IDENTIFICATION OF NONLINEAR SYSTEMS - A SURVEY

by

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Abstract

A survey of nonlinear system identification algorithms and related topics is presented by extracting significant results from the literature and presenting these in an organised and systematic way. Algorithms based on the functional expansions of Wiener and Volterra, the identification of block oriented and bilinear systems, the selection of input signals, structure detection, parameter estimation and recent results from catastrophe theory are included. The limitations, relationships and applicability of the methods are discussed throughout.

1. Introduction

Most control systems encountered in practice are nonlinear to some extent and although it may be possible to represent systems which are perturbed over a restricted operating range by a linear model, in general, nonlinear processes can only be adequately characterized by a nonlinear model. Since a mathematical description of a process is often a prerequisite to analysis and controller design the study of system identification techniques has become an established branch of control theory. However, whereas system identification techniques for linear systems are now well established and have been widely applied, the identification of nonlinear systems has not received such attention or exposure. This can of course be attributed to the inherent complexity of nonlinear systems and the difficulty of deriving identification algorithms that can be applied to a reasonably large class of nonlinear systems.

The objective of the present study is to survey the available methods of nonlinear system identification by extracting significant results from the published literature and presenting these in an organised and systematic fashion. Wherever possible the limitations, applicability and relationships between the various algorithms are discussed. Previous surveys on this topic have concentrated on particular approaches to the problem , Arnold and Narendra[2] considered the orthogonal expansion methods, Aleksandrovskii and Deich[3], and Hung and Stark[6] reviewed the kernel identification algorithms, Simpson and Power[4] investigated correlation techniques and Mehra[7] considered parameter estimation algorithms. All these approaches are considered in the present survey together with numerous alternatives and related topics which have been developed over the last decade or so.

The survey begins by considering the functional series of Volterra and Wiener and the identification algorithms developed by Wiener, Bose and Barrett. The confusion between the two approaches studied by Wiener is clarified and the relationship between the methods is examined. The Lee and Schetzen algorithm and its derivatives, which have found wide application particularly to biological systems, is described and the limitations of these methods which have recently been reported in the literature are discussed.

Identification of the Volterra kernels is considered by reviewing the algorithms based on correlation analysis, orthogonal expansions and frequency domain methods. The selection of input signals including pseudorandom, constant switching pace symmetric and compound inputs are studied and the use of dither signals in the identification of systems containing multiple valued non-linearities is investigated.

Identification algorithms for block oriented systems, which can be represented by various interconnections of linear dynamic and static nonlinear elements, and the advantages and limitations of these methods compared to the functional series expansions are examined. Structure detection algorithms, identification of bilinear systems and parameter estimation methods using both linear and nonlinear in the parameter models are analysed. Throughout the advantages and disadvantages of the algorithms are examined and wherever possible applications of the methods are cited.

Survey articles on the related topics of quasi-linearization, the analysis of nonlinear systems and polynomic systems theory have been published by Lawrence[5], Barrett[1] and Crouch[9] respectively.

A comprehensive bibliography on Volterra series techniques, functional expansions and related subjects has been compiled by Barrett[8].

2. Functional Series Methods

The study of nonlinear functionals

$$y(t) = F[u(t'); \quad t' \leq t] \tag{1}$$

began in 1887 when Volterra[10] investigated Taylor series expansions and introduced the representation

$$y(t) = \sum_{n=1}^{\infty} \int\cdots\int_{\Omega} h_n(\tau_1,\tau_2\ldots\tau_n) \prod_{i=1}^{n} u(t-\tau_i)d\tau_i = \sum_{n=1}^{\infty} w_i(t) \tag{2}$$

which has become widely known as the Volterra series. Frechet[11] considered this representation and generalised the Weierstrass polynomial approximation theorem to show that every continuous functional F on a set of functions u which are continuous on a finite interval (a,b) can be represented by the functional series eqn (2). The functions $h_i(\tau_1,\tau_2\ldots\tau_i)$ are referred to as the Volterra kernels. The kernels are bounded and continuous in each τ_j, symmetric in their arguments and for causal systems $h_i(\tau_1,\tau_2\ldots\tau_i) = 0$ for any $\tau_i<0$. Convergence of the Volterra series for both deterministic and stochastic inputs has been studied in the literature[12-14]. Identification of nonlinear systems based on the Volterra representation requires the measurement of the kernels $h_i(\tau_1,\tau_2\ldots\tau_i)$.

2.1 Wiener method and related algorithms

Wiener[16] was one of the first authors to consider the identification of nonlinear systems and two distinct approaches were developed. There appears to be some confusion in the literature regarding these methods which in the present report will be referred to as Wiener I and Wiener II[2]. In Wiener I, Wiener applied Cameron and Martins[15] idea of representing each functional term by a Fourier-Hermite series. For the Fourier or memory portion of the expansion Wiener used Laguerre functions $\ell_p(t)$ which form a complete orthonormal set on $[0,\infty)$. Using these functions the past of the input can be represented by the coefficients

$$V_p(t) = \int_0^\infty \ell_p(\tau)u(t-\tau)d\tau \quad , \qquad p = 0,1... \tag{3}$$

This is followed by a Hermite expansion in the Laguerre coefficients to yield the system output

$$y(t) = \lim_{n\to\infty} \sum_{m_o=0}^{\infty} \sum_{m_1=0}^{\infty} \cdots \sum_{m_n=0}^{\infty} C_{m_o m_1 \ldots m_n} H_{m_o}(V_o(t)) H_{m_1}(V_1(t)) \cdots H_{m_n}(V_n(t)) \tag{4}$$

where $H_k(\cdot)$ denotes the partially normalised Hermite polynomials defined by $H_k(u) = (-1)^k 2^{-k/2} \frac{e^{u^2}}{\sqrt{k!}} \frac{d^k}{du^k}\{e^{-u^2}\}$ and the Wiener coefficients $C_{m_o \ldots m_n}$ are determined by computing the time average[16,17]

$$C_{m_o m_1 \ldots m_n} = \overline{y(t) H_{m_o}(V_o(t)) \ldots H_{m_n}(V_n(t))} \tag{5}$$

for a white Gaussian input u(t). The synthesis of a nonlinear system

using Wieners method can be visualised as a linear system with multiple outputs representing the expansion of the past of the input in terms of Laguerre functions, in cascade with a nonlinear no-memory system representing the Hermite polynomials followed by a network of amplifiers and adders representing the coefficients as illustrated in Fig.1. Although Wiener does not mention Volterra's work anywhere in his book[16] it is easy to show that Wiener's characterization is equivalent to an expansion of the Volterra type[20].

Whilst Wiener's formulation is very elegant theoretically it is impractical and difficult to apply because of the excessive number of coefficients required. If n Laguerre coefficients are used to describe the past of the input and p coefficients to expand the system functional in Hermite polynomials then p^n coefficients are required to characterise the system[18]. Thus identification of even a simple system containing a second order nonlinearity would require the evaluation of typically 10^{10} coefficients.

As an alternative to the Hermite polynomials in the Wiener I method Bose[19] partitioned the function space of the past of the input into nonoverlapping cells by introducing gate functions

$$Q_k(v) = \begin{cases} 1 & \text{if } v \text{ is in the k'th subinterval} \\ 0 & \text{otherwise} \end{cases}$$

Analogous to eqn (4), Bose's representation then becomes

$$y(t) = \lim_{\substack{n\to\infty \\ m\to\infty}} \sum_{k_o=1}^{m} \sum_{k_1=1}^{m} \cdots \sum_{k_n=1}^{m} D_{k_o k_1 \ldots k_n} Q_{k_o}(V_o(t)).Q_{k_1}(V_1(t)) .Q_{k_n}(V_n(t)) \qquad (6)$$

where the coefficients can be identified as

$$D_{k_o k_1 \ldots k_n} = \frac{\overline{y(t) Q_{k_o}(V_o(t)) \ldots Q_{k_n}(V_n(t))}}{\overline{Q_{k_o}(V_o(t)) Q_{k_1}(V_1(t)) \ldots Q_{k_n}(V_n(t))}} \qquad (7)$$

for any ergodic process with suitable bandwidth as input. Unfortunately the Bose series is not a true orthogonal series since terms cannot be added to the series and the coefficients evaluated without changing the previously obtained coefficient values. An interesting application of Bose's method has been reported by Pincock and Atherton[21].

Barrett[22] used the multivariable Hermite functionals $\mathcal{H}^n[u]$ ($\mathcal{H}^{(0)}[u(t)] = 1$; $\mathcal{H}^{(1)}[u(t)] = u(t)$; $\mathcal{H}^{(2)}[u(t_1), u(t_2)] = u(t_1)u(t_2) - \delta(t_2 - t_1)$, etc), derived from the Hermite polynomials given by Grad, to represent the system eqn (1) by the functional series

$$y(t) = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{-\infty}^{\infty} \ldots \int k_n(t; \tau_1, \tau_2 \ldots \tau_n) \mathcal{H}^{(u)}[u; \tau_1, \tau_2 \ldots \tau_n] d\tau_1 d\tau_2 \ldots d\tau_n \qquad (8)$$

where the kernels can be determined by the averages

$$k_n(t; \tau_1, \ldots \tau_n) = \overline{y(t) \mathcal{H}^{(n)}[u; \tau_1, \tau_2 \ldots \tau_n]} \qquad (9)$$

for a white Gaussian input process. Barretts method is essentially the same as the Wiener II algorithm outlined below. The method can be generalised to coloured Gaussian inputs but the resulting set of simultaneous integral equations is extremely difficult to solve.

