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Conference Paper title: Parametric Representations of Facial Expressions on PDE-Based Surfaces

Publication year: 2008

Publication title: Proceedings of the 8th IASTED International Conference on Visualization, Imaging and Image Processing

ISBN: 978-0-88986-759-8

Publisher: ACTA Press

Publisher's site: *http://www.actapress.com*

Original online publication is available at:
http://www.actapress.com/Content_Of_Proceeding.aspx?ProceedingID=494

PARAMETRIC REPRESENTATION OF FACIAL EXPRESSIONS ON PDE-BASED SURFACES

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ABSTRACT

Parameterisation of facial expressions on PDE surface representations of human faces are presented in this work. Taking advantage of the boundary-value approach inherent to Bloor-Wilson PDE method, facial expressions are achieved by manipulating the original boundary curves. Such curves are responsible for generating a surface representation of a human face in its neutral configuration, so that regions on these curves represent a given facial expression in a fast and realistic manner. Additionally, the parameterisation proposed here is carried out by applying different mathematical transformations to the affected curves according to the corresponding facial expression. Full analytic expressions parameterising some of the most common facial expressions such as smiling and eyebrow raising are in this work. Some graphical examples of these facial expressions are used to illustrate the results obtained using Bloor-Wilson PDE method as the foundations of the parameterisation scheme proposed here. Thus, it is shown that an efficient, intuitive and realistic parameterisation of facial expressions is attainable using Bloor-Wilson PDE method in along with a suitable mathematical expression.

KEY WORDS

Geometric modelling, PDE method, boundary representation, facial expressions, animation.

1 Introduction

Non-verbal communication between humans highly relies on facial expression and, given its importance, research has been done on this area under different perspectives ranging from psychology to virtual reality. For instance, one of the most important works carried out in this area from a qualitative point of view is known as FACS (Facial Action Coding System) [8], which describes a vast number of facial expressions as action units or combinations of them (depending on the complexity of the facial expression). The relevance of this work has two aspects: The first, the fact that it qualitatively details the effects of each action unit has on the face and second, it states the potential variations that may occur under different age conditions.

From a geometric modeling perspective, the modelling of both, a human face and facial expressions, with realistic outputs is paramount since there is a natural familiarization with them that obviates any subtle deviation from the outputs regarded as normal. Furthermore, the ever increasing demand for accurate, fast and realistic facial expression modelling in applications such as computer games, film and virtual communications enhances the need for developing a technique for the geometric modelling of facial expressions fulfilling these requirements.

The first geometric model of a human face used for animation purposes is presented in [9], which uses cosine interpolation to achieve morphing between two different expressions, belonging to a collection of pre-determined expressions. Since then, a number of work relevant to this area have been published; for instance, the work presented in [7] proposes a biomechanical model for facial animation emphasising the importance of the skin-skull interface in the achievement of realistic animation. Additionally, this work describes the different approaches undertaken to achieve facial expression on mesh-based models, some them are: control points, texture-map assembly, feature tracking and finite element representations. For further details on such techniques, the reader is referred to this work where additional references for each methodology are enounced.

Recent advances in facial expression modelling include applications such as sketching facial expressions in a two-dimensional environment and then transferring them to the three-dimensional model as in [4] and [5]. Additionally, facial expression has also been achieved by using pre-processed motion caption sequences as shown in [3] where video-recorded two-dimensional animation control signals are translated into three-dimensional facial expressions. The visual results obtained using these techniques are excellent; however, these techniques are still highly dependent on either databases or computational resources. Thus, parametric models have been developed with the aim of addressing these issues. For example, the work portrayed in [11] uses a set of approximately one hundred independent parameters acting on a connected set of vertices composing the face and offers the possibility of creating and animating charicaturesque models. Another example of the advances in this are is represented in [10] where a system for creating, editing, animating and rendering human faces is developed. For further details, the reader is referred to [6], which comprises a literature review on the advances of facial modelling and animation.

