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A note on the use of supply-use tables in impact analyses*

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Abstract

Little attention has so far been paid to the problems inherent in interpreting the meaning of results from standard impact analyses using symmetric input-output tables. Impacts as well as drivers of these impacts must be either of the product type or of the industry type. Interestingly, since supply-use tables distinguish products and industries, they can cope with product impacts driven by changes in industries, and vice versa. This paper contributes in two ways. Firstly, the demand-driven Leontief quantity model, both for industry-by-industry as well as for product-by-product tables, is formalised on the basis of supply-use tables, thus leading to impact multipliers, both for industries and products. Secondly, we demonstrate how the supply-use formulation can improve the incorporation of disparate satellite data into input-output models, by offering both industry and product representation. Supply-use blocks can accept any mix of industry and product satellite data, as long as these are not overlapping.

MSC: 91B76–Environmental economics (natural resource models, harvesting, pollution, etc.)

Keywords: Technology assumptions, supply-use framework, multipliers.

1. Background

An input-output matrix of technical coefficients (**A**) generally depicts either the direct requirements of commodity i needed to produce one physical unit of commodity j or, alternatively, the direct inputs from industry i needed to produce one physical unit of industry j. The former is built up with a product-by-product input-output table and the latter, with an industry-by-industry input-output table. Both are called symmetric input-

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output tables for having products or industries in both rows and columns; and the choice between them has only been addressed so far in detail by Rueda-Cantuche (2011).

Before the SNA-68 (UN, 1968), national statistical institutes almost exclusively constructed industry-by-industry input-output tables instead of commodity-by-commodity tables[1] and they used to set up the so-called transaction tables (ten Raa, 1994). In such tables, each element displayed the input requirements of industry i per unit of industry j's production, as well as the final demand compartments (household and government consumption, investment and exports net of imports). Ten Raa (1994) noted that an input-output transaction table reduced the construction of a matrix of technical coefficients **A** just to a matter of dividing each element by their corresponding total output.

However, there were three different problems identified here. Firstly, products and industries cannot always be classified in the same way. Secondly, in addition to a multitude of inputs, industries may also have a multitude of outputs. Thirdly, products contained in each row and column of an industry-by-industry table are not homogeneous in terms of production (see e.g. Rainer, 1989).

To address these complications, the Systems of National Accounts proposed by the United Nations (1968, 1993), first established the concepts of use and make matrices within an input-output framework. Demand (use) and supply (make) of commodities were described by industries. This new framework provided a more accurate description of product flows and at the same time, made economists face a new problem in the construction of technical coefficients. Basically, the construction of technical coefficients was reduced to a matter of treatment of secondary products. Many establishments produce only one group of commodities, which are the primary products of the industry to which they are classified. However, some establishments produce commodities that are not among the primary products of the industry to which they belong. As a result, non-zero off-diagonal elements would appear in the make matrix. Alternative treatments of secondary products rest upon the separation of outputs and inputs associated with secondary products so that they can be added to the outputs and inputs of the industry in which the secondary product is a characteristic output. Assumptions on these inputs structures imply an **A**-matrix of technical coefficients as a function of the use and make matrices. The reader should be aware that a make matrix (industry by product) is merely the transposition of a supply matrix (product by industry) and we may use both indistinctly.

The matrix of technical coefficients has been used for economic analysis by means of the so-called Leontief quantity model and the Leontief price model, which are based on the following two equations: $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y}$ and $\mathbf{p} = \mathbf{p}\mathbf{A} + \mathbf{v}$.

Here, $\mathbf{x}$ is a column vector of total output; $\mathbf{y}$, a column vector of final demand; $\mathbf{p}$, a row vector of prices; and $\mathbf{v}$, a row vector of value-added coefficients. The standard

1. In what follows, we will refer to the "commodity-by-commodity input-output tables" and "product-by-product input-output tables" as fully equivalent.

Leontief quantity model would be given by $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$, and the standard Leontief price model as $\mathbf{p} = \mathbf{v}(\mathbf{I} - \mathbf{A})^{-1}$.

The first equation is used for national or regional economic planning; in accordance, there will be a direct effect over the output levels which will depend on the final demand variations ($\Delta\mathbf{y}$) and additional indirect effects that will be determined by the so called Leontief inverse matrix, $(\mathbf{I} - \mathbf{A})^{-1}$. The second equation can be used to assess the price effects resulting from an energy shock, which surely will bring about variations in the value-added shares of a product, to mention an example. For the time being and for the sake of clarification, we have deliberately omitted the identification of the outputs as products or industries. We will introduce this distinction later on.

Within this context, two major trade-offs were recently indentified concerning the choice of type of symmetric input-output tables to be used in input-output analyses (Rueda-Cantuche, 2011). The main limitation of these tables relates to their underlying symmetry, which implies that they must be defined as either product-by-product or industry-by-industry.

