part of the impact of investment in long-lived assets. They ignore the effects of lags in the investment process on the capital stock. If we know that investment affects production technology with certain time lags, then our initial choice of capital and the timing of investments should take these lags into account. The dynamic efficiency model presented in this thesis captures inter - temporal effects including lags in the impact of investment in capital. Therefore this analysis should enable us to examine better the influences of capital stock on the efficiency and productivity of OECD countries during the period studied.

FGNZ computed productivity indexes for OECD countries and decomposed them into 'efficiency catch up' and 'technology change' (see Chapter 2 for the definition of these terms). Their analysis covered the time period 1979 to 1988. We shall compute these same measures of efficiency and productivity using dynamic efficiencies. This will make it possible to compare the static and dynamic efficiency-based results and highlight the additional insights offered by using dynamic efficiency. Thus we introduce here a *dynamic* Malmquist productivity index and its decomposition. The chapter unfolds as follows. Section (7.2) introduces the dynamic productivity index and its decomposition into technical change and efficiency catch-up. Section (7.3) sets up the models we need to calculate the required measures. Section (7.4) examines the efficiency and productivity of OECD countries in the dynamic context and compares the results with those previously reported for the same data set in the static context. Conclusions are drawn in section (7.5).

7.2 Productivity index under the dynamic model

The basic Malmquist index computed under static DEA is presented in Chapter 2. To calculate each index under static DEA we use the data of two periods (e.g. two consecutive years) assessing efficiency in each period separately. The approach does not explicitly take into account the past or the future of invested inputs and does not take into account any intermediate production. The indexes are exclusively based on the input output of two consecutive periods. Our dynamic Malmquist index avoids this problem.

7.2.1 A Dynamic Malmquist index for productivity change: Methodology

The Malmquist non-parametric productivity index introduced by Färe *et al.* (1992 and 1995a) is based on linear programming and can be decomposed in several ways to give various indexes of productivity changes from one period to another. See, for example, FGNZ (1994), Caves, Christensen and Diewert (1982a,b). The conventional methodology used to derive the non-parametric Malmquist index can be extended in a straightforward way to a dynamic Malmquist Index using assessment paths.

The calculation of the new productivity measure using dynamic production possibility sets requires an estimate of the dynamic efficiency measure for two adjacent windows. For simplicity we use W_t for the window ending in period t, e.g. periods t-4, t-3, t-2, t-1 and t where the length of window is 5 periods, and we use PPS(W_t) for the Dynamic Production Possibility Set in window W_t as defined in Chapter 4.

Assume (X^{Wt},Y^{Wt}) is an input-output path in window W_t, and $F_i(X^{Wt}, Y^{Wt})$ denotes for "dynamic input - oriented" measure of technical efficiency of path (X^{Wt}, Y^{Wt}) as defined in Chapter 5. It is obvious that $F_i(X^{Wt}, Y^{Wt}) \leq 1$. Following Shephard (1970) and Färe (1988) the input distance function for window W_t can be defined as:

$$D_i(X^{Wt}, Y^{Wt}) = (F_i(X^{Wt}, Y^{Wt}))^{-1}.$$

This function is the reciprocal of the "minimum" proportional shrink of input path X^{Wt}, given output path Y^{Wt}. Note that $D_i(X^{Wt}, Y^{Wt}) \geq 1$ if and only if $(X^{Wt}, Y^{Wt}) \varepsilon PPS(W_t)$. In addition $D_i(X^{Wt}, Y^{Wt}) = 1$ if and only if (X^{Wt}, Y^{Wt}) is dynamically efficient. The output distance function (Shephard (1970)) is defined similarly and under constant returns to scale.

Output distance function = (input distance function)$^{-1}$ (See Chapter 1).

The time reference of the technology can be different from the time reference of the input-output path assessed. For example $D_i^{Wt1}(X^{Wt2}, Y^{Wt2})$ is a distance function where the time superscript on the distance function indicates the reference technology's time window; the time superscript on the input output path indicates the window of operation for the observation whose efficiency is being assessed. If the observation and the technology relate to the different windows then a cross-window evaluation is performed and the resulting efficiency score will be in the range of 0 to ∞.

Linear programming models introduced in earlier chapters for dynamic efficiency, first envelop the observed input output paths, for the purpose of defining best practice frontier, and then measure a path's distance from the frontier, yielding a technical efficiency score. Applied to the cross-window data, these models produce a dynamic measure of a path's productive efficiency relative to paths of a time window other than its own. Thus we introduce a dynamic Malmquist index, which can recognise sources of productivity change across windows. A dynamic Malmquist index and its decompositions are an extension of the static Malmquist index. (Färe *et al.* (1992 and 1997)).

Let us now define two cross-window distance functions, $D_i^{Wt}(X^{Wt+1},Y^{Wt+1})$ and $D_i^{Wt+1}(X^{Wt},Y^{Wt})$. We do not assume that (X^{Wt+1},Y^{Wt+1}) necessarily belongs to PPS(W_t) or that (X^{Wt}, Y^{Wt}) belongs to PPS(W_{t+1}). With these distance functions and following the standard definition of a Malmquist index (Caves *et al.* (1982a,b)) we are able to define and provide a basic decomposition of the Malmquist productivity index under dynamic efficiency as;

$$M_i^{Wt}(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1}) = \frac{D_i^{Wt}(X^{Wt+1}, Y^{Wt+1})}{D_i^{Wt}(X^{Wt}, Y^{Wt})}. \quad (7.1)$$

$M_i^{Wt}(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1})$ provides an index to compare (X^{Wt+1}, Y^{Wt+1}) to (X^{Wt}, Y^{Wt}) by using W_t technology as a reference technology. Although $D_i^{Wt}(X^{Wt}, Y^{Wt}) \geq 1$ but $D_i^{Wt}(X^{Wt+1}, Y^{Wt+1})$ and $D_i^{Wt+1}(X^{Wt}, Y^{Wt})$ may or may not be greater than or equal to 1 since W_{t+1} input-output paths may or may not be feasible with the technology of the W_t window. Similarly W_t input-output path may or may not be feasible within the technology of the W_{t+1} window.

Thus $M_i^{Wt}(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1}) < = > 1$ depending on whether productivity between t and t+1 has respectively become worse, is constant or has risen.

Alternatively, one could define window W_{t+1} technology as reference technology in a dynamic Malmquist index; i.e.

$$M_i^{Wt+1}(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1}) = \frac{D_i^{Wt+1}(X^{Wt+1}, Y^{Wt+1})}{D_i^{Wt+1}(X^{Wt}, Y^{Wt})}. \quad (7.2)$$

Färe *et al.* (1992) define the Malmquist index as the geometric mean of the above two indexes. Similarly the dynamic Malmquist index can be defined as:

$$M_i(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1})$$
$$= \left(M_i^{Wt}(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1}) \times M_i^{Wt+1}(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1})\right)^{1/2}$$
$$= \left(\left[\frac{D_i^{Wt}(X^{Wt+1}, Y^{Wt+1})}{D_i^{Wt}(X^{Wt}, Y^{Wt})}\right] \times \left[\frac{D_i^{Wt+1}(X^{Wt+1}, Y^{Wt+1})}{D_i^{Wt+1}(X^{Wt}, Y^{Wt})}\right]\right)^{1/2} \quad (7.3)$$

Then we define efficiency change between window W_t and W_{t+1} as

$$\Delta EFF(W_t, W_{t+1}) = \left[\frac{D_i^{Wt+1}(X^{Wt+1}, Y^{Wt+1})}{D_i^{Wt}(X^{Wt}, Y^{Wt})}\right] \quad (7.4)$$

and technical change as

$$TECH(W_t, W_{t+1}) =$$
$$= \left(\left[\frac{D_i^{Wt}(X^{Wt+1}, Y^{Wt+1})}{D_i^{Wt+1}(X^{Wt+1}, Y^{Wt+1})}\right] \times \left[\frac{D_i^{Wt}(X^{Wt}, Y^{Wt})}{D_i^{Wt+1}(X^{Wt}, Y^{Wt})}\right]\right)^{1/2} \quad (7.5)$$

ΔTECH(W_t, W_{t+1}) measures the relative distance between the production frontier in window W_t and window W_{t+1} and thus how much the best-practice technology shifts from one window to the next. This index captures the shift in technology between the two windows W_t and W_{t+1}.

The change in productive efficiency is given by ΔEFF(W_t, W_{t+1}), that is , the ratio of two own-window productive efficiency scores calculated relative to best practice in window W_{t+1} and window W_t respectively. It indicates whether a path has moved closer to or further from one window to another.

As can be seen in the above definitions the product of the above two indexs, $\Delta EFF(W_t, W_{t+1})$ and $\Delta TECH(W_t, W_{t+1})$, is equal to Malmquist index, $M_i(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1})$. i.e.

$$M_i(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1}) = \Delta EFF(W_t, W_{t+1}) \times \Delta TECH(W_t, W_{t+1}). \quad (7.6)$$

Values of $M_i(X^{Wt}, Y^{Wt}, X^{Wt+1}, Y^{Wt+1})$, $\Delta EFF(W_t, W_{t+1})$ and $\Delta TECH(W_t, W_{t+1})$ greater that one indicate that performance in that area has worsened from one window to another; values less than one indicate a progress in performance.

These productivity indexes under dynamic efficiency enable us to compare our results with FGNZ since they used similar indexes for the decomposition of the productivity indexes of 17 OECD countries.

7.3 Setting up the assessment model

7.3.1 The data

Our data on GDP levels, labour and capital stocks comes from the most recent version of the Penn World Tables (Summers and Heston (1991) - version 5.6). The Penn World Tables display a set of national accounts covering a large number of countries. A unique feature of the tables is that expenditure entries are denominated in a common set of prices in a common currency so that real international quantity comparisons can be made both between countries and over time (Summers and Heston (1991)). These data are built from the benchmark studies of the "International Comparison Progress of the United Nations and National Accounts data". The procedures used to create the data set are discussed in some detail in Summers and Heston (1991).

7.3.2 The efficiency model

The model used to calculate the dynamic efficiency of each country is Model 5-5. Similar to FGNZ we use Gross Domestic Product (GDP) as our single measure of aggregate output and capital stock and employment as our aggregate input proxies.

In each window we cover a period of five years. For example for the dynamic efficiency of the window ending 1980 we used the input output data paths from 1976 to 1980. We included the capital stock in 1975 as initial capital stock to the model and the total capital stock at the end of 1980 as end stock of capital. Therefore, our model constructs a best practice frontier from the data over long windows of time, covering five years, which enables us to compare the long-term performance of different countries. Thus the dynamic efficiency score for each country depends on the input output levels within the window, the initial level of capital stock and the level of capital stock at the end of the window. Hereafter, we refer to each dynamic efficiency score with its last year in the window under assessment, for example dynamic efficiency score 1980 is the outcome of the comparison of input output paths over periods ending in 1980 using the dynamic efficiency model with length of 5 years.

7.3.3 The productivity indexes

In this section we use the productivity approach described in the previous section to calculate the dynamic Malmquist index and its components using the OECD data referred to above. The distance functions needed were computed using Model 5-5 to compute cross-window distance functions, e.g. for $D_i^{Wt}(X^{Wt+1},Y^{Wt+1})$ we solved Model 5-5 using for the country under assessment the input output path of window W_{t+1} within a reference set including all countries with their input output paths of window W_t.

We decompose the Malmquist index as shown in (7.6).

7.4 Results and discussion

All calculations were done in SAS version 6.12 (see SAS Institute (1989)), using PROC LP in SAS/OR as explained in Emrouznejad (2000). A result sheet for each country is presented in Appendix B. In each sheet we provide dynamic efficiency scores, technical change, efficiency change, Malmquist index and its decomposition. Some graphs are presented for better visual comparison of the results.

For instance take the USA. Its efficiency scores appear in the bottom of the sheet. Its efficiency graph clearly shows that the dynamic efficiency of the USA is lower during the 1970s than the 1980s. The USA efficiency trend is very stable with its highest rate in the last 6 windows. The low efficiency trend of the USA in the 1970s has been confirmed by other researchers too, (see for example Abramovitz (1986, 1990), Baumol (1986) and Baumol *et al.* (1989)).

One should bear in mind that we set up the dynamic efficiency model for 5 years in each assessment window. This means that we implicitly assume that the bulk of the impact on GDP due the capital invested at one point in time would be seen within 5 years. However this assumption may not always be correct. For example, Maddison (1982, 1989) in his the study of the world economy provides evidence that incomes have been converging over a fairly long period and Maudos *et al.* (1999) in their OECD assessment model assumed 25 years age as a proxy of the per capita endowment of human capital.

The investment of the USA for enhancing its productivity has been articulated by many researchers including Abramovitz (1986, 1990), Baumol (1986) and Baumol *et al.* (1989).

The efficiency change, technical change, dynamic Malmquist productivity index and its decomposition for the USA are presented on the top of its sheet in Appendix B. Overall averages of each index in two decades are provided in the right top table on the sheet. For the USA, productivity progress (1.00601 average Malmquist productivity index over all period) is more due to efficiency change (1.01051 on average over all periods) than technical change (0.99711 on average over all periods). The decomposition of the Malmquist index to technical change and efficiency change is figured in the left top table and it is illustrated in the middle graph in the sheet.

The aim of this Chapter is not to go to a discussion of the results for all countries. Rather to comment on the comparison of our results with those based on the static DEA model, as presented in FGNZ. We focus on those where the two studies differ substantially.

7.4.1 Comparison of dynamic efficiency with static efficiency

Using the dynamic efficiency approach, JAPAN and the UK are consistently efficient. This is different from the FGNZ results. Table 7-1 provides the comparison of our results with those of FGNZ. To make our results more comparable with those of FGNZ we report, in Table 7-1, the dynamic efficiency for window 1988 and the average of FGNZ efficiencies for 1988 and 1983, using efficiencies rather than distance functions. The last two columns show the efficiency rank of OECD countries under the two different approaches.

Table 7-1: The average efficiency of each country

	Average from dynamic efficiency (1984-88)	Average from FGNZ	Dynamic efficiency rank	FGNZ rank
AUSTRALIA	0.9206	0.8117	14	6
AUSTRIA	0.9474	0.7437	12	11
BELGIUM	0.9075	0.7633	17	9
CANADA	0.9821	0.8883	6	3
DENMARK	0.9780	0.6387	7	16
FINLAND	1.0000	0.6548	1 joint	14
FRANCE	0.9545	0.7612	11	10
GERMANY	0.9983	0.7195	5	12
GREECE	0.9083	0.8836	16	4
IRELAND	0.9204	0.6113	15	17
ITALY	0.9208	0.7917	13	8
JAPAN	1.0000	0.6525	1 joint	15
NORWAY	0.9642	0.7993	10	7
SPAIN	0.9700	0.6681	9	13
SWEDEN	0.9722	0.8777	8	5
U.K.	1.0000	0.9343	1 joint	2
U.S.A	1.0000	1.0000	1 joint	1
Average	0.9614	0.7765		

As can be seen our scores are relatively high and this is because of the nature of the dynamic DEA model, covering a larger number of inputs and outputs. We find JAPAN and FINLAND very efficient while FGNZ found them very inefficient. In contrast FGNZ found GREECE in fourth place after the UK and CANADA while we find that GREECE is one of the least efficient countries.

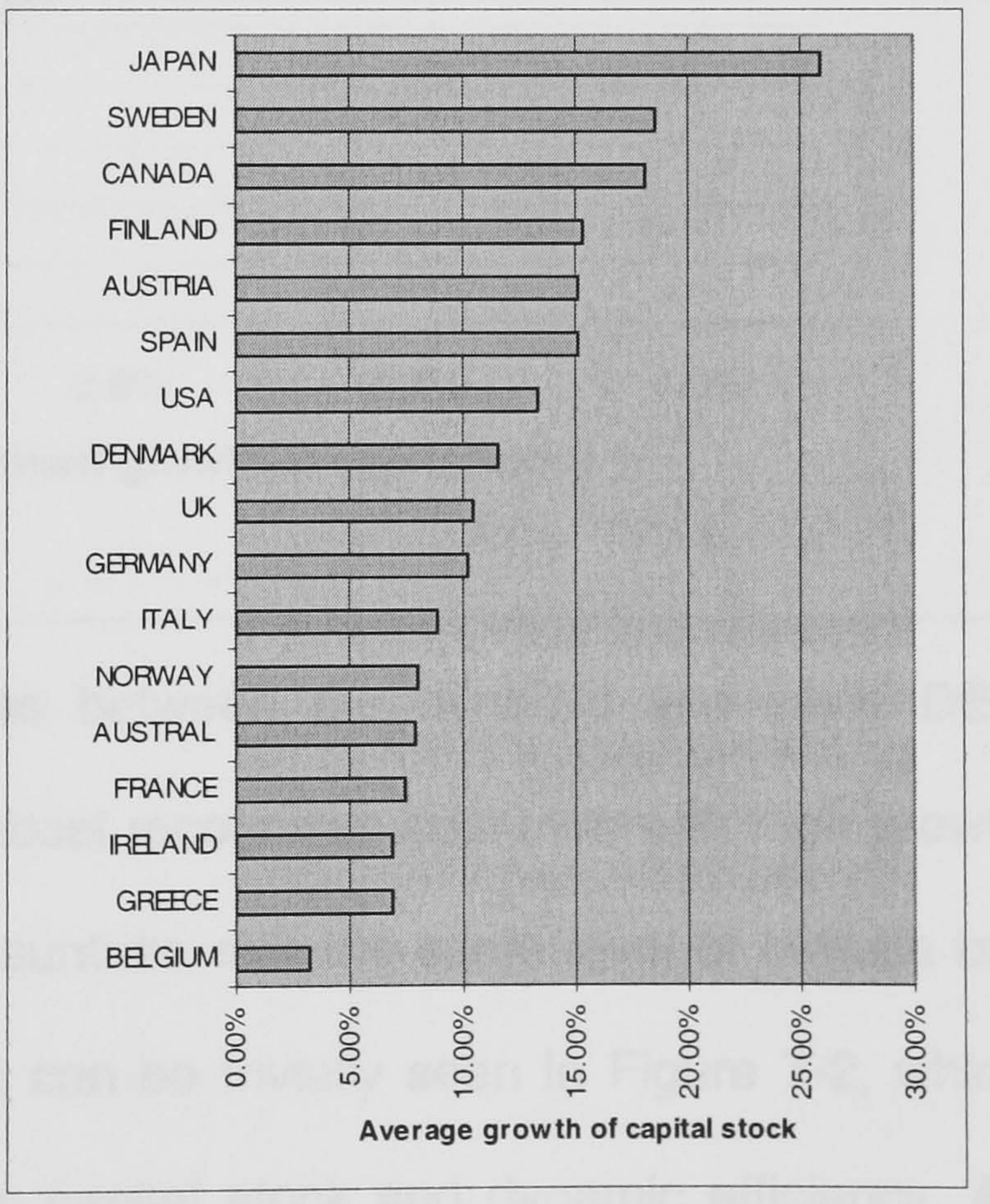
Why does dynamic efficiency make such a big difference for JAPAN and GREECE? Probably the main difference in modelling dynamic and static efficiency measures is due to the effect of capital stock. The initial capital stock in each window is given to the dynamic efficiency model as an extra input. The end stock of capital is treated as an extra output. Therefore the level of growth of capital stock can affect the dynamic efficiency scores.

The growth in capital stock from 1983 to 1988 is presented in Table 7-2.

Table 7-2: Growth in capital, from 1983 to 1988, for OECD countries

	Growth in Capital stock
AUSTRALIA	7.90%
AUSTRIA	15.14%
BELGIUM	3.25%
CANADA	18.12%
DENMARK	11.59%
FINLAND	15.36%
FRANCE	7.44%
GERMANY	10.22%
GREECE	6.85%
IRELAND	6.92%
ITALY	8.85%
JAPAN	25.89%
NORWAY	8.03%
SPAIN	15.13%
SWEDEN	18.53%
UK	10.43%
USA	13.37%
Average	11.94%

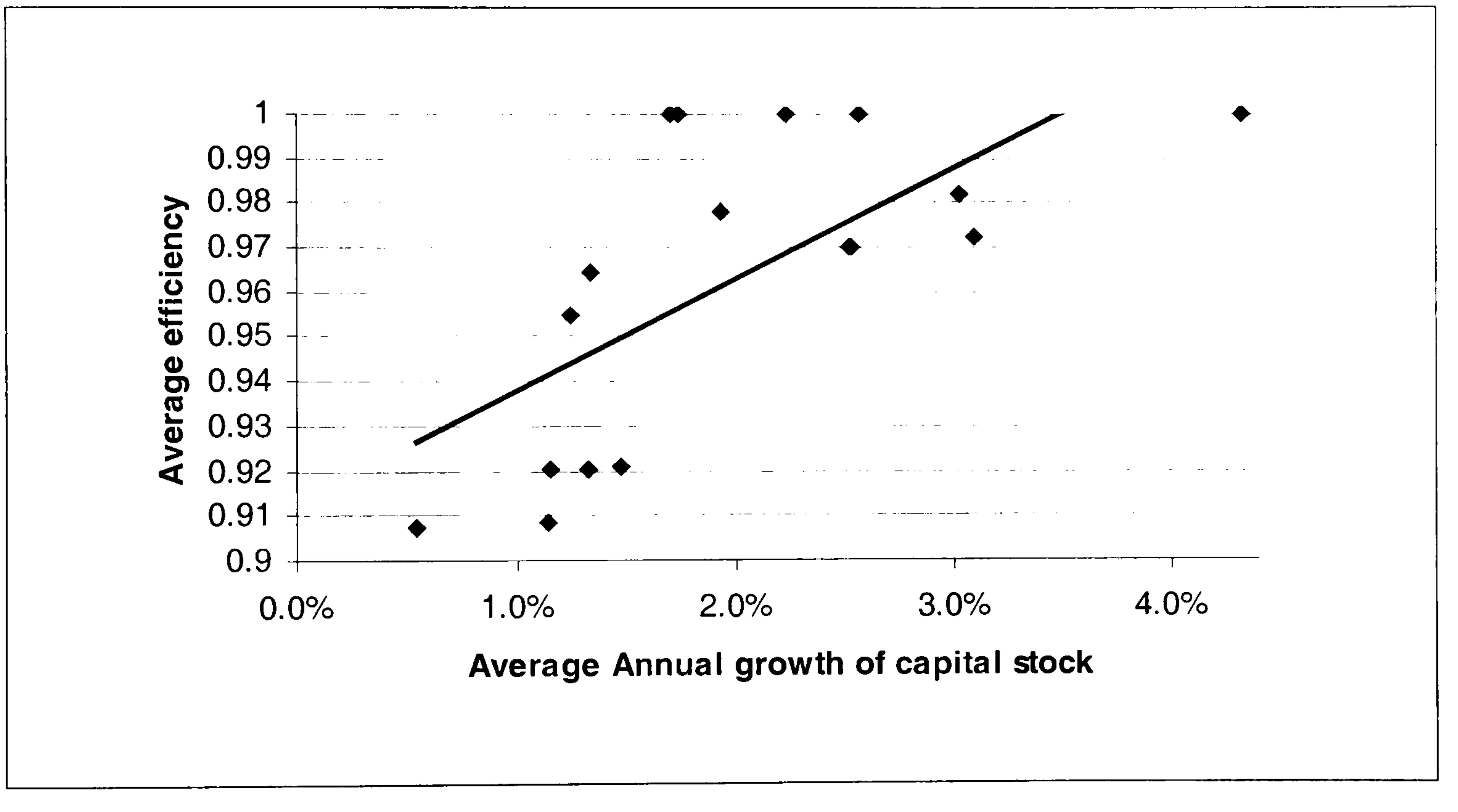
Figure 7-1. OECD in the order of capital growth



The overall average of annual capital growth is nearly 2% (11.94% increase from 1983 to 1988). JAPAN with the capital growth of about 4.3%

each year (25.8% increase from 1983 to 1988) is the OECD country with the highest level of capital growth. In contrast, GREECE is one of the countries with the least growth in capital stock of about 1.1% per annum. So its level of capital at the end of the window is not improved compared with the other OECD countries.

Figure 7-2. Dynamic efficiency rises when capital growth rises



This is one of the differences between the dynamic and static DEA models. The dynamic efficiency model recognises countries with high growth of capital as more efficient than countries with the same level of outputs but lower level of capital growth. This can be trivially seen in Figure 7-2, which shows the correlation of growth in capital stock and dynamic efficiency. As this figure shows, dynamic efficiency rises, generally, when the capital growth rises. This is the important factor of the dynamic efficiency model which can not be captured with a static DEA model in terms of calculating the efficiency.

Maudos *et al.* (1999) have also analysed the OECD countries but they included human capital as an extra input to the model. What they found is that human capital is an important factor in the efficiency and productivity measures. They found that the position of JAPAN improves considerably in terms of efficiency (and productivity) when we consider human capital as an extra input. Our analysis confirms their finding in that capital growth is an important factor in efficiency (and productivity) measurement.

7.4.2 Comparison of productivity indexes with those of FGNZ

For the comparison of productivity indexes we report, in Table 7-3, the average of the Malmquist index, technical change and efficiency change for windows ending 1984 to 1988 covering assessment periods 1979 to 1988. These results are the closest comparable with the averages of the same indexes reported by FGNZ over similar periods. This is drawn from disaggregated results in Appendix B.

The indexes in Table 7-3 are computed using efficiencies rather than distance functions and so an index value of over 1 represents productivity gain while under 1 productivity regress. A comparison of the foregoing results with those in table 6 in FGNZ, reproduced here in Table 7-4 for ease of reference, shows that there is agreement overall that technical change increased slightly over these periods.

Table 7-3. Average of productivity indexes over for 1984 – 1988 under the dynamic DEA model

	Malmquist	Technical Change	Efficiency Change
AUSTRALIA	0.97573	0.97276	1.00299
AUSTRIA	1.02029	1.02127	0.99902
BELGIUM	0.99914	0.99589	1.00358
CANADA	1.00794	1.00805	0.99986
DENMARK	1.03442	1.02610	1.00839
FINLAND	1.00718	1.00718	1.00000
FRANCE	0.98418	0.99907	0.98507
GERMANY	0.99833	1.00000	0.99833
GREECE	0.95888	0.98190	0.97660
IRELAND	0.96716	0.98443	0.98254
ITALY	0.99797	1.01321	0.98473
JAPAN	1.00000	1.00000	1.00000
NORWAY	0.99266	1.00809	0.98478
SPAIN	1.00056	0.99188	1.00892
SWEDEN	0.99804	0.99725	1.00062
UK	1.00000	1.00000	1.00000
USA	1.00015	1.00015	1.00000
Average	0.99662	1.00042	0.99620

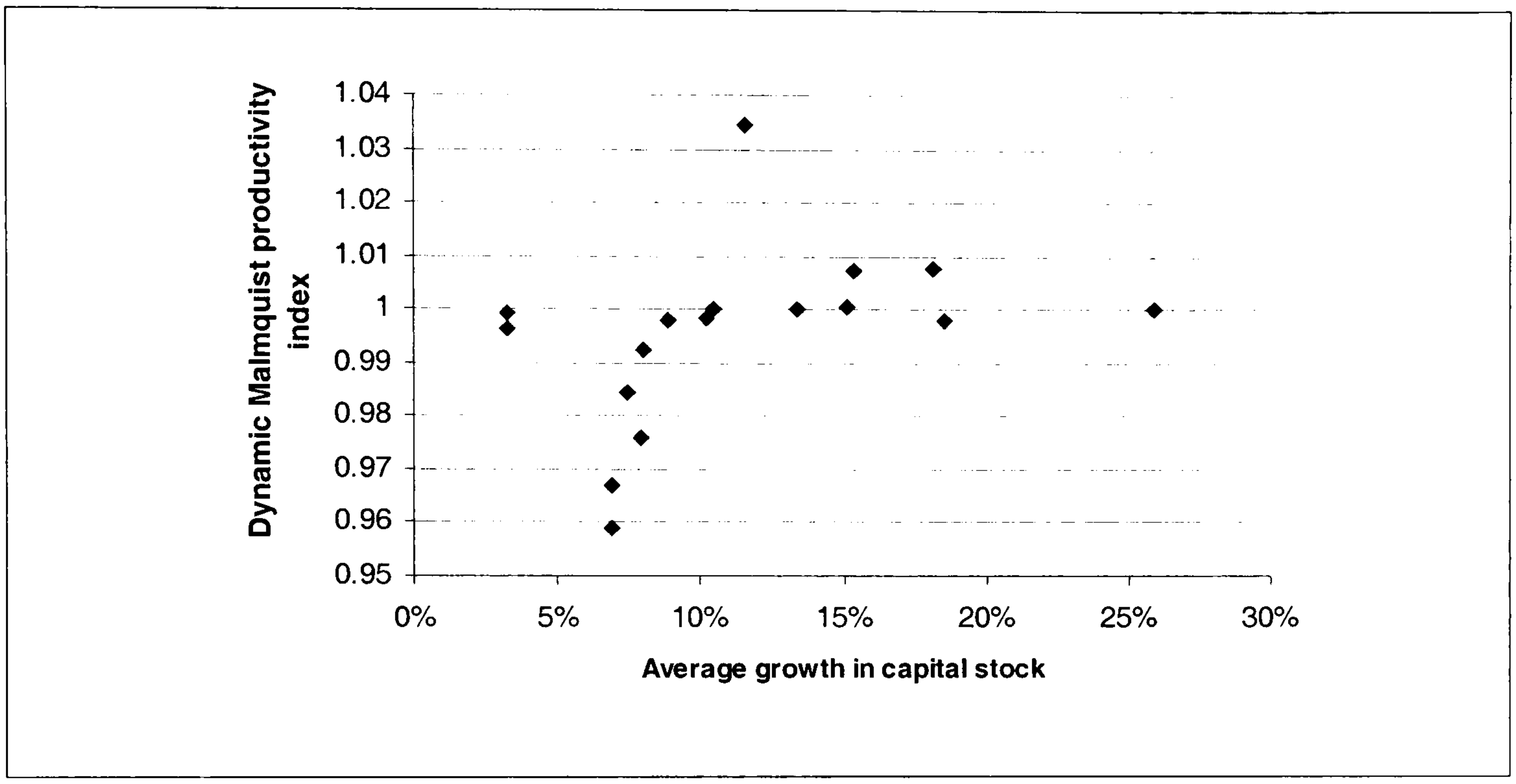
Therefore average productivity enhancement is due to innovation (technical change) than improving in efficiency (note average technical change>1 while average efficiency change<1). However, our Malmquist index components are different from those in FGNZ for some countries. Why is there a difference between the two approaches? Again we relate this to the level of capital growth as the capital stock is one of the important factors that has been taken into account in dynamic but not in static efficiency.

Table 7-4. Average of productivity indexes from FGNZ

	Malmquist	Technical Change	Efficiency Change
AUSTRALIA	0.9973	1.0009	0.9964
AUSTRIA	0.9981	1.0009	0.9972
BELGIUM	1.0092	1.0161	0.9932
CANADA	1.0151	1.0161	0.9990
DENMARK	1.0026	1.0009	1.0017
FINLAND	1.0272	1.0161	1.0108
FRANCE	1.0081	1.0161	0.9921
GERMANY	1.0117	1.0161	0.9956
GREECE	0.9962	1.0009	0.9953
IRELAND	0.9821	1.0009	0.9813
ITALY	1.0195	1.0161	1.0033
JAPAN	1.0287	1.0161	1.0124
NORWAY	1.0236	1.0161	1.0073
SPAIN	0.9898	1.0009	0.9890
SWEDEN	1.0019	1.0009	1.0010
UK	1.0012	1.0009	1.0003
USA	1.0085	1.0085	1.0000
Average	1.0070	1.0085	0.9986

Figure 7-3 illustrates the correlation of the dynamic Malmquist productivity index and the level of annual capital growth in OECD countries. This figure clearly shows that an increase in capital stock can improve the productivity index. A comparison of average annual growth with technical change and efficiency change also shows the same picture.

Figure 7-3. Correlation of capital growth with dynamic Malmquist indexes



7.5 Conclusion

In this chapter we first developed a dynamic productivity index and decomposed it. Then we provided both efficiency and productivity indexes for a set of 17 industrialised countries.

Results at country level are presented in Appendix B. We focused on the difference between static and dynamic results. The comparison of our results shows that static models, ignore the important factor of the capital stock. We concluded that dynamic efficiency increases when capital stock rises. A similar result was obtained for the productivity index and its components. This confirms similar results obtained by Maudos *et al.* (1999). They found that the inclusion of human capital has a significant effect on the accurate measurement of total factor productivity. We both recognise the higher rate of efficiency gains in JAPAN, for example, are due to higher growth of capital in Japan. It is reasonable to expect that, since capital stock has effects which spread over several years. The dynamic efficiency results should reflect reality better than those based on static DEA models.

CHAPTER 8: Alternative measures of dynamic efficiency and interpretation of DEA weights

8.1 Introduction

This chapter extends further the dynamic DEA model developed in this thesis. It examines alternative efficiency measures and it offers an interpretation of the dual to the model.

In essence the approach developed in this thesis constructs a PPS and our dynamic DEA Model 5-5 can identify whether a DMU path is Pareto efficient over time or not. How far a DMU path is from its peer(s) on the frontier is another question which will be addressed in this chapter. The chapter also discusses the insights offered by the dual to the dynamic DEA model. The chapter unfolds as follows.

Section (8.2) lays out some alternative measures of dynamic efficiency. In this section we define a radial measure then we introduce a dynamic model to deal with non - discretionary inputs and outputs in some periods. A more general measure of dynamic efficiency for when the length of sub - periods under assessment are not equal is also presented in this section. The formulation of the dual dynamic model with the interpretation of dual variables as input - output prices is explored in section (8.3). Section (8.4) concludes.

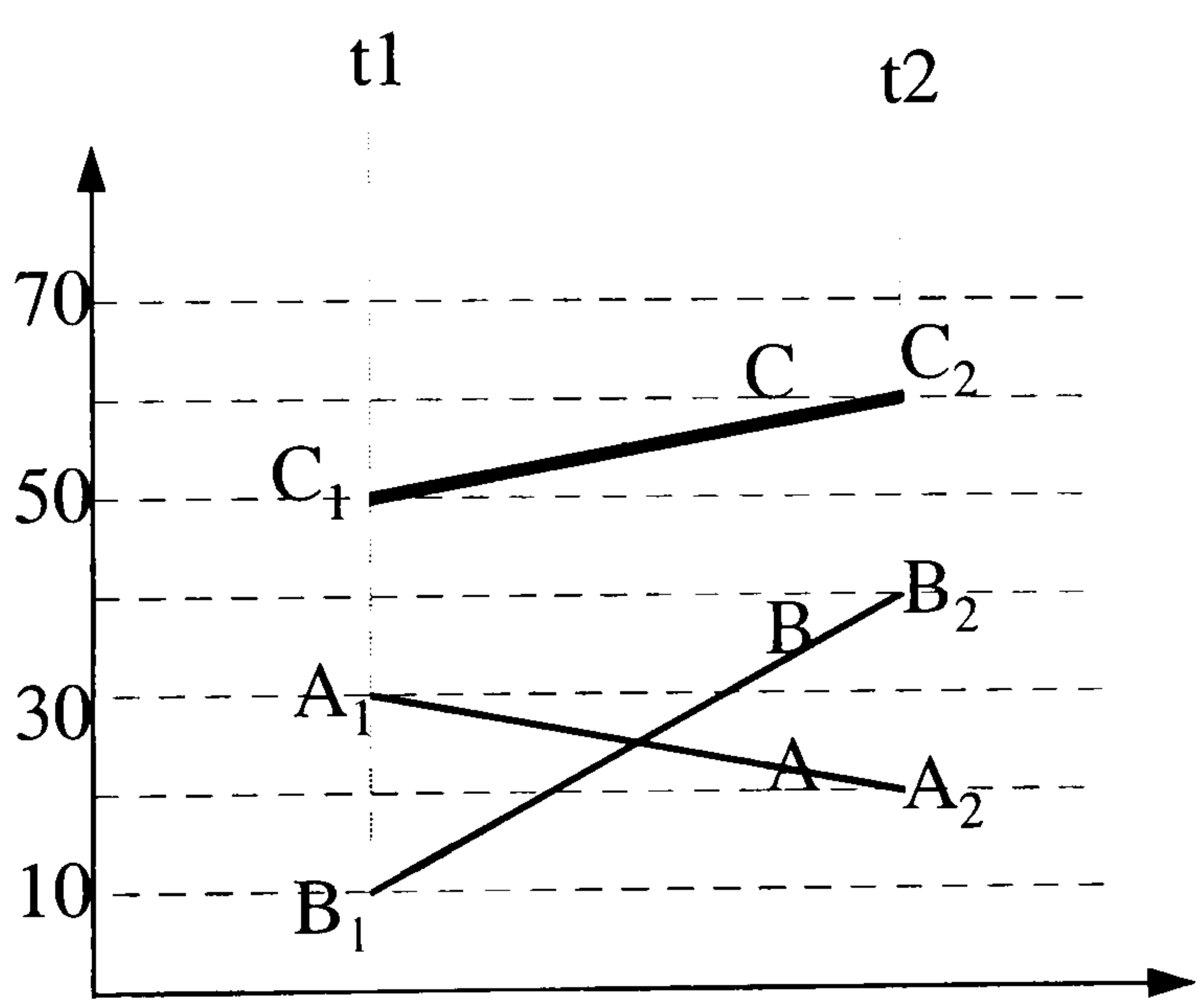
8.2 Alternative measures of dynamic efficiency

As noted above the dynamic Model 5-5 can be used to identify whether or not a DMU - path is Pareto efficient. How far a DMU - path is from its peer(s) on the frontier is another question. Just as under static DEA there is no unique measure of the distance of a DMU from the PPS frontier so here too there is no unique measure of distance from the frontier. Normally a radial measure of this distance is used but other measures are also possible. Two such measures are discussed next.

8.2.1 Defining an efficiency measure of radial reduction across all periods within the assessment window

The measure of dynamic efficiency in Model 5-5 is the average of the lowest proportional contraction of the input levels of a path, contraction being the smallest in each period but not necessarily across all periods. To see better the meaning of this consider a simple case of three paths in two periods associated with a single input per unit of output as illustrated in Figure 8-1.

Figure 8-1. Three paths in two periods associated with a single input per unit of output



	t1	t2
A	30	20
B	10	40
C	50	60

Obviously paths A and B are efficient paths and C is an inefficient path. Model 5-5 provides a different rate of reduction for path C in different periods of time. In t1 the efficiency is $\frac{B_1}{C_1}$ = 20% and in t2 it is $\frac{A_2}{C_2}$ = 33% and Model

5-5 gives min $\{(\frac{A_1}{C_1}+\frac{A_2}{C_2})/2,\ (\frac{B_1}{C_1}+\frac{B_2}{C_2})/2\}$ = 43%. An alternative measure of dynamic efficiency can be defined which does not permit different contraction ratios over time. In this new model the efficiency rate of each inefficient path can be calculated by projecting it to the PPS radially over time. Model 5-5 is modified to Model 8-1 to yield this measure of efficiency.

Model 8-1. Equal radial contraction in all periods within a window

$t=\tau,\ \tau+1,\ \ldots,\ \tau+T$

$$\text{Min}\,\alpha = k_0 - \varepsilon\Big(\sum_{t=1}^{T}\sum_{i\in I_2} S_i^{t-} + \sum_{t=1}^{T}\sum_{i\in I_2} \delta_i^{t-} + \sum_{t=1}^{T}\sum_{r=1}^{s} S_r^{t+} + \sum_{i\in I_2}\gamma_i^{-} + \sum_{i\in I_2}\gamma_i^{+}\Big)$$

s.t.

$$\sum_j^N \lambda_j x_{ij}^t = k_0 x_{ij_0}^t - S_i^{t-} \quad ;i\in I_1,\ t=\tau,\ldots,\tau+T$$

$$\sum_j^N \lambda_j z_{ij}^t = k_0 z_{ij_0}^t - \delta_i^{t-} \quad ;i\in I_2,\ t=\tau,\ldots,\tau+T$$

and constraints sets C3, C4 and C5 in Model 5 - 5.

Variables are as in Model 5 - 5.

In this model the efficiency rate of path C in Figure 8-1 is 45.5% in each period and the target path is (22.73, 27.27) which is a convex combination of the two efficient paths A and B and hence it belongs to the PPS frontier. (Note that (22.73, 27.27)= 0.635 $\times$ A + 0.365 $\times$ B and and $\frac{22.73}{C_1}=\frac{27.27}{C_2}=0.455$)

Comparing Model 8-1 with Model 5-5, it is obvious that the measure in Model 8-1 is never lower than those obtained from Model 5-5. This is because Model 5-5 is less constrained than Model 8-1. In fact Model 8-1 is the same as Model 5-5 with the additional constraint of

$$a^{t} = k_0 \quad \forall t .$$

8.2.2 Defining an efficiency measure when some inputs - outputs are non discretionary

In computing the dynamic efficiency of DMU paths Model 5-5 estimates the projection of "inefficient paths" onto "efficient paths". These projections involve input reduction. However inputs may not be controllable by management to the same degree over time. Therefore Model 5-5 may not yield an appropriate measure of efficiency in certain cases. For example a unit may have external inputs such as changing market size over time which the manager has no control. Such a DMU can not improve its efficiency by reducing an input level in all periods.

In order to deal with the problems associated with non - discretionary inputs, in static DEA, several alternative models have been suggested (see Banker and Morey (1986a)). Dynamic efficiency models can also deal with non - discretionary inputs in some, if not all, periods of time. This will lead us to define an alternative measure of dynamic efficiency based on the reduction of inputs in specific periods of time and holding input levels constant in other periods. A model for this purpose is formulated in (8.2).

Model 8-2. Period non - discretionary measure of dynamic efficiency within window $t = \tau,\ \tau+1,\ ...,\ \tau+T$

$$\text{Min}\ \alpha = \frac{1}{N_1}\sum_{t\in T_1}\alpha^t - \varepsilon\left(\sum_{t=1}^{T}\sum_{i\in I_2} S_i^{t-} + \sum_{t=1}^{T}\sum_{i\in I_2}\delta_i^{t-} + \sum_{t=1}^{T}\sum_{r=1}^{s} S_r^{t+} + \sum_{i\in I_2}\gamma_i^- + \sum_{i\in I_2}\gamma_i^+\right)$$

s.t.

$$\sum_{j}^{N}\lambda_j x_{ij}^t = \alpha^t x_{ij_0}^t - S_i^{t-} \quad ; i\in I_1, t\in T_1$$

$$\sum_{j}^{N}\lambda_j z_{ij}^t = \alpha^t z_{ij_0}^t - \delta_i^{t-} \quad ; i\in I_2, t\in T_1$$

$$\sum_{j}^{N}\lambda_j x_{ij}^t = x_{ij_0}^t - S_i^{t-} \quad ; i\in I_1, t\in T_2$$

$$\sum_{j}^{N}\lambda_j z_{ij}^t = z_{ij_0}^t - \delta_i^{t-} \quad ; i\in I_2, t\in T_2$$

and C3, C4 and C5 in Model 5-5,

Variables are as described in Model 5-5.

where $T = T_1 \cup T_2$ and N_1 is the number of periods in T_1.

This model is based on the assumption that managers are interested in holding the input levels in periods $t\in T_2$ as non - discretionary and examine the possibility of reducing input levels in periods $t\in T_1$. The model will measure the (in)efficiency according to the possibility of reducing input in $t\in T_1$ while not increasing inputs in other periods.

To illustrate the efficiency measure with non - discretionary input in Figure 8-1 assume we are interested in holding input level as it is in period t1 and examine any possible reduction in period t2. Therefore the efficiency of path C is 0.33 $(=\frac{20}{60})$.

8.2.3 Defining a dynamic efficiency model when periods under assessment are not of equal length

In all models presented in this thesis we assumed that the assessment periods are divided to sub – periods with equal length. Generally one is that assessing the performance of organisations when the data are available over not - equivalent length of periods. Assume, a window of length of K is divided to T sub – periods, K_1, K_2, ..., K_T (not necessarily equivalent length) such that $K_1+K_2+\ldots+K_T=K$. Assume further that x, z and y are the same notations as used in Model 5-5 with reference to the sub – periods K_1, K_2, ..., K_T. i.e. the input output paths are: $(x^{K1, K2, \ldots, KT}, z^{K1, K2, \ldots, KT}, y^{K1, K2, \ldots, KT})$. Let Z^0 and Z are respectively the initial input to the assessment window and the final capital input at the end of assessment window. Therefore we could define a similar model to Model 5-5 for assessment of organisations with variant sub – periods. This is presented in Model 8-3.

The main difference in this model is that the definition of efficiency measure is now adjusted by the length of sub – periods. In other words since the length of the sub - periods under assessment are not equivalent instead of minimising the simple average we minimise the weighted average, weighted by length of sub - periods. It is obvious, if we assume that the window is equality divided to T sub - period the Model 8-3 will collapse to Model 5-5.

This can be easily seen with replacing $K_1, K_2, \ldots, K_T$ with $\frac{K}{T}$ in Model 8-3.

Model 8-3. Dynamic efficiency for unequivalent sub - periods

$$\text{Min}\,\alpha = \frac{\sum_{t=K_1}^{K_T} t\alpha^t}{K} - \varepsilon\Big(\sum_{t=K_1}^{K_T}\sum_{i\in I_1} S_i^{t-} + \sum_{t=K_1}^{K_T}\sum_{i\in I_2} \delta_i^{t-} + \sum_{t=K_1}^{K_T}\sum_{r=1}^{s} S_r^{t+} + \sum_{i\in I_2}\gamma_i^- + \sum_{i\in I_2}\gamma_i^+\Big)$$

s.t.

$$\text{C1}: \quad \sum_{j=1}^{N}\lambda_j x_{ij}^t = \alpha^t x_{ij_0}^t - S_i^{t-} \quad ; i\in I_1,\ t = K_1, \ldots, K_T$$

$$\text{C2}: \quad \sum_{j=1}^{N}\lambda_j z_{ij}^t = \alpha^t z_{ij_0}^t - \delta_i^{t-} \quad ; i\in I_2,\ t = K_1, \ldots, K_T$$

$$\text{C3}: \quad \sum_{j=1}^{N}\lambda_j y_{rj}^t = y_{rj_0}^t + S_r^{t+} \quad ; r=1, \ldots, s,\ t = K_1, \ldots, K_T$$

$$\text{C4}: \quad \sum_{j}^{N}\lambda_j Z_{ij} = Z_{ij_0} + \gamma_i^+ \quad ; i\in I_2$$

$$\text{C5}: \quad \sum_{j=1}^{N}\lambda_j Z_{ij}^0 = Z_{ij_0}^0 - \gamma_i^- \quad ; i\in I_2$$

$$\lambda_j \ge 0; \forall j,\ S_i^{t-} \ge 0,\ \delta_i^{t-} \ge 0\ (\forall t, \forall i\in I_1),\ S_r^{t+} \ge 0 (\forall r, \forall t),\ \gamma_i^+ \ge 0,\ \gamma_i^- \ge 0 (\forall i\in I_2)$$

where;

$I_1 \subset \{1, \ldots, m\}$ are flow inputs,

$I_2 \subset \{1, \ldots, m\}$ are those inputs that their end - stock will be converted, directley or indirectley, into more output some type at some future period.

Z_{ij}^0 is the initial - stock of capital of type i for DMU j; $i\in I_2$,

Z_{ij} is the end - stock capital of type i for DMU j; $i\in I_2$.

The next section uses the original model to discuss the insight its dual offers.

8.3 *Dual dynamic efficiency model*

As it is known in the static DEA context the dual to the envelopment model gives implicit values to inputs and outputs (Thanassoulis (1995)). In the dynamic efficiency context we also have similar information from the dual to Model 5-5.

The aim of this section is to derive an economic interpretation of the dual to the dynamic DEA Model 5-5. This can be of value in practical applications as in static DEA.

8.3.1 Economic interpretation of dual variables - static DEA model

As developed by Charnes *et al.* (1978) the DEA model in which input vector x is related to a vector of outputs y can be written as follows:

Model 8-4. Static DEA model

$$\mathbf{Min}_{\lambda, h}\ h$$
$$s.t\ \Sigma_j \lambda_j x_j \leq h\, x_{j0}$$
$$\Sigma_j \lambda_j y_j \geq y_{j0}\ ,\ \lambda_j \geq 0\ .$$

where λ is the intensity vector, $(x_j\ ,\ y_j)$ is the input - output vector of DMU j, j_0 is the DMU being assessed and, h is the efficiency rate. Thus (1-h) is the inefficiency rate or the failure of the DMU j_0 to use minimum input given its output levels.

Let us assume the DMUs sell the output at price p, and their objective is to maximise revenue. This problem can be formulated as in Model 8-5 (see *Lovell (1993)*).

Model 8-5. Revenue maximisation DEA

$$
\begin{aligned}
&\mathbf{Max}_{p,\lambda}\ \Sigma_r\, p_{rjo}\, y_{rjo} \\
&s.t\ \Sigma_j\, \lambda_j\, x_{ij} \le x_{ijo}\ ;\ \forall i \\
&\Sigma_j\, \lambda_j\, y_{rj} \ge y_{rjo}\ ;\ \forall r \\
&\lambda_j \ge 0,\ y_r \ge 0\ ;\ \forall j,\ r.
\end{aligned}
$$

where $p_{rj} = (p_{1j}, \ldots, p_{mj})$ are the output prices for DMU j.

Now let u_r and v_i be dual variables associated with the constraints to Model 8-5. Then the dual to Model 8-5 becomes:

Model 8-6. Dual revenue maximisation DEA

$$
\begin{aligned}
&\mathbf{Min}_{v,\,u}\ \Sigma_i\, v_I\, x_{ijo} \\
&s.t.\ \Sigma_r u_r y_{rj} - \Sigma_i v_i\, x_{ij} \le 0\ ;\forall j \\
&u_r \ge p_{rjo}\ ;\forall r \\
&u_r \ge 0,\ v_i \ge 0\ ;\forall i,\ \forall r
\end{aligned}
$$

At optimality, the two objective functions of the primal and the dual Model 8-5 and Model 8-6 are equal. Thus $\Sigma_i v_i^* x_{ijo} = \Sigma_r p_{rjo} y_{r\,jo}^*$, where a superscript * denotes an optimal value of the corresponding variable.

Based upon the complementary slackness condition of linear programming (see Thrall (1996)) for the optimum solution to Model 8-5 and Model 8-6 it is obtained that,

$$(p_{rj0} - u_r^*)\, y_{r\,j0}^* = 0\ ,\ \forall r.$$

Thus if $y_{r\,j0}^* > 0$ then $p_{rj0} = u_r^*$, $\forall r$.

This indicates that if the r^{th} component of the revenue maximisation output vector is positive then its corresponding dual variable may be interpreted as the imputed r^{th} output price.

In an analogous way the corresponding dual variables to input constraints may be interpreted as imputed input prices in a cost minimisation primal DEA model (see Sueyoshi (1995)).

This interpretation of dual variables indicates that v_i and u_r respectively can be seen as input - output price in the following DEA model which is dual to Model 8-4.

Model 8-7. Dual to Model 8-2.

$$\begin{array}{ll}
\mathbf{Max}_{u,v}\ \sum_r u_r y_{rj0} & \\
\text{s.t. } \sum_r u_r y_{rj} - \sum_i v_i x_{ij} \le 0\ ;\forall j & \\
\sum_i v_i x_{ij0} = 1\ ; & \forall j \\
u_r \ge 0,\ v_i \ge 0\ ; & \forall i\ ,\ \forall r
\end{array}$$

8.3.2 Economic interpretation of dual variables - dynamic DEA model

Dynamic DEA models can also be discussed along the same lines. In the context of dynamic DEA, the total cost for path j can be calculated as the $\Sigma_t\Sigma_i v^t_i\, x^t_{ij}$ where v^t_i is the price of input i at period t. Similarly, the total revenue for path j can be calculated as $\Sigma_t\Sigma_r u^t_r\, y^t_{rj}$ where u^t_r is the price of output r at period t. Thus any changes over time due to dynamic properties of production will be reflected in the price variables u^t_r and v^t_i. We can identify the following revenues and costs for DMU j.

Table 8-1. Cost and revenue notations - DMU j

Notation	Interpretation	Calculation
R_{rj}^t	Revenue to path j from output r at period t	$u_r^t y_{rj}^t$.
R_j^t	Revenue to path j from its outputs at t	$\Sigma_r R_{rj}^t = \Sigma_r u_r^t y_{rj}^t$.
$R_j^{1,\ldots,t}$	Revenue - path of DMU j	$(R_j^1, \ldots, R_j^t) = (\Sigma_r u_r^1 y_{rj}^1, \ldots, \Sigma_r u_r^t y_{rj}^t)$.
R_j	Total revenue to DMU j from its output paths	$\Sigma_t \Sigma_r R_{rj}^t = \Sigma_r \Sigma_t u_r^t y_{rj}^t$.
C_{ij}^t	Cost to path j from input i at period t	$v_i^t x_{ij}^t$.
C_j^t	Cost to path j from its inputs at t	$\Sigma_r C_{ij}^t = \Sigma_r v_i^t x_{ij}^t$.
$C_j^{1,\ldots,t}$	Cost - path of DMU j	$(C_j^1, \ldots, C_j^t) = (\Sigma_r v_i^1 x_{ij}^1, \ldots, \Sigma_r v_i^t x_{ij}^t)$.
C_j	Total cost to DMU j from its input paths	$\Sigma_t \Sigma_i C_{ij}^t = \Sigma_t \Sigma_i v_i^t x_{ij}^t$.

We have two different orientations to efficiency measurement of DMUs under dynamic DEA. First, the course of the revenue - path $R_j^{1,\ldots,t}$ can be maximised while the cost - path remains constant over the entire life of DMU j_0.

Second, cost - paths $C_j^{1, \ldots, t}$ can be minimised for a given revenue - path $R_j^{1, \ldots, t}$ (= constant).

Let us consider the revenue maximisation orientation. The dynamic efficiency of DMU j_0 with cost - path of $C_{j0}^{1, \ldots, T}$ and revenue path of $R_{j0}^{1, \ldots, T}$ can be assessed using the following model:

Model 8-8. Dual dynamic DEA -Model 1

$$\mathbf{Max}_{R,\,C}\ \Sigma_t R^t_{j0}$$

$$s.t.\ \Sigma_t R^t_j - \Sigma_t C^t_j \leq 0\ ; \forall j$$

$$C^t_{j0} = \frac{1}{T}\ (or\ 1)\ ; \forall t$$

R and C as defined in Table 8-1.

The model estimates the maximum total revenue DMU j_0 could generate over the assessment window given its costs incurred. Its total cost over the period has been shared equally across the periods of the assessment window, that is $C^t_{j0} = \frac{1}{T}$ for all t. We have normalised the level of the total cost through the life of DMU j_0 to 1 since $\sum_{t=1}^{T} C^t{}_{j0} = \sum_{t=1}^{T} \frac{1}{T} = 1$. Therefore $\Sigma_t R^t_{j0} \leq 1$ and so the optimum value of the objective function in Model 8-8 is less than 1 which can be interpreted as an efficiency rate.

Using the notation in Table 8-1 we see that Model 8-8 can be written as Model 8-9 below.

Model 8-9. Dual dynamic DEA - Model 2

$$\Theta_{j0} = \mathbf{Max}_{u,\,v}\ \Sigma_{r,t} u_r^t y_{rj0}^t$$

$$s.t.\ \Sigma_{r,t} u_r^t y_{rj}^t - \Sigma_{i,t} v_i^t x_{ij}^t \le 0\ ;\forall j$$

$$\Sigma_i v_i^t x_{ij0}^t = \frac{1}{T}\ (or\ 1)\ ;\forall t$$

$$u_r^t \ge 0,\ v_i^t \ge 0\ ;\forall i,\ \forall r,\ \forall t$$

From duality theory of linear programming the dual to Model 8-9 is Model 8-10 below.

Model 8-10. Primal dynamic DEA

$$\text{Min}\ \phi_0 = \frac{1}{T}\sum_t \phi_t$$

s.t.

$$\sum_j \lambda_j x_{ij}^t \le \phi^t x_{ij_0}^t \qquad ; i = 1 \ldots m,\ t = 1 \ldots T$$

$$\sum_j \lambda_j y_{rj}^t \ge y_{rj_0}^t \qquad ;\ r=1 \ldots s,\ t=1 \ldots T$$

$$\lambda_j \ge 0\ ;\forall \text{j}$$

This is the envelopment dynamic DEA Model 5-1 presented in Chapter 5.

An inspection of the optimal set of weights for a DMU in dual Model 8-9 would reveal which of its inputs and outputs contribute to its efficiency rating in each period. Thus the dual dynamic DEA Model 8-9 gives the value-based measure of the efficiency of DMU j_0 (see Thanassoulis (1995)). The variables

$u_r^{1,2,...,T}$, $v_r^{1,2,...,T}$ in Model 8-9 are respectively the dual variables relating to the constraints in Model 8-10 corresponding to the path of output r and the path of input i. The dual variable - paths $u_r^{1,2,...T}$ and $v_i^{1,2,...T}$ can be seen respectively as a virtual marginal value of output - path r and an implicit marginal value of input - path i. The efficiency measure of DMU j_0 yielded by the dual DEA Model 8-9 is the ratio of the total virtual value of its output levels to the total virtual value of its input levels over successive periods of time. The total virtual input value at each period is always fixed at some arbitrary level, usually $\frac{1}{T}$ as in Model 8-9. Hence the total virtual output is restricted the range of [0, 1].

Similarly the dual to Model 5-5 is presented in Model 8-11.