The Wiener II method[16,23] will be described by interpreting Wiener's differential of a Brownian motion function as a sample function from a white Gaussian process. Wiener used a Gram-Schmidt orthogonalisation procedure to construct a new functional series

$$y(t) = F[u(t');t' \leq t] = \sum_{n=0}^{\infty} [G_n(k_n,u(t))] \tag{10}$$

where the functionals $\{G_n\}$ are orthogonal for a Gaussian white stimulus. The first few terms of the Wiener functional series are

$$\begin{aligned}
G_1[k_1,u(t)] &= \int_{-\infty}^{\infty} k_1(\tau)u(t-\tau)d\tau \\
G_2[k_2,u(t)] &= \iint_{-\infty}^{\infty} k_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 - P\int_{-\infty}^{\infty} k_2(\tau_1,\tau_1)d\tau_1 \\
G_3[k_3,u(t)] &= \iiint_{-\infty}^{\infty} k_3(\tau_1,\tau_2,\tau_3)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 \\
&\quad - 3P\iiint_{-\infty}^{\infty} k_3(\tau_1,\tau_1,\tau_2)u(t-\tau_2)d\tau_1 d\tau_2
\end{aligned} \tag{11}$$

where P is the power spectral density of the white noise input. The n'th order Wiener functional is not homogeneous and thus for instance the linear Wiener kernel does not represent the whole linear part of the system. In general the Wiener kernels are not equal to the Volterra kernels and relationships between the two expansions have been studied by Yasui[24].

Identification of a nonlinear system based on Wiener's series involves the measurement of the kernels k_n in eqn (11). Wiener[6,16] expanded the kernels in a series of Laguerre functions

$$k_n(\tau_1,\tau_2\ldots\tau_n) = \sum_{m_o=0}^{\infty} \cdots \sum_{m_n=0}^{\infty} C_{m_o\ldots m_n} \ell_{m_o}(\tau_1)\ldots\ell_{m_n}(\tau_n) \tag{12}$$

where

$$C_{m_o m_1 .. m_n} = \int_{-\infty}^{\infty} .. \int k_n(\tau_1, \tau_2 .. \tau_n) \ell_{m_o}(\tau_1) ... \ell_{m_n}(\tau_n) d\tau_1 .. d\tau_n \tag{13}$$

To determine the coefficients $C_{m_o m_1 .. m_n}$ a system A is constructed in such a manner that its output $y_a(t)$ for a white Gaussian input is $G_n[\kappa_n, u(t)]$ in which the kernel of the leading term is $\kappa_n(\tau_1, \tau_2 .. \tau_n) = \ell_{m_o}(\tau_1) ... \ell_{m_n}(\tau_n)$. This system is connected to the unknown system B with output $y_b(t)$ as illustrated in Fig.2. The average value of the product of the output can be shown to be

$$\overline{y_a(t) y_b(t)} = n! P^n \int_{-\infty}^{\infty} .. \int k_n(\tau_1, .. \tau_n) \ell_{m_o}(\tau_1) .. \ell_{m_n}(\tau_n) d\tau_1 .. d\tau_n \tag{14}$$

Since the integral in eqn (14) is in the form of eqn (13) the coefficient $C_{m_o .. m_n}$ is given by

$$C_{m_o .. m_n} = \frac{1}{n! P^n} \overline{y_a(t) y_b(t)} \tag{15}$$

This procedure is repeated for all values of $m_o .. m_n = 0,1,2...$ The coefficients in Wiener's functional series eqn (11) are the coefficients of the Hermite polynomials such that eqn (10) can be rewritten as

$$y(t) = \sum_{n=0}^{\infty} [G_n(k_n, u(t))] = \sum_{n=0}^{\infty} H_n[k_n, u(t)] \tag{16}$$

With this interpretation, and using Laguerre functions as base functions eqn (16) becomes equal to eqn (4) and the equivalence of both the Wiener methods is established. The excessive computations associated with the Wiener I method are therefore retained. This coupled with the fact that even a linear system is characterized in

a very cumbersome way, the difficulty of incorporating a priori information and the problems of using the complex identified model for prediction have resulted in very few applications[17,25,26] of Wiener's technique.

French and Butz[27] have developed an algorithm based on the expansion of the Wiener kernels in terms of Walsh functions. The nonlinear system is described in terms of a set of kernels which contain the dyadic convolution operation and identification is performed using the fast Walsh-Fourier transform.

The functional series representation and Wiener's ideas have been studied by several authors notably, Barrett[1], Bose[19], Brilliant[28], Flake[29], George[30], Harris[31], Singleton[32], Yasui[24,33] and Zadeh[34]. Numerous other contributions are contained in the bibliography compiled by Barrett[8]. Other integral operators including the Hammerstein and Uryson operators[2] are discussed in section 4.

An alternative method of measuring the Wiener kernels k_n of a nonlinear system was developed by Lee and Schetzen[35,70] using correlation techniques and a white Gaussian input process. The procedure consists of computing multidimensional correlation functions between the white Gaussian input and the system output to yield

$$k_n(\tau_1,\ldots\tau_n) = \frac{1}{n!P}\{y(t) - \sum_{m=0}^{n-1} G_m[k_m,u(t)]\}u(t-\tau_1)\ldots u(t-\tau_n)$$

$$\tau_1,\tau_2\cdots\tau_n \qquad (17)$$

The second term on the rhs of eqn (17) only has a value on the diagonal and is included to remove impulse functions which would otherwise appear when $\tau_1 = \tau_2 \cdot\cdot = \tau_n$. The algorithm is illustrated

schematically in Fig.3. Although the method removes many of the difficulties associated with the Wiener formulation the amount of computation required can still be excessive. For a single input system the n'th order kernel must be estimated at $((n+m-1)!)/(n!(m-1)!)$ points where $m = \mu/\Delta t$, μ is the system memory and Δt the sampling interval[36]. Computing time therefore increases almost exponentially with the order of the kernel to be evaluated.

The estimation error associated with eqn (17) will be more severe at the diagonal points because of the presence of low order integral terms and it has been suggested that more accurate kernel estimates at these points can be obtained by interpolation between the nondiagonal points rather than direct estimation. The method has been widely applied[6] notably to biological[36-43] and structural[44,45] systems. Most of the practical applications have however been restricted to quadratic systems and it has recently been suggested[46-48] that the excessive errors associated with the diagonal kernel estimates in the continuous time formulation introduce fundamental difficulties in the identification of third and higher order kernels. These problems can however be alleviated by using appropriate discrete stochastic inputs. Choi and Warren[49] have recently derived a discrete formulation of the Lee and Schetzen algorithm for discrete input processes in both the time and frequency domain. Palm and Poggio[46-48] provide a rigorous analysis of the Lee and Schetzen algorithm and have shown that Wiener's original formulation based on Brownian motion inputs includes a larger class of systems than the Lee and Schetzen method but the difficulty can easily be avoided. A comprehensive study of the Lee and Schetzen method including a full analysis of estimation errors has been given by Marmarelis and Marmarelis[37].

Krause[50] developed an identification method based on the Lee and Schetzen procedure but using a random impulse train (poisson process) as input rather than a white Gaussian stimulus. The method is especially applicable to neuronal synaptic systems but would be very difficult to apply to an industrial process.

2.2 Volterra series methods

Solution of the identification problem based on the Volterra series involves measurement of the Volterra kernels[2]. To illustrate the approach consider the identification of a system which can be described by just the first two Volterra kernels

$$y(t) = \int_0^\infty h_1(\tau_1)u(t-\tau_1)d\tau_1 + \int_0^\infty\int h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \tag{18}$$

Defining the mean squared error as $E\{(z(t)-y(t))^2\}$ where $z(t)$ is the measured output and applying Calculus of Variations yields

$$\bar{z}(t) = \int_0^\infty h_1(\tau_1)\overline{u(t-\tau_1)} + \int_0^\infty\int h_2(\tau_1,\tau_2)\overline{u(t-\tau_1)u(t-\tau_2)}d\tau_1 d\tau_2 \tag{19}$$

$$\overline{z(t)u(t-\sigma)} = \int_0^\infty h_1(\tau_1)\overline{u(t-\tau_1)u(t-\sigma)}d\tau_1 + \int_0^\infty\int h_2(\tau_1,\tau_2)\overline{u(t-\tau_1)u(t-\tau_2)u(t-\sigma)}d\tau_1 d\tau_2 \tag{20}$$

$$\overline{z(t)u(t-\sigma_1)u(t-\sigma_2)} = \int_0^\infty h_1(\tau_1)\overline{u(t-\tau_1)u(t-\sigma_1)u(t-\sigma_2)}d\tau_1 + \int_0^\infty\int h_2(\tau_1,\tau_2)\overline{u(t-\tau_1)u(t-\tau_2)u(t-\sigma_1)u(t-\sigma_2)}d\tau_1 d\tau_2 \tag{21}$$

The solution of this set of equations for a general stochastic input is extremely difficult. Katzenelson and Gould[51] describe an iterative procedure of optimisation and successive substitution. Hsieh[52] proposed a gradient technique and Alper[53] and Eykhoff[54] considered the discrete-time version. If the system input can be selected to be white Gaussian then the system of integral eqns reduces to

$$\begin{aligned} \bar{z}(t) &= \int_0^\infty h_2(\tau,\tau)\,d\tau \\ \overline{z(t)u(t-\sigma_1)} &= h_1(\sigma_1) \\ \overline{z(t)u(t-\sigma_1)u(t-\sigma_2)} &= \bar{z}\delta(\sigma_1-\sigma_2)+2h_2(\sigma_1,\sigma_2) \end{aligned} \tag{22}$$

and the solution is direct providing the mean level $\bar{z}$ is removed. For systems which involve higher than second order kernels even this specialised case offers considerable difficulties. Identification of the kernels using multidimensional step responses has been examined by Schetzen[61].