On the one hand, the Leontief quantity model, which is driven by demand for products, presents a trade-off whenever the impact analysis relates to external accounts (environment, employment, etc) pre-multiplying the Leontief inverse matrix and which are only available at industry detail. Then, either one could incorrectly assume that these external accounts reflect product detail, and employ a product–by-product input-output table in order to assess the effects of a unit change in final demand of a single product; or, alternatively, correctly take the external accounts as industry-specific information and use industry-by-industry tables. The latter practice, however, would preclude calculating effects of changes in the final demand of single products, because in an industry-by-industry table, final demand only exists as mixed bundles of goods and services produced by particular industries.

On the other hand, the Leontief price model, which is driven by industry supply, imposes trade-offs whenever the impact analysis relates to external accounts that are only available at industry detail. In this case, key questions such as the fuel price effects generated by an increase in the labour costs of the petroleum refining industry cannot really be answered by input-output price models as it may be generally thought. Either one could incorrectly assume that variations in the primary costs (labour) happen within homogenous branches of activity rather than in industries and thus, employ product-by-product tables or instead, one could correctly assume that price changes of labour costs effectively occur within entire industries and therefore, use industry-by-industry tables. In the latter case, the reported price impacts will refer to the fuel industry rather than to the fuel product itself.

Rueda-Cantuche (2011) proposed the use of supply and use tables instead of input-output tables for resolving the different trade-offs efficiently. Indeed, supply and use tables are defined and compiled at product-by-industry detail and do not require the symmetries causing the trade-offs described above. However, this author did not go beyond the mere statement and discussion of the convenience of extending the use of supply and use tables in input-output analysis. Therefore, this paper is aimed at formalizing Rueda-Cantuche's argument concerning the trade-offs that he identified.

In the next section, we will introduce the basics of the construction of symmetric input-output tables, which will be further described under a common schematic representation in Section 3. In what follows, Section 4 will generalize the calculation of impact multipliers for industries and commodities, separately. Section 5 describes the main empirical findings for the Brazilian economy in 2005 and finally, the last section will draw the main conclusions of this paper.

2. Introduction

Amongst other textbooks, the *United Nations Handbook on Input-Output Table Compilation* (UN 1999) distinguishes two basic technology assumptions for the construction of symmetric product-by-product input-output tables: in the *industry technology assumption*, the production recipe is unique to an industry, while products' input recipe is a weighted sum over industries' production recipes; in the *commodity technology assumption*[2], the input recipe is unique to a product, while industries' production recipes are a weighted sum over their primary and secondary outputs.

In practice, both assumptions are known to have drawbacks: Applying the commodity technology assumption can lead to negative elements during table construction, and requires the supply matrix to be square, which could lead to loss of detail in rectangular accounts.[3] The commodity technology assumption has proven to be theoretically superior while the industry technology assumption has been shown to be implausible (Kop Jansen and Ten Raa 1990). Comparative advantages of these perspectives are however not the concern of this work.

The construction of industry-by-industry input-output tables[4] requires two main assumptions stating that when product output is translated into industry output, the pattern

2. In what follows, we will refer to the "commodity technology assumption" and the "product technology assumption" as fully equivalent.

3. Konijn and Steenge (1995), Almon (2000), Bohlin and Widell (2006) and Smith and McDonald (2011) suggest ways of getting around the problems associated with the technology assumption. Konijn and Steenge (1995) suggest an input allocation procedure that uses activities of industries in their production of products. However, the data necessary to make this method operational are generally not available. Almon (2000) suggests a balancing algorithm that explicitly deals with cases where the subtraction of inputs of from a secondary production recipe would generate negative entries. Bohlin and Widell (2006) (extended by Smith and McDonald, 2011) apply an optimisation calculus, where they define the technology assumptions in terms of process coefficients that are both industry- and product-specific (see Ten Raa and Rueda-Cantuche 2007), and then minimize the variance of these process coefficients subject to summation rules.

4. In this context, Yamano and Ahmad (2006) argue that the "description of the conversion (industry-technology assumption) is inaccurate where industry-by-industry tables are concerned, and is better described as *a fixed product sales structure* assumption. In other words the conversion merely assumes that the proportion of domestically produced commodity A bought by industry B from industry C is proportional to industry C's share of the total (domestic) economy production of commodity A. Put this way, it is clear that this is a far less demanding assumption than that implied by the equivalent, but differently named, "industry technology" assumption.

of sales will remain the same. This is the so-called sales structure approach that only admits two options: 1) where industry supply is independent of the products delivered (*fixed industry sales structure*), and 2) where industry supply is independent of the producing industry (*fixed product sales structure*). Employing arguments similar to those used in discussing the industry and product technology assumptions for the construction of product-by-product input-output tables (Ten Raa and Rueda-Cantuche 2007), Rueda-Cantuche and Ten Raa (2009) proved that the fixed industry sales approach is theoretically superior.