The virtual input - output paths attributable to each input - output show exactly how the efficiency rating of the corresponding DMU is derived. Dual Model 8-11 allows each DMU to select the weighting structure over successive periods for the inputs - outputs which would make the DMU appears at its most efficient in comparison to the other DMUs.

Model 8-11. Dual to Model 5-5

$$\theta = Max \sum_{t=\tau}^{\tau+T} \sum_{r=1}^{s} u_r^t y_{rj0}^t + \sum_{i \in I_2} (w_i^+ Z_{ij0}^{\tau+T} - w_i^- Z_{ij0}^{\tau-1})$$

$s.t.$

$$\sum_{t=\tau}^{\tau+T} \left(\sum_{r=1}^{s} u_r^t y_{rj}^t - \sum_{i \in I_1} v_i^t x_{ij}^t - \sum_{i \in I_2} v_i^t z_{ij}^t \right) + \sum_{i \in I_2} (w_i^+ Z_{ij}^{\tau+T} - w_i^- Z_{ij}^{\tau-1}) \le 0 \quad \forall j$$

$$\sum_{i \in I_1} v_i^t x_{ij0}^t + \sum_{i \in I_2} v_i^t z_{ij0}^t = \frac{1}{T}; \qquad \forall t = \tau, \ldots \tau + T.$$

$$v_i^t \ge \varepsilon \;\; ; \forall i \in 1, \ldots, m, t = \tau, \ldots, \tau + T$$

$$u_i^t \ge \varepsilon \;\; ; \forall r = 1, \ldots, s, t = \tau, \ldots, \tau + T$$

$$w_i^+ \ge \varepsilon \;\; ; \forall i \in I_2$$

$$w_i^- \ge \varepsilon \;\; ; \forall i \in I_2$$

8.4 Conclusion

In this chapter alternative measures of dynamic efficiency were examined and the dual to dynamic efficiency model was explored.

Two alternative measures of dynamic efficiency were introduced. One defines an efficiency measure of equal radial contraction across all periods within the assessment window. In a second measure non - discretionary variables are handled. The measure is based on the assumption that managers wish not to raise the input levels in some periods and examine the possibility of reducing input levels in other periods.

Further, the interpretation of the dual to the dynamic efficiency model was given, arriving as a value - based dynamic DEA model. This model offers valuable insights on the performance of DMUs being assessed. In the next chapter we use this model for the assessment of higher education institutions.

CHAPTER 9: The assessment of higher education institutions using dynamic DEA: A case study in UK universities

9.1 Introduction

This chapter compares dynamic DEA, static DEA and performance indicators as alternative tools for assessing the performance of organisational units such as higher education institutions (HEIs). Such units typically use one or more resources in one or several years to secure outputs in the same or future years. The assessment of UK universities is used as a base for comparing three assessment methods, dynamic DEA, static DEA and performance indicators. The comparison focuses on how well the three methods agree on the performance of an institution relative to the HEI sector. Performance indicators (PIs) are normally used to assess organisations and each one is set up as a ratio of one input to an output, or of one output to an input. PIs are widely used in both public and private sectors. In particular they are adopted by the UK Government for assessing the performance of governmental bodies like National Health Service (NHS), Local education Authorities (LEA) and Higher Educational Institutions. Probably the main advantage of using PIs for representing the performance of organisations is that they are easy to understand since in each PI we deal with single input single output.

However, various studies have suggested that PIs are not suitable measures for the case of multiple input multiple output. The problem will arise from the fact that a PI reflects only one input and one output level and so it is difficult to gain an overall view of the performance of a DMU when not all of its PIs indicate a similar level of performance. This has been addressed in several studies including Barrow and Wagstaff (1989), Greenberg and Nunamaker (1987) and Thanassoulis, Boussofiane and Dyson (1996).

On the other hand since the seminal paper of DEA by Charnes *et al.* (1978) there have been numerous enhancement to the methodology (See Seiford (1997)) and increasing number of applications of the method particularly in assessment of public sector organisations (See for example

Thanassoulis *et al.* (1995) Thanassoulis and Dunstan (1994) and Thanassoulis (1995)). However, the problem with single period (i.e. static) DEA is the fact that static contemporaneous DEA reflects only one period of time so it is difficult to gain an overall view of the performance of a DMU operating over several periods. We would normally expect dynamic DEA and contemporaneous static DEA based assessment of the performance of a DMU not to agree for some institutions. Hence, the three methods may disagree substantially on the relative performance of an individual institution. Dynamic DEA, unlike static DEA and PIs, considers simultaneously all aspects of the performance of a DMU which may therefore be deemed a good performer even when its performance on individual PIs or on a specific period static DEA is not outstanding.

The prime purpose in this chapter is to explore the difference between the three approaches dynamic DEA, static DEA and PIs and to show what the dynamic DEA methodology proposed in this thesis can add to the static DEA and PI based analyses that higher education funding councils might have undertaken. The chapter suggests that they complement rather than replace one another in assessment of performance. The chapter is structured as follows.

It begins with an overview of the assessment of teaching and research in the UK higher education sector in section (9.2). Then it sets up a dynamic DEA model for assessing UK universities over periods 1995 to 1998 in section

(9.3). The availability of data and selecting suitable input output variables are also discussed in this section. Section (9.4) compares the results of the three methodologies and comment on the differences. Some further results from dynamic DEA for individual institutions are presented in section (9.5). These include target setting, peers and variable returns to scale scores. Conclusions are drawn in section (9.6).

9.2 Background

The assessment of teaching and research outcomes in UK higher education institutions have been central to both Government and institutions in the last two decades. As mentioned earlier due to simplicity of the use of PIs, they are widely accepted by UK Government for assessing public bodies. For example PIs are used in higher education funding bodies to help managers to assess the efficiency of service for which they are responsible. Earlier work of specific performance indicators in the UK was done in the late 1970s as part of the OECD's Institutional Management in higher education programme (see Sizer (1979)). Further developments at a national level were limited until the Jarratt Report (1985) on university efficiency and the Green paper (1985) on higher education, which recommended the introduction of PIs. Following that a Joint working group was established. Their first report was published in 1986, having considered a range of PIs for teaching and research. In 1986, the University Grants Committee (UGC) also published the results of the research selectivity exercises which was to influence research funding. This exercise was repeated in 1989 taking into account the quality of research output per member of staff. However there are many difficulties including the problem of weighting different types of outputs; for example different type of publications (see for example Gillett (1989)).

Following these but specifically for the purpose of research outcomes the most comprehensive assessment of research in UK universities is undertaken by the Research Assessment Exercises (RAE) (see for example HEFCE (1996)). The RAE in UK universities aims to produce a quality rating as a basis for the allocation of research grant from funding bodies. (The funding bodies in UK include the Higher Education Funding Council for England (HEFCE), Scottish Higher Education Funding Council (SHEFC), Higher Education Funding Council For Wales (HEFCW) and Department of Education Northern Ireland (DENI)). The first research assessment exercise was carried out in 1986 followed by those in 1989 and 1992. The 1996 RAE was the latest.

Immediately after the publication of the results of the latest Research Assessment Exercise the Higher (see for example the Times Higher Education Supplement (1996)) and other newspapers published a tabulation of universities in a league table. The league table is based on a simple procedure of converting the RAE grades to 1 to 7 to then produce a score by multiplying up by the number of research active staff in a given unit of assessment and taking the average grade for all research active staff in the University.

The most recent development of PIs at a national level has been published by HEFCE in 1999 (see HEFCE (1999b)). The main reason of the development of PIs by HEFCE was the Government's concern with ensuring value for money, increasing accountability and the strengthening of institutional management. Therefore the development of PIs may help HEFCE in distributing the right funds to institutions in terms of their scores obtained from various PIs or to help the institutions with lower scores to improve them to national level relative to the other institutions.

However the main criticism of performance indicators is that they are taking into account only single input and single output at a time. A public sector organisation like a university usually provides a mix of outputs which can not easily be aggregated into a single index of output. In particular some output may be the outcome of several years' investment both in teaching and research. Therefore with using PIs one must produce a set of indicators to over come this problem. Some studies attach weights to mulitple inputs and outputs and take weighted outputs and weighted inputs, but the weights must be given prior to the calculation of PIs. Readers interested in performance indicators in higher education are referred to Cave *et al.* (1991) or Johnes and Taylor (1990).

Data Envelopment Analysis (DEA) when applied to the evaluation of universities has the advantage that there is no need to assign prior weights to inputs and outputs. DEA is attaching the 'best' weights possible for each institution's profile of input-output values. For example, Bessent *et al.* (1983) used the CCR model to analyse the performance of technical colleges. Ahn *et al.* (1988) used DEA to compare the efficiencies of private and public institutions in the USA. Beasley (1989) used DEA for comparing university departments. Readers interested in DEA in higher education could refer to Sarrico (1999).

However, in almost all DEA studies in higher education, data for one year is used. Some authors have indicated that the efficiency of a university could not be captured by analysis of one year's data only. For example Tomkins and Green (1988) in the assessment of UK universities pointed out that "ideally one needs data over more years for some of the variables used". Beasley (1989) used data for one year to analyse the performance of university departments but he has noted that "it is clear departments should be compared over a number of years (e.g. equipment expenditure in one year will affect research output in future years)".

In this chapter we demonstrate how the dynamic DEA model developed in this thesis could be used for evaluating efficiency in higher education. In particular we assess the UK universities for the period 1995 to 1998. The next section identifies input and output variables and sets up the different models which can be used to assess HEIs.

9.3 Setting up the assessment model

9.3.1 Input output variables

The determination of input output variables is difficult in an educational organisation and in particular in university assessments. The main products of a university are its teaching and research outcomes. Therefore in order to assess HEIs on their responsibility of delivering knowledge it is necessary to identify input output variables pertaining to this function.

The inputs should represent all the resources used and the outputs the corresponding activity levels of the research and teaching as main objectives of the HEIs. However, following publication of HEFCE PIs, we want to use inputs and outputs as close to those of HEFCE as possible to make the comparison of the dynamic DEA results with this set of PIs easier. In this set, HEFCE (1999a and 1999b) has used two inputs and two outputs as follows:

Inputs	**Outputs**
Academic staff cost	Number of PhDs awarded
Funding council allocation for research	Income from research grants and contracts

The HEFCE indicators therefore look on the number of PhDs awarded and income of research grants and contracts relative to the academic staff cost and to the funding council allocation for research to that institution. The main advantage of HEFCE PIs is that they took into account the different patterns of input to output in different cost centres and then combined them to give the single indicator. However in our DEA analysis, and due to lack of data over all the assessment period, we ignore the difference in different cost centres and treat all cost centres similarly.

The main criticism of the use of the above input output variables is that academic staff cost is used for training of both undergraduate as well as postgraduates, including PhDs. An indicator of number of PhDs to academic staff cost may be incorrect and perhaps misleading when they are used solely for interpretation of university performance. A university that efficiently uses its resources on the academic staff cost for undergraduate purposes and does not produce a high number of PhDs may be given a very low score. Yet, a university which is not using its resource on the academic staff cost efficiently on teaching but produces a large number of PhDs will be given a higher score. However this indicator would be more acceptable if it used only *research* academic staff cost as input but unfortunately disaggregated data for academic staff cost by research and teaching is not available. The same problem applies to the indicator of research grants and contracts relative to academic staff cost. The numerator of this indicator covers the income from research activity while its denominator covers both research and teaching academic staff cost. To avoid this problem we have to recognise what other output should be involved when we include academic staff cost in a multiple input output model like DEA.

As a proxy of output of academic staff cost we also include in the model number of undergraduates and other postgraduates awarded degrees in addition to the number of PhDs awarded. With these three outputs we need to include other staff cost as well as academic staff cost. Therefore it would be probably better to use total funding council grants for input purposes. This includes both academic and non-academic staff cost as well as any other cost in the institution.

The funding council grant can be generally categorised into recurrent and capital cost (Ahn *et al.* (1988)). Disaggregation of total funding council grant to current and capital enables us to define a dynamic model and to distinguish between current and capital input. Therefore on the input side the two main inputs in our model are capital and recurrent grants allocated by Funding Councils (HEFCE, SHEFC, HEFCW and DENI). The recurrent grants are the block grant for teaching and research and include academic and other staff cost. The capital grants include all non-recurrent grants from the funding council to support special initiatives and capital grants in respect of buildings and equipment. Therefore the inputs are:

- REC : Total recurrent grants.

- CAP : Total capital grants.

On the output side and following the above discussion we consider, for each academic year 4 output measures as follows.

- RGC : Income from research grants and contracts.
- PhDs : Number of PhDs awarded.
- PGs : Number of other postgraduate degrees awarded, not including PhDs.
- UGs : Number of undergraduate degrees awarded.

It must be noted that our model is mainly for comparison with the HEFCE PIs. Both assessments ignore other research outputs such as papers or the quality of research.

Note that we regard research income as an output measure. In HEFCE PIs also it is considered as output but this contrasts with some previous work, for example Beasley (1989) who used research income as input measure. Tomkins and Green (1988) point out that there is confusion over the role of research income. They noted that "some conceptual development is needed regarding income generation as a measure of output. Where income is generated to further academic research that income is an intermediate measure of output." Overall also some have used research income as an input measure and others used it as an output measure but research income is output in some stages and input at another stages. Therefore a static analysis will not be able to capture the role of the research income in educational organisations like universities. However we believe that our dynamic DEA model will capture the role of research income better than static DEA would since in a dynamic model we assess a university over a longer period.

9.3.2 Data

The assessment periods we are examining in this chapter are the academic years 1994-1995, 1995-1996, 1996-1997 and 1997-1998. For simplicity hereafter we refer to each of these academic years to 1994, 1995, 1996 and 1997 respectively. The data we used in this study are derived from the publication of Higher Education Statistics Agency (HESA). The Higher education Statistics Agency is the official agency of the collection, analysis and dissemination of quantitative information between the relevant government departments, the higher education funding councils and universities and colleges.

REC, CAP and RGC are derived from HESA (1996), HESA (1997a), HESA (1998a) and HESA (1999a). UGs, PGs and PhDs are derived from HESA (1997b), HESA (1998b) and HESA (1999b). We include 102 Institutions in our analysis which data is available over the assessment periods.

9.3.3 Assessment by standard DEA

In order to formulate the DEA model for the academic year t we denote;

- REC_j^t : Total recurrent grants in year t for the j^{th} university.
- CAP_j^t : Total capital grants in year t for the j^{th} university.
- RGC_j^t: Income from research grants and contracts in year t for the j^{th} university.
- $PhDs_j^t$: Number of PhDs awarded in year t for the j^{th} university.
- PGs_j^t: Number of other postgraduates awarded in year t for the j^{th} university.
- UGs_j^t: Number of undergraduates awarded in year t for the j^{th} university.

It is assumed that constant returns to scale hold in the DEA analysis. Therefore the DEA model solved, in academic year t, to estimate the relative efficiency of university j_0 is Model 9-1. This is the weights based version of the CRS DEA model. The weights that Mode 9-1 determines are:

- Input weights: $\mathbf{v}^t_{REC}, \mathbf{v}^t_{CAP}$.
- Output weights: $\mathbf{u}^t_{UGs}, \mathbf{u}^t_{PGs}, \mathbf{u}^t_{PhDs}, \mathbf{u}^t_{RGC}$.

These weights are called "virtual multipliers". The weighted output in each year is the "virtual output" in the reference year, t; i.e.

$$WO^t = (\mathbf{u}^t_{UGs} \times UGs) + (\mathbf{u}^t_{PGs} \times PGs) + (\mathbf{u}^t_{PhDs} \times PhDs) + (\mathbf{u}^t_{RGC} \times RGC).$$

The weighted input in each year is the “virtual input” in the reference year, t:

$$WI^t = (\mathbf{v}^t_{REC} \times REC) + (\mathbf{v}^t_{CAP} \times CAP).$$

It is arguable that weights attached to PhDs should be no less than weights attached to PGs, and the weights attached to PGs should be no less than the weights attached to UGs. Therefore a simple weight restriction can be added to the model as follows:

$$\mathbf{u}^t_{UGs} \leq \mathbf{u}^t_{PGs} \leq \mathbf{u}^t_{PhDs.}$$

Beasley (1989) in the analysis of efficiency of UK higher education accounting departments used similar constraints but he restricted them more, ensuring that the weight associated with a PhD is at least 25% greater than the weight associated with a taught postgraduate and a weight associated with a taught postgraduate is at least 25% greater than the weight associated with an undergraduate student. Obviously setting up such weight restrictions would affect the results but for the purpose of our model we admit the concept of his weight restrictions and set up

$$1.25\ \mathbf{u}^t_{PGs} \leq \mathbf{u}^t_{PhDs.}$$

$$1.25\ \mathbf{u}^t_{UGs} \leq \mathbf{u}^t_{PGs.}$$

As Beasley (1989) also mentioned, it is clear that policy makers might have set up their own preferred weights and run the model again.

Therefore for university j_0 Model 9-1 finds the best weights for inputs and outputs so that its efficiency measure is maximised. In other words the model maximises the sum of the ratio of the virtual output to the virtual input in the reference year t. i.e;

$$\text{Maximise} \frac{WO^t}{WI^t}.$$

Therefore each university is assigned the highest possible efficiency score that the constraints allow from the given data by choosing the appropriate virtual multipliers (weights) for the outputs and inputs in the reference year t.

The constraints ensure that none of the HEIs register an efficiency measure greater than 1. If the optimum value of the objective function is 1 then university j_0 is relatively efficient in the sense that it cannot improve the level of any one output without at the same time shrinking the level of some other output or input.

We run the static Model 9-1 for each academic year 1995-96, 1996-97, 1997-98. As an overall static DEA efficiency we calculated the average of the efficiency scores obtained by each institution in these academic years. This average is more suitable for comparison with our dynamic DEA scores over the same years.

Model 9-1. A DEA model for assessing HEIs in academic year t.

$$
\begin{aligned}
& Max\ (u^{t}_{UGs} \times UGs^{t})_{j0} + (u^{t}_{PGs} \times PGs^{t})_{j0} + (u^{t}_{PhDs} \times PhDs^{t})_{j0} + (u^{t}_{RGC} \times RGC^{t})_{j0} \\
& s.t. \\
& \left[(u^{t}_{UGs} \times UGs^{t})_{j} + (u^{t}_{PGs} \times PGs^{t})_{j} + (u^{t}_{PhDs} \times PhDs^{t})_{j} + (u^{t}_{RGC} \times RGC^{t})_{j} \right] \\
& -\left[(v^{t}_{REC} \times REC^{t})_{j} + (v^{t}_{CAP} \times CAP^{t})_{j} \right] \leq 0 \qquad ;\forall j \\
& (v^{t}_{REC} \times REC^{t})_{j0} + (v^{t}_{CAP} \times CAP^{t})_{j0} = 1 \\
& 1.25 u^{t}_{PGs} \leq u^{t}_{PhDs} \\
& 1.25 u^{t}_{UGs} \leq u^{t}_{PGs} \\
& u^{t}_{CAPOUT} \leq u^{t}_{PhDs} \\
& \text{All u and v} > 0.
\end{aligned}
$$

Table 1 (in Appendix C) shows the average efficiency scores obtained. The distribution of institutions over the range of efficiency rating obtained is shown in Table 9-1. The results indicate that in comparative terms all but 5 institutions could reduce some of their source in these academic years.

Table 9-1. Distribution of average relative efficiency obtained from static contemporaneous DEA in 1995-96, 1996-1997 and 1997-98.

Efficiency range	Number of Institutions
Efficiency < 39.99	6
40-49.99	18
50-59.99	28
60-69.99	24
70-79.99	12
80-89.99	5
90-99.99	4
100	5

Therefore, an institution with an efficiency range of 80 to 90 percent should be able to reduce resource levels by between 10 to 20% across the board, and so on for the remaining institutions with efficiency rating below 100%. As already noted, the sole data used in each academic year may mean that the efficiency estimates are incorrect or, alternatively, that an institution which appears relatively inefficient may be able to justify its lower activity levels for its resource levels by investing them for future purposes. This is in particular correct when high level of capital input, for example, could increase the level of future output but it can not be captured in static DEA, then the institution becomes less efficient in the reference assessment period.

The next section assess the performance of institutions over a longer period of time using the dynamic DEA model introduced in this thesis.

9.3.4 Assessment by dynamic DEA

We use the dynamic DEA Model 5-5 for assessing the HEIs on the same data set and for the same academic years as above. For this, we need an initial and a final capital input. Obviously it is very difficult to estimate the initial capital but as a proxy measure the capital input in 1994 is considered as initial capital for assessment periods 1995 to 1997, thought this is only part of the larger underlying capital prior to 1995. We are not capturing changes to base capital prior to 1995. The capital output is the total capital during the periods under assessment and includes initial capital. We have not assigned any depreciation (or appreciation) though this is quite possible once appreciation or depreciation rates are decided upon.

In order to formulate the model mathematically let;

- $CAPIN_j$ be the level of initial capital for the j^{th} university.
- $CAPOUT_j$ be the level of capital output as of the last year of the assessment period for the j^{th} university.

The other variables needed are REC_j^t, CAP_j^t, RGC_j^t, $PhDs_j^t$, PGs_j^t and UGs_j^t which are as defined earlier in this chapter.

In setting dynamic DEA we could use Model 5-5 but we prefer to set up a weights version of the model (see Model 8-11), as it is better for presenting the weights restrictions. Therefore for each university we find the best weights for inputs and outputs in each academic year:

- Input weights: $\mathbf{v}_{CAPIN}$, $\mathbf{v}^t_{REC}$, $\mathbf{v}^t_{CAP}$.
- Output weights: $\mathbf{u}^t_{Ugs}$, $\mathbf{u}^t_{PGs}$, $\mathbf{u}^t_{PhDs}$, $\mathbf{u}^t_{RGC}$, $\mathbf{u}_{CAPOUT}$

We also find the sum of maximum ratio of the weighted output to the weighted input as in Model 9-2.

Like static DEA the output and input weights are called “virtual multipliers”. The weighted output in each year is the “virtual output” in the reference year, t:

$$WO^t = (\mathbf{u}^t_{UGs} \times UGs) + (\mathbf{u}^t_{PGs} \times PGs) + (\mathbf{u}^t_{PhDs} \times PhDs) + (\mathbf{u}^t_{RGC} \times RGC).$$

The virtual output over three years is the sum of the virtual outputs of the three years in the assessment window plus the virtual output for capital output in the last period, i.e.

$$WO = WO^{1995} + WO^{1996} + WO^{1997} + (\mathbf{u}_{CAPOUT} \times CAPOUT)$$

The weighted input in each year is the "virtual input" in the reference year, t:

$$WI^{t} = (\mathbf{v}^{t}_{REC} \times REC) + (\mathbf{v}^{t}_{CAP.} \times CAP).$$

The virtual input is the sum of the virtual inputs of the three years in the assessment window plus virtual input from initial capital input in the first year, i.e.

$$WI = (\mathbf{v}_{CAPIN} \times CAPIN) + WI^{1995} + WI^{1996} + WI^{1997}.$$

The model maximises the average of the ratio of the total virtual output to the total virtual input over periods under consideration subject to holding the virtual input of the institution under assessment equal to unity at each time and make sure that the total virtual output would not be greater than the total virtual input for all institutions in the assessment set.

Therefore each university is assigned the highest possible efficiency score that the constraints allow from the given data by choosing the appropriate virtual multipliers (weights) for the outputs and inputs over assessment periods.

Like static DEA to avoid attaching equal weights for UGs, PGs and PhDs we use the weight restrictions that we set up already. i.e.

$1.25\ \mathbf{u}^t_{PGs} \leq \mathbf{u}^t_{PhDs}\ ;\ \forall t$

$1.25\ \mathbf{u}^t_{UGs} \leq \mathbf{u}^t_{PGs.}\ ;\ \forall t.$

Moreover, with respect to the capital output at the end of assessment periods we felt that the weight associated with it should be related to the weight associated with other outputs within the assessment window. Essentially the purpose of considering capital output at the end of the assessment window is that it can potentially be used to produce outputs in future. However arguably output after the assessment window is less certain than that observed during the window and so the terminal output cannot be more valuable than the output during the window itself. Therefore for the purposes of this study we set up the following weights.

$\mathbf{u}^t_{CAPOUT} \leq \mathbf{u}^t_{UGs}$

$\mathbf{u}^t_{CAPOUT} \leq \mathbf{u}^t_{PGs}$

$\mathbf{u}^t_{CAPOUT} \leq \mathbf{u}^t_{PhDs.}$

An alternative way would be to restrict the weight for terminal capital output in relation to the weight attached to the initial capital. Our initial capital only related to 1994 and so we did not pursue this approach.

With the above specification the model we solved for dynamic DEA is Model 9-2 which is an instance of weights Model 8-11 with extra constraints for weight restrictions.

Model 9-2. Dynamic DEA model for assessing HE institutions in 1995 to 1997.

$$
\begin{aligned}
&Max\ \frac{1}{3}\sum_{t=1995}^{1997}\left[(u^t_{UGs}\times UGs^t)_{j0}+(u^t_{PGs}\times PGs^t)_{j0}+(u^t_{PhDs}\times PhDs^t)_{j0}+(u^t_{RGC}\times RGC^t)_{j0}\right]\\
&\qquad +(u_{CAPOUT}\times CAPOUT)_{j0}-(v_{CAPIN}\times CAPIN)_{j0}\\
&s.t.\\
&(u_{CAPOUT}\times CAPOUT)_j+\\
&\sum_{t=1995}^{1997}\left[\ (u^t_{UGs}\times UGs^t)_j+(u^t_{PGs}\times PGs^t)_j+(u^t_{PhDs}\times PhDs^t)_j+(u^t_{RGC}\times RGC^t)_j\ \right]-\\
&(v_{CAPIN}\times CAPIN)_j-\sum_{t=1995}^{1997}\left[\ (v^t_{REC}\times REC^t)_j+(v^t_{CAP}\times CAP^t)_j\ \right]\le 0 \quad \text{for } j=1,\ldots 102.\\
&\left.\begin{aligned}
&(v^t_{REC}\times REC^t)_{j0}+(v^t_{CAP}\times CAP^t)_{j0}=1\\
&1.25u^t_{PGs}\le u^t_{PhDs}\\
&1.25u^t_{UGs}\le u^t_{PGs}\\
&u^t_{CAPOUT}\le u^t_{PhDs}\\
&u^t_{CAPOUT}\le u^t_{PGs}\\
&u^t_{CAPOUT}\le u^t_{UGs}
\end{aligned}\right\}\text{for } t=1995, 1996 \text{ and } 1997.\\
&\text{All } u \text{ and } v>0.
\end{aligned}
$$

Using this model the total of 102 institutions were assessed using data for the academic years 1994 to 1997. The efficiency score and institutions' ranking are presented in Table 2 of Appendix C. The distribution of HEIs over the range of efficiency ratings obtained are shown in Table 9-2.

Table 9-2. Distribution of relative efficiency obtained from dynamic DEA.

Efficiency range	Number of Institutions
Efficiency < 40	3
40-49.99	9
50-59.99	27
60-69.99	22
70-79.99	18
80-89.99	13
90-99.99	2
100	8

The model indicates that 8 institutions are dynamically efficient while the remaining 94 institutions are dynamically inefficient. It means that 94 inefficient institutions are able to reduce their resource used within years 1995 to 1997 and without reducing any of their output levels. Obviously we did not take into account the quality of output, notably the quality of the research output. This may affect the accuracy of the efficiency scores obtained. But this applies to static DEA and the HEFCE PIs too.

The next section assesses the performance of HEIs using performance indicators.

9.3.5 Assessment HEIs by Performance indicators

In order to compare the three approaches, Dynamic DEA, Static DEA and PIs we constructed PIs defined as the ratios of each output to each input variable used within the dynamic DEA for each academic year. Then the PIs constructed are:

- UGs/ CAP; Ratio of undergraduate degrees awarded to capital cost.
- PhDs / CAP; Ratio of PhDs awarded to capital cost.
- PGs/ CAP; Ratio of other postgraduate degrees awarded to capital cost.
- RGC/ CAP; Ratio of income from research grant and contracts to capital cost.
- UGs/ REC; Ratio of undergraduate degrees awarded to recurrent cost.
- PhDs / REC; Ratio of PhDs awarded to recurrent cost.
- PGs/ REC; Ratio of other postgraduate degrees awarded to recurrent cost.
- RGC/ REC; Ratio of income from research grants and contracts to recurrent cost.

Each one of these ratios is calculated for the three academic year 1995-1996, 1996-1997, 1997-1998. Some of these PIs are very similar to those defined and published by HEFCE (1999b). However we decided to reproduce them here. There are several reasons for this. First, the published PIs are available only for the academic year 1997-98. However our dynamic DEA model covers three academic years. Hence we need to produce PIs to cover the additional academic years. Secondly, in the publication there are only 4 PIs while the number of inputs and outputs in our model gives rise to 8 PIs. Thirdly, our sector comprises 102 institutions hence our dynamic DEA and static DEA scores are represent the efficiency respect to the best frontier within the set of 102 institutions while HEFCE published PIs for 170 institution in academic year 1997-98. The data for three years is available for these institutions only. Finally, the published PIs are adjusted by cost centres neither our DEA models take into account the factor of different cost centres.

We reported in Table 3 (in Appendix C) the average of each PI obtained over the three academic years. This means the higher the PI value, the better the performance of the institutions on that PI. The main difference of this ratios with those of HEFCE comes from the fact that some of our inputs and outputs are defined slightly different from HEFCE and that our PIs are averages of three years while the HEFCE ones cover only the academic year 1997-98.

The PIs do not generally agree on the performance of an institution. Table 9-3 shows the pair - wise correlation of eight PIs.

Table 9-3. Pair - wise correlation PIs

	UGs/ CAP	PGs/ CAP	PhDs / CAP	RGC/ CAP	UGs/ REC	PGs/ REC	PhDs / REC	RGC/ REC
UGs/ CAP	1							
PGs/ CAP	0.112326	1						
PhDs / CAP	-0.37091	0.443588	1					
RGC/ CAP	-0.53604	0.307881	0.869438	1				
UGs/ REC	0.846854	-0.10698	-0.58161	-0.66861	1			
PGs/ REC	-0.16753	0.867131	0.386919	0.320694	-0.20319	1		
PhDs / REC	-0.52272	0.299444	0.939361	0.862777	-0.61271	0.373416	1	
RGC/ REC	-0.70412	0.154068	0.781952	0.953973	-0.71601	0.283883	0.855585	1

The correlation coefficients are generally low in most cases and are negative in some (in 19 pairs out of 28 pairs, the correlation coefficients are negative or less than 0.50) . The negative correlation coefficients are not really surprising and they are fully consistent with different objective of the institutions in the comparison set. For example the negative correlation between UGs/ REC and PhDs / REC (=-0.61271) is showing that universities with high level of research may fail in the undergraduate training and so universities with high level of teaching may not gain high level of the research output. Large correlation coefficients in Table 9-3 are due to the highly correlated corresponding activities captured in those PIs. For example, the coefficient of 0.855585 between PhDs / REC with RGC/ REC indicates that both number of PhDs awarded and income from research grants and contracts are very correlated.

We are not, in this chapter, aiming to discuss the individual results obtained from the PIs but rather to focus on the main difference between PIs and dynamic DEA. In order to make the comparison we need to summarise, in some way, the eight PIs obtained for each institution to one score as overall PI score for the institution over three academic years.

One way usually used for summarising a set of PIs into one indicator is weighting combination of them (see for example Johnes and Taylor (1990)). Generally the weights are given and they may be subjective. Differently, Thanassoulis, Boussofiane and Dyson (1996) used a ranking method to summarise 25 PIs in their study of heath care into 4 PIs. The advantage of the ranking method is that we do not need to give any weights hence the final rank will be less subjective than the weighting method. We adopted, here, their method and summarise the PIs into a set of four indicators. The procedure is as follows.

First we ranked all 102 institutions in each of the overall average of PI values (Table 3 in Appendix C). These ranks are shown in Table 4 (Appendix C). These values mean the lower the PI rank, the better the performance of the institutions on that PI (i.e. rank of 1 is used for the best performing institution on that PI). These ranks are then used to construct four summary PIs as follows:

- Mean Rank (MRank): This is the average of the ranks for each institution across eight PIs in Table 4 (Appendix C).
- Rank of Mean Rank (RMRank): This indicates the ranks of the institution on MRank, 1 is the rank of the institution with the lowest mean rank value in MRank.
- Favourite Rank (FRank): This holds the lowest value of the rank of an institution on all eight PIs. Hence, FRank takes the most favourable view of the performance of the institution as conveyed by its best rank on any one of eight PIs.
- Rank Favourite Rank (RFRank): This indicates the ranks of FRank, the rank 1 means the best performing institution in one of the eight PIs.

All these summary ranks appear in Table 5 in Appendix C. Hence in the next section we comment on the comparison of the scores obtained from the three different approaches.

9.4 *Comparison of dynamic DEA scores with static DEA and PIs*

9.4.1 Difference between the three approaches

We begin with a comparison of dynamic DEA ranks, overall static DEA ranks and PI ranks three different methods for ranking higher education institutions.

The dynamic DEA efficiency measure of institution j_0 maximises the sum of the ratio of the virtual output to virtual input over three academic years (See Model 9-2). The virtual input and virtual output are determined based on the optimal weight value that Model 9-2 assigned to the institution under assessment. These weights values are determined so as the maximise the efficiency score of the institution assessed simultaneously over three academic years and with consideration of what level of capital the institution had before the time horizon and what level of the capital the institution will have at the end of the horizon for the future. The static DEA scores in each year can be seen as being a particular instance of the corresponding dynamic DEA efficiency score. Let us for example assume that in the context of our dynamic DEA model we consider only the academic year of 1996-1997 and ignore the initial and capital constraints in the dynamic DEA. Therefore the dynamic DEA model will collapse to static DEA for the reference year 1996-97. In terms of similarity, hence, it can be seen that static DEA can be thought of as an instance of dynamic DEA. Dynamic DEA thus gives more realistic scores to units where they are operating over several years. One disadvantage of dynamic DEA occurs when there are large numbers of periods in the assessment horizon. In this case the weight flexibility of dynamic DEA may well lead to little discrimination between various units, while static DEA models may well represent the differences between DMUs better.

Similar to the above discussion each PI, in the reference period, can be seen as an instance of the static DEA model in that period with inclusion of the one input and one output as indicated in the PI and replacing all other input outputs to zero. Again the difference between static DEA and PIs will arise from the fact that DEA will examine the performance of a unit with reference to possibility of increasing all outputs, or decreasing all input, simultaneously while PIs consider the maximum gain in a single output from a single input, but both static DEA and PI within the reference year.

Next we see in which cases the three approaches are consistent.

9.4.2 Consistency of the three approaches

We provide in, each approach, an overall rank for institutions. See Table 1 for static efficiency overall rank, Table 2 for dynamic efficiency rank, Table 4 for PIs rank and Table 5 for PIs summary rank (all in Appendix C). These make all three approaches comparable on the performance of HEIs in the sense that in each case a lower rank value represents better performance. Table 9-4 shows the correlation coefficients between dynamic DEA and the other two approaches; PIs and static DEA.

Table 9-4. Correlation of dynamic DEA with PIs and static DEA

Indicator	Correlation
PIs	
UGs/ CAP	0.26
PGs/ CAP	0.54
PhDs / CAP	0.37
RGC/ CAP	0.32
UGs/ REC	0.13
PGs/ REC	0.34
PhDs / REC	0.27
RGC/ REC	0.21
PI based summary measures of performance	
Mean Rank	0.59
Rank of Mean Rank	0.57
Favourite Rank	0.79
Rank Favourite Rank	0.86
DEA	
Overall static DEA	0.92

The correlation coefficients in Table 9-4 show that there are always positive but generally very poor agreement between individual PIs and dynamic DEA. We obviously expect a higher correlation between dynamic DEA and PI-based summary ranks. Table 9-4 shows that the correlation coefficients are relatively higher than with individual PIs. The maximum correlation between dynamic DEA and PIs is with Rank Favourite Rank at 0.86.

We see from Table 9-4 that the overall rank of dynamic DEA is highly correlated to static DEA rank. This suggests that generally dynamic DEA and static DEA are in the same direction with very high association. The more general practical significance of this finding is that we will get a similar view on performance from a period - specific static DEA and from dynamic DEA. Dynamic DEA takes into account more general performance by an institution over several years simultaneously and thus it conveys a broader view of the institution's efficiency.

Despite the overall agreement between the static and dynamic DEA the two approaches disagree substantially in some institutions. This can be seen by looking at the actual ranks of the two approaches in Table 1 and Table 2 (in Appendix C). According these two tables 62 institutions are ranked very closely (Absolute deviation of the two ranks <10), 27 institution are ranked with difference between 10 to 20, the remaining 13 institutions are ranked very differently in the two static and dynamic DEA approaches.

The main reason that dynamic DEA gives different scores to these institutions is that, firstly, dynamic DEA assesses the institutions by examining them over three academic years simultaneously and secondly, the variation of capital input affects much more the dynamic than the static efficiency. We, would argue that the three approaches complement each other rather to replace one another. Each gives a different insight to the efficiency of organisations like universities.

9.5 Further results obtained using dynamic DEA (Supper efficiency, weak efficiency, Peers, target and VRS)

As can be seen in Table 2 (Appendix C) eight Institutions are dynamically efficient (i.e. efficiency score = 1) which determine the dynamic efficient frontier. These include: Cranfield University, Institute of Education, Keele University, London Business School, London School of Economics & Political Science, The London Institute, University College London and University of London. All other institutions were inefficient.

From the dual formulation we could get the source of inefficiency, the target of input output and the peers to each inefficient Institution.

In order to find the institution rank we ordered all institutions by their efficiency score. In this manner we could only distinguish between inefficient institutions as all efficient institutions are ranked the same. It would be useful to distinguish between efficient institutions. Anderson and Peterson (1993) have set up a procedure for ranking efficient units in DEA. This is called supper efficiency. In order to rank the efficient units under the dynamic DEA model we used the same procedure developed by Anderson and Peterson (1993) excluding the institution under assessment from the reference set. The results are presented in table below.

Table 9-5. Supper efficiency and rank of efficient units

Institution	Supper efficiency	Rank under dynamic DEA super-efficiency*
Cranfield University	241.83	1
Keele University	198.77	2
University of London	156.32	3
University College London	150.32	4
The London Institute	149.15	5
London Sch of Economics & Political Sci	117.46	6
London Business School	122.62	7

*Institute of Education has undetermined super efficiency score and so cannot be ranked in this context. See below.

As seen in this table Cranfield University is the most efficient institution.

In order to view where the source of inefficiency (or efficiency) comes from we could analyse the weights obtained under the dynamic efficiency model. For Cranfield University the weights suggest that its efficiency score mainly comes from its high level of income from research grants and contracts, in particular, in 1997.

Our dynamic efficiency scores show 94 institutions are below the dynamic envelope boundary, that is, with efficiency score less than 1. They can achieve higher number of degree awarded (UGs, PGs and PhDs) and / or higher income from research grant and contracts with the same levels of their inputs. Like static DEA, from the dynamic model we can find the peers to each inefficient institution (see Chapter 8). The peers to each institution are presented in Table 6 (Appendix C).

An odd but interesting efficient university is Institute of HE. It is not peer to any non-efficient institution. An analysis of its weights in Model 9-2 and its dual variables shows that this institution is weak efficient. In other words, when we use two phase solutions it gains the objective function of 1 in the first phase but it has non-zero slacks. This can also be seen in Table 9-8 where in the super efficiency calculation the institution becomes unbounded. The reason is probably because this university is specialist in the postgraduate training. So its number of postgraduate is relatively high and this would cause DEA to put it on the frontier, but on the inefficient part of the envelope.

There are numerous options that an inefficient institution can choose for moving itself closer to the efficiency frontier. Like static DEA, an institution can become efficient by increasing its outputs while keeping the inputs at their current levels, or by decreasing its inputs while maintaining the current output levels. This requires simultaneous changes in the input output levels over 3 years. From the weight assigned by dynamic DEA to non - efficient institutions it is possible to calculate virtual inputs, virtual outputs and target for each institution. In particular the target value for inputs, for an inefficient institutions that its slacks in envelope dynamic DEA model are zero, can be obtained using their efficiency scores $\times$ the actual input level.

In earlier chapters we mentioned that it is possible to extend the CRS dynamic DEA to VRS dynamic DEA by adding an extra convexity constraint, i.e,

$\sum_j \lambda_j=1$;

Here we recalculated the dynamic efficiency scores adding the above constraint to Model 9-2. As we expected, like static DEA, the efficiency scores in VRS dynamic DEA generally are greater than the efficiency scores in CRS dynamic DEA. The VRS efficiencies are presented in Table 7 in Appendix C. The deviation between CRS and VRS dynamic efficiency scores differs by institution. There is no change to the scores of 57 institutions. The deviation scores for the rest of the institutions range from 1 to 49.

9.6 *Conclusion*

In this chapter we compared dynamic DEA, static DEA and performance indicators as alternative tools for assessing the performance of higher education institutions in the UK. Such institutions use resources to secure outputs over several years. We commented on the recent publication of HEFCE PIs (1999b) and extended it to cover several more PIs which could complement each other. Then we analysed the same data set using static contemporaneous technology. Static DEA is trying to find the best frontier in each year and ignores the possibility of using previous resources or the possibility of enhancing the resources left for future output. The issue is addressed by setting up a dynamic DEA model.

We then attached a rank to each institution on performance using each of above three assessments to make our results by the three alternative approaches reasonably comparable. The study showed that there is consistency as well as diversion between the three approaches. We concluded that the three approaches complement each other, rather than replace one by another, in the sense that each one offers a different perspective of the performance of each institution.

Further to the above, additional information traditionally obtained in static DEA assessments was also obtained using the dynamic efficiency model on a real data set. Such information includes peers and targets which now can be used to guide an inefficient institution to improved performance over time rather than at one point in time. The next chapter gives a summary of the methods developed in this thesis and comments on potential future research.

CHAPTER 10: Summary, conclusions and further exploration

In this thesis we propose a DEA based approach for assessing the comparative efficiencies of units operating production processes where input - output levels are inter - temporally dependent. One cause of inter - temporal dependence between input and output levels is capital stock which influences output levels over many production periods. Such units cannot be assessed by traditional or 'static' DEA. The method developed in the thesis overcomes the problem of inter - temporal input - output dependence by using input - output 'paths' mapped out by operating units over time as the basis of assessing them.

The aim of this thesis was therefore to deal with the problem that traditional or "static" DEA fails to capture the performance of DMUs with inter – temporally dependent input – output levels. The proposed approach extended static PPS to a dynamic PPS, capturing longer periods of the life of DMUs.

In dynamic PPS one important issue is to capture initial and terminal stock of input. Therefore extra constraints were included in the definition of the PPS to take into account the initial level of stock and the capability of enhancing product from the DMU's terminal stock of input.

The dynamic PPS used to develop a new DEA model for measuring the dynamic efficiency of DMUs.

Using simulated data, we illustrated how snap - shot static efficiencies can fail to capture true performance when there is inter - temporal dependence of input - output levels. The dynamic efficiency model captured better the performance of DMUs in such cases.

The possibility to define alternative measures of dynamic efficiency was examined. As another possibility we defined a non - discretionary period measure. The measure is based on the assumption that managers wish not to raise the input levels in some periods and examine the possibility of reducing input levels in other periods.

Moreover, the use of dual variables in the new approach as input - output price was argued. The interpretation of the dual to the dynamic efficiency model was given, arriving as a value - based dynamic DEA model. This model offers valuable insights on the performance of DMUs being assessed.

Further to these, the conventional methodology used to derive the non-parametric Malmquist index was extended in a straightforward way to a dynamic Malmquist Index using assessment paths. The methodology then was used to examine the efficiency and productivity of OECD countries in the dynamic context and to compare the results with those previously reported for the same data set in the static context. The comparison showed that there is overall consistency but individual diversion in both static and dynamic results. Some individual countries were scored very differently by dynamic vs. static DEA model and we concluded that dynamic efficiency increases when capital stock rises. A similar result was obtained for the productivity index and its components.

A further application used to compare dynamic DEA, static DEA and performance indicators as alternative managerial tools for assessing the performance of organisational units such as higher education institutions. In particular we used data for 102 UK universities to illustrate the differences in the above three methodologies. Such institutions use resources to secure outputs over several years. We also commented on the recent publication of HEFCE PIs (1999b) and extended it to cover several more PIs which could complement each other. Then we analysed the same data set using static contemporaneous technology. Static DEA is trying to find the best frontier in each year and ignores the possibility of using previous resources or the possibility of future enhancement. The issue is addressed by setting up a dynamic DEA model. The study showed that there is consistency as well as diversion between the three approaches. We concluded that the three approaches complement each other, rather replace one by another, in the sense that each one offers a different perspective of the performance of each institution. However the variation of capital input affects much more the dynamic than the static efficiency.

Further to the above, additional information traditionally obtained in static DEA assessments was also obtained using the dynamic efficiency model on a real data set. Such information includes peers and targets which now can be used to guide an inefficient institution to improved performance over time rather than at one point in time.

However further extensions of the dynamic efficiency model are needed. In particular it would be useful to extend the other static DEA models such as additive model to dynamic DEA. In term of definition of the dynamic efficiency measure further it would be useful to extend our measure to a non - radial measure over a sequence of time periods. Also further investigation is needed of the impact of the length of window in dynamic efficiency assessments. Since DMUs are accumulating capital input for further use, the capital may be incorporated into the risk averse behaviour of DMUs, hence it would be interesting to extend the dynamic DEA model to stochastic dynamic DEA.

Appendix A: The simulation results

Table A1. Flow input generated in simulation (I)

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
U1	12	24	46	45	86	16	6	42	48	44	100	91	7	21	72	44
U2	65	2	30	33	17	58	87	99	36	40	54	89	19	50	87	51
U3	74	82	4	96	17	4	24	9	97	6	89	88	65	74	97	55
U4	20	95	39	47	8	11	75	26	91	44	83	98	36	83	3	51
U5	72	88	5	94	59	75	8	76	85	17	73	34	83	34	59	57
U6	77	6	39	42	90	13	66	34	39	61	48	73	57	41	82	51
U7	25	15	99	6	84	68	93	14	19	92	14	42	48	8	13	43
U8	51	95	87	90	23	3	74	63	31	52	80	69	66	78	6	58
U9	43	97	28	37	9	40	31	77	15	20	51	70	77	5	89	46
U10	48	43	75	88	84	42	32	22	81	44	90	19	65	80	75	59
U11	6	22	55	50	10	44	100	11	15	35	6	67	10	19	71	35
U12	39	19	16	13	10	57	38	75	42	95	47	78	3	91	47	45
U13	58	16	51	35	7	77	89	3	5	52	60	96	28	93	5	45
U14	39	23	94	12	13	43	23	23	8	35	45	74	46	68	88	42
U15	29	71	89	46	66	20	7	77	90	66	80	13	42	56	23	52
U16	5	46	21	82	37	94	81	29	46	10	45	92	48	62	93	53
U17	91	36	95	74	92	40	55	88	42	18	40	81	16	58	92	61
U18	49	8	69	48	45	34	49	48	7	47	80	47	90	51	74	50
U19	15	46	6	96	57	82	12	87	75	93	76	31	67	84	23	57
U20	69	92	53	79	25	29	84	84	27	6	10	51	89	65	77	56
U21	38	62	41	98	47	90	82	13	22	93	36	30	52	13	4	48
U22	4	47	41	79	82	12	72	11	2	72	83	55	57	72	4	46
U23	98	28	10	66	4	89	28	2	44	73	24	77	3	33	89	44
U24	27	85	57	57	86	42	96	58	77	16	49	64	47	35	52	57
U25	20	19	83	55	52	55	20	95	41	78	18	63	46	75	89	54
U26	76	87	3	52	53	54	34	23	93	50	74	71	71	51	62	57
U27	40	43	82	97	61	63	79	14	58	36	56	38	81	70	48	58
U28	40	67	78	24	34	45	90	64	25	17	12	94	60	8	3	44
U29	57	87	15	14	25	65	74	46	31	45	39	69	22	61	18	45
U30	74	15	39	58	94	53	81	10	59	43	19	72	29	80	100	55
U31	58	67	32	98	60	92	92	17	70	89	76	65	38	95	62	67
U32	93	12	47	60	54	39	22	9	69	93	40	74	91	91	67	57
U33	71	23	60	80	53	87	28	47	7	9	97	69	15	6	85	49
U34	54	83	48	50	48	27	85	38	98	100	89	47	5	70	31	58
U35	30	86	12	23	52	51	26	58	23	84	68	65	11	52	71	48
U36	15	60	28	67	4	25	29	84	46	38	3	19	99	27	14	37
U37	69	43	93	87	23	62	77	73	86	35	5	54	55	77	65	60
U38	94	98	55	97	36	74	3	31	33	31	67	73	54	32	22	53
U39	89	20	40	51	45	13	4	67	81	36	95	55	85	46	24	50
U40	99	20	45	80	8	85	42	42	54	64	92	94	92	34	13	58

U41	84	43	95	66	78	89	98	44	32	29	13	17	45	70	29	56
U42	90	10	53	37	40	25	65	64	48	88	34	26	31	28	85	48
U43	59	79	100	19	9	87	9	13	21	5	64	6	25	84	52	42
U44	52	54	57	25	24	81	11	72	39	29	97	8	81	20	62	48
U45	15	21	46	93	81	67	95	6	82	88	32	58	33	20	71	54
U46	98	65	83	33	58	60	65	22	75	87	55	25	75	73	5	59
U47	52	91	66	91	41	38	8	73	27	38	81	99	7	10	52	52
U48	61	12	10	58	64	88	45	86	93	31	44	66	56	53	6	51
U49	37	72	34	13	21	6	45	34	38	88	75	87	14	47	70	45
U50	3	10	84	7	58	35	12	97	11	48	92	82	18	29	97	46
U51	66	52	26	24	77	14	46	87	12	46	8	28	18	52	15	38
U52	45	5	25	76	10	21	74	26	57	92	88	72	13	54	88	50
U53	29	53	71	24	49	62	98	70	37	76	50	76	31	74	33	56
U54	19	50	24	40	99	96	22	23	81	90	46	57	52	62	66	55
U55	27	38	43	11	28	64	91	55	17	49	16	26	18	70	2	37
U56	75	4	63	14	31	64	96	95	72	82	90	17	43	27	16	53
U57	99	36	86	39	79	75	48	62	17	37	55	9	36	8	30	48
U58	12	89	43	16	55	25	27	16	22	76	79	73	7	20	26	39
U59	91	35	44	66	18	55	72	66	98	45	66	9	38	97	76	58
U60	8	53	26	42	57	58	52	34	90	87	23	19	25	74	22	45
U61	42	87	95	5	32	68	13	39	85	20	96	80	61	34	95	57
U62	55	44	28	6	79	88	61	23	3	18	14	25	89	82	57	45
U63	64	22	57	38	35	62	81	21	63	36	10	84	16	78	97	51
U64	45	57	10	75	85	62	78	33	66	4	49	81	7	25	50	49
U65	5	89	26	80	65	4	58	83	85	66	68	82	65	30	80	59
U66	88	53	38	60	82	34	13	47	71	58	47	100	20	59	93	58
U67	42	75	100	3	47	64	92	25	68	87	16	12	26	2	98	50
U68	37	68	1	80	75	41	84	14	59	40	45	26	45	13	37	44
U69	13	13	78	76	62	41	17	29	96	57	21	32	24	32	39	42
U70	13	86	3	12	46	14	85	41	18	32	89	73	28	74	67	45
U71	47	12	30	23	93	3	62	15	28	22	61	73	25	15	37	36
U72	67	14	21	33	42	95	95	62	62	6	73	39	82	32	89	54
U73	23	86	10	46	31	92	97	47	50	2	76	80	98	61	41	56
U74	76	62	23	59	24	69	83	55	62	15	63	81	35	12	35	50
U75	54	40	65	39	11	34	61	10	56	42	53	43	60	32	50	43
U76	24	85	96	84	69	88	12	55	71	84	97	38	15	18	76	61
U77	53	56	32	29	77	31	65	59	94	78	85	60	77	46	82	62
U78	96	83	80	27	96	67	43	16	61	79	67	25	96	77	7	61
U79	70	60	96	75	9	48	9	31	89	94	33	14	55	16	7	47
U80	84	48	82	96	65	84	37	50	46	61	43	58	46	74	91	64
U81	55	95	94	42	4	33	84	30	91	73	3	7	7	52	90	51
U82	63	3	97	65	91	14	29	34	21	52	46	44	17	36	99	47
U83	82	58	73	3	51	68	62	87	9	44	32	59	30	49	75	52
U84	42	67	97	6	88	45	12	87	37	2	88	71	77	90	34	56
U85	85	35	54	12	10	15	3	73	90	13	84	6	18	54	96	43
U86	70	25	48	15	53	34	30	4	27	99	71	69	26	83	39	46
U87	28	36	8	76	43	8	23	86	15	84	33	98	71	68	12	46
U88	90	32	5	70	52	17	7	84	55	56	7	59	28	71	55	46
U89	40	94	87	54	31	60	61	43	48	35	58	18	63	49	78	55
U90	25	58	66	68	81	54	49	10	76	31	66	26	16	5	21	44
U91	92	42	74	74	3	13	13	43	69	54	98	44	4	83	82	53

U92	85	68	11	77	44	41	18	48	52	82	18	27	31	35	6	43
U93	99	78	74	90	95	75	13	23	37	92	84	19	66	65	27	62
U94	77	21	43	58	2	66	53	90	22	50	88	76	30	3	80	51
U95	12	21	40	44	9	46	91	59	56	89	18	94	75	57	21	49
U96	13	8	90	36	74	18	14	5	86	57	93	88	2	87	52	48
U97	66	15	85	95	45	76	43	96	29	52	76	7	96	51	63	60
U98	26	11	68	28	2	14	47	70	3	45	6	86	23	19	49	33
U99	64	62	7	96	90	54	67	52	76	67	50	47	21	61	42	57
U100	77	24	26	61	78	70	81	68	43	90	66	22	35	27	89	57
Average	52	48	51	52	48	50	51	47	51	53	55	55	43	50	53	51
Stdv.	28	29	30	29	29	27	31	28	28	28	29	28	28	26	31	29

Table A2. Change in stock input generated in simulation (I)

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
U1	10	42	51	50	88	24	15	47	53	49	100	92	15	29	75	49
U2	69	11	36	39	25	62	88	99	42	45	59	90	26	55	88	56
U3	76	84	13	97	24	13	31	18	97	14	90	89	68	77	97	59
U4	27	95	45	52	16	20	77	33	92	49	85	98	42	85	12	55
U5	74	89	13	94	62	77	16	79	87	25	75	40	84	40	63	61
U6	79	14	45	47	91	21	69	40	45	65	52	75	61	47	83	56
U7	32	22	100	14	85	71	94	22	26	93	22	48	52	17	21	48
U8	55	95	88	91	30	12	76	66	37	56	82	71	69	80	15	62
U9	48	97	35	43	17	45	37	79	23	27	55	73	79	14	90	51
U10	53	48	77	89	85	47	38	29	83	49	91	26	68	82	77	63
U11	14	29	59	55	18	50	100	19	23	41	15	70	18	27	74	41
U12	44	27	24	21	18	61	43	77	47	95	52	80	12	92	52	50
U13	61	23	55	41	15	79	90	12	14	56	63	96	35	94	13	50
U14	45	30	94	20	21	48	30	30	16	41	50	77	51	71	89	48
U15	36	74	90	51	69	27	16	79	91	69	82	21	48	60	30	56
U16	13	50	28	84	43	95	83	36	51	19	50	93	53	65	94	57
U17	92	42	96	76	93	46	59	89	47	26	46	82	23	62	93	65
U18	54	16	72	53	50	40	53	53	15	52	82	52	91	55	77	54
U19	22	51	15	97	61	84	20	88	77	93	78	37	70	86	30	61
U20	71	93	57	81	32	36	86	86	34	15	18	56	90	68	79	60
U21	43	66	46	98	51	91	84	21	29	93	42	36	56	21	13	53
U22	13	51	46	81	84	20	74	19	11	75	85	59	61	75	13	51
U23	98	35	18	69	13	90	35	11	49	76	31	79	11	39	90	50
U24	34	87	61	61	88	48	97	62	79	23	54	67	52	41	57	61
U25	28	27	85	59	57	59	28	95	47	80	25	66	51	77	90	58
U26	78	89	12	57	57	58	40	30	94	54	76	73	74	56	65	61
U27	45	48	84	97	64	66	80	22	61	42	60	44	83	72	53	61
U28	46	70	80	31	40	50	91	67	32	24	20	95	64	16	11	49
U29	61	88	23	22	32	68	76	51	37	50	44	72	29	64	26	50
U30	76	23	45	62	94	57	82	18	63	48	27	75	36	82	100	59
U31	62	70	38	99	63	93	93	24	72	90	78	68	44	95	66	70
U32	94	20	52	64	59	44	29	17	72	93	46	77	91	92	70	61

