Classical Horizontal Inequity and Reranking: an Integrated Approach

by

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Abstract

The last 20 years have seen a significant evolution in the literature on horizontal inequity (HI) and have generated two major and "rival" methodological strands, namely, classical HI and reranking. We propose in this paper a class of ethically flexible tools that integrate these two strands. This is achieved using a measure of inequality that merges the well-known Gini coefficient and Atkinson indices, and that allows a decomposition of the total redistributive effect of taxes and transfers into a vertical equity effect and a loss of redistribution due to either classical HI or reranking. An inequality-change approach and a money-metric cost-of-inequality approach are developed. The latter approach makes aggregate classical HI decomposable across groups. As in recent work, equals are identified through a nonparametric estimation of the joint density of gross and net incomes. An illustration using Canadian data from 1981 to 1994 shows a substantial, and increasing, robust erosion of redistribution attributable both to classical HI and to reranking, but does not reveal which of reranking or classical HI is more important since this requires a judgement that is fundamentally normative in nature.

Keywords Horizontal inequity, reranking, tax equity, inequality, Canadian tax system.

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1 Introduction

The assessment of tax systems draws on two fundamental principles: efficiency and equity. The former relates to the presence of distortions in the economic behaviour of agents, while the latter focuses on distributive justice. Persistent conflicts in the application of these principles often mean that an optimal tax system must strive to balance them.

In this paper we examine in more detail a specific aspect of the notion of equity: horizontal equity (HE) in taxation. Two main approaches to the measurement of HE are found in the literature, which has evolved substantially in the last twenty years. The classical formulation of the HE principle prescribes the equal treatment of individuals who share the same level of welfare before government intervention. HE may also be viewed as implying the absence of reranking: for a tax to be horizontally equitable, the ranking of individuals on the basis of pre-tax welfare should not be altered by a fiscal system.

This paper draws on this conceptual duality by proposing a new set of tools that allow one to study simultaneously both classical horizontal inequity (HI) and reranking. For this, we use a class of social welfare functions that display aversion to riskiness in net incomes[1] as well as aversion to rank inequality and relative deprivation. The associated class of inequality indices combines the popular Gini coefficient and Atkinson index of inequality, and is related to functions found in rank-dependent expected utility theory. This dual functional structure of our social welfare functions allows us to measure jointly classical HI and reranking on fundamentally separate normative bases, a new feature in the literature. As in several recent papers[2], we also integrate the measurement of vertical equity (VE) and HE, and decompose the redistributive effect of taxes and transfers into a VE effect, a classical HI effect, and a reranking effect. Unlike previous work, however, we allow for different parameters of aversion to classical HI and to reranking[3], and we evaluate classical HI and reranking using separate and specific "functional" bases. Separate normative weights on reranking, classical HI and VE seem justified by the different normative underpinnings of these concepts, as argued in the next section, and can also lead to optimal tax outcomes that are different from when these criteria are identically weighted. Ethical flexibility in the weighting of these criteria can also enable sensitivity checks on comparisons of classical HI and reranking across time and/or distributions.

A change-in-inequality approach and a cost-of-inequality approach to the decompo-

[1]Gross incomes are market incomes (pre-tax and pre-transfer incomes), and net incomes are disposable incomes (total incomes after taxes and transfers).

[2]See for instance Aronson, Johnson and Lambert (1994), Aronson and Lambert (1994) and Aronson, Lambert and Trippeer (1997).

[3]This is in the spirit of King (1983) and Chakravarty (1985) for VE and reranking, and Auerbach and Hasset (1999) for VE and classical HI.

sition of total redistribution are developed. The change-in-inequality approach has been hithertho the most popular in the literature, probably because of its close link with the now conventional use of inequality indices in distributive analysis. The cost-of-inequality approach has, however, the advantage of being money-metric: it gives the amount of money that society would be willing to give up to eliminate inequality, to preserve VE or to eliminate classical HI and reranking, and therefore provides indicators that are comparable to other money-metric indicators of government performance (such as equivalent or compensating variations for the assessment of efficiency criteria). As we will see, classical HI indicators based on the cost-of-inequality approach are also additively decomposable across percentiles and subgroups for wide ranges of ethical parameter values, and with local weights that are "uncontaminated" by vertical equity considerations[4].

The empirical estimation of these indices must also be tackled. In part, this is because (as noted in sections 2 and 5) estimation difficulties were deemed in the past to limit severely the practical relevance of some approaches to measuring VE and HI. For our change-in-inequality approach, natural estimators for the proposed indices are easily derived and computed and involve at most no more than the use of a non-parametric regression of net income against gross income. For the cost-of-inequality indices, straightforward estimators are again available for the assessment of the total redistributive and reranking effects, but an estimation of the variability of net incomes around predicted net incomes is now needed to estimate VE and the local and global welfare costs of the presence of classical HI. We again suggest estimating this non-parametrically. For the cost-of-inequality approach, for instance, we use a kernel estimation of the conditional distribution of net incomes at various percentiles of gross incomes (in the fashion of Duclos and Lambert (2000)). This makes estimators of classical HI statistically consistent and it also avoids the need for somewhat arbitrary normative assumptions on the treatment of "near-equals".

An illustration using Canadian data from 1981 to 1994 shows a substantial, and increasing, erosion of redistribution attributable to both HI and reranking. We also notice a greater problem with HI among low income households. It is not possible to say, however, whether reranking is a greater problem than classical HI, since this also involves a *normative and not solely an empirical* judgement. These results can be explained reasonably well by macroeconomic shocks (particularly the two recessions that struck Canada during that period), by socio-demographic changes, and by reforms in the tax and transfer system that increased its bite and its complexity.

The rest of the paper runs as follows. Section 2 reviews in some detail the evolution of the concept and measurement of horizontal inequity. This review is important since it justifies the methodological approach followed subsequently in the paper. Section 3

[4] Lambert and Ramos (1997) implemented this idea first, though in an environment with ethical "rigidity".

introduces the notation and our dual-parameter social welfare function. Section 4 develops the two sets of measures of total redistribution, VE, classical HI and reranking. The estimation procedure is presented in section 5. Section 6 discusses the paper's links with previous work. Our methodology is illustrated, using Canadian data, in section 7. The last section briefly concludes.

2 Classical horizontal equity and reranking

Why should concerns for horizontal equity influence the design of an optimal tax system? Several answers have been provided, using either of two approaches. The traditional or "classical" approach defines HE as the equal treatment of equals (see Musgrave (1959)). While this principle is generally well accepted, different rationales are advanced to support it. First, a tax which discriminates between comparable individuals is liable to create resentment and a sense of insecurity, possibly also leading to social unrest. This is supported by the socio-psychological literature which shows that exclusion and discrimination have an impact both on individual well-being and on social cohesion and welfare. For instance, status/role structure theory indicates that one's relative socio-economic position (and its variability) "may give rise to definable and measurable social and psychological reactions, such as different types of alienation" (Durant and Christian (1990), p.210).

Second, the principles of progressivity and income redistribution, which are key elements of most tax and transfer systems, are generally undermined by HI (as we shall see in our own treatment below). This has indeed been one of the main themes in the development of the reranking approach in the last decades (see for instance Atkinson (1979) and Jenkins (1988)). Hence, a desire for HE may simply derive from a general aversion to inequality, without any further appeal to other normative criteria. Feldstein (1976) also notes that when utility functions are identical across individuals, a utilitarian social welfare function is maximized when equal incomes are taxed equally. This result then makes the principle of HE become a corollary of the principle of VE. A separate justification of HE would, however, generally be required when preferences are heterogeneous (they usually are), and because in some circumstances a random tax can otherwise be found to be optimal (Stiglitz (1982)). HI may moreover suggest the presence of imperfections in the operation of the tax and transfer system, such as an imperfect delivery of social welfare benefits, attributable to poor targeting or to incomplete take-up (see Duclos (1995b)). It can also signal tax evasion, which can *inter alia* cost the government significant losses of tax revenue (see Bishop *et al.* (1994)).

Third, HE can be argued to be an ethically more robust principle than VE (VE relates to the reduction of welfare gaps between unequal individuals). The HE principle is often seen as a consequence of the fundamental moral principle of the equal worth of human beings, and as a corollary of the equal sacrifice theories of taxation. Depending on the retained specification of distributive fairness, the requirements of vertical justice can vary considerably, while the principle of horizontal equity remains essentially invariant (Musgrave (1990)). Plotnick (1982) also supports this view by arguing that HI in the redistributive process would cause a loss of social welfare relative to an horizontally equitable tax, regardless of any VE value judgments on the final distribution. This has led several authors (including Stiglitz (1982), Balcer and Sadka (1986) and Hettich (1983)) to advocate that HE be treated as a separate principle from VE, and thus to form one of the objectives between which an optimal trade-off must be sought in the setting of tax policy[5].

