Kent Business School

ISSN 1748-7595 (Online)

Working Paper Series

Should You Stop Investing in a Sinking Fund When it's Sinking?

John Mingers
Kent Business School

Kim T Parker
Kent Business School

University of Kent

Working Paper No.183
December 2008

Should you stop investing in a sinking fund when it's sinking?

John Mingers[a]

[a] *Kent Business School, University of Kent, Canterbury CT7 2PE, UK*
phone: 01227 824008, e-mail: j.mingers@kent.ac.uk (corresponding author)

Kim T. Parker[b]

[b] *Kent Business School, University of Kent, Canterbury CT7 2PE, UK*
phone: 01227 823604

Abstract

Many people invest regularly in sinking funds that track stock market indices. When stock markets themselves sink significantly, as in the current credit crunch, investors face a decision as to whether they should continue paying into a falling fund, or switch payment to a risk-free deposit account until the market recovers. Most financial advice is to keep investing on the grounds that as the unit price falls more units can be purchased and that this is ultimately beneficial (dollar-cost averaging, DCA) However, most academic studies show that DCA is sub-optimal, at least to a lump sum strategy. In this paper we consider a specific, tax-free fund – the Individual Savings Account (ISA). We demonstrate, both analytically and numerically, that in a situation of perfect information a stop and restart policy can beat DCA. From these results we test some heuristics that could be used by an everyday investor under real-world conditions of uncertainty and volatility.

Keywords:
Investment Analysis, Dollar-Cost Averaging, Index-Linked Funds, Stop and Restart

V1, November 2008

Submitted to EJOR

Should you stop investing in a sinking fund when it's sinking?

1. Introduction

Many people invest a fixed amount each month in a sinking fund that is linked to a stock-market index such as the FTSE100. This may be purely for savings or may be intended to pay off a mortgage in the future. In periods when the stock market is undergoing a downturn, such as the early 2000s or now with the credit crunch, it is tempting to stop the investments which appear to be losing value and instead invest in a risk free alternative such as a cash deposit until the market recovers. However, the advice given almost unanimously by financial advisors is to keep the investments going. The reasoning being that as the market falls the unit value also falls and so the fixed payment will buy more units, generating a greater return in the long run when the market recovers. This investment strategy is generally known as "dollar-cost averaging" (DCA).

The academic literature, however, argues that generally DCA is not an optimal investment strategy and it is rarely recommended. But virtually all of these studies investigate the situation where the whole of the investment capital is available at the beginning of the investment period and the main choice is between DCA and a lump sum investment of the whole amount. In these circumstances the lump sum method generates higher returns although it is also riskier. The situation we wish to investigate is different in that the money only becomes available as it is earned period by period. The alternative to DCA is to switch the investment to a secure cash deposit during the downturn and then reinvest the whole amount accrued in the fund at some point during the upturn. The question is, under what circumstances, if any, will this generate a greater return than DCA for the ordinary private investor?

After a review of the literature we consider two situations – first we investigate whether this "stop and start" policy can outperform DCA with perfect information about how the market actually behaved, and second whether we can use these insights to construct simple rules for investors who cannot predict market behaviour. We consider the specific case of the UK's Individual Savings Account (ISA) which is a tax-free investment vehicle in which a named individual may invest up to a maximum of £3,600 per year in each of an equity fund and a cash fund.

2. Literature review

Brennan *et al* (2005, p. 514), in a recent review of DCA, state that "despite the fact that contemporary *academic* texts no longer contain any discussion of DCA, it is still a strategy that is widely advocated in more popular publications". Investment manuals such as Orman (2001), Schwab (2002) and Malkiel (1996) all recommend DCA and Greenhut (2006) found over 250 websites with discussions and illustrations of it.