Another common approach is to approximate the kernels by an expansion of orthogonal functions

$$h_n(\tau_1,\ldots\tau_n) = \sum_{n=0}^{N}\sum_{m=0}^{N}\cdots\sum_{k=0}^{N} a_{nm..k}\phi_n(\tau_1)\,..\,\phi_k(\tau_n) \tag{23}$$

Methods of solution for the coefficients $a_{nm..k}$ include gradient type algorithms[55-59] and pattern recognition techniques[60].

Korenberg[62] has considered the identification of differential systems having a Volterra series expansion using a slowly exponentially decaying sum of sinusoids as input. The terms of the differential expansion are determined orthogonally using linear regression and simple averaging procedures. A method of directly identifying the Volterra kernels using an input $u(t) = e^{-at}r(t)$ where $r(t)$ is a

bounded zero mean independent process was also developed by Korenberg[63]. The identification is orthogonal and the kernels are obtained by an averaging procedure. Fakhouri[64] has developed an algorithm for the identification of the discrete Volterra kernels in terms of multidimensional z-transforms using high order correlation functions and coloured Gaussian inputs.

An excellent review of the theory and applications of the kernel identification methods has been compiled by Hung and Stark[6]. Interpretation of the identified kernels has been analysed by Hung, Stark and Eykhoff[65].

Volterra series have been widely applied in the analysis of communication systems and several authors, Bedrosian and Rice[66], Brilliant[28], Barrett[1], Bussgang, Ehrman and Graham[67], Narayanan[68] and Zames[69] have contributed to this area. The use of functionals in nonlinear analysis has also been studied extensively[1,71-75].

2.3 Frequency Domain Techniques

The kernels in the Volterra series eqn (2) can be expressed in terms of multidimensional Laplace[30] or Fourier transfer functions. Brillinger[76] showed that an asymptotically unbiased estimate of the isolated n'th degree frequency domain transfer function is given by

$$\hat{S}_n(\lambda_1,\ldots\lambda_n) = \frac{\hat{f}_{u\ldots uy}(-\lambda_1,\ldots-\lambda_n)}{n!\hat{f}_{uu}(\lambda_1)\ldots\hat{f}_{uu}(\lambda_n)} \qquad (24)$$

where $f_{uu}(\lambda)$ is the power spectral density of the input and $f_{u\ldots uy}(-\lambda_1,..-\lambda_n)$ is the cumulant spectrum of order n+1.

Feueruerger and Huber et al[78] have also contributed to the theory and computation of higher order spectra. French and Butz[79] developed a frequency domain method of measuring the Wiener kernels, by substituting complex exponential filters in place of the Laguerre functions and, using an FFT algorithm. The method is analogous to the Lee and Schetzen algorithm in the time domain but results in a considerable reduction in the computational requirement. A schematic diagram of the estimation procedure for the second degree kernel is illustrated in Fig.4. Recently Barker and Davy[80] have shown that estimates of the first two Volterra kernels can be computed using Fourier transform techniques with a pseudo-random ternary input.

2.4 Input Signals

In the search for methods of simplifying the measurement techniques and reducing the number of data points and computations required to identify either the Volterra or Wiener kernels numerous authors have investigated quasi-white and pseudo-random inputs as alternatives to white noise. Almost without exception all the authors consider the identification of just the first two kernels and only this case will be studied here. The technique used is based on computing the first and second degree correlation functions to yield expressions analogous to eqn's (20) and (21). If the input is antisymmetric the odd order averages tend to zero and eqn's (20) and (21) reduce to

$$\phi_{uz}(\sigma) = \int_0^\infty h_1(\tau_1)\overline{u(t-\tau_1)u(t-\sigma)}d\tau_1 \qquad (25)$$

$$\phi_{uuz}(\sigma_1,\sigma_2) = \iint_0^\infty h_2(\tau_1,\tau_2)\overline{u(t-\tau_1)u(t-\tau_2)u(t-\sigma_1)u(t-\sigma_2)}\, d\tau_1 d\tau_2 \qquad (26)$$

Estimates of the kernels $h_1(\sigma)$, $h_2(\sigma_1,\sigma_2)$ can be obtained directly for a Gaussian white simulus u(t) since eqn's (25) and (26) reduce to the form of eqn (22) for this input.

Hooper and Gyftopoulos[81] first described the practical measurement of a second order Volterra kernel by cross-correlation using a ternary m-sequence. Although the identification time using these inputs was reduced by a factor of 70, compared with a Gaussian white input, anomalies appeared in the fourth order autocorrelation functions, eqn (26), which Hooper and Gyftopoulos could not explain. Similar anomalies were also observed by Simpson[82] in the fourth order autocorrelation function of pseudo-random inverse repeat sequences. Ream[83] investigated these anomalies, and Barker and Pradisthayon[84] showed that the non-zero values in the higher than second order autocorrelation functions of m-sequences are due to deterministic characteristics of these sequences. Barker, Obidegwu and Pradisthayon[25] noted that some pseudorandom signals are more suitable than others for the measurement of the second order Volterra kernel and proposed a criterion for input selection. The selection of suitable input sequences was later studied by Kadri and Lamb[86], and Barker et al[87,88]. The problem of anomalies can however by completely avoided even for prbs inputs if the compound input method[89] is used. Consider the identification of a system which can be described by just the first two terms of the Volterra series eqn (2). For a compound pseudorandom input $u(t) = x_1(t)+x_2(t)$ where $\phi_{x_i x_i}(\lambda) = \delta(\lambda)$, $\phi_{x_1 x_2}(\lambda) = 0 \forall \lambda$ unbiased estimates of the system kernels can be determined by computing

$$\phi_{x_1 x_2 y}(\sigma_1,\sigma_2) = h_2(\sigma_1,\sigma_2) \tag{27}$$

$$\phi_{x_1 z}(\sigma) = h_1(\sigma) \tag{28}$$

where $z(t) = y(t)-\hat{w}_2(t)$ and $\hat{w}_2(t)$ the output of the second order kernel is computed from the estimate of eqn (27). The technique can be applied to estimate the kernels in a general n'th order Volterra expansion[89] but this involves the use of multilevel compound inputs for n>2. Most authors avoid this problem by treating only finite second order Volterra systems. However, if the system kernels are factorable[90] all the kernels can be identified sequentially from a single level compound input[91]. For example if the input to the factorable Volterra system illustrated in Fig.5 is the compound signal $u(t) = \sum_{j=1}^{k} x_j(t)$ where $\bar{x}_i = 0$, $\phi_{x_i x_i}(\lambda) = \beta_i \delta(\lambda)$ and the x_i's are independent then

$$\phi_{x_1 \ldots x_k y'}(\sigma_1,\sigma \ldots \sigma) = \overline{(y(t)-\overline{y(t)})x_1(t-\sigma_1)} \prod_{i=2}^{k} x_i(t-\sigma)$$

$$= \phi_{x_1 \ldots x_k w'_k}(\sigma_1,\sigma)$$

$$= (k-1)!\left(\prod_{n=1}^{k} \beta_n\right) \sum_{i=1}^{k} \{h_{i,k}(\sigma_i) \prod_{\substack{j=1 \\ j\neq i}}^{k} h_{j,k}(\sigma)\} \tag{29}$$

Hence the second degree correlation function $\phi_{x_1 \ldots x_k y'}(\sigma_1,\sigma \ldots \sigma)$ between the input and output reduces to yield an estimate of the highest order kernel dynamics. This result holds exactly even for a compound prbs input. Estimates of the linear subsystems $h_{j,i}(t)$ can be readily computed using a Marquardt algorithm[91].

The linear systems associated with the remaining kernels can be identified by continuing this procedure.

Recently Marmarelis[37,92] has introduced CRNS (constant switching pace symmetric random signals) as an alternative to band limited Gaussian white noise and has applied these inputs to the identification of various biological systems. Although the autocorrelation functions of these signals exhibit fluctuations over the whole argument space, Marmarelis has shown that CRNS result in a more acceptable fourth order autocorrelation function compared to pseudorandom inputs[93].