The value of studying classical HI has nonetheless been questioned by a few authors, among whom figures Kaplow (1989, 1995), who rejects the premise that the initial distribution is necessarily just (see also Atkinson (1979) and Lerman and Yitzhaki (1995)) and adds that utilitarianism and the Pareto principle may justify the unequal treatment of equals (as seen above)[6]. A number of authors have also expressed dissatisfaction with the classical approach to HE because of the implementation difficulties it was seen to present. Indeed, since no two individuals are ever exactly alike in a finite sample, it was argued (see *inter alia* Feldstein (1976) and Plotnick (1982,1985)) that analysis of equals had to proceed on the basis of groupings of unequals which were ultimately arbitrary and which represented "an artificial way to salvage empirical applicability" (Plotnick (1985), p. 241). The proposed alternative was then to link HI and reranking and to note that the absence of reranking *implies* the classical requirement of HE: "the tax system should preserve the utility order, implying that if two individuals would have the same utility level in the absence of taxation, they should also have the same utility level if there is a tax" (Feldstein (1976), p.94)[7].

Various other ethical justifications have also been suggested for the requirement of no-reranking. For normative consistency, King (1983) argues for adding the qualification "and treating unequals accordingly" to the classical definition of HE, by which it then becomes clear that classical HE also *implies* the absence of reranking. Indeed, if two un-

[5]As in all trade-offs, it is clear that violations of HE are often inevitable (although still regrettable), such as when some forms of behaviour are encouraged for efficiency or VE reasons. This seems to be the case, for instance, for the HI created by the tax breaks granted to encourage charitable giving, for mortgage interest tax relief to encourage owner-occupied housing, for the tax deductibility of political contributions to encourage political participation, for indirect tax rate differentiation to promote VE, etc... Other forms of HI may, however, be less intentional and be a sign of truly sub-optimal tax policy.

[6]The same is true for the reranking of individuals, which is discussed below. See also King(1983), who sees this implication as a flaw of strict utilitarianism since it ignores the fairness of the redistributive process.

[7]The requirement of no reranking further implies that marginal tax rates should not exceed 100%, which can be taken as a basic economic requirement for incentive preservation and efficiency. On this, see *inter alia* Lambert and Yitzhaki (1995).

equals are reranked by some redistribution, then it could be argued at a conceptual level that at a particular point in that process of redistribution, these two unequals became equals and were then made unequal (and reranked), thus violating classical HE. Hence, from the above, “a necessary and sufficient condition for the existence of horizontal inequity is a change in ranking between the ex ante and the ex post distributions” (King (1983), p. 102). King (1983) then proposes an additively separable social welfare function that is characterized by a parameter of aversion to horizontal inequity and vertical inequality. This function decreases with the distance of each net income from its order-preserving level of net income. Chakravarty (1985) also argues that reranking causes an individual’s utility to differ from what it would have been otherwise, and moreover suggests that this difference reduces the final level of individual utility, creating a loss of social welfare as measured again by a utilitarian welfare function.

The theory of relative deprivation (which is well documented in the socio-psychological literature) also suggests that people often specifically compare their relative individual fortune with that of others in similar or close circumstances[8]. The first to formalize the theory of relative deprivation, Davis (1959), expressly allowed for this by suggesting how comparisons with similar *vs* dissimilar others lead to different kinds of emotional reactions; he used the expression “relative deprivation" for “in-group" comparisons (*i.e.*, for HI), and “relative subordination" for “out-group" comparisons (*i.e.*, for VE) (Davis (1959), p.283). Moreover, in the words of Runciman (1966), another important contributor to that theory, “people often choose reference groups closer to their actual circumstances than those which might be forced on them if their opportunities were better than they are" (p.29).

In a discussion of the post-war British welfare state, Runciman also notes that “the reference groups of the recipients of welfare were virtually bound to remain within the broadly delimited area of potential fellow-beneficiaries. It was anomalies within this area which were the focus of successive grievances, not the relative prosperity of people not obviously comparable" (p.71). Finally, in his theory of social comparison processes, Festinger (1954) also argues that “given a range of possible persons for comparison, someone close to one’s own ability or opinion will be chosen for comparison" (p.121). In an income redistribution context, it is thus plausible to assume that comparative reference groups are established on the basis of similar gross incomes and proximate pre-tax ranks, and that individuals subsequently make comparisons of post-tax outcomes across these groups. Individuals would then assess their relative redistributive ill-fortune in reference groups of comparables by monitoring *inter alia* whether they are overtaken by or overtake these

[8] Relative deprivation has also been linked to rank-dependent measures of inequality. See Sen (1973), Yitzhaki (1979) and Hey and Lambert (1980) for the link between relative deprivation and the popular Gini coefficient; see also Duclos (2000) for the links to other rank-dependent measures, and for the material on which this paragraph draws.

comparables in income status, thus providing a plausible "micro-foundation" for the use of no-reranking as a normative criterion.

This suggests that comparisons with close individuals (but not necessarily exact equals) would be at least as important in terms of social and psychological reactions as comparisons with dissimilar individuals, and thus that analysis of HI and reranking in that context should be at least as important as considerations of VE. It also says that, although classical HI and reranking are both necessary and sufficient signs of HI, they are (and will be perceived as) different manifestations of violations of the HE principle[9]. Hence, it seems reasonable that VE and the no-reranking requirement be assessed separately from the classical requirement of equal treatment of equals when assessing the impact of taxes and transfers on social welfare[10]. We now turn formally to this task.

3 Social Welfare Functions and Inequality

Let $F_{X,N}(\cdot,\cdot)$ be the joint cumulative distribution function (cdf) of gross (X) and net (N) income, with support contained in the positive real orthant. Let $p = F_X(\cdot)$ be the marginal cdf for gross incomes, and let $X(p)$ be the quantile function for gross incomes, formally defined as $X(p) = \inf\{s > 0 | F_X(s) \geq p\}$ for $p \in [0,1]$. Mean gross income is then $\mu_X = \int_0^1 X(p)dp$. The $N(q)$ q-quantile function for net incomes and average net income μ_N can be defined analogously.

To interpret $X(p)$, Yaari (1988) suggests thinking of a population of a given total mass being uniformly distributed on the unit interval. "Then, for every $p \in [0,1]$, we can think of $X(p)$ as the income density at p, or, more loosely, as individual p's income" (Yaari (1988), p.382). Working with quantiles makes one's analysis automatically consistent with Dalton's population principle, and leads to comparisons of population-normalised income profiles. It also simplifies significantly the exposition, and will help clarify later some of the more subtle differences between our proposed methodology and that of previous work. Note also that for the methodological contribution of this paper, we do not need to assume that the joint or marginal cdf's are continuous. Empirical implementation of the methodology (as for the empirical implementation of all approaches that purport to measure classical HI), will require, however, such continuity assumptions. We will come back to this point again in section 5 when we discuss estimation procedures using a finite number of population observations.

[9]This is nicely discussed in the survey by Jenkins and Lambert (1999) and by Plotnick's (1999) comments on it. See also Kaplow (1989), who observes that the goal of limiting reranking may conflict with that of limiting the unequal treatment of equals.

[10]In the words of Feldstein (1976): "The problem for tax design is therefore to balance the desire for horizontal equity against the utilitarian principle of welfare maximisation. Balancing these two goals requires an explicit measure of the departure from horizontal equity." (p.83).

A Yaari (1988) social welfare function is additive and linear in income levels $X(p)$, with weights determined by a general non-negative function $w(p)$ of individuals' positions in the income parade:

$$W_X = \int_0^1 X(p))w(p)\,dp. \tag{1}$$

Without loss of generality, the weights $w(p)$ can be be normalised such that

$$\int_0^1 w(p)\,dp = 1. \tag{2}$$

This rank-dependent formulation will form the basis of our measure of reranking. The specification of the social welfare function (1) can also be derived from Mehran's (1976) general class of linear measures of income inequality, which are given by [11]

$$I_X = \int_0^1 \frac{\mu_X - X(p)}{\mu_X} w(p)\,dp. \tag{3}$$

We then have that $W_X = \mu_X(1 - I_X)$.

For W to obey the Pigou-Dalton principle of transfers and to be concave in incomes, the weights must be such that:

$$w(p_i) \leq w(p_j) \quad \text{if} \quad p_i \geq p_j. \tag{4}$$

There are many specifications of $w(p)$ which obey this restriction [12]. Throughout the rest of this paper, we focus on a continuous single-parametrization of $w(p)$ which has been the most popular in the literature, the S-Ginis of Donaldson and Weymark (1983) or, equivalently, the extended Ginis of Yitzhaki (1983) (see also Kakwani (1980)). This single parametrization for $w(p)$, which obeys conditions (2) and (4) and which we denote by $w(p,v)$, is given by:

$$w(p,v) = v(1-p)^{(v-1)}, \quad v \geq 1, \tag{5}$$

where v is a parameter of inequality aversion which uses differences in ranks to differentiate the ethical weights granted to individuals in assessing social welfare (this will be discussed further below). When $v = 2$, this gives the standard Gini social welfare function.