Many studies have compared DCA with a range of other investment strategies including lump sum, buy and hold, value averaging, and mixed strategies such as 50:50 between the market linked fund and a riskless investment (Brennan, Li et al. 2005; Leggio and Lien 2003; Thorley 1994; Williams and Bacon 1993). The main results are that DCA is clearly a sub-optimal strategy, especially in comparison with lump sum (LS) investing, at least under conditions where the returns are assumed to be *iid* (Constantinides 1979); and it is also sub-optimal after adjusting for risk within a mean variance framework (Rozeff 1994). However, most of these studies assumed that the stock market behaved as a random walk, a hypothesis now generally rejected (De Bondt and Thaler 1985; Jegadeesh and Titman 1993; Summers 1986).

This has led to behavioural research that tries in take into account psychological reasons for its continued popularity (Statman 1995) although it still appears sub-optimal. Despite these generally negative results there is still some support for DCA: for example, Brennan et al (2005) argue that the critical studies make assumptions about the market (i.e., that it is a random walk) that are in fact false, and show that it is a reasonable strategy for an uninformed investor who is adding to an existing portfolio; and Greenhut (2006) argues that the superiority of LS comes about because over long time periods the markets have consistently risen which favours LS over DCA.

All of these studies, however, contrast DCA with other strategies in the situation where all the investment capital is available at the start of the investment period. We are interested in the situation where the payments can only occur monthly, as the money is earned, and we are specifically concerned with the possibility of stopping the investments in a downturn; investing in a riskless, interest-bearing account; and then reinvesting the total accumulated once the market is rising - a stop-restart (SR) strategy. We will examine this possibility with the example of UK ISA accounts: tax-free funds that can be stocks and shares or cash but are limited to an investment of £3600 in any year. We are particularly concerned to see if there are simple rules that could be used by an ordinary investor rather than mathematically or computationally sophisticated ones (Spronk and Hallerbach 1995). There are other approaches for portfolio optimization, for example using mathematical programming (Barro and Canestrelli 2005) or multi-criteria (Ballestero et al. 2007), but these are not appropriate for an ordinary investor in a real situation.

3. Stop and restart given perfect information

In this section we will consider the situation where we know what actually happened in the market in order to see if it is possible to beat DCA with stop and restart (SR) under perfect conditions. We will then use the results to design decision rules for investors in an uncertain market situation. In all the following both transactions costs and tax implication are ignored. This is justifiable as ISAs are non taxable, and with the index-tracker used in the example the costs of buying units are zero. We do make the assumption that the payments can be switched from the equity ISA to the cash ISA at will. This would reduce the amount that the investor could put in the cash ISA in any year (as they could invest up to £3600 in both types) but if necessary the payment could be switched to a non-ISA deposit account with a consequent reduction of interest rate because of tax.

3.1 An empirical example

Figure 1 shows the UK's FTSE100 index from 2000 to 2008, a period with a strong downturn followed by a more gradual recovery.

Figure 1 FTSE100, 2000-2008 about here

We will use the example of a stocks and shares index-tracking ISA over the downturn and upturn. The cost of a unit was available at various points during the period and this was regressed against the index to give: cost = 0.1976 x index (R^2 = 99.99%). This enabled the unit cost for each period to be estimated and from that the number of units bought each month is calculated for a monthly investment of £100. The units were accumulated until the peak in October 2007 at which point their value was £12,295. This was the base value of following a DCA strategy.

Two periods were then selected somewhat arbitrarily to see if the stop and restart strategy could beat DCA. The results are shown in Table 1 and on Figure 1. In both cases periodic investment was stopped in month 7 of 2001 when the downturn seemed well established. In case A it was restarted in month 4 of 2005, and in B it was restarted earlier in month 12 of 2003. In each case the full amount that had been accumulated at a given interest rate was used to buy units at the prevailing price. In case A we can see that even assuming an net interest rate of 4% SR does not match the DCA total. However, in case B we can see that the DCA total is exceeded for interest rates of 2% and above. This shows that it is possible for SR to beat DCA, and also shows the importance of restarting the investment as early as possible after the beginning of the upturn in order to buy more units at a cheap price.

