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Monograph:

IDENTIFICATION OF NON-LINEAR SYSTEMS

USING THE WIENER MODEL

by

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ABSTRACT

An algorithm for the identification of non-linear systems
which can be described by a Wiener model consisting of a linear
system followed by a single-valued non-linearity is presented.
Cross-correlation techniques are employed to decouple the
identification of the linear dynamics from the characterization
of the non-linear element.

INTRODUCTION

Although the functional series expansion of Volterra provides an adequate representation for a large class of non-linear systems, practical identification schemes based on this description often result in an excessive computational burden. It is for this reason that several authors[1,2] have considered the identification of specific configurations of non-linear systems, notably cascade systems composed of linear subsystems with memory and continuous zero-memory non-linear elements.

The Wiener model, illustrated in Fig.1, consists of a linear system followed by a continous no-memory non-linear element. The model is a much simplified version of Wiener's original non-linear system characterization[3] and belongs to the class of models studied by Cameron and Martin[4], and Bose[5]. In the present study, correlation analysis is used to decouple the identification of the linear and non-linear component subsystems when the input is a white Gaussian process. The results of a simulation study are included to illustrate the validity of the algorithm.

IDENTIFICATION OF THE LINEAR SUBSYSTEM

Consider the Wiener model, Fig.1, where the linear time-invariant system has an impulse response $h(t)$ and the continuous single-valued non-linear element can be represented by a finite polynomial of the form

$$y(t) = \gamma_1 q(t) + \gamma_2 q^2(t) + \ldots + \gamma_k q^k(t) \tag{1}$$

The measured system output, $z(t)$, can then be expressed as

$$z(t) = \gamma_1 \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau + \gamma_2 \iint_{-\infty}^{\infty} h(\tau_1) h(\tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1 d\tau_2$$

$$+ \ldots + \int_{-\infty}^{\infty} \ldots \int h(\tau_1) \ldots h(\tau_k) x(t-\tau_1) \ldots x(t-\tau_k) d\tau_1 \ldots d\tau_k$$

$$+ n(t) \qquad\qquad (2)$$

which has the form of a Volterra series with the special property that the kernals are separable.

If the input signal $x(t)$ is a zero mean white Gaussian process with a spectral density of 1 watt/cycle, then its i'th dimensional autocorrelation function is given by[6]

$$\overline{x(t_1) x(t_2) \ldots x(t_i)} = 0 \qquad \text{for i odd}$$
$$= \sum_i \prod_{n \neq m} \delta(t_n - t_m) \qquad \text{for i even} \qquad (3)$$

where the summation is over all ways of dividing 'i' objects into pairs.

If the input to the Wiener model comprises a Gaussian white process $x(t)$ with a mean level 'b', then from eqn (1) the measured system output $z(t)$ is given by

$$z(t) = w_1(t) + w_2(t) + \ldots + w_k(t) + n(t) \qquad (4)$$

where
$$w_i(t) = \gamma_i \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} h(\tau_1) \ldots h(\tau_i) \{x(t-\tau_1)+b\}$$

$$\ldots \{x(t-\tau_i)+b\} d\tau_1 \ldots d\tau_i \qquad (5)$$

The first order cross-correlation function between the Gaussian white input $x(t)$ and the measured system output is defined as

$$\phi_{xz}(\sigma) = E\{z(t)x(t-\sigma)\}$$

$$= \overline{w_1(t)x(t-\sigma)} + \overline{w_2(t)x(t-\sigma)} + \ldots$$

$$\ldots + \overline{w_k(t)x(t-\sigma)} + \overline{n(t)x(t-\sigma)} \tag{6}$$

Evaluating the first term on the rhs of eqn (6)

$$\overline{w_1(t)x(t-\sigma)} = \gamma_1 \int_{-\infty}^{\infty} h(\tau_1)\{\overline{x(t-\tau_1)+b\}x(t-\sigma)}\,d\tau_1$$

$$= \gamma_1 h(\sigma) \tag{7}$$

Considering the second term on the rhs of eqn (6)

$$\overline{w_2(t)x(t-\sigma)} = \gamma_2 \iint_{-\infty}^{\infty} h(\tau_1)h(\tau_2)\overline{\{x(t-\tau_1)+b\}\{x(t-\tau_2)+b\}x(t-\sigma)}\,d\tau_1\,d\tau_2$$

$$= 2b\gamma_2 h(\sigma) \int_{-\infty}^{\infty} h(\tau_1)\,d\tau_1 \tag{8}$$

Similarly, for the third term

$$\overline{w_3(t)x(t-\sigma)} = 3\gamma_3 h(\sigma)\{\int_{-\infty}^{\infty} h^2(\tau_1)\,d\tau_1 + b^2 \iint_{-\infty}^{\infty} h(\tau_2)h(\tau_3)\,d\tau_2\,d\tau_3 \tag{9}$$

Higher order terms are evaluated in a similar manner.