U33	74	30	64	82	57	88	34	51	16	17	98	72	23	14	86	54
U34	59	85	53	55	53	33	86	44	98	100	90	52	14	72	37	62
U35	36	87	20	30	56	55	33	62	30	85	71	69	19	57	73	52
U36	22	63	34	70	13	32	36	85	51	43	12	27	99	34	22	43
U37	72	49	94	88	30	66	79	75	87	41	14	58	59	79	68	64
U38	95	98	59	97	42	76	12	37	39	38	70	76	59	38	29	58
U39	90	28	45	55	50	21	13	70	83	42	95	59	86	51	31	55
U40	100	27	50	82	16	86	47	47	58	67	93	94	93	40	21	61
U41	85	48	95	69	80	90	98	49	38	35	21	24	50	73	36	59
U42	91	18	57	42	46	32	68	67	53	89	40	33	37	35	86	53
U43	63	81	100	26	17	88	17	21	28	14	67	14	32	85	57	47
U44	56	59	61	32	31	83	19	75	45	35	98	17	83	27	65	52
U45	23	28	51	94	83	70	95	15	84	89	39	62	39	27	74	58
U46	98	68	84	39	62	64	68	29	77	88	60	32	77	76	13	62
U47	57	91	69	92	46	43	17	75	34	43	83	99	16	18	56	56
U48	65	20	18	62	68	89	50	87	94	37	49	69	60	57	14	56
U49	42	75	40	21	28	15	50	40	44	89	77	88	21	52	73	50
U50	12	18	86	15	62	41	20	98	19	53	93	84	26	36	97	51
U51	69	57	33	31	79	22	51	88	20	51	16	35	25	57	23	44
U52	50	13	32	78	18	28	76	33	61	93	89	75	20	58	89	54
U53	35	57	74	31	54	66	98	73	42	78	54	79	38	76	39	60
U54	26	55	31	45	100	96	29	30	83	91	51	61	56	66	69	59
U55	34	44	48	19	35	67	92	59	25	54	24	33	26	73	11	43
U56	77	13	66	22	37	67	96	95	75	84	91	25	48	33	23	57
U57	99	42	87	45	81	77	53	65	25	43	59	18	42	17	37	53
U58	20	90	48	24	59	32	34	24	29	78	81	76	15	27	32	45
U59	92	40	49	69	25	59	74	69	98	50	69	17	44	98	78	62
U60	17	57	33	48	61	62	56	40	91	88	30	26	32	76	29	50
U61	48	88	96	13	38	71	21	45	86	27	97	82	64	40	95	61
U62	59	49	34	15	81	89	65	30	12	26	22	31	90	84	61	50
U63	67	29	61	44	41	65	83	28	67	42	18	86	24	80	97	55
U64	50	61	18	77	86	65	80	39	69	13	54	83	16	32	55	53
U65	14	90	33	82	68	13	62	85	86	69	71	83	68	36	82	63
U66	89	57	44	63	84	40	21	52	74	62	52	100	27	63	93	61
U67	47	77	100	12	52	67	93	32	71	88	24	20	32	11	98	55
U68	42	71	10	82	77	46	86	21	63	46	50	33	50	21	42	49
U69	20	21	80	78	66	46	25	35	96	61	28	38	31	39	44	47
U70	21	88	12	20	51	22	87	47	25	38	90	75	34	77	70	50
U71	52	20	37	30	94	12	66	23	35	29	65	75	32	23	43	42
U72	70	22	28	39	47	95	95	65	65	14	76	45	84	38	90	58
U73	30	87	18	51	37	93	98	52	55	11	78	81	98	64	47	60
U74	78	66	30	63	31	71	84	60	65	23	66	82	41	20	41	55
U75	58	45	69	45	19	40	64	19	60	47	58	48	64	38	55	49
U76	31	87	97	86	72	89	20	59	74	85	97	44	23	26	78	65
U77	57	60	38	36	79	38	68	63	94	80	86	64	79	51	84	65
U78	96	85	82	33	96	70	48	24	64	81	70	32	96	79	15	65
U79	73	64	97	77	18	53	18	37	90	95	39	22	59	24	15	52
U80	85	53	84	97	68	85	43	54	51	64	48	62	51	76	92	68
U81	59	95	95	48	13	39	85	36	92	75	12	15	16	56	91	55
U82	67	12	97	68	91	22	35	40	28	56	51	49	25	42	99	52
U83	84	62	75	11	56	71	66	88	17	49	38	62	36	54	77	56

U84	47	70	97	14	89	50	20	88	43	11	89	74	79	91	40	60
U85	87	41	58	20	18	23	12	75	91	21	86	15	25	58	96	48
U86	73	32	53	23	57	40	36	12	33	99	73	72	33	84	44	51
U87	34	42	17	78	48	16	30	87	23	85	39	98	74	71	20	51
U88	91	38	14	73	56	25	16	85	59	60	16	63	35	73	59	51
U89	46	94	88	59	37	64	64	48	53	40	62	26	66	54	80	59
U90	32	62	69	71	83	58	54	18	78	37	69	33	23	14	29	49
U91	93	47	77	77	12	20	21	48	72	58	98	49	13	84	84	57
U92	86	71	19	79	49	47	25	53	57	83	26	34	37	41	14	48
U93	99	80	76	91	96	78	21	30	43	92	86	27	69	68	34	66
U94	79	29	48	62	11	70	57	91	29	55	89	78	37	12	82	55
U95	20	28	45	49	17	51	92	63	60	90	25	95	77	61	28	53
U96	21	17	91	42	77	25	22	13	88	61	93	89	10	88	56	53
U97	70	23	87	95	50	78	48	96	35	56	78	16	96	55	66	63
U98	33	19	71	34	11	22	51	72	12	50	15	88	30	26	53	39
U99	67	65	16	97	91	58	70	57	78	70	55	52	28	65	48	61
U100	79	31	33	64	80	73	83	71	48	91	69	29	41	34	90	61
Average	57	53	56	57	53	55	56	52	55	57	59	59	49	54	58	55
Stdv.	26	26	27	26	26	24	28	25	26	26	26	25	25	24	28	26

Table A3. Stock input generated in simulation (I)

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15
U1	10	52	103	153	241	265	280	327	380	429	529	621	636	665	740
U2	69	80	116	155	180	242	330	429	471	516	575	665	691	746	834
U3	76	160	173	270	294	307	338	356	453	467	557	646	714	791	888
U4	27	122	167	219	235	255	332	365	457	506	591	689	731	816	828
U5	74	163	176	270	332	409	425	504	591	616	691	731	815	855	918
U6	79	93	138	185	276	297	366	406	451	516	568	643	704	751	834
U7	32	54	154	168	253	324	418	440	466	559	581	629	681	698	719
U8	55	150	238	329	359	371	447	513	550	606	688	759	828	908	923
U9	48	145	180	223	240	285	322	401	424	451	506	579	658	672	762
U10	53	101	178	267	352	399	437	466	549	598	689	715	783	865	942
U11	14	43	102	157	175	225	325	344	367	408	423	493	511	538	612
U12	44	71	95	116	134	195	238	315	362	457	509	589	601	693	745
U13	61	84	139	180	195	274	364	376	390	446	509	605	640	734	747
U14	45	75	169	189	210	258	288	318	334	375	425	502	553	624	713
U15	36	110	200	251	320	347	363	442	533	602	684	705	753	813	843
U16	13	63	91	175	218	313	396	432	483	502	552	645	698	763	857
U17	92	134	230	306	399	445	504	593	640	666	712	794	817	879	972
U18	54	70	142	195	245	285	338	391	406	458	540	592	683	738	815
U19	22	73	88	185	246	330	350	438	515	608	686	723	793	879	909
U20	71	164	221	302	334	370	456	542	576	591	609	665	755	823	902
U21	43	109	155	253	304	395	479	500	529	622	664	700	756	777	790
U22	13	64	110	191	275	295	369	388	399	474	559	618	679	754	767
U23	98	133	151	220	233	323	358	369	418	494	525	604	615	654	744
U24	34	121	182	243	331	379	476	538	617	640	694	761	813	854	911
U25	28	55	140	199	256	315	343	438	485	565	590	656	707	784	874

U26	78	167	179	236	293	351	391	421	515	569	645	718	792	848	913
U27	45	93	177	274	338	404	484	506	567	609	669	713	796	868	921
U28	46	116	196	227	267	317	408	475	507	531	551	646	710	726	737
U29	61	149	172	194	226	294	370	421	458	508	552	624	653	717	743
U30	76	99	144	206	300	357	439	457	520	568	595	670	706	788	888
U31	62	132	170	269	332	425	518	542	614	704	782	850	894	989	1055
U32	94	114	166	230	289	333	362	379	451	544	590	667	758	850	920
U33	74	104	168	250	307	395	429	480	496	513	611	683	706	720	806
U34	59	144	197	252	305	338	424	468	566	666	756	808	822	894	931
U35	36	123	143	173	229	284	317	379	409	494	565	634	653	710	783
U36	22	85	119	189	202	234	270	355	406	449	461	488	587	621	643
U37	72	121	215	303	333	399	478	553	640	681	695	753	812	891	959
U38	95	193	252	349	391	467	479	516	555	593	663	739	798	836	865
U39	90	118	163	218	268	289	302	372	455	497	592	651	737	788	819
U40	100	127	177	259	275	361	408	455	513	580	673	767	860	900	921
U41	85	133	228	297	377	467	565	614	652	687	708	732	782	855	891
U42	91	109	166	208	254	286	354	421	474	563	603	636	673	708	794
U43	63	144	244	270	287	375	392	413	441	455	522	536	568	653	710
U44	56	115	176	208	239	322	341	416	461	496	594	611	694	721	786
U45	23	51	102	196	279	349	444	459	543	632	671	733	772	799	873
U46	98	166	250	289	351	415	483	512	589	677	737	769	846	922	935
U47	57	148	217	309	355	398	415	490	524	567	650	749	765	783	839
U48	65	85	103	165	233	322	372	459	553	590	639	708	768	825	839
U49	42	117	157	178	206	221	271	311	355	444	521	609	630	682	755
U50	12	30	116	131	193	234	254	352	371	424	517	601	627	663	760
U51	69	126	159	190	269	291	342	430	450	501	517	552	577	634	657
U52	50	63	95	173	191	219	295	328	389	482	571	646	666	724	813
U53	35	92	166	197	251	317	415	488	530	608	662	741	779	855	894
U54	26	81	112	157	257	353	382	412	495	586	637	698	754	820	889
U55	34	78	126	145	180	247	339	398	423	477	501	534	560	633	644
U56	77	90	156	178	215	282	378	473	548	632	723	748	796	829	852
U57	99	141	228	273	354	431	484	549	574	617	676	694	736	753	790
U58	20	110	158	182	241	273	307	331	360	438	519	595	610	637	669
U59	92	132	181	250	275	334	408	477	575	625	694	711	755	853	931
U60	17	74	107	155	216	278	334	374	465	553	583	609	641	717	746
U61	48	136	232	245	283	354	375	420	506	533	630	712	776	816	911
U62	59	108	142	157	238	327	392	422	434	460	482	513	603	687	748
U63	67	96	157	201	242	307	390	418	485	527	545	631	655	735	832
U64	50	111	129	206	292	357	437	476	545	558	612	695	711	743	798
U65	14	104	137	219	287	300	362	447	533	602	673	756	824	860	942
U66	89	146	190	253	337	377	398	450	524	586	638	738	765	828	921
U67	47	124	224	236	288	355	448	480	551	639	663	683	715	726	824
U68	42	113	123	205	282	328	414	435	498	544	594	627	677	698	740
U69	20	41	121	199	265	311	336	371	467	528	556	594	625	664	708
U70	21	109	121	141	192	214	301	348	373	411	501	576	610	687	757
U71	52	72	109	139	233	245	311	334	369	398	463	538	570	593	636
U72	70	92	120	159	206	301	396	461	526	540	616	661	745	783	873
U73	30	117	135	186	223	316	414	466	521	532	610	691	789	853	900
U74	78	144	174	237	268	339	423	483	548	571	637	719	760	780	821
U75	58	103	172	217	236	276	340	359	419	466	524	572	636	674	729
U76	31	118	215	301	373	462	482	541	615	700	797	841	864	890	968

U77	57	117	155	191	270	308	376	439	533	613	699	763	842	893	977
U78	96	181	263	296	392	462	510	534	598	679	749	781	877	956	971
U79	73	137	234	311	329	382	400	437	527	622	661	683	742	766	781
U80	85	138	222	319	387	472	515	569	620	684	732	794	845	921	1013
U81	59	154	249	297	310	349	434	470	562	637	649	664	680	736	827
U82	67	79	176	244	335	357	392	432	460	516	567	616	641	683	782
U83	84	146	221	232	288	359	425	513	530	579	617	679	715	769	846
U84	47	117	214	228	317	367	387	475	518	529	618	692	771	862	902
U85	87	128	186	206	224	247	259	334	425	446	532	547	572	630	726
U86	73	105	158	181	238	278	314	326	359	458	531	603	636	720	764
U87	34	76	93	171	219	235	265	352	375	460	499	597	671	742	762
U88	91	129	143	216	272	297	313	398	457	517	533	596	631	704	763
U89	46	140	228	287	324	388	452	500	553	593	655	681	747	801	881
U90	32	94	163	234	317	375	429	447	525	562	631	664	687	701	730
U91	93	140	217	294	306	326	347	395	467	525	623	672	685	769	853
U92	86	157	176	255	304	351	376	429	486	569	595	629	666	707	721
U93	99	179	255	346	442	520	541	571	614	706	792	819	888	956	990
U94	79	108	156	218	229	299	356	447	476	531	620	698	735	747	829
U95	20	48	93	142	159	210	302	365	425	515	540	635	712	773	801
U96	21	38	129	171	248	273	295	308	396	457	550	639	649	737	793
U97	70	93	180	275	325	403	451	547	582	638	716	732	828	883	949
U98	33	52	123	157	168	190	241	313	325	375	390	478	508	534	587
U99	67	132	148	245	336	394	464	521	599	669	724	776	804	869	917
U100	79	110	143	207	287	360	443	514	562	653	722	751	792	826	916
Average	57	110	165	222	275	329	385	437	492	549	608	667	715	770	827
Stdv.	26	35	45	53	60	68	71	72	79	81	85	80	88	93	96

Table A4. Efficiency scores generated in simulation (I)

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
U1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U11	0.96	1.00	0.76	0.97	0.75	0.69	0.90	0.95	0.92	0.93	0.74	1.00	0.91	0.62	0.83	0.86
U12	0.74	1.00	0.74	0.91	0.69	0.96	1.00	1.00	0.76	0.88	1.00	0.97	0.96	0.98	0.77	0.89
U13	0.92	0.90	0.98	1.00	1.00	0.81	0.69	0.79	0.62	0.79	1.00	0.66	0.89	0.87	1.00	0.86
U14	0.85	0.60	0.83	1.00	0.83	1.00	0.91	0.71	1.00	1.00	0.64	0.79	0.86	1.00	0.72	0.85
U15	1.00	0.70	0.78	1.00	0.67	0.76	0.98	0.95	0.69	0.87	0.71	0.62	0.80	0.63	0.72	0.79
U16	1.00	0.74	0.94	0.72	0.89	0.91	0.78	0.92	0.90	0.61	1.00	0.73	0.96	1.00	1.00	0.87
U17	0.69	0.90	0.93	0.76	0.97	0.74	0.75	1.00	0.80	0.84	0.98	0.75	0.97	0.73	0.89	0.85

U18	0.89	0.93	0.78	0.80	0.62	0.93	0.65	0.74	0.61	0.88	0.93	0.78	0.75	0.80	0.98	0.80
U19	0.85	0.68	1.00	0.94	0.74	0.78	0.99	0.94	1.00	0.96	0.83	0.75	0.80	0.83	1.00	0.87
U20	0.90	0.85	0.70	0.76	0.78	0.94	0.95	0.87	0.85	0.64	0.66	0.76	0.90	0.77	0.87	0.81
U21	1.00	0.98	0.67	0.86	0.63	0.72	0.82	0.72	0.63	0.98	0.86	1.00	0.71	1.00	0.78	0.82
U22	0.86	0.81	0.75	0.72	0.82	0.71	0.69	0.90	0.79	0.90	0.95	1.00	0.88	0.87	0.80	0.83
U23	0.74	0.98	0.91	0.72	1.00	0.80	0.93	0.98	0.64	1.00	0.79	1.00	0.87	0.76	0.63	0.85
U24	0.93	0.60	0.78	0.75	0.95	0.92	1.00	0.62	0.68	0.67	0.76	0.88	0.66	0.62	0.77	0.77
U25	0.82	0.67	0.65	1.00	0.84	0.61	0.89	0.83	0.63	0.93	0.66	0.62	0.71	1.00	0.84	0.78
U26	0.91	0.83	0.89	0.92	0.66	0.79	0.85	1.00	1.00	0.85	0.99	0.79	0.82	0.99	0.71	0.87
U27	0.72	0.96	0.76	1.00	0.68	0.97	1.00	0.91	0.61	0.85	0.90	0.66	0.76	0.92	0.72	0.83
U28	1.00	0.81	0.89	0.92	1.00	1.00	0.75	0.70	0.87	0.67	0.91	0.68	0.73	1.00	0.71	0.84
U29	0.93	0.94	0.93	1.00	1.00	0.82	0.81	0.94	0.66	0.98	0.71	0.72	1.00	0.68	0.64	0.85
U30	0.64	0.66	0.91	0.80	0.71	1.00	0.83	0.82	0.65	0.96	0.87	0.75	1.00	0.67	0.66	0.80
U31	0.70	0.73	1.00	0.64	0.69	0.76	0.84	0.71	0.86	0.69	1.00	0.82	0.88	0.66	0.63	0.77
U32	1.00	0.61	1.00	0.70	0.75	1.00	0.72	0.87	0.87	0.80	0.93	0.76	0.99	0.71	0.61	0.82
U33	0.72	0.82	0.70	0.93	0.61	0.95	0.65	0.68	0.62	0.97	0.84	0.67	0.80	1.00	0.83	0.79
U34	1.00	0.66	0.88	1.00	1.00	0.90	0.97	0.66	0.84	0.87	0.72	0.90	0.73	0.79	0.82	0.85
U35	0.65	0.79	0.82	0.88	0.70	0.76	0.88	0.90	0.67	0.76	1.00	0.77	0.97	1.00	0.80	0.82
U36	0.72	0.67	0.63	0.84	0.85	0.68	0.87	0.87	0.93	0.65	1.00	0.63	0.75	0.86	1.00	0.80
U37	1.00	1.00	0.70	0.98	0.70	1.00	0.72	0.98	0.88	0.95	0.63	0.68	1.00	0.91	0.91	0.87
U38	0.74	1.00	0.90	0.93	0.93	0.94	0.69	0.86	0.85	0.89	0.63	0.92	0.96	0.70	1.00	0.86
U39	0.62	1.00	0.74	0.92	0.98	1.00	0.97	0.64	0.64	0.95	0.66	0.64	0.82	1.00	0.98	0.84
U40	0.70	0.90	0.95	0.72	1.00	1.00	1.00	0.85	0.83	0.88	0.96	0.78	0.73	0.78	0.62	0.85
U41	0.93	0.82	0.73	0.62	0.82	0.89	1.00	0.93	0.79	0.82	0.88	0.98	0.96	0.96	0.74	0.86
U42	0.91	0.68	0.67	0.61	0.82	0.70	0.98	0.91	0.91	0.97	1.00	0.91	0.81	0.65	0.96	0.83
U43	0.72	1.00	1.00	0.77	0.78	0.67	0.67	0.79	0.91	1.00	0.61	1.00	0.95	0.76	0.88	0.83
U44	1.00	1.00	0.78	1.00	0.62	0.71	0.82	0.75	1.00	0.95	0.63	0.60	0.98	0.93	0.93	0.85
U45	0.94	0.93	0.87	0.81	0.73	0.68	0.94	0.74	0.94	0.72	0.87	0.72	0.98	0.71	0.94	0.83
U46	0.89	0.62	0.84	0.65	0.78	0.83	0.83	0.90	0.77	1.00	0.90	0.87	1.00	0.76	1.00	0.84
U47	1.00	0.85	1.00	1.00	0.96	0.77	0.97	1.00	0.82	0.94	0.63	0.65	0.95	0.78	1.00	0.89
U48	1.00	0.73	0.99	1.00	0.90	1.00	0.80	1.00	0.83	0.70	0.67	0.62	0.87	0.84	0.88	0.86
U49	0.99	0.94	0.72	0.66	0.91	0.94	0.75	0.88	0.77	1.00	0.62	0.74	0.62	0.81	0.95	0.82
U50	0.82	0.64	0.96	0.84	0.82	0.64	0.63	0.63	0.69	0.60	0.61	1.00	1.00	0.62	0.83	0.76
U51	0.69	0.85	0.94	0.78	0.71	0.97	0.63	0.69	0.77	0.77	0.92	1.00	0.76	0.97	0.89	0.82
U52	0.91	0.67	0.73	1.00	1.00	0.81	0.71	0.87	0.72	0.97	0.64	0.63	0.64	1.00	0.65	0.80
U53	0.64	0.63	0.62	0.64	0.76	0.92	0.75	0.62	0.82	0.76	1.00	0.81	0.79	0.90	0.69	0.76
U54	0.63	0.98	0.90	0.74	0.73	0.91	0.86	0.75	1.00	0.80	0.95	0.73	0.83	0.97	0.62	0.83
U55	0.63	0.73	0.93	0.75	0.85	0.85	0.73	0.68	1.00	0.95	0.71	0.85	0.66	0.95	0.62	0.79
U56	0.79	0.85	0.87	0.72	0.60	0.99	0.99	1.00	1.00	0.64	0.74	0.60	0.98	0.68	0.98	0.83
U57	0.94	0.75	0.87	0.74	0.90	0.73	1.00	0.88	0.83	0.97	0.92	0.72	0.99	0.95	0.89	0.87
U58	0.72	0.76	1.00	0.69	0.62	0.62	0.90	0.88	0.89	0.81	0.68	0.62	1.00	0.71	0.70	0.77
U59	0.83	0.80	0.75	0.86	0.61	0.67	0.63	0.80	0.87	0.86	0.75	0.83	0.60	1.00	0.71	0.77
U60	0.89	0.93	1.00	0.90	0.97	0.70	0.95	1.00	1.00	0.66	0.93	1.00	0.80	0.67	1.00	0.89
U61	0.73	0.65	0.80	0.74	0.74	0.87	0.63	1.00	0.93	0.64	1.00	0.87	0.72	0.92	0.95	0.81
U62	1.00	0.77	0.75	0.61	0.97	1.00	0.68	0.75	0.72	0.68	0.93	0.62	0.81	0.69	0.91	0.79
U63	1.00	0.81	0.93	0.87	0.60	0.82	0.93	0.75	0.71	0.73	0.92	0.98	0.82	0.98	0.64	0.83
U64	0.78	0.71	0.66	0.85	0.88	0.92	0.70	0.81	1.00	1.00	0.93	1.00	1.00	0.62	1.00	0.86
U65	1.00	0.79	0.77	0.90	0.74	0.85	0.68	0.62	1.00	0.91	0.86	0.88	1.00	0.83	0.92	0.85
U66	0.81	0.94	1.00	0.75	0.66	0.89	1.00	0.95	1.00	0.90	0.92	0.78	0.75	0.92	0.91	0.88
U67	0.98	1.00	0.72	0.65	1.00	1.00	0.67	1.00	0.70	0.87	0.73	0.68	1.00	0.98	0.82	0.85
U68	0.65	0.99	1.00	0.98	1.00	0.60	1.00	0.78	0.73	1.00	0.96	0.81	0.84	0.80	0.72	0.86

U69	0.73	1.00	0.81	0.78	0.74	0.78	0.97	0.91	0.74	0.94	0.71	0.70	0.96	0.98	0.86	0.84
U70	0.66	0.93	0.76	0.94	1.00	0.63	0.83	1.00	0.87	0.69	0.86	0.77	0.92	0.77	0.87	0.83
U71	0.93	0.72	0.91	0.97	0.83	0.77	1.00	1.00	0.87	0.64	0.89	0.93	0.79	0.76	1.00	0.87
U72	0.86	0.97	0.88	0.74	1.00	0.69	1.00	1.00	0.75	0.88	1.00	0.73	0.92	0.71	0.83	0.86
U73	0.74	1.00	0.92	0.82	0.98	0.64	0.97	0.82	1.00	0.96	0.76	0.99	1.00	0.85	0.64	0.87
U74	0.89	0.98	1.00	0.90	1.00	0.92	1.00	0.99	0.87	0.71	0.66	0.61	0.78	0.89	0.98	0.88
U75	0.66	0.71	0.77	0.69	0.73	1.00	1.00	0.62	0.79	1.00	0.92	0.71	1.00	0.88	0.78	0.82
U76	0.71	0.75	0.89	1.00	0.91	0.90	0.89	0.88	0.91	0.60	0.73	1.00	1.00	0.75	0.99	0.86
U77	0.72	0.96	0.93	0.93	0.65	0.76	1.00	0.77	0.90	1.00	1.00	0.85	0.84	0.91	0.96	0.88
U78	1.00	0.77	1.00	0.83	0.90	0.87	0.64	0.66	0.65	0.72	0.75	1.00	0.84	0.70	0.90	0.82
U79	0.69	0.86	0.84	0.67	0.65	0.65	0.65	0.72	0.69	1.00	0.95	1.00	1.00	0.83	0.85	0.80
U80	0.60	0.85	1.00	0.94	0.70	0.63	0.66	0.67	0.64	0.86	0.88	1.00	0.86	0.80	1.00	0.81
U81	0.94	1.00	0.82	0.90	0.62	0.94	1.00	0.94	0.77	1.00	0.63	0.66	0.93	0.63	0.78	0.84
U82	0.84	0.65	0.82	1.00	0.91	0.79	0.95	1.00	0.64	0.93	0.63	1.00	0.99	1.00	0.75	0.86
U83	1.00	1.00	0.72	0.81	0.94	0.61	0.88	1.00	0.94	1.00	0.74	0.74	0.75	1.00	0.74	0.86
U84	0.96	1.00	0.61	0.82	0.72	1.00	0.90	0.94	0.95	0.71	0.87	0.84	0.82	0.84	0.91	0.86
U85	0.88	0.97	0.81	1.00	0.65	0.91	0.69	0.80	1.00	0.76	0.63	0.69	0.78	1.00	0.87	0.83
U86	0.82	0.95	0.86	0.91	0.97	0.87	0.67	0.67	0.63	0.73	0.82	0.91	0.97	0.86	0.68	0.82
U87	0.66	0.78	1.00	1.00	0.70	1.00	0.69	0.61	0.68	0.89	0.73	0.91	0.77	1.00	0.81	0.82
U88	0.70	0.64	0.87	0.90	1.00	0.81	0.78	0.81	0.74	0.94	1.00	0.98	1.00	0.66	1.00	0.86
U89	0.80	0.90	0.61	0.85	0.95	0.82	0.69	0.88	0.92	1.00	0.95	0.86	0.79	0.89	0.93	0.86
U90	0.64	0.90	1.00	0.69	0.94	0.61	0.68	0.78	0.81	0.77	0.64	0.86	1.00	0.86	1.00	0.81
U91	0.92	0.64	0.95	0.69	0.84	0.77	0.64	0.88	0.82	0.63	1.00	0.72	0.98	1.00	0.73	0.81
U92	0.86	0.73	0.64	0.80	0.75	0.69	0.94	0.81	0.90	0.87	0.63	0.98	0.94	0.80	0.64	0.80
U93	0.64	0.65	0.73	0.82	0.94	1.00	0.93	0.75	0.90	0.96	0.75	0.96	0.83	0.70	1.00	0.84
U94	0.68	0.95	0.85	0.96	1.00	0.79	0.98	1.00	0.96	0.96	0.90	0.73	0.94	0.62	1.00	0.89
U95	0.90	0.96	0.88	0.73	0.67	0.91	0.90	0.76	0.67	1.00	0.76	0.96	0.94	0.60	0.79	0.83
U96	0.98	1.00	0.88	0.62	0.76	0.85	0.67	0.86	0.85	1.00	1.00	0.82	0.70	0.85	0.83	0.84
U97	0.92	1.00	0.79	0.86	0.88	1.00	0.98	0.76	1.00	0.93	1.00	0.88	0.72	0.99	0.68	0.89
U98	0.97	0.76	1.00	0.91	1.00	0.75	0.73	0.84	0.74	0.70	0.95	1.00	0.98	0.87	0.94	0.88
U99	0.90	0.93	0.89	0.65	0.64	0.66	1.00	0.85	0.72	0.91	0.62	0.81	0.81	0.65	0.83	0.79
U100	0.68	0.85	0.73	0.96	0.64	0.76	0.94	0.73	1.00	0.95	0.80	0.93	0.76	0.76	0.77	0.82
Average	0.85	0.85	0.86	0.85	0.84	0.85	0.86	0.85	0.84	0.87	0.84	0.84	0.88	0.85	0.85	0.85
Stdv.	0.13	0.13	0.12	0.13	0.14	0.13	0.13	0.12	0.13	0.13	0.14	0.14	0.11	0.13	0.13	0.06

Table A5. Efficiency scores generated in simulation (II)

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
U1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

U10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U11	0.33	1.00	0.67	0.64	0.36	0.60	0.99	0.37	0.40	0.54	0.34	1.00	0.36	0.43	0.79	0.59
U12	0.56	1.00	0.40	0.38	0.37	0.69	1.00	1.00	0.59	0.96	1.00	0.84	0.31	0.93	0.62	0.71
U13	0.69	0.40	0.65	1.00	1.00	0.83	0.91	0.31	0.33	0.66	1.00	0.96	0.49	0.94	1.00	0.74
U14	0.56	0.45	0.95	1.00	0.38	1.00	0.45	0.45	1.00	1.00	0.61	0.81	0.62	1.00	0.91	0.75
U15	1.00	0.79	0.91	1.00	0.75	0.43	0.34	0.83	0.92	0.75	0.85	0.38	0.59	0.68	0.45	0.71
U16	1.00	0.61	0.44	0.87	0.55	0.95	0.86	0.50	0.62	0.37	1.00	0.94	0.63	1.00	1.00	0.76
U17	0.93	0.54	0.96	0.81	0.93	0.57	0.68	1.00	0.59	0.42	0.57	0.85	0.40	0.70	0.93	0.73
U18	0.64	0.35	0.78	0.63	0.61	0.53	0.63	0.63	0.34	0.62	0.85	0.62	0.92	0.65	0.81	0.64
U19	0.39	0.61	1.00	0.97	0.69	0.86	0.38	0.90	1.00	0.94	0.82	0.51	0.76	0.88	1.00	0.78
U20	0.77	0.94	0.66	0.84	0.47	0.50	0.88	0.88	0.48	0.34	0.36	0.65	0.91	0.75	0.83	0.68
U21	1.00	0.73	0.58	0.98	0.62	0.92	0.87	0.38	0.45	0.94	0.55	1.00	0.65	1.00	0.32	0.73
U22	0.32	0.62	0.58	0.85	0.87	0.38	0.79	0.37	0.31	0.80	0.87	1.00	0.69	0.80	0.32	0.64
U23	0.97	0.49	0.36	0.75	1.00	0.91	0.49	0.31	0.60	1.00	0.46	1.00	0.31	0.52	0.91	0.67
U24	0.48	0.89	0.69	0.69	0.89	0.59	1.00	0.70	0.83	0.40	0.64	0.74	0.62	0.54	0.66	0.69
U25	0.44	0.43	0.87	1.00	0.66	0.67	0.43	0.95	0.58	0.84	0.42	0.73	0.62	1.00	0.91	0.70
U26	0.82	0.90	0.31	0.66	0.66	0.67	0.53	1.00	1.00	0.64	0.81	0.78	0.79	0.65	0.72	0.73
U27	0.57	0.59	0.87	1.00	0.72	0.73	1.00	0.39	0.69	0.54	0.69	0.56	0.86	0.78	0.63	0.71
U28	1.00	0.76	0.84	0.46	1.00	1.00	0.92	0.74	0.47	0.41	0.37	0.95	0.71	1.00	0.31	0.73
U29	0.69	0.90	0.40	1.00	1.00	0.75	0.81	0.61	0.51	0.61	0.56	0.77	1.00	0.72	0.42	0.72
U30	0.81	0.40	0.56	0.70	0.95	1.00	0.85	0.36	0.71	0.59	0.43	0.80	1.00	0.85	0.99	0.73
U31	0.70	0.76	1.00	0.98	0.71	0.93	0.94	0.41	0.78	0.92	1.00	0.75	0.56	0.95	0.73	0.81
U32	1.00	0.37	1.00	0.71	0.67	1.00	0.44	0.35	0.77	0.94	0.57	0.81	0.92	0.93	0.76	0.75
U33	0.79	0.45	0.71	0.85	0.66	0.90	0.49	0.62	0.34	0.35	0.97	0.78	0.40	1.00	0.88	0.68
U34	1.00	0.87	0.63	1.00	1.00	0.48	0.89	0.56	0.97	0.99	0.92	0.62	0.33	0.78	0.51	0.77
U35	0.50	0.89	0.38	0.46	0.66	0.65	0.48	0.70	0.45	0.88	1.00	0.75	0.37	1.00	0.79	0.66
U36	0.39	0.71	0.49	0.76	0.32	0.47	0.50	0.88	0.62	0.56	1.00	0.43	0.98	0.48	1.00	0.64
U37	1.00	1.00	0.94	0.90	0.45	1.00	0.83	0.80	0.89	0.54	0.33	0.67	1.00	0.83	0.75	0.80
U38	0.95	1.00	0.68	0.97	0.54	0.81	0.31	0.51	0.53	0.51	0.76	0.80	0.67	0.52	1.00	0.70
U39	0.91	1.00	0.57	0.65	0.61	1.00	0.32	0.76	0.86	0.54	0.95	0.67	0.88	1.00	0.46	0.75
U40	0.99	0.43	0.60	0.85	1.00	1.00	1.00	0.59	0.67	0.74	0.94	0.95	0.94	0.53	0.39	0.77
U41	0.88	0.59	0.95	0.76	0.84	0.92	1.00	0.60	0.52	0.49	0.39	0.41	0.60	0.78	0.50	0.68
U42	0.92	0.36	0.66	0.55	0.57	0.47	0.74	0.74	0.63	0.91	1.00	0.47	0.51	0.49	0.88	0.66
U43	0.71	1.00	1.00	0.42	0.35	0.90	0.35	0.39	0.44	1.00	0.74	1.00	0.47	0.88	0.66	0.69
U44	1.00	1.00	0.69	1.00	0.46	0.86	0.37	0.80	1.00	0.49	0.97	0.35	0.86	0.43	0.73	0.73
U45	0.40	0.44	0.61	0.94	0.86	0.76	0.95	0.34	0.87	0.91	0.52	0.70	0.52	0.43	0.79	0.67
U46	0.98	0.75	0.87	0.52	0.70	0.71	0.74	0.45	0.81	1.00	0.68	0.47	1.00	0.80	1.00	0.77
U47	1.00	0.92	1.00	1.00	0.58	0.56	0.35	1.00	0.48	0.55	0.86	0.99	0.34	0.36	1.00	0.73
U48	1.00	0.38	0.36	1.00	0.74	1.00	0.61	1.00	0.94	0.51	0.60	0.75	0.68	0.66	0.33	0.70
U49	0.55	0.80	0.53	0.39	0.44	0.34	0.61	0.53	0.56	1.00	0.82	0.90	0.39	0.62	0.78	0.62
U50	0.32	0.36	0.88	0.34	0.70	0.54	0.38	0.97	0.37	0.63	0.94	1.00	1.00	0.50	0.97	0.66
U51	0.75	0.66	0.48	0.46	0.83	0.39	0.61	0.90	0.38	0.61	0.35	1.00	0.42	0.66	0.40	0.59
U52	0.60	0.32	0.47	1.00	1.00	0.44	0.81	0.48	0.69	0.94	0.91	0.80	0.38	1.00	0.91	0.72
U53	0.49	0.66	0.79	0.46	0.63	0.73	0.98	0.78	0.55	0.82	1.00	0.83	0.51	0.81	0.52	0.70
U54	0.42	0.64	0.46	0.57	0.99	0.96	0.45	0.45	1.00	0.92	0.62	0.69	0.65	0.73	0.75	0.69
U55	0.48	0.56	0.59	0.37	0.49	0.74	0.93	0.68	1.00	0.64	0.41	0.48	0.42	0.78	0.31	0.59
U56	0.81	0.32	0.73	0.39	0.51	0.74	0.96	1.00	1.00	0.86	0.92	0.41	0.59	0.48	0.40	0.67
U57	0.98	0.54	0.89	0.57	0.84	0.82	1.00	0.72	0.41	0.55	0.68	0.36	0.54	0.35	0.50	0.65
U58	0.38	0.91	1.00	0.41	0.67	0.47	0.48	0.40	0.44	0.82	0.84	0.81	1.00	0.43	0.47	0.64
U59	0.93	0.53	0.60	0.75	0.42	0.67	0.79	0.75	0.97	0.60	0.76	0.35	0.56	1.00	0.82	0.70
U60	0.35	0.66	1.00	0.59	0.69	0.70	0.65	1.00	1.00	0.90	0.45	1.00	0.47	0.81	1.00	0.75

U61	0.59	0.90	0.96	0.33	0.52	0.77	0.38	1.00	0.88	0.43	1.00	0.85	0.72	0.53	0.95	0.72
U62	1.00	0.60	0.49	0.34	0.84	1.00	0.72	0.45	0.32	0.42	0.39	0.46	0.91	0.87	0.69	0.63
U63	1.00	0.44	0.69	0.56	0.54	0.72	0.86	0.44	0.73	0.55	0.36	0.88	0.40	0.84	0.97	0.67
U64	0.60	0.69	0.36	0.82	0.88	0.73	0.84	0.52	1.00	1.00	0.63	1.00	1.00	0.47	1.00	0.77
U65	1.00	0.91	0.48	0.85	0.74	0.32	0.70	0.87	1.00	0.75	0.77	0.86	1.00	0.50	0.85	0.77
U66	0.91	0.66	1.00	0.71	0.87	0.53	1.00	0.62	1.00	0.70	0.62	0.99	0.43	0.71	0.94	0.78
U67	0.58	1.00	0.99	0.31	1.00	1.00	0.93	1.00	0.77	0.90	0.40	0.38	1.00	0.31	0.98	0.77
U68	0.55	0.77	1.00	0.85	1.00	0.58	1.00	0.39	0.71	1.00	0.60	0.47	0.61	0.39	0.55	0.70
U69	0.38	1.00	0.84	0.82	0.73	0.58	0.41	0.49	0.96	0.69	0.44	0.51	0.46	0.52	0.56	0.63
U70	0.38	0.90	0.31	0.38	1.00	0.39	0.89	1.00	0.42	0.52	0.91	0.80	0.49	0.81	0.76	0.66
U71	0.62	0.37	0.50	0.45	0.94	0.31	1.00	1.00	0.49	0.44	0.72	0.80	0.47	0.40	1.00	0.63
U72	0.76	0.39	0.44	0.52	1.00	0.95	1.00	1.00	0.72	0.33	1.00	0.57	0.87	0.52	0.91	0.73
U73	0.45	1.00	0.36	0.61	0.51	0.93	0.97	0.62	1.00	0.31	0.82	0.85	1.00	0.72	0.58	0.72
U74	0.82	0.73	1.00	0.71	1.00	0.77	1.00	0.68	0.72	0.40	0.73	0.85	0.54	0.37	0.53	0.72
U75	0.67	0.57	0.75	0.57	0.37	1.00	1.00	0.37	0.68	1.00	0.66	0.59	1.00	0.52	0.64	0.69
U76	0.46	0.89	0.96	1.00	0.77	0.91	0.37	0.68	0.79	0.88	0.97	1.00	1.00	0.42	0.82	0.79
U77	0.66	0.69	0.51	0.50	0.83	0.51	1.00	0.70	0.94	1.00	1.00	0.71	0.83	0.61	0.87	0.76
U78	1.00	0.87	1.00	0.48	0.96	0.76	0.59	0.41	0.72	0.84	0.76	1.00	0.96	0.83	0.34	0.77
U79	0.78	0.71	0.96	0.81	0.36	0.63	0.36	0.51	0.91	1.00	0.52	1.00	1.00	0.41	0.34	0.69
U80	0.88	0.63	1.00	0.96	0.75	0.88	0.55	0.64	0.61	0.71	0.59	1.00	0.61	0.81	1.00	0.77
U81	0.68	1.00	0.95	0.59	0.32	0.52	1.00	0.50	0.93	1.00	0.32	0.34	0.34	0.66	0.92	0.67
U82	0.73	0.31	0.97	1.00	0.92	0.39	0.50	1.00	0.44	0.66	0.61	1.00	0.41	1.00	0.98	0.73
U83	1.00	1.00	0.80	0.31	0.65	0.77	0.73	1.00	0.35	1.00	0.52	0.70	0.50	1.00	0.82	0.74
U84	0.59	1.00	0.97	0.33	0.91	1.00	0.38	0.90	0.55	0.31	0.91	0.79	0.83	0.92	0.53	0.73
U85	0.89	0.54	0.67	1.00	0.37	0.40	0.31	0.80	1.00	0.39	0.88	0.34	0.42	1.00	0.96	0.66
U86	0.78	0.47	0.63	0.40	0.66	0.53	0.50	0.32	0.48	0.98	0.79	0.78	0.47	0.87	0.56	0.61
U87	0.49	0.55	1.00	1.00	0.59	1.00	0.46	0.89	0.40	0.88	0.52	0.98	0.79	1.00	0.38	0.73
U88	0.92	0.51	0.33	0.78	1.00	0.41	0.34	0.88	0.67	0.68	1.00	0.71	1.00	0.79	1.00	0.73
U89	0.57	0.95	0.90	0.67	0.51	0.71	0.72	0.59	0.63	1.00	0.70	0.42	0.73	0.64	0.83	0.70
U90	0.47	0.70	1.00	0.76	0.86	0.67	0.64	0.36	0.82	0.51	0.75	0.47	1.00	0.33	1.00	0.69
U91	0.93	0.58	0.81	0.81	0.32	0.38	0.38	0.59	0.78	0.67	1.00	0.60	0.32	1.00	0.87	0.67
U92	0.89	0.76	0.37	0.83	0.60	0.58	0.42	0.63	0.66	0.86	0.42	0.48	0.51	0.54	0.33	0.59
U93	0.99	0.84	0.81	0.92	0.96	1.00	0.38	0.45	0.55	0.93	0.88	0.43	0.75	0.75	1.00	0.78
U94	0.83	0.44	0.59	0.70	1.00	0.76	0.66	1.00	0.44	0.64	0.91	0.82	0.50	0.31	1.00	0.71
U95	0.38	0.44	0.57	0.60	0.36	0.61	0.93	0.70	0.68	1.00	0.42	0.95	0.82	0.69	0.44	0.64
U96	0.38	1.00	0.92	0.55	0.81	0.42	0.39	0.33	0.89	1.00	1.00	0.91	0.30	0.90	0.65	0.70
U97	0.76	1.00	0.89	0.95	0.61	1.00	0.59	0.96	1.00	0.65	1.00	0.35	0.96	0.65	0.73	0.81
U98	0.48	0.37	1.00	0.49	1.00	0.39	0.62	0.78	0.31	0.61	0.33	1.00	0.45	0.43	0.63	0.59
U99	0.74	0.72	0.34	0.96	0.92	0.67	1.00	0.66	0.82	0.76	0.64	0.62	0.44	0.72	0.59	0.71
U100	0.83	0.46	0.48	0.72	0.83	0.78	0.86	0.76	1.00	0.92	0.75	0.44	0.54	0.48	0.91	0.72
Average	0.75	0.72	0.75	0.74	0.74	0.74	0.72	0.70	0.72	0.75	0.74	0.76	0.70	0.73	0.76	0.73
Stdev	0.23	0.23	0.23	0.23	0.22	0.22	0.25	0.24	0.23	0.23	0.23	0.22	0.24	0.22	0.23	0.10

Table B1a: Average efficiency in simulation (I) for technology TEC1

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.852	0.760	0.599	0.580	0.612	0.572	0.606	0.545	0.562	0.566	0.569	0.612	0.625	0.585	0.631	0.618
Dyn-2		0.852	0.858	0.855	0.757	0.740	0.757	0.725	0.760	0.702	0.728	0.751	0.735	0.746	0.748	0.765
Dyn-3			0.852	0.858	0.899	0.902	0.834	0.825	0.837	0.823	0.851	0.808	0.812	0.830	0.831	0.843
Dyn-4				0.852	0.858	0.899	0.928	0.924	0.884	0.890	0.892	0.880	0.905	0.869	0.866	0.887
Dyn-5					0.941	0.946	0.924	0.927	0.932	0.926	0.945	0.916	0.916	0.924	0.936	0.930
Dyn-6						0.957	0.963	0.952	0.947	0.956	0.958	0.966	0.947	0.943	0.952	0.954
Dyn-7							0.972	0.976	0.969	0.969	0.972	0.978	0.977	0.965	0.967	0.972
Dyn-8								0.983	0.986	0.981	0.982	0.986	0.985	0.986	0.979	0.983
Dyn-9									0.991	0.992	0.990	0.989	0.992	0.993	0.992	0.991
Dyn-10										0.995	0.995	0.995	0.992	0.993	0.996	0.995
Dyn-11											0.997	0.998	0.998	0.994	0.996	0.997
Dyn-12												0.998	0.999	0.999	0.996	0.998
Dyn-13													0.999	0.999	0.999	0.999
Dyn-14														0.999	0.999	0.999
Dyn-15															1.000	1.000

Table B1b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC1

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.009	0.099	0.266	0.289	0.236	0.281	0.260	0.316	0.291	0.304	0.284	0.234	0.269	0.277	0.243	0.244
Dyn-2		0.137	0.115	0.117	0.167	0.190	0.172	0.192	0.164	0.231	0.215	0.182	0.207	0.175	0.189	0.175
Dyn-3			0.135	0.114	0.118	0.129	0.156	0.144	0.138	0.157	0.158	0.162	0.170	0.157	0.166	0.147
Dyn-4				0.127	0.126	0.131	0.125	0.101	0.119	0.130	0.137	0.140	0.113	0.153	0.131	0.128
Dyn-5					0.121	0.122	0.133	0.116	0.134	0.127	0.137	0.142	0.114	0.130	0.131	0.128
Dyn-6						0.131	0.129	0.128	0.142	0.113	0.146	0.155	0.106	0.125	0.126	0.130
Dyn-7							0.130	0.125	0.146	0.124	0.143	0.148	0.113	0.137	0.121	0.132
Dyn-8								0.131	0.151	0.123	0.154	0.158	0.111	0.144	0.142	0.139
Dyn-9									0.155	0.125	0.154	0.156	0.115	0.146	0.145	0.142
Dyn-10										0.127	0.155	0.160	0.113	0.147	0.144	0.141
Dyn-11											0.156	0.163	0.118	0.147	0.147	0.146
Dyn-12												0.164	0.119	0.150	0.143	0.144
Dyn-13													0.119	0.150	0.146	0.138
Dyn-14														0.150	0.146	0.148
Dyn-15															0.147	0.147

Table B2a: Average efficiency in simulation for technology TEC2

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.852	0.782	0.636	0.606	0.634	0.593	0.627	0.565	0.583	0.597	0.604	0.632	0.648	0.607	0.649	0.641
Dyn-2		0.852	0.857	0.856	0.775	0.757	0.770	0.741	0.774	0.722	0.743	0.768	0.747	0.760	0.763	0.777
Dyn-3			0.852	0.857	0.900	0.905	0.844	0.837	0.847	0.836	0.860	0.821	0.828	0.840	0.841	0.851
Dyn-4				0.852	0.857	0.900	0.930	0.926	0.893	0.897	0.901	0.890	0.912	0.878	0.875	0.893
Dyn-5					0.942	0.949	0.931	0.933	0.940	0.934	0.950	0.923	0.924	0.931	0.941	0.936
Dyn-6						0.959	0.966	0.959	0.951	0.962	0.964	0.969	0.952	0.948	0.959	0.959
Dyn-7							0.973	0.979	0.974	0.972	0.976	0.981	0.979	0.968	0.970	0.975
Dyn-8								0.984	0.988	0.985	0.984	0.989	0.988	0.987	0.981	0.986
Dyn-9									0.992	0.994	0.993	0.989	0.993	0.994	0.993	0.993
Dyn-10										0.996	0.996	0.997	0.992	0.994	0.997	0.996
Dyn-11											0.997	0.998	0.998	0.995	0.997	0.997
Dyn-12												0.999	0.999	0.999	0.996	0.998
Dyn-13													1.000	0.999	1.000	1.000
Dyn-14														1.000	0.999	1.000
Dyn-15															1.000	1.000

Table B2b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC2

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.009	0.076	0.229	0.264	0.213	0.259	0.239	0.295	0.270	0.273	0.253	0.213	0.245	0.254	0.225	0.221
Dyn-2		0.137	0.115	0.116	0.160	0.179	0.162	0.182	0.157	0.216	0.206	0.172	0.197	0.167	0.179	0.167
Dyn-3			0.135	0.114	0.118	0.129	0.151	0.140	0.135	0.150	0.155	0.157	0.163	0.151	0.162	0.143
Dyn-4				0.127	0.125	0.131	0.123	0.101	0.121	0.130	0.135	0.140	0.112	0.148	0.128	0.127
Dyn-5					0.121	0.123	0.132	0.116	0.135	0.125	0.138	0.142	0.113	0.129	0.130	0.128
Dyn-6						0.132	0.129	0.127	0.144	0.113	0.145	0.156	0.107	0.126	0.127	0.131
Dyn-7							0.130	0.127	0.148	0.124	0.145	0.151	0.113	0.138	0.122	0.133
Dyn-8								0.132	0.153	0.123	0.155	0.160	0.112	0.144	0.142	0.140
Dyn-9									0.156	0.126	0.154	0.157	0.115	0.146	0.145	0.143
Dyn-10										0.127	0.156	0.162	0.113	0.147	0.145	0.142
Dyn-11											0.156	0.163	0.118	0.147	0.146	0.146
Dyn-12												0.164	0.119	0.150	0.143	0.144
Dyn-13													0.120	0.150	0.147	0.139
Dyn-14														0.151	0.146	0.148
Dyn-15															0.147	0.147

Table B3a: Average efficiency in simulation for technology TEC3

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Ave rage
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.837	0.737	0.596	0.577	0.602	0.562	0.594	0.540	0.548	0.566	0.571	0.598	0.580	0.549	0.624	0.605
Dyn-2		0.837	0.863	0.856	0.761	0.735	0.750	0.702	0.742	0.692	0.717	0.746	0.731	0.733	0.733	0.757
Dyn-3			0.837	0.863	0.893	0.904	0.829	0.822	0.831	0.805	0.845	0.800	0.809	0.823	0.825	0.837
Dyn-4				0.837	0.863	0.893	0.923	0.919	0.877	0.884	0.880	0.870	0.897	0.861	0.862	0.880
Dyn-5					0.934	0.938	0.918	0.919	0.923	0.919	0.937	0.914	0.909	0.918	0.928	0.923
Dyn-6						0.949	0.956	0.945	0.940	0.948	0.953	0.960	0.942	0.935	0.948	0.948
Dyn-7							0.966	0.969	0.960	0.965	0.965	0.973	0.974	0.961	0.962	0.966
Dyn-8								0.977	0.979	0.975	0.977	0.981	0.981	0.983	0.975	0.979
Dyn-9									0.985	0.988	0.986	0.987	0.988	0.989	0.990	0.987
Dyn-10										0.992	0.992	0.994	0.991	0.991	0.993	0.992
Dyn-11											0.994	0.996	0.996	0.993	0.994	0.995
Dyn-12												0.997	0.997	0.998	0.994	0.997
Dyn-13													0.998	0.998	0.999	0.998
Dyn-14														0.998	0.998	0.998
Dyn-15															0.999	0.999

Table B3b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC3

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Ave rage
Static	0.014	0.117	0.271	0.293	0.245	0.291	0.272	0.321	0.304	0.304	0.283	0.246	0.308	0.308	0.248	0.255
Dyn-2		0.137	0.115	0.108	0.162	0.189	0.175	0.201	0.170	0.239	0.219	0.177	0.205	0.178	0.199	0.177
Dyn-3			0.134	0.115	0.117	0.128	0.158	0.140	0.137	0.163	0.157	0.159	0.172	0.152	0.161	0.145
Dyn-4				0.127	0.125	0.133	0.127	0.099	0.115	0.128	0.132	0.140	0.114	0.151	0.133	0.127
Dyn-5					0.117	0.120	0.133	0.113	0.132	0.127	0.133	0.138	0.115	0.123	0.126	0.125
Dyn-6						0.127	0.129	0.124	0.136	0.113	0.145	0.151	0.104	0.124	0.125	0.128
Dyn-7							0.131	0.121	0.140	0.124	0.140	0.146	0.110	0.137	0.120	0.130
Dyn-8								0.126	0.147	0.124	0.151	0.155	0.109	0.142	0.139	0.136
Dyn-9									0.150	0.124	0.152	0.155	0.112	0.144	0.142	0.140
Dyn-10										0.126	0.154	0.160	0.112	0.146	0.142	0.140
Dyn-11											0.155	0.162	0.117	0.147	0.145	0.145
Dyn-12												0.163	0.117	0.149	0.142	0.143
Dyn-13													0.118	0.149	0.146	0.138
Dyn-14														0.149	0.145	0.147
Dyn-15															0.146	0.146

Table B4a: Average efficiency in simulation for technology TEC4

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.88	0.849	0.853	0.851
Static	0.837	0.737	0.596	0.577	0.602	0.562	0.594	0.54	0.548	0.566	0.571	0.598	0.58	0.549	0.624	0.605
Dyn-2		0.837	0.863	0.856	0.761	0.735	0.75	0.702	0.742	0.692	0.717	0.746	0.731	0.733	0.733	0.757
Dyn-3			0.837	0.863	0.893	0.904	0.829	0.822	0.831	0.805	0.845	0.8	0.809	0.823	0.825	0.837
Dyn-4				0.837	0.863	0.893	0.923	0.919	0.877	0.884	0.88	0.87	0.897	0.861	0.862	0.88
Dyn-5					0.934	0.938	0.918	0.919	0.923	0.919	0.937	0.914	0.909	0.918	0.928	0.923
Dyn-6						0.949	0.956	0.945	0.94	0.948	0.953	0.96	0.942	0.935	0.948	0.948
Dyn-7							0.966	0.969	0.96	0.965	0.965	0.973	0.974	0.961	0.962	0.966
Dyn-8								0.977	0.979	0.975	0.977	0.981	0.981	0.983	0.975	0.979
Dyn-9									0.985	0.988	0.986	0.987	0.988	0.989	0.99	0.987
Dyn-10										0.992	0.992	0.994	0.991	0.991	0.993	0.992
Dyn-11											0.994	0.996	0.996	0.993	0.994	0.995
Dyn-12												0.997	0.997	0.998	0.994	0.997
Dyn-13													0.998	0.998	0.999	0.998
Dyn-14														0.998	0.998	0.998
Dyn-15															0.999	0.999

Table B4b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC4

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.014	0.117	0.271	0.293	0.245	0.291	0.272	0.321	0.304	0.304	0.283	0.246	0.308	0.308	0.248	0.255
Dyn-2		0.137	0.115	0.108	0.162	0.189	0.175	0.201	0.17	0.239	0.219	0.177	0.205	0.178	0.199	0.177
Dyn-3			0.134	0.115	0.117	0.128	0.158	0.14	0.137	0.163	0.157	0.159	0.172	0.152	0.161	0.145
Dyn-4				0.127	0.125	0.133	0.127	0.099	0.115	0.128	0.132	0.14	0.114	0.151	0.133	0.127
Dyn-5					0.117	0.12	0.133	0.113	0.132	0.127	0.133	0.138	0.115	0.123	0.126	0.125
Dyn-6						0.127	0.129	0.124	0.136	0.113	0.145	0.151	0.104	0.124	0.125	0.128
Dyn-7							0.131	0.121	0.14	0.124	0.14	0.146	0.11	0.137	0.12	0.13
Dyn-8								0.126	0.147	0.124	0.151	0.155	0.109	0.142	0.139	0.136
Dyn-9									0.15	0.124	0.152	0.155	0.112	0.144	0.142	0.14
Dyn-10										0.126	0.154	0.16	0.112	0.146	0.142	0.14
Dyn-11											0.155	0.162	0.117	0.147	0.145	0.145
Dyn-12												0.163	0.117	0.149	0.142	0.143
Dyn-13													0.118	0.149	0.146	0.138
Dyn-14														0.149	0.145	0.147
Dyn-15															0.146	0.146

Table B5a: Average efficiency in simulation for technology TEC5

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.842	0.790	0.741	0.712	0.699	0.682	0.726	0.660	0.699	0.696	0.730	0.704	0.724	0.711	0.705	0.721
Dyn-2		0.842	0.830	0.830	0.800	0.778	0.801	0.789	0.800	0.778	0.814	0.801	0.807	0.798	0.785	0.804
Dyn-3			0.842	0.830	0.857	0.869	0.845	0.850	0.848	0.855	0.869	0.850	0.867	0.857	0.857	0.853
Dyn-4				0.842	0.830	0.857	0.888	0.890	0.895	0.894	0.889	0.903	0.901	0.892	0.894	0.881
Dyn-5					0.910	0.920	0.924	0.921	0.924	0.932	0.936	0.928	0.926	0.923	0.926	0.925
Dyn-6						0.930	0.942	0.942	0.941	0.946	0.954	0.958	0.950	0.944	0.947	0.945
Dyn-7							0.950	0.955	0.955	0.960	0.961	0.972	0.969	0.964	0.965	0.961
Dyn-8								0.962	0.966	0.971	0.973	0.979	0.980	0.979	0.976	0.973
Dyn-9									0.972	0.980	0.981	0.987	0.986	0.988	0.986	0.983
Dyn-10										0.985	0.986	0.991	0.991	0.990	0.991	0.989
Dyn-11											0.988	0.993	0.995	0.993	0.992	0.992
Dyn-12												0.994	0.995	0.996	0.994	0.995
Dyn-13													0.996	0.996	0.997	0.997
Dyn-14														0.997	0.997	0.997
Dyn-15															0.998	0.998