The generality of the functional series methods and the wide class of systems which can be characterised by the expansions are distinct advantages of this approach and several authors[6,17,25,26,36-45] have applied these methods to hydrological, physiological, structural and electrical systems. Most of these authors have however considered the identification of just the first two kernels and this indicates the major disadvantage of the functional series approach. For example to obtain a reasonable approximation to the first order kernel requires the estimation of approximately 40 points. The second order kernel will require the estimation of (40x40)/2 points, taking account of symmetry, the third order kernel (40x40x40)/3 etc. Considering that the number of input/output pairs recorded from the process must be well in excess of the points to be estimated if any reasonable amount of smoothing is to be obtained then it is easy to see the impracticalities of this approach irrespective of which algorithm is implemented. Additional problems include the difficulty of applying the Lee and Schetzen[35] method to estimate third order kernels as discussed above, and

the necessity of using multilevel inputs to isolate the non-orthogonal Volterra kernels when systems containing higher than second order kernels are considered. Identification of factorable Volterra systems[91] using compound inputs alleviates this latter problem but only if independent inputs which are difficult to generate[94] are used. The difficulty of incorporating a priori information, of interpreting the kernels estimated and relating the results to the physical system are further disadvantages of the functional series methods.

3. Multiple-valued Nonlinearities

The difficult problem of identifying systems which contain double-valued nonlinearities such as hysteresis have been studied by only a few authors. It is easy to see that the Volterra series eqn (2) cannot represent systems in this class since the characteristic subharmonics associated with memory type nonlinearities are not generated by the Volterra expansion. Identification of simple systems in this class has been achieved by superimposing a dither or high frequency signal on the desired input to quench undesirable jump phenomena and induce continuity[95,96]. If dither can be injected into the system to change the effective characteristic of the memory-type nonlinearity to a single valued function the identification techniques described above can be applied under certain conditions. Alternatively special inputs must be devised to traverse around the characteristic in some pre-determined manner[97]. Conditions under which dither extinguishes or quenches jump phenomena have been rigorously studied by Zames and Schneydor[98].

4. Block Oriented Systems

In an attempt to reduce the computational burden associated with the functional series methods various authors have considered the identification of block oriented systems which can be represented by interconnections of linear dynamic systems and static nonlinear elements. The basic philosophy underlying this approach has been to avoid a black box description by identifying these systems in terms of the individual elements in a manner which preserves the system structure and provides valuable information for control. Probably the most studied system within this class is the cascade system composed of a linear system followed by a nonlinear element in cascade with another linear system known as the general model illustrated in Fig.6. In all the systems considered it is assumed that all the internal signals, such as $x(t)$ and $q(t)$ in Fig.6 are not accessible for measurement. Gardiner[99] and Economakos[100] have suggested methods of identifying the linear kernel associated with this system by injecting multilevel inputs and isolating the kernel outputs. Identification of the higher order kernels was considered by Webb[101] using multilevel single frequency tests and by Sandor and Williamson[102] using tensor techniques. Unfortunately these methods will often result in an excessive experimentation time which may be prohibitive in an industrial environment.

Korenberg[104] expanded the output $y_2(t)$ in a Volterra series and showed that

$$\phi_{u_2y_2}(\sigma) = C_1 \int_0^\infty h_2(\alpha)h_1(\sigma-\alpha)d\alpha \tag{30}$$

$$\phi_{u_2u_2y_2}(\sigma_1,\sigma_2) = C_2 \int_0^\infty h_2(\alpha)h_1(\sigma_1-\alpha)h_1(\sigma_2-\alpha)d\alpha + \overline{y(t)}\ (\sigma_1-\sigma_2) \tag{31}$$

for a white Gaussian input $u_2(t)$. By taking Fourier transforms of eqn (30) and (31) Korenberg solved for the gain and phase characteristics of $h_1(t)$ and $h_2(t)$ and used these estimates to graph the nonlinearity. If the nonlinearity is even $\phi_{u_2y_2}(\sigma) = 0$, and if it is odd $\phi_{u_2u_2y_2}(\sigma_1,\sigma_2) = 0$ and in these cases the higher order correlations $\overline{u_2(t-\sigma)u_2(t-1)y_2(t)}$ and $\overline{u_2(t-\sigma_1)u_2(t-\sigma_2)u(t-1)y_2(t)}$ respectively, must be computed. Korenberg[104] also considered the identification of higher order cascades using multidimensional correlation functions and an exponentially decaying random input. De Boer[105] investigated the result of eqn (30) and developed an algorithm using a regression function method[106] based on a Hermite polynomial expansion and correlation analysis. Goldberg and Durling[107] used a conjugate gradient search to identify the elements in a cascade connection of a linear subsystem sandwiched between two nonlinear elements.

Other authors have considered subclasses of the general model referred to as the Wiener[108,109] model obtained by setting $h_2(t) = \delta(t)$ in Fig.6, and the Hammerstein model[110] obtained by setting $h_1(t) = \delta(t)$ in Fig.6. The Hammerstein model represents a realization of the Hammerstein operator

$$H_h\left[u(t)\right] = \int h(t,\tau)F\left[\tau,u(\tau)\right]d\tau \tag{32}$$

and was originally proposed by Narendra and Gallman[110]. The input/output relationship for this model is particularly simple

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})} F[u(k)] \tag{33}$$

and numerous authors have developed identification algorithms by extending linear estimation techniques[110-115].

The Uryson model is a realisation of the Uryson integral operator

$$\mathcal{U}[u(t)] = \int K[t,\tau,u(t)]d\tau \tag{34}$$

and was introduced by Gallman[116] who used Hermite polynomials $H_i(\cdot)$ to represent the nonlinearity and reduced the identification procedure to a multi-input single output problem as illustrated in Fig.7. The Uryson model consists of several Hammerstein models in parallel.

Many authors have developed specialised identification algorithms for other systems within this class; Brown[117] and Simpson and Power[118] considered feedforward systems, Lawrence[119] and Economakos[120] analysed feedback systems, Godfrey and Briggs[121] studied processes with direction dependent responses, and Baumgartner and Rugh[122], Wysocki and Rugh[123] developed algorithms for the S_m model.

Recently the separable class of random processes which were introduced by Nuttall[124] and studied by Balasubramanian and Atherton[125], West[126], Douce[127] and Yuen[128] were used to formulate a unified identification procedure for most of the system structures

mentioned above[129]. To illustrate the procedure consider the general model in Fig.6. Billings and Fakhouri[130,131] have extended the results of separable processes to show that if $u_1(t)$ is separable with respect to $u_2(t)$ (the Gaussian processes, and sine wave process are separable) and separability is preserved under linear and double nonlinear transformation then

$$\phi_{u_1 y_2}(\sigma) = C_{FG}\int\int h_2(\theta)h_1(\tau_1)\phi_{u_1 u_2}(\sigma-\theta-\tau_1)d\theta d\tau_1 \tag{35}$$

$$\phi_{u_1^2 y_2}(\sigma) = C_{FFG}\int\int\int h_2(\theta)h_1(\tau_1)h_1(\tau_2)$$

$$\overline{u_2(t-\theta-\tau_1)u_2(t-\theta-\tau_2)u_1^2(t-\sigma)}d\tau_1 d\tau_2 d\theta \tag{36}$$

For the special case when $u_1(t) = u(t)$, $u_2(t) = u(t)+b$ where $u(t)$ is a zero mean white Gaussian process and b is a non-zero mean level eqn's (35) and (36) reduce to

$$\phi_{u y_2'}(\sigma) = C_{FG}\int h_1(\tau_1)h_2(\sigma-\tau_1)d\tau_1 \tag{37}$$

$$\phi_{u^2 y_2'}(\sigma) = C_{FFG}\int h_2(\tau_1)h_1^2(\sigma-\tau_1)d\tau_1 \tag{38}$$

where C_{FG} and C_{FFG} are constants. Equations (35) to (38)[130,131] represent a generalisation of Korenbergs[103] results eqn's (30) and (31). The first order correlation functions of eqn's (37) and (38) exist for all continuous single valued nonlinearities and estimates of the linear subsystems $h_1(t)$ and $h_2(t)$ can be obtained independently of $F[\cdot]$ using a multistage least squares algorithm operating directly on these estimates[130]. Because the system is identified in terms of the individual elements $h_1(t)$, $h_2(t)$ and $F[\cdot]$ even systems containing very violent nonlinearities such as

saturation and deadzone can be readily identified[132]. Eqn's (37) and (38) in fact represent estimates of the first two Volterra kernels showing that under the theory of separable processes all the terms in the Volterra expansion collapse to the form of eqn's (37) and (38). The results can be applied to the identification of the Hammerstein[115], Wiener[108], feedforward[115], feedback[133] and S_m systems[134], are unbiased in the presence of noise and can be implemented for Gaussian non-white inputs using typically 4000 data pairs[135].

Identification of multiplicative connections of linear dynamic systems[91] have been discussed in section 4 under the heading of factorable Volterra systems. Haber and Keviczky[136] give a comprehensive summary of many nonlinear model structures.