To assess inequality and inequity, we use (1) and (5) replacing $X(p)$ in (1) by a utility function of income. Since we wish our social welfare function to be homothetic (in order

[11] For more on this (including the link with the popular Lorenz curves), see for instance Zoli (1999).

[12] For recent examples, see Aaberge (2000) and Wang and Tsui (2000), which extend *inter alia* the I-Ginis of Donaldson and Weymark (1980).

to generate relative inequality indices, see Blackorby and Donaldson (1978)), we choose the following isoelastic function (Atkinson (1970)), $U_\epsilon(y)$, with $\epsilon \geq 0$:

$$U_\epsilon(y) = \begin{cases} \frac{y^{1-\epsilon}}{1-\epsilon}, & \text{when} \quad \epsilon \neq 1, \\ \ln(y), & \text{when} \quad \epsilon = 1. \end{cases} \tag{6}$$

$U_\epsilon(y)$ being concave (strictly so for $\epsilon > 0$), individuals will be averse to uncertainty in their net income level, with ϵ being their parameter of relative risk aversion. This will form the basis of our measure of classical HI. The overall social welfare function then aggregates these utilities across the population by using the rank-dependent ethical weights, $w(p, v)$. For the distribution of gross incomes, this function can therefore be expressed as:

$$W_X(\epsilon, v) = \int_0^1 U_\epsilon(X(p))w(p, v)\, dp. \tag{7}$$

$W_N(\epsilon, v)$ is similarly defined by replacing $X(p)$ by $N(p)$ in (7).

The functions (7) have in fact already been proposed by Berrebi and Silber (1981) in a discrete setting. When $\epsilon = 0$, they collapse to the S-Gini social welfare functions (when $v = 2$, to the standard Gini social welfare function); alternatively, when $v = 1$, they collapse to the utilitarian Atkinson social welfare functions. In the context of social welfare and inequality measurement, ϵ is the well-known parameter of relative inequality aversion introduced by Atkinson (1970). As an expositional shortcut, we will refer to v as a parameter of aversion to "rank inequality". In this regard, note that Yaari (1988) defines "an indicator for the policy maker's degree of equality mindedness at p" as $-w'(p)/w(p)$. This indicator thus captures the speed at which the weights $w(p)$ decrease with the ranks p. For $w(p, v)$, it gives:

$$\frac{-\partial w(p, v)/\partial p}{w(p, v)} = (v-1)(1-p)^{-1}. \tag{8}$$

Thus, the local degree of "equality mindedness" for $w(p, v)$ is a proportional function of the single parameter v. As definition (8) makes clear, this degree of inequality aversion is defined at a particular rank p in the distribution of income, independently of the precise value that *income* takes at that rank. The larger the value of v, the larger the local degree of equality mindedness, and the faster the fall of the weights $w(p, v)$ with an increases in the rank p. Therefore, the greater the value of v, the more sensitive is the social decision-maker to differences and inequality in ranks when it comes to granting ethical weights to individuals. We use this effect and this interpretation of v to depict it in this paper as a parameter of aversion to "rank inequality".

Araar and Duclos (1998) show that the social welfare functions in (7) can be interpreted as average utility corrected for relative utility deprivation in the population. Individual relative deprivation is the expected shortfall from the well-being of others in society (see again Runciman (1966) for an influential definition). For an integer $v \geq 2$, relative deprivation in the population is then the expected individual deprivation of those who would find themselves the most deprived in a group of $v-1$ randomly selected individuals[13]. An increase in v gives greater weight to the relative deprivation of the poorer in the aggregation of individual relative deprivation.

As Sen (1973, p.39) argues, $U_\epsilon(y_i)$ can be an individual utility function, or it can be the "component of social welfare corresponding to person i, being itself a strictly concave function of individual utilities". Sen also adds that "it is fairly restrictive to think of social welfare as a sum of individual welfare components" (p.39), and that one might feel that "the social value of the welfare of individuals should depend crucially on the levels of welfare (or incomes) of others" (p.41). He also expressed concerns that the utilitarian Atkinson form was insensitive to the distribution of utilities. As suggested above, these concerns are all addressed by the form (7). Moreover, as Ben Porath and Gilboa (1994, p.445) note, "the most salient drawback of linear measures [e.g., the traditional S-Gini's] is that the effect on the social welfare of a transfer of income from one individual to another depends only on the ranking of the incomes but not on their absolute levels". Equation (7) escapes this drawback, since $U(y)$ does not have to be affine in incomes[14].

Now denote by $F_{N|X=x}(\cdot)$ the cdf of N conditional on $X=x$. The q-quantile function for net incomes conditional on a p-quantile value for gross incomes is then defined as $N(q|p) = \inf\{s > 0 | F_{N|X=X(p)}(s) \geq q\}$ for $q \in [0,1]$. $N(q|p)$ thus gives the net income of the individual whose net income rank (or percentile) is q, among all those whose rank is p in the distribution of gross incomes. The expected net income of those at rank p in the distribution of gross income is given by:

$$\bar{N}(p) = \int_0^1 N(q|p)dq. \tag{9}$$

Hence, if the tax system were horizontally equitable in the classical sense and if individuals at rank p in the distribution of gross income were granted $\bar{N}(p)$ in net incomes, social welfare would equal:

$$W_N^E(\epsilon, v) = \int_0^1 U_\epsilon(\bar{N}(p)) w\left(p, v\right) dp. \tag{10}$$

The expected net income utility of those at rank p in the distribution of gross income is, however, equal to:

[13]This interpretation of v was originally given in another context by Muliere and Scarsini (1989).

[14]Equation (7) is also linked to rank-dependent expected utility theory, as noted in Chew and Epstein (1989) and Ben Porath and Gilboa (1994).

$$\bar{U}_\epsilon(p) = \int_0^1 U_\epsilon\left(N(q|p)\right) dq. \tag{11}$$

If, instead of $U_\epsilon(\bar{N}(p))$, individuals at rank p were granted their expected net income utility $\bar{U}_\epsilon(p)$, social welfare would equal:

$$W_N^P(\epsilon, v) = \int_0^1 \bar{U}_\epsilon(p) w\left(p, v\right) dp. \tag{12}$$

Note that, compared to W_N, W_N^P can be interpreted as the value of a social evaluation function that uses *ex ante* expected utility instead of *ex post* utility to assess social welfare. By the concavity of the utility function, we have that $U_\epsilon(\bar{N}(p)) \geq \bar{U}_\epsilon(p)$, which captures the local welfare cost at p of net income uncertainty. Hence, we also have that $W_N^E(\epsilon, v) \geq W_N^P(\epsilon, v)$, a feature which will later capture the global cost of classical HI in the form of the cost of net income uncertainty over the entire range of p.

Let $\xi_X(\epsilon, v)$ be the *equally distributed equivalent* income for a distribution of gross incomes X: if $\xi_X(\epsilon, v)$ were enjoyed by all, it would generate the same social welfare as that generated by the current distribution of gross incomes. By definition, $\xi_X(\epsilon, v)$ is thus given by:

$$W_X\left(\epsilon, v\right) = \int_0^1 U_\epsilon\left(\xi_X(\epsilon, v)\right) w\left(p, v\right) dp = U_\epsilon(\xi_X(\epsilon, v)). \tag{13}$$

For expositional simplicity, we will often not mention explicitly in what follows the dependence of ξ_X and other functions on ϵ and v. Denoting the inverse of the utility function $U_\epsilon(\cdot)$ by

$$U_\epsilon^{-1}\left(y\right) = \begin{cases} (1-\epsilon)y^{\frac{1}{1-\epsilon}}, & \text{when} \quad \epsilon \neq 1, \\ \exp\left(y\right), & \text{when} \quad \epsilon = 1, \end{cases} \tag{14}$$

it then follows that $\xi_X = U_\epsilon^{-1}\left(W_X(\epsilon, v)\right)$. Similar definitions obtain for ξ_N, ξ_N^E and ξ_N^P using W_N, W_N^E, and W_N^P, respectively. Finally, it is conventional since Atkinson (1970) to measure inequality as the difference between ξ_X and μ_X as a proportion of μ_X:

$$I_X = 1 - \frac{\xi_X}{\mu_X}. \tag{15}$$

I_X then measures the cost of inequality as a proportion of total income: it is the percentage of total income that could be spent in removing inequality with no resulting loss in social welfare. The indices I_N, I_N^E and I_N^P are defined analogously.

4 Income Inequality and Tax Equity

4.1 Change in Inequality Approach

The redistributive change in inequality that results from the effect of taxes and transfers can be expressed as:

$$\Delta I = I_X - I_N. \tag{16}$$

Note by (15) that this is equivalent to $\xi_N - \xi_X$ when the means of X and N are the same.