Table B5b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC5

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.010	0.065	0.122	0.157	0.142	0.171	0.143	0.199	0.149	0.176	0.124	0.138	0.169	0.146	0.160	0.138
Dyn-2		0.136	0.119	0.114	0.130	0.151	0.144	0.136	0.123	0.177	0.147	0.122	0.135	0.141	0.146	0.137
Dyn-3			0.132	0.117	0.114	0.125	0.134	0.114	0.115	0.127	0.136	0.129	0.120	0.127	0.124	0.124
Dyn-4				0.125	0.129	0.126	0.127	0.097	0.119	0.114	0.127	0.133	0.104	0.128	0.124	0.121
Dyn-5					0.107	0.120	0.128	0.106	0.121	0.108	0.130	0.124	0.095	0.119	0.110	0.115
Dyn-6						0.123	0.131	0.114	0.128	0.110	0.136	0.139	0.092	0.126	0.118	0.122
Dyn-7							0.131	0.116	0.137	0.114	0.138	0.146	0.101	0.134	0.125	0.127
Dyn-8								0.116	0.143	0.118	0.145	0.151	0.106	0.136	0.132	0.131
Dyn-9									0.145	0.122	0.148	0.155	0.110	0.142	0.136	0.137
Dyn-10										0.125	0.151	0.159	0.113	0.144	0.140	0.139
Dyn-11											0.152	0.160	0.115	0.147	0.141	0.143
Dyn-12												0.161	0.116	0.149	0.143	0.142
Dyn-13													0.117	0.149	0.145	0.137
Dyn-14														0.149	0.145	0.147
Dyn-15															0.146	0.146

Table B6a: Average efficiency in simulation for technology TEC6

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.843	0.826	0.720	0.754	0.706	0.661	0.721	0.658	0.654	0.708	0.678	0.692	0.697	0.703	0.737	0.717
Dyn-2		0.843	0.854	0.862	0.828	0.832	0.798	0.791	0.824	0.780	0.798	0.813	0.794	0.802	0.812	0.816
Dyn-3			0.843	0.854	0.890	0.909	0.868	0.876	0.861	0.850	0.879	0.862	0.867	0.867	0.877	0.869
Dyn-4				0.843	0.854	0.890	0.922	0.917	0.907	0.917	0.898	0.897	0.923	0.904	0.902	0.898
Dyn-5					0.929	0.936	0.939	0.940	0.930	0.940	0.950	0.939	0.941	0.943	0.945	0.939
Dyn-6						0.947	0.957	0.958	0.953	0.956	0.961	0.968	0.963	0.956	0.964	0.958
Dyn-7							0.964	0.970	0.967	0.973	0.970	0.978	0.981	0.975	0.975	0.973
Dyn-8								0.976	0.980	0.981	0.983	0.983	0.986	0.987	0.984	0.982
Dyn-9									0.984	0.988	0.987	0.991	0.990	0.991	0.992	0.989
Dyn-10										0.991	0.991	0.995	0.995	0.993	0.994	0.993
Dyn-11											0.993	0.996	0.997	0.996	0.996	0.995
Dyn-12												0.996	0.998	0.998	0.997	0.997
Dyn-13													0.998	0.998	0.999	0.998
Dyn-14														0.998	0.998	0.998
Dyn-15															0.999	0.999

Table B6b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC6

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.010	0.031	0.145	0.131	0.137	0.192	0.152	0.201	0.191	0.165	0.175	0.157	0.197	0.157	0.136	0.145
Dyn-2		0.137	0.114	0.104	0.130	0.129	0.141	0.138	0.126	0.171	0.159	0.120	0.145	0.141	0.139	0.135
Dyn-3			0.132	0.111	0.114	0.127	0.138	0.112	0.121	0.128	0.136	0.135	0.128	0.119	0.128	0.125
Dyn-4				0.127	0.123	0.130	0.128	0.099	0.116	0.116	0.118	0.136	0.110	0.121	0.124	0.121
Dyn-5					0.113	0.119	0.126	0.110	0.123	0.112	0.133	0.136	0.096	0.120	0.116	0.119
Dyn-6						0.126	0.130	0.119	0.132	0.113	0.138	0.149	0.099	0.125	0.123	0.125
Dyn-7							0.129	0.123	0.140	0.118	0.141	0.151	0.109	0.137	0.127	0.131
Dyn-8								0.125	0.147	0.122	0.149	0.155	0.110	0.140	0.138	0.136
Dyn-9									0.149	0.126	0.151	0.158	0.113	0.144	0.140	0.140
Dyn-10										0.127	0.153	0.161	0.115	0.145	0.143	0.141
Dyn-11											0.154	0.162	0.117	0.148	0.144	0.145
Dyn-12												0.163	0.118	0.150	0.144	0.144
Dyn-13													0.118	0.150	0.146	0.138
Dyn-14														0.149	0.146	0.148
Dyn-15															0.146	0.146

Table B7a: Average efficiency in simulation for technology TEC7

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.841	0.738	0.644	0.555	0.600	0.558	0.606	0.533	0.575	0.570	0.608	0.594	0.604	0.594	0.612	0.615
Dyn-2		0.841	0.838	0.841	0.763	0.725	0.730	0.706	0.732	0.692	0.735	0.749	0.745	0.730	0.728	0.754
Dyn-3			0.841	0.838	0.869	0.882	0.825	0.812	0.818	0.800	0.836	0.799	0.817	0.824	0.820	0.829
Dyn-4				0.841	0.838	0.869	0.897	0.903	0.876	0.872	0.870	0.869	0.883	0.857	0.863	0.870
Dyn-5					0.917	0.926	0.915	0.906	0.915	0.912	0.928	0.913	0.906	0.911	0.918	0.915
Dyn-6						0.933	0.946	0.938	0.930	0.939	0.948	0.954	0.939	0.930	0.941	0.940
Dyn-7							0.955	0.957	0.952	0.956	0.957	0.968	0.967	0.959	0.959	0.959
Dyn-8								0.966	0.969	0.969	0.971	0.977	0.977	0.980	0.971	0.972
Dyn-9									0.975	0.982	0.982	0.984	0.985	0.986	0.986	0.983
Dyn-10										0.987	0.987	0.991	0.990	0.989	0.990	0.989
Dyn-11											0.990	0.994	0.995	0.992	0.991	0.992
Dyn-12												0.995	0.996	0.996	0.993	0.995
Dyn-13													0.997	0.996	0.997	0.997
Dyn-14														0.997	0.997	0.997
Dyn-15															0.998	0.998

Table B7b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC7

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.011	0.115	0.222	0.310	0.244	0.293	0.259	0.327	0.279	0.299	0.249	0.248	0.286	0.266	0.254	0.244
Dyn-2		0.136	0.116	0.107	0.152	0.192	0.187	0.191	0.166	0.236	0.200	0.163	0.187	0.175	0.194	0.172
Dyn-3			0.132	0.116	0.113	0.124	0.151	0.134	0.135	0.162	0.153	0.153	0.163	0.143	0.148	0.140
Dyn-4				0.125	0.128	0.127	0.125	0.098	0.117	0.127	0.127	0.136	0.113	0.148	0.129	0.125
Dyn-5					0.109	0.120	0.133	0.112	0.125	0.118	0.131	0.129	0.111	0.118	0.120	0.121
Dyn-6						0.123	0.131	0.118	0.128	0.111	0.140	0.144	0.098	0.124	0.119	0.124
Dyn-7							0.131	0.117	0.134	0.117	0.137	0.144	0.105	0.135	0.121	0.127
Dyn-8								0.119	0.143	0.120	0.147	0.152	0.107	0.139	0.132	0.132
Dyn-9									0.146	0.122	0.150	0.153	0.110	0.142	0.139	0.137
Dyn-10										0.125	0.153	0.159	0.112	0.144	0.139	0.139
Dyn-11											0.154	0.161	0.115	0.147	0.142	0.144
Dyn-12												0.162	0.116	0.148	0.142	0.142
Dyn-13													0.117	0.149	0.145	0.137
Dyn-14														0.149	0.145	0.147
Dyn-15															0.146	0.146

Table B8a: Average efficiency in simulation for technology TEC8

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.842	0.839	0.752	0.739	0.723	0.703	0.734	0.684	0.693	0.724	0.720	0.723	0.730	0.723	0.742	0.738
Dyn-2		0.842	0.856	0.871	0.842	0.819	0.819	0.807	0.825	0.798	0.825	0.824	0.807	0.818	0.822	0.827
Dyn-3			0.842	0.856	0.886	0.911	0.875	0.873	0.867	0.874	0.887	0.872	0.881	0.872	0.878	0.875
Dyn-4				0.842	0.856	0.886	0.916	0.919	0.915	0.915	0.906	0.913	0.922	0.911	0.906	0.901
Dyn-5					0.929	0.939	0.940	0.939	0.937	0.946	0.951	0.941	0.939	0.940	0.943	0.940
Dyn-6						0.947	0.956	0.958	0.954	0.960	0.965	0.968	0.963	0.956	0.961	0.959
Dyn-7							0.963	0.968	0.968	0.973	0.972	0.980	0.978	0.974	0.973	0.972
Dyn-8								0.973	0.978	0.982	0.982	0.985	0.986	0.985	0.983	0.982
Dyn-9									0.981	0.988	0.988	0.991	0.991	0.991	0.990	0.989
Dyn-10										0.991	0.991	0.995	0.995	0.993	0.994	0.993
Dyn-11											0.993	0.996	0.997	0.996	0.995	0.995
Dyn-12												0.997	0.997	0.998	0.996	0.997
Dyn-13													0.998	0.998	0.998	0.998
Dyn-14														0.998	0.998	0.998
Dyn-15															0.999	0.999

Table B8b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC8

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.010	0.017	0.114	0.134	0.119	0.149	0.136	0.176	0.156	0.148	0.134	0.120	0.161	0.136	0.126	0.122
Dyn-2		0.136	0.115	0.104	0.126	0.132	0.137	0.130	0.117	0.160	0.150	0.119	0.138	0.139	0.131	0.131
Dyn-3			0.132	0.113	0.114	0.129	0.134	0.111	0.118	0.123	0.134	0.133	0.121	0.121	0.123	0.124
Dyn-4				0.125	0.124	0.129	0.128	0.100	0.120	0.114	0.124	0.135	0.104	0.122	0.124	0.121
Dyn-5					0.113	0.122	0.127	0.109	0.125	0.113	0.132	0.132	0.096	0.120	0.113	0.118
Dyn-6						0.127	0.131	0.117	0.134	0.114	0.138	0.147	0.096	0.125	0.121	0.125
Dyn-7							0.130	0.122	0.142	0.117	0.141	0.152	0.106	0.136	0.127	0.130
Dyn-8								0.123	0.147	0.122	0.148	0.155	0.110	0.140	0.136	0.135
Dyn-9									0.148	0.125	0.151	0.158	0.113	0.144	0.139	0.140
Dyn-10										0.126	0.153	0.161	0.115	0.145	0.142	0.140
Dyn-11											0.154	0.162	0.117	0.148	0.143	0.145
Dyn-12												0.163	0.117	0.150	0.144	0.143
Dyn-13													0.118	0.149	0.146	0.138
Dyn-14														0.149	0.145	0.147
Dyn-15															0.146	0.146

Table B9a: Average efficiency in simulation for technology TEC9

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.88	0.849	0.853	0.851
Static	0.843	0.722	0.651	0.595	0.603	0.583	0.637	0.558	0.613	0.609	0.657	0.622	0.616	0.626	0.637	0.638
Dyn-2		0.843	0.801	0.791	0.705	0.694	0.716	0.719	0.728	0.708	0.751	0.733	0.756	0.734	0.724	0.743
Dyn-3			0.843	0.801	0.837	0.832	0.786	0.792	0.8	0.786	0.824	0.8	0.816	0.814	0.816	0.811
Dyn-4				0.843	0.801	0.837	0.865	0.862	0.851	0.851	0.847	0.863	0.863	0.848	0.859	0.849
Dyn-5					0.887	0.894	0.893	0.882	0.894	0.898	0.907	0.901	0.896	0.894	0.897	0.895
Dyn-6						0.908	0.924	0.915	0.913	0.917	0.931	0.938	0.926	0.915	0.927	0.921
Dyn-7							0.936	0.937	0.932	0.934	0.938	0.957	0.953	0.944	0.947	0.942
Dyn-8								0.947	0.951	0.949	0.953	0.966	0.967	0.968	0.961	0.958
Dyn-9									0.959	0.967	0.966	0.974	0.976	0.978	0.98	0.971
Dyn-10										0.975	0.976	0.984	0.982	0.982	0.985	0.981
Dyn-11											0.981	0.988	0.989	0.987	0.987	0.986
Dyn-12												0.99	0.992	0.992	0.989	0.991
Dyn-13													0.994	0.993	0.994	0.994
Dyn-14														0.995	0.994	0.995
Dyn-15															0.995	0.995

Table B9b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC9

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.01	0.133	0.216	0.273	0.237	0.268	0.231	0.301	0.24	0.263	0.198	0.22	0.276	0.233	0.227	0.222
Dyn-2		0.136	0.125	0.126	0.173	0.21	0.195	0.177	0.163	0.225	0.181	0.157	0.17	0.178	0.192	0.172
Dyn-3			0.132	0.124	0.115	0.134	0.163	0.136	0.133	0.161	0.15	0.149	0.153	0.144	0.139	0.141
Dyn-4				0.126	0.133	0.132	0.13	0.099	0.121	0.132	0.131	0.139	0.113	0.144	0.131	0.127
Dyn-5					0.106	0.12	0.138	0.115	0.122	0.115	0.135	0.121	0.108	0.116	0.119	0.119
Dyn-6						0.123	0.135	0.116	0.122	0.114	0.141	0.133	0.099	0.125	0.117	0.123
Dyn-7							0.135	0.112	0.129	0.115	0.137	0.139	0.099	0.133	0.118	0.124
Dyn-8								0.111	0.137	0.118	0.143	0.145	0.103	0.133	0.126	0.127
Dyn-9									0.139	0.122	0.145	0.148	0.105	0.139	0.134	0.133
Dyn-10										0.124	0.15	0.155	0.109	0.141	0.136	0.136
Dyn-11											0.152	0.158	0.112	0.143	0.138	0.141
Dyn-12												0.16	0.114	0.145	0.139	0.139
Dyn-13													0.115	0.147	0.144	0.135
Dyn-14														0.148	0.144	0.146
Dyn-15															0.145	0.145

Table B10a: Average efficiency in simulation for technology TEC10

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.849	0.853	0.859	0.853	0.835	0.846	0.856	0.854	0.837	0.869	0.843	0.835	0.880	0.849	0.853	0.851
Static	0.842	0.751	0.702	0.655	0.641	0.627	0.676	0.594	0.642	0.649	0.690	0.656	0.669	0.661	0.653	0.674
Dyn-2		0.842	0.816	0.820	0.785	0.749	0.765	0.761	0.771	0.746	0.787	0.786	0.776	0.762	0.758	0.780
Dyn-3			0.842	0.816	0.857	0.866	0.836	0.829	0.835	0.834	0.851	0.835	0.854	0.848	0.837	0.842
Dyn-4				0.842	0.816	0.857	0.885	0.891	0.893	0.885	0.881	0.892	0.893	0.882	0.886	0.875
Dyn-5					0.909	0.920	0.922	0.915	0.921	0.925	0.931	0.923	0.919	0.918	0.920	0.920
Dyn-6						0.928	0.943	0.942	0.936	0.943	0.953	0.954	0.945	0.938	0.944	0.943
Dyn-7							0.950	0.954	0.955	0.958	0.962	0.970	0.966	0.963	0.962	0.960
Dyn-8								0.962	0.966	0.971	0.972	0.979	0.978	0.978	0.974	0.972
Dyn-9									0.972	0.979	0.982	0.985	0.986	0.987	0.986	0.982
Dyn-10										0.985	0.986	0.990	0.990	0.990	0.991	0.989
Dyn-11											0.989	0.993	0.994	0.993	0.992	0.992
Dyn-12												0.994	0.995	0.996	0.994	0.995
Dyn-13													0.996	0.996	0.997	0.997
Dyn-14														0.997	0.997	0.997
Dyn-15															0.998	0.998

Table B10b: Average of absolute deviation with true efficiency in simulation (I) for technology TEC10

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
Static	0.010	0.102	0.163	0.215	0.198	0.225	0.193	0.265	0.208	0.222	0.164	0.186	0.219	0.193	0.210	0.185
Dyn-2		0.137	0.124	0.113	0.139	0.171	0.162	0.155	0.137	0.195	0.162	0.128	0.155	0.150	0.164	0.149
Dyn-3			0.132	0.122	0.116	0.122	0.138	0.119	0.121	0.138	0.140	0.133	0.133	0.127	0.133	0.129
Dyn-4				0.126	0.133	0.129	0.125	0.097	0.117	0.121	0.126	0.133	0.109	0.132	0.123	0.122
Dyn-5					0.107	0.121	0.129	0.109	0.120	0.108	0.129	0.122	0.101	0.118	0.112	0.116
Dyn-6						0.123	0.133	0.115	0.127	0.109	0.136	0.138	0.094	0.125	0.119	0.122
Dyn-7							0.132	0.116	0.136	0.114	0.138	0.145	0.101	0.135	0.124	0.127
Dyn-8								0.117	0.142	0.118	0.145	0.151	0.106	0.136	0.130	0.131
Dyn-9									0.145	0.121	0.149	0.155	0.110	0.141	0.137	0.137
Dyn-10										0.124	0.152	0.159	0.113	0.143	0.140	0.138
Dyn-11											0.152	0.160	0.115	0.146	0.141	0.143
Dyn-12												0.161	0.116	0.149	0.142	0.142
Dyn-13													0.117	0.149	0.145	0.137
Dyn-14														0.149	0.145	0.147
Dyn-15															0.146	0.146

Table C1: Average efficiency in simulation (II) for data set SET1

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.737	0.701	0.729	0.737	0.730	0.721	0.710	0.684	0.687	0.718	0.725	0.732	0.665	0.693	0.732	0.713
Dyn-2		0.745	0.730	0.719	0.722	0.742	0.753	0.726	0.701	0.698	0.732	0.738	0.735	0.685	0.675	0.722
Dyn-3			0.741	0.727	0.743	0.735	0.771	0.775	0.755	0.736	0.724	0.738	0.756	0.769	0.724	0.746
Dyn-4				0.741	0.736	0.757	0.779	0.775	0.804	0.809	0.786	0.767	0.763	0.780	0.786	0.774
Dyn-5					0.823	0.826	0.837	0.836	0.811	0.815	0.815	0.805	0.797	0.793	0.765	0.811
Dyn-6						0.853	0.858	0.857	0.841	0.837	0.840	0.838	0.817	0.817	0.817	0.837
Dyn-7							0.882	0.874	0.869	0.876	0.875	0.880	0.857	0.853	0.822	0.865
Dyn-8								0.908	0.886	0.888	0.899	0.912	0.873	0.871	0.859	0.887
Dyn-9									0.903	0.903	0.915	0.922	0.912	0.898	0.886	0.906
Dyn-10										0.927	0.941	0.939	0.918	0.943	0.904	0.929
Dyn-11											0.948	0.945	0.939	0.942	0.931	0.941
Dyn-12												0.959	0.955	0.961	0.946	0.955
Dyn-13													0.967	0.973	0.956	0.965
Dyn-14														0.971	0.966	0.968
Dyn-15															0.965	0.965

Table C2: Average efficiency in simulation (II) for data set SET2

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.736	0.689	0.720	0.719	0.730	0.727	0.709	0.688	0.702	0.729	0.723	0.730	0.663	0.693	0.721	0.712
Dyn-2		0.736	0.712	0.722	0.736	0.730	0.731	0.707	0.690	0.703	0.723	0.737	0.740	0.692	0.678	0.717
Dyn-3			0.736	0.712	0.743	0.753	0.771	0.755	0.733	0.716	0.722	0.737	0.764	0.763	0.740	0.742
Dyn-4				0.736	0.712	0.743	0.777	0.788	0.788	0.775	0.744	0.751	0.753	0.780	0.787	0.761
Dyn-5					0.808	0.807	0.802	0.790	0.783	0.776	0.799	0.812	0.805	0.801	0.798	0.798
Dyn-6						0.830	0.832	0.818	0.815	0.809	0.822	0.836	0.832	0.833	0.827	0.825
Dyn-7							0.852	0.847	0.840	0.839	0.855	0.854	0.853	0.854	0.851	0.849
Dyn-8								0.866	0.868	0.862	0.882	0.883	0.871	0.871	0.873	0.872
Dyn-9									0.883	0.890	0.903	0.911	0.900	0.888	0.896	0.896
Dyn-10										0.906	0.918	0.926	0.921	0.915	0.912	0.916
Dyn-11											0.929	0.940	0.937	0.932	0.934	0.934
Dyn-12												0.949	0.948	0.945	0.947	0.947
Dyn-13													0.957	0.956	0.960	0.958
Dyn-14														0.963	0.965	0.964
Dyn-15															0.965	0.965

Table C3: Average efficiency in simulation (II) for data set SET3

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.744	0.674	0.724	0.714	0.720	0.729	0.708	0.689	0.707	0.724	0.727	0.725	0.660	0.692	0.720	0.710
Dyn-2		0.744	0.706	0.713	0.742	0.725	0.726	0.715	0.689	0.705	0.726	0.737	0.744	0.696	0.682	0.718
Dyn-3			0.744	0.706	0.741	0.748	0.760	0.758	0.738	0.721	0.725	0.743	0.769	0.769	0.745	0.743
Dyn-4				0.744	0.706	0.741	0.778	0.779	0.791	0.777	0.750	0.754	0.757	0.788	0.795	0.763
Dyn-5					0.808	0.807	0.806	0.793	0.788	0.781	0.805	0.819	0.813	0.809	0.806	0.803
Dyn-6						0.832	0.834	0.822	0.819	0.813	0.831	0.841	0.835	0.839	0.834	0.830
Dyn-7							0.856	0.848	0.846	0.843	0.859	0.860	0.855	0.859	0.859	0.854
Dyn-8								0.869	0.869	0.867	0.886	0.885	0.874	0.874	0.877	0.875
Dyn-9									0.886	0.892	0.908	0.915	0.905	0.892	0.900	0.900
Dyn-10										0.908	0.923	0.931	0.927	0.919	0.917	0.921
Dyn-11											0.934	0.944	0.944	0.938	0.937	0.939
Dyn-12												0.952	0.951	0.951	0.951	0.951
Dyn-13													0.961	0.959	0.964	0.961
Dyn-14														0.966	0.968	0.967
Dyn-15															0.968	0.968

Table C4: Average efficiency in simulation (II) for data set SET4

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.745	0.702	0.723	0.735	0.739	0.729	0.714	0.683	0.690	0.724	0.721	0.724	0.664	0.700	0.726	0.715
Dyn-2		0.739	0.717	0.718	0.734	0.741	0.746	0.717	0.695	0.690	0.722	0.747	0.734	0.694	0.690	0.720
Dyn-3			0.747	0.728	0.757	0.732	0.778	0.782	0.764	0.742	0.722	0.747	0.756	0.764	0.718	0.749
Dyn-4				0.744	0.736	0.744	0.777	0.776	0.817	0.803	0.787	0.773	0.761	0.786	0.786	0.774
Dyn-5					0.813	0.821	0.832	0.822	0.812	0.815	0.811	0.803	0.797	0.787	0.773	0.808
Dyn-6						0.847	0.853	0.867	0.850	0.840	0.851	0.849	0.825	0.811	0.805	0.840
Dyn-7							0.869	0.872	0.859	0.862	0.878	0.867	0.858	0.834	0.826	0.858
Dyn-8								0.894	0.895	0.884	0.891	0.911	0.882	0.878	0.861	0.887
Dyn-9									0.902	0.905	0.918	0.922	0.901	0.894	0.872	0.902
Dyn-10										0.929	0.927	0.929	0.936	0.928	0.901	0.925
Dyn-11											0.952	0.960	0.937	0.946	0.943	0.948
Dyn-12												0.951	0.943	0.958	0.953	0.951
Dyn-13													0.966	0.964	0.953	0.961
Dyn-14														0.970	0.967	0.968
Dyn-15															0.959	0.959

Table C5: Average efficiency in simulation (II) for data set SET5

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.742	0.690	0.719	0.720	0.730	0.727	0.710	0.688	0.702	0.729	0.724	0.729	0.662	0.693	0.723	0.712
Dyn-2		0.742	0.713	0.723	0.736	0.731	0.731	0.708	0.690	0.703	0.722	0.738	0.740	0.692	0.679	0.718
Dyn-3			0.742	0.713	0.744	0.754	0.773	0.755	0.733	0.716	0.722	0.737	0.764	0.762	0.740	0.743
Dyn-4				0.742	0.713	0.744	0.778	0.789	0.789	0.775	0.744	0.751	0.753	0.780	0.788	0.762
Dyn-5					0.810	0.807	0.803	0.790	0.783	0.776	0.799	0.812	0.806	0.802	0.798	0.799
Dyn-6						0.830	0.833	0.818	0.816	0.809	0.822	0.837	0.832	0.833	0.826	0.826
Dyn-7							0.853	0.848	0.841	0.839	0.855	0.854	0.853	0.854	0.851	0.850
Dyn-8								0.866	0.868	0.862	0.882	0.883	0.871	0.871	0.873	0.872
Dyn-9									0.883	0.890	0.903	0.910	0.900	0.889	0.896	0.896
Dyn-10										0.906	0.919	0.926	0.921	0.915	0.911	0.916
Dyn-11											0.930	0.940	0.937	0.932	0.934	0.934
Dyn-12												0.949	0.948	0.945	0.947	0.947
Dyn-13													0.958	0.956	0.960	0.958
Dyn-14														0.963	0.965	0.964
Dyn-15															0.965	0.965

Table C6: Average efficiency in simulation (II) for data set SET6

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.739	0.680	0.719	0.720	0.716	0.726	0.712	0.686	0.706	0.729	0.719	0.724	0.662	0.692	0.712	0.709
Dyn-2		0.746	0.705	0.714	0.747	0.718	0.728	0.708	0.689	0.701	0.735	0.747	0.740	0.705	0.686	0.719
Dyn-3			0.734	0.715	0.734	0.752	0.751	0.748	0.730	0.726	0.725	0.743	0.775	0.771	0.742	0.742
Dyn-4				0.751	0.711	0.751	0.785	0.785	0.796	0.769	0.742	0.756	0.761	0.781	0.786	0.765
Dyn-5					0.818	0.806	0.796	0.791	0.784	0.773	0.807	0.810	0.821	0.803	0.810	0.802
Dyn-6						0.823	0.828	0.816	0.812	0.814	0.830	0.833	0.828	0.831	0.843	0.826
Dyn-7							0.864	0.853	0.842	0.846	0.856	0.868	0.848	0.859	0.864	0.856
Dyn-8								0.879	0.872	0.872	0.886	0.892	0.880	0.877	0.873	0.879
Dyn-9									0.895	0.887	0.898	0.924	0.902	0.883	0.903	0.899
Dyn-10										0.909	0.922	0.930	0.936	0.928	0.926	0.925
Dyn-11											0.935	0.943	0.935	0.943	0.939	0.939
Dyn-12												0.944	0.949	0.960	0.958	0.953
Dyn-13													0.970	0.966	0.956	0.964
Dyn-14														0.973	0.972	0.973
Dyn-15															0.970	0.970

Table C7: Average efficiency in simulation (II) for data set SET7

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.746	0.694	0.720	0.728	0.734	0.727	0.714	0.687	0.696	0.720	0.725	0.724	0.668	0.695	0.725	0.714
Dyn-2		0.746	0.727	0.711	0.729	0.745	0.748	0.723	0.702	0.693	0.722	0.746	0.730	0.685	0.684	0.721
Dyn-3			0.746	0.727	0.750	0.738	0.776	0.777	0.764	0.738	0.730	0.745	0.757	0.765	0.724	0.749
Dyn-4				0.746	0.727	0.750	0.776	0.780	0.807	0.803	0.789	0.776	0.771	0.786	0.782	0.774
Dyn-5					0.813	0.818	0.834	0.827	0.815	0.808	0.808	0.813	0.790	0.796	0.772	0.809
Dyn-6						0.845	0.852	0.857	0.846	0.846	0.847	0.840	0.821	0.817	0.811	0.838
Dyn-7							0.878	0.876	0.868	0.869	0.879	0.874	0.851	0.843	0.830	0.863
Dyn-8								0.898	0.892	0.890	0.899	0.906	0.875	0.872	0.854	0.886
Dyn-9									0.910	0.909	0.914	0.919	0.910	0.892	0.879	0.905
Dyn-10										0.926	0.931	0.936	0.926	0.936	0.899	0.926
Dyn-11											0.947	0.950	0.942	0.944	0.941	0.945
Dyn-12												0.960	0.953	0.955	0.948	0.954
Dyn-13													0.961	0.963	0.961	0.962
Dyn-14														0.969	0.968	0.968
Dyn-15															0.968	0.968

Table C8: Average efficiency in simulation (II) for data set SET8

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.744	0.666	0.732	0.724	0.727	0.736	0.715	0.694	0.714	0.718	0.722	0.720	0.661	0.696	0.726	0.713
Dyn-2		0.736	0.702	0.707	0.748	0.723	0.727	0.714	0.695	0.699	0.723	0.746	0.751	0.689	0.674	0.717
Dyn-3			0.739	0.709	0.745	0.751	0.760	0.750	0.748	0.724	0.724	0.751	0.773	0.768	0.749	0.745
Dyn-4				0.743	0.713	0.745	0.768	0.781	0.797	0.783	0.760	0.749	0.759	0.796	0.797	0.766
Dyn-5					0.810	0.814	0.798	0.802	0.780	0.789	0.796	0.817	0.814	0.812	0.809	0.804
Dyn-6						0.830	0.842	0.815	0.818	0.805	0.829	0.850	0.825	0.836	0.838	0.829
Dyn-7							0.848	0.838	0.854	0.852	0.863	0.868	0.862	0.867	0.868	0.858
Dyn-8								0.869	0.869	0.872	0.877	0.884	0.870	0.882	0.874	0.875
Dyn-9									0.880	0.893	0.910	0.916	0.908	0.891	0.900	0.900
Dyn-10										0.901	0.926	0.924	0.931	0.912	0.910	0.917
Dyn-11											0.942	0.947	0.939	0.935	0.928	0.938
Dyn-12												0.946	0.942	0.955	0.952	0.949
Dyn-13													0.967	0.951	0.969	0.962
Dyn-14														0.965	0.968	0.967
Dyn-15															0.968	0.968

Table C9: Average efficiency in simulation (II) for data set SET9

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.739	0.696	0.711	0.724	0.736	0.736	0.721	0.687	0.690	0.713	0.729	0.721	0.662	0.694	0.719	0.712
Dyn-2		0.737	0.718	0.707	0.739	0.741	0.740	0.726	0.698	0.699	0.716	0.743	0.724	0.682	0.687	0.718
Dyn-3			0.741	0.727	0.758	0.730	0.786	0.781	0.763	0.731	0.736	0.751	0.757	0.772	0.723	0.750
Dyn-4				0.751	0.722	0.760	0.776	0.773	0.797	0.796	0.784	0.774	0.763	0.794	0.784	0.773
Dyn-5					0.808	0.816	0.843	0.820	0.806	0.802	0.800	0.817	0.786	0.797	0.777	0.807
Dyn-6						0.852	0.847	0.858	0.841	0.837	0.840	0.832	0.829	0.822	0.816	0.837
Dyn-7							0.882	0.881	0.861	0.879	0.873	0.874	0.859	0.838	0.822	0.863
Dyn-8								0.902	0.893	0.883	0.905	0.903	0.870	0.869	0.859	0.885
Dyn-9									0.909	0.917	0.918	0.912	0.915	0.901	0.872	0.906
Dyn-10										0.917	0.935	0.945	0.918	0.942	0.906	0.927
Dyn-11											0.938	0.944	0.945	0.947	0.938	0.943
Dyn-12												0.956	0.953	0.947	0.948	0.951
Dyn-13													0.956	0.961	0.952	0.956
Dyn-14														0.970	0.976	0.973
Dyn-15															0.969	0.969

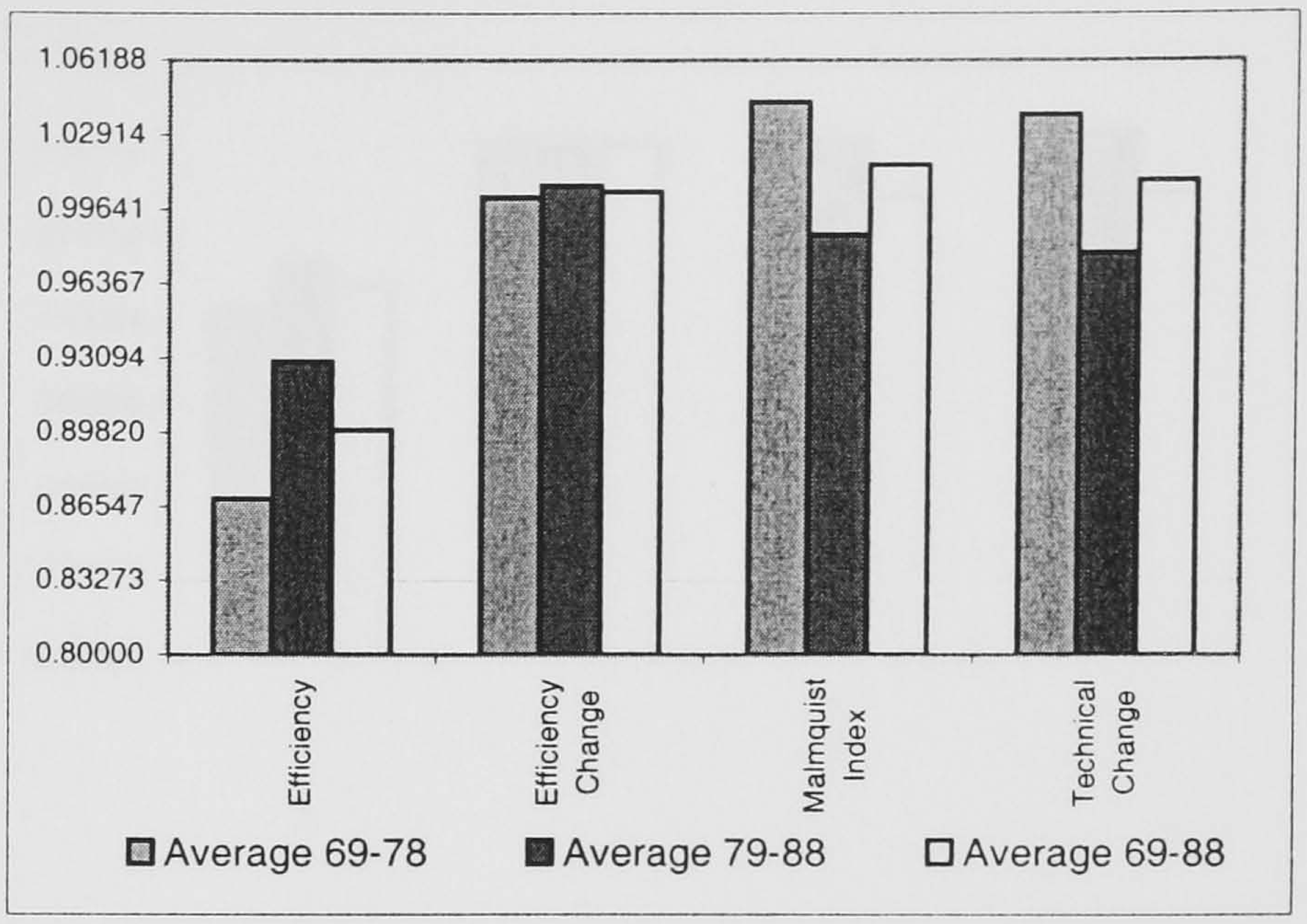
Table C10: Average efficiency in simulation (II) for data set SET10

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	Average
TRUE	0.746	0.716	0.746	0.738	0.742	0.738	0.722	0.703	0.721	0.748	0.743	0.756	0.696	0.731	0.759	0.734
Static	0.750	0.701	0.713	0.726	0.737	0.733	0.719	0.694	0.692	0.710	0.718	0.731	0.676	0.690	0.716	0.714
Dyn-2		0.744	0.736	0.716	0.730	0.738	0.739	0.716	0.709	0.687	0.724	0.741	0.732	0.682	0.681	0.720
Dyn-3			0.746	0.737	0.747	0.740	0.767	0.769	0.764	0.744	0.739	0.753	0.764	0.768	0.716	0.750
Dyn-4				0.750	0.724	0.758	0.770	0.786	0.808	0.812	0.793	0.770	0.779	0.794	0.779	0.777
Dyn-5					0.816	0.821	0.827	0.831	0.823	0.801	0.817	0.803	0.796	0.804	0.771	0.810
Dyn-6						0.839	0.843	0.854	0.841	0.841	0.848	0.832	0.822	0.810	0.816	0.834
Dyn-7							0.885	0.880	0.862	0.876	0.887	0.868	0.855	0.839	0.823	0.864
Dyn-8								0.894	0.887	0.899	0.898	0.898	0.878	0.862	0.853	0.884
Dyn-9									0.919	0.906	0.916	0.924	0.919	0.895	0.883	0.909
Dyn-10										0.934	0.932	0.945	0.926	0.942	0.891	0.928
Dyn-11											0.946	0.951	0.937	0.949	0.950	0.947
Dyn-12												0.959	0.957	0.956	0.956	0.957
Dyn-13													0.952	0.961	0.954	0.956
Dyn-14														0.974	0.964	0.969
Dyn-15															0.958	0.958

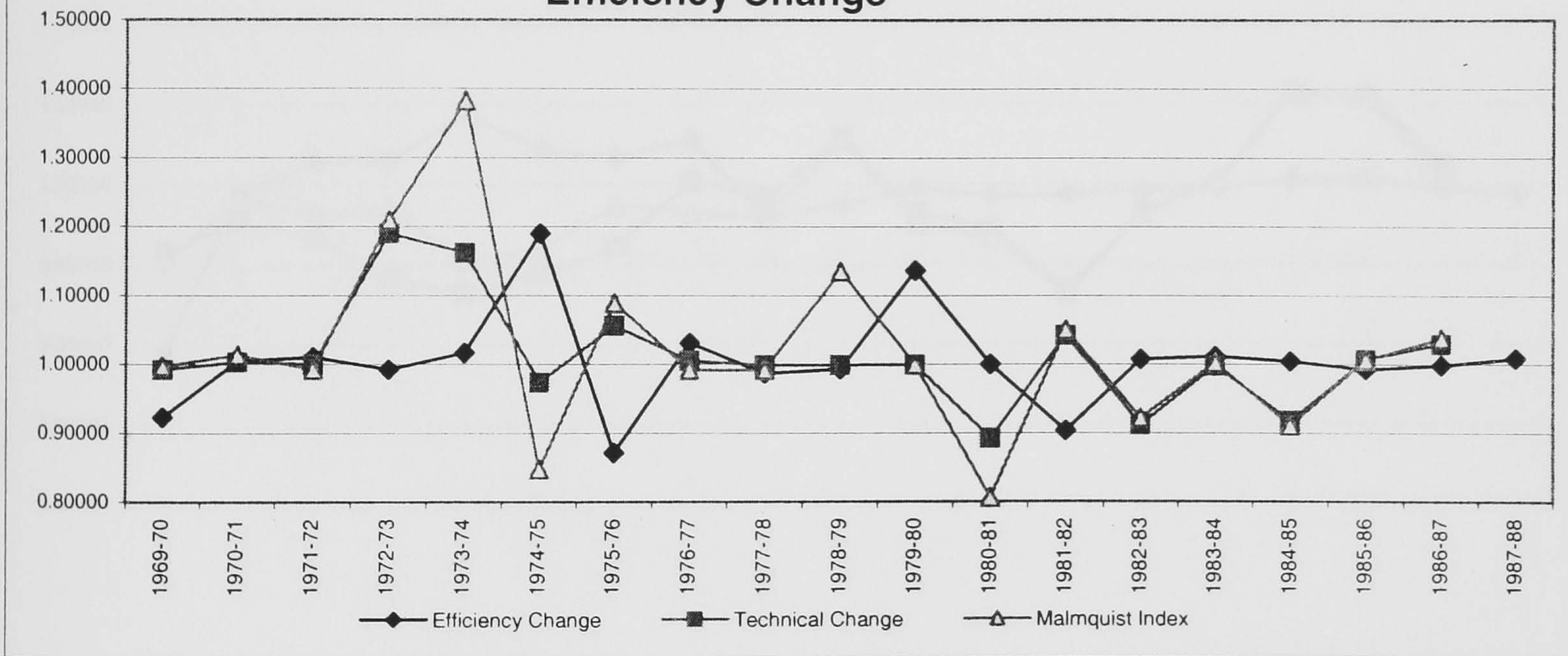
Appendix B: A report of efficiency and productivity of Industrialised counties, OECD.

	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.92289	1.00000	0.92289
1970-71	1.00446	0.99236	0.99678
1971-72	1.01053	1.00307	1.01362
1972-73	0.99245	1.00000	0.99245
1973-74	1.01630	1.18989	1.20929
1974-75	1.18983	1.16212	1.38272
1975-76	0.87090	0.97304	0.84743
1976-77	1.03121	1.05601	1.08897
1977-78	0.98687	1.00465	0.99146
1978-79	0.99271	0.99868	0.99140
1979-80	1.13658	0.99878	1.13519
1980-81	1.00000	0.99976	0.99976
1981-82	0.90352	0.89247	0.80636
1982-83	1.00749	1.04191	1.04971
1983-84	1.01139	0.91173	0.92212
1984-85	1.00450	0.99778	1.00227
1985-86	0.99201	0.91773	0.91039
1986-87	0.99856	1.00751	1.00606
1987-88	1.00850	1.02904	1.03779

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.86910	0.92965	0.89938
Efficiency Change	1.00181	1.00695	1.00425
Malmquist Index	1.04370	0.98552	1.01614
Technical Change	1.03798	0.97741	1.00929

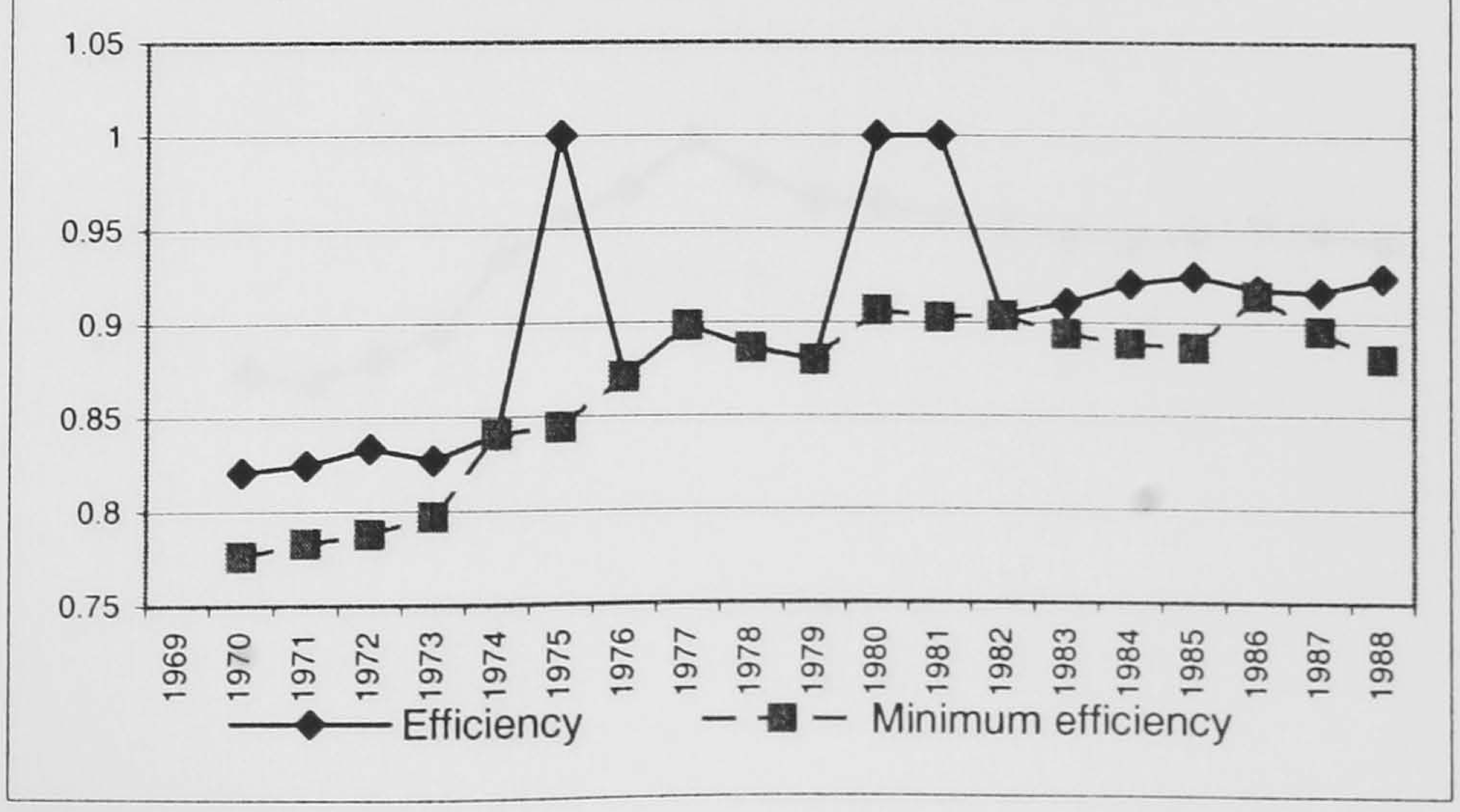


Decomposition of Productivity Index to Technical Change and Efficiency Change



Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.88952	1979	0.87983
1970	0.82093	1980	1.00000
1971	0.82459	1981	1.00000
1972	0.83327	1982	0.90352
1973	0.82698	1983	0.91029
1974	0.84046	1984	0.92066
1975	1.00000	1985	0.92481
1976	0.87090	1986	0.91742
1977	0.89808	1987	0.91609
1978	0.88629	1988	0.92388

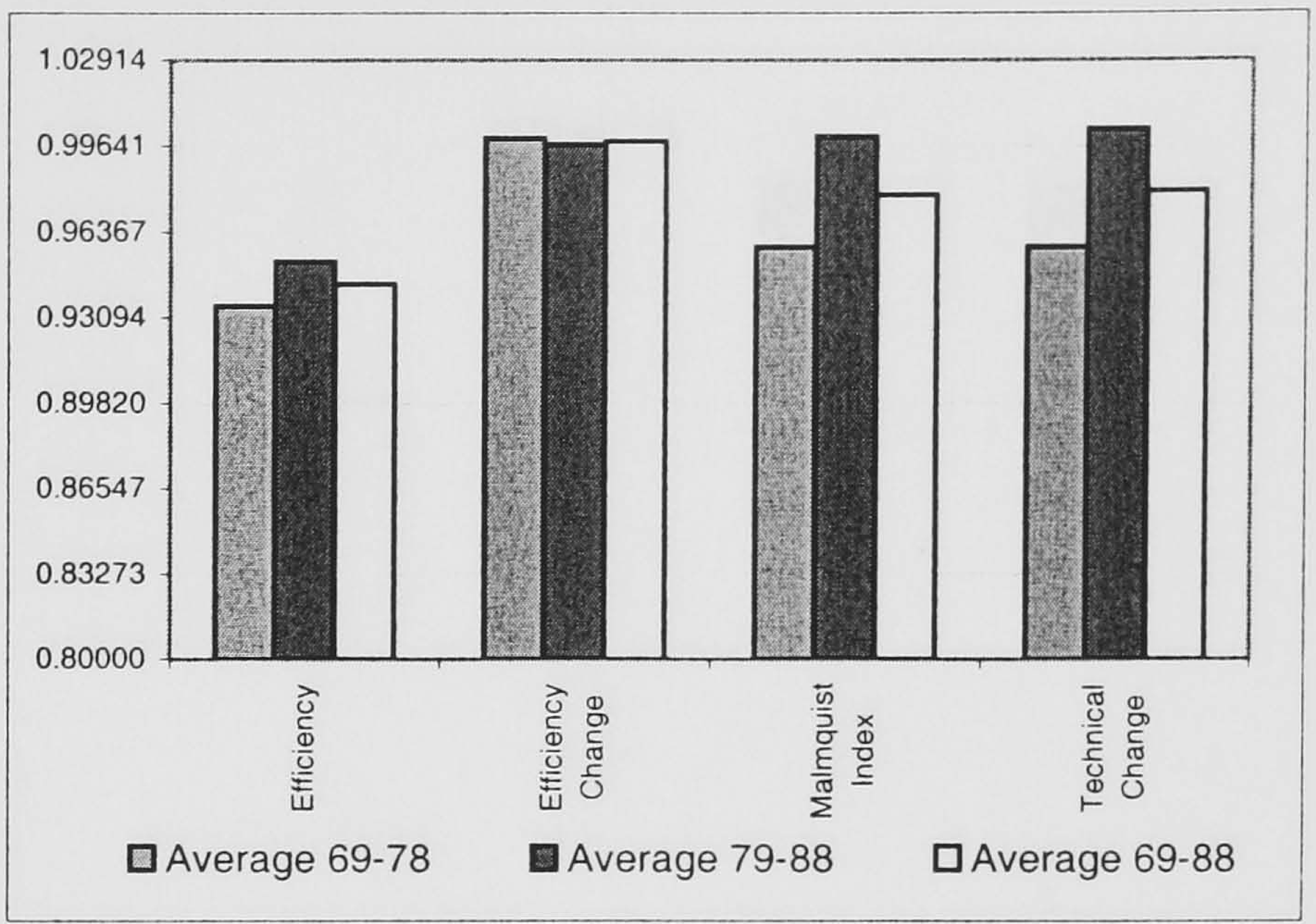
A comparison of efficiency with least efficient country

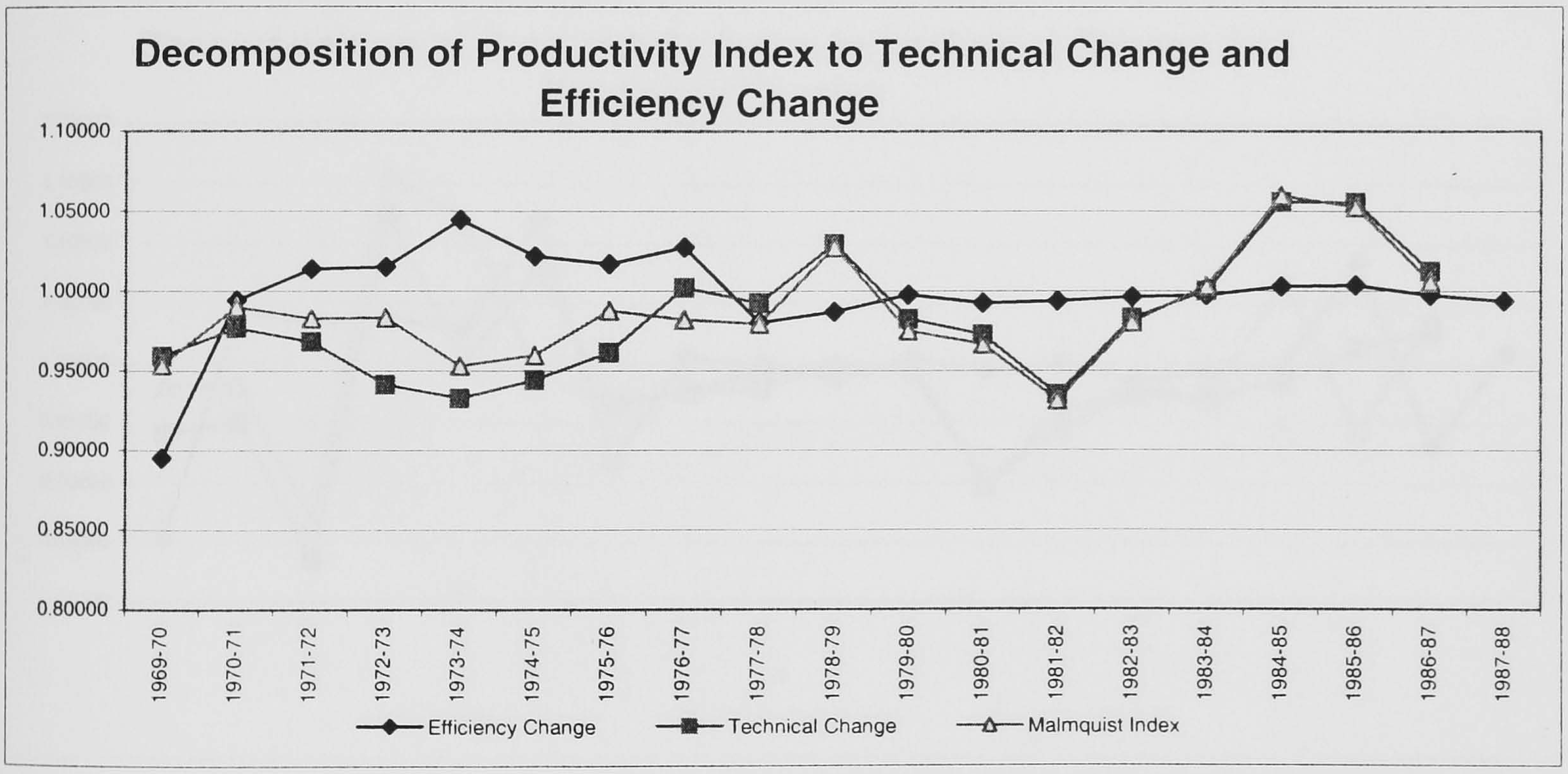


Summary of dynamic efficiency, productivity and its decomposition for AUSTRIA

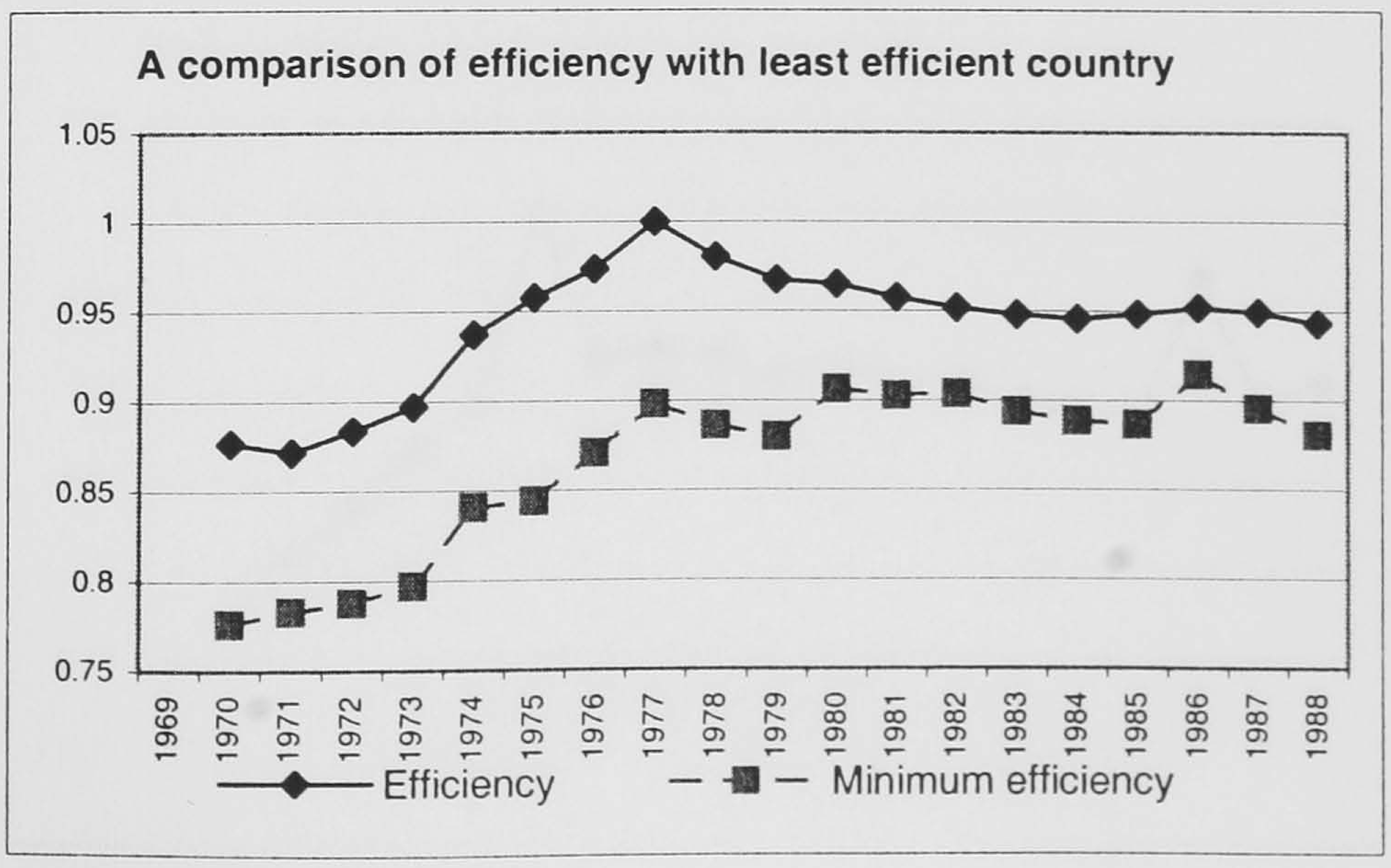
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.89509	0.90410	0.80925
1970-71	0.99412	0.95909	0.95345
1971-72	1.01370	0.97674	0.99012
1972-73	1.01516	0.96807	0.98274
1973-74	1.04459	0.94136	0.98334
1974-75	1.02188	0.93272	0.95312
1975-76	1.01673	0.94401	0.95980
1976-77	1.02740	0.96132	0.98766
1977-78	0.98005	1.00197	0.98197
1978-79	0.98693	0.99237	0.97940
1979-80	0.99766	1.02970	1.02729
1980-81	0.99227	0.98249	0.97489
1981-82	0.99365	0.97264	0.96647
1982-83	0.99621	0.93494	0.93140
1983-84	0.99740	0.98285	0.98030
1984-85	1.00291	1.00007	1.00297
1985-86	1.00359	1.05577	1.05956
1986-87	0.99741	1.05532	1.05259
1987-88	0.99377	1.01232	1.00601