Applications of the algorithms described above are difficult[137,138] to find although most of the authors have considered the identification of simulated systems. It is interesting to note however that many of the applications of the functional series methods have involved the identification of systems which are block oriented, and which can be represented by simple cascade connections of linear and nonlinear blocks.

5. Structure Detection

If the algorithms for restricted classes of nonlinear systems outlined above are to be implemented it is often necessary to determine the structural form, or type of model representation which best approximates to the process prior to the identification. Information regarding the structure of simple cascade systems is inherent in the identification results based on separable

processes[129,131] eqns (37) and (38). If the system under investigation is linear then $\phi_{uy_2'}(\sigma)$ is given by eqn (37) and $\phi_{u^2y_2'}(\sigma) = 0 \forall \sigma$. Thus the first degree correlation function yields an estimate of $h_1(t)*h_2(t)$ and the second degree correlation function provides a convenient test of linearity. If the first and second degree correlation functions, eqn (37) and (38) are equal except for a constant of proportionality the system must have the structure of the Hammerstein model ($h_1(t) = \delta(t)$). However, if the second degree correlation function is the square of the first degree correlation function, except for a constant of proportionality, the system has the structure of a Wiener model ($h_2(t) = \delta(t)$). The structure of higher order cascade systems can be detected using an algorithm by Douce[139] illustrated in Fig.8 for the general model. Douce has shown that if a linear model $\hat{g}(t)$ is fitted between u(t) and y(t), Fig.8, the input u(t) and residual e(t) are uncorrelated[126] when the input is a separable process even though e(t) contains components due to noise and the distortion in $F[\cdot]$. The nonlinear distortion can however be detected by correlating the residual e(t) with a test signal $z_1(t)$ obtained by passing the Gaussian input u(t) through selected Hermitian polynomials $H_i(\cdot)$. If the correlation $\phi_{z_1e}(\sigma)$ is significant then the coefficients associated with the Hermitian representation of the nonlinear characteristic can be measured and the position of the nonlinearity or structure of the system can be detected by a simple ordering process.

A measure of the degree of nonlinearity was introduced by Rajbman[140,141] using dispersion functions. The cross-dispersion function which is defined in terms of the conditional mean

$$\theta_{yu}(t_1,t_2) = E\left[\{E\left[y(t_1)\middle|u(t_2)\right] - E\left[y(t_1)\right]\}^2\right] \qquad (39)$$

was introduced to measure the nonlinear relationship between signals which cannot be detected using linear correlation methods. Unfortunately dispersion functions are difficult to compute and similar information can be obtained by evaluating $\phi_{u^2y^2}(\sigma)$[131] as defined in eqn's (36) and (38) or using Douce's algorithm[139], both of which are much simpler measures of nonlinear effects.

Saridis and Hofstadter[142,143] have investigated the classification of nonlinear stochastic systems using pattern recognition techniques based on auto and cross covariance functions. Experimental results indicate that classification of unknown nonlinear systems with respect to basic structural properties can be accomplished.

6. Bilinear Systems

Bilinear systems

$$\begin{aligned} \underline{x}^{o} &= A\underline{x} + B\underline{x}u + \underline{C}u \\ y &= \underline{d}x \end{aligned} \qquad (40)$$

have received a great deal of attention recently and algorithms for the identification of both stochastic and deterministic bilinear systems have been proposed. Balakrishnan[144] and Bruni, Di Pillo and Koch[145] derived algorithms using maximum likelihood techniques. Baheti and Mohler[146] applied correlation analysis in conjunction with least squares, Baghelli and Guidorzi[147] used a simple least squares

estimator based on an input/output expansion and Karanam, Frick and Mohler[148] developed an algorithm using Walsh functions. Baheti, Mohler and Spang[149] considered the identification of the first two kernels in the Volterra series expansion of a bilinear system

$$y(k) = \Delta t \sum_{i=1}^{\infty} \omega_1(i)u(k-i) + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \omega_2(i,j)u(k-i)u(k-j) + \text{higher order terms} \tag{41}$$

$$\omega_1(i) = \frac{1}{\Delta t} \int_{(i-1)\Delta t}^{i\Delta t} \underline{d} e^{At_1} \underline{c} dt_1 \tag{42}$$

$$\omega_2(i,j) = \frac{1}{(\Delta t)^2} \int_{(i-1)\Delta t}^{i\Delta t} \int_{(j-1)\Delta t}^{j\Delta t} \underline{d}\, e^{At_1} B_{(i-1)\Delta t (j-1)\Delta t} e^{A(t_2-t_1)} \underline{c}\, \mathcal{S}(t_2-t_1) dt_2 dt_1 \tag{43}$$

where $\mathcal{S}(\cdot)$ is the unit step function. Estimates of the first two kernels were obtained using correlation analysis and a pseudo-random ternary input

$$\hat{\omega}_1(i) = \frac{3}{2\Delta t} \phi_{yu}(i) \tag{44}$$

$$\hat{\omega}_2(i,j) = \frac{9}{4(\Delta t)^2} \phi_{yuu}(i,j) \tag{45}$$

Errors in the estimate of the second order kernel eqn (45) were attributed to the presence of higher order terms but no mention was made of the anomalies associated with the higher order auto-correlation functions of the ternary input[82-88] (section 2.4).

Identification of time invariant bilinear systems with zero initial conditions and directly observable states was also considered using the Wiener-Hopf type equation

$$\underline{x}(t) = \int_0^\infty e^{At_1} \{\underline{c}\, u(t-t_1)\}dt_1 + \int_0^\infty \{e^{At_1} B\underline{x}(t-t_1)u(t-t_1)\}dt_1 \qquad (46)$$

and correlation methods[149]. Unfortunately, most of these algorithms appear to be either very difficult to implement or apply only to a simplified class of bilinear systems[150] and further research is needed in this area.

7. Parameter Estimation

Parameter estimation methods for nonlinear systems can be classified according to the model structure and are based on either linear or nonlinear-in-the-parameter models. The choice between these two approaches is often dictated by the process under investigation. If the structural form of the describing differential equation is known parameter estimation algorithms can be applied directly to estimate the unknown parameters. When little a priori information is available and the process is treated as a black-box the usual approach is to expand the input/output using a suitable model representation which is usually selected to be nonlinear in the input and output variables but linear in the parameters.

A review of estimation methods for nonlinear-in-the-parameter models

$$\dot{x} = f(u,x,\beta)$$
$$y = h(u,x) \tag{47}$$

has been compiled by Seinfeld[151]. Recent developments in this field include a combined smoothing and parameter estimation algorithm[152] which compensates for uncertain model structure and external disturbances by introducing time varying parameters into the model, and a recursive algorithm which can include data reuse[153].

Estimation using linear in the parameter models has been based largely on the Hammerstein model[110-115] and discrete Volterra series[53-55] discussed in Sections 2.2 and 4 above. Other authors have considered polynomic expansions of the system states. Garg and Boziuk[154] assumed that all the system states are measured without noise and used a least squares algorithm to estimate the parameters C_{n_i} in the expansion

$$y(t) = \sum_{i=1}^{f} \sum_{n=n_1}^{n_m} x_i^{n}(t) C_{n_i} \tag{48}$$

Netravali and De Figuerredo[155] assumed that all the system states are observable and applied a stochastic approximation algorithm using the expansion

$$x_{k+1} = g(x_k) + u_k$$
$$y_{k+1} = x_{k+1} + v_{k+1} \tag{49}$$

where v_{k+1} represents additive noise and

$$g(x_k) = (x_k^{\,2},\ldots,x_k^{\,n},\psi(x_k))^T$$

$$\psi(x_k) = \sum_{i_1=1}^{n} a_{i_1} x_k^{\,i_1} + \sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1} a_{i_1 i_2} x_k^{\,i_1} x_k^{\,i_2} + \ldots$$

$$\ldots + \sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1}\cdots\sum_{i_{p-1}=1}^{i_{p-2}}\sum_{i_p=1}^{i_{p-1}} a_{i_1 i_2 \ldots i_p} x_k^{\,i_1} x_k^{\,i_2} \ldots x_k^{\,i_p} \qquad (50)$$

The input/output description

$$y(m+1) = q\left[y(m-1),..y(m-N),u(m-k)..u(m-k-N)\right] \qquad (51)$$

was considered by Hsia and Ghandi[156] who developed an identification algorithm assuming that the form of the nonlinear difference equation expansion, eqn (51), and all moments of the noise corrupting the system measurements are known a priori. A polynomic input/output expansion similar to eqn (51) has recently been introduced by Billings and Leontaritis[157]. Assuming no a priori information is available extended least squares and recursive maximum likelihood algorithms have been developed to yield unbiased estimates of the system parameters in the presence of multiplicative and bilinear noise terms. Methods of detecting the system structure and selecting significant nonlinear terms in the model to achieve a parsimonious description have been derived[157].