Interest rate	**0%**	**1%**	**2%**	**3%**	**4%**
A: 7/2001-4/2005	11,678	11,794	11,914	12,037	12,162
B: 7/2001-12/2003	12,220	12,274	12,330	12,387	12,444

Table 1 Gross returns for various interest rates

The next section will investigate this situation in more detail.

3.2 Detailed numerical and analytical results

In order to investigate this situation in more detail we will simplify it slightly as shown in Figure 2.

Figure 2 Idealised version of figure 1 about here

The main assumption that we make is that the index, and thus cost of units, has constant slopes during it downturn and upturn. Thus, the unit cost starts at 1 and falls at a rate d until it reaches its lowest value, a, after t_b periods. It then rises at a rate u until it reaches a value of 1 again at period t_e. We can formulate the following equations for the cumulative returns.

<u>Under DCA</u>

During the downturn, the cost will vary as $1 + (i\text{-}1)d$, where i is the period (d will be negative). So in each period the amount invested (P) will purchase $P/(1 + (i\text{-}1)d)$ units.

So, the cumulative units purchased by t_b will be:

$$\sum_{i=1}^{i=tb} \frac{P}{1+(i-1)d} \tag{1}$$

Similarly for the upturn the cumulative units will be:

$$\sum_{i=2}^{i=te-tb+1} \frac{P}{a+(i-1)u} \tag{2}$$

Adding these gives the total units available at the end of the period, and also the value since the unit cost started as 1 and finished as 1.

Under SR

For the period 0 to t_s:

$$\sum_{i=1}^{i=ts} \frac{P}{1+(i-1)d} \tag{3}$$

For the period t_r to t_e:

$$\sum_{i=tr-tb+1}^{i=te-tb+1} \frac{P}{a+(i-1)u} \tag{4}$$

For the period t_s +1 to t_r there will be (t_r - t_s -1) periods of interest-earning investment at rate r so the future value will be:

$$P\left(\frac{(1+r)^{tr-ts-1}-1}{r}\right) \tag{5}$$

This lump sum will then be used to buy units in the fund at time t_r.

The price at t_r will be a + (t_r - t_b)u, so the number of units bought will be:

$$P\left(\frac{(1+r)^{tr-ts-1}-1}{r(a+(t_r-t_b)u}\right) \tag{6}$$

From these expressions we can calculate the difference between SR and DCA and could potentially optmise it with respect to the only two decision variables t_r and t_s. In fact, since the periods 1 - t_s and t_r - te are common to both we actually just need to compare the number of units accumulated between t_s and t_r under DCA with the number of units purchased with the lump sum at time t_r. Thus the increased value (IV) of the SR policy (which may be negative) is:

$$IV = P\left(\frac{(1+r)^{tr-ts-1}-1}{r(a+(t_r-t_b)u}\right) - \sum_{i=ts+1}^{i=tb} \frac{P}{1+(i-1)d} - \sum_{i=2}^{i=tr-tb} \frac{P}{a+(i-1)u} \tag{7}$$

The finite sums can be expressed using the following:

$$\sum_{i=1}^{i=n} \frac{1}{a+(i-1)d} = \left[\psi(n+a/d)-\psi(a/d)\right]/d$$

Where ψ is the digamma function, that is the logarithmic derivative of the gamma function. Its derivative is another transcendental function – the trigamma function.

Theoretically, this expression should be differentiated with respect to t_s and t_r to find the values that maximise it. However, these expressions are not very tractable and do not yield easy to interpret results, so we will begin by using them to explore the situation numerically and then consider a continuous approximation.

Table 2 shows the results obtained for a selection of stop (t_s) and restart (t_r) periods after the downturn and upturn respectively. The parameters, based on the data from Figure 1, are: initial cost per unit = 1, low cost per unit, a = 0.576, t_b = 38, te = 86, down slope, d = -0.01116, upslope, u = 0.00883. With this scenario, the DCA value is £11,289. We assume that the risk free rate of interest is a constant 3% which would seem to be generally obtainable especially given that ISAs are tax-free. Combinations that better the DCA return are shaded.