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.93553	0.95261	0.94407
Efficiency Change	0.99956	0.99721	0.99845
Malmquist Index	0.95809	1.00016	0.97802
Technical Change	0.95817	1.00290	0.97936



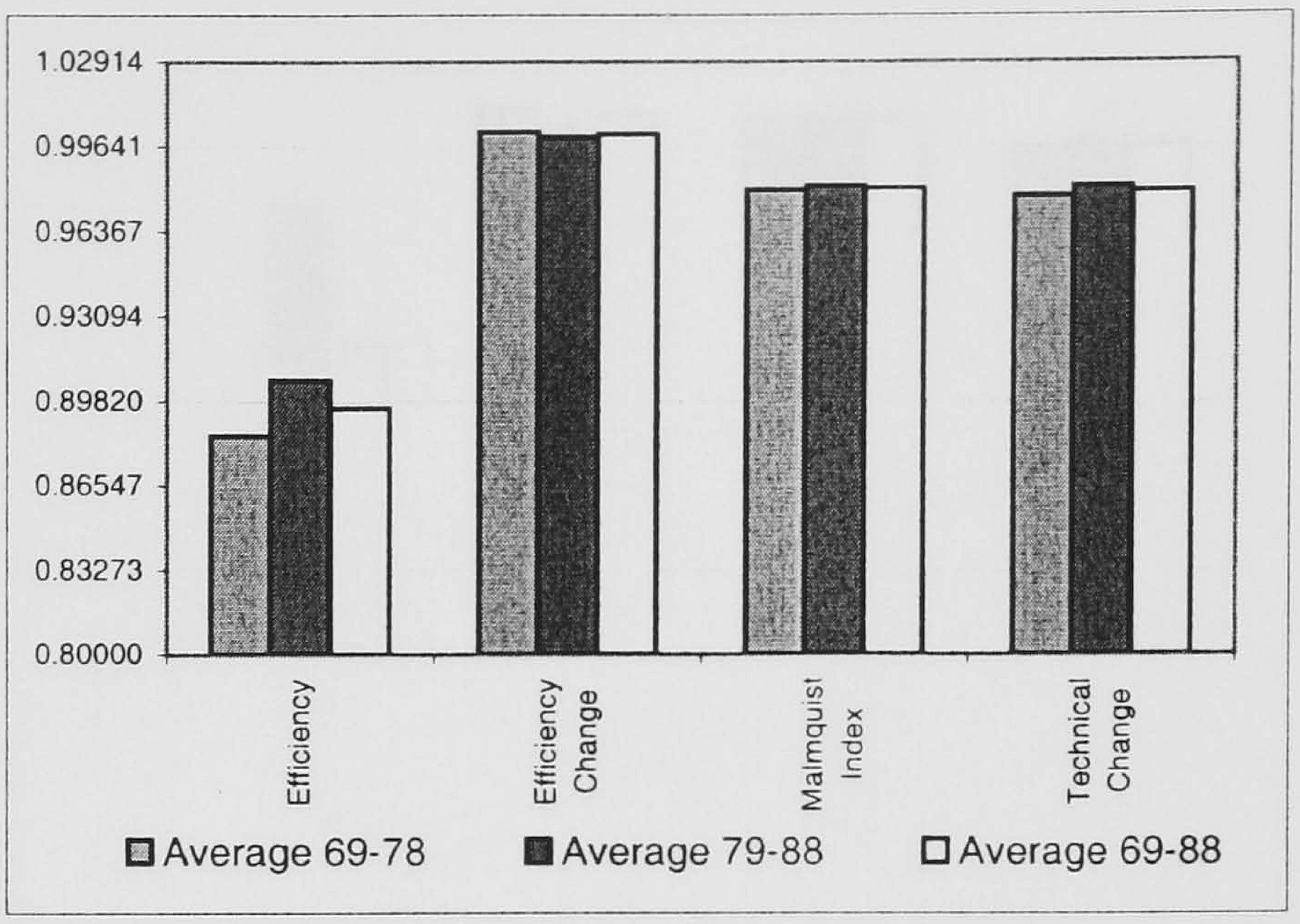


Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.97940	1979	0.96723
1970	0.87665	1980	0.96497
1971	0.87149	1981	0.95751
1972	0.88344	1982	0.95143
1973	0.89683	1983	0.94782
1974	0.93682	1984	0.94536
1975	0.95732	1985	0.94811
1976	0.97333	1986	0.95151
1977	1.00000	1987	0.94905
1978	0.98005	1988	0.94313

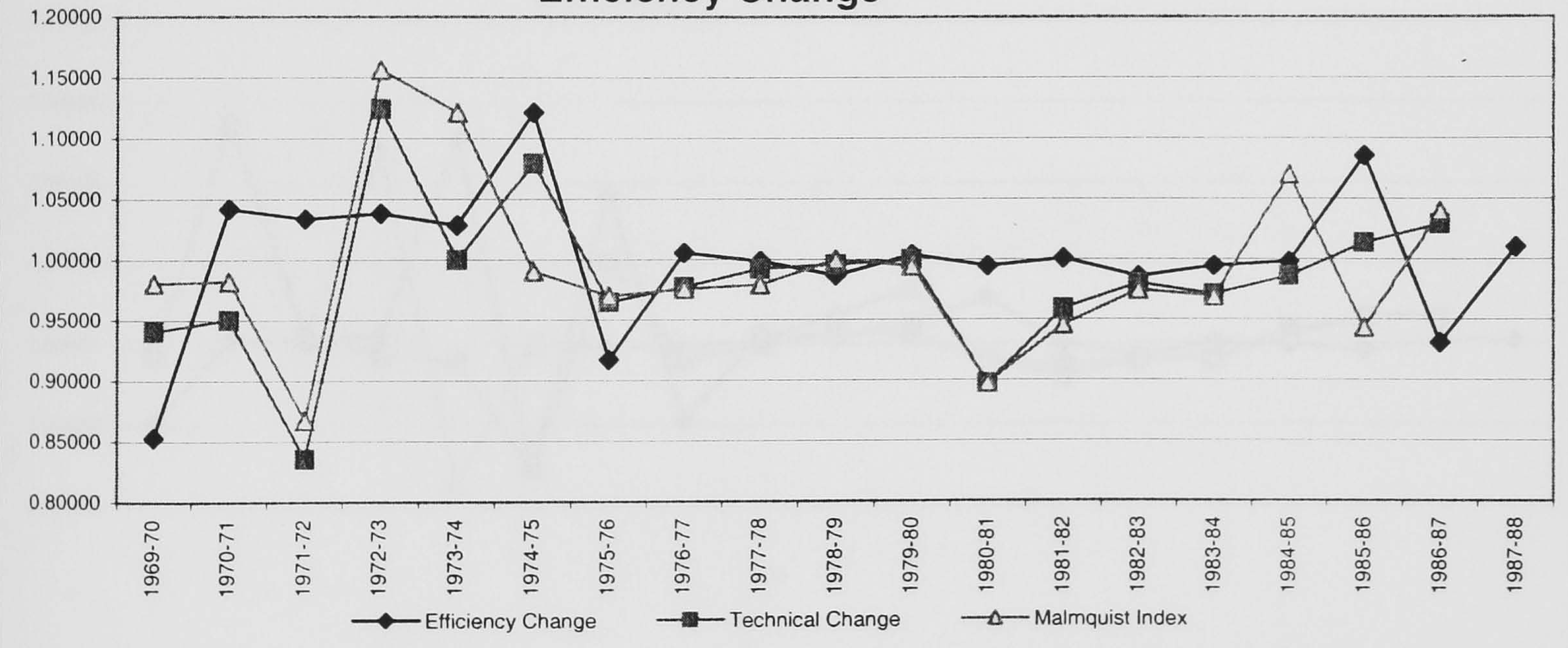


	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.85334	0.91087	0.77728
1970-71	1.04182	0.94089	0.98023
1971-72	1.03340	0.95032	0.98207
1972-73	1.03810	0.83554	0.86738
1973-74	1.02846	1.12423	1.15623
1974-75	1.12102	1.00000	1.12102
1975-76	0.91726	1.07953	0.99020
1976-77	1.00551	0.96464	0.96995
1977-78	0.99826	0.97737	0.97567
1978-79	0.98651	0.99221	0.97883
1979-80	1.00381	0.99568	0.99948
1980-81	0.99424	1.00000	0.99424
1981-82	1.00036	0.89721	0.89753
1982-83	0.98568	0.95928	0.94554
1983-84	0.99424	0.98009	0.97445
1984-85	0.99767	0.97108	0.96882
1985-86	1.08527	0.98580	1.06986
1986-87	0.93069	1.01350	0.94325
1987-88	1.01006	1.02898	1.03933

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.88492	0.90648	0.89570
Efficiency Change	1.00237	1.00022	1.00135
Malmquist Index	0.97989	0.98139	0.98060
Technical Change	0.97756	0.98129	0.97933

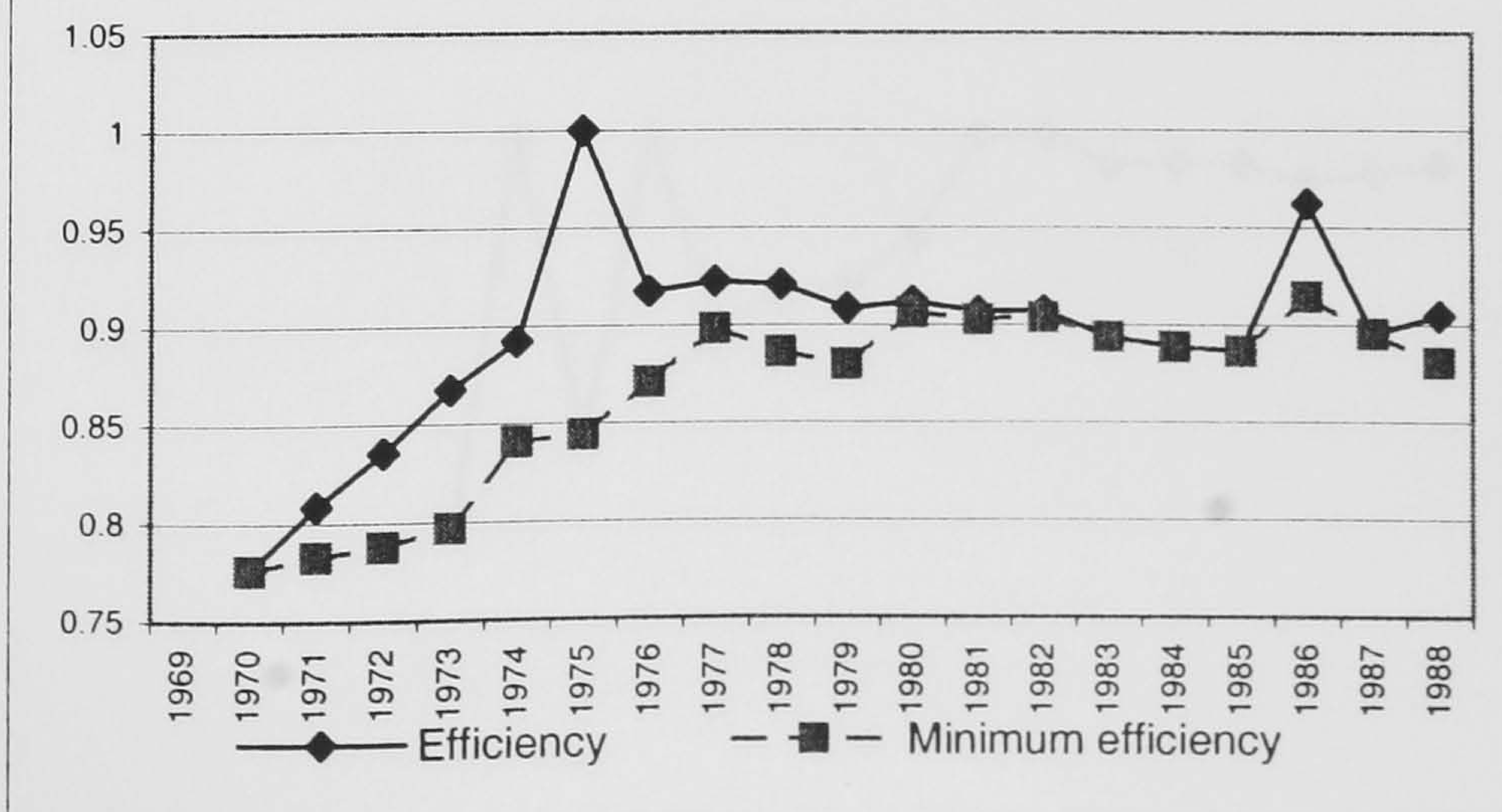


Decomposition of Productivity Index to Technical Change and Efficiency Change



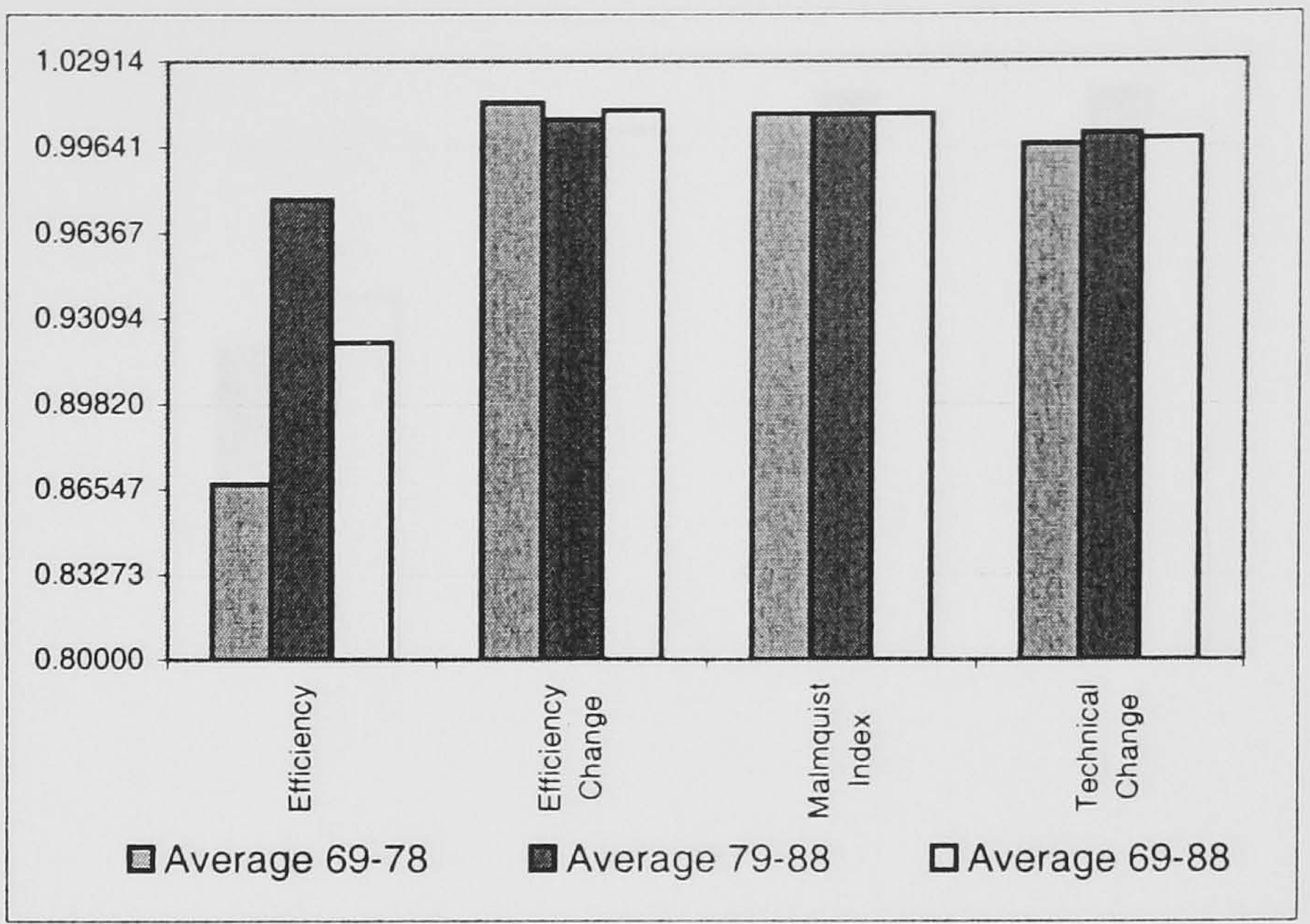
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.90944	1979	0.90829
1970	0.77606	1980	0.91175
1971	0.80851	1981	0.90650
1972	0.83552	1982	0.90682
1973	0.86736	1983	0.89384
1974	0.89205	1984	0.88869
1975	1.00000	1985	0.88662
1976	0.91726	1986	0.96222
1977	0.92231	1987	0.89552
1978	0.92071	1988	0.90453

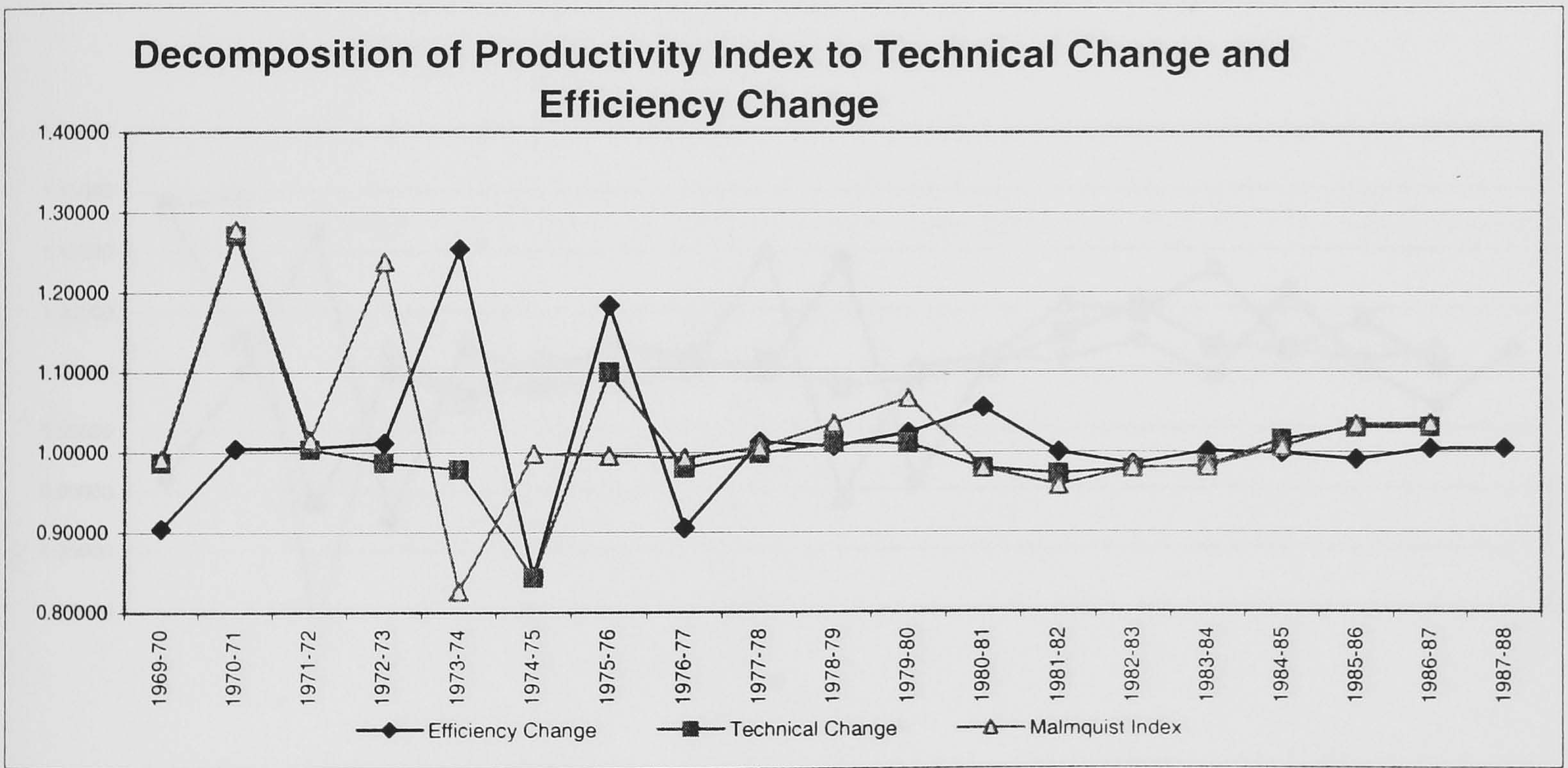
A comparison of efficiency with least efficient country



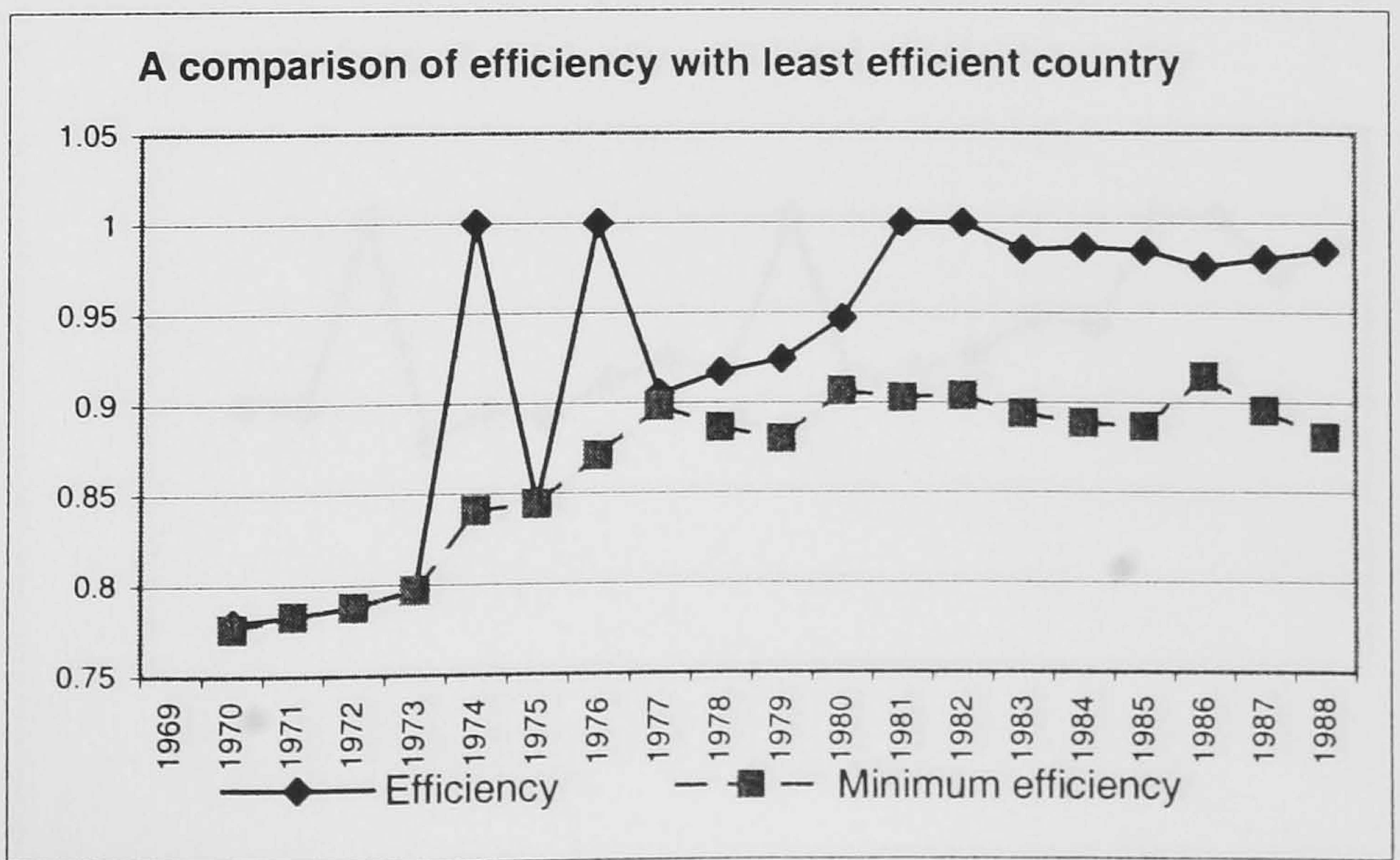
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.90480	0.82639	0.74772
1970-71	1.00491	0.98766	0.99250
1971-72	1.00593	1.27107	1.27861
1972-73	1.01179	1.00385	1.01569
1973-74	1.25465	0.98692	1.23823
1974-75	0.84436	0.97875	0.82642
1975-76	1.18433	0.84314	0.99855
1976-77	0.90505	1.10078	0.99626
1977-78	1.01305	0.98035	0.99314
1978-79	1.00771	0.99842	1.00611
1979-80	1.02457	1.01139	1.03624
1980-81	1.05639	1.01116	1.06818
1981-82	1.00000	0.98053	0.98053
1982-83	0.98508	0.97312	0.95861
1983-84	1.00121	0.97942	0.98060
1984-85	0.99840	0.98292	0.98135
1985-86	0.99079	1.01604	1.00669
1986-87	1.00418	1.03041	1.03473
1987-88	1.00474	1.03146	1.03635

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.86747	0.97663	0.92205
Efficiency Change	1.01366	1.00726	1.01063
Malmquist Index	1.00932	1.00925	1.00929
Technical Change	0.99773	1.00183	0.99967





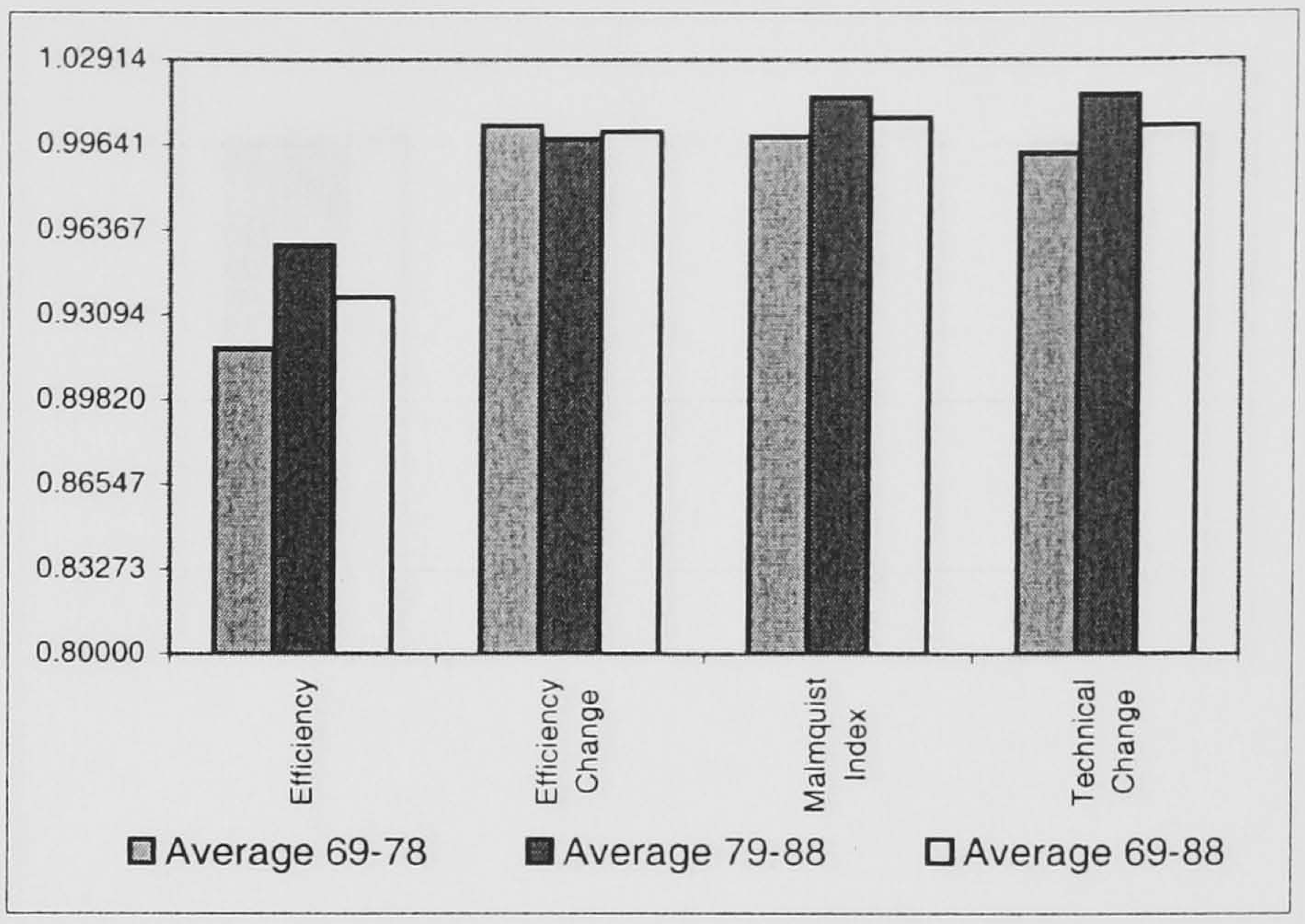
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.86127	1979	0.92392
1970	0.77928	1980	0.94662
1971	0.78311	1981	1.00000
1972	0.78775	1982	1.00000
1973	0.79703	1983	0.98508
1974	1.00000	1984	0.98627
1975	0.84436	1985	0.98470
1976	1.00000	1986	0.97563
1977	0.90505	1987	0.97971
1978	0.91686	1988	0.98435

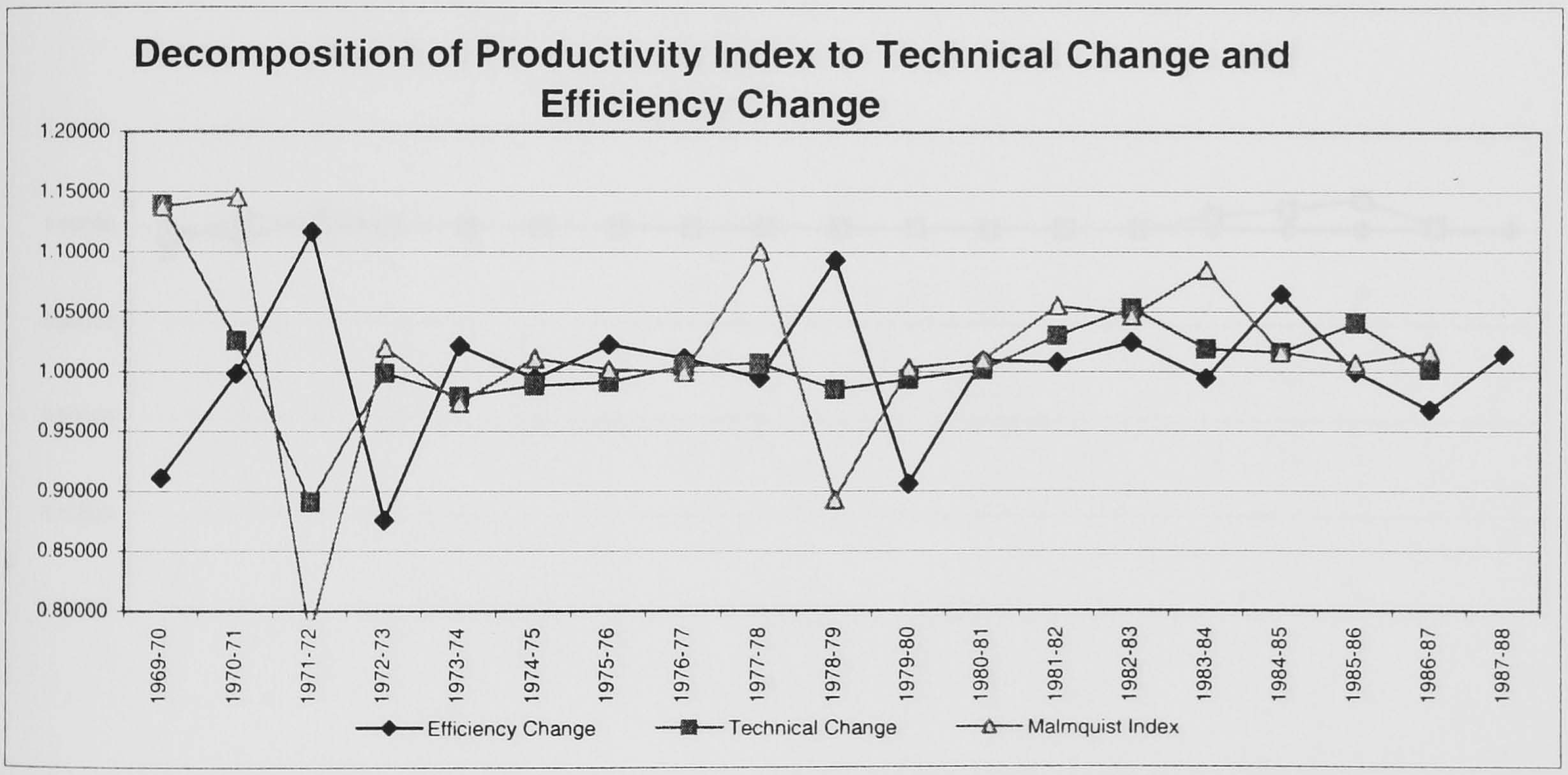


Summary of dynamic efficiency, productivity and its decomposition for DENMARK

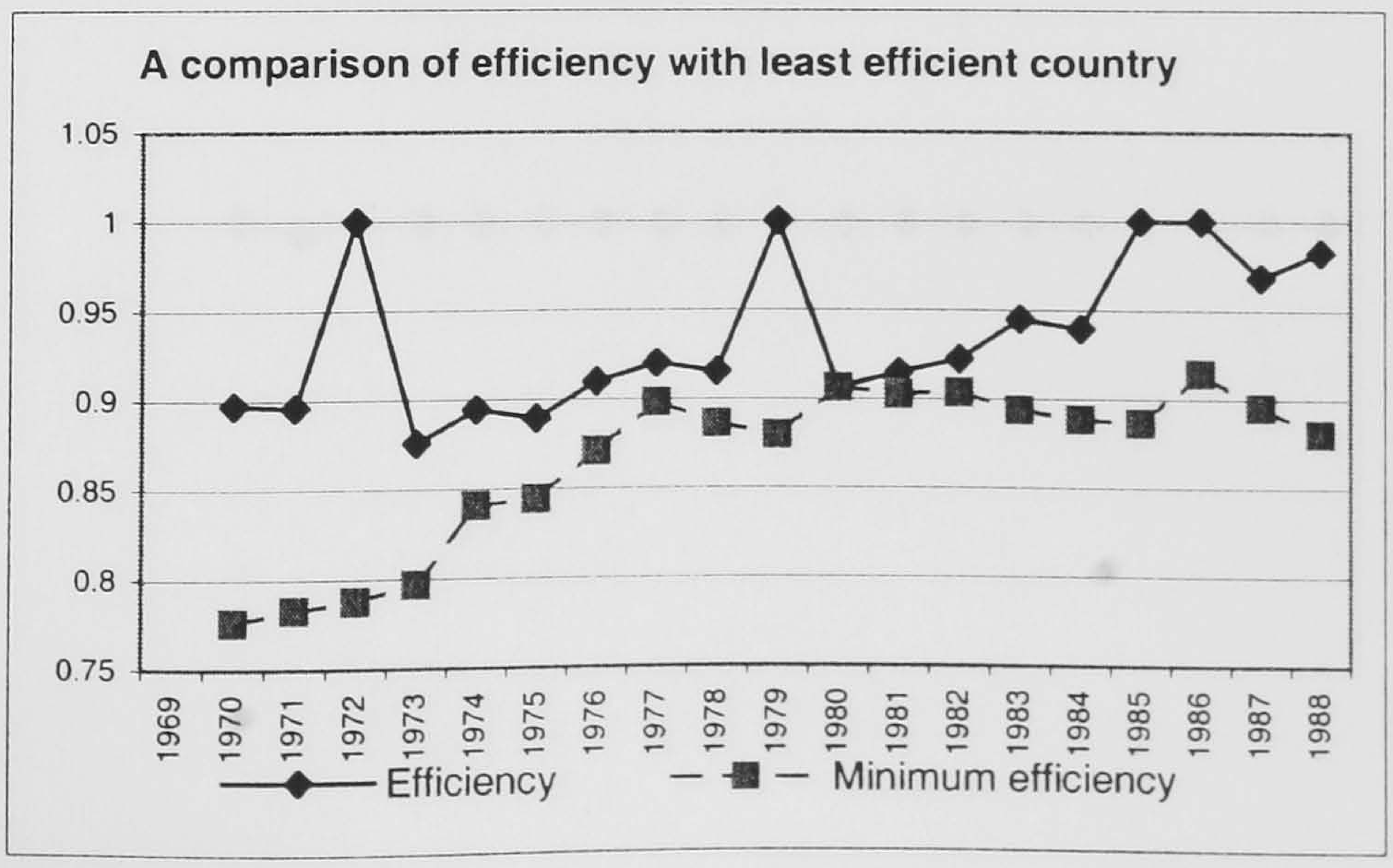
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.91132	0.90877	0.82818
1970-71	0.99838	1.13923	1.13739
1971-72	1.11655	1.02584	1.14540
1972-73	0.87571	0.89055	0.77987
1973-74	1.02137	0.99875	1.02009
1974-75	0.99440	0.97939	0.97391
1975-76	1.02287	0.98828	1.01089
1976-77	1.01112	0.99104	1.00206
1977-78	0.99475	1.00463	0.99936
1978-79	1.09283	1.00727	1.10077
1979-80	0.90615	0.98509	0.89263
1980-81	1.00968	0.99362	1.00323
1981-82	1.00798	1.00165	1.00964
1982-83	1.02436	1.03014	1.05523
1983-84	0.99419	1.05285	1.04674
1984-85	1.06475	1.01901	1.08498
1985-86	1.00000	1.01618	1.01618
1986-87	0.96826	1.04049	1.00746
1987-88	1.01473	1.00195	1.01671

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.91813	0.95779	0.93796
Efficiency Change	1.00393	0.99890	1.00155
Malmquist Index	0.99979	1.01476	1.00688
Technical Change	0.99338	1.01566	1.00393





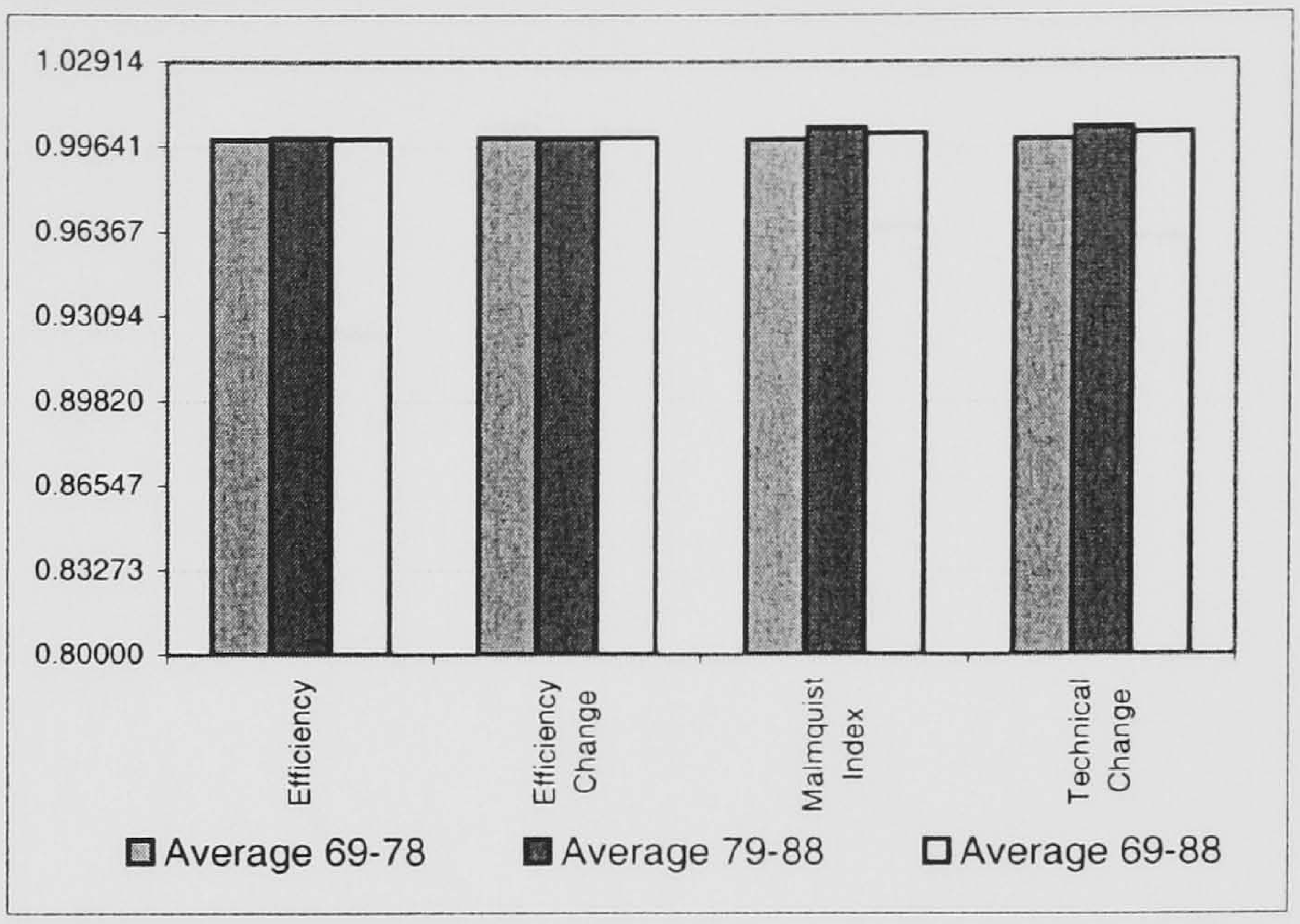
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.98436	1979	1.00000
1970	0.89707	1980	0.90615
1971	0.89561	1981	0.91491
1972	1.00000	1982	0.92221
1973	0.87571	1983	0.94468
1974	0.89443	1984	0.93919
1975	0.88942	1985	1.00000
1976	0.90976	1986	1.00000
1977	0.91988	1987	0.96826
1978	0.91506	1988	0.98252

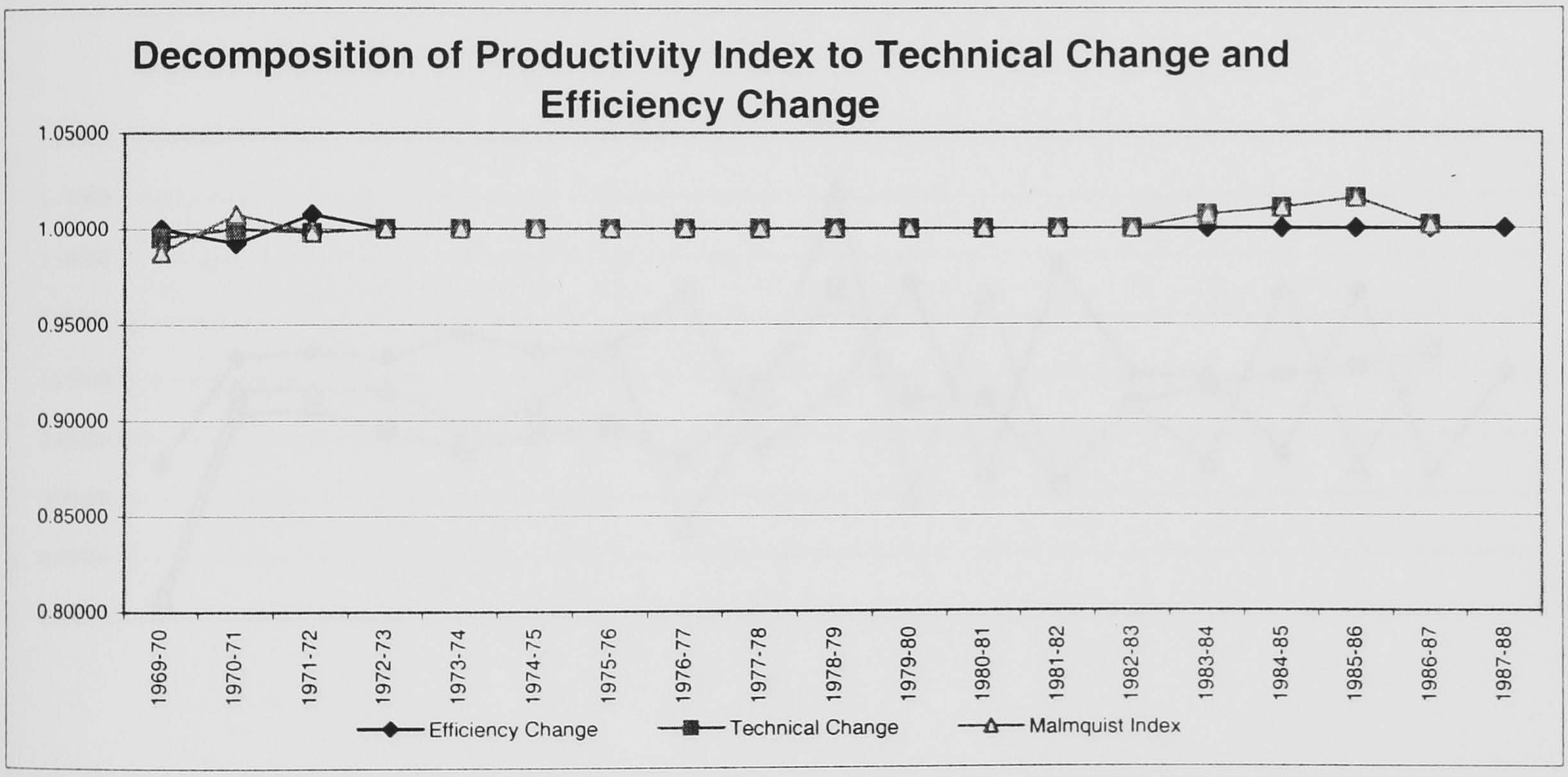


Summary of dynamic efficiency, productivity and its decomposition for FINLAND

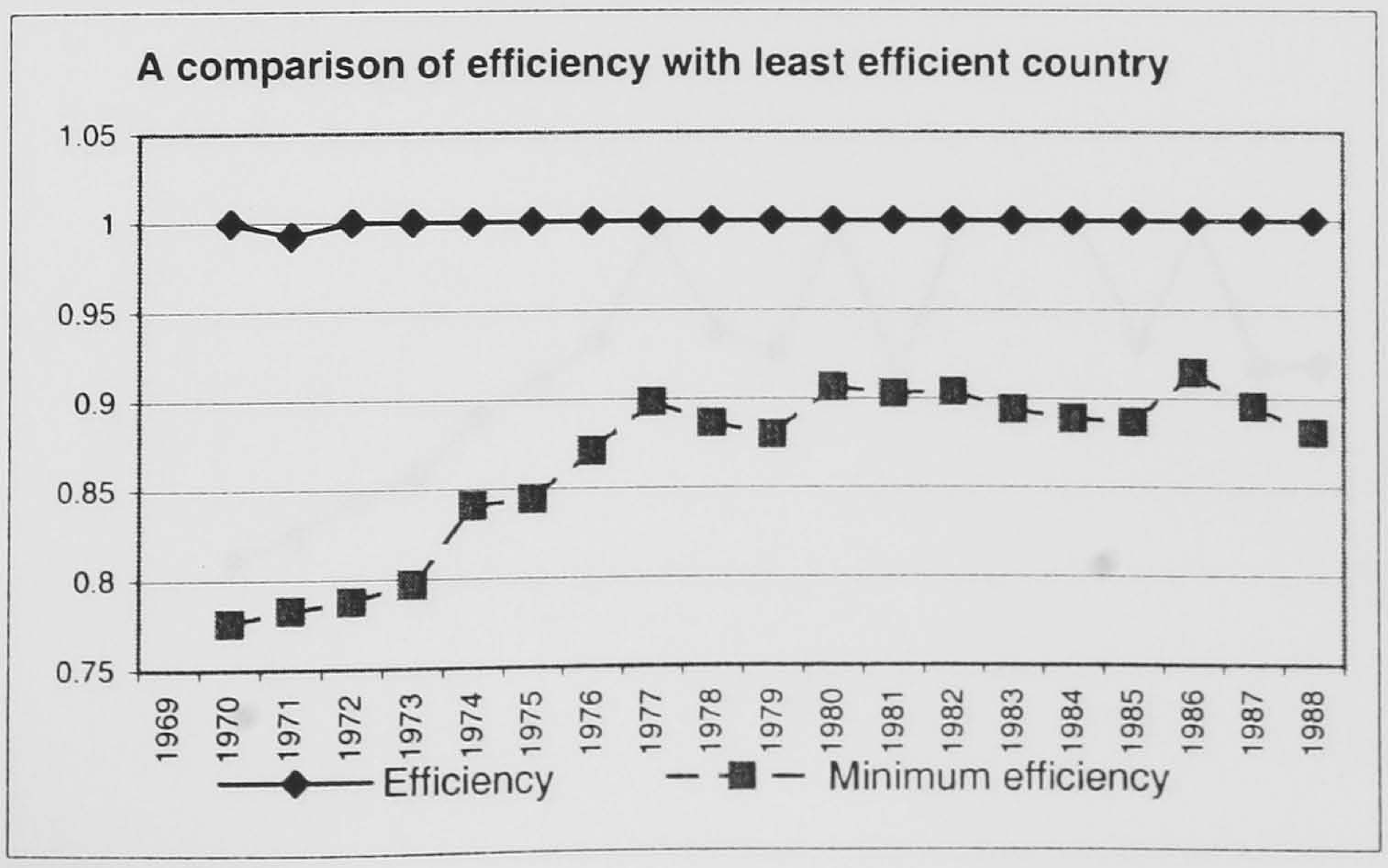
	Efficiency Change	Technical Change	Malmquist Index
1969-70	1.00000	1.00000	1.00000
1970-71	0.99233	0.99535	0.98772
1971-72	1.00773	1.00000	1.00773
1972-73	0.99996	0.99811	0.99806
1973-74	1.00004	1.00000	1.00004
1974-75	1.00000	1.00000	1.00000
1975-76	1.00000	1.00000	1.00000
1976-77	1.00000	1.00000	1.00000
1977-78	1.00000	1.00000	1.00000
1978-79	1.00000	1.00000	1.00000
1979-80	1.00000	1.00000	1.00000
1980-81	1.00000	1.00000	1.00000
1981-82	1.00000	1.00000	1.00000
1982-83	1.00000	1.00000	1.00000
1983-84	1.00000	1.00000	1.00000
1984-85	1.00000	1.00713	1.00713
1985-86	1.00000	1.01074	1.01074
1986-87	1.00000	1.01597	1.01597
1987-88	1.00000	1.00205	1.00205

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.99923	1.00000	0.99961
Efficiency Change	1.00001	1.00000	1.00000
Malmquist Index	0.99935	1.00399	1.00155
Technical Change	0.99935	1.00399	1.00154





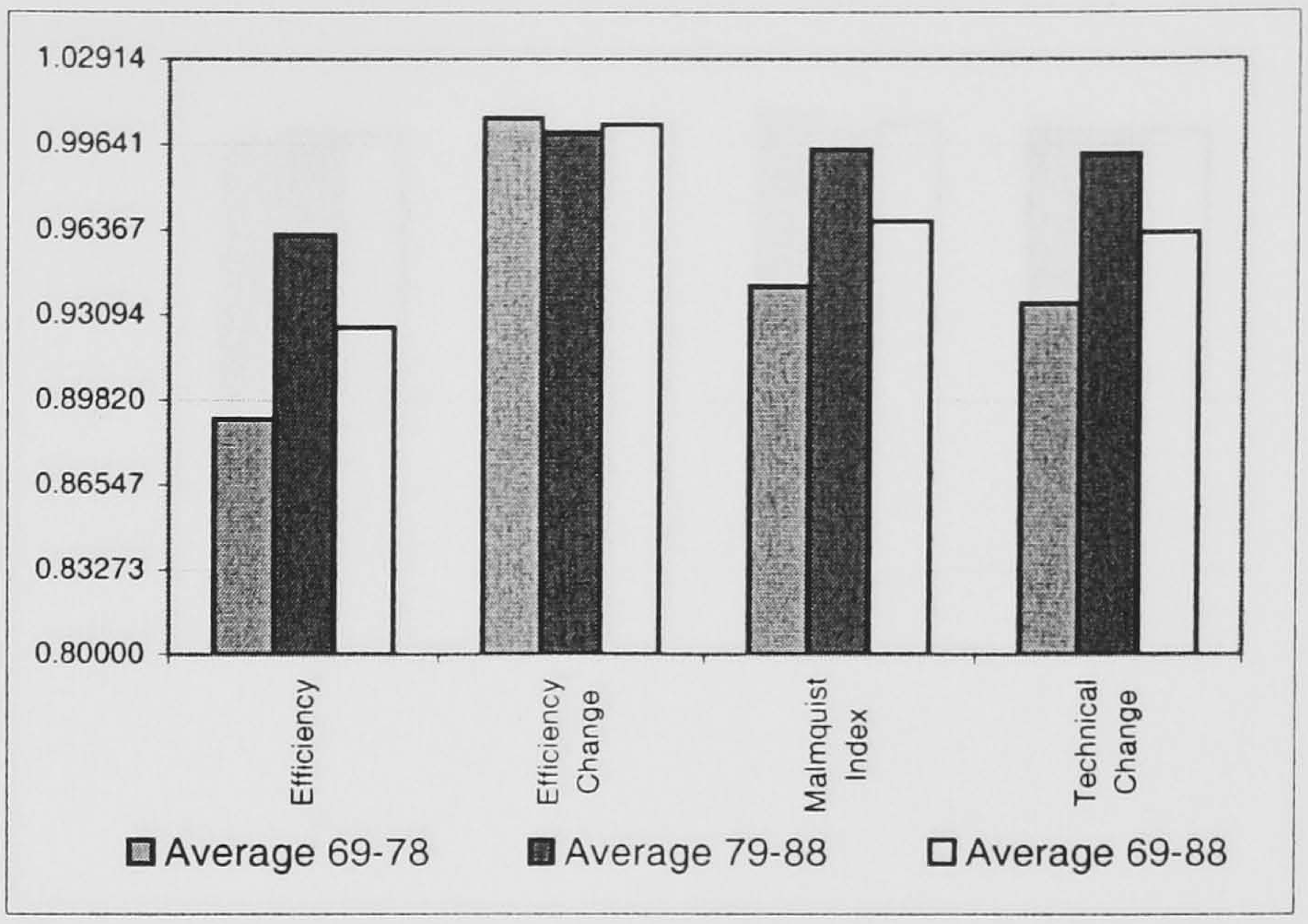
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	1.00000	1979	1.00000
1970	1.00000	1980	1.00000
1971	0.99233	1981	1.00000
1972	1.00000	1982	1.00000
1973	0.99996	1983	1.00000
1974	1.00000	1984	1.00000
1975	1.00000	1985	1.00000
1976	1.00000	1986	1.00000
1977	1.00000	1987	1.00000
1978	1.00000	1988	1.00000

