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Summary

The problem of controller design for multivariable systems with uncertain dynamics is aggravated by the obvious difficulty of making precise theoretical predictions based on uncertain data. The paper reviews some recent attempts to formalise this problem and describes how asymptotic analysis can play an important role in such studies.

Introduction

It is generally true that control theoretic studies of controller design assume the existence of a precise model of plant dynamics. In fact, given the existence of a 'precise' linear model, control theory has reached some maturity. It is however, frequently the case in industrial or other applications that a model of the plant is not available or that the available model of plant dynamics is too complex to make normal design calculations (e.g. calculation of poles, zero, transfer matrix, canonical forms etc.) feasible. In both cases, the plant model is, from the point of view of design work, effectively unknown - yet controller design must still be undertaken! In such situations it is clearly possible to consider the use of adaptive or self-tuning controllers [1] or standard identification procedures [2]. It may, however be considered that the inevitable complexities implicit in adaptive control are not necessary in the given application and hence that fixed structure process controllers (with their known simplicity and robustness) are to be preferred. Such controllers could be designed on the basis of a low order, approximate model deduced by identification of input-output data but (i) this needs access to computing facilities that may not be available or regarded as necessary and (ii) there is always the uncertainty present concerning the behaviour of the real plant in the presence of the controller designed on the basis of the identified model. The construction of a viable alternative would clearly be a useful addition to the designers armoury.

It is the purpose of this paper to describe a conceptual framework and related theoretical concepts relevant to the construction of a viable theory of non-adaptive unknown systems control and to illustrate the form of result possible using asymptotic analysis. It should be noted that the possibility of constructing viable control theories for systems with unknown model has only recently been identified by the pioneering work of Davison [3] with its modification and generalizations [4], [5] and the more detailed work of the author and his collaborators [6] - [12] and Åstrom [13]. All of this material can, at least in part, be regarded as an attempt to generalize the classical on-line tuning procedures (such as that due to Ziegler and Nichols [14]) to the multivariable case. One thing is most certainly clear [15] - the problem of controller design for systems with uncertain dynamics precludes the possibility of making precise theoretical predictions of closed-loop performance under some (and probably all) conditions of interest! At the minimal level of plant information required, it appears to be necessary [15] to have some structural information (e.g. stability, minimum-phase, rank,...) together with some numerical information (e.g. steady-state data, rise-times,...) and theoretical work should turn its attention to the general questions:

(i) Given the structural information known, does a stabilizing controller of the required structure exist,
(ii) is the paramtric information required to construct such a controller available and
(iii) what is a suitable paramerization of the controller?

If the answers to questions (i) and (ii) are in the affirmative and the solution to (iii) is known, then the choice of parameters could be undertaken by on-line tuning.

The form of result possible can be illustrated by the work in [3] - [5] where, roughly speaking, a stable unknown system with no invariant zeros at the origin of the complex plane can be stabilized by a P+I controller with transfer function matrix,

$$K(s) = k\left[K_1 + \frac{1}{s}K_2\right] \qquad (1)$$

by choosing, for example, $K_1 = G(o)$ (D.C. gain of plant deduced from open-loop step responses), $K_2 = \varepsilon\, G(o)$ (where $\varepsilon \geqslant o$) and the 'overall gain' k in a (proved) non-empty range

$$o \leqslant k < k^*(\varepsilon) \qquad (2)$$

The existence of a stabilizing k is the most the theory can provide (unless other information is invoked [12]) but a suitable value can always be found on-line.

The above example illustrates the general 'low-gain philosophy' [15] based upon the intuitive notion that a stable system will retain its stability under low gain feedback. There is, however an inverse approach (tentatively termed the 'high-gain philosophy' [15]) that is suggested by the recently developed theory of multivariable root-loci (see, for example, [7] or the review in [16]) and the argument that an unknown plant can be stabilized by the controller (1) by suitable choice of K_1 and K_2 and choice of k in a non-empty range

$$k^* < k \tag{3}$$

if the plant is minimum-phase and has asymptotes in the open left-half complex plane. The proof of existence of k^* in (3) is clearly a problem in asymptotic analysis!

The general details of such a theoretical approach are under consideration. Precise results are now available, however, to cover design of P+I controllers for unknown m-input/m-output, linear, time-invariant systems in R^n,

$$\dot{x}(t) = A\,x(t) + Bu(t)$$

$$y(t) = C\,x(t) \tag{4}$$

possessing the (assumed known) structural properties of being minimum-phase and $|CB| \neq 0$. They are described in the following sections where previous work is extended to produce stability results and transient performance assessment in the presence of measurement nonlinearities.