Table 2 Gross returns at 3% interest for various stop/restart periods about here

The results show that it is relatively easy to better the DCA return so long as one makes the switches relatively soon after the change in direction if the index. The greatest gains come with the quickest changes – the maximum with these parameters being £12,717 – a 13% improvement. The response surface is also relatively flat showing that it is fairly insensitive to changes in period although the marginal change in the up periods is nearly twice as great as in the down period. The results are only moderately sensitive to the interest rate – at 4% the maximum return is £12,812 while even at 0% it is £12,444. This latter result shows that it is possible to benefit even if the monthly payments are just held as cash, not in a deposit account.

These results are for an idealised version of the situation with perfect information and so are not directly applicable to a real investor with uncertainty about the market but they do suggest that simple rules of the form "if after x periods the index has fallen by y% then switch out" are worth evaluating.

In terms of the optimal values of t_s and t_r, we can consider the mean price that units are purchased at under DCA. With reference to Figure 2, with the DCA approach units will be purchased at falling prices from t_s to t_b. At the start the price will be relatively high; at the bottom it will be low. Purchase of units continues, at a rising price now, until t_r. We could develop a formula for the mean price over the period t_s to t_r. Under the alternative SR system, the payments will be accumulated as a lump-sum (assuming no interest at the moment) until t_r at which point it will be used to buy units at the prevailing price. SR will be better than DCA provided that the price at t_r is lower than the mean price under DCA since in total more units will be able to be purchased. Gaining interest would simply increase the lump-sum available and thus the number of units that could be bought.

To maximise the advantage to SR we need to minimise the lump-sum price and maximise the DCA mean price. The former will be achieved when t_r is as small as possible (subject to being greater than t_b) as this will be the lowest price. The latter will be achieved if t_s (subject to being greater than 1) is as small as possible as that would lead to the highest mean DCA price. Thus the optimal strategy is to stop the payments in the first period of the downturn and restart them in the first period of the upturn.

We can now consider some analytic results approximating the discrete case with a continuous model.

Although the investment flows and accrual of interest occur discretely, the time step between monetary transfers (monthly) is quite short compared with the time constants that govern the behaviour of the whole system which are typically measured in years (e.g. the reciprocal of interest rate). Consequently a good approximation to the behaviour of the system can be obtained by assuming that investment is a continuous process.
During the first period from 0 to t_s, the number of index-linked units, x, increases according to the formula

$$x_s = \int_0^{t_s} \frac{P}{\pi(t)} dt = P\int_0^{t_s} \frac{dt}{\pi(t)} \tag{8}$$

where $\pi(t)$ is the price of index-linked units at time t. (The subscripts s, r and e are used to indicate values of variables at times t_s, t_r and t_e). Between t_s and t_r the amount in the deposit account, D, grows according to the formula

$$\frac{dD}{dt} = P + rD \quad . \tag{9}$$

This differential equation has a simple solution which gives the value of the deposit at time t_r as

$$D_r = \frac{P}{r}\left(e^{r(t_r - t_s)} - 1\right) \quad . \tag{10}$$

At t_r, the money on deposit will be used to buy index-linked units at the price π_r, hence

$$x_r = x_s + \frac{D_r}{\pi_r} \quad . \tag{11}$$

From then on, P will be used to buy new units until the end of the investment period. The final number of units held will therefore be

$$x_e = P\int_0^{t_s} \frac{dt}{\pi(t)} + \frac{P}{\pi_r r}\left(e^{r(t_r - t_s)} - 1\right) + P\int_{t_r}^{t_e} \frac{dt}{\pi(t)} \quad . \tag{12}$$

P is clearly a constant factor and so maximising x_e will be achieved by maximising the final holding per unit of investment flow. We may now consider the objective: what values of t_s and t_r will maximise our final holding? Necessary conditions are that:

$$\frac{\partial x}{\partial t_s} = 0, \quad \frac{\partial x}{\partial t_r} = 0 \quad . \tag{13}$$