To decompose this total redistributive effect, recall that we have previously provided a measure of social welfare for gross and net incomes as well as for *two* locally horizontally-equitable tax systems: one in which each individual at rank p in the distribution of gross incomes receives $\bar{N}(p)$ (see equation (10)), and one in which each of these individuals receives $\bar{U}_\epsilon(p)$ (see equation (12)). Note that in this second case, there is HE locally, but individuals nevertheless suffer the cost of local net income uncertainty (since $\bar{U}_\epsilon(p) \leq U_\epsilon(\bar{N}(p))$). Because of this, we have that $W_N \leq W_N^P \leq W_N^E$. Mean income is nevertheless the same under all of these three distributions of net income utility, since $\mu_N = \int_0^1 \bar{N}(p)dp = \int_0^1 \int_0^1 N(q|p)dq\ dp$. Hence, by (15), we have that $I_N \geq I_N^P \geq I_N^E$, which in conjunction with (16) leads to the following decomposition of ΔI :

$$\Delta I = I_X - I_N = \underbrace{I_X - I_N^E}_{V} - \underbrace{\left(I_N^P - I_N^E\right)}_{H \geq 0} - \underbrace{\left(I_N - I_N^P\right)}_{R \geq 0}. \tag{17}$$

The decomposition has three parts. V represents the decrease in inequality yielded by a tax which treats equals equally. Thus, V measures the underlying vertical equity or progressivity of a tax $X(p) - \bar{N}(p)$ at rank p. H measures the increase in overall income inequality attributable to the unequal post-tax treatment of pre-tax equals. The excess of I_N^P over I_N^E is due to the appearance of post-tax utility and income inequality within groups of pre-tax equals (recall the definition of W_N^P and W_N^E). This is the classical HI effect set in a utilitarian framework.

Finally, R measures the extent of reranking. Although for W_N^P and I_N^P, the effect of classical HI on inequality and welfare has already been borne, all individuals at p are still assumed to enjoy the same post-tax utility, $\bar{U}_\epsilon(p)$, under W_N^P. This distribution of net income utility is thus different for the actual, *a posteriori*, distribution of net income utilities, where in the presence of HI some of those at p will end up with a higher and some with a lower net income utility than $\bar{U}_\epsilon(p)$. This will lead to reranking among the pre-tax equals, and also across pre-tax unequals if the variations in net income utilities are significant enough. As is well known from the literature on reranking (see the early work by Atkinson (1979) and Plotnick (1981) for instance), taking into account reranking when using rank-dependent inequality indices increases measured inequality and decreases the redistributive effect of taxation, and this explains why I_N generally exceeds I_N^P, and also why the difference can be interpreted as the impact of reranking on the net redistributive effect of taxation.

Interpreted in the light of Duclos (2000), moving from pre-tax-ordered to post-tax-ordered net income utility increases population relative deprivation, since in the latter ordering some relatively poor individuals find out their true (lower) relative standing in the distribution of net living standards. Consequently, ξ_N^P is an overestimate of ξ_N, and I_N^P underestimates I_N. Another interpretation of R is to think of individuals as assessing whether they are overtaken by or whether they overtake others in the redistributive process. We may think of individuals resenting being outranked by others, but enjoying outranking others, and then assessing their net feeling of resentment by the amount by which the utility of the richer (than themselves) actually exceeds what the utility of the richer class would have been had no "new rich" displaced 'old rich' in the distribution of net incomes. We can then show that $W_N^P(\epsilon, v) - W_N(\epsilon, v)$ is the expected net utility resentment of the poorest person in samples of $v-1$ randomly selected individuals, and thus that R is an ethically-weighted indicator of such net resentment in the population (see again Duclos (2000)).

4.2 Cost of Inequality Approach

We now perform a decomposition similar to that previously presented, though with a slight difference in the implied distribution of income. In the above change-in-inequality approach, average income is (implicitly) kept the same while comparing distributions, but social welfare and inequality vary across W_X, W_N, W_N^E and W_N^P. In the cost of inequality approach, social welfare is kept the same across the distributions being compared, but the mean income required to attain this level of welfare varies since income inequality varies across I_X, I_N, I_N^E and I_N^P. Each element of the decomposition in this section 4.2 will thus correspond to a difference in means at *equal social welfare*, which is set to $W_N(\epsilon, v)$.

As in Atkinson (1970), the cost of inequality in the distribution of net income can be expressed as:

$$C_N = \mu_N - \xi_N = \mu_N I_N. \tag{18}$$

C represents the level of *per capita* income that society could use for the elimination of inequality with no loss of social welfare. This amount could also be recuperated by the government as supplementary tax revenue, again with no loss of social welfare if inequality were eliminated in the process. Notice that, unlike I, C is homogeneous of degree one in income.

Let C_F represent the cost of inequality subsequent to a flat (or proportional, and thus inequality neutral) tax on gross incomes that generates the same level of social welfare as the distribution of net incomes. Denote the average income under this welfare-neutral flat tax by μ_F. The net effect of redistribution on the cost of inequality then becomes:

$$\Delta C = C_F - C_N. \tag{19}$$

Since $\xi_N = \mu_N - C_N = \mu_F - C_F$ (recall that social welfare is kept the same in all distributions), we also have:

$$\Delta C = \mu_F - \mu_N. \tag{20}$$

Moreover, since $\xi_N = \mu_N(1 - I_N) = \mu_F(1 - I_X)$, the mean of incomes subsequent to this flat tax can be calculated as

$$\mu_F = \frac{(1 - I_N)}{(1 - I_X)}\mu_N. \tag{21}$$

Thus:

$$\Delta C = \frac{(I_X - I_N)}{(1 - I_X)}\mu_N \tag{22}$$

which is positive if $I_X > I_N$. Since a proportional tax does not alleviate inequality, for identical social welfare the government cannot collect as much revenue as it can under a redistributive tax system. Consequently, more money must be left in the hands of taxpayers to compensate for the absence of redistribution, and $\mu_F > \mu_N$. This expression has a close connection with the tax progressivity index of Blackorby and Donaldson (1984), which is simply $\Delta C/\mu_N$. It is also related to an index of tax performance developed in Duclos (1995a)[15]. The larger the redistributive decrease in the cost of inequality, the more performant is the tax system. Since HI tends to decrease ΔC, it will also reduce tax performance. Duclos (1995a,1997a) also shows that, for an additive social welfare function U_ϵ, the more progressive the tax system, the greater its tax performance, and that, for a progressive tax system, performance is increasing in the degree of inequality aversion ϵ [16]. This is also valid in the context of the more general social welfare functions $W(\epsilon, v)$, since they can be interpreted as weighted averages of the U_ϵ. Thus, the more

[15]This was also noted in Duclos and Lambert (2000) in the context of purely utilitarian and additive social welfare functions. To see this, let τ be the excess proportional tax revenue that the government could collect on net incomes N to make social welfare equal to that yielded by an inequality neutral tax regime with average income μ_N. τ is thus implicitly defined as:

$$(1 - \tau)(1 - I_N)\mu_N \equiv (1 - I_X)\,\mu_N \tag{23}$$

and it can then be shown using (23) that:

$$\Delta C = \frac{\tau}{(1 - \tau)}\mu_N. \tag{24}$$

[16]A tax system without HI is progressive if the elasticity of $\bar{N}(p)$ with respect to $X(p)$ is everywhere below 1. The lower the elasticity, the more progressive the tax system.

progressive the net tax system, $N(p) - X(p)$, the greater the value of ΔC. If the tax system is progressive, the greater the value of ϵ, the greater the redistributive fall in the cost of inequality.

We then write the decomposition of the total variation in the cost of inequality as:

$$\Delta C = C_F - C_N = \underbrace{C_F - C_N^H}_{V^*} - \underbrace{\left(C_N^R - C_N^H\right)}_{H^* \geq 0} - \underbrace{\left(C_N - C_N^R\right)}_{R^* \geq 0}. \tag{25}$$

Thus, the redistributive index again decomposes into three effects.

Let us examine the reranking effect first. C_N^R measures the cost of inequality in a social-welfare-neutral distribution in which every one at pre-tax rank p is granted an identical utility that is proportional to expected (or *ex ante*) post-tax utility $\bar{U}_\epsilon(p)$. This scaling of $\bar{U}_\epsilon(p)$ is necessary to maintain social welfare equal to W_N. By the isoelasticity of the utility function U_ϵ, the utility scaling is equivalent to multiplying all net incomes by a welfare-equalising constant γ, and then granting everyone his *ex ante* expected net income utility. To see this more precisely, denote social welfare under that new distribution as W_N^R. It is such that

$$W_N^R(\epsilon, v) = \int_0^1 \left[\int_0^1 U_\epsilon(\gamma N(q|p))\ dq\right]\ w(p,v)\ dp. \tag{26}$$

Since by construction social welfare with this distribution is the same as W_N, and since relative income inequality associated to W_N^P and W_N^R are also the same (net incomes are just scaled down by a constant factor γ in W_N^R), average income under the distribution defining W_N^R equals:

$$\mu_N^R = \mu_N \frac{(1 - I_N)}{(1 - I_N^P)}. \tag{27}$$

Note that $\mu_N^R \leq \mu_N$ since $I_N \geq I_N^P$ (as discussed just above equation (17)). Equalisation of social welfare then also implies that $\gamma = \mu_N^R/\mu_N \leq 1$. γ can therefore be interpreted as 1 minus the money-metric cost of reranking as a percentage of net income. Again, since $W_N^P = W_N$ in the cost-of-inequality approach, the reranking effect can also be expressed as:

$$R^* = C_N - C_N^R = \mu_N - \mu_N^R, \tag{28}$$

where R^* represents the additional *per capita* income that would accrue to the government (with no change in social welfare) if it could treat equals equally but could yet not eliminate the local net income uncertainty cost of classical HI. This treatment would avoid reranking among pre-tax equals and unequals[17] but would not yet therefore eliminate the welfare cost of classical HI.