Notwithstanding the above theoretical considerations, statistical offices construct national input-output tables based on hybrid technology or combined fixed sales structure assumptions. So, in what follows, we will simply take what statistical office publish as given, and start with a formulation of their different assumptions using a supply-use framework; then, we will show how the supply-use blocks can be useful in simultaneously generating multipliers both for industries and for products (and thus solving the trade-offs caused by the symmetry of input-output tables).

We will show in the next sections how the industry technology and the fixed product sales structure assumptions can be jointly formulated in a common framework that allows carrying out impact analyses simultaneously in terms of products and industries. The same will apply for the product technology and the fixed industry sales structure assumptions.

3. Schematic representation of the assumptions made in the construction of input-output tables

In the *United Nations Handbook on Input-Output Table Compilation* (UN 1999) and the *Eurostat Manual of Supply, Use and Input-Output Tables* (Eurostat, 2008), there are various assumptions to be used for the construction of industry by industry or product-by-product symmetric input-output tables.[5] In the following, we will show that at least the ones referred to in the last section can be represented in one unified supply-use formulation. We will use the standard Eurostat Manual notation (UN 1999). Notice that the supply matrix, which we will denote $\mathbf{V}^{\mathsf{T}}$ corresponds to the transposition of the so called "make matrix".

5. A number of authors suggest further alternatives such as mixed technology and activity technology assumptions (see Gigantes (1970); Schinnar (1978); Konijn and Steenge (1995) but for a comprehensive list, see also Ten Raa and Rueda-Cantuche (2003)) For a generalized formulation of the industry and product technology assumptions, see Ten Raa and Rueda-Cantuche (2007).

3.1. Industry-related assumptions

Let a single-region *supply-use transaction block* $\mathbf{T}$ be represented by:

$$\mathbf{T} = \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \quad , \tag{1}$$

with $\mathbf{U}$ being a product-by-industry *use* matrix, showing the input U_{ij} of product i into industry j, and $\mathbf{V}$ being a industry-by-product *make* matrix, with V_{ij} showing the output by industry i of product j. This block formulation is well known in the input-output literature[6] and it is already proposed as an example by Eurostat (2008).

Let $\mathbf{T}$ satisfy the national accounting identity:

$$\begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_c \\ \mathbf{e}_i \end{bmatrix} + \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \mathbf{g} \end{bmatrix} \quad , \tag{2}$$

where $[\mathbf{e}_c\,\mathbf{e}_i]^{\mathsf{T}}$ is the row summation vector formed by two summation sub-vectors corresponding to commodities ($\mathbf{e}_c$) and industries ($\mathbf{e}_i$), superscript $^{\mathsf{T}}$ denotes transposition, $\mathbf{y}_c$ is a vector of *final demand* of products, and $\mathbf{q}$ and $\mathbf{g}$ are vectors of *total product and industry outputs*, respectively. Equation 2 includes the product balance $\mathbf{U}\mathbf{e}_i + \mathbf{y}_c = \mathbf{q}$, and the industry balance $\mathbf{V}\mathbf{e}_c = \mathbf{g}$. Therefore, it can be transformed into:

$$\begin{aligned}
&\begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \mathbf{g} \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_c \\ \mathbf{e}_i \end{bmatrix} = \left\{ \begin{bmatrix} \hat{\mathbf{q}} & 0 \\ 0 & \hat{\mathbf{g}} \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \right\} \begin{bmatrix} \mathbf{e}_c \\ \mathbf{e}_i \end{bmatrix} = \\
&\quad \left\{ \begin{bmatrix} \hat{\mathbf{q}} & 0 \\ 0 & \hat{\mathbf{g}} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{q}}^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{q}}^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} \right\} \begin{bmatrix} \hat{\mathbf{q}} & 0 \\ 0 & \hat{\mathbf{g}} \end{bmatrix} \begin{bmatrix} \mathbf{e}_c \\ \mathbf{e}_i \end{bmatrix} = \\
&\quad \left\{ \begin{bmatrix} \mathbf{I}_c & 0 \\ 0 & \mathbf{I}_i \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{q}}^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{q} \\ \mathbf{g} \end{bmatrix} \\
&\Leftrightarrow \begin{bmatrix} \mathbf{q} \\ \mathbf{g} \end{bmatrix} = \left\{ \mathrm{I} - \begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{q}}^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} \right\}^{-1} \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \left\{ \mathbf{I} - \begin{bmatrix} 0 & \mathbf{B} \\ \mathbf{D} & 0 \end{bmatrix} \right\}^{-1} \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix}
\end{aligned} \quad , \tag{3}$$

where $\mathbf{D}$ and $\mathbf{B}$[7] form the *supply* and *use coefficient matrices*, $\mathbf{I}$ is an identity matrix, and the hat symbol (^) denotes a diagonalised vector. $\mathbf{B} = \mathbf{U}\hat{\mathbf{g}}^{-1}$ is called the (product-by industry) *use coefficients* matrix (input structures), and $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1}$ is called the (industry-by-product) *market share* matrix.