Collecting terms

$$\phi_{xz}(\sigma) = h(\sigma)\{\gamma_1 + 2b\gamma_2 \int_{-\infty}^{\infty} h(\tau_1)\,d\tau_1 + 3\gamma_3 \int_{-\infty}^{\infty} h^2(\tau_1)\,d\tau_1$$

$$+ 3\gamma_3 b^2 \iint_{-\infty}^{\infty} h(\tau_2)h(\tau_3)\,d\tau_2\,d\tau_3 + \ldots\} + \overline{n(t)x(t-\sigma)} \tag{10}$$

Assuming that the input signal and the noise process are statistically

independent, and providing the linear subsystem is stable, bounded-inputs bounded outputs, the first order cross-correlation function eqn (10) becomes directly proportional to the impulse response of the linear system

$$\phi_{xz}(\sigma) = \beta h(\sigma) \tag{11}$$

This represents an application of a result due to Nuttall[7], who showed that for a wide class of signals the input-output cross-correlation function for a non-linear no-memory device is proportional to the input autocorrelation function.

If the identification is performed with the aid of a digital computer, the cross-correlation function eqn (11) will be in sampled data form and estimates of the coefficients in the pulse transfer function representation of the linear system

$$Z\{\beta h(\sigma)\} = \frac{B(z^{-1})}{A(z^{-1})} \tag{12}$$

can be obtained using a least squares algorithm.

IDENTIFICATION OF THE NON-LINEAR ELEMENT

Consider the schematic diagram of the identification procedure illustrated in Fig.1. The error $e(i)$ between the sampled process output $z(i)$ and the output of the Wiener model $\hat{y}(i)$ can be defined as

$$e(i) = z(i) - \hat{y}(i) \tag{13}$$

where
$$\hat{y}(i) = \gamma_1^1 \hat{q}(i) + \gamma_2^1 \hat{q}^2(i) \ldots + \gamma_k^1 \hat{q}^k(i) \tag{14}$$

$$\hat{q}(i) = -a_1 \hat{q}(i-1) \ldots -a_\ell \hat{q}(i-\ell) + b_1 x(i-1)$$
$$\ldots + b_\ell x(i-\ell) \tag{15}$$

and $\gamma_t = \beta^t \gamma_t^1$, $t = 1,2..k$. Combining eqn's (13), (14) and (15), and considering $(N+\ell)$ measurements of the sampled process input and output gives the matrix equation

$$
\begin{bmatrix} Z(\ell+1) \\ Z(\ell+2) \\ \vdots \\ Z(\ell+N) \end{bmatrix} = \begin{bmatrix} \hat{q}(\ell+1) , \hat{q}^2(\ell+1) & \cdots & \hat{q}^k(\ell+1) \\ \hat{q}(\ell+2) , \hat{q}^2(\ell+2) & \cdots & \hat{q}^k(\ell+2) \\ \vdots & \vdots & \vdots \\ \hat{q}(\ell+N) , \hat{q}^2(\ell+N) & \cdots & \hat{q}^k(\ell+N) \end{bmatrix} \begin{bmatrix} \gamma_1^1 \\ \gamma_2^1 \\ \vdots \\ \gamma_k^1 \end{bmatrix} + \begin{bmatrix} e(\ell+1) \\ e(\ell+2) \\ \vdots \\ e(\ell+N) \end{bmatrix}
$$

or $\qquad Z = \phi\theta + E \qquad\qquad\qquad$ (16)

Since all the elements of the matrices Z and ϕ can either be measured or estimated, a least squares estimate of the coefficients γ_j^1, $j = 1,2\ldots k$ associated with the non-linear element can be readily computed

$$
\hat{\theta} = (\phi^T\phi)^{-1}\phi^T Z \qquad\qquad (17)
$$

and the identification is complete.

SIMULATION RESULTS

The identification procedure outlined above was used to identify the parameters in a Wiener model consisting of a linear system with transfer function

$$
G(s) = \frac{1}{s^2 + 6s + 25} \qquad\qquad (18)
$$

in cascade with a non-linear element of the form

$$
y(t) = 5.0q(t) + 50.0q^2(t) + 500.0q^3(t) \qquad (19)
$$

To provide a realistic simulation study, the model was simulated on an Applied Dynamics 4 analogue computer with an input signal produced from the summation of a dc level and the output of a white Gaussian noise generator. Samples of the input-output signals were processed using a CONPAC 4020 process computer to provide an estimate of the sample cross-correlagram illustrated in Fig.2.

Least squares estimates of the parameters in the linear pulse
transfer function model and the polynomial representation of the
non-linear element are summarised in Table 1.