[17] Assuming that $\bar{N}(p)$ is increasing in p.

Let us now turn to V^* and H^*. C_N^H is the cost of inequality under a welfare-neutral *ex post* horizontally equitable tax, that is, one in which everyone at rank p receives a proportion γ of the certainty-equivalent level of net income at p. This certainty-equivalent net income is given by $\xi_N(p) = U_\epsilon^{-1}\left(\bar{U}_\epsilon(p)\right)$ (see (11) and (14)). Hence, for constant social welfare, an horizontally-equitable tax system corresponds to a distribution of $\gamma\xi_N(p)$ to each individual at pre-tax percentile p[18]. Note that this procedure maintains expected utility locally the same as for W_N^R, and that it therefore also maintains social welfare globally the same as W_N. Under this welfare-neutral distribution, average income is given by:

$$\mu_N^H = \int_0^1 \gamma\xi_N(p)dp, \tag{29}$$

and since social welfare is kep constant, the measure of classical horizontal inequity H^* corresponds to:

$$H^* = C_N^R - C_N^H = \mu_N^R - \mu_N^H. \tag{30}$$

H^* is then a money-metric measure of the welfare cost caused by classical HI.

Finally, we measure vertical equity as follows:

$$V^* = C_F - C_N^H = \mu_F - \mu_N^H. \tag{31}$$

This measures the difference in the cost of inequality of two horizontally equitable tax systems, the first being a flat tax system, and the second granting everyone a proportion γ of his certainty equivalent level of net income, with both systems yielding the same level of social welfare W_N. V^* is positive if the tax system is progressive in an *ex ante*, certainty-equivalent, sense: that is, if the distribution across percentiles of the certainty-equivalent net incomes is more equal (and thus less inequality costly) than the distribution of gross incomes. At constant social welfare, V^* represents the additional *per capita* revenue that (relative to a flat tax) would be available if the tax system were *ex post* horizontally equitable, and thus created neither classical HI nor reranking.

Using (25), (28), (30) and (31), we see that ΔC can also be alternatively expressed as:

$$\Delta C = \mu_F - \mu_N = \underbrace{\mu_F - \mu_N^H}_{V^*} - \underbrace{\left(\mu_N^R - \mu_N^H\right)}_{H^* \geq 0} - \underbrace{\left(\mu_N - \mu_N^R\right)}_{R^* \geq 0}. \tag{32}$$

4.3 Decomposition of classical HI

We may wish to know at which percentile or for which population group HI is more pronounced, and by how much it contributes to total HI. In the case of reranking, this

[18]Recall that for the change-in-inequality approach, we instead attributed $\bar{N}(p)$ to each p-ranked individual.

exercise is of limited use, since this concept is ill-suited to separability of the measure across exclusive groups. To obtain a decomposition of the classical HI term, we define the local cost of classical violations of HE, at p, as:

$$H^*(p) = \gamma \left[\bar{N}(p) - \xi_N(p) \right], \tag{33}$$

where $H^*(p)$ is the risk-premium of net income uncertainty at percentile p, once the cost of reranking ($\gamma = \mu_N^R / \mu_N$) has been accounted for. $H^*(p)$ is thus the money-metric cost of local classical HI at p. It represents the supplementary *per capita* amount local taxpayers would be willing to contribute, with no loss of expected utility, to be subject to a horizontally equitable local tax schedule. We can aggregate $H^*(p)$ using population weights, which by (29), (30) and (33) yields H^*, the index of total classical HI:

$$\int_0^1 H^*(p)dp = \int_0^1 \gamma \left[\bar{N}(p) - \xi_N(p) \right] dp = \gamma\mu_N - \mu_N^H = \mu_N^R - \mu_N^H = H^*. \tag{34}$$

As in Duclos and Lambert (2000), when the local measure $H^*(p)$ is aggregated into the global index H^*, the weights used ensure that the significance attributed to local inequity depends only on population weights, and not upon the standard of living at which HI is experienced[19].

5 Estimation

The classical HI literature has long stressed the importance of identifying exact pre-tax equals in a distribution of gross income. As we rarely find two such identical individuals in a finite sample, empirical implementation of the classical HI concept was seen to pose important difficulties, which led in large part to the development of the reranking approach, as mentioned in section 2. Until recently, most authors attempted to solve this identification problem by fixing income bands and using them to group individuals as "equals". In addition to being somewhat arbitrary, this approach also has the failing of yielding results that can be quite sensitive to the size of the bands [20].

Estimation issues can thus matter significantly in assessing the practicality of any methodology for the measurement of income redistribution. Since the methodology proposed in this paper attempts to separate total redistribution into a sum of VE, classical HI

[19]Again, as in Duclos and Lambert (2000, theorem 3), an additional decomposition of (34) could be performed across socio-demographic groups and percentiles, yielding a sum of classical HI within and across socio-demographic groups.

[20]See Aronson, Johnson and Lambert (1994) for an illustration of this; as the size of the income bands shrinks, classical HI gradually disappears, leaving only an index of reranking.

and reranking using two different approaches, these estimation issues would seem particularly relevant in our context, and we therefore now turn to the estimation of objects such as those that appear in equations (17), (25) and (33).

Using sample observations, estimating W_X, W_N and W_N^P (and therefore I_X, I_N, I_N^P, ΔI, ΔC, C_F, C_N, μ_F and R) is relatively straightforward: we simply replace the population distribution by the empirical or sample distribution of incomes, effectively replacing the integral signs in the expressions of the kind of (7) by a summation across sample observations. To see this more precisely, denote by $(x_j, n_j), j = 1, ..., J$, the joint observations of gross (x_j) and net (n_j) incomes of a sample of J observations, ordered such that $x_j \leq x_{j+1}, j = 1, ..., J-1$. An estimator $\hat{W}_X$ of W_X in (7) is then given by replacing the population quantiles $X(p)$ by the sample quantiles x_j, and integrating over the rank-dependent weights:

$$\hat{W}_X(\epsilon, v) = \sum_{j=1}^{J} \int_{(j-1)/J}^{j/J} U_\epsilon(x_j)\ w(p)\ dp \tag{35}$$

$$= -\sum_{j=1}^{J} U_\epsilon(x_j)\ (1-p)^v|_{(j-1)/J}^{j/J} \tag{36}$$

$$= \sum_{j=1}^{J} U_\epsilon(x_j) \left[\left(\frac{J-j+1}{J} \right)^v - \left(\frac{J-j}{J} \right)^v \right]. \tag{37}$$

Note that the weights on the right of (37) are precisely those found in Donaldson and Weymark's (1980) formulation of the S-Gini indices for discrete distributions. An analogous estimator for W_N is found by ordering the sample observations in increasing values of n_j, and applying (37) with a subsitution of $U_\epsilon(x_j)$ by $U_\epsilon(n_j)$. For $\hat{W}_N^P$, we keep the same ranking as for $\hat{W}_X$, but replace again $U_\epsilon(x_j)$ by $U_\epsilon(n_j)$. By (27) and (28), this procedure also allows estimating μ_N^R, C_N^R and R^*.

For samples of *iid* observations, the asymptotic sampling distribution of such rank-dependent estimators can be obtained from the results of Davidson and Duclos (1997) and Duclos (1997b) by replacing incomes by utility functions of income. Note that for $\hat{W}_N^P$ this procedure is analogous to computing the sampling distribution of a concentration index, by which the ordering of the sample observations generally differs from that of the variable that is cumulated (here, the ordering of the $U_\epsilon(n_j)$ typically differs from that of the $U_\epsilon(x_j)$).

As for $\hat{W}_N^E$, we first replace each net income observation in the x_j-ordered sample by the predicted level of net income, $\hat{\hat{n}}_j$, at the x_j gross income value of that observation. $\hat{W}_N^E$ is then given by substituting the x_j in (37) by these predicted net incomes. $\hat{\hat{n}}_j$ can be estimated using non-parametric regressions, as we do in the illustration that follows, and as outlined for instance in Silverman (1986) and Härdle (1990). This requires that the population $\bar{N}(p)$ be a continuous and "smooth" function of p. $\hat{W}_N^E$ then leads to natural estimators for I_N^E, V and H.