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6. Note that the supply-use-block formulation requires the make matrix $\mathbf{V}$ to be defined as industry-by-product, and not as product-by-industry.

7. Our B matrix is equivalent to the Z matrix in Eurostat (2008).

Using the supply-use-block formulation in Equation (3), a compound Leontief inverse can be written as:

$$\mathrm{L}_{\mathrm{I}}^{*} = \begin{bmatrix} \mathbf{I} & -\mathbf{B} \\ -\mathbf{D} & \mathbf{I} \end{bmatrix}^{-1} \quad . \tag{4}$$

Applying the partitioned inverse of Miyazawa (1968), Equation (4) can be written as:

$$\mathbf{L}_{\mathrm{I}}^{*} = \begin{bmatrix} \mathbf{I}+\mathbf{BL}_{\mathrm{I,ii}}\mathbf{D} & \mathbf{BL}_{\mathrm{I,ii}} \\ \mathbf{L}_{\mathrm{I,ii}}\mathbf{D} & \mathbf{L}_{\mathrm{I,ii}} \end{bmatrix} \quad , \tag{5}$$

where $\mathbf{L}_{\mathrm{I,ii}} = (\mathbf{I}-\mathbf{DB})^{-1}$ is precisely the Leontief inverse of the industry-by-industry type of a technical coefficient matrix constructed on the basis of the *fixed product sales structure* (see Eurostat, 2008, p. 349). Considering the series expansion of $\mathbf{L}_{\mathrm{I,ii}} = (\mathbf{I}+\mathbf{DB}+(\mathbf{DB})(\mathbf{DB})+\cdots)$, we find:

$$\begin{aligned}\mathbf{BL}_{\mathrm{I,ii}}\mathbf{D} &= \mathbf{B}\left(\mathbf{I}+\mathbf{DB}+(\mathbf{DB})^{2}+\cdots\right)\mathbf{D} = \mathbf{BD}+\mathbf{B}(\mathbf{DB})\mathbf{D}+\mathbf{B}(\mathbf{DBDB})\mathbf{D}+\cdots = \\ &= \mathbf{BD}+(\mathbf{BD})(\mathbf{BD})+(\mathbf{BD})(\mathbf{BD})(\mathbf{BD})+\cdots,\end{aligned}$$

which leads to:

$$\mathbf{I}+\mathbf{BL}_{\mathrm{I,ii}}\mathbf{D} = \mathbf{I}+\mathbf{BD}+(\mathbf{BD})(\mathbf{BD})+\cdots = (\mathbf{I}-\mathbf{BD})^{-1} = \mathbf{L}_{\mathrm{I,cc}},$$

and which is identical to the series expansion of the Leontief inverse of a product-by-product type technical coefficients matrix constructed with the *industry technology model* (see Eurostat, (2008), p. 349). Equation (5) can be simplified to

$$\mathbf{L}_{\mathrm{I}}^{*} = \begin{bmatrix} \mathbf{L}_{\mathrm{I,cc}} & \mathbf{L}_{\mathrm{I,cc}}\mathbf{B} \\ \mathbf{L}_{\mathrm{I,ii}}\mathbf{D} & \mathbf{L}_{\mathrm{I,ii}} \end{bmatrix}. \tag{6}$$

Regarding the off-diagonal elements, the reader may find easily that $\mathbf{L}_{\mathrm{I,ii}}\mathbf{D} = \mathbf{DL}_{\mathrm{I,cc}}$ and $\mathbf{BL}_{\mathrm{I,ii}} = \mathbf{L}_{\mathrm{I,cc}}\mathbf{B}$. The matrices of market shares **D** and of input structures **B** are clearly used to convert the resulting impacts of industries into those of products, and impacts of products into those of industries, respectively.

Hence, and this is the first result of this paper, when supply and use matrices are handled under an integrated supply-use framework, the compound Leontief inverse elegantly reproduces the product-by-product type model under the industry technology assumption and the industry-by-industry model under the fixed product sales structure assumption.

3.2. Product-related assumptions

Product technology assumes an input recipe that is characteristic for a certain product. Here, we use the relationships $\mathbf{V}\mathbf{e}_c = \hat{\mathbf{g}}\mathbf{e}_i = \mathbf{g}$ and $\mathbf{V}^{\mathsf{T}}\mathbf{e}_i = \mathbf{q}$, and re-write the national accounting identity in Equation (2) as:

$$\begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_c \\ \mathbf{e}_i \end{bmatrix} + \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{U}\mathbf{e}_i + \mathbf{y}_c \\ \mathbf{V}\mathbf{e}_c \end{bmatrix} = \begin{bmatrix} \mathbf{U}\mathbf{e}_i + \mathbf{y}_c \\ \hat{\mathbf{g}}\mathbf{e}_i \end{bmatrix} = \tag{7}$$

$$= \begin{bmatrix} 0 & \mathbf{U} \\ \hat{\mathbf{g}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_i \\ \mathbf{e}_i \end{bmatrix} + \begin{bmatrix} \mathbf{y}_\mathrm{c} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{V}^{\mathsf{T}}\mathbf{e}_i \\ \mathbf{g} \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \mathbf{g} \end{bmatrix},$$