Looking at the first of these,

$$\frac{\partial x}{\partial t_s} = \frac{1}{\pi_s} - \frac{1}{\pi_r} e^{r(t_r - t_s)} = 0 \quad . \tag{14}$$

We next differentiate x with respect to t_r. From equation 12,

$$\frac{\partial x}{\partial t_r} = \frac{1}{\pi_r}\left(e^{r(t_r - t_s)} - 1\right)\left(1 - \frac{\dot{\pi}_r / \pi_r}{r}\right) \tag{15}$$

where the "dot" notation denotes differentiation with respect to time. Assuming that $t_r > t_s$, the only way the derivative in (10) can be zero is for

$$\frac{\dot{\pi}_r}{\pi_r} = r \quad . \tag{16}$$

Thus the maximum occurs when the growth rate of the units' price (after the upturn) becomes equal to the deposit-account interest rate. For the moment, we shall assume that there is a time when this condition holds. If so, it determines a value for t_r. Once t_r is known, it is then possible to use (13) to find the value of t_s.

The forms of equations (14) and (16) are such that there is a very simple graphical technique for identifying the optimal values. Consider a graph of the logarithm of the unit price plotted against time. Equation (16) is equivalent to the condition

$$\frac{d \ln \pi_r}{dt} = r \quad . \tag{17}$$

Equation (14) can also be rewritten in logarithmic form

$$\ln \pi_r - \ln \pi_s = r(t_r - t_s) \quad . \tag{18}$$

From (17) we see that t_r can be identified as the point where the gradient of the curve of logarithm of price plotted against time is equal to the interest rate r. Equation (18) shows that the straight line joining the point $(t_s, \ln \pi_s)$ to the point $(t_r, \ln \pi_r)$ also has a gradient of r. These two facts together mean that t_s and t_r are related as shown on the graph in Figure 3.

The analysis so far has only identified necessary conditions for maximising return but they can be shown to be sufficient as well.

Figure 3 ln(price) vs time about here

The preliminary indication is that one should switch back from the deposit account into index-linked units when the growth rate of the unit price is equal to the deposit-account interest rate. It is interesting to note that, as shown in Figure 3, the investor should switch out of the units and into the deposit account *before* the price begins to turn down. This may seem a counter-intuitive result but we should remember that in this model we have perfect information and so, in particular, we know that there *will* be a downturn coming. Given this, it is better *not* to carry on buying units at an increasing price as we will be able to buy them more cheaply in the future.

In the real world, of course, we do not have this certainty and we will now move to consider if these idealised results can be of help to the ordinary investor.

4. Stop and restart in the real world

Both the numerical example and the analytic results show that if the behaviour of the index is known that we should switch out of the equity fund as soon as (or indeed before) the downturn has started and then restart as soon as the upturn has begun. In this section we will investigate two simple strategies, derived from the above, to see if they could work in a real world situation of uncertainty.

From the numerical example in the previous section it appeared as though we might investigate a simple rule for detecting the changes of slope in the index. This would be of the form "stop/restart investing when the index has fallen/risen by x% over y periods". From the analytic results the suggestion would be to crudely estimate the underlying rate of change of the index price and then compare it with the prevailing rate of interest. There are of course more sophisticated techniques for detecting changes in slope but these would be beyond the capabilities of an everyday investor and so are not considered in this paper.

To investigate the effects of these rules we obtained the monthly FTSE index since January 1970 – a period of 38 years which is significantly longer than even mortgage repayment periods. This time span includes several periods with major downturns, e.g., 1973-5 and 2001-3 as well as long periods of growth such as 1982-7 and 1995-2001. There were also many smaller downturns of less than a year. We estimated the unit cost of the ISA for each period using the regression equation established above. Investing £100 every month yielded a total return of £290,368 – the DCA value.