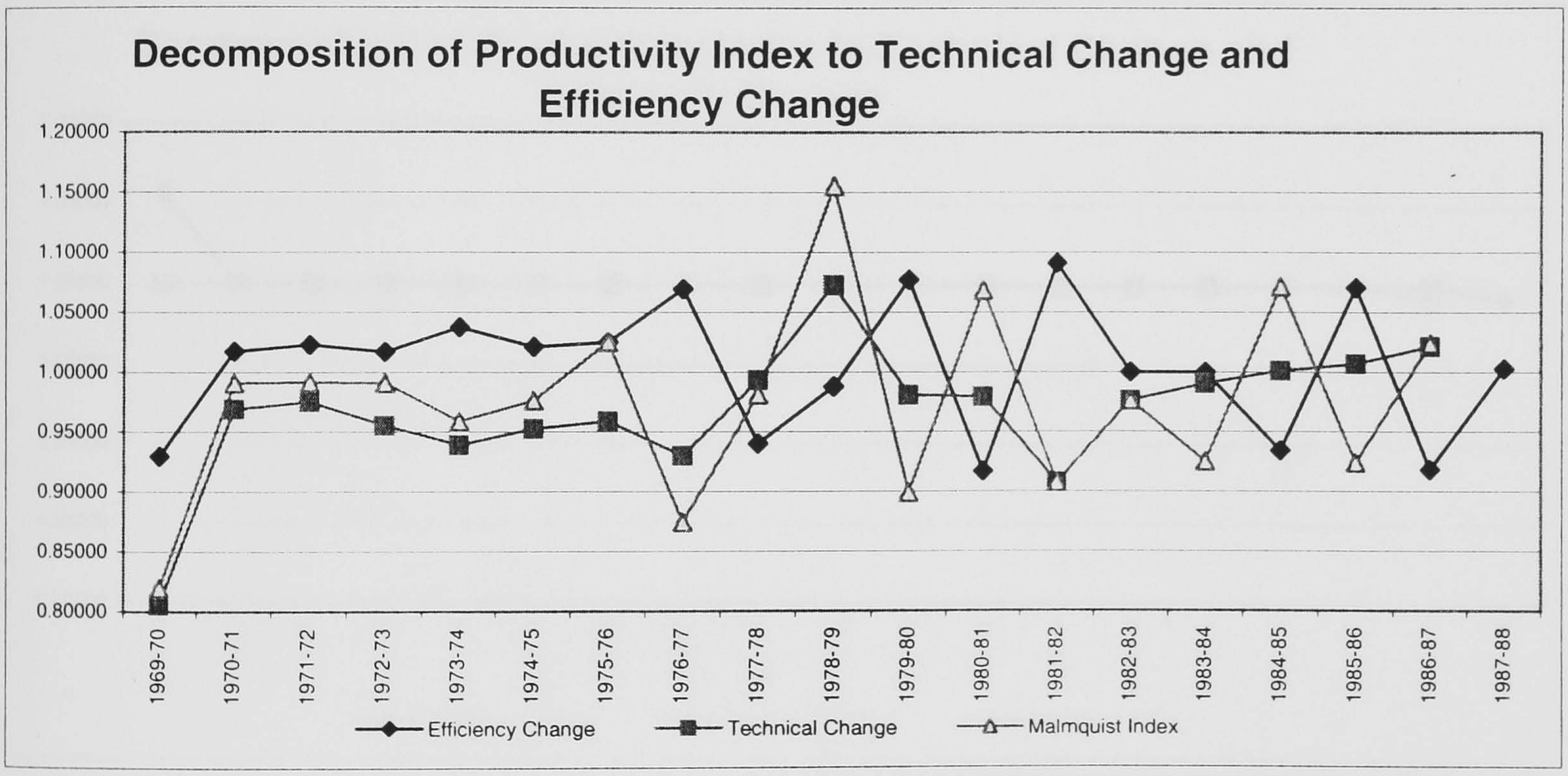


Summary of dynamic efficiency, productivity and its decomposition for FRANCE

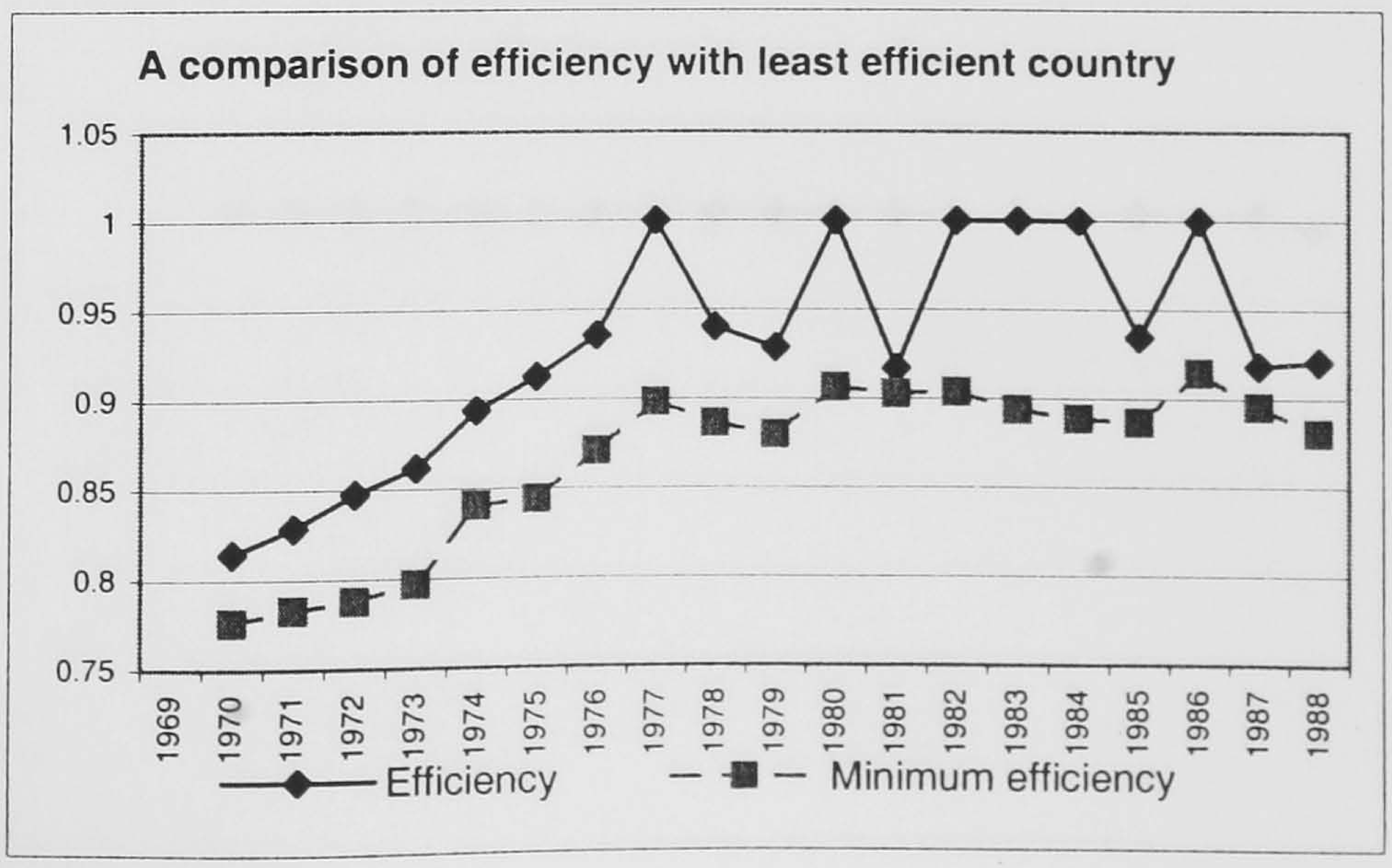
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.92970	0.87474	0.81325
1970-71	1.01728	0.80572	0.81965
1971-72	1.02287	0.96875	0.99090
1972-73	1.01676	0.97509	0.99144
1973-74	1.03728	0.95518	0.99079
1974-75	1.02099	0.93918	0.95889
1975-76	1.02490	0.95255	0.97626
1976-77	1.06923	0.95888	1.02526
1977-78	0.94006	0.93009	0.87433
1978-79	0.98773	0.99336	0.98117
1979-80	1.07698	1.07262	1.15519
1980-81	0.91670	0.98076	0.89907
1981-82	1.09086	0.97934	1.06833
1982-83	1.00000	0.90785	0.90785
1983-84	1.00000	0.97613	0.97613
1984-85	0.93414	0.99050	0.92526
1985-86	1.07050	1.00085	1.07141
1986-87	0.91810	1.00662	0.92417
1987-88	1.00261	1.02127	1.02393

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.89096	0.96180	0.92638
Efficiency Change	1.00668	1.00110	1.00404
Malmquist Index	0.94219	0.99459	0.96701
Technical Change	0.93535	0.99288	0.96260





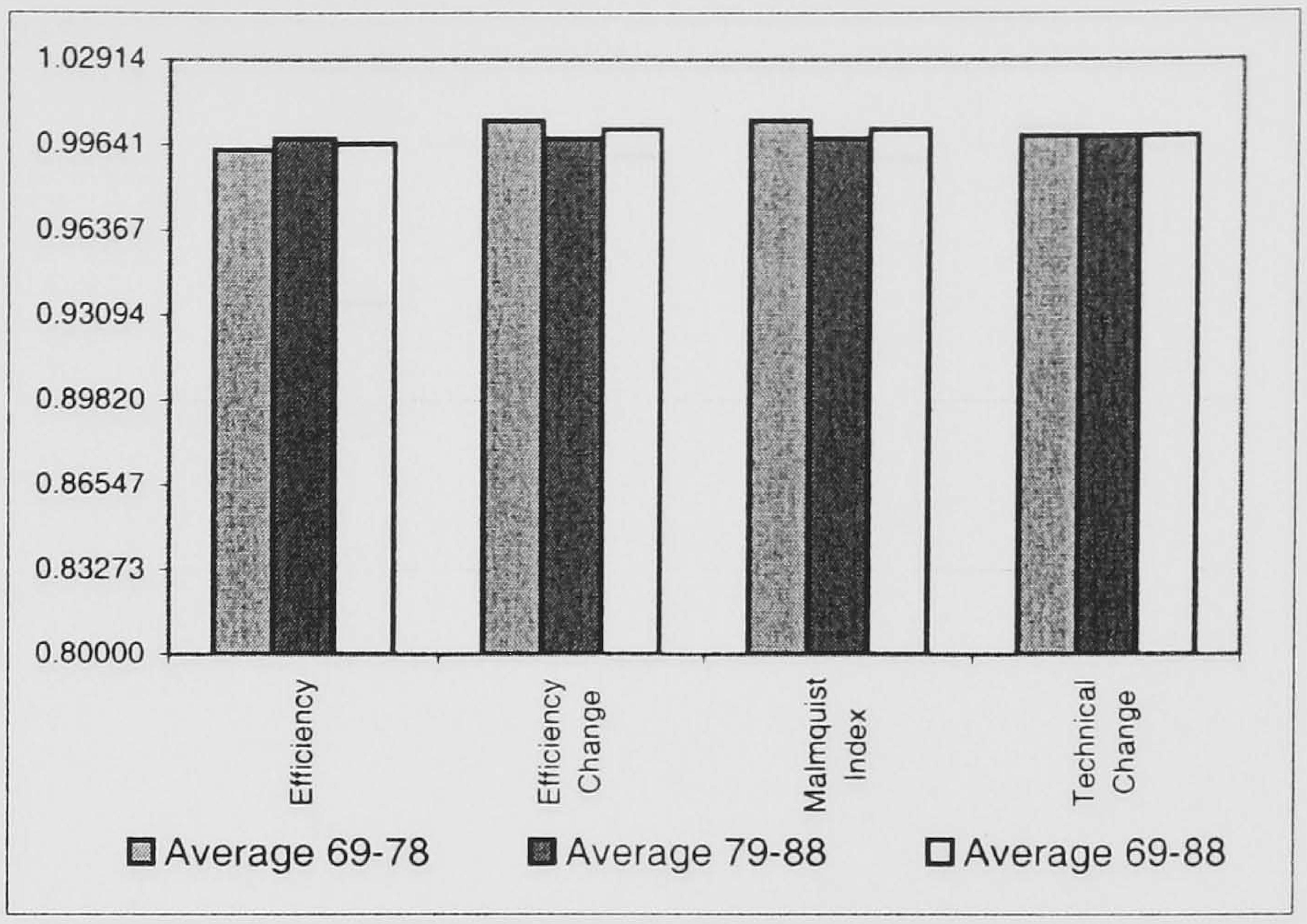
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.87601	1979	0.92852
1970	0.81442	1980	1.00000
1971	0.82850	1981	0.91670
1972	0.84745	1982	1.00000
1973	0.86165	1983	1.00000
1974	0.89377	1984	1.00000
1975	0.91253	1985	0.93414
1976	0.93525	1986	1.00000
1977	1.00000	1987	0.91810
1978	0.94006	1988	0.92049

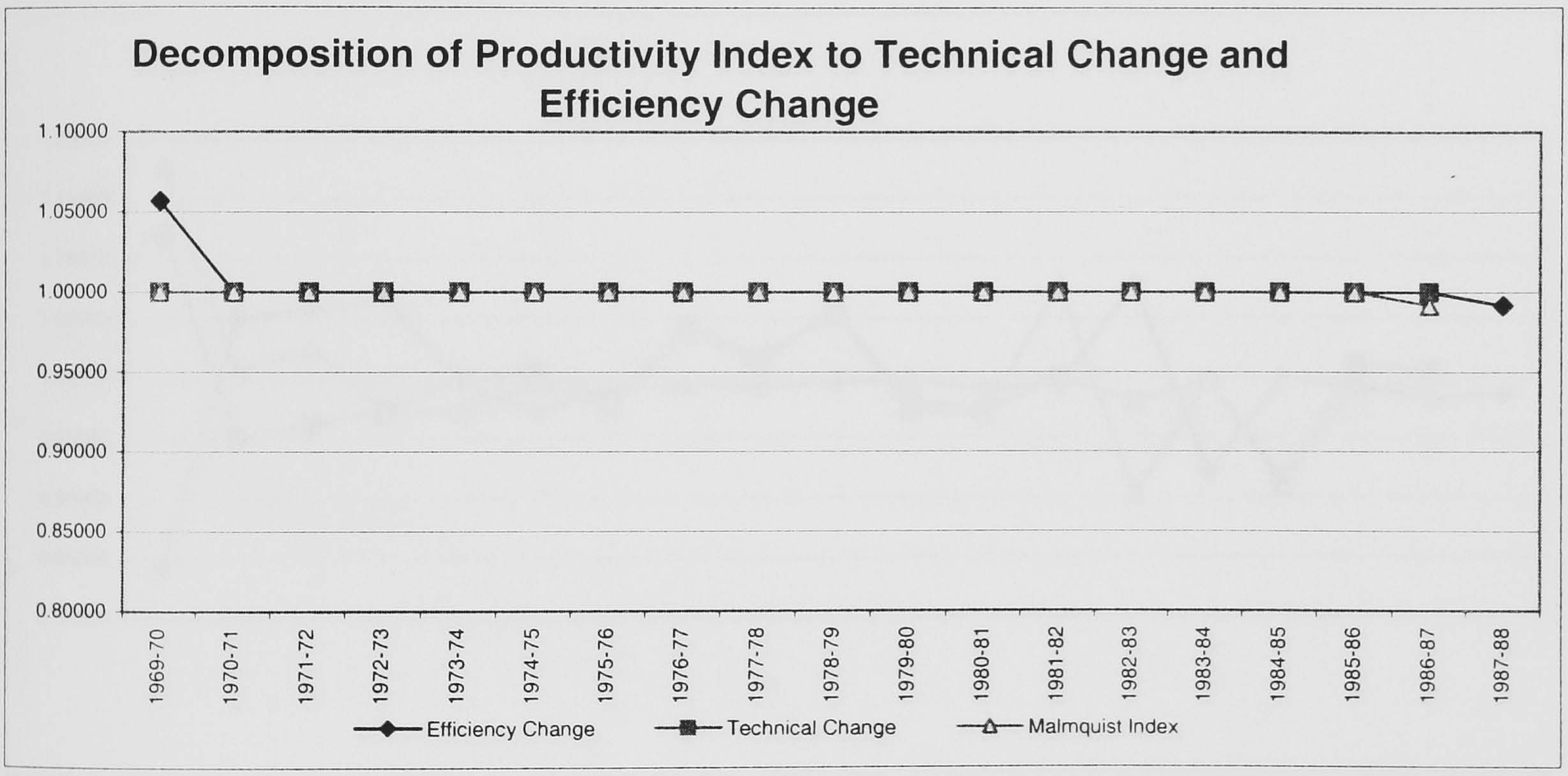


Summary of dynamic efficiency, productivity and its decomposition for GERMANY

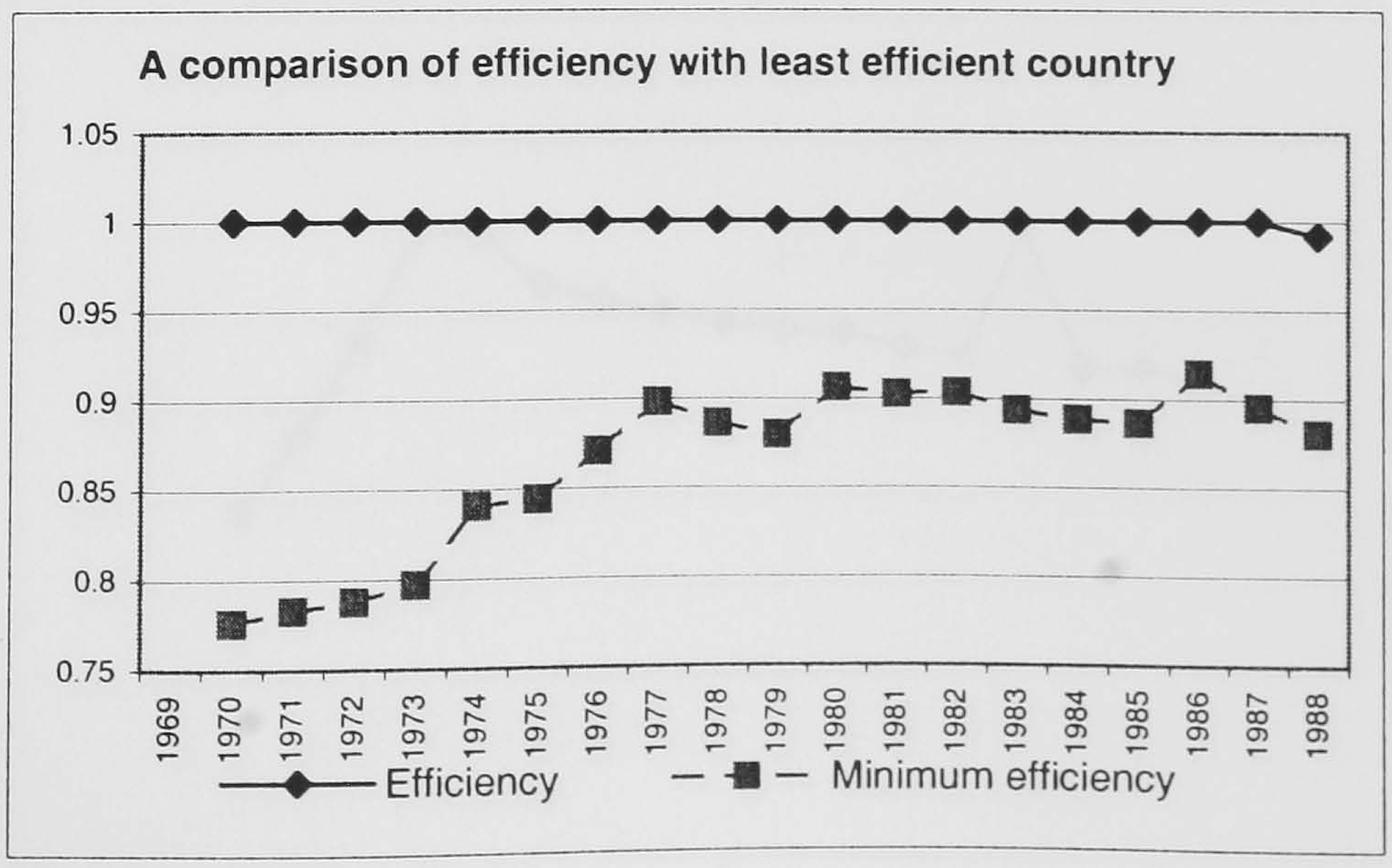
	Efficiency Change	Technical Change	Malmquist Index
1969-70	1.05661	1.00000	1.05661
1970-71	1.00000	1.00000	1.00000
1971-72	1.00000	1.00000	1.00000
1972-73	1.00000	1.00000	1.00000
1973-74	1.00000	1.00000	1.00000
1974-75	1.00000	1.00000	1.00000
1975-76	1.00000	1.00000	1.00000
1976-77	1.00000	1.00000	1.00000
1977-78	1.00000	1.00000	1.00000
1978-79	1.00000	1.00000	1.00000
1979-80	1.00000	1.00000	1.00000
1980-81	1.00000	1.00000	1.00000
1981-82	1.00000	1.00000	1.00000
1982-83	1.00000	1.00000	1.00000
1983-84	1.00000	1.00000	1.00000
1984-85	1.00000	1.00000	1.00000
1985-86	1.00000	1.00000	1.00000
1986-87	1.00000	1.00000	1.00000
1987-88	0.99166	1.00000	0.99166

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.99464	0.99917	0.99690
Efficiency Change	1.00566	0.99907	1.00254
Malmquist Index	1.00566	0.99907	1.00254
Technical Change	1.00000	1.00000	1.00000



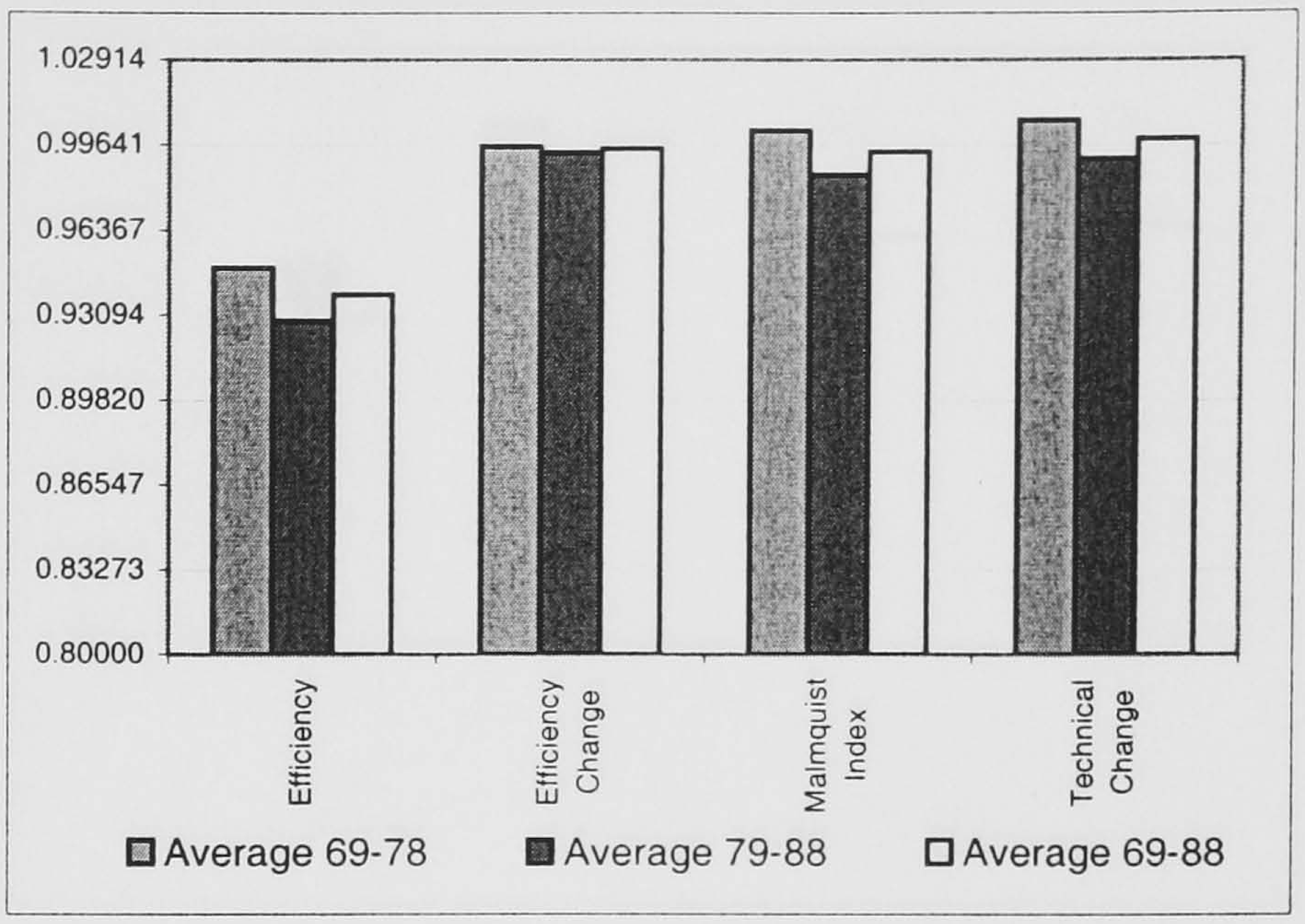


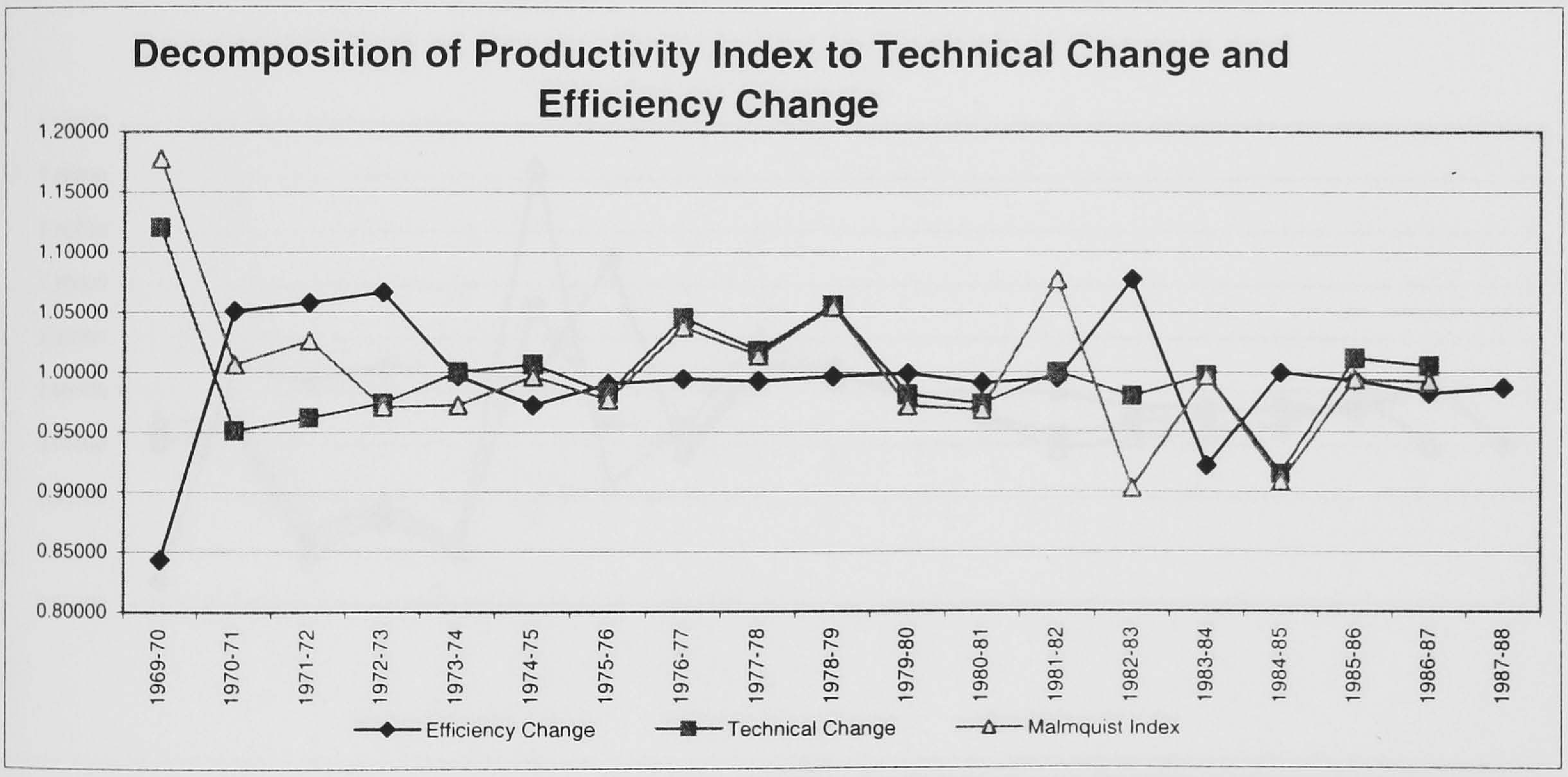
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.94642	1979	1.00000
1970	1.00000	1980	1.00000
1971	1.00000	1981	1.00000
1972	1.00000	1982	1.00000
1973	1.00000	1983	1.00000
1974	1.00000	1984	1.00000
1975	1.00000	1985	1.00000
1976	1.00000	1986	1.00000
1977	1.00000	1987	1.00000
1978	1.00000	1988	0.99166



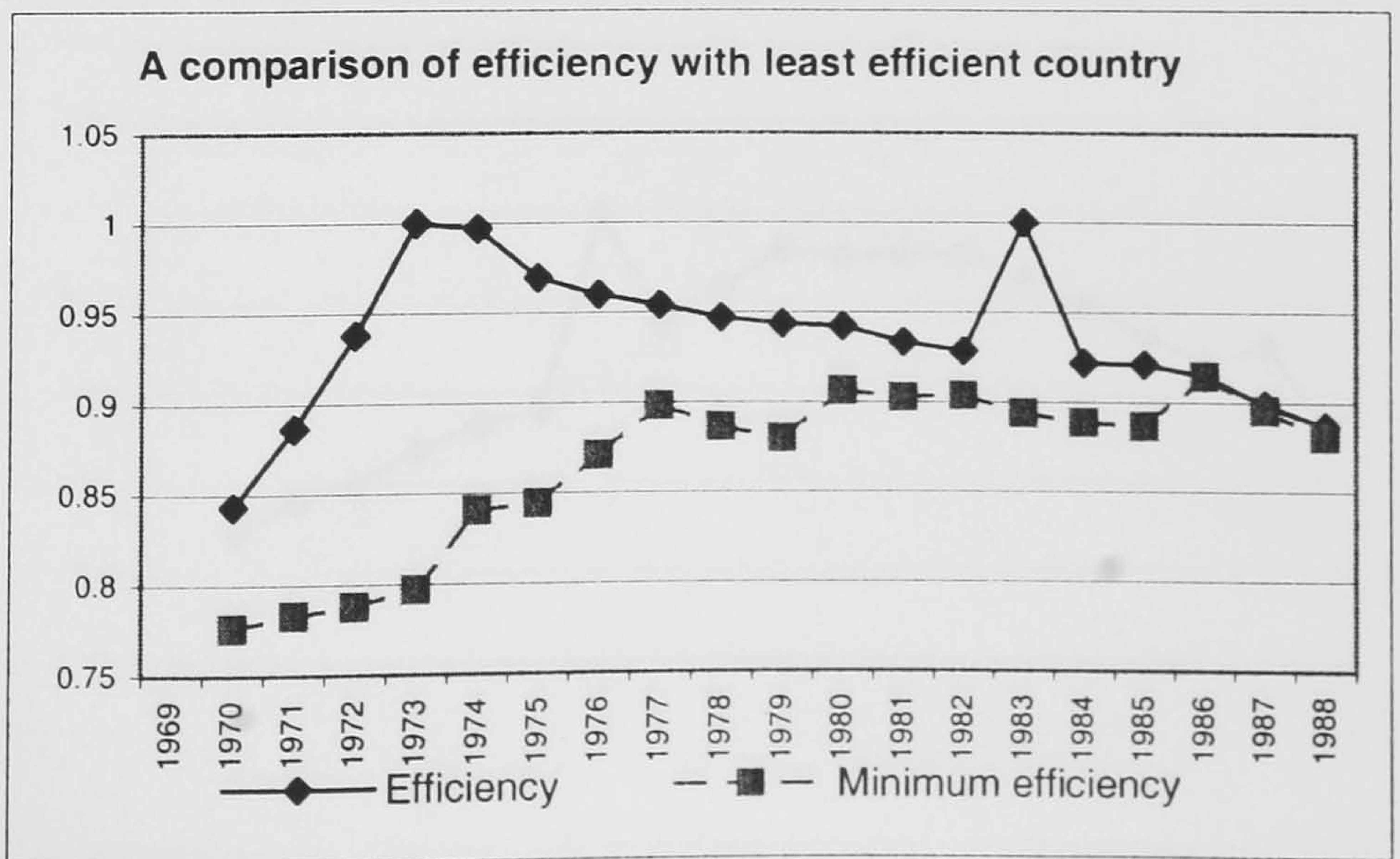
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.84342	1.00000	0.84342
1970-71	1.05079	1.12036	1.17726
1971-72	1.05787	0.95126	1.00631
1972-73	1.06662	0.96205	1.02614
1973-74	0.99677	0.97416	0.97101
1974-75	0.97249	1.00000	0.97249
1975-76	0.99006	1.00646	0.99646
1976-77	0.99414	0.98254	0.97678
1977-78	0.99237	1.04532	1.03734
1978-79	0.99635	1.01785	1.01414
1979-80	0.99865	1.05576	1.05434
1980-81	0.99070	0.98090	0.97178
1981-82	0.99455	0.97355	0.96824
1982-83	1.07731	1.00000	1.07731
1983-84	0.92153	0.97986	0.90297
1984-85	0.99932	0.99728	0.99660
1985-86	0.99288	0.91528	0.90876
1986-87	0.98245	1.01168	0.99392
1987-88	0.98684	1.00539	0.99215

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.94939	0.92886	0.93912
Efficiency Change	0.99609	0.99380	0.99501
Malmquist Index	1.00213	0.98512	0.99408
Technical Change	1.00600	0.99108	0.99893





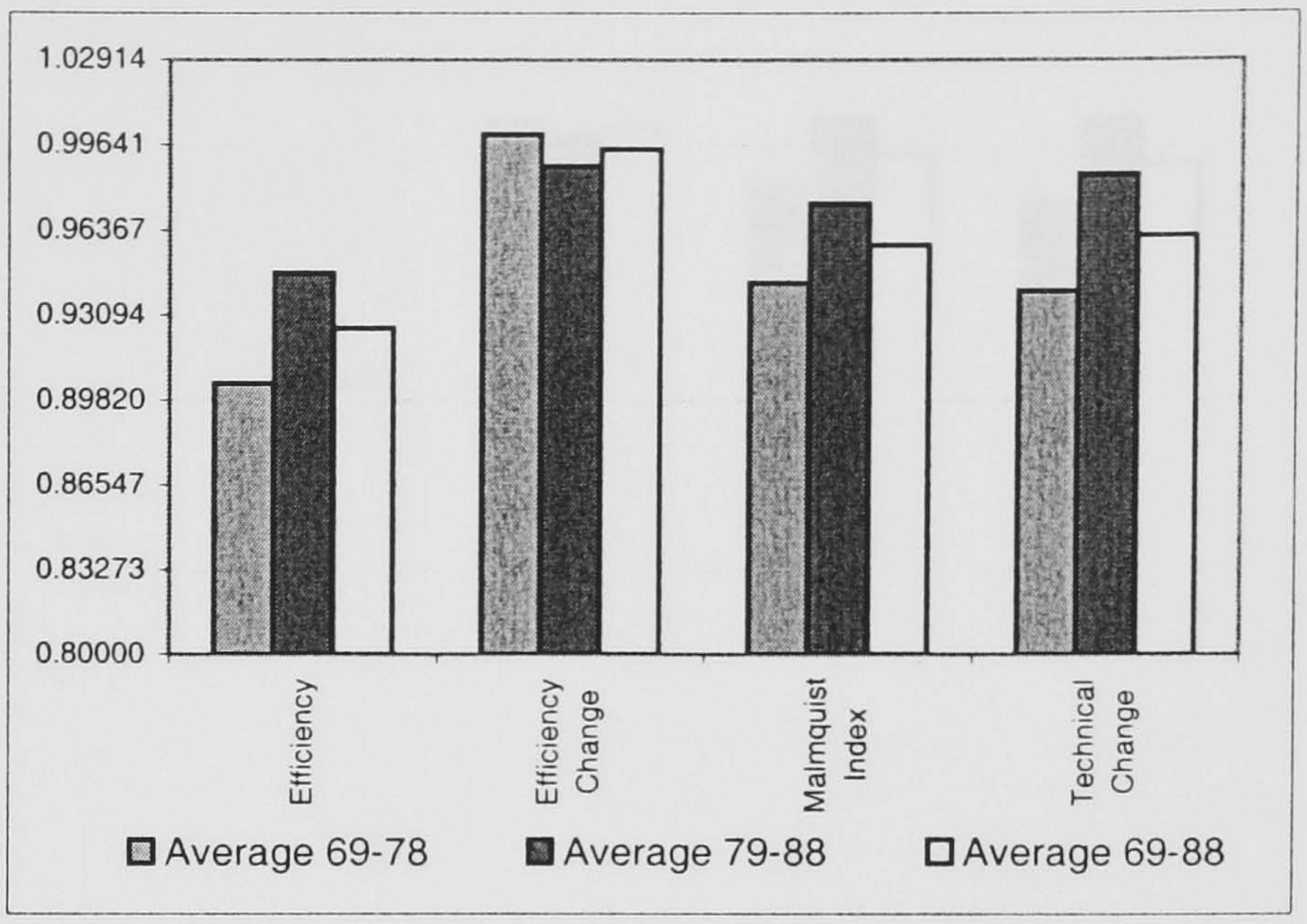
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	1.00000	1979	0.94335
1970	0.84342	1980	0.94208
1971	0.88626	1981	0.93332
1972	0.93754	1982	0.92823
1973	1.00000	1983	1.00000
1974	0.99677	1984	0.92153
1975	0.96935	1985	0.92090
1976	0.95971	1986	0.91435
1977	0.95409	1987	0.89830
1978	0.94681	1988	0.88648

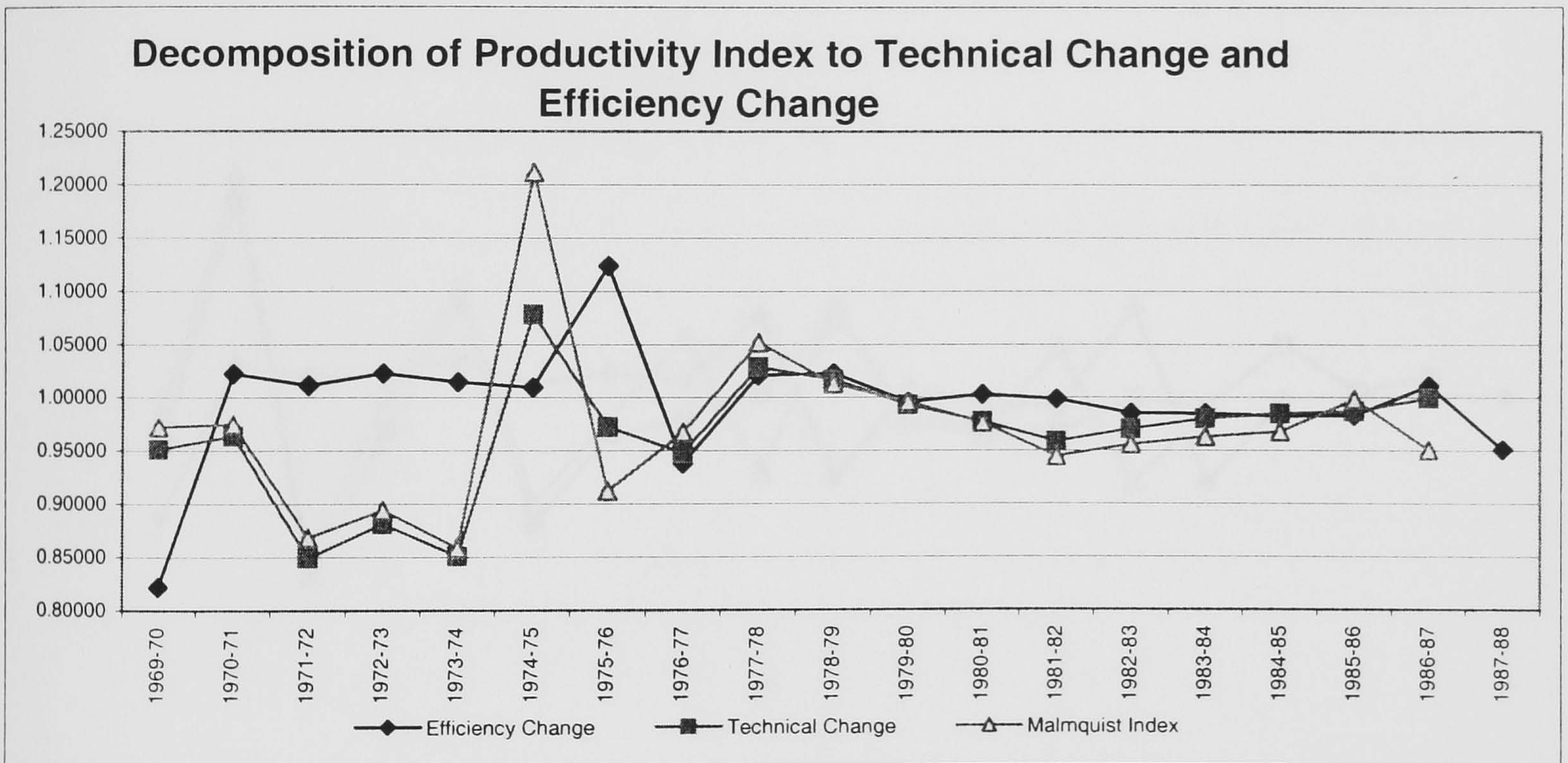


Summary of dynamic efficiency, productivity and its decomposition for IRELAND

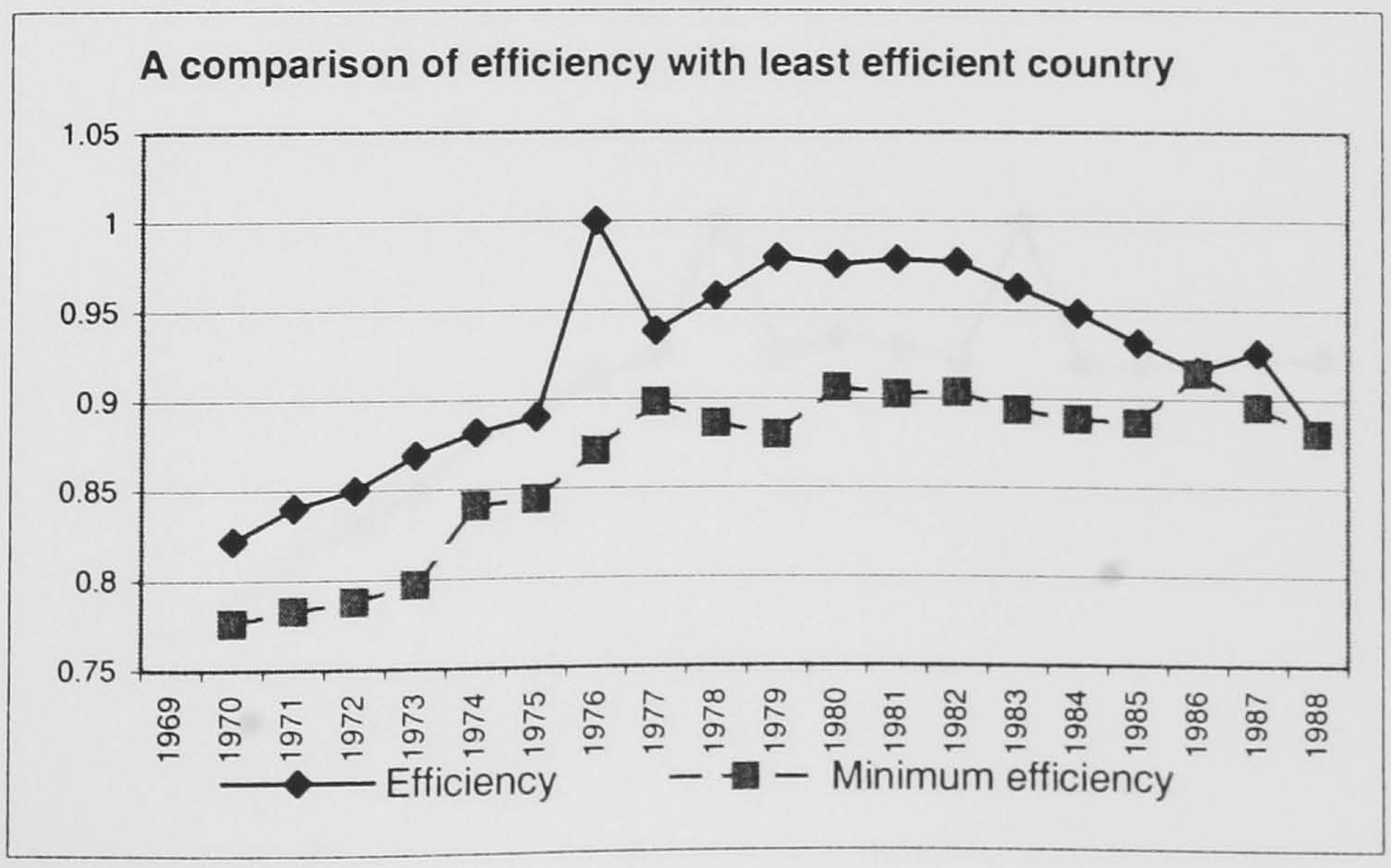
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.82177	0.88099	0.72397
1970-71	1.02236	0.95127	0.97254
1971-72	1.01137	0.96388	0.97483
1972-73	1.02274	0.84929	0.86861
1973-74	1.01459	0.88119	0.89404
1974-75	1.00939	0.85082	0.85881
1975-76	1.12363	1.07840	1.21172
1976-77	0.93812	0.97242	0.91224
1977-78	1.02073	0.94760	0.96724
1978-79	1.02270	1.02861	1.05196
1979-80	0.99597	1.01704	1.01294
1980-81	1.00292	0.99304	0.99593
1981-82	0.99862	0.97749	0.97615
1982-83	0.98540	0.95866	0.94466
1983-84	0.98511	0.97059	0.95614
1984-85	0.98251	0.98032	0.96318
1985-86	0.98284	0.98476	0.96786
1986-87	1.01126	0.98694	0.99805
1987-88	0.95099	0.99954	0.95056

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.90480	0.94745	0.92612
Efficiency Change	1.00074	0.98840	0.99490
Malmquist Index	0.94360	0.97394	0.95797
Technical Change	0.94045	0.98538	0.96173





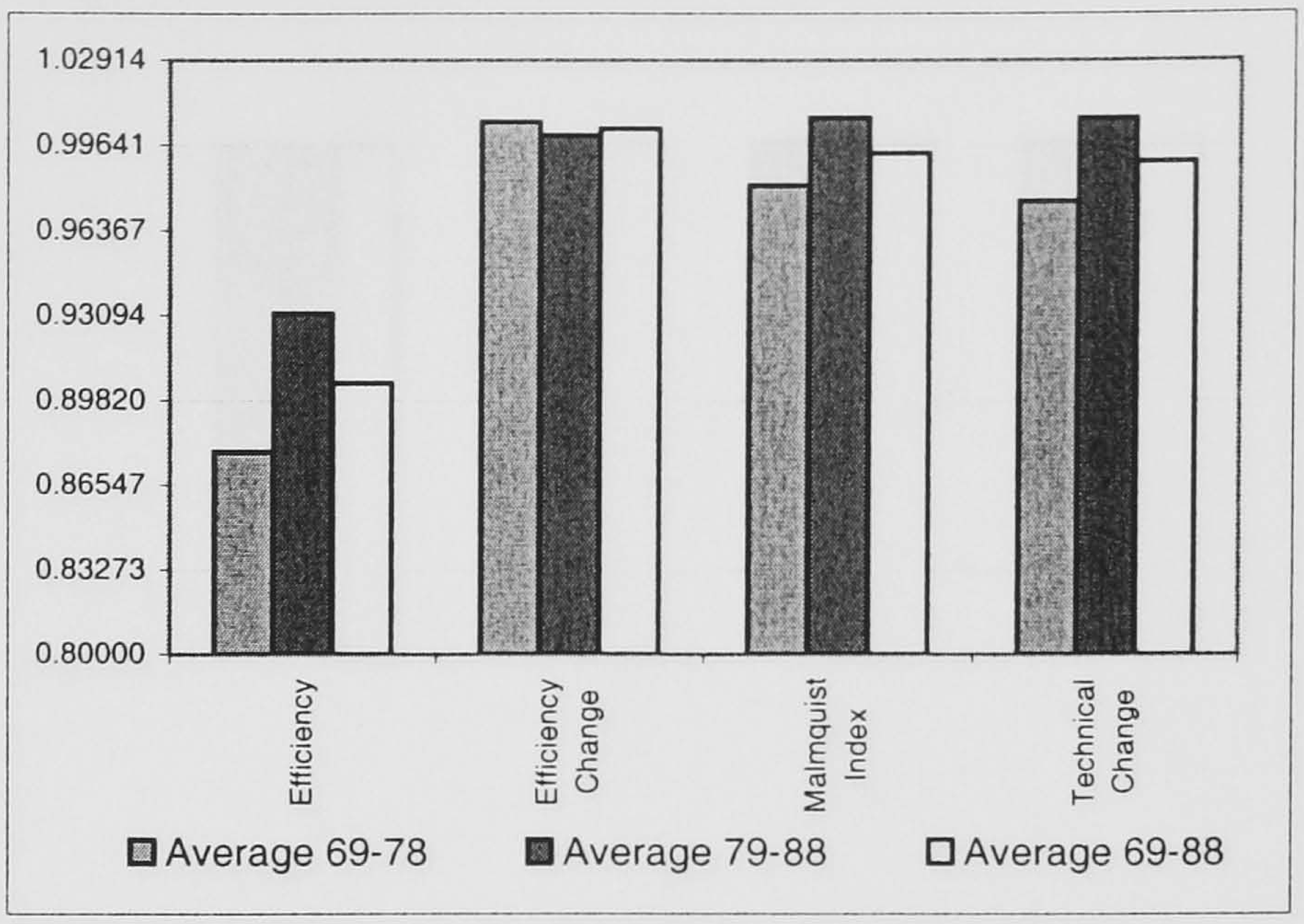
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	1.00000	1979	0.97930
1970	0.82177	1980	0.97535
1971	0.84015	1981	0.97820
1972	0.84969	1982	0.97685
1973	0.86902	1983	0.96259
1974	0.88170	1984	0.94825
1975	0.88997	1985	0.93167
1976	1.00000	1986	0.91568
1977	0.93812	1987	0.92599
1978	0.95756	1988	0.88061

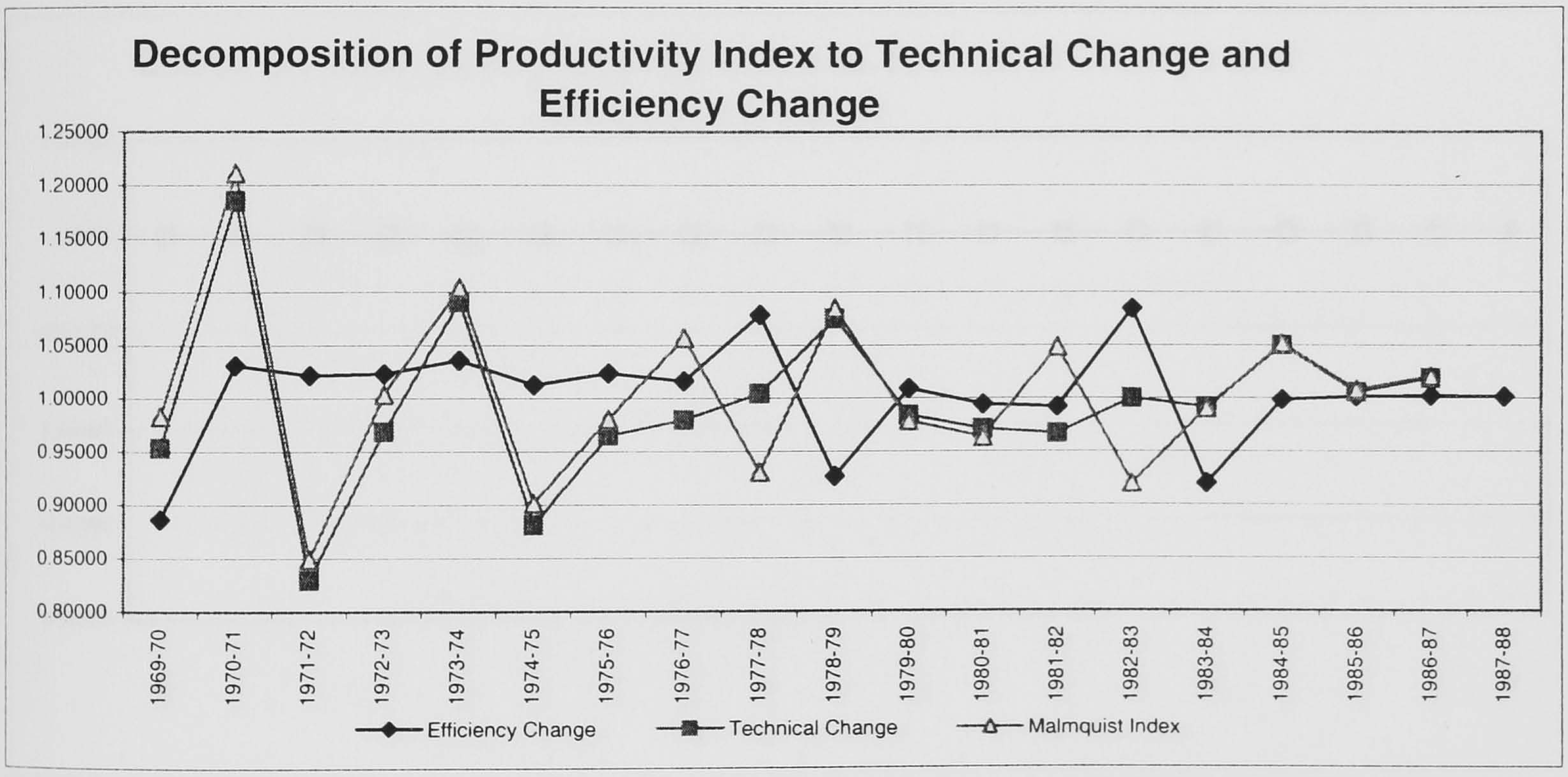


Summary of dynamic efficiency, productivity and its decomposition for ITALY

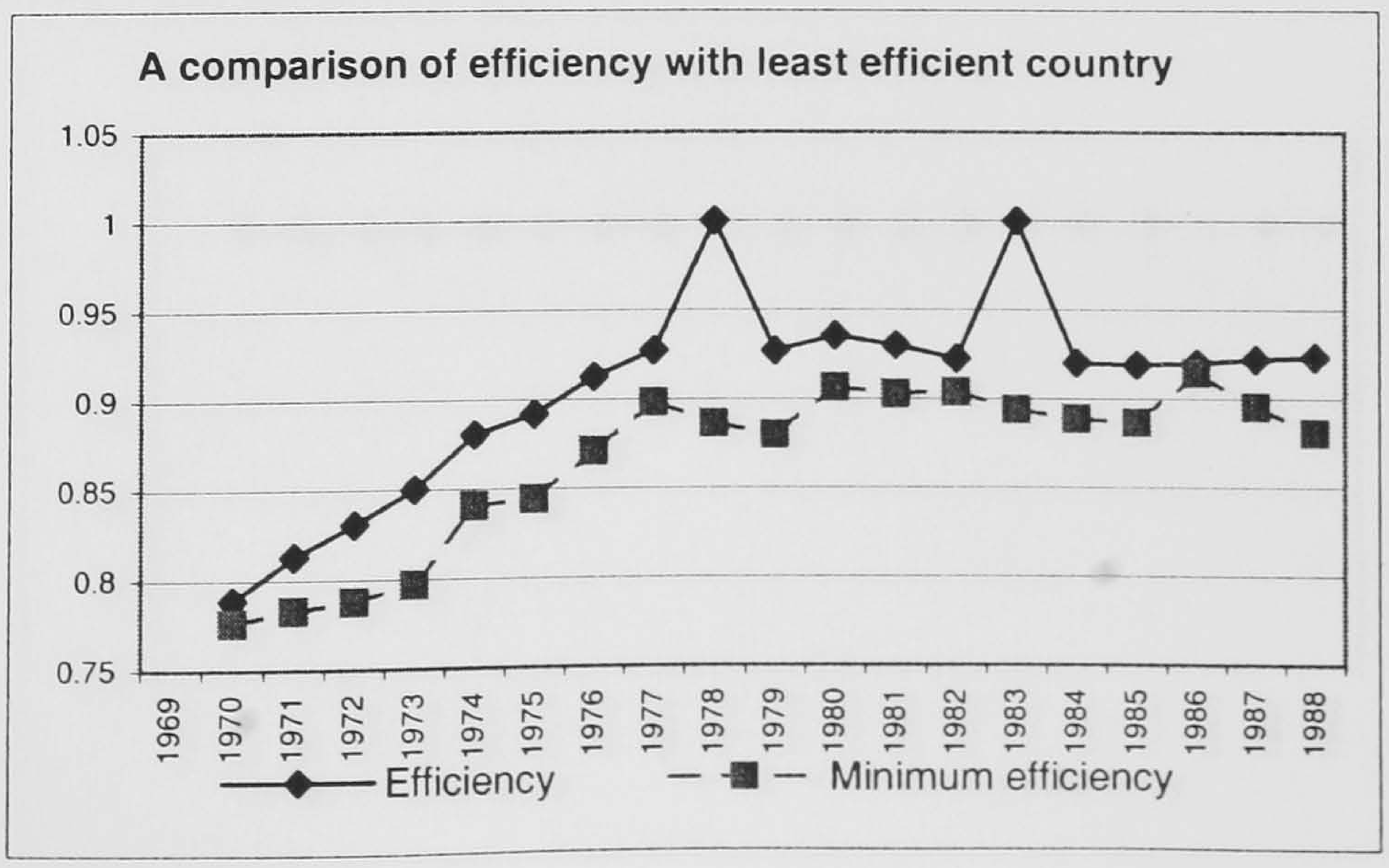
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.88630	0.89095	0.78965
1970-71	1.03093	0.95380	0.98330
1971-72	1.02174	1.18556	1.21134
1972-73	1.02339	0.82978	0.84918
1973-74	1.03571	0.96901	1.00362
1974-75	1.01252	1.09049	1.10415
1975-76	1.02344	0.88114	0.90180
1976-77	1.01611	0.96508	0.98063
1977-78	1.07853	0.97993	1.05689
1978-79	0.92668	1.00434	0.93070
1979-80	1.00932	1.07476	1.08477
1980-81	0.99419	0.98434	0.97861
1981-82	0.99202	0.97107	0.96332
1982-83	1.08405	0.96709	1.04838
1983-84	0.91992	1.00000	0.91992
1984-85	0.99881	0.99155	0.99037
1985-86	1.00127	1.04993	1.05126
1986-87	1.00220	1.00571	1.00792
1987-88	1.00148	1.01887	1.02038