Then, Equation (7) can be transformed into:

$$\begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{V}^{\mathsf{T}}\mathbf{e}_i \\ \mathbf{g} \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \hat{\mathbf{g}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_i \\ \mathbf{e}_i \end{bmatrix} = \left\{ \begin{bmatrix} \mathbf{V}^{\mathsf{T}} & 0 \\ 0 & \hat{\mathbf{g}} \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \hat{\mathbf{g}} & 0 \end{bmatrix} \right\} \begin{bmatrix} \mathbf{e}_i \\ \mathbf{e}_i \end{bmatrix} =$$

$$\left\{ \begin{bmatrix} \left(\mathbf{V}^{\mathsf{T}}\right) & 0 \\ 0 & \hat{\mathbf{g}} \end{bmatrix} \begin{bmatrix} \left(\mathbf{V}^{\mathsf{T}}\right)^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \hat{\mathbf{g}} & 0 \end{bmatrix} \begin{bmatrix} \left(\mathbf{V}^{\mathsf{T}}\right)^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} \right\} \begin{bmatrix} \left(\mathbf{V}^{\mathsf{T}}\right) & 0 \\ 0 & \hat{\mathbf{g}} \end{bmatrix} \begin{bmatrix} \mathbf{e}_i \\ \mathbf{e}_i \end{bmatrix} =$$

$$\left\{ \begin{bmatrix} \mathbf{I}_i & 0 \\ 0 & \mathbf{I}_i \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{U} \\ \hat{\mathbf{g}} & 0 \end{bmatrix} \begin{bmatrix} \left(\mathbf{V}^{\mathsf{T}}\right)^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{V}^{\mathsf{T}}\mathbf{e}_i \\ \mathbf{g} \end{bmatrix} \tag{8}$$

$$\Leftrightarrow \begin{bmatrix} \mathbf{V}^{\mathsf{T}}\mathbf{e}_i \\ \mathbf{g} \end{bmatrix} = \left\{ \mathbf{I} - \begin{bmatrix} 0 & \mathbf{U} \\ \hat{\mathbf{g}} & 0 \end{bmatrix} \begin{bmatrix} \left(\mathbf{V}^{\mathsf{T}}\right)^{-1} & 0 \\ 0 & \hat{\mathbf{g}}^{-1} \end{bmatrix} \right\}^{-1} \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \left\{ \mathbf{I} - \begin{bmatrix} 0 & \mathbf{B} \\ \mathbf{C}^{-1} & 0 \end{bmatrix} \right\}^{-1} \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix},$$

where $\mathbf{C} = \mathbf{V}^{\mathsf{T}}\hat{\mathbf{g}}^{-1}$ and $\mathbf{B}$ form the *supply* and *use coefficients* blocks, respectively.

Using the supply-use-block formulation in Equation (8), a new compound Leontief inverse can be written as:

$$\mathbf{L}_\mathrm{C}^* = \begin{bmatrix} \mathbf{I} & -\mathbf{B} \\ -\mathbf{C}^{-1} & \mathbf{I} \end{bmatrix}^{-1}. \tag{9}$$

Applying the partitioned inverse of Miyazawa 1968), Equation (9) can be written as:

$$\mathbf{L}_\mathrm{C}^* = \begin{bmatrix} \mathbf{I} + \mathbf{B}\mathbf{L}_{\mathrm{C,ii}}\mathbf{C}^{-1} & \mathbf{B}\mathbf{L}_{\mathrm{C,ii}} \\ \mathbf{L}_{\mathrm{C,ii}}\mathbf{C}^{-1} & \mathbf{L}_{\mathrm{C,ii}} \end{bmatrix}, \tag{10}$$

where $\mathbf{L}_{\mathrm{C,ii}} = \left(\mathbf{I} - \mathbf{C}^{-1}\mathbf{B}\right)^{-1}$ is the Leontief inverse of the industry-by-industry technical coefficient matrix constructed on the basis of the *fixed industry sales structure* (Eurostat, (2008), p. 349). Considering the series expansion of $\mathbf{L}_{\mathrm{C,ii}} = \left(\mathbf{I} + \mathbf{C}^{-1}\mathbf{B} + \left(\mathbf{C}^{-1}\mathbf{B}\right)\left(\mathbf{C}^{-1}\mathbf{B}\right) + \cdots\right)$, we find:

$$\begin{aligned}\mathbf{B}\mathbf{L}_{\mathrm{C,ii}}\mathbf{C}^{-1} &= \mathbf{B}\left(\mathbf{I}+\mathbf{C}^{-1}\mathbf{B}+\left(\mathbf{C}^{-1}\mathbf{B}\right)^{2}+\cdots\right)\mathbf{C}^{-1} = \\ &= \mathbf{B}\mathbf{C}^{-1}+\mathbf{B}\left(\mathbf{C}^{-1}\mathbf{B}\right)\mathbf{C}^{-1}+\mathbf{B}\left(\mathbf{C}^{-1}\mathbf{B}\right)\left(\mathbf{C}^{-1}\mathbf{B}\right)\mathbf{C}^{-1}+\cdots = \\ &= \mathbf{B}\mathbf{C}^{-1}+\left(\mathbf{B}\mathbf{C}^{-1}\right)\left(\mathbf{B}\mathbf{C}^{-1}\right)+\left(\mathbf{B}\mathbf{C}^{-1}\right)\left(\mathbf{B}\mathbf{C}^{-1}\right)\left(\mathbf{B}\mathbf{C}^{-1}\right)+\cdots\end{aligned}$$

and yields:

$$\mathbf{I}+\mathbf{B}\mathbf{L}_{\mathrm{C,ii}}\mathbf{C}^{-1} = \mathbf{I}+\mathbf{B}\mathbf{C}^{-1}+\left(\mathbf{B}\mathbf{C}^{-1}\right)\left(\mathbf{B}\mathbf{C}^{-1}\right)+\cdots = \left(\mathbf{I}-\mathbf{B}\mathbf{C}^{-1}\right)^{-1} = \mathbf{L}_{\mathrm{C,cc}},$$

which is the series expansion of the Leontief inverse of a product-by-product technical coefficient matrix using the *product technology model* (see Eurostat, (2008), p. 349). Then, Equation (10) can be reduced to:

$$\mathbf{L}_{\mathrm{C}}^{*} = \begin{bmatrix} \mathbf{L}_{\mathrm{C,cc}} & \mathbf{L}_{\mathrm{C,cc}}\mathbf{B} \\ \mathbf{L}_{\mathrm{C,ii}}\mathbf{C}^{-1} & \mathbf{L}_{\mathrm{C,ii}} \end{bmatrix}. \tag{11}$$

It is easy to show that the off-diagonal terms transform as $\mathbf{L}_{\mathrm{C,ii}}\mathbf{C}^{-1} = \mathbf{C}^{-1}\mathbf{L}_{\mathrm{C,cc}}$ and $\mathbf{B}\mathbf{L}_{\mathrm{C,ii}} = \mathbf{L}_{\mathrm{C,cc}}\mathbf{B}$. The matrices $\mathbf{C}^{-1}$ and $\mathbf{B}$ are used to convert impacts of industries into those of products, and impacts of products into those of industries, respectively.

Hence, as the second result of this paper, when supply and use matrices are integrated in a supply-use framework, the compound Leontief inverse elegantly reproduces the product-by-product type model assuming the product technology assumption and the industry-by-industry model assuming the fixed industry sales structure assumption.

It is interesting to note that these two models provide negative elements in the resulting technical coefficient matrices, while the models dealt with in the subsection 3.1 always provide non-negative terms.

4. Generalized input-output calculations

It was always the intention of Leontief to combine the input-output table with external, physical information, for example in order to examine questions relating to environmental impacts or the labour market (Leontief and Ford, 1970; Leontief and Duchin, 1986). Since Leontief's work there have been numerous publications of what Miller and Blair (2009) call *generalized* input-output analyses. For example, Kagawa and Suh (2009), and Suh *et al.* (2010) (see also references therein) use make and use matrices in environmental Life Cycle Assessment. When applied to a supply-use framework, the generalised calculus elegantly reproduces industry and product multipliers in one single shot. Assume for example that external physical information $\mathbf{f}_{\mathrm{i}}$ is available only at the industry level. Invoking the industry technology assumption, as in Equation (6), multipliers can be written as:

$$\begin{bmatrix} 0 & \mathbf{f}_\mathrm{i} \end{bmatrix} \mathbf{L}_\mathrm{I}^* = \begin{bmatrix} \mathbf{f}_\mathrm{i}\,\mathbf{L}_\mathrm{I,ii}\,\mathbf{D} & \mathbf{f}_\mathrm{i}\,\mathbf{L}_\mathrm{I,ii} \end{bmatrix}, \tag{12}$$

with $\mathbf{f}_\mathrm{i}\,\mathbf{L}_\mathrm{I,ii}$ representing industries, and $\mathbf{f}_\mathrm{i}\,\mathbf{L}_\mathrm{I,ii}\,\mathbf{D}$ representing commodities. This feature was applied in a generalized multi-region analyses of embodied CO_2 for Denmark and its trading partners (Lenzen *et al.* 2004). Alternatively, if physical information $\mathbf{f}_\mathrm{c}$ is available only for products, multipliers are then defined as:

$$\begin{bmatrix} \mathbf{f}_\mathrm{c} & 0 \end{bmatrix} \mathbf{L}_\mathrm{I}^* = \begin{bmatrix} \mathbf{f}_\mathrm{c}\,\mathbf{L}_\mathrm{I,cc} & \mathbf{f}_\mathrm{c}\,\mathbf{L}_\mathrm{I,cc}\,\mathbf{B} \end{bmatrix}. \tag{13}$$