Notwithstanding, given that most of the techniques developed to date are based on the use of mesh models, an underlying restriction has been imposed to the parameterisation of facial expression. Thus, the use of parametric geometric models represents an alternative for achieving an efficient parametric description of facial expressions. Particularly, parametric PDE surfaces obtained by Bloor-Wilson PDE method may prove useful as the foundations of an efficient parameterisation of facial expressions capable of producing fast and realistic results.

Therefore, this work presents a technique for parameterising different facial expressions based on such a PDE method by adjusting specific region of the original boundary curves, which generate the neutral pose of a given human face. The outline of such work is divided as follows: Section 2 provides a brief description of the mathematical theory sustaining Bloor-Wilson PDE method, followed by some technical guidelines accounting for the geometric model of a human face in use, which are given in Section 3. Section 4 outlines the parameterisation of some facial expressions and its corresponding graphical output. To finalise, Section 5 presents the conclusions and future directions of this work.

2 Bloor-Wilson PDE Method

Bloor-Wilson PDE method generates parametric surfaces that are defined as the solution to an elliptic partial differential equation (PDE) in a parametric domain, which then is mapped into the physical space; that is, $x(u,v)$, $y(u,v)$, $z(u,v)$). The general form of the generating PDE is given by

$$\left(\frac{\partial^2}{\partial u^2} + a^2 \frac{\partial^2}{\partial v^2}\right)^2 \mathbf{X}(u,\, v) = 0\,, \tag{1}$$

where $\mathbf{X}(u,\, v)$ denotes the function representing the parametric surface, u and v are the parametric surface coordinates and $a \geq 1$ is a parameter inherent to the PDE.

Equation (1) is a fourth order PDE and, as such, its particular solution requires four boundary conditions, which are generally responsible for determining the value of $\mathbf{X}(u,\, v)$ and some of the derivatives at determined positions. However, the order of the PDE does not represent a limitation in the implementation of this PDE method as it may be seen in [2] where Equation (1) was adapted to represent a sixth order PDE as means for achieving fast surface modelling. Moreover, note that if $a = 1$, corresponds to the biharmonic equation, which is widely known in physical sciences and several techniques for its solution have been developed and that the use of periodic boundary conditions leads to a Fourier Series type solution.

For the purposes of this work and for the sake of brevity, the approximate solution to Equation (1) employed to generate the results presented throughout this work is outlined below; however, further details on the mathematical aspects of this method can be found in [1] and [12]. The approximate solution is then given by,

$$\begin{aligned} \mathbf{X}(u,\, v) &= \mathbf{A_0}(u) + \sum_{n=1}^{N} [\mathbf{A_n} \cos(nv) + \mathbf{B_n} \sin(nv)] \\ &+ \mathbf{R}(u,\, v)\,, \end{aligned} \tag{2}$$

where,

$$\begin{aligned} \mathbf{A_0} &= \mathbf{a_{00}} + \mathbf{a_{01}} u + \mathbf{a_{02}} u^2 + \mathbf{a_{03}} u^3\,, & (3)\\ \mathbf{A_n} &= (\mathbf{a_{n1}} + \mathbf{a_{n3}} \mathbf{u})\, e^{anu} + (\mathbf{a_{n2}} + \mathbf{a_{n4}} u)\, e^{-anu}\,, & (4)\\ \mathbf{B_n} &= (\mathbf{b_{n1}} + \mathbf{b_{n3}} \mathbf{u})\, e^{anu} + (\mathbf{b_{n2}} + \mathbf{b_{n4}} u)\, e^{-anu}\,. & (5) \end{aligned}$$

and $\mathbf{R}(u,\, v)$ is a function defined as,

$$\begin{aligned} \mathbf{R}(u,\, v) &= \mathbf{r_1}(u) e^{wu} + \mathbf{r_2}(u) e^{-wu} \\ &+ \mathbf{r_3}(u) u e^{wu} + \mathbf{r_4}(u) u e^{-wu}\,, \end{aligned} \tag{6}$$