An Approximation Philosophy

The theoretical approach taken by the authors for the design of the controller K for the unknown plant G in the feedback configuration of Fig. 1(a) with the (assumed known) memoryless measurement nonlinearity N is to consider the problem of controller design for a simple approximate model G_A of the plant in the configuration of Fig. 1(b) with a linear approximation F to the nonlineary N. Theoretical studies can then be directed to the generation of computable conditions for the stability of the real feedback system and the search for estimates of the closed-loop response y in terms of the approximating response y_A.

The basic result can be deduced from recent robustness studies [17] if we regard that the plant G and its approximation G_A as mappings from a vector space U into a vector space Y. As we are interested in square systems we will take Y = U. The space Y is assumed to contain a subspace Y_o endowed with a suitable norm topology with respect to which it is complete. A system is said to be stable if it maps Y_o into itself. Causality structures are easily included but are ignored here for simplicity. Suppose that the plant G and its approximation G_A are related by the feedback structure of Fig. 2 or, in algebraic form,

$$G = (I + G_A H)^{-1} G_A \tag{5}$$

where $H: Y \to Y$ is stable and that the nonlinearity N satisfies the condition

$$N - F = N_1 + N_2 \tag{6}$$

where N_1 has finite incremental gain k_1 such that ($||.||$ being the norm in Y_o)

$$||N_1 y - N_1 z|| \leq k_1\, ||y - z|| \tag{7}$$

for all $y, z \in Y_o$ and N_2 is bounded in the sense that there exists $q \geq o$ such that

$$||N_2 y|| \leq \frac{q}{2}\ \forall y \in Y \tag{8}$$

The following result can immediately be deduced from [17] by regarding G as a perturbation of its approximation G_A and describing Fig. 1(b) by the linear operator L_c in Y:

Theorem 1: If the feedback system of Fig. 1(b) is stabilized in the presence of K and

(a) the control mapping K has an inverse on Y,
(b) $L_c K^{-1} H$ maps Y_o into itself with

$$\lambda \triangleq ||L_c K^{-1} H|| < 1 \tag{9}$$

(c) the nonlinearity satisfies

$$\mu \triangleq (1-\lambda)^{-1} ||L_c|| k_1 < 1 \tag{10}$$

then the controller K stabilizes the plant G in the configuration of Fig. 1(a). Moreover, under these conditions, the responses y and y_A are related by the inequality,

$$||y - y_A|| \leq \left\{\frac{\mu}{1-\mu} + \lambda\right\} \frac{1}{(1-\lambda)}\, ||y_A|| + ||L_c||\, \frac{q}{2}\, \frac{1}{(1-\lambda)(1-\mu)} \tag{11}$$

Note that the RHS of (11) contains only known quantities and hence, in principle, can be computed to bound the error (in norm) $y - y_A$ in predicted response. Its use in general design studies is being considered at the present time. We will turn our attention however to the specific situation when G has a minimum-phase state-space form (4) with CB nonsingular and the use of a multivariable first order model G_A with transfer function matrix [6],[7],[11],[15] with inverse

$$G_A^{-1}(s) = s\,A_o + A_1 \tag{12}$$

where $A_o = (CB)^{-1}$ is estimated using initial rate data from open-loop plant unit step responses [6], [11] and A_1 is, in principle [15], any mxm real, constant matrix e.g. choosing A_1 equal to the inverse d.c. gain matrix (deduced, perhaps, using steady state open-loop step responses data [6],[11]) would ensure that the plant and its approximation have identical rise-time and steady state data. An equivalent state-space model is

$$\dot{y}(t) = -A_o^{-1}A_1\,y(t) + A_o^{-1}u(t) \qquad (13)$$

Throughout the remainder of the paper we will also assume that Y_o is simply the product space $L^m_\infty(o,+\infty)$ and that Y is its natural extended space. For simplicity however we will not use the implicit causality structure.