In a study on Brazil by Wachsmann *et al.* (2009), physical information on energy consumption was generally available for industries as $\mathbf{f}_\mathrm{i}$, but for some industries, detailed commodity information was available as $\mathbf{f}_\mathrm{c}$. Hence, a vector $\begin{bmatrix} \mathbf{f}_\mathrm{c} & \mathbf{f}_\mathrm{i}^* \end{bmatrix}$ was constructed with $\mathbf{f}_\mathrm{i}^*$ representing the industry data $\mathbf{f}_\mathrm{i}$ and setting the industries represented in $\mathbf{f}_\mathrm{c}$ to zero. The industry and product multipliers are then:

$$\begin{bmatrix} \mathbf{f}_\mathrm{c} & \mathbf{f}_\mathrm{i}^* \end{bmatrix} \mathbf{L}_\mathrm{I}^* = \begin{bmatrix} \mathbf{f}_\mathrm{i}^*\,\mathbf{L}_\mathrm{I,ii}\,\mathbf{D} + \mathbf{f}_\mathrm{c}\,\mathbf{L}_\mathrm{I,cc} & \mathbf{f}_\mathrm{i}^*\,\mathbf{L}_\mathrm{I,ii} + \mathbf{f}_\mathrm{c}\,\mathbf{L}_\mathrm{I,cc}\,\mathbf{B} \end{bmatrix}. \tag{14}$$

In Equations (12–14), the matrices $\mathbf{D}$ and $\mathbf{B}$ are used to convert industry data into product data ($\mathbf{f}_\mathrm{i}\,\mathbf{L}_\mathrm{I,ii}$), and vice versa ($\mathbf{f}_\mathrm{c}$). Similar relationships can be derived for models assuming product-related assumptions.

5. Empirical application

Provided rectangular supply-use frameworks with more products than industries, the calculation of total energy intensities (energy multipliers) can differentiate between products and industries, and thus add value over conventional mutlipliers based on input-output tables. Take for instance, the petrol and coke refining industry, which may produce petrol, fuel oil and diesel oil, amongst other products.

In order to prove the utility of supply-use tables in impact analysis and the theoretical framework presented before, we will run two experiments aiming to compare supply-use-based with input-output-based energy multipliers for the Brazilian economy in 2005. In particular, we will first determine simultaneously industry and commodity multipliers as in (12), assuming that energy data are only available at the industry level ($\mathbf{f}_\mathrm{i}$). Second, we will discuss the difference between supply-use-based commodity multipliers and input-output-based industry multipliers when a mix of energy industry data $\mathbf{f}_\mathrm{i}$ and energy commodity data $\mathbf{f}_\mathrm{c}$ is used.

The Brazilian supply-use tables for 2005 issued by the Instituto Brasileiro de Geografia e Estatística (IBGE, 2008) distinguish 110 commodities, but only 55 industries. Commodity detail is higher than industry detail especially for agriculture, food manufacturing, and refining. Whilst energy data are not available at the high commodity detail

Table 1: *Energy multipliers (in units of kt oil equivalent per million 2005 Reais–ktoe/mR\$, where 1 ktoe/mR\$ = 102 terajoules per million US\$) for Brazilian cropping and forestry industries and commodities.*

	Industry data only	Industry & commodity data
SUT industry multipliers	$\mathbf{f}_i \mathbf{L}_{I,ii}$	$\mathbf{f}_i^* \mathbf{L}_{I,ii} + \mathbf{f}_c \mathbf{L}_{I,cc} \mathbf{B}$
Cropping and forestry	0.1999	0.1595
Grazing and fishing	0.1024	0.1051
SUT commodity multipliers	$\mathbf{f}_i \mathbf{L}_{I,ii} \mathbf{D}$	$\mathbf{f}_i^* \mathbf{L}_{I,ii} \mathbf{D} + \mathbf{f}_c \mathbf{L}_{I,cc}$
Rice in the husk	0.1960	0.1136
Corn	0.1721	0.1114
Wheat	0.1999	0.1140
Sugar cane	0.1981	0.1138
Soy beans	0.1972	0.1136
Other crops	0.1910	0.1129
Manioc	0.1948	0.1135
Tobacco	0.1992	0.1139
Cotton	0.1956	0.1136
Citrus fruit	0.1974	0.1137
Coffee	0.1970	0.1137
Forestry products	0.1973	1.3943

for the agriculture and food manufacturing sectors, energy data on refining distinguishes diesel oil, fuel oil, petrol, and LPG (EPE 2011).