The estimation of μ_N^H is slightly more difficult, as it requires an estimate of $F_{N|X=X(p)}$ at various pre-tax quantiles $X(p)$ in order to estimate $\xi_N(p)$. This can be done using techniques of non-parametric density estimation, which essentially require that the conditional population distribution $F_{N|X=X(p)}(n)$ be a "sufficiently smooth" function of n and $X(p)$. In practice, this implies that the joint pdf (and thus the joint cdf) be continuous in both n and in p. As alluded to above, although continuity is not required for the methodological approach to measuring classical HI, it thus becomes a quasi-essential condition for its empirical applicability. In our illustration below (as in Duclos and Lambert (2000)), we simplify the computation involved by assuming a normal distribution for $F_{N|X=X(p)}$, and by using kernel estimation to assess its variance around the (previously) estimated expected net income $\hat{\bar{n}}_j$ at each sample value x_j. We then simulate a number of (conditional) net incomes around the $\hat{\bar{n}}_j$, which allows us to compute an estimate of $\xi_N(p)$ for the various empirical quantiles of gross incomes. Estimating μ_N^H then leads to natural estimators for C_N^H, μ_N^H, V^*, H^* and $H^*(p)$ for the decomposition of classical HI.

It is important to stress that this exercise does not amount to a normative treatment of "near equals" as equals in estimating classical HI. Here, the statistical and normative procedures are separated. The first step is a purely statistical exercise: we estimate the conditional expectation or the conditional density of net incomes through kernel estimation. In this statistical exercise, no assumptions on the normative treatment of near equals are needed; all that is required are some statistical assumptions on the smoothness and continuity of the joint distribution of gross and net incomes. The second step then computes normative indices of inequality for the (conditional) net income distributions of exact pre-tax equals. All expected and simulated net incomes then come from the same conditional level of gross income. Thus, this second normative step does not involve treating pre-tax unequals as pre-tax equals.

6 Discussion

The methodological approach developed above is linked to and generalises a number of other measures of HI. When $\epsilon = 0$, R yields for $v = 2$ the reranking indices of Atkinson (1979), Plotnick (1981), and Aronson *et al.* (1994) (when their "near-equal" bandwidths approach 0), and for $v \geq 1$ it gives the class of reranking indices of Duclos (1993). The vertical equity indices V are those of Reynolds-Smolensky (1977) and Kakwani (1977, 1984) for $v = 2$ and of Pfähler (1987) for $v \geq 1$. When $v = 1$, H^* yields the class of classical HI indices introduced in Duclos and Lambert (2000).

Setting $\epsilon = 0$ implies a null classical HI effect: $H = H^* = 0$. Increases in ϵ will increase V and V^* if, respectively, the expected tax system and the welfare-equivalent horizontally equitable tax system are progressive (as discussed above). Increases in ϵ will definitely increase H^*, and each of the $H^*(p)$. This is because the cost of inequality in the distribution $N(q|p)$ necessarily increases with ϵ, and that therefore $\xi_N(p)$ will decrease with ϵ. From (33) and (34), it follows that $H^*(p)$ and $H(p)$ increase with ϵ. Thus, ϵ can be treated as an index of aversion to classical HI [21].

Setting $v = 1$ results in the weight $w(p, v)$ being equal to 1 for all individuals; consequently, W now becomes a mean of utilities, the calculation of which no longer requires rankings. Thus, the reranking effect becomes nil when $v = 1$: $R = R^* = 0$. The larger the parameter v, the more weight we give to the reranking resentment of the poorest, which may or may not increase R and R^*.

The values of v and ϵ most representative of social preferences can be obtained by a "leaky bucket" experiment, if we interpret them in the light of vertical equity preferences. The underlying idea is to measure society's tolerance to costs incurred while transferring income from a rich to a poor individual—be they administrative costs or forgone efficiency (see King (1983)). These experiences suggest that values for ϵ situated between 0.25 and 1.0, and for v between 1 and 4, seem reasonable (see for instance Duclos (2000)), and it is therefore on these ranges of ethical parameter values that the illustration below will focus.

The classical HI indices H and H^* are such that they do not depend on local measures of inequality that use rank information to assess income inequality. This is different from what is done in Aronson *et al.* (1994) and Aronson and Lambert (1994) (referred to below as "AJL"). To separate the measurement of classical HI and reranking, AJL follow Lambert and Aronson's (1993) decomposition of the Gini coefficient as a sum of within-group and between-group inequality components. The within-group indices of inequality are computed in a "lexicographic" manner, by assigning each group member a weight that is independent of the distribution of income in the other groups. For the AJL measurement of classical HI, this idea translated into assigning to members of groups of pre-tax equals ranks and ethical weights that depend on the members' unequal position in the net income distribution within their respective groups of pre-tax equals. Since all equals had the same rank before tax, but had different incomes and different ranks after tax, the "within-group-

[21] In our formulation, the value of ϵ also affects (slightly) the assessment of reranking. This is because we transform incomes into utilities, then measure the impact of reranking in utility-metrics, before transforming back to money-metric equally-distributed-equivalent incomes. Since the utility and inverse utility transformations are not linear, the reranking indices are not invariant to the choice of ϵ. As shown in Figure 3 below, however, in the Canadian illustration at least this dependence on the choice of ϵ is very small for rather wide ranges of ethical parameter values. We have no reason to suspect otherwise in other applications, but this can obviously be investigated on a case-by-case basis.

of-pre-tax-equals" inequality was used to reveal classical HI.

Note that AJL's reranking term is computed only after pre-tax equals have been reranked among themselves, comparing this counter-factual situation to the situation obtained when everyone has been reranked. However, if they were truly equal before tax, pre-tax equals should conceivably not be reranked after tax *before* computing the reranking effect. Indeed, in the now clsssical Atkinson and Plotnick reranking index, this within-equals reranking could be argued to contribute instead to the total reranking index. To see this more clearly, consider a distribution of only two pre-tax equals, who are made unequal after taxation. In the AJL approach, the reranking index is zero, although there has clearly been a change of ranks.

This last result for the AJL procedure is essentially due to the choice of the same rank-dependent social welfare function for the measurement of *both* classical HI and reranking. Although obviously influenced by AJL's important contribution, this paper uses instead a separate measure of classical HI that is "rank-independent" since it only depends on the dispersion of post-tax income *levels* among pre-tax equals. Because of this, we can capture classical HI without altering the initial (identical) ranking of pre-tax equals. When $\epsilon = 0$, this also implies that it is the sum of the reranking and classical HI indices in AJL that gives our index of reranking R. As noted above, Aronson *et. al.* (1994) consider this last result to be true only when their "near-equals bandwith" approaches zero.

Using an estimated post-tax distribution of truly pre-tax equals solves one difficulty of the AJL approach (as it was first and has since been *applied*, not as it was *conceptualised* in the original paper), which has been little noticed. Because pre-tax equals in AJL's empirical illustration are only "close-equals", it is possible that post-tax inequality among pre-tax equals is in fact smaller than their actual pre-tax inequality, which would suggest the presence of *negative* classical HI. That is, the tax system would be seen to reduce inequality among equals, which of course cannot seem right. This problem disappears when we proceed (using either AJL's or this paper's methodology) to the estimation of the distribution of post-tax inequality among pre-tax equals.

In both the AJL and our own approaches, the reranking indices clearly depend on a rank-dependent formulation for the social welfare function. This would seem more natural than a dependence on somewhat artificial adjustments to an additive formulation for social welfare functions, as is found for instance in King (1983) and Chakravarty (1985)).

The ethical parameters ϵ and v thus distinguish the assessment of classical HI and reranking. We could also specify a separate ethical parameter for the assessment of vertical equity, by having a social welfare function for aggregation across percentiles that differs from the sum of the individual utility functions used for the groups of equals (as is done for instance in Auerbach and Hassett (1999)). This could involve replacing $\bar{U}_\epsilon(p)$

by $U_\alpha(\xi(p))$ in equation (12), where α would be a parameter of vertical inequity aversion. This parameter would serve to aggregate the certainty equivalent incomes across percentiles, these incomes themselves being found through the use of the classical HI aversion parameter, ϵ. The particular details of this are, however, outside the scope of this paper and are left for future research.

7 Illustration

7.1 Data

We now illustrate briefly the use of our methodology using Canadian data drawn from Statistics Canada's *Survey of Consumer Finances* of 1981, 1985, 1990 and 1994. These surveys, covering between 36,000 and 46,000 Canadian households, include over one hundred variables, including gross income, net income, the sum of taxes paid and assorted transfers received. The size and composition of families, including the number of adults and children, are also recorded. Each family present in the survey is assigned a statistical weight corresponding to the number of Canadian households it represents, which we multiply by the number of household members to obtain "grossing-up" weights. To transform household income data into adult-equivalent units, we use the equivalence scale of the OECD, which assigns a weight of 1 to the first adult, 0.7 to additional adults, and 0.5 to each child under 17 years of age. Finally, in order to facilitate the interpretation of the results, we normalize gross and net incomes by their means, which has however no substantive effect on inequality and cost of inequality comparisons, since all indices are homogeneous of either degree 0 or 1.