4.1 The "x% over y periods" strategy

Given this strategy, there are four parameters to investigate – the period and % for both stopping and restarting. However, it was observed that the index is quite volatile at times and combinations of particularly low and high values could trigger stop/restart when there was actually little evidence of a significant change. To overcome this, we applied simple exponential smoothing to the index but allowed the smoothing constant to be one of the parameters that could altered. With a value of 1 it is equivalent to no smoothing.

We began with fairly conservative parameters of a 10% change over 10 periods for both stop and restart, and no smoothing. We used a risk free interest rate of 3% unless otherwise stated. This combination resulted in a total return of £283,064, some 3%

below the DCA value. During the time span investment was stopped only 8 times, although on occasions for only 4 or 5 periods. The best value of the smoothing constant was 0.9, below that the return fell off marginally.

The system was made more sensitive by reducing both periods to 5 and both % to 5%. The generated a better return of £302,503, an improvement of 4% over DCA. There were 18 stops, some for only 1 period. Reducing the smoothing constant to 0.5 reduced the number of times the investment was stopped to 10, but significantly reduced the total yield thus showing that it is worthwhile switching even for short periods.

Experimenting will all possible values showed that the best that could be achieved was 5, 3%, 4, 7% with no smoothing which gave a total return of £312, 686, a gain of 7.7% over the DCA value. This process involved 17 switches, not that many over a 38 year period.

4.2 The "greater than the interest rate" strategy

To implement this we took a simple estimation of the underlying price change over x periods. x could be different for detecting the downturn and the upturn but in practice the same value for both was best. The optimal value of x was 4, giving a total return of £312, 237, very similar to the other method. However, there were over 40 switches, often lasting for only one period.

5. Conclusions

In this paper we have considered a specific, but very common, investment – the tax-free ISA linked to a stock market index. We have asked the question, when the market is falling should one carry on investing each month in the hope that the extra units that can be bought will, in the long term, compensate for the falling value, or is it better to switch the investments to a risk-free deposit account until the market rises? Conventional wisdom, especially from financial advisors, suggests the former although previous academic investigation in other situations suggests that DCA is generally not optimal.

By looking at a situation with perfect knowledge of market movements we have shown, both analytically and numerically, that it is certainly possible to do better than DCA. The optimal strategy is to switch out of the investment as early as possible after the downturn (indeed, with *perfect* future knowledge one should switch *before* the downturn) and then restart as soon as the upturn has begun.

Based on this theory we have been able explore two simple heuristics that would enable ordinary investors to improve on dollar cost averaging even within a volatile and uncertain market. Over a long period, provided that the start and restart strategy is rigorously followed, gains of up to 7% can be made with quite modest alternative rates of interest. These results assume zero transaction costs and no tax implications but this is the case in the ISA example used. There is scope to investigate more sophisticated methods of detecting market shifts but these may be infeasible for the everyday investor.

However, these results can be seen in a different light. If we consider the viewpoint of an investor who does not want to become actively engaged with their investment, will they actually be losing much? The answer is not a huge amount. If the best that could be achieved with the long-run and rigorous implementation of the strategy is only 7%, then simply carrying on investing in the tracker is not a bad option.