The Group Method of Data Handling (GMDH)[158-160] has been developed by Ivakhnenko using the principles of heuristic self-organisation to solve complex problems with large dimensionality and short data sequences. A schematic diagram of the method is illustrated in Fig.9. The method fits a polynomic model

$$y = f(x_1, x_2, x_3, x_4) \tag{52}$$

using a heuristic approach. The data is split into a training set and a testing set to avoid over fitting and at any stage only two variables x_j, x_k say, are considered and the coefficients in a quadratic expansion P_2 of these variables are estimated using least squares. When all the pairwise combinations of variables have been considered the outputs y_i of the quadratic filters are passed through a selection layer. The outputs (y_1, y_3 and y_6 in Fig.9) which satisfy the selection criterion proceed to the second layer and the same procedure is repeated until only one output remains or a predetermined degree of polynomial is reached. Although the method ignors most of the basic principles of estimation theory it does considerably reduce the computations associated with fitting polynomic models and appears to work well in nonlinear prediction applications.

Data splitting algorithms, cluster analysis, input design, model validation and the application of results from catastrophe theory to parameter estimation methods for nonlinear systems have been investigated by Mehra[161-163]. Catastrophe theory provides a library of generic models with polynomial nonlinearities, whose global bifurcational behaviours are well understood and which are suitable candidates for choosing canonic model structures in the identification of nonlinear systems. The catastrophe surfaces also give immediate information regarding the selection of inputs[163] which should be designed to excite the system such that the nonlinear response predominates. This can be achieved by applying control inputs to drive the system beyond the jump threshold values such that a catastrophe in the system response is observed.

The centre manifold or reduction theorem[161,162] provides guidelines for deciding whether a nonlinear model is required. The theorem shows that the nonlinear character of the system is exhibited in terms of those states which have purely imaginary eigenvalues associated with them. For example, suppose an unknown system is perturbed around an equilibrium point and the linearised dynamics are identified. If the 95% confidence limits around the identified eigenvalues enclose the imaginary axis then by the centre manifold theorem the addition of non-linearities in the model is indicated. The applications of catastrophe theory to the global stability and control analysis of aircraft at high angles of attack have been studied extensively by Mehra et al[164].

Applications of parameter estimation methods to systems where the structural form of the model is known a priori are fairly numerous[151,165,166]. In contrast there are only a few applications of techniques based on linear-in-the-parameter expansions to systems which are treated as a black-box[17,167].

8. Conclusions

Considerable progress has been made in the identification of nonlinear systems over the last two decades. Properties of the functional expansions, design of algorithms, selection and properties of inputs, identification of specific nonlinear model structures and parameter estimation methods have all been studied. The choice between these various approaches to nonlinear system identification will often be dictated by the process, the amount of a priori information and the purpose of the identification.

The functional series methods can be applied to a wide class of processes and work well for systems with "mild" nonlinearities. Identification of systems which contain higher than second order kernels is however very difficult because of the excessive amount of computation required. The difficulty of incorporating a priori information, and of interpreting the final estimates and relating them to the physical characteristics of the process are additional disadvantages. These problems are fundamental to the functional series methods and represent the price paid for the generality of this approach.

Although the functional series methods can be applied to block oriented systems it is often advantageous to use one of the specially designed algorithms for this class of processes. Determination of the structure of the process prior to identification is obviously very important for this class of systems and although some results are available further research is needed on this topic. Whilst some of the algorithms for block oriented systems are very restrictive others can be applied to various model structures. The methods which decouple the identification of the linear subsystems from the identification of the nonlinear elements are particularly attractive because even systems with very violent nonlinearities can be considered. Moreover, the structure of the original process is preserved and this provides a very concise description of the process which can be related to the characteristics of the system under study and provides valuable insight and information for control. Although several algorithms for block oriented systems are available further research is needed to simplify and extend the current methods.

If the form of the differential equation which represents the system is known then estimates of the unknown parameters can be readily obtained using any of a number of well established parameter estimation methods. When there is little or no a priori information available however, the problem of selecting an input/output expansion and estimating the unknown parameters must be investigated. This approach can lead directly back to the functional series expansions with the only advantage that the constraints on the input, such as Gaussian white noise, are less restrictive. The major difficulty is the selection of an expansion which can represent a broad class of systems but does not involve an excessive number of terms. The results from catastrophe theory and expansions based on both the system input and output may offer some simplification in this area.

Identification of nonlinear systems is a very difficult problem and no one technique can be recommended as providing an acceptable solution. All the algorithms considered have their advantages and disadvantages and each must be judged according to the problem under investigation. This inevitably leaves the experimenter with some difficult decisions and compromises, and further research is required to develop improved identification and structure detection algorithms to reduce the number of data points and simplify the measurement techniques.

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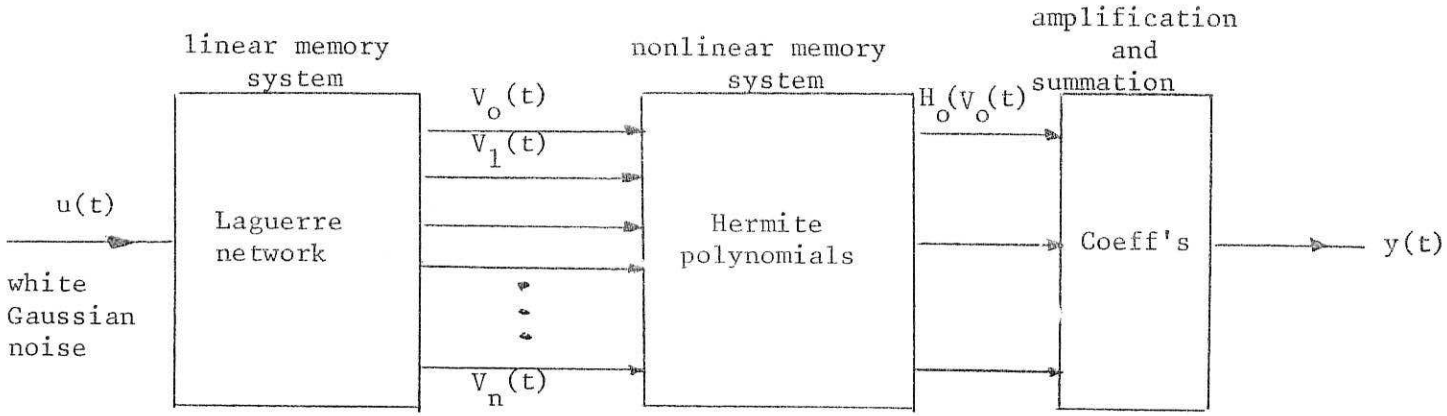