7.2 Change-in-inequality results

We first consider the change-in-inequality approach. Table 1 first shows estimates of the inequality of gross and net incomes, calculated for the four years of observations, and for $v = 1.5$ and $\epsilon = 0.4$, values which appear reasonable in light of the experiences mentioned in Section 6. Bootstrap standard deviations for the estimators are shown in parentheses, and their theoretical asymptotic standard deviations appear within brackets[22]. Table 1 indicates a fairly consistent increase in the inequality of gross incomes between 1981 and 1994. Inequality in net incomes also rose between 1981 and 1994, but most of this increase occurred between 1990 and 1994, as the variation in inequality was either negative or statistically insignificant between 1981 and 1990. The explanation for

[22]The asymptotic standard errors were computed using the software DAD, which can be freely downloaded from www.mimap.ecn.ulaval.ca.

this becomes clear upon examination of ΔI, the decrease in inequality attributable to the tax system. Indeed, the absolute value of this measure increases with time, compensating in large part for the increase in I_X. Notice that the proportional redistributive effect $\Delta I/I_X$ follows essentially the same trend, increasing strongly over time. Many of the estimates and differences across years are statistically significant, given the large sample sizes, and the asymptotic and bootstrap standard errors are very close to each other.

Table 1: Inequality and redistribution, 1981 to 1994 ($\epsilon = 0.4, v = 1.5$)

Bootstrap standard deviations in parentheses
Asymptotic standard errors in brackets

Year	I_X	I_N	ΔI	$\Delta I/I_X$
1981	0.3323	0.2211	0.1112	0.3346
	(0.0022)	(0.0015)	(0.0013)	(0.0027)
	[0.0022]	[0.0013]	[0.0012]	[0.0023]
1985	0.3680	0.2334	0.1346	0.3658
	(0.0029)	(0.0021)	(0.0016)	(0.0030)
	[0.0029]	[0.0022]	[0.0014]	[0.0029]
1990	0.3719	0.2267	0.1452	0.3904
	(0.0030)	(0.0020)	(0.0017)	(0.0029)
	[0.0027]	[0.0020]	[0.0017]	[0.0029]
1994	0.4415	0.2677	0.1738	0.3937
	(0.0028)	(0.0021)	(0.0016)	(0.0027)
	[0.0028]	[0.0021]	[0.0015]	[0.0027]

Figure 1 presents 1994 gross $X(p)$ and expected net incomes $\bar{N}(p)$ as well as sample net incomes, represented by points and ordered by their corresponding gross income value. The fact that these points are dispersed around the curve of expected net incomes indicates the presence of classical HI and reranking. Indeed, net incomes do not always increase with the rank of gross income—indicating that changes of rank have occurred. Moreover, even though the sample does not include any two gross incomes that are perfectly identical, we observe that some points are practically situated on a vertical line, a symptom of classical HI.

The information shown in Figure 1, combined with the estimation techniques described in Section 5, allows the computation of V, H, and R for different values of ϵ and v. The indices, computed for $\epsilon = 0.4$ and $v = 1.5$, are presented in Table 2. As predicted by the theoretical model, the vertical effect V dominates ΔI, with non-negligible losses in redistribution attributable to classical HI and reranking. The point estimates of V, H, and R all increased during each of the periods under observation (except for H between 1985 and 1990) and these increases are almost everywhere statistically significant. In all three cases the greatest change was between 1990 and 1994. These results agree with the Canadian results of Duclos and Lambert (2000) on classical HI as well as with Duclos and Tabi (1999) on reranking in Canada. In contrast, Aronson, Lambert, and Trippeer (1997), using U.S. and British data, observe the opposite effects between 1981 and 1990. Lambert and Ramos (1997) find a decline for V and H, along with an increase in R between 1985 and 1990, for Spain.

Table 2: Vertical equity, horizontal inequity and reranking, 1981 to 1994

$\epsilon = 0.4, v = 1.5$
Bootstrap standard deviations in parentheses

Year	ΔI	V	H	R	$V/\Delta I$	$H/\Delta I$	$R/\Delta I$
1981	0.1112	0.1280	0.0084	0.0083	1.1507	0.0756	0.0751
	(0.0013)	(0.0008)	(0.0002)	(0.0000)	(0.0023)	(0.0017)	(0.0008)
1985	0.1346	0.1557	0.0105	0.0106	1.1566	0.0779	0.0787
	(0.0016)	(0.0012)	(0.0002)	(0.0001)	(0.0025)	(0.0016)	(0.0009)
1990	0.1452	0.1673	0.0103	0.0119	1.1525	0.0707	0.0818
	(0.0017)	(0.0015)	(0.0002)	(0.0001)	(0.0027)	(0.0015)	(0.0010)
1994	0.1738	0.2097	0.0216	0.0143	1.2069	0.1246	0.0823
	(0.0016)	(0.0010)	(0.0003)	(0.0002)	(0.0028)	(0.0019)	(0.0009)

We are also interested in relative values, i.e., in values normalised by the total redistributive change in ΔI. $V/\Delta I$ represents the proportion of the observed redistribution obtained in the absence of horizontal inequity and reranking, *i.e.*, the tax system's "potential" for redistribution. This measure has not varied significantly between 1981 and 1990, but has very significantly increased during the period 1990-1994. $H/\Delta I$ represents the adverse redistribution generated by classical HI as a share of the observed total redistribution. This measure did not vary significantly from 1981 to 1990, but grew significantly between 1990 and 1994, so that the net effect was a statistically significant increase in horizontal inequity between 1981 and 1994. As to reranking as a proportion of net redistribution $(R/\Delta I)$, it witnessed no statistically significant change throughout the period[23].

[23] Once again our results are in opposite direction to those obtained by Aronson, Lambert and Trippeer (1997) for the United States and Great Britain, and those of Lambert and Ramos (1997) for Spain.

The combined effect of classical HI and reranking is not negligible – amounting to between 15% and 21% of the net redistributive effect.

Our results show divergences in the evolution of classical HI and reranking over time. This phenomenon may seem surprising at first in light of the strong conceptual link between the two concepts hitherto highlighted in the previous literature. For a given level of HI, however, the incidence of reranking depends on the proximity of the groups of equals. Thus, even if H grew rapidly between two periods, a substantial increase in the inequality in gross income could attenuate the impact of HI on on reranking (by "spreading" groups of equals further from each others). This actually occurred between 1990 and 1994. The opposite effect would occur subsequent to a fall in I_X.

7.3 Sensitivity of results

As discussed above, measures of classical HI and reranking are sensitive to the ethical parameters ϵ and v. Figures 2 and 3 confirm, however, that the measure of classical HI is considerably more sensitive to the choice of ϵ than of v, and conversely for the index of reranking. When ϵ increases from 0 to 0.8, classical HI increases from 0 to close to 0.1, which says that classical HI alone can decrease net redistribution by up to 0.1 (which is high, considering that the inequality indices considered here are bounded by 0 and 1). Changes in v have little effects on H, but an increase in v from 1.0 to 3.0 raises R from 0 to 0.05, with a corresponding increase in redistributive costs.

But what about the relative magnitude of classical HI and reranking? Which of these two manifestations of horizontal inequity is more detrimental to redistribution? Our results cannot furnish a clear answer to this question, which requires a normative judgement. This is illustrated in Table 3, where the H/R ratio varies considerably with the choice of ϵ and v, ranging from 0 when $\epsilon = 0$ to infinity when $v = 1$. The Table also confirms that H/R increases with ϵ/v, which is consistent with the definition of the two parameters. Table 4 shows, however, that the H/R ratio fell between 1981 and 1990, but grew significantly between 1990 and 1994 and ended higher than its 1981 value and this, for all three pairs of values of ϵ and v shown. This is in fact observed regardless of the choices of ϵ and v (except of course for a choice of $\epsilon = 0$, when $H = 0$, or for a choice of $v = 1$, for which $R = 0$). Thus, as a proportion of reranking, classical HI seems considerably higher in 1994 than in 1981.