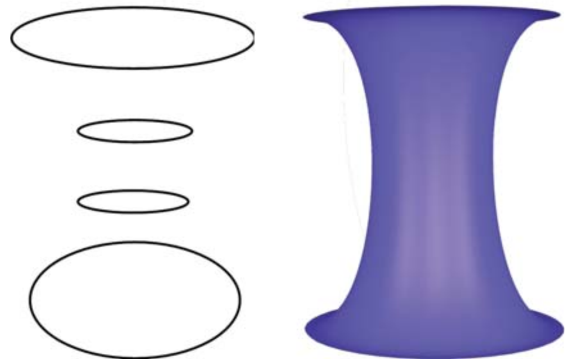
The value of the constant vectors $\mathbf{a_{ij}}$ and $\mathbf{b_{ij}}$ are determined by the specified boundary conditions, which for this purpose, have to be expressed in terms of a Fourier series, w has been conveniently chosen as $w = a(N+1)$ and, $\mathbf{r_1}(v)$, $\mathbf{r_2}(v)$, $\mathbf{r_3}(v)$ and $\mathbf{r_4}(v)$ are vector functions guaranteeing that the approximate solution satisfies the originally imposed boundary conditions. Each of the components of these vectors corresponds to a direction in the cartesian three-dimensional space.

The results of this work have been produced using all the boundary curves as positional boundary conditions; that is, the value of $\mathbf{X}(u,\, v)$ is determined at four different values of u. Such a modification does not affect the core of the PDE method and without losing generality, a PDE surface representation with these curves can be found. This alteration to the traditional formulation is motivated on the fact that for the particular case of face modelling, the user is mainly concerned with obtaining a surface passing through all the generating curves since they may be associated with key features of the face. Figure 1 presents the resulting PDE surface when the modified formulation of Bloor-Wilson PDE method is used to calculate such a surface using the boundary curves shown in this Figure.

3 PDE Surface Representation of a Human Face

In the interest of preserving an acceptable level of detail on the features of the face, the geometric model representing a human face used throughout this work has been obtained using twenty-eight generating curves. Thus, the complete surface is composed of the solution of nine different PDEs, using four consecutive boundary curves accordingly; that is, all the nine PDEs share one boundary curve with either one or two different PDE so that continuity is guaranteed along the entire surface representation.

The generating curves were extracted using MAYA software from a high resolution laser-scanned model of a



(a) (b)

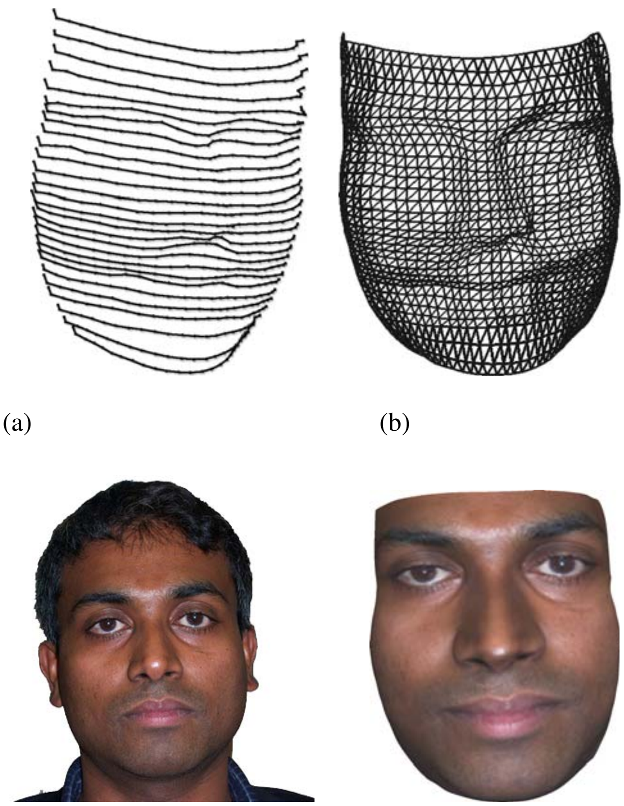
Figure 1. Example of a PDE surface generated by the modified version of the Bloor-Wilson PDE method whereby the resulting PDE passes through all the generating boundary curves. The generating boundary curves are shown in (a) the resulting surface after using the traditional and modified formulations is presented in (b).