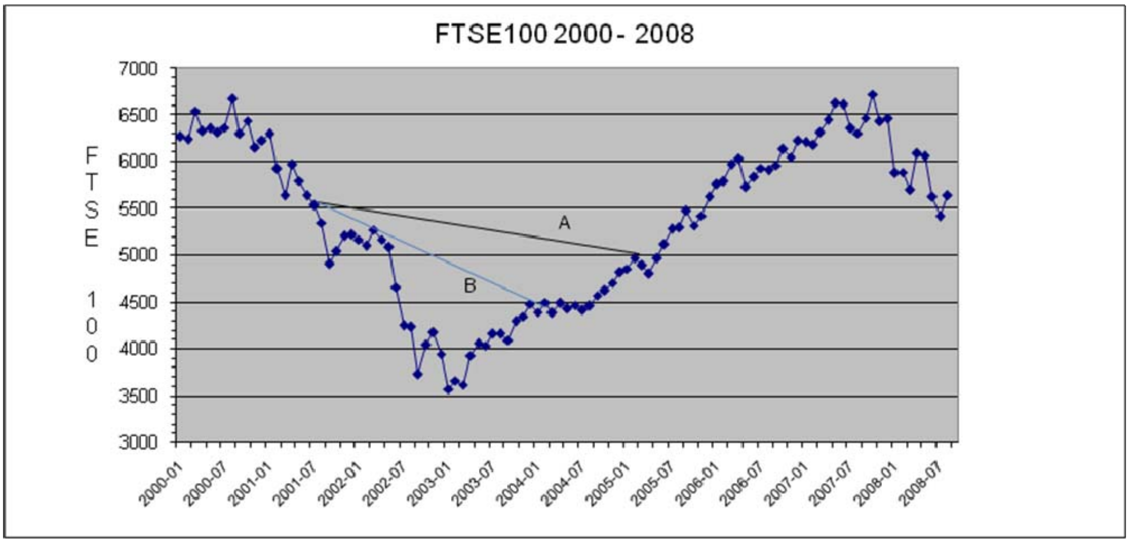

Figure 1 FTSE100, 2000-2008

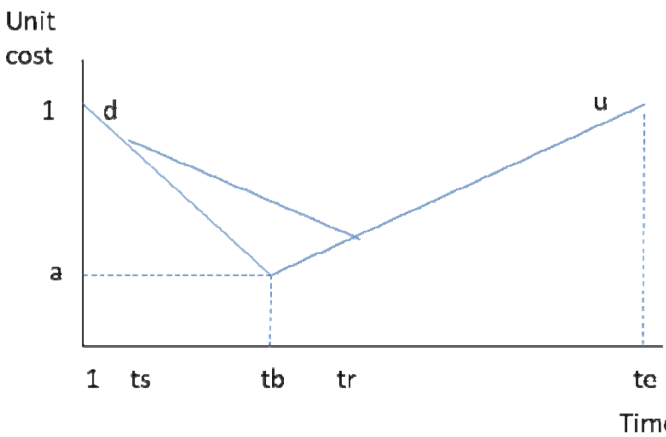

Figure 2 Idealised version of figure 1

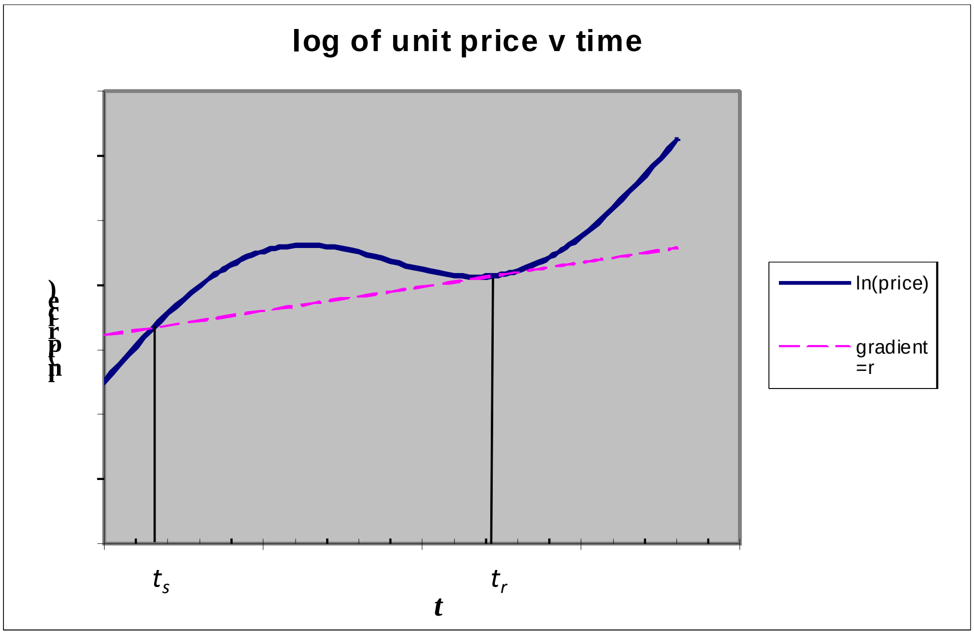

Figure 3 ln(price) vs time

Periods after start of upturn	Periods after start of downturn						
	4	8	12	16	20	24	28
2	£12,717	£12,416	£12,144	£11,903	£11,696	£11,527	£11,398
6	£12,410	£12,143	£11,904	£11,697	£11,524	£11,387	£11,291
10	£12,108	£11,871	£11,663	£11,486	£11,342	£11,234	£11,167
14	£11,812	£11,603	£11,422	£11,272	£11,154	£11,073	£11,032
18	£11,524	£11,339	£11,183	£11,056	£10,963	£10,906	£10,888
22	£11,243	£11,081	£10,946	£10,842	£10,770	£10,734	£10,738
26	£10,969	£10,828	£10,713	£10,629	£10,577	£10,560	£10,583
30	£10,703	£10,580	£10,484	£10,418	£10,384	£10,385	£10,426
34	£10,445	£10,339	£10,260	£10,210	£10,193	£10,211	£10,267
38	£10,194	£10,104	£10,040	£10,006	£10,004	£10,037	£10,108

Table 2 Gross returns at 3% interest for various stop/restart periods

References

Ballestero, E., Gunther, M., Pla-Santamaria, D., and Stummer, C. Portfolio selection under strict uncertainty: A multi-criteria methodology and its application to the Frankfurt and Vienna Stock Exchanges. European J. of Operational Research 2007; 181; (3) 1476-1487.

Barro, D., and Canestrelli, E. Dynamic portfolio optimization: Time decomposition using the Maximum Principle with a scenario approach. European. J. Operational Research 2005; 163; 217-229.

Brennan, M., Li, F., and Torous, W. Dollar cost averaging. Review of Finance 2005; 9; 509-535.

Constantinides, G. A note on the sub-optimality of dollar-cost averaging as an investment policy. J. of Financial and Quantitative Analysis 1979; 14; 443-450.