Fig.1. Schematic of Wiener's Method

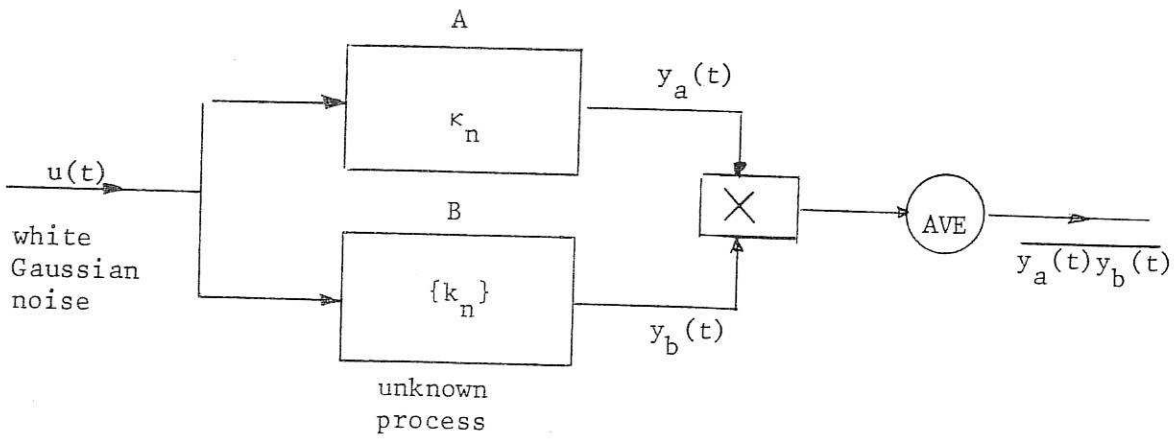

Fig.2. Identification of the G-functionals

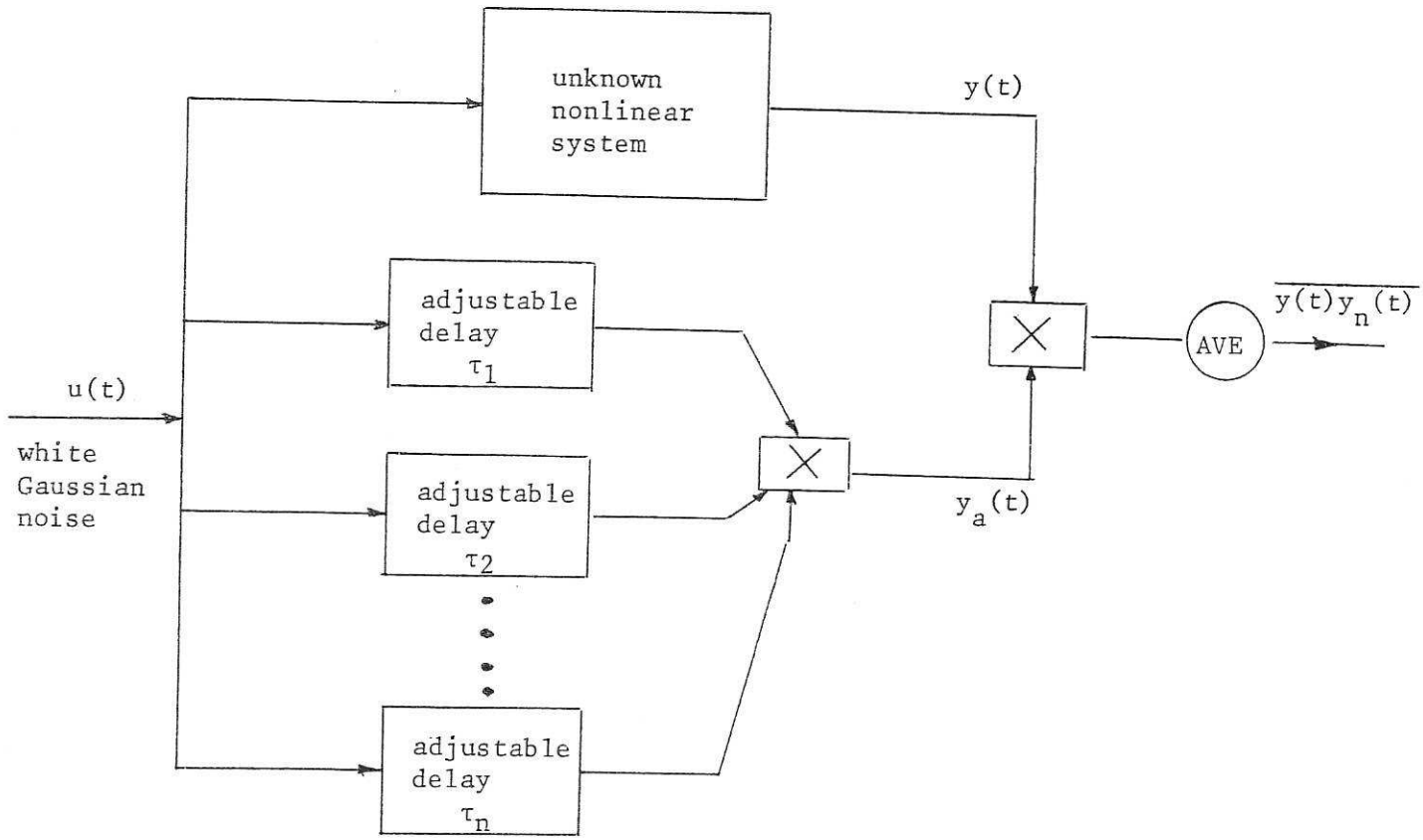

Fig.3. The Lee and Schetzen procedure

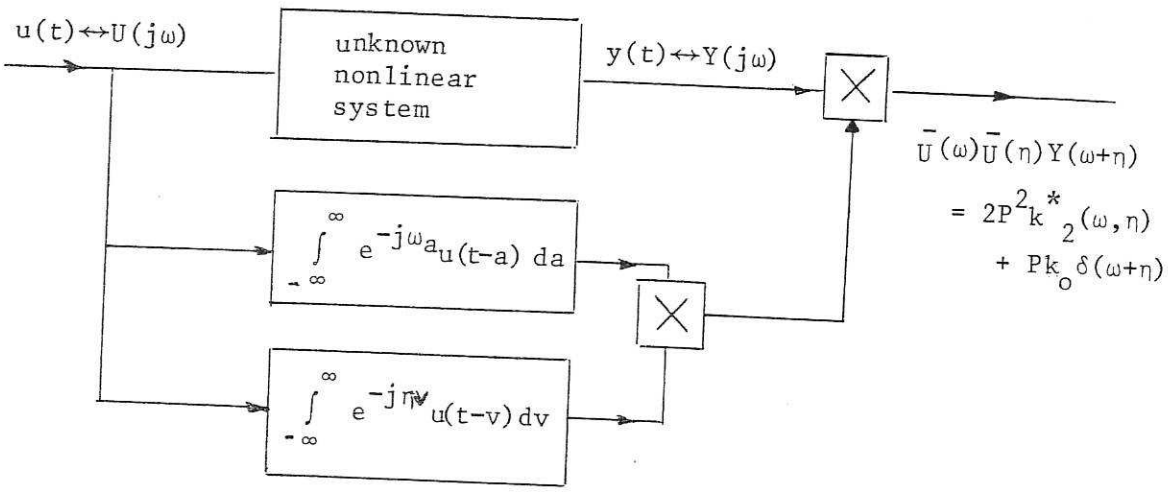

Fig.4. The French and Butz algorithm

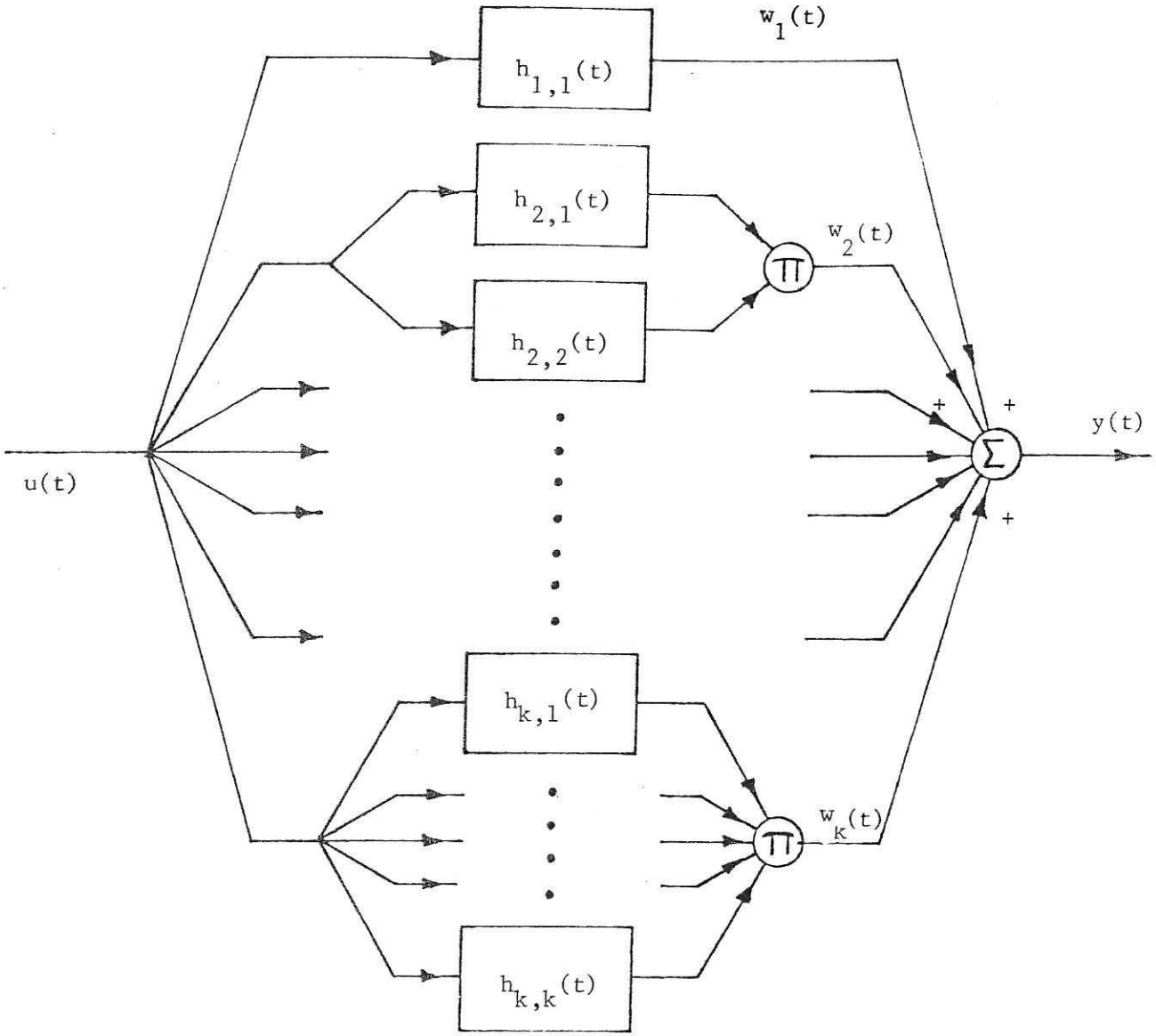