Parameter	a_1	a_2	b_1	b_2	γ_1	γ_2	γ_3
True value	-1.58	0.67	0.215	0.0	5.0	50.0	500.0
Estimate	-1.57	0.66	0.216	-0.004	5.14	52.42	518.12

Table 1 A Summary of the Identification Results

CONCLUSIONS

A procedure for the identification of systems having the structure
of the Wiener model has been presented.   Provided the non-linear
system can be excited by a Gaussian white process with a non-zero
mean, the impulse response of the linear system can be identified
independently of the non-linear element.   This effectively decouples
the identification procedure and simplifies considerably the
identification of this class of non-linear system.
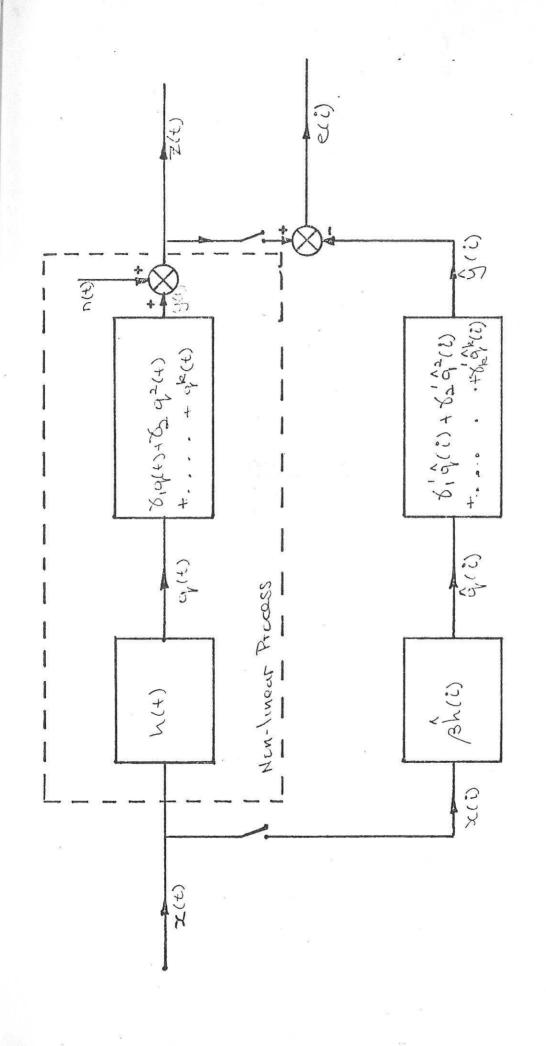
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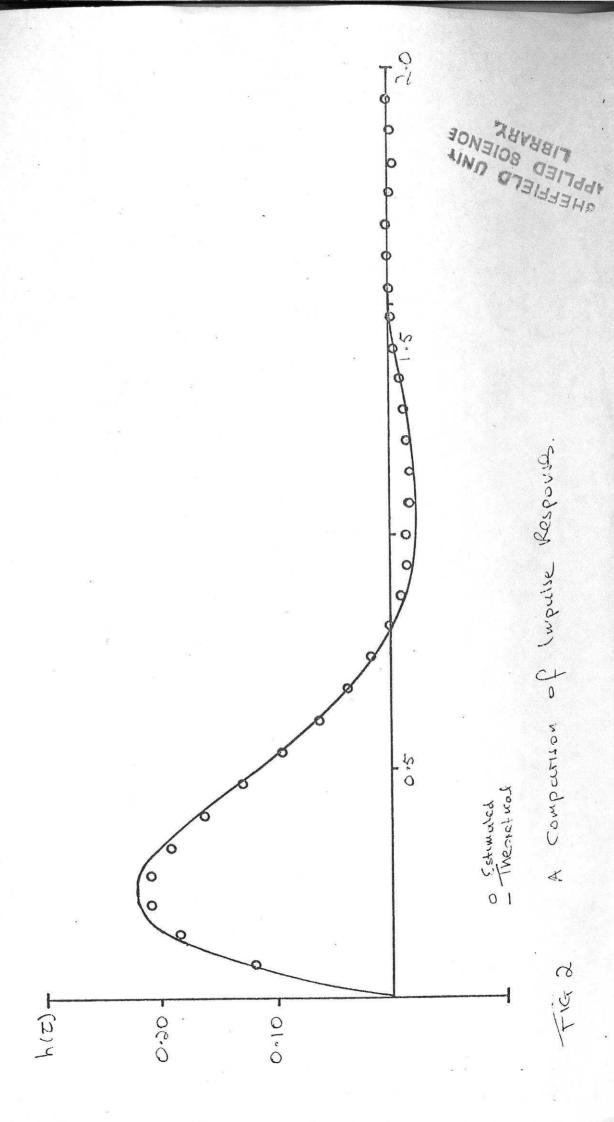
Figure Captions

FIG. 1.   Schematic diagram of the identification

procedure for the Wiener model.

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o   Estimated

-   Theoretical

FIG. 2.   A comparison of impulse responses.

FIG 1. Schematic diagram of the identification procedure for the reservoir model.

FIG 2

A Comparison of Impulse Response.

o Estimated
— Theoretical