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.87844	0.93185	0.90515
Efficiency Change	1.00554	1.00036	1.00308
Malmquist Index	0.98112	1.00722	0.99348
Technical Change	0.97501	1.00703	0.99018





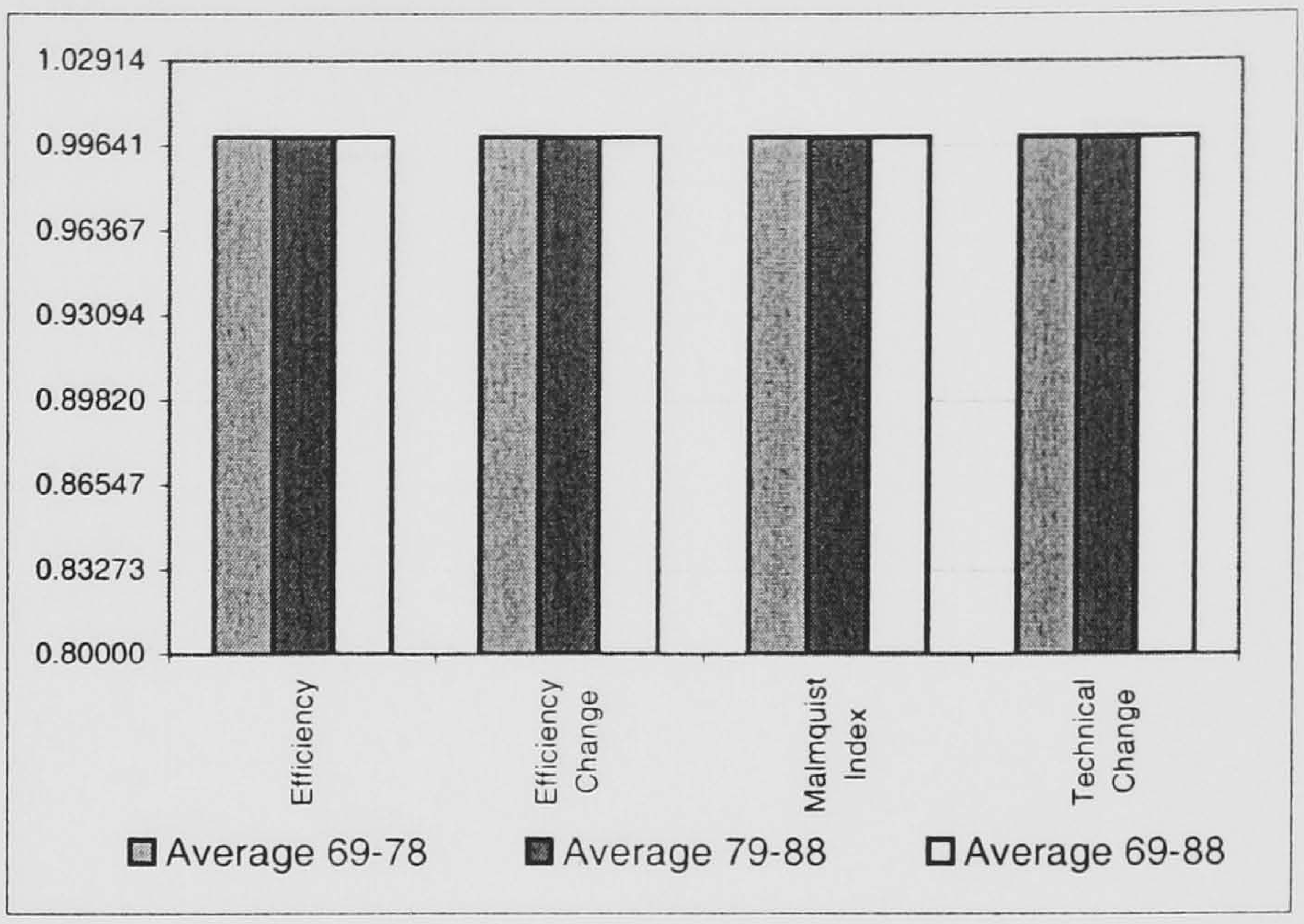
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.88988	1979	0.92668
1970	0.78870	1980	0.93532
1971	0.81309	1981	0.92988
1972	0.83077	1982	0.92247
1973	0.85019	1983	1.00000
1974	0.88056	1984	0.91992
1975	0.89158	1985	0.91882
1976	0.91249	1986	0.91999
1977	0.92719	1987	0.92201
1978	1.00000	1988	0.92338

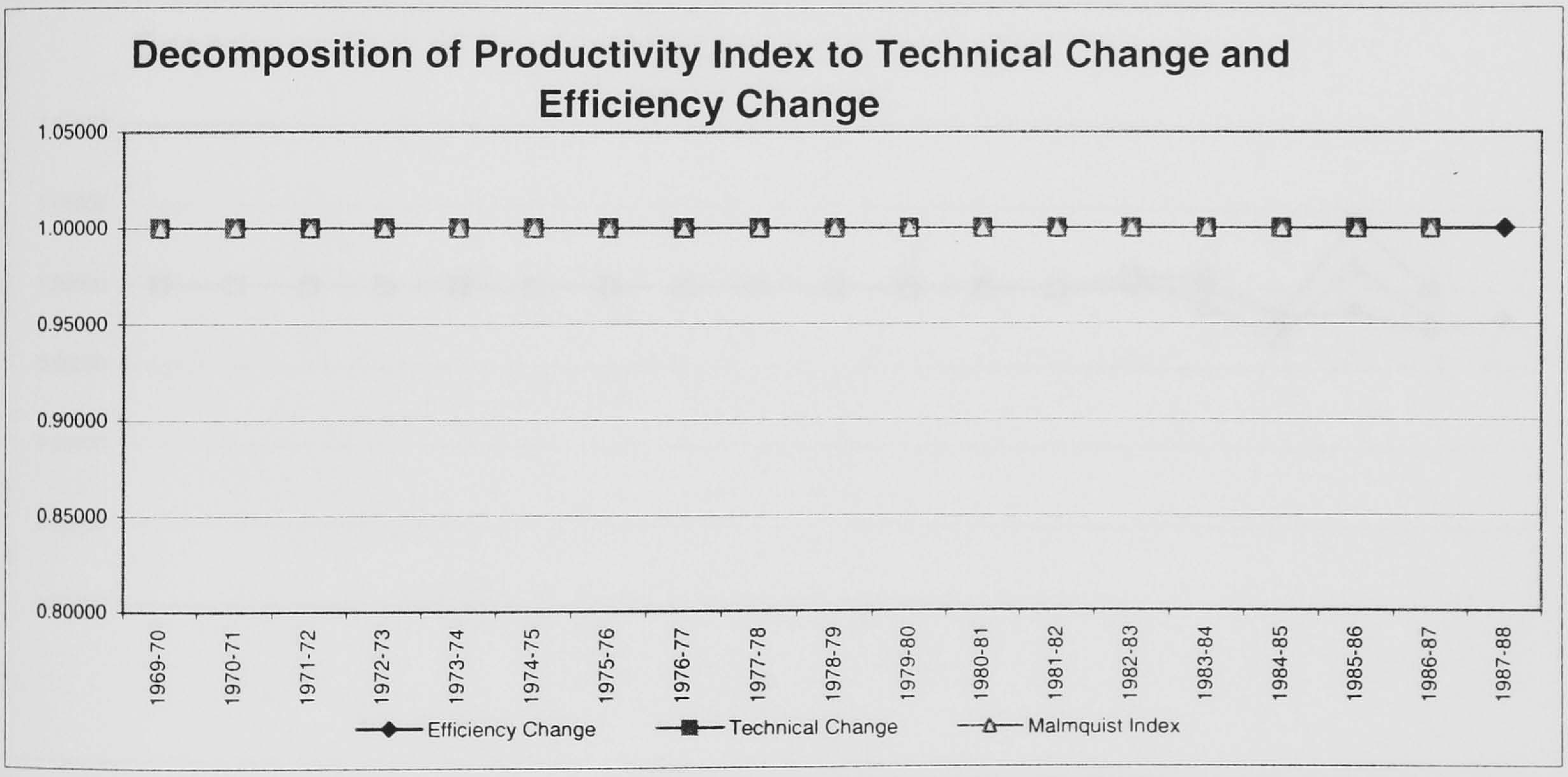


Summary of dynamic efficiency, productivity and its decomposition for JAPAN

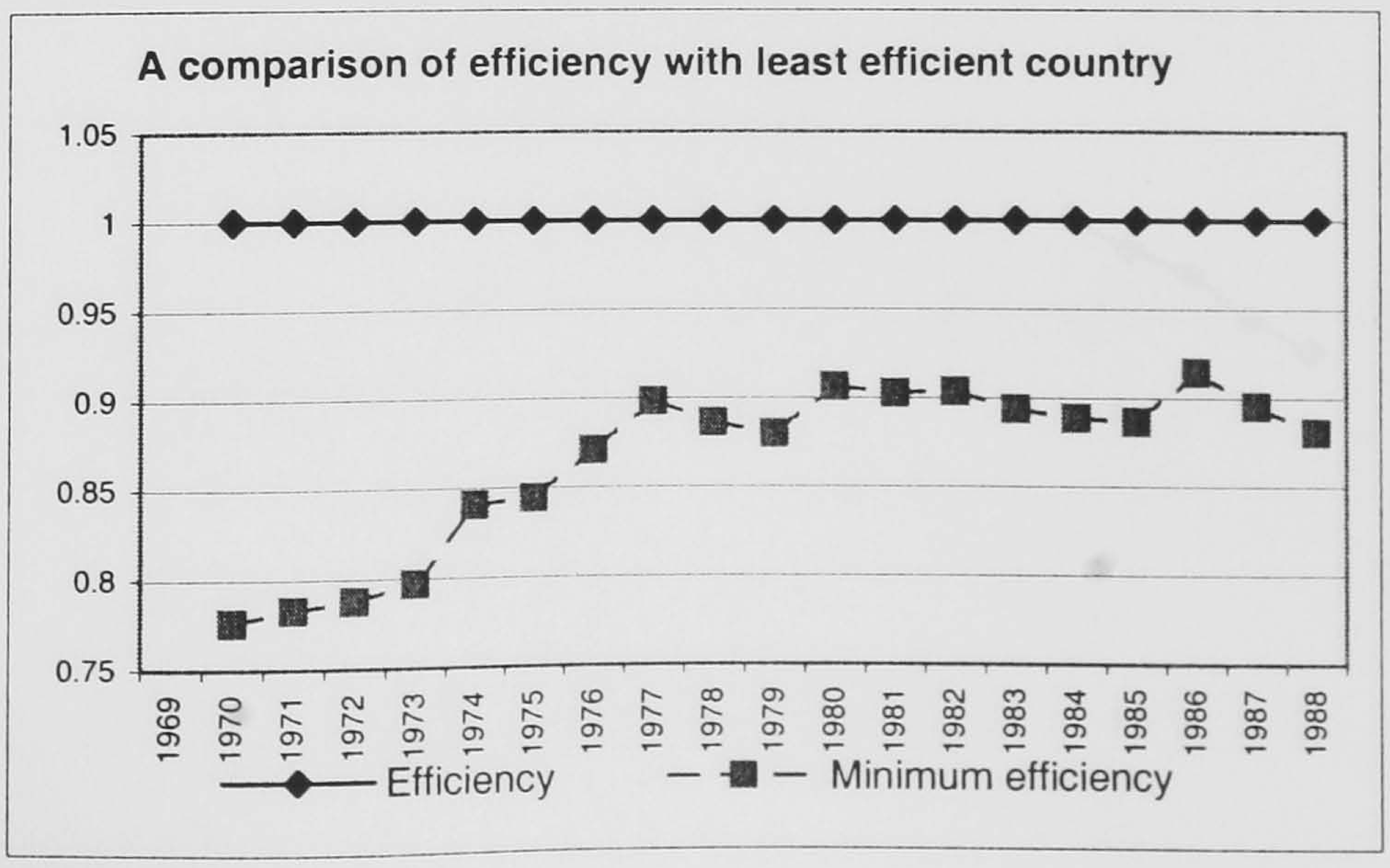
	Efficiency Change	Technical Change	Malmquist Index
1969-70	1.00000	1.00000	1.00000
1970-71	1.00000	1.00000	1.00000
1971-72	1.00000	1.00000	1.00000
1972-73	1.00000	1.00000	1.00000
1973-74	1.00000	1.00000	1.00000
1974-75	1.00000	1.00000	1.00000
1975-76	1.00000	1.00000	1.00000
1976-77	1.00000	1.00000	1.00000
1977-78	1.00000	1.00000	1.00000
1978-79	1.00000	1.00000	1.00000
1979-80	1.00000	1.00000	1.00000
1980-81	1.00000	1.00000	1.00000
1981-82	1.00000	1.00000	1.00000
1982-83	1.00000	1.00000	1.00000
1983-84	1.00000	1.00000	1.00000
1984-85	1.00000	1.00000	1.00000
1985-86	1.00000	1.00000	1.00000
1986-87	1.00000	1.00000	1.00000
1987-88	1.00000	1.00000	1.00000

	Average 69-78	Average 79-88	Average 69-88
Efficiency	1.00000	1.00000	1.00000
Efficiency Change	1.00000	1.00000	1.00000
Malmquist Index	1.00000	1.00000	1.00000
Technical Change	1.00000	1.00000	1.00000





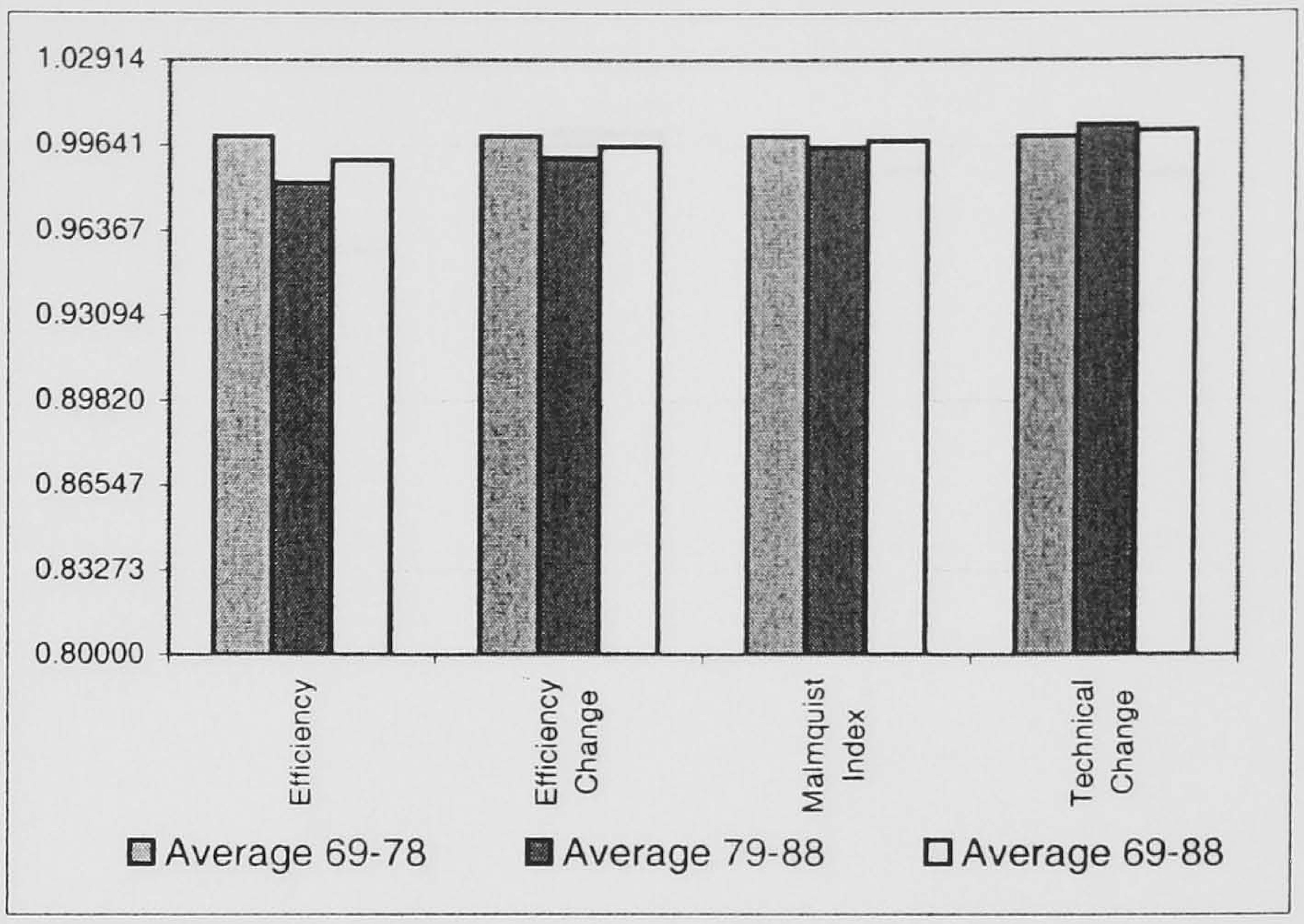
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	1.00000	1979	1.00000
1970	1.00000	1980	1.00000
1971	1.00000	1981	1.00000
1972	1.00000	1982	1.00000
1973	1.00000	1983	1.00000
1974	1.00000	1984	1.00000
1975	1.00000	1985	1.00000
1976	1.00000	1986	1.00000
1977	1.00000	1987	1.00000
1978	1.00000	1988	1.00000

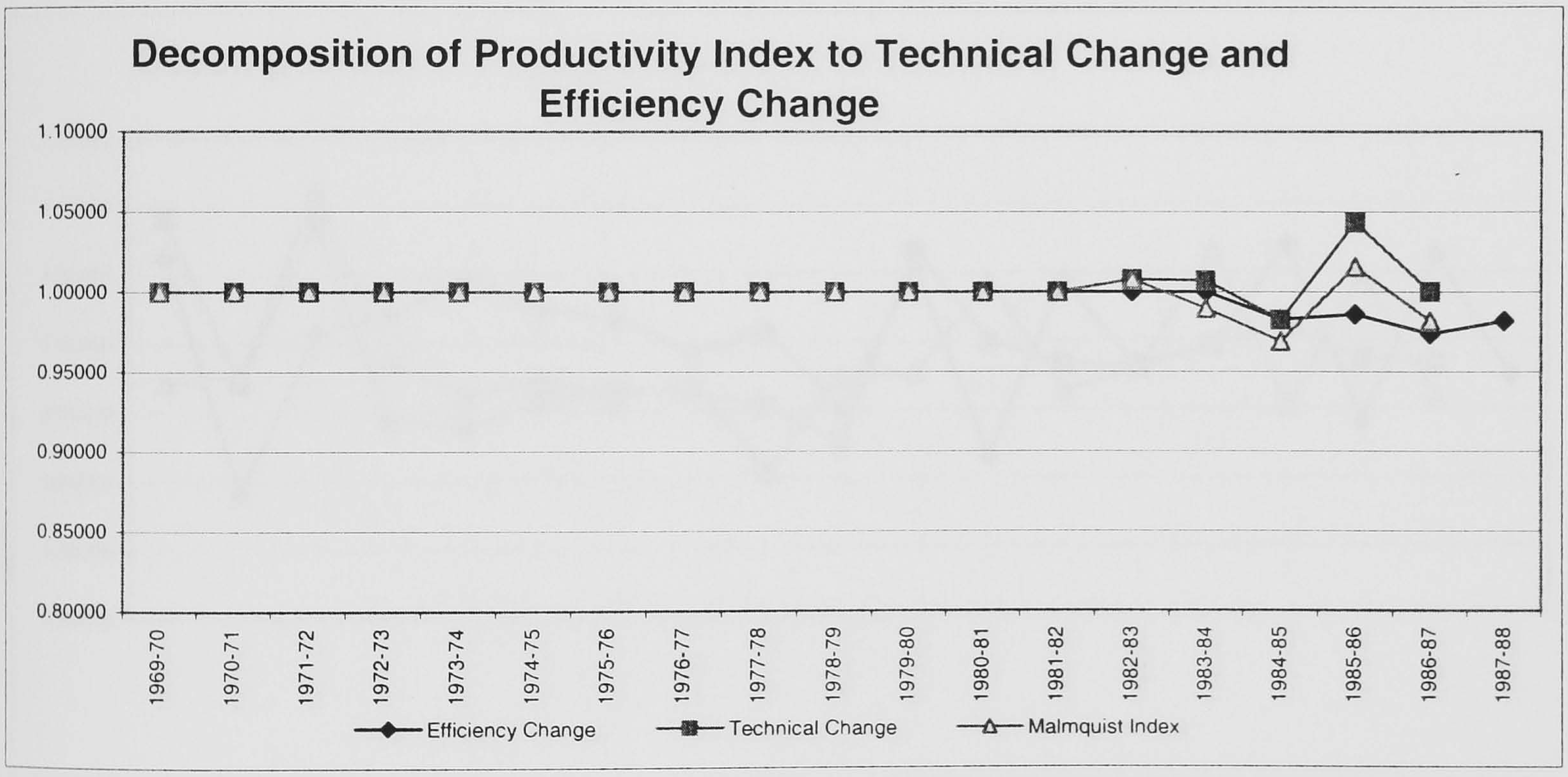


Summary of dynamic efficiency, productivity and its decomposition for NORWAY

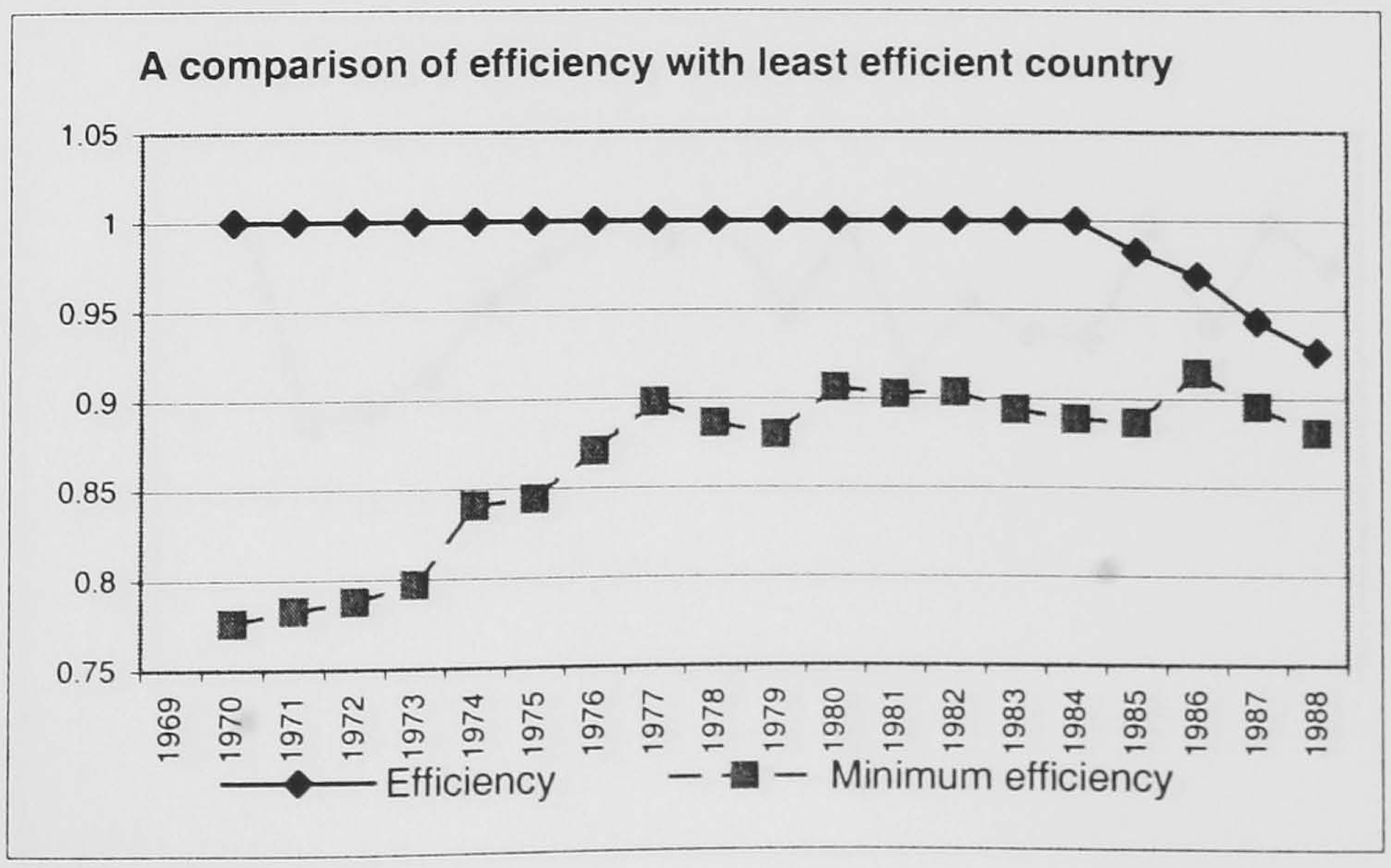
	Efficiency Change	Technical Change	Malmquist Index
1969-70	1.00000	1.00000	1.00000
1970-71	1.00000	1.00000	1.00000
1971-72	1.00000	1.00000	1.00000
1972-73	1.00000	1.00000	1.00000
1973-74	1.00000	1.00000	1.00000
1974-75	1.00000	1.00000	1.00000
1975-76	1.00000	1.00000	1.00000
1976-77	1.00000	1.00000	1.00000
1977-78	1.00000	1.00000	1.00000
1978-79	1.00000	1.00000	1.00000
1979-80	1.00000	1.00000	1.00000
1980-81	1.00000	1.00000	1.00000
1981-82	1.00000	1.00000	1.00000
1982-83	1.00000	1.00000	1.00000
1983-84	1.00000	1.00778	1.00778
1984-85	0.98265	1.00702	0.98954
1985-86	0.98597	0.98256	0.96877
1986-87	0.97372	1.04307	1.01565
1987-88	0.98156	1.00000	0.98156

	Average 69-78	Average 79-88	Average 69-88
Efficiency	1.00000	0.98209	0.99104
Efficiency Change	1.00000	0.99154	0.99599
Malmquist Index	1.00000	0.99592	0.99807
Technical Change	1.00000	1.00449	1.00213





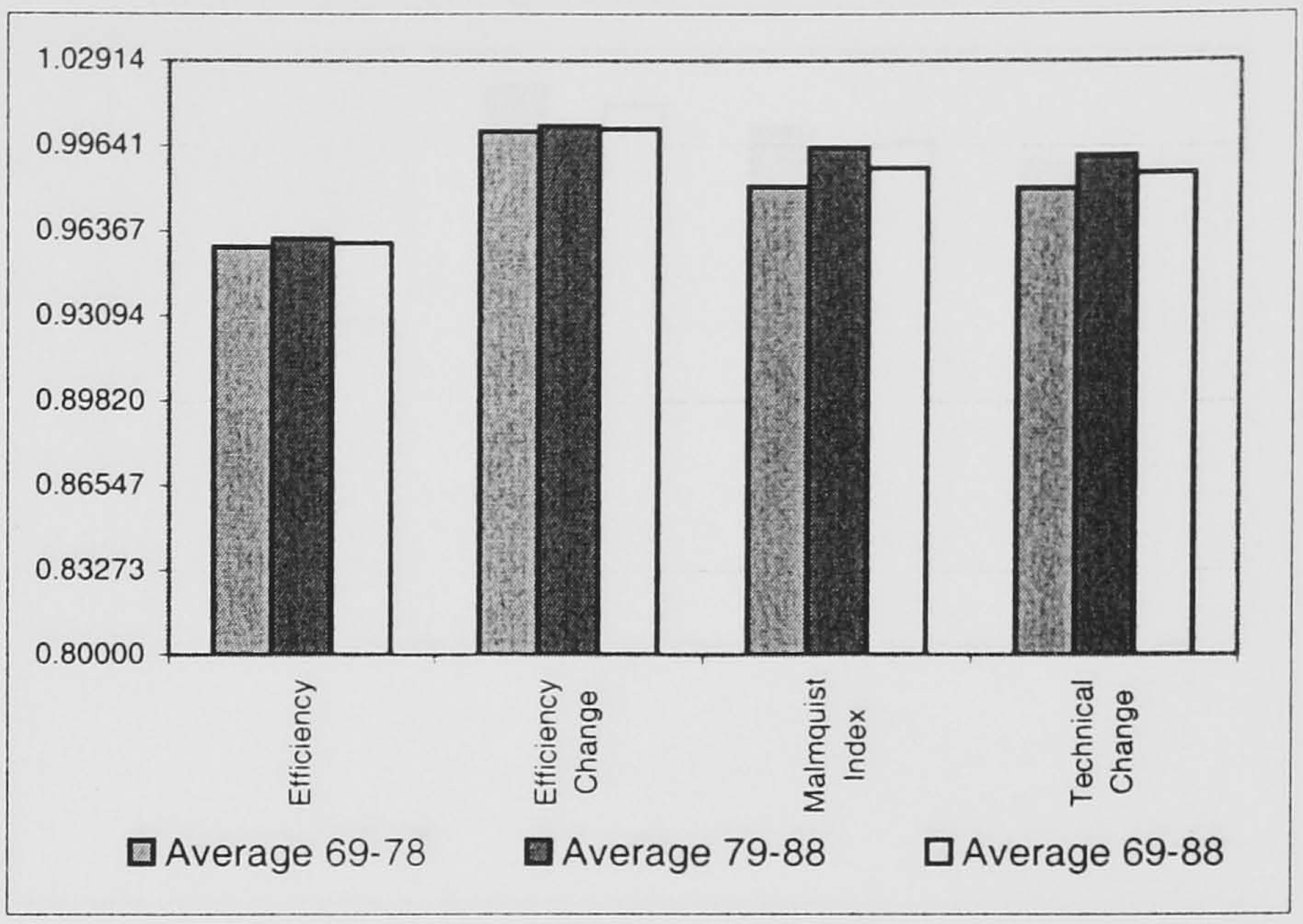
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	1.00000	1979	1.00000
1970	1.00000	1980	1.00000
1971	1.00000	1981	1.00000
1972	1.00000	1982	1.00000
1973	1.00000	1983	1.00000
1974	1.00000	1984	1.00000
1975	1.00000	1985	0.98265
1976	1.00000	1986	0.96886
1977	1.00000	1987	0.94339
1978	1.00000	1988	0.92599

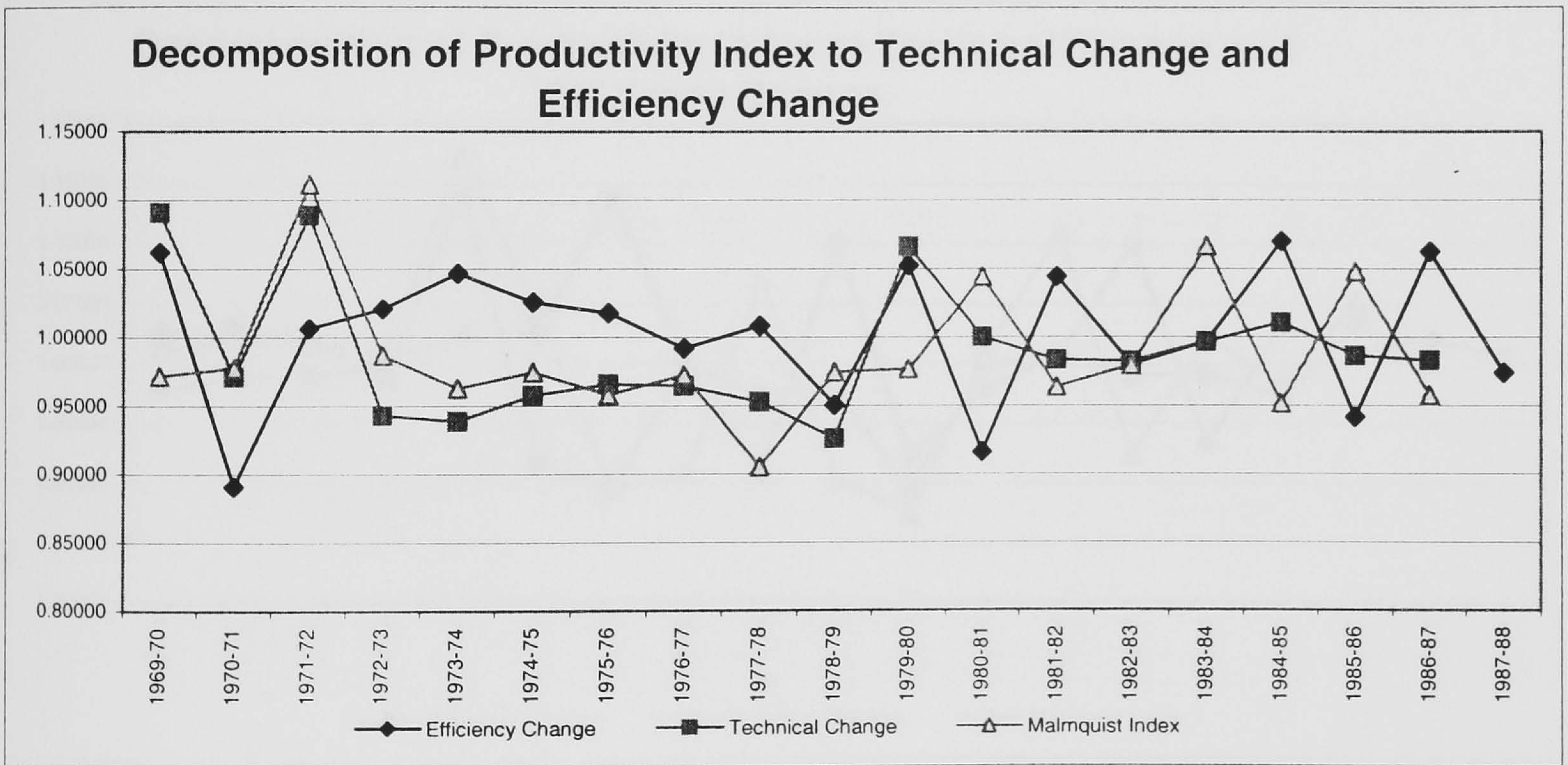


Summary of dynamic efficiency, productivity and its decomposition for SPAIN

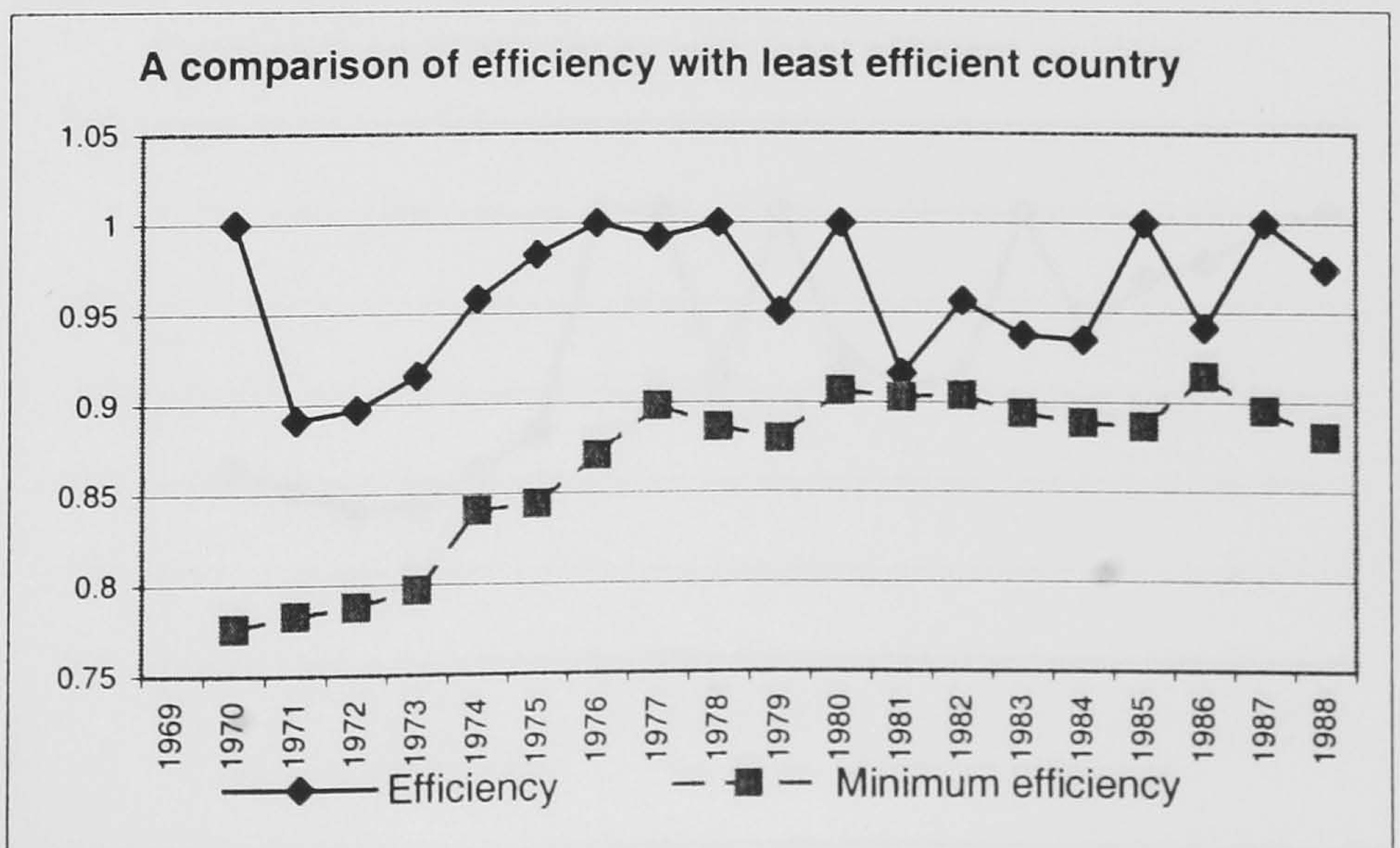
	Efficiency Change	Technical Change	Malmquist Index
1969-70	1.06217	0.92934	0.98712
1970-71	0.89083	1.09092	0.97183
1971-72	1.00661	0.97130	0.97772
1972-73	1.02081	1.08888	1.11153
1973-74	1.04649	0.94284	0.98667
1974-75	1.02567	0.93853	0.96262
1975-76	1.01778	0.95756	0.97459
1976-77	0.99187	0.96580	0.95795
1977-78	1.00820	0.96447	0.97237
1978-79	0.95010	0.95273	0.90519
1979-80	1.05252	0.92590	0.97452
1980-81	0.91593	1.06648	0.97682
1981-82	1.04425	1.00000	1.04425
1982-83	0.98011	0.98328	0.96372
1983-84	0.99685	0.98250	0.97940
1984-85	1.07012	0.99670	1.06659
1985-86	0.94136	1.01058	0.95132
1986-87	1.06229	0.98647	1.04791
1987-88	0.97398	0.98314	0.95756

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.95767	0.96097	0.95932
Efficiency Change	1.00205	1.00416	1.00305
Malmquist Index	0.98076	0.99579	0.98788
Technical Change	0.98024	0.99278	0.98618





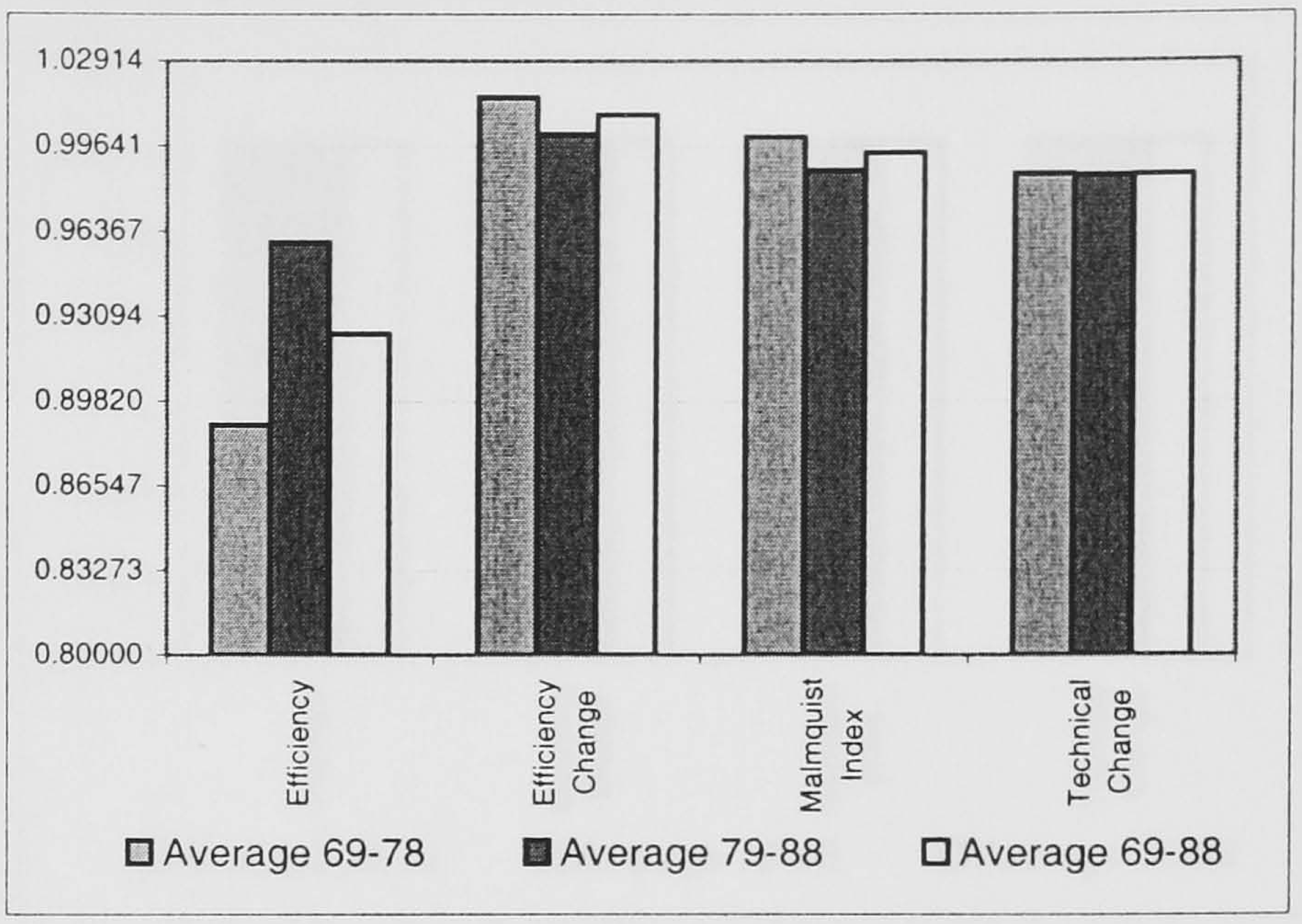
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.94146	1979	0.95010
1970	1.00000	1980	1.00000
1971	0.89083	1981	0.91593
1972	0.89672	1982	0.95646
1973	0.91538	1983	0.93743
1974	0.95794	1984	0.93448
1975	0.98253	1985	1.00000
1976	1.00000	1986	0.94136
1977	0.99187	1987	1.00000
1978	1.00000	1988	0.97398

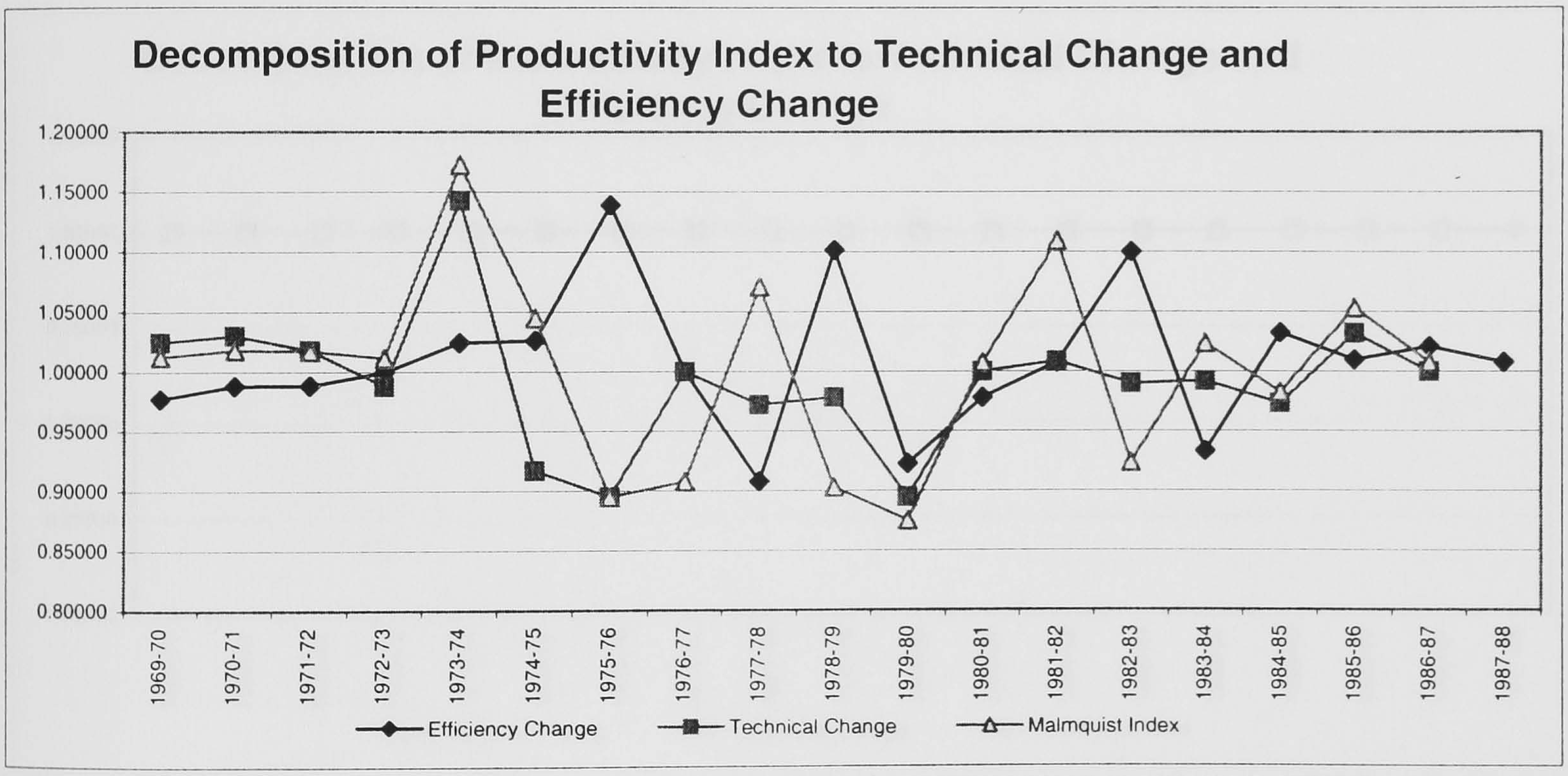


Summary of dynamic efficiency, productivity and its decomposition for SWEDEN

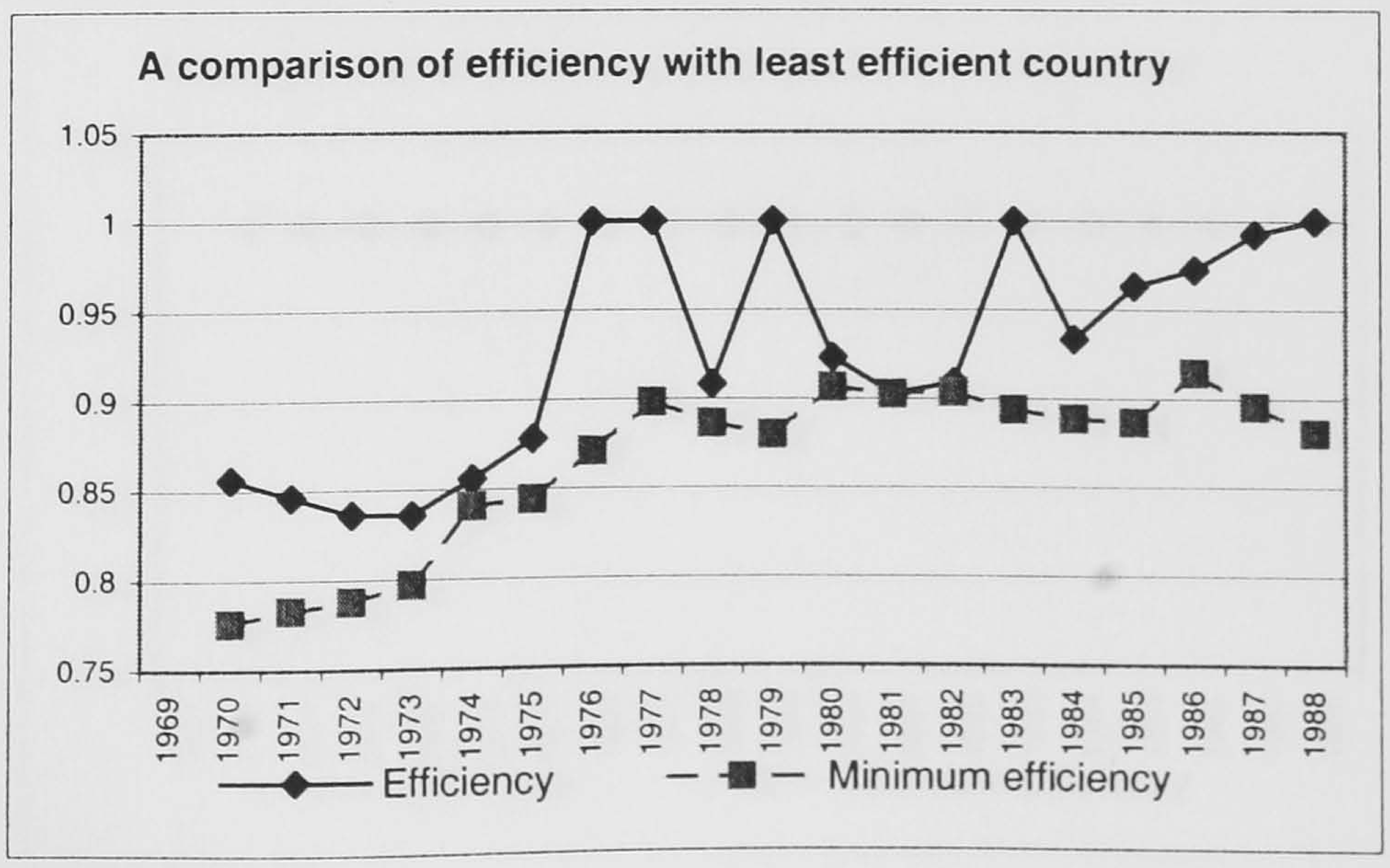
	Efficiency Change	Technical Change	Malmquist Index
1969-70	0.97735	0.87034	0.85063
1970-71	0.98804	1.02448	1.01224
1971-72	0.98840	1.02999	1.01805
1972-73	0.99935	1.01802	1.01736
1973-74	1.02406	0.98755	1.01131
1974-75	1.02597	1.14282	1.17251
1975-76	1.13892	0.91729	1.04472
1976-77	1.00000	0.89505	0.89505
1977-78	0.90788	1.00000	0.90788
1978-79	1.10146	0.97210	1.07073
1979-80	0.92282	0.97840	0.90288
1980-81	0.97804	0.89478	0.87512
1981-82	1.00734	1.00000	1.00734
1982-83	1.09990	1.00841	1.10915
1983-84	0.93324	0.98973	0.92365
1984-85	1.03201	0.99167	1.02341
1985-86	1.00961	0.97312	0.98248
1986-87	1.02065	1.03175	1.05305
1987-88	1.00761	1.00000	1.00761

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.88921	0.95957	0.92439
Efficiency Change	1.01514	1.00125	1.00856
Malmquist Index	1.00005	0.98719	0.99396
Technical Change	0.98576	0.98532	0.98555





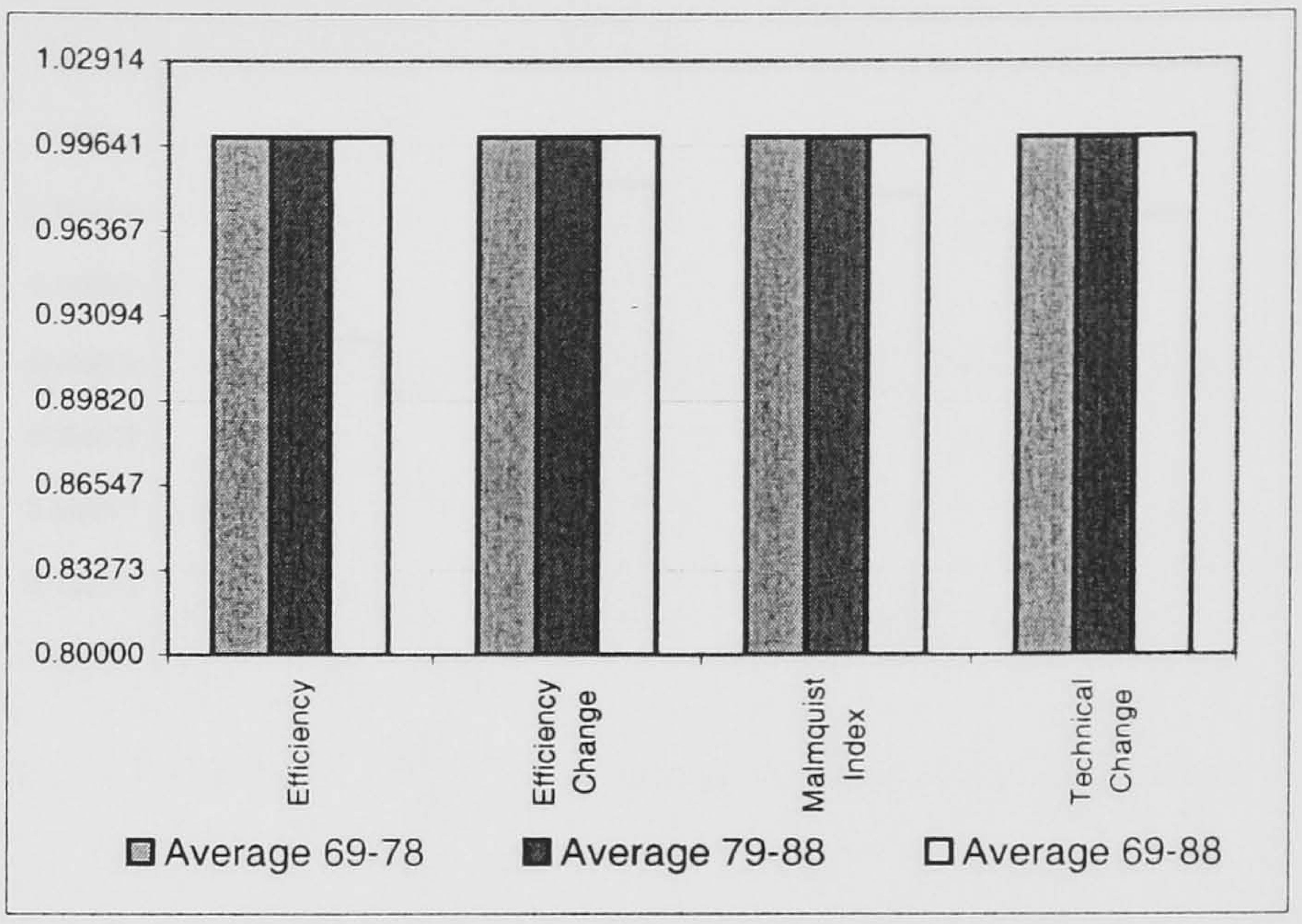
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.87613	1979	1.00000
1970	0.85628	1980	0.92282
1971	0.84605	1981	0.90255
1972	0.83623	1982	0.90917
1973	0.83569	1983	1.00000
1974	0.85580	1984	0.93324
1975	0.87803	1985	0.96311
1976	1.00000	1986	0.97237
1977	1.00000	1987	0.99244
1978	0.90788	1988	1.00000

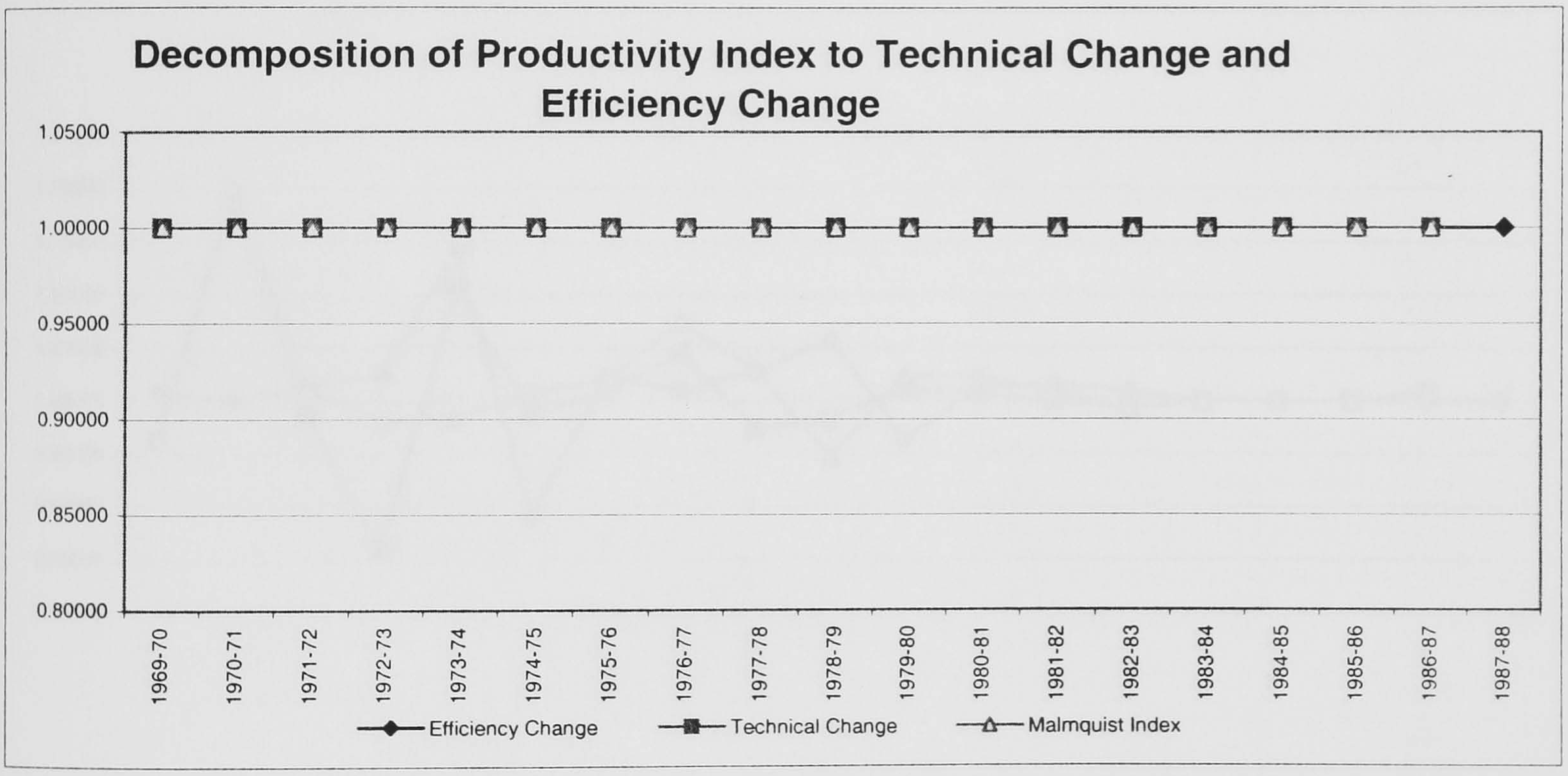


Summary of dynamic efficiency, productivity and its decomposition for U.K.