human head where the face is kept in its neutral pose; i.e. eyes opened, mouth closed and showing no emotion. The curves were extracted so that each of them was transverse to the head and fairly parallel to each other. Additionally, each curve is composed of 47 points, the first 23 outlines the face, whereas the next 23 are merely a reflection of the first 23 points in a given plane in order to define such curves from zero to 2π and the last point ensures that each of these curves is closed properly.

Bloor-Wilson PDE method has been utilised for computing the corresponding PDE surface representation associated with the formerly extracted curves. To this end, nine different PDEs have been solved by finding the coefficients of Equation 2 respectively and using three Fourier modes; i.e. $N = 3$. Since, the region of interest is the face, the parametric domain in the v direction is restricted to the region varying from 0 to π. The resolution of the mesh associated with the complete surface representation in all the results presented through this work is composed of 41 x 31 points and, the solution was calculated from 0 to π.

The texturing process has been carried out by taking advantage of Bloor-Wilson PDE method since the two-dimensional parametric domain used to find the solution to the PDE can also be used to determine the coordinates of the texture mapping. Thus, for each (u, v) point in the parametric domain a texture element from a two-dimensional image can be associated with a point in the resulting PDE surface are found leading to fairly straightforward methodology. The two-dimensional texture image is previously warped so that key feature points of the face remain invariant and any unnecessary background is removed. The details associated with this particular warping technique are omitted for they can be regarded beyond the scope of this particular application. Moreover, the major advantage given by this texturing technique is that since texture coordinates are based on the parametric domain used to solve the PDE, the same two-dimensional image can be used to texture any of the surfaces whereby facial expression is intended.

Figure 2 shows the process of obtaining a PDE surface representation of a human face with the methodology proposed here. The frontal region of the generating boundary curves that have been used to generate the PDE surface representation of a face used throughout this work are shown in Figure 2.a, the mesh representation of the resulting PDE surface is outlined in Figure 2.b, whereas the two-dimensional image used to texture the results and the textured PDE surface representation of a face in a neutral state are presented in Figure 2.c and Figure 2.d on each respective case.



(a) (b)

(c) (d)

Figure 2. Process followed for obtaining a PDE surface representation of a human face. The generating boundary curves are shown in (a), the resulting mesh associated with the corresponding surface is outlined in (b), the two-dimensional image of the texture is presented in (c) and the textured PDE surface is shown in (d).

4 Parameterisation of Facial Expressions

The parameterisation of facial expressions developed here consists of identifying the boundary curves and points in these curves that are affected by each expression so that a suitable mathematical transformation can be apllied in order to obtain the wanted results. For the sake of brevity, full details on some the expressions namely, smiling, frowning and eyebrow raising are given below; however, some graphical results of a wider collection of expressions are presented.

4.1 Smiling and frowning

Smiling and frowning are facial expressions characterized by arching the lips upwards or downwards respectively. Thus, the generating boundary curves number 3 to eleven of the geometric model in use (the curves are numbered sequentially starting at the bottom of the chin to the top of the forehead), are the curves defining the lips and the surrounding area, whereas the points delimiting the mouth corners correspond to the sixth and eighteenth. The mathematical transformation needs to produce the effect of curving the mouth and this is achieved by applying a weighted sine function to all the points between the mouth corners in each of the affected boundary curves. Given that the effect of arching the mouth is more pronounced either in the upper part of the mouth when smiling or in the lower part when frowning, a weighted intensity factor is associated with each of these curves so that this effect is enhanced in both cases respectively.