Fig.5. Factorable Volterra systems

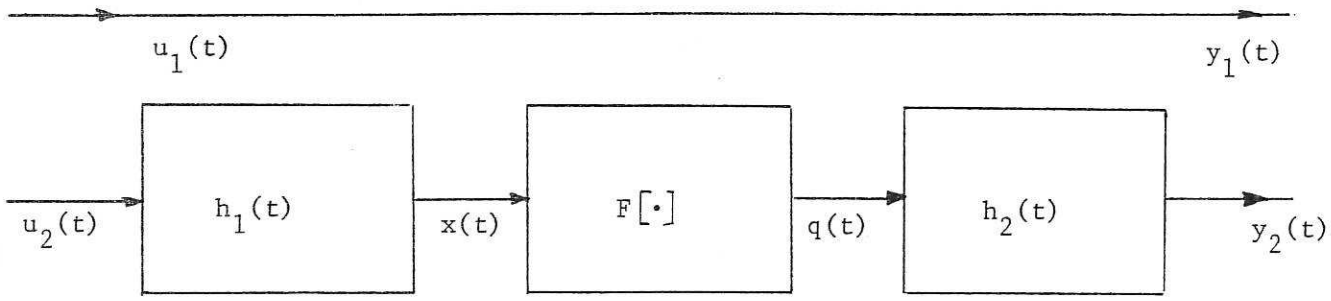

Fig.6. The general model

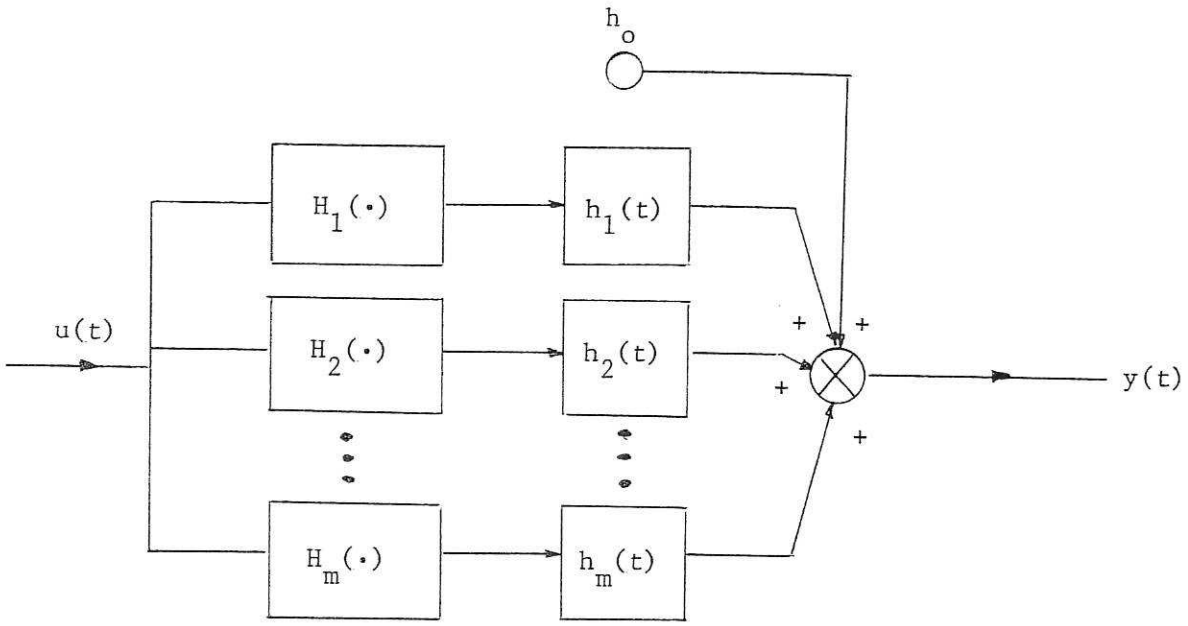

Fig.7. The Uryson model

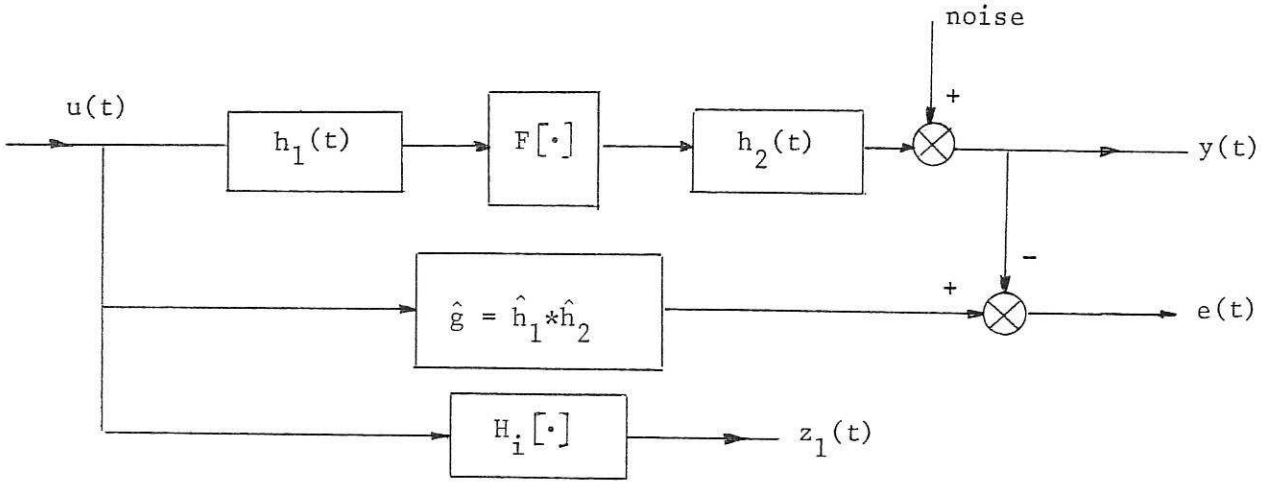

Fig.8. Douce's algorithm

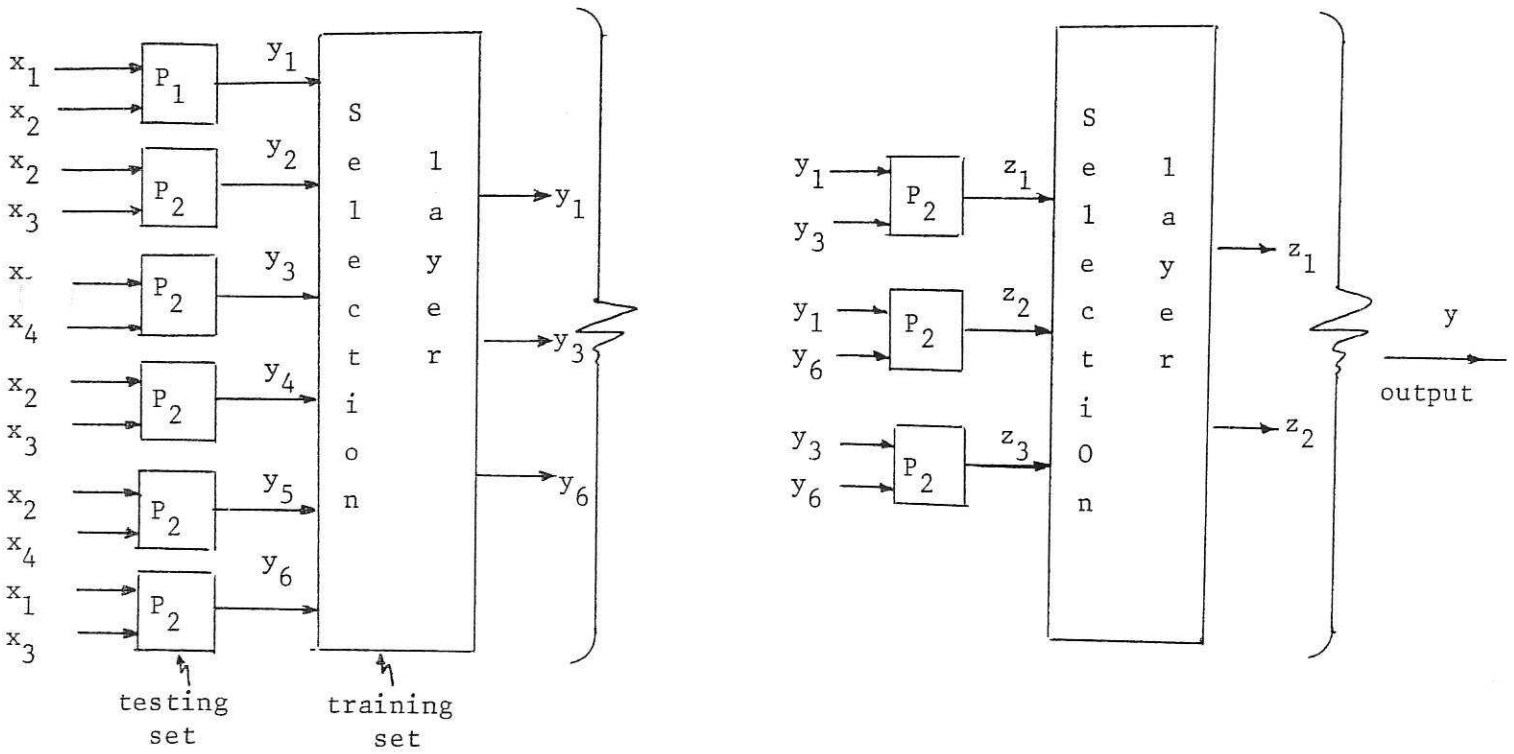

Fig.9. The GMDH