Control of Unknown Plant

The application of the above theory to design begins by the construction of a suitable controller for G_A. We will suppose that F is an m×m nonsingular matrix and write K as a minimal realization of the transfer function matrix

$$K(s) = \left(A_o \,\mathrm{diag}\left\{k_j + c_j + \frac{k_j c_j}{s}\right\}_{1\le j\le m} - A_1\right)F^{-1} \qquad (14)$$

where $\{k_j\}$ and $\{c_j\}$ are available proportional and integral tuning parameters respectively. A simple calculation yields the fact that L_c has transfer function matrix

$$(I+G_A KF)^{-1}G_A K = \mathrm{diag}\left\{\frac{1}{(s+k_j)(s+c_j)}\right\}_{1\le j\le m}\left(\mathrm{diag}\{(k_j+c_j)s + k_j c_j\}_{1\le j\le m} - s\,A_o^{-1}A_1\right)F^{-1} \qquad (15)$$

and hence that the approximate feedback system is stable if $k_j > o$, $c_j \geq o$ and that response speeds and reset times in loop j are of the order of k_j^{-1} and c_j^{-1} respectively. Defining the matrix norm $||M||_m = \max_i \sum_j |M_{ij}|$ and the 'overall gain'

$$k \triangleq \min_j k_j \qquad (16)$$

then we obtain the following asymptotic results:

Lemma 1:

$$\lim_{k\to+\infty} ||L_c|| \leq ||F^{-1}||_m \qquad (17)$$

equality holding if $F = I_m$.

Proof: For simplicity let $k_j = k > o$ and $c_j = c > o$, $1\le j\le m$, and write (15) in the form

$$\frac{k}{s+k}\cdot\frac{k}{(k-c)}\cdot(I_m - k^{-1}A_o^{-1}A_1)F^{-1} + \frac{c}{s+c}\cdot\frac{c}{(c-k)}\cdot(I_m - c^{-1}A_o^{-1}A_1)F^{-1} \qquad (18)$$

Clearly the second term has norm tending to zero as $k\to+\infty$ and the first term has norm $\leq ||F^{-1}||_m$, equality holding if $F = I_m$.

Lemma 2: There exists $k^* \geq 0$ such that K has a stable inverse for $k \geq k^*$ with norm in Y_o satisfying

$$\lim_{k\to+\infty} ||K^{-1}|| = 0 \qquad (19)$$

Proof: It is a simple problem in algebra to show that K has an inverse system with a stable transfer function matrix for large k. The result can then be deduced by, say, partial fraction expansion or construction of a minimal realization [11].

There application in prediction of the behaviour of the unknown plant G in the presence of K requires the following structural result:

Lemma 3: If S(A,B,C) is minimum-phase with $|CB| \neq 0$ then G has a decomposition (5) with H proper and stable.

Proof: Simply note that G(s) has inverse

$$G^{-1}(s) = s\,A_o + H(s) \qquad (20)$$

where H is proper and stable and write $H = \tilde{H} - A_1$.

We can now state the main result of this paper:

Theorem 2: Suppose that an m×m unknown plant S(A,B,C) is known to be minimum-phase with $|CB| \neq 0$ and that the nonlinearity N is such that $\gamma \triangleq ||F^{-1}||_m\, k_1 < 1$ for suitable choice of F and k_1. Then, for each choice of integral tuning parameters $c_1 \geq o, c_2 \geq o, \ldots, c_m \geq o$, there exists $k^* \geq o$ such that, for all values of overall gain $k \triangleq \min_j k_j \geq k^*$, the controller (14) with $A_o = (CB)^{-1}$ will stabilize the unknown plant in the configuration shown in Fig. 1(a). Moreover, under these conditions, the responses of the real and approximating feedback systems from zero initial conditions are related asymptotically by the relation

$$\limsup_{k\to+\infty} ||y - y_A|| \leq \frac{\gamma}{1-\gamma}\limsup_{k\to+\infty} ||y_A|| + ||F^{-1}||_m \frac{q}{2}\frac{1}{(1-\gamma)} \qquad (21)$$

Proof: Simply note that the conditions of theorem 1 are satisfied as $k_j > o$, $c_j \geq o$, $1\le j\le m$, indicate that L_c is stable whilst lemmas 1-3 indicate that $L_c K^{-1} H$ is bounded, $\lim_{k\to\infty} \lambda = 0$ and $\lim_{k\to+\infty} \mu \leq \gamma < 1$. Equation (21) follows directly from this information, equation (11) and the observation that both $\mu/(1-\mu)$ and $(1-\mu)^{-1}$ are monotonically increasing in $[0,1)$.