The mathematical transformation responsible for producing either a smile or a frown is given by,

$$\begin{aligned} c_j(x_i) &= c_j(x_i) \\ &+ \epsilon_x(j)\sin\left(\frac{\pi*(18-i)}{12.0}\right) * \mid c_{j+1}(x_i) - c_j(x_i) \mid, \\ c_j(y_i) &= c_j(y_i) \\ &\pm \epsilon_y(j)\sin\left(\frac{\pi*(18-i)}{12.0}\right) * \mid c_{11}(y_i) - c_j(y_i) \mid, \\ c_j(z_i) &= c_j(z_i) + \epsilon_z(j)\frac{\mid 12-i \mid}{6.0} c_j(z_i), \end{aligned} \tag{7}$$

where $j = 4, \ldots, 10$ denotes the number of the curves to be transformed, $i = 6 \ldots, 18$ is the point of the curve to which the transformation is applied, whereas $\epsilon_x(j)$, $\epsilon_y(j)$, and $\epsilon_z(j)$ represent the intensity factors with which each curve is modified, which in the case of smiling, $\epsilon_z(j) = 0.1$ for all the curves, whilst $\epsilon_x(j)$ and $\epsilon_y(j)$ vary from one generating curve to another with,

$$\epsilon_x(j) = \{0.2,\ 0.25,\ 0.35,\ 0.40,\ 0.45,\ 0.60,\ 0.75\}$$

and

$$\epsilon_y(j) = \{0.2,\ 0.25,\ 0.35,\ 0.40,\ 0.45,\ 0.60,\ 0.75\}\ .$$

The values of such intensity factors used in frowning are $\epsilon_z(j) = -0.05$ for all the curves and,

$$\epsilon_x(j) = \{0.55,\ 0.50,\ 0.45,\ 0.40,\ 0.35,\ 0.25,\ 0.20\}$$

and

$$\epsilon_y(j) = \{0.55,\ 0.50,\ 0.45,\ 0.40,\ 0.35,\ 0.25,\ 0.20\}\ .$$

The $\pm$ sign in the transformation applied along the y direction is the one determining is such a transformation consists of smiling (the plus sign is employed) or frowning.

Figure 3 presents the results obtained after applying the transformation associated with both, smile and frown, to the original neutral face. Figure 3.a clearly shows how the lips are arched upwards producing a smile whilst Figure 3.c, schematize the opposite effect producing a frown.

(a) (b)

Figure 3. Example of a PDE surface representation of a face smiling and frowning.

These expressions can be performed on one side of the face and thus, the respective mathematical transformation can be applied unilaterally to the required side, leaving the other one on its neutral position. Figure 4 presents the results obtained after applying both expressions on each side of the face.

4.2 Mouth Opening

The action of opening the mouth is characterized by separating the lips as a consequence of moving the lower part of the face, from the bottom of the chin to the top of the lower lip, downwards. The parameterisation of such a movement is undertaken by shifting the affected boundary curves downwards in a non-uniform manner so that the effect is more noticeable at the lower part, fading noticeable towards the lower lip. The mathematical transformation is given by,

$$\begin{aligned} c_j(x_i) &= c_j(x_i)\,, \\ c_j(y_i) &= c_j(y_i) \\ &- \epsilon_y(j)\sin\left(\frac{\pi*(23-i)}{22.0}\right) * \mid c_{j+1}(y_i) - c_j(y_i) \mid, \\ c_j(z_i) &= c_j(z_i)\,, \end{aligned}$$

where $j = 1 \ldots 7$, represents the number of the curve being modified, i denotes the point to be transformed, and $\epsilon_y(j)$ specifies the intensity factors applied to each curve, which for this particular case are,

$$\epsilon_y(j) = \{0.95,\ 0.90,\ 0.85,\ 0.80,\ 0.75,\ 0.70\}.$$

Figure 4. PDE surface representations showing unilateral versions of the smiling and frowning.