De Bondt, W., and Thaler, R. Does the stock market overreact? Journal of Finance 1985; 40; 793-805.

Greenhut, J. Mathematical illusion:why dollar-cost averaging does not work. J. of Financial Planning 2006; 19; (10) 76-83.

Jegadeesh, N., and Titman, S. Returns to buying winners and selling losers. Journal of finance 1993; 48; 65-91.

Leggio, K., and Lien, D. An empirical examination of the effectiveness of dollar-cost averaging using downside risk performance measures. J. of Economics and Finance 2003; 27; (2) 211-223.

Malkiel, B. A Random Walk Down Wall Street: The Best Investment Advice for the New Century, Norton & Co: New York; 1996.

Orman, S. The Road to Wealth: A Comprehensive Guide to your Money - Everything you Need to Know in Good and Bad Times, Riverhead Books: New York; 2001.

Rozeff, M. Lump-sum investing versus dollar-averaging. J. of Portfolio Management 1994; 4; (45-50).

Schwab, C. You're 50 - Now What? Investing for the Second Half of Your Life, Three rivers Press: New York; 2002.

Spronk, J., and Hallerbach, W. Financial modelling: Where to go? With an illustration for portfolio management. European J. Operational Research 1995; 99; (1) 113-125.

Statman, M. A behavioral framework for dollar-cost averaging. J. of Portfolio Management 1995; 3; 70-78.

Summers, L. Does the stock market rationally reflect fundamental values? Journal of Finance 1986; 41; 591-601.

Thorley, S. The fallacy of dollar-cost averaging. Financial Practice and Education 1994; 4; 138-143.

Williams, R., and Bacon, P. Lump sum beats dollar-cost averaging. Journal of Financial Planning 1993; 6; (2) 64-67.

University of Kent

http://www.kent.ac.uk/kbs/research-information/index.htm