The practical interpretation of the result is that, given the structural information and the numerical value of CB for the unknown plant, and the data for the nonlinearity, the controller parameterization (14) can be easily constructed and the parameters adjusted to produce the desired response from the approximating linear feedback systems. The resultant trial controller can then be hooked up to the real plant in the presence of the nonlinearity when theorem 2 states that, provided the computed value of γ is less than unity and the overall gain k is greater than its unknown bound k^*, the resultant feedback system will be i/o stable. More than this, the theorem states that, if instability is found, one need only increase the gains to achieve stability nd the performance degradation can be represented approximately by

$$||y(t) - y_A(t)||_m \leq \frac{\gamma}{1-\gamma} \sup_{t \geq o} ||y_A(t)||_m + ||F^{-1}||_m \frac{q}{2} \frac{1}{(1-\gamma)} \qquad (22)$$

the RHS containing known quantities only. In particular, if the nonlinearity N is the identity (i.e. unit feedback), then we can choose $F = I_m$, $k_1 = 0$, $q = 0$ and (21) reduces to

$$\lim_{k \to \infty} \sup ||y - y_A|| = 0 \qquad (23)$$

indicating that y(t) is very close to $y_A(t)$, $t \geq o$.

Finally, although stability is guaranteed and the real system behaves in a similar manner to the approximation in the closed-loop, it is important to recognize that suitable choice of tuning parameters leads to fast responses, exact tracking of steps, rejection of step disturbances and low closed-loop interaction if F is diagonal. For example, for simplicity choose $c_j = o$, $1 \leq j \leq m$, and $k_j = k$, $1 \leq j \leq m$, then the closed-loop transfer function matrix of the approximating feedback system reduces to the matrix $k/(s+k)(I_m - k^{-1}A_o^{-1}A_1)F^{-1}$ which is approximately diagonal if k is large.

Conclusions

The problem of controller design in the presence of severe plant uncertainty is a problem of immense practical importance that has traditionally been examined using differential sensitivity methods, and more recently, by the ideas of adaptive and self-tuning control. In situations where adaptive control is not desirable yet the plant model is uncertain in both dimension and parameters, the theoretical situation is still relatively underdeveloped due (presumably) to the conceptual problem of making precise predictions based on uncertain data. It is clear however [15] that 'low-gain' [3]-[5], [12], [13] and 'high-gain' [6]-[11] theories can be constructed to cover some (but not all?) situations of practical interest. Work continues in these areas.

The use of high-gain feedback systems as a means of combating uncertainty was pioneered by Bode [18] and Horowitz [19] and the use of asymptotic analysis for root-loci [7], [16] and singular perturbations [20] is now established. The contribution of this paper is to describe a framework under development by the author for the formal description and development of such notions as a systematic design tool. Theorem 2 has the typical structure obtained, providing existence, parameterization of stabilizing controllers together with an asymptotic characterization of performance that can be used to guide design work.

Finally, we note that similar results are available for sampled-data systems [8], [9] and that the results have strong connections with the parallel work of Porter [21], [22].

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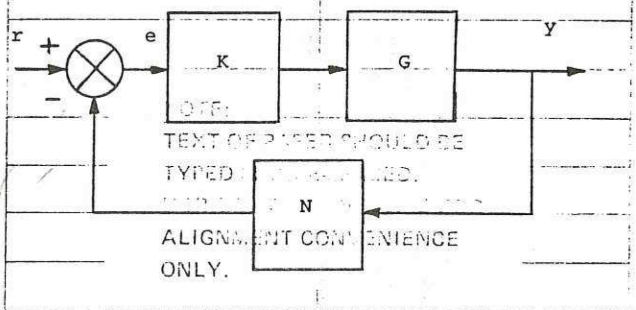

Fig. 1(a) Real Feedback System

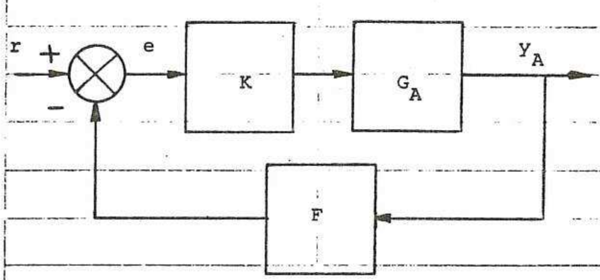

Fig. 1(b) Approximated Feedback System

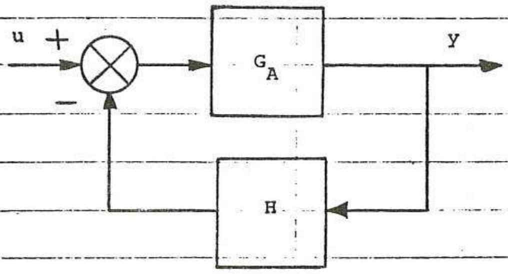

Fig. 2 Plant Decomposition