Note that the x and z coordinates are left unchanged. A qualitative comparison between the PDE surface representation of a neutral face and the one obtained after applying the transformation responsible for opening the mouth is presented in Figure 5.

4.3 Eyebrow Raising

The effect produced on the face by raising an eyebrow consists of arching the eyebrow whilst bringing it upwards towards the forehead. To that end, a sinusoidal function is applied to the region bounded by the points delimiting each respective eyebrow in each of the affected generating curves. The full mathematical formulation for raising the right eyebrow is given by,

$$\begin{aligned} c_j(x_i) &= c_j(x_i)\,, \\ c_j(y_i) &= c_j(y_i) \\ &+ \epsilon_y(j)\sin\left(\frac{\pi*(10-i)}{5.0}\right)*\mid c_{j+1}(y_i)-c_j(y_i)\mid, \\ c_j(z_i) &= c_j(z_i)+\epsilon_z(j)\frac{10-i}{5.0}c_j(z_i)\,. \end{aligned} \tag{8}$$

Again, $j = 23, \ldots, 26$ represents the curve that is being modified, $i = 5, \ldots, 10$ denotes the points affected by the transformation and $\epsilon_y(j) = 0.85$ together with $\epsilon_z(j) = 0.01$ represent the constant intensity factors applied over the y and z direction respectively. The expression associated with the left eyebrow is similar to the one stated in Equation 8 with very subtle differences such as

(a) (b)

Figure 5. Comparison between the neutral face and the same face with the mouth open, both are PDE surface representations. The neutral state of the face is shown in (a) whilst the face with the mouth open is presented in (b).

the values i can take. The graphical results are shown in Figure 6 where the resulting surfaces after raising each eyebrow separately are presented.

(a) (b)

Figure 6. PDE surface representation of a face raising each eyebrow separately.

4.4 Graphical examples

For the purposes of illustration, some graphical examples of some other facial expressions are presented. Figure 7 presents different expressions such as eyes closed , lateral shift of the lower lip to the right and some combination such as eyebrow raising with either smiling or frowning.

5 Conclusions

A methodology for generating and manipulating PDE surface representations of a human face using Bloor-Wilson PDE method as its foundation has been developed. The manipulation is carried out through the definition of a set

Figure 7. PDE surfaces representing different facial expressions including, closing ayes, lateral shift of lower lip and different combinations.

of parameters which are responsible for the characterization of facial expressions. The process consists of identifying the generating points and the points on these curves that are affected by a given facial expression. Then a suitable mathematical transformation is applied to these points so that the desired expression is portrayed realistically on the resulting PDE surface representation. Thus, this methodology has taken full advantage of Bloor-Wilson PDE method creating different facial expressions from a neutral configuration in virtually real time in a realistic manner. Graphical results for expressions namely, smiling, frowning, mouth opening, lateral shift of the lower lip, eyebrow raising and eye closing; however, due to space constraints, the corresponding mathematical formulation is not included for all of them and only the ones associated with smiling, frowning and eyebrow raising are detailed in this work.

The algorithms developed is this work can be further extended to achieve facial animation by either incorporating a parameter controlling the evolution of any facial expression by regulating the intensity in which such an expression is performed or by morphing among facial expressions. The authors would like to stress that the methodology proposed here may represent an excellent alternative for applications requiring real-time results or those with limited memory resources since only the original information related to the neutral configuration of the face required so that the parametric transformation can be carried out.

6 Acknowledgements

This work has been carried out through the support received from The UK Engineering and Physical Sciences Research Council grants EP/C015118/01 and EP/D000017/01.

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