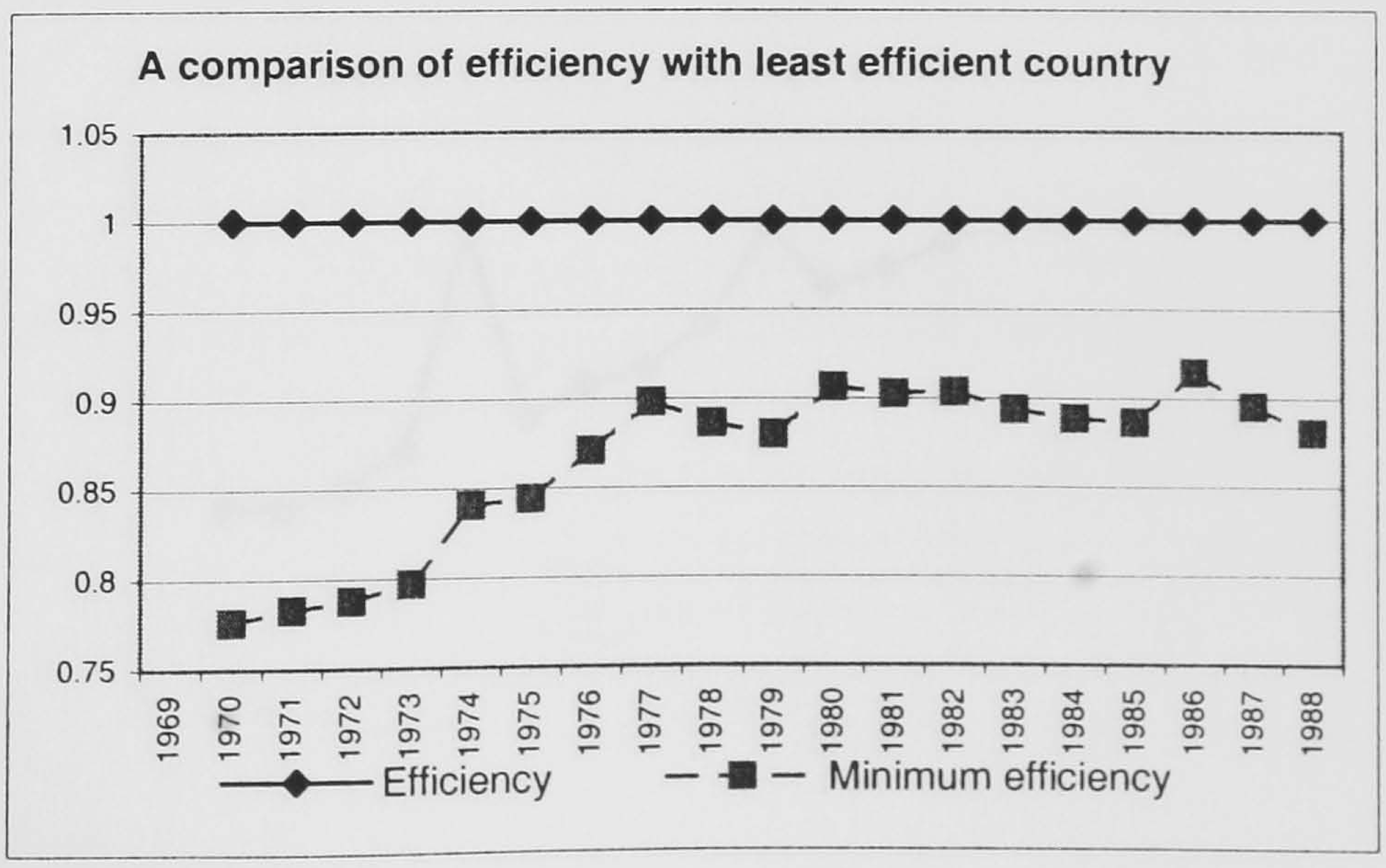
	Efficiency Change	Technical Change	Malmquist Index
1969-70	1.00000	1.00000	1.00000
1970-71	1.00000	1.00000	1.00000
1971-72	1.00000	1.00000	1.00000
1972-73	1.00000	1.00000	1.00000
1973-74	1.00000	1.00000	1.00000
1974-75	1.00000	1.00000	1.00000
1975-76	1.00000	1.00000	1.00000
1976-77	1.00000	1.00000	1.00000
1977-78	1.00000	1.00000	1.00000
1978-79	1.00000	1.00000	1.00000
1979-80	1.00000	1.00000	1.00000
1980-81	1.00000	1.00000	1.00000
1981-82	1.00000	1.00000	1.00000
1982-83	1.00000	1.00000	1.00000
1983-84	1.00000	1.00000	1.00000
1984-85	1.00000	1.00000	1.00000
1985-86	1.00000	1.00000	1.00000
1986-87	1.00000	1.00000	1.00000
1987-88	1.00000	1.00000	1.00000

	Average 69-78	Average 79-88	Average 69-88
Efficiency	1.00000	1.00000	1.00000
Efficiency Change	1.00000	1.00000	1.00000
Malmquist Index	1.00000	1.00000	1.00000
Technical Change	1.00000	1.00000	1.00000





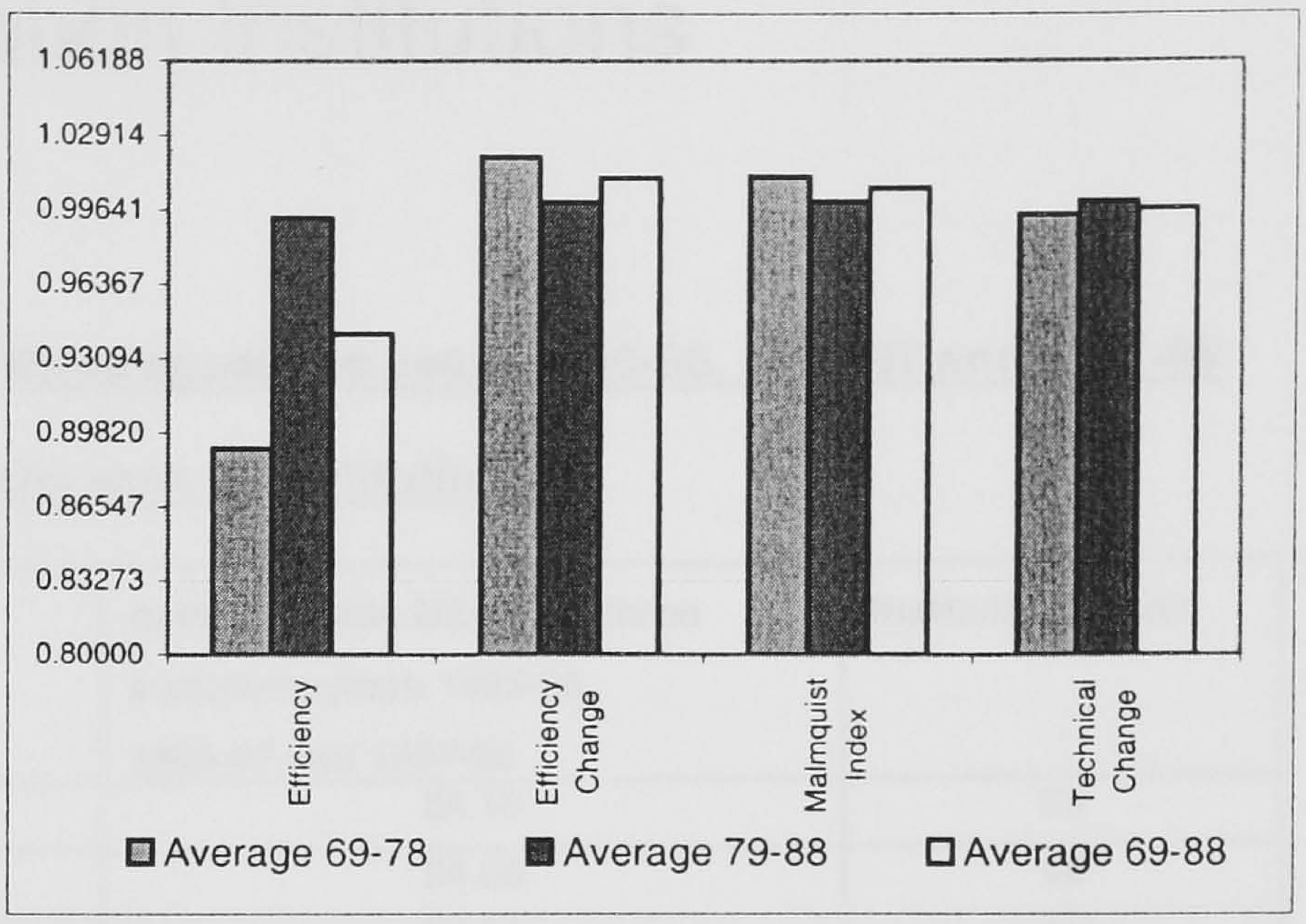
Year	Dynamic efficiency	Year	Dynamic efficiency
1969	1.00000	1979	1.00000
1970	1.00000	1980	1.00000
1971	1.00000	1981	1.00000
1972	1.00000	1982	1.00000
1973	1.00000	1983	1.00000
1974	1.00000	1984	1.00000
1975	1.00000	1985	1.00000
1976	1.00000	1986	1.00000
1977	1.00000	1987	1.00000
1978	1.00000	1988	1.00000

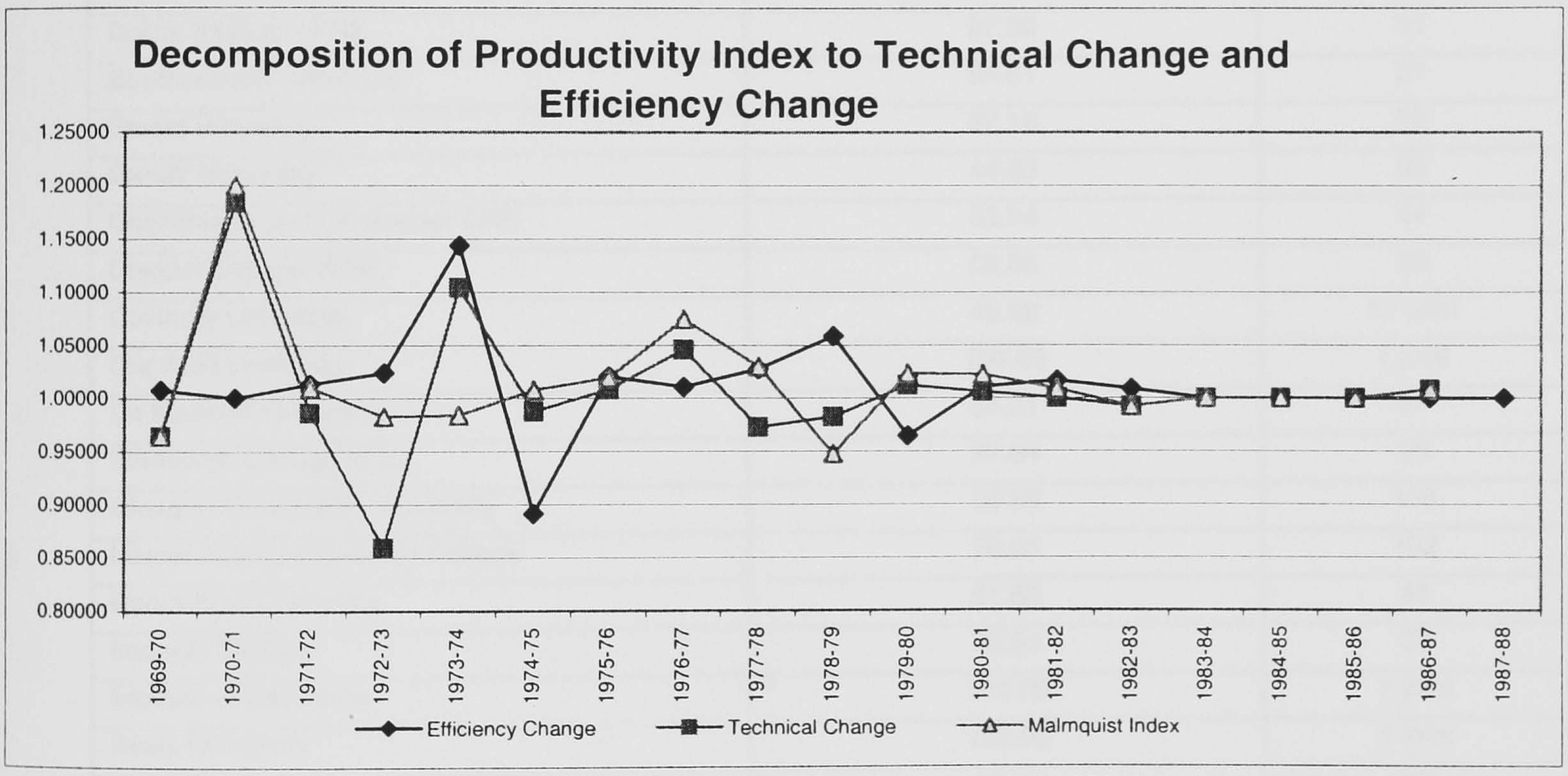


Summary of dynamic efficiency, productivity and its decomposition for U.S.A.

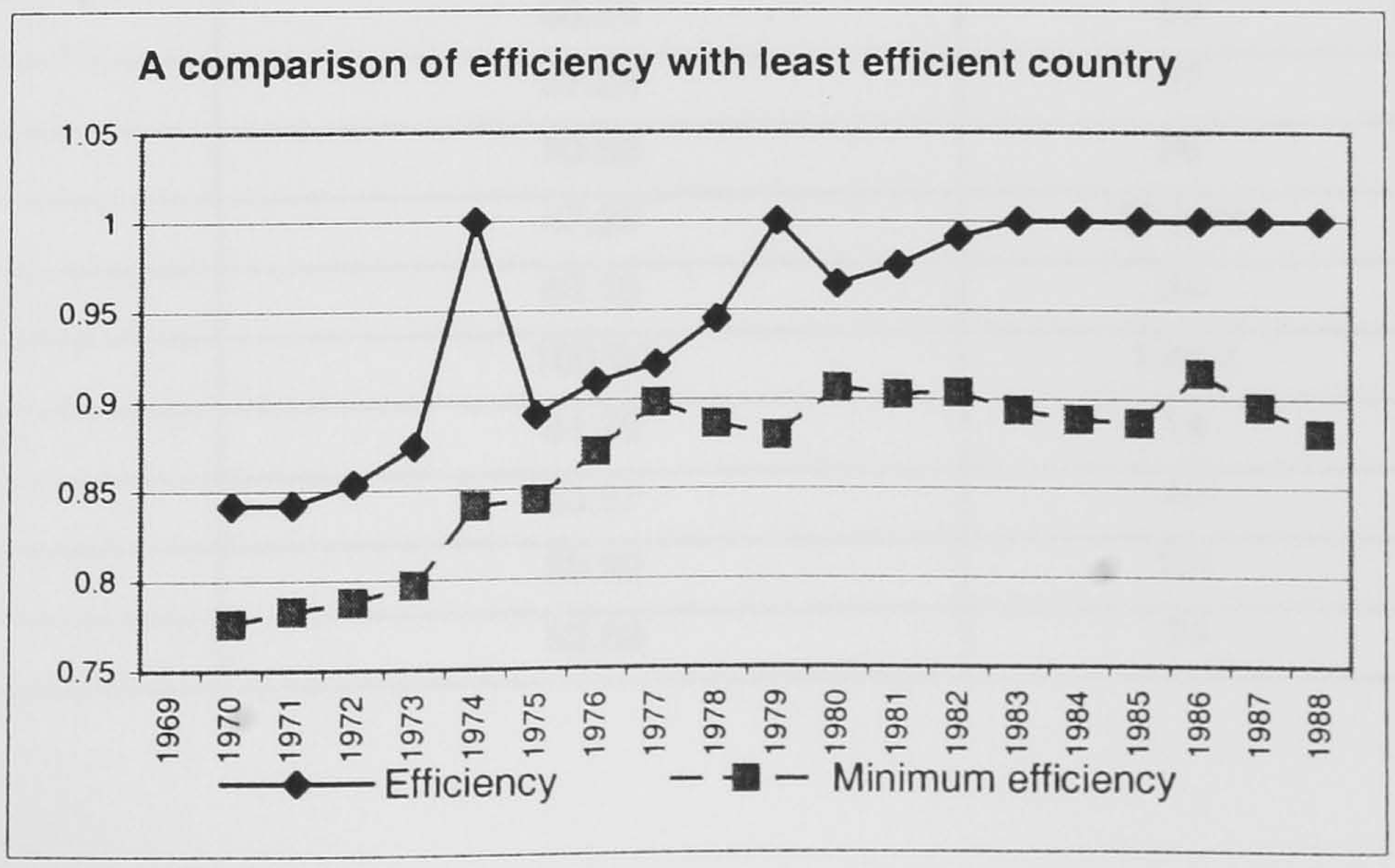
	Efficiency Change	Technical Change	Malmquist Index
1969-70	1.00817	0.83035	0.83714
1970-71	1.00045	0.96460	0.96503
1971-72	1.01350	1.18379	1.19977
1972-73	1.02421	0.98653	1.01041
1973-74	1.14383	0.85929	0.98288
1974-75	0.89141	1.10419	0.98428
1975-76	1.02063	0.98760	1.00797
1976-77	1.01080	1.00841	1.01930
1977-78	1.02744	1.04581	1.07450
1978-79	1.05836	0.97299	1.02978
1979-80	0.96465	0.98257	0.94783
1980-81	1.01088	1.01224	1.02325
1981-82	1.01651	1.00593	1.02253
1982-83	1.00884	1.00000	1.00884
1983-84	1.00000	0.99248	0.99248
1984-85	1.00000	1.00000	1.00000
1985-86	1.00000	1.00000	1.00000
1986-87	1.00000	1.00000	1.00000
1987-88	1.00000	1.00825	1.00825

	Average 69-78	Average 79-88	Average 69-88
Efficiency	0.89126	0.99310	0.94218
Efficiency Change	1.01988	1.00010	1.01051
Malmquist Index	1.01111	1.00035	1.00601
Technical Change	0.99435	1.00016	0.99711





Year	Dynamic efficiency	Year	Dynamic efficiency
1969	0.83501	1979	1.00000
1970	0.84184	1980	0.96465
1971	0.84222	1981	0.97514
1972	0.85359	1982	0.99124
1973	0.87426	1983	1.00000
1974	1.00000	1984	1.00000
1975	0.89141	1985	1.00000
1976	0.90979	1986	1.00000
1977	0.91962	1987	1.00000
1978	0.94485	1988	1.00000



Appendix C: The assessment of Higher Education Institutions

Table 1: Average static DEA over three academic years 1995-96, 1996-97 and 1997-98 and the rank of institutions

Institution	Average static DEA over three academic years 1995-96, 1996-97 and 1997-98	Institutions' rank
Anglia Polytechnic University	54.10	68
Aston University	54.05	69
Birkbeck College	65.30	35
Bolton Institute of HE	67.59	31
Bournemouth University	64.61	37
Brunel University	57.16	58
Cardiff University	44.53	92
Cheltenham and Gloucester CHE	63.04	44
Chester College of HE	69.05	28
Coventry University	48.88	81 joint
Cranfield University	100.00	1 joint
De Montfort University	54.31	67
Edinburgh College of Art	37.84	98
Glasgow Caledonian University	35.39	100
Harper Adams University College	29.24	102
Heriot-Watt University	61.63	48
Imperial College	73.90	20
Institute of Education	100.00	1 joint
Keele University	100.00	1 joint
King Alfred's College, Winchester	60.19	50
Kingston University	57.25	57
Lancaster University	70.55	26
Leeds Metropolitan University	47.29	87 joint
Liverpool ohn Moores University	66.15	34
London Business School	100.00	1 joint
London Sch of Economics & Political Sci	81.70	14
Loughborough University	63.57	42
Napier University	29.92	101
North East Wales Institute	52.68	74

Nottingham Trent University	48.75	84
Oxford Brookes University	76.25	17
Queen Margaret College	47.64	86
Royal Holloway, University of London	45.78	90
Sheffield Hallam University	42.28	95
South Bank University	39.22	97
St George's Hospital Medical School	74.63	19
St Mary's College	50.77	77
Staffordshire University	43.21	93 joint
The London Institute	99.65	6
The Queen's University of Belfast	68.55	29
UMIST	94.90	7
University College London	100.00	1 joint
University College Northampton	63.93	39
University of Aberdeen	55.66	63
University of Bath	49.76	79
University of Birmingham	69.92	27
University of Bradford	55.86	61
University of Brighton	36.39	99
University of Bristol	59.28	51
University of Cambridge	89.66	10
University of Central England in Birmingham	62.46	46
University of Central Lancashire	59.06	53
University of Derby	62.16	47
University of Dundee	51.48	75
University of Durham	55.77	62
University of East Anglia	72.30	23
University of East London	43.21	93 joint
University of Edinburgh	53.38	72
University of Essex	82.82	12
University of Exeter	57.56	56
University of Glasgow	48.02	85
University of Greenwich	67.15	32
University of Hertfordshire	63.77	41
University of Huddersfield	63.83	40
University of Hull	64.62	36
University of Kent at Canterbury	68.14	30
University of Leeds	67.05	33
University of Leicester	76.97	16
University of Lincolnshire and Humberside	58.57	55
University of Liverpool	53.04	73
University of London	90.50	9
University of Manchester	51.43	76
University of Newcastle upon Tyne	55.47	64
University of North London	59.25	52

University of Northumbria at Newcastle	64.17	38
University of Nottingham	71.79	24
University of Oxford	93.20	8
University of Paisley	41.27	96
University of Plymouth	45.73	91
University of Portsmouth	53.60	71
University of Reading	81.77	13
University of Salford	50.02	78
University of Sheffield	63.19	43
University of Southampton	62.80	45
University of St Andrews	46.95	89
University of Stirling	48.79	83
University of Strathclyde	61.60	49
University of Sunderland	54.52	66
University of Surrey	71.66	25
University of Sussex	83.34	11
University of Teesside	75.90	18
University of Ulster	54.55	65
University of Wales Institute, Cardiff	55.91	60
University of Wales, Aberystwyth	48.88	81 joint
University of Wales, Bangor	53.92	70
University of Wales, Lampeter	47.29	87 joint
University of Wales, Swansea	72.75	22
University of Warwick	72.79	21
University of West of England, Bristol	49.04	80
University of Westminster	58.70	54
University of Wolverhampton	77.87	15
University of York	57.09	59

Table 2: Table of dynamic efficiency and the rank of each institution

Institution	Dynamic efficiency score	Institutions' Rank
Anglia Polytechnic University	60.02	63
Aston University	64.68	51
Birkbeck College	70.42	40
Bolton Institute of HE	89.41	11
Bournemouth University	79.41	24
Brunel University	55.66	74
Cardiff University	45.74	96
Cheltenham and Gloucester CHE	72.67	34 joint
Chester College of HE	76.97	27
Coventry University	51.39	86
Cranfield University	100	1 joint
De Montfort University	75.31	31
Edinburgh College of Art	40.46	99
Glasgow Caledonian University	36.87	100
Harper Adams University College	35.68	101
Heriot-Watt University	68.12	45
Imperial College	83.25	20
Institute of Education	100	1 joint
Keele University	100	1 joint
King Alfred's College, Winchester	71.57	39
Kingston University	60.58	62
Lancaster University	68.63	43
Leeds Metropolitan University	55.16	75
Liverpool John Moores University	77.29	26
London Business School	100	1 joint
London Sch of Economics & Political Sci	100	1 joint
Loughborough University	63.51	52
Napier University	33.89	102
North East Wales Institute	62.99	54
Nottingham Trent University	50.82	90
Oxford Brookes University	84.81	18
Queen Margaret College	54.74	77
Royal Holloway, University of London	58.65	67
Sheffield Hallam University	44.58	97
South Bank University	46.16	94
St George's Hospital Medical School	79.21	25
St Mary's College	50.95	88
Staffordshire University	50.84	89
The London Institute	100	1 joint

The Queen's University of Belfast	80.41	22
UMIST	93.3	9
University College London	100	1 joint
University College Northampton	80.66	21
University of Aberdeen	53.68	80
University of Bath	52.55	82 joint
University of Birmingham	62.16	56
University of Bradford	51.87	85
University of Brighton	41.09	98
University of Bristol	66.66	49
University of Cambridge	86.17	14
University of Central England in Birmingham	61.59	59
University of Central Lancashire	68.19	44
University of Derby	63.43	53
University of Dundee	57.3	70
University of Durham	52.55	82 joint
University of East Anglia	64.89	50
University of East London	58.46	68
University of Edinburgh	52.42	84
University of Essex	88	12
University of Exeter	61.97	57
University of Glasgow	55.07	76
University of Greenwich	80.08	23
University of Hertfordshire	84.91	17
University of Huddersfield	74.19	32
University of Hull	67.29	47
University of Kent at Canterbury	71.6	38
University of Leeds	72.67	34 joint
University of Leicester	76.93	28
University of Lincolnshire and Humberside	68.07	46
University of Liverpool	59.01	66
University of London	100	1 joint
University of Manchester	52.91	81
University of Newcastle upon Tyne	59.19	64
University of North London	71.75	37
University of Northumbria at Newcastle	72.68	33
University of Nottingham	84.31	19
University of Oxford	76.52	29
University of Paisley	57.08	71
University of Plymouth	49.73	91
University of Portsmouth	60.64	61
University of Reading	70.32	41
University of Salford	56.42	72
University of Sheffield	75.47	30
University of Southampton	67.05	48

University of St Andrews	51.18	87
University of Stirling	47.66	93
University of Strathclyde	62.22	55
University of Sunderland	59.09	65
University of Surrey	69.94	42
University of Sussex	85.36	16
University of Teesside	90.84	10
University of Ulster	54.37	79
University of Wales Institute, Cardiff	56.11	73
University of Wales, Aberystwyth	46.1	95
University of Wales, Bangor	61.2	60
University of Wales, Lampeter	49	92
University of Wales, Swansea	85.59	15
University of Warwick	72.07	36
University of West of England, Bristol	54.47	78
University of Westminster	57.55	69
University of Wolverhampton	87.1	13
University of York	61.7	58

Table 3: Average of PIs over three academic years 1995-1996, 1996-1997 and 1997-1998

Institution	UGs/ CAP	PGs/ CAP	PhDs / CAP	RGC/ CAP	UGs/ REC	PGs/ REC	PhDs / REC	RGC/ REC
Anglia Polytechnic University	1.43	0.70	0.04	0.07	1.56	0.77	0.05	0.07
Aston University	1.21	1.76	2.07	0.95	0.67	0.96	1.24	0.54
Birkbeck College	1.45	1.59	1.13	0.69	1.25	1.22	0.87	0.51
Bolton Institute of HE	2.27	1.95	0.29	0.10	1.38	1.19	0.17	0.05
Bournemouth University	2.45	0.98	0.06	0.13	1.75	0.70	0.04	0.09
Brunel University	0.85	1.34	1.04	0.61	0.86	1.37	1.03	0.62
Cardiff University	0.55	0.72	0.58	0.66	0.70	0.91	0.72	0.83
Cheltenham and Gloucester CHE	1.74	0.94	0.08	0.16	1.53	0.80	0.09	0.14
Chester College of HE	2.67	0.91	0.04	0.07	1.20	0.41	0.03	0.03
Coventry University	1.53	0.74	0.21	0.12	1.32	0.63	0.19	0.11
Cranfield University	0.17	3.16	2.14	4.74	0.16	2.85	1.97	4.31
De Montfort University	0.97	0.32	0.02	0.13	1.65	0.53	0.03	0.21
Edinburgh College of Art	0.47	0.68	0.11	0.14	0.73	1.09	0.21	0.22
Glasgow Caledonian University	0.85	0.71	0.18	0.14	0.85	0.72	0.17	0.14
Harper Adams University College	0.68	0.04	0.19	0.21	0.91	0.05	0.26	0.29
Heriot-Watt University	0.33	0.94	0.51	0.63	0.68	1.97	1.06	1.30
Imperial College	0.21	0.47	1.39	2.77	0.23	0.52	1.55	3.09
Institute of Education	0.20	5.23	1.25	0.98	0.20	5.08	1.23	0.93
Keele University	2.95	2.56	1.46	1.50	2.27	1.93	1.11	1.13
King Alfred's College, Winchester	1.94	0.33	0.06	0.01	1.81	0.32	0.07	0.01
Kingston University	1.55	1.69	0.10	0.07	1.20	1.30	0.08	0.06
Lancaster University	1.17	1.31	1.29	0.89	1.23	1.40	1.37	0.94
Leeds Metropolitan University	1.41	0.76	0.04	0.08	1.30	0.69	0.04	0.07
Liverpool ohn Moores University	2.54	1.05	0.35	0.24	1.20	0.50	0.17	0.11
London Business School	0.00	6.25	1.53	3.52	0.00	5.48	1.33	3.09
London Sch of Economics & Political Sci	0.99	5.93	2.37	2.20	0.56	2.94	1.21	1.27
Loughborough University	1.15	1.25	1.74	1.49	0.90	0.99	1.33	1.16
Napier University	0.62	0.34	0.02	0.09	0.84	0.42	0.03	0.12
North East Wales Institute	2.16	0.40	0.10	0.21	1.37	0.25	0.06	0.13
Nottingham Trent University	1.66	0.64	0.07	0.15	1.35	0.50	0.06	0.12
Oxford Brookes University	2.04	1.91	0.34	0.27	1.61	1.52	0.30	0.21
Queen Margaret College	1.90	0.21	0.09	0.47	1.16	0.13	0.06	0.29
Royal Holloway, University of London	1.27	0.88	1.17	1.13	0.87	0.57	0.76	0.76
Sheffield Hallam University	1.28	0.85	0.09	0.24	1.08	0.74	0.08	0.20

South Bank University	0.81	0.89	0.18	0.21	0.86	0.95	0.19	0.22
St George's Hospital Medical School	0.31	0.29	0.76	3.75	0.23	0.21	0.55	2.73
St Mary's College	1.23	1.18	0.00	0.01	1.17	1.08	0.00	0.01
Staffordshire University	1.06	0.45	0.03	0.22	1.24	0.54	0.05	0.25
The London Institute	3.75	0.98	0.02	0.02	3.21	0.83	0.02	0.01
The Queen's University of Belfast	2.04	1.77	1.70	1.21	1.25	1.12	1.07	0.77
UMIST	1.09	2.35	5.84	2.84	0.54	1.08	2.75	1.41
University College London	1.29	1.49	3.59	3.73	1.13	1.25	3.15	2.66
University College Northampton	2.42	0.60	0.07	0.05	1.73	0.44	0.06	0.04
University of Aberdeen	0.62	0.53	1.13	0.91	0.80	0.69	1.50	1.19
University of Bath	0.30	0.55	0.83	0.72	0.53	0.97	1.47	1.27
University of Birmingham	0.68	1.32	1.82	1.58	0.68	1.31	1.87	1.58
University of Bradford	1.05	1.21	1.56	0.76	0.88	1.01	1.31	0.64
University of Brighton	1.27	0.45	0.12	0.18	1.06	0.38	0.10	0.15
University of Bristol	0.65	0.98	1.88	1.86	0.59	0.86	1.67	1.70
University of Cambridge	0.95	0.87	3.05	2.81	0.88	0.82	2.86	2.63
University of Central England in Birmingham	1.47	2.26	0.06	0.08	1.07	1.68	0.04	0.06
University of Central Lancashire	2.07	0.65	0.43	0.09	1.52	0.49	0.31	0.06
University of Derby	1.90	0.92	0.02	0.13	1.67	0.81	0.02	0.12
University of Dundee	0.40	0.42	0.55	0.95	0.70	0.72	0.95	1.64
University of Durham	0.87	1.22	1.54	1.17	0.75	1.06	1.33	1.00
University of East Anglia	0.94	1.29	1.68	1.02	1.02	1.40	1.82	1.10
University of East London	1.61	1.09	0.09	0.12	0.98	0.64	0.04	0.07
University of Edinburgh	0.47	0.43	1.21	1.29	0.59	0.54	1.53	1.62
University of Essex	1.61	2.38	2.92	1.63	1.07	1.59	1.91	1.08
University of Exeter	1.22	2.21	1.42	0.96	0.80	1.44	0.91	0.63
University of Glasgow	0.57	0.09	0.33	0.84	0.95	0.16	0.56	1.41
University of Greenwich	1.90	2.18	0.17	0.76	1.25	1.43	0.11	0.50
University of Hertfordshire	1.94	1.25	0.48	0.26	1.58	1.01	0.38	0.21
University of Huddersfield	1.93	1.63	0.28	0.13	1.47	1.24	0.21	0.10
University of Hull	0.93	2.06	1.17	0.68	0.86	1.82	1.06	0.63
University of Kent at Canterbury	0.90	1.18	1.13	0.90	1.11	1.44	1.39	1.10
University of Leeds	1.10	1.04	1.99	1.69	0.98	0.93	1.75	1.51
University of Leicester	0.65	1.91	1.10	1.41	0.81	2.37	1.36	1.74
University of Lincolnshire and Humberside	2.38	0.46	0.06	0.06	1.52	0.30	0.04	0.04
University of Liverpool	0.74	0.99	1.81	1.74	0.57	0.76	1.35	1.33
University of London	0.34	2.38	3.29	6.08	0.13	0.95	1.26	2.32
University of Manchester	0.54	0.80	1.30	1.30	0.59	0.89	1.45	1.44
University of Newcastle upon Tyne	0.75	1.05	1.19	1.58	0.73	1.02	1.12	1.51
University of North London	1.81	1.18	1.44	0.10	0.89	0.56	0.62	0.05

University of Northumbria at Newcastle	1.47	1.38	0.07	0.12	1.54	1.43	0.07	0.14
University of Nottingham	1.09	0.99	1.96	1.84	1.07	0.92	1.82	1.72
University of Oxford	0.46	0.40	2.34	2.74	0.53	0.46	2.69	3.14
University of Paisley	0.69	0.39	0.13	0.09	1.12	0.64	0.20	0.13
University of Plymouth	1.28	0.59	0.51	0.26	1.22	0.55	0.48	0.25
University of Portsmouth	1.49	0.54	0.17	0.20	1.51	0.57	0.19	0.20
University of Reading	1.20	1.77	2.31	1.21	1.06	1.58	2.04	1.07
University of Salford	0.99	0.76	0.55	0.39	1.16	0.89	0.64	0.45
University of Sheffield	1.07	1.47	2.20	2.19	0.73	1.02	1.47	1.53
University of Southampton	0.73	0.79	1.41	1.96	0.70	0.77	1.38	1.87
University of St Andrews	0.46	0.21	0.79	0.69	0.81	0.37	1.33	1.23
University of Stirling	0.61	1.14	0.67	0.49	0.73	1.33	0.80	0.57
University of Strathclyde	0.60	1.84	1.01	0.69	0.68	2.01	1.11	0.77
University of Sunderland	1.67	0.74	0.09	0.14	1.46	0.62	0.08	0.13
University of Surrey	0.50	1.12	1.13	0.95	0.79	1.78	1.81	1.49
University of Sussex	1.93	1.30	2.16	1.35	1.62	1.10	1.81	1.15
University of Teesside	2.59	1.65	0.24	0.13	1.61	0.98	0.14	0.08
University of Ulster	0.95	1.71	0.42	0.29	0.94	1.71	0.42	0.29
University of Wales Institute, Cardiff	1.36	1.00	0.01	0.04	1.46	1.07	0.02	0.05
University of Wales, Aberystwyth	0.81	1.16	0.95	0.66	0.76	1.10	0.91	0.62
University of Wales, Bangor	1.23	0.97	0.78	0.76	1.20	0.93	0.71	0.74
University of Wales, Lampeter	1.64	0.23	0.93	0.19	0.99	0.15	0.59	0.12
University of Wales, Swansea	2.04	1.94	2.64	1.23	0.93	0.92	1.28	0.60
University of Warwick	0.87	2.95	1.63	1.63	0.65	2.22	1.25	1.23
University of West of England, Bristol	1.45	1.26	0.10	0.23	1.12	1.00	0.08	0.18
University of Westminster	1.16	1.88	0.15	0.14	1.05	1.76	0.13	0.14
University of Wolverhampton	2.66	1.15	0.20	0.06	2.01	0.86	0.16	0.05
University of York	0.70	0.94	0.42	1.97	0.70	0.92	0.45	1.97

Table 4: Overall rank of PIs

Institution	UGs/ CAP	PGs/ CAP	PhDs / CAP	RGC/ CAP	UGs/ REC	PGs/ REC	PhDs / REC	RGC/ REC
Anglia Polytechnic University	37	76	94	95	13	66	89	87
Aston University	48	22	13	38	87	48	34	53
Birkbeck College	35	27	40	49	27	30	47	54
Bolton Institute of HE	10	14	64	86	22	31	72	93
Bournemouth University	7	57	92	82	5	71	92	85
Brunel University	70	31	43	54	64	24	43	49
Cardiff University	87	74	52	51	80	56	50	41
Cheltenham and Gloucester CHE	24	59	85	72	15	64	78	71
Chester College of HE	3	63	95	94	36	92	97	99
Coventry University	31	72	67	83	25	76	68	83
Cranfield University	101	4	12	2	100	4	6	1
De Montfort University	61	96	100	80	8	84	98	65
Edinburgh College of Art	91	77	77	76	79	35	65	63
Glasgow Caledonian University	69	75	71	74	67	70	70	72
Harper Adams University College	79	102	69	67	58	102	63	59
Heriot-Watt University	96	61	55	53	86	8	41	25
Imperial College	99	85	30	8	97	85	15	4
Institute of Education	100	3	33	34	99	2	35	40
Keele University	2	6	26	22	2	9	38	33
King Alfred's College, Winchester	17	95	91	102	4	95	83	102
Kingston University	30	24	78	93	33	27	82	92
Lancaster University	50	33	32	41	31	23	23	39
Leeds Metropolitan University	38	71	93	92	26	73	95	89
Liverpool ohn Moores University	6	50	61	62	35	87	71	82
London Business School	102	1	25	5	102	1	29	3
London Sch of Economics & Political Sci	60	2	7	10	93	3	36	26
Loughborough University	52	38	19	23	59	45	26	31
Napier University	82	94	97	89	68	91	96	80
North East Wales Institute	11	91	80	68	23	97	85	76
Nottingham Trent University	26	79	87	73	24	86	86	79
Oxford Brookes University	14	17	62	59	11	17	62	64
Queen Margaret College	20	99	84	56	39	101	87	57
Royal Holloway, University of London	44	65	36	32	63	79	49	44

Sheffield Hallam University	41	67	81	63	44	68	81	67
South Bank University	71	64	70	66	66	50	67	62
St George's Hospital Medical School	97	97	50	3	98	98	56	5
St Mary's College	45	43	102	101	37	36	102	101
Staffordshire University	57	87	96	65	30	83	90	60
The London Institute	1	55	98	100	1	61	99	100
The Queen's University of Belfast	13	21	20	29	28	32	40	42
UMIST	54	9	1	6	94	37	3	23
University College London	40	28	2	4	40	28	1	6
University College Northampton	8	80	88	98	6	90	88	98
University of Aberdeen	83	84	39	39	71	72	17	30
University of Bath	98	82	47	46	96	47	18	27
University of Birmingham	78	32	17	21	84	26	8	16
University of Bradford	58	40	23	44	61	42	30	46
University of Brighton	43	88	76	71	48	93	77	70
University of Bristol	80	56	16	14	89	59	14	13
University of Cambridge	63	66	4	7	62	62	2	7
University of Central England in Birmingham	33	10	90	91	47	14	91	91
University of Central Lancashire	12	78	58	88	17	88	61	90
University of Derby	22	62	99	79	7	63	100	81
University of Dundee	94	90	53	36	83	69	44	14
University of Durham	68	39	24	31	75	39	28	38
University of East Anglia	64	35	21	33	51	22	10	34
University of East London	28	48	83	85	53	74	93	88
University of Edinburgh	90	89	34	27	91	82	16	15
University of Essex	29	8	5	19	46	15	7	36
University of Exeter	47	11	28	35	72	18	45	47
University of Glasgow	86	101	63	42	55	99	55	22
University of Greenwich	21	12	73	43	29	20	76	55
University of Hertfordshire	16	37	57	61	12	43	60	66
University of Huddersfield	19	26	65	78	19	29	64	84
University of Hull	65	13	37	50	65	10	42	48
University of Kent at Canterbury	66	42	38	40	43	19	21	35
University of Leeds	53	51	14	17	54	52	13	18
University of Leicester	81	16	42	24	69	5	24	11
University of Lincolnshire and Humberside	9	86	89	97	16	96	94	97
University of Liverpool	74	53	18	16	92	67	25	24
University of London	95	7	3	1	101	49	32	8
University of Manchester	88	68	31	26	90	57	20	21

University of Newcastle upon Tyne	73	49	35	20	78	41	37	19
University of North London	23	41	27	87	60	80	53	94
University of Northumbria at Newcastle	34	30	86	84	14	21	84	73
University of Nottingham	55	54	15	15	45	53	9	12
University of Oxford	92	92	8	9	95	89	4	2
University of Paisley	77	93	75	90	41	75	66	75
University of Plymouth	42	81	56	60	32	81	57	61
University of Portsmouth	32	83	72	69	18	78	69	68
University of Reading	49	20	9	30	49	16	5	37
University of Salford	59	70	54	57	38	58	52	56
University of Sheffield	56	29	10	11	77	40	19	17
University of Southampton	75	69	29	13	81	65	22	10
University of St Andrews	93	100	48	48	70	94	27	29
University of Stirling	84	46	51	55	76	25	48	52
University of Strathclyde	85	19	44	47	85	7	39	43
University of Sunderland	25	73	82	77	21	77	80	77
University of Surrey	89	47	41	37	73	11	11	20
University of Sussex	18	34	11	25	9	34	12	32
University of Teesside	5	25	66	81	10	46	74	86
University of Ulster	62	23	59	58	56	13	59	58
University of Wales Institute, Cardiff	39	52	101	99	20	38	101	96
University of Wales, Aberystwyth	72	44	45	52	74	33	46	50
University of Wales, Bangor	46	58	49	45	34	51	51	45
University of Wales, Lampeter	27	98	46	70	52	100	54	78
University of Wales, Swansea	15	15	6	28	57	55	31	51
University of Warwick	67	5	22	18	88	6	33	28
University of West of England, Bristol	36	36	79	64	42	44	79	69
University of Westminster	51	18	74	75	50	12	75	74
University of Wolverhampton	4	45	68	96	3	60	73	95
University of York	76	60	60	12	82	54	58	9

Table 5: Overall rank of PIs

Institution	Mean Rank	Rank of Mean Rank	Favourite Rank	Rank Favourite Rank
Anglia Polytechnic University	70	89	13	49
Aston University	43	32	13	49
Birkbeck College	39	26	27	82
Bolton Institute of HE	49	45	10	35
Bournemouth University	61	70	5	17
Brunel University	47	40	24	75
Cardiff University	61	70	41	95
Cheltenham and Gloucester CHE	59	66	15	57
Chester College of HE	72	96	3	12
Coventry University	63	74	25	79
Cranfield University	29	8	1	1
De Montfort University	74	99	8	28
Edinburgh College of Art	70	90	35	91
Glasgow Caledonian University	71	93	67	101
Harper Adams University College	75	101	58	100
Heriot-Watt University	53	52	8	28
Imperial College	53	51	4	15
Institute of Education	43	34	2	7
Keele University	17	1	2	7
King Alfred's College, Winchester	74	98	4	15
Kingston University	57	62	24	75
Lancaster University	34	16	23	72
Leeds Metropolitan University	72	95	26	81
Liverpool ohn Moores University	57	61	6	23
London Business School	34	14	1	1
London Sch of Economics & Political Sci	30	9	2	7
Loughborough University	37	21	19	63
Napier University	87	102	68	102
North East Wales Institute	66	83	11	41
Nottingham Trent University	68	84	24	75
Oxford Brookes University	38	25	11	41
Queen Margaret College	68	85	20	67
Royal Holloway, University of London	52	49	32	87

Sheffield Hallam University	64	76	41	95
South Bank University	65	80	50	99
St George's Hospital Medical School	63	73	3	12
St Mary's College	71	92	36	92
Staffordshire University	71	93	30	86
The London Institute	64	79	1	1
The Queen's University of Belfast	28	6	13	49
UMIST	28	7	1	1
University College London	19	2	1	1
University College Northampton	70	88	6	23
University of Aberdeen	54	55	17	60
University of Bath	58	63	18	61
University of Birmingham	35	20	8	28
University of Bradford	43	33	23	72
University of Brighton	71	91	43	98
University of Bristol	43	30	13	49
University of Cambridge	34	19	2	7
University of Central England in Birmingham	58	65	10	35
University of Central Lancashire	62	72	12	45
University of Derby	64	78	7	26
University of Dundee	60	68	14	55
University of Durham	43	31	24	75
University of East Anglia	34	15	10	35
University of East London	69	87	28	85
University of Edinburgh	56	57	15	57
University of Essex	21	3	5	17
University of Exeter	38	23	11	41
University of Glasgow	65	81	22	71
University of Greenwich	41	27	12	45
University of Hertfordshire	44	35	12	45
University of Huddersfield	48	42	19	63
University of Hull	41	29	10	35
University of Kent at Canterbury	38	24	19	63
University of Leeds	34	16	13	49
University of Leicester	34	16	5	17
University of Lincolnshire and Humberside	73	97	9	31
University of Liverpool	46	38	16	59
University of London	37	22	1	1
University of Manchester	50	47	20	67

University of Newcastle upon Tyne	44	35	19	63
University of North London	58	64	23	72
University of Northumbria at Newcastle	53	53	14	55
University of Nottingham	32	10	9	31
University of Oxford	49	44	2	7
University of Paisley	74	99	41	95
University of Plymouth	59	67	32	87
University of Portsmouth	61	69	18	61
University of Reading	27	5	5	17
University of Salford	56	57	38	94
University of Sheffield	32	12	10	35
University of Southampton	46	37	10	35
University of St Andrews	64	75	27	82
University of Stirling	55	56	25	79
University of Strathclyde	46	38	7	26
University of Sunderland	64	76	21	70
University of Surrey	41	27	11	41
University of Sussex	22	4	9	31
University of Teesside	49	46	5	17
University of Ulster	49	43	13	49
University of Wales Institute, Cardiff	68	86	20	67
University of Wales, Aberystwyth	52	50	33	89
University of Wales, Bangor	47	41	34	90
University of Wales, Lampeter	66	82	27	82
University of Wales, Swansea	32	10	6	23
University of Warwick	33	13	5	17
University of West of England, Bristol	56	60	36	92
University of Westminster	54	54	12	45
University of Wolverhampton	56	57	3	12
University of York	51	48	9	31

Table 6: Dynamic Efficiency and peers

	Peers to non efficient institutions							
Institution	**Cranfield University**	**The London Institute**	**Keele University**	**Institute of Education**	**London Business School**	**London Sch of Economics & Political Sci**	**University College London**	**University of London**
Anglia Polytechnic University		Y	Y					
Aston University		Y	Y			Y	Y	
Birkbeck College			Y					
Bolton Institute of HE		Y					Y	
Bournemouth University		Y	Y					
Brunel University	Y	Y			Y		Y	
Cardiff University			Y		Y		Y	
Cheltenham and Gloucester CHE			Y					
Chester College of HE		Y	Y					
Coventry University		Y	Y					
Cranfield University	Y							
De Montfort University		Y	Y					
Edinburgh College of Art		Y	Y		Y			
Glasgow Caledonian University		Y	Y		Y			
Harper Adams University College		Y	Y		Y			
Heriot-Watt University	Y	Y			Y			
Imperial College	Y							
Institute of Education				Y				
Keele University			Y					
King Alfred's College, Winchester		Y	Y					
Kingston University		Y	Y					
Lancaster University	Y	Y	Y		Y			
Leeds Metropolitan University		Y	Y					
Liverpool ohn Moores University		Y	Y					
London Business School					Y			
London Sch of Economics & Political						Y		

Sci								
Loughborough University	Y	Y			Y		Y	Y
Napier University		Y	Y		Y			
North East Wales Institute		Y	Y					
Nottingham Trent University		Y	Y					
Oxford Brookes University		Y	Y					
Queen Margaret College		Y	Y					
Royal Holloway, University of London		Y	Y		Y		Y	Y
Sheffield Hallam University		Y	Y					
South Bank University	Y	Y			Y			
St George's Hospital Medical School	Y							Y
St Mary's College		Y	Y					
Staffordshire University		Y	Y					
The London Institute		Y						
The Queen's University of Belfast			Y		Y	Y	Y	
UMIST	Y		Y			Y	Y	Y
University College London							Y	
University College Northampton		Y	Y					
University of Aberdeen	Y	Y						
University of Bath	Y	Y						
University of Birmingham	Y	Y	Y					
University of Bradford			Y		Y		Y	
University of Brighton		Y	Y				Y	
University of Bristol	Y				Y		Y	
University of Cambridge	Y		Y		Y		Y	
University of Central England in Birmingham		Y	Y					
University of Central Lancashire		Y	Y				Y	
University of Derby		Y	Y					
University of Dundee	Y	Y			Y			
University of Durham		Y	Y		Y		Y	
University of East Anglia	Y	Y	Y		Y			
University of East London		Y				Y		
University of Edinburgh	Y	Y			Y		Y	
University of Essex		Y	Y			Y	Y	
University of Exeter		Y	Y		Y	Y		Y
University of Glasgow	Y	Y			Y		Y	
University of Greenwich		Y	Y					
University of Hertfordshire		Y					Y	
University of Huddersfield		Y	Y					
University of Hull	Y	Y	Y					

University of Kent at Canterbury	Y	Y						
University of Leeds			Y		Y		Y	
University of Leicester		Y	Y		Y		Y	
University of Lincolnshire and Humberside		Y	Y					
University of Liverpool		Y			Y		Y	Y
University of London								Y
University of Manchester	Y	Y						
University of Newcastle upon Tyne	Y	Y			Y		Y	
University of North London		Y	Y					
University of Northumbria at Newcastle		Y	Y					
University of Nottingham	Y	Y			Y		Y	
University of Oxford	Y							
University of Paisley		Y	Y					
University of Plymouth		Y	Y			Y	Y	
University of Portsmouth		Y	Y					
University of Reading		Y	Y		Y		Y	
University of Salford			Y				Y	
University of Sheffield					Y	Y	Y	
University of Southampton		Y			Y			
University of St Andrews	Y	Y			Y		Y	
University of Stirling		Y	Y		Y			
University of Strathclyde	Y	Y						
University of Sunderland			Y					
University of Surrey	Y	Y			Y			
University of Sussex		Y	Y				Y	
University of Teesside		Y	Y					
University of Ulster		Y	Y					
University of Wales Institute, Cardiff		Y	Y		Y			
University of Wales, Aberystwyth		Y	Y		Y		Y	
University of Wales, Bangor		Y	Y		Y		Y	
University of Wales, Lampeter		Y	Y					
University of Wales, Swansea		Y	Y		Y			Y
University of Warwick	Y		Y					
University of West of England, Bristol		Y	Y					
University of Westminster		Y	Y					
University of Wolverhampton		Y	Y					
University of York	Y				Y			

Table 7: Table of dynamic efficiency and the rank of each institution (variable returns to scale)

Institution	Dynamic efficiency score (VRS)	Institutions' Rank
Anglia Polytechnic University	60.13	69
Aston University	70.43	46
Birkbeck College	72.88	40
Bolton Institute of HE	97.13	12
Bournemouth University	83.01	27
Brunel University	55.79	79
Cardiff University	45.74	98
Cheltenham and Gloucester CHE	78.39	31
Chester College of HE	100.00	1 joint
Coventry University	51.64	90
Cranfield University	100.00	1 joint
De Montfort University	75.31	37
Edinburgh College of Art	62.86	59
Glasgow Caledonian University	37.41	101
Harper Adams University College	63.73	57
Heriot-Watt University	68.12	51
Imperial College	83.25	26
Institute of Education	100.00	1 joint
Keele University	100.00	1 joint
King Alfred's College, Winchester	92.35	14
Kingston University	60.80	66
Lancaster University	68.63	50
Leeds Metropolitan University	55.16	80
Liverpool John Moores University	77.29	33
London Business School	100.00	1 joint
London Sch of Economics & Political Sci	100.00	1 joint
Loughborough University	63.51	58
Napier University	35.55	102
North East Wales Institute	78.70	30
Nottingham Trent University	50.83	93
Oxford Brookes University	84.81	24
Queen Margaret College	100.00	1 joint
Royal Holloway, University of London	58.89	74
Sheffield Hallam University	44.58	99
South Bank University	46.16	96
St George's Hospital Medical School	86.23	19
St Mary's College	77.87	32
Staffordshire University	51.53	91

The London Institute	100.00	1 joint
The Queen's University of Belfast	80.41	28
UMIST	94.07	13
University College London	100.00	1 joint
University College Northampton	87.47	17
University of Aberdeen	53.68	84
University of Bath	52.77	86
University of Birmingham	62.16	62
University of Bradford	51.87	89
University of Brighton	41.80	100
University of Bristol	66.66	55
University of Cambridge	86.17	20
University of Central England in Birmingham	62.23	60
University of Central Lancashire	69.12	49
University of Derby	66.85	54
University of Dundee	57.30	77
University of Durham	52.55	87
University of East Anglia	64.89	56
University of East London	60.65	67
University of Edinburgh	52.42	88
University of Essex	88.43	16
University of Exeter	61.97	63
University of Glasgow	55.07	81
University of Greenwich	80.08	29
University of Hertfordshire	84.91	23
University of Huddersfield	74.19	39
University of Hull	67.38	52
University of Kent at Canterbury	71.66	44
University of Leeds	72.67	42
University of Leicester	76.93	34
University of Lincolnshire and Humberside	70.99	45
University of Liverpool	59.01	73
University of London	100.00	1 joint
University of Manchester	52.91	85
University of Newcastle upon Tyne	59.19	70
University of North London	74.96	38
University of Northumbria at Newcastle	72.68	41
University of Nottingham	84.31	25
University of Oxford	76.52	35
University of Paisley	59.06	72
University of Plymouth	49.75	94
University of Portsmouth	60.64	68
University of Reading	70.32	47
University of Salford	56.42	78
University of Sheffield	75.48	36

University of Southampton	67.05	53
University of St Andrews	51.18	92
University of Stirling	47.66	95
University of Strathclyde	62.22	61
University of Sunderland	59.09	71
University of Surrey	69.94	48
University of Sussex	85.36	22
University of Teesside	91.61	15
University of Ulster	54.40	83
University of Wales Institute, Cardiff	57.61	75
University of Wales, Aberystwyth	46.10	97
University of Wales, Bangor	61.20	65
University of Wales, Lampeter	97.58	11
University of Wales, Swansea	85.60	21
University of Warwick	72.07	43
University of West of England, Bristol	54.47	82
University of Westminster	57.55	76
University of Wolverhampton	87.10	18
University of York	61.70	64

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