In a first experiment, we re-classified the Brazilian raw energy data into the 55-industry classification $\mathbf{f}_i$. As (12) shows, supply-use-based and input-output-based industry multipliers are the same: $\mathbf{f}_i \mathbf{L}_{I,ii}$. However, the supply-use framework allows the simultaneous determination of commodity multipliers $\mathbf{f}_i \mathbf{L}_{I,ii} \mathbf{D}$ (see (12)). Except for wheat, which is solely produced by the 'Cropping and forestry' industry, commodity multipliers are lower than industry multipliers for all crops (Table 1, industry data only column). This is because some crops are partly produced by mixed-business broadacre farms in the less energy-intensive 'Grazing and fishing' sector. Such co-product detail is only available in supply-use tables, and lost in input-output tables. The error associated with this loss of detail is 16% for corn, and 1-3% for other crops.[8]

In a second experiment, we re-classified only the raw energy data for the petroleum and coke refining sector into the 110-commodity classification $\mathbf{f}_c$, and deleted the entry for petroleum and coke refining in the industry data $\mathbf{f}_i^*$. As Equation (14) shows, both supply-use-based industry and commodity multipliers are now different from input-output-based industry multipliers. Once again, the supply-use framework allows the simultaneous determination of commodity multipliers. Only now, the distinction of 'Forestry products' as an energy-intensive industry becomes apparent (Table 1, industry and commodity data column). This is because wood charcoal operations that are only

8. Relative errors are calculated as $|\mathbf{f}_i \mathbf{L}_{I,ii} \mathbf{D} - \mathbf{f}_i^* \mathbf{L}_{I,ii} \mathbf{D} + \mathbf{f}_c \mathbf{L}_{I,cc}| / \mathbf{f}_i \mathbf{L}_{I,ii} \mathbf{D}$, in this case $|0.1721 - 0.1114|/0.1721 \approx 16\%$.

Table 2: *Energy multipliers (in units of kt oil equivalent per million 2005 Reais–ktoe/mR$) for Brazilian petroleum and coke refining industries and commodities.*

	Industry & commodity data	
	IO industry multipliers	SUT multipliers
Petroleum and coke refining	0.9859	0.8406
LPG		1.1012
Petrol		0.4727
Gasoalcohol		0.4693
Fuel oil		1.6410
Diesel oil		1.2405
Other petroleum and coke refining products		0.8804

part of the 'Forestry products' sub-sector consume much more energy than cropping. Once, again, such detail is only available in supply-use tables, since forestry and cropping is aggregated in input-output tables. The error associated with this aggregation is in the order of 80%.

Similar errors between 70% and 90% can be observed when comparing the one input-output-based multiplier for the 'Petroleum and coke refining' industry, and the supply-use-based commodity multipliers for the six refining sub-sectors (Table 2). Here, LPG, fuel oil, and diesel oil appear more energy-intensive than petrol and gasoalcohol, which once again cannot be discerned from input-output industry multipliers.

Our results bear significant implications for real-world policy. Assume for example that the Brazilian Government debated the impact of a 90 R$/toe energy tax (about 5% on top of the price of petrol, for example) on agricultural commodities, and in turn on different food products. Such a policy question would be rather mis-informed by any analysis using only industry-specific energy data (see Table 1). Opponents of such energy taxes could base their arguments on multipliers derived from industry data, and warn that if the government went ahead with the tax, households (who consumed 46.5 bR$ of crop sector output in 2008) would be short by 90 R$/toe $\times$ 0.19 ktoe/mR$ $\times$ 46.5 bR$ = 800 mR$. However, upon using mixed industry and commodity data in a SUT framework, it would become clear that some of this tax impact would in reality affect forestry products (charcoal), and not crop-based products, and that the real adverse impact on households would be significantly lower at 90 R$/toe $\times$ 0.11 ktoe/mR$ $\times$ 46.5 bR$ = 450 mR$.

6. Conclusions

We believe that the unnoticed drawback underlying the use of input-output tables in impact analyses is their symmetry, in the sense that they must be defined either on a product-by-product or on an industry-by-industry basis. Rueda-Cantuche (2011) identified two major trade-offs in the calculation of impact multipliers when using symmetric input-output tables. However, the author only stated that supply-use tables would overcome this undesirable effect but without formalising his argument. This note extends Rueda-Cantuche's reasoning and shows that the use of supply-use tables in a common framework concerning product- and industry-related assumptions may overcome the undesirable limitations of symmetric input-output tables. We show that the industry technology and the fixed product sales structure assumptions can be jointly formulated in a common framework that allows us to carry out impact analyses simultaneously in terms of products and industries. The same applies for the product technology and the fixed industry sales structure assumptions. As we have proven for the empirical example of Brazilian energy multipliers, using rectangular supply-use tables has significant advantages for real-world impact analyses whenever physical satellite data (environmental, socio-economic, tourism, etc.) are available.

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