The measurement of HI must also tackle the issue of household heterogeneity in needs, which affects the pre-tax ranking of individuals as well as the impact of taxes and transfers on their economic well-being. To check the sensitivity of inequality, classical HI and reranking indices to the choice of equivalence scales, I, H, R and V were computed using three alternative equivalence scales. The OECD scale was already described above. The Cutler and Katz scale divides household incomes by the factor $(n_a + \theta n_c)^{\phi}$, where

Table 3: H/R, 1994

	v						
ϵ	1	1.5	2	2.5	3	3.5	4
0.0	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	-	0.6619	0.3918	0.2974	0.2490	0.2168	0.1931
0.4	-	1.5105	0.8942	0.6759	0.5592	0.4864	0.4346
0.6	-	2.7162	1.5845	1.1927	0.9830	0.8520	0.7587
0.8	-	4.8187	2.7934	2.0866	1.7145	1.4831	1.3211
1.0	-	7.3706	4.2250	3.1476	2.5833	2.2391	1.9975

Table 4: H/R, 1981 to 1994

	H/R		
year	$\epsilon = 0.4, v = 1.5$	$\epsilon = 0.8, v = 2$	$\epsilon = 1, v = 4$
1981	1.0120	1.5789	1.1892
1985	0.9906	1.5025	1.0996
1990	0.8655	1.2545	0.9029
1994	1.5105	2.7934	1.9975

n_a is the number of adults and n_c is the number of children. Parameters θ and ϕ can generally take any value between 0 and 1, but for the purposes of this illustration we set $\theta = \phi = 0.5$. This scale therefore assigns a decreasing weight to each additional household member. The Statistics Canada equivalence scale is implicitly based on estimates of low-income thresholds, which depend on family size as well as population density in the region of residence[24]. Table 5 shows results based on these three equivalence scales. I_X, I_N, ΔI and V appear to be relatively insensitive to the choice of equivalence scales. This is not the case for H and R, however, which as a proportion of ΔI vary from 8.8% to 12.5% for H and from 5.9% to 9.6% for R. The overall redistributive costs of horizontal inequity can thus vary from 14.7% to 21.7% of net redistribution, depending on the choice of equivalence scales.

[24]See the Statistics Canada web page http://www.statcan.ca for further information.

Table 5: Inequality and inequity 1994 ($\epsilon = 0.4, v = 1.5$)

	Scale		
	OECD	Cutler and Katz $\theta = \phi = 0.5$	Statistics Canada
I_X	0.4415	0.4501	0.4082
I_N	0.2677	0.2657	0.2245
ΔI	0.1738	0.1844	0.1837
V	0.2097	0.2114	0.2236
H	0.0216	0.0163	0.0222
R	0.0143	0.0108	0.0176
$V/\Delta I$	1.2069	1.1470	1.2169
$H/\Delta I$	0.1246	0.0883	0.1211
$R/\Delta I$	0.0823	0.0587	0.0958

7.4 Cost of inequality results

Estimates of the cost-of-inequality indices are shown in Table 6, again for $v = 1.5$ and $\epsilon = 0.4$. Observations on the absolute value of the measures are basically the same as previously, except that we can now usefully interpret these values as money-metric indicators of the benefits of VE and of the costs of classical HI and reranking. The estimates of V^*, H^*, and R^* increase in each sub-period, except for H^* between 1985 and 1990, although the increases are often not statistically significant between 1981 and 1990. The increases in VE, classical HI and reranking between 1990 and 1994 are, however, large and statistically significant. Over 15 years, the cost of classical HI rises from 0.6% to 1.3% of per capita income, and the cost of reranking increases from 1.1% to 1.9% of average income. These increases are, in fact, well correlated with the increases in net redistribution, whose money-metric benefit rises from 16.7% to 30.9% of per capita income. As a proportion of ΔC, H^* and R^* are indeed often statistically indistinguishable across the four years.

Examination of Table 7 and Figure 4 (using $\epsilon = 0.4$ and $v = 1.5$) reveals that the local measure of classical HI, $H^*(p)$, generally declines with the rank of gross income. In particular, the values of $H^*(p)$ are particularly high for the 20% of the population with the lowest incomes. Thus, discrimination in the tax system between equals appears most pronounced among the poorest, particularly in 1994. This observation agrees with the conclusions in Duclos and Lambert (2000). This phenomenon is in fact observed for all positive values of ϵ.

Table 6: Vertical equity, classical horizontal inequity, and reranking based on cost of inequality, 1981 to 1994; $\epsilon = 0.4, v = 1.5$; Bootstrap standard deviations in parentheses

Year	ΔC	V^*	H^*	R^*	$V^*/\Delta C$	$H^*/\Delta C$	$R^*/\Delta C$
1981	0.1670	0.1836	0.0060	0.0106	1.0994	0.0359	0.0636
	(0.0017)	(0.0010)	(0.0001)	(0.0001)	(0.0016)	(0.0007)	(0.0010)
1985	0.2139	0.2351	0.0075	0.0138	1.0993	0.0350	0.0644
	(0.0029)	(0.0029)	(0.0001)	(0.0002)	(0.0015)	(0.0006)	(0.0010)
1990	0.2306	0.2531	0.0073	0.0152	1.0978	0.0318	0.0659
	(0.0031)	(0.0031)	(0.0002)	(0.0002)	(0.0018)	(0.0007)	(0.0013)
1994	0.3090	0.3415	0.0128	0.0187	1.1052	0.0446	0.0606
	(0.0029)	(0.0029)	(0.0002)	(0.0002)	(0.0015)	(0.0007)	(0.0009)

7.5 Discussion of results

Reforms in the Canadian tax and transfer system, macroeconomic shocks, and socio-demographic changes have all strongly affected the distribution and redistribution of income since the early 1980's, and can help explain our main results or important increases in redistribution and in the HI costs to that redistribution[25]. The 1981 and 1987 tax reforms introduced a broadening of the tax base through restrictions in certain tax preferences (*e.g.*, higher capital gain inclusion rate, restrictions on tax deferral mechanisms, and the repeal of certain tax shelter provisions). Although such base-broadening measures tend to favour horizontal equity, some selective and targetted measures implemented during the 1981-1994 period have had the opposite effect; examples include the replacement of personal exemptions by personal tax credits, the move from a family-based to an individual-based tax system, the means-testing of child tax credits, and the introduction of a lifetime exemption for capital gains. Besides, despite sometimes lower marginal tax rates, income was increasingly subjected between 1981 and 1994 to higher average tax rates, partly prompted by increasing public deficits. Thus, the observed progression in classical HI and reranking can be imputed to a changing tax system and also to stronger fiscal intervention.

On the transfer side, HI stems in large part from the unemployment insurance, old-age benefit and social assistance programmes, whose parameters and size have evolved significantly over the last twenty years. These programmes often discriminate on the basis of sex, age, household type, region of residence, marital status, monthly income variability, etc., in a way which can cause reranking and classical HI, although they will also typically

[25]For more detailed evidence on this, see Duclos and Lambert (2000).

Table 7: Classical Horizontal inequity by percentile, 1981 to 1994

($\epsilon = 0.4, v = 1.5$)

Rank	1981	1985	1990	1994
0.05	0.0228	0.0227	0.0231	0.0418
0.15	0.0106	0.0145	0.0151	0.0400
0.25	0.0056	0.0084	0.0094	0.0208
0.35	0.0040	0.0058	0.0070	0.0120
0.45	0.0034	0.0044	0.0054	0.0082
0.55	0.0033	0.0037	0.0045	0.0060
0.65	0.0029	0.0035	0.0039	0.0051
0.75	0.0030	0.0031	0.0034	0.0043
0.85	0.0025	0.0032	0.0034	0.0037
0.95	0.0039	0.0033	0.0038	0.0030

generate an increase in net redistribution. The higher classical HI observed among low income households can be explained by the fact that most transfers are aimed at this income group. Socio-demographic and economic changes have also affected income redistribution, mainly through transfers. As a consequence of population aging, more Canadians depend on old-age benefits. The growing number of divorces has resulted in the rise of single-parent families, many of which depend on social welfare. Regional and industrial shocks have made the unemployment system look more like a social assistance than a social insurance system for many regions and industries. These changes have typically led to greater government intervention, resulting in more significant, but also more imperfect, redistribution.

8 Conclusion

This paper presents a decomposition of the redistributive change in income inequality as a sum of vertical equity, classical horizontal inequity (HI) and reranking components. We separate the measurement of the last two components since we argue that although classical HI and reranking are both necessary and sufficient signs of violations of the principle of horizontal equity, they are different manifestations of those violations. The decomposition uses a non-additive social welfare function which combines the features of the well-known Atkinson and Gini social welfare indices. This allows classical HI and reranking to be assessed jointly, though on fundamentally separate functional bases. The index of reranking is based on a social welfare aversion to rank inequality and relative deprivation; the index of classical horizontal inequity is based on an aversion to net income riskiness. This dual formulation also allows for the specification of different ethical parameter values for the measurement of classical HI and reranking. Two measurement approaches are developed: one in terms of changes in indices of inequality, and the other in terms of changes in the costs of inequality. Both approaches help us evaluate the cost of classical HI and reranking, either in terms of forgone redistributive effects or in terms of foregone tax revenues for the government. An illustration on Canadian data indicates that these losses can be significant, and that they have generally tended to grow between 1981 and 1994. Finally, although comparing classical HI and reranking at a given point in time requires a normative judgement, the two phenomena have clearly gone through a divergent evolution in Canada between 1981 and 1994, with classical HI as a proportion of reranking being considerably higher in 1994 than in 1981.

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Figure 1: Gross, net and expected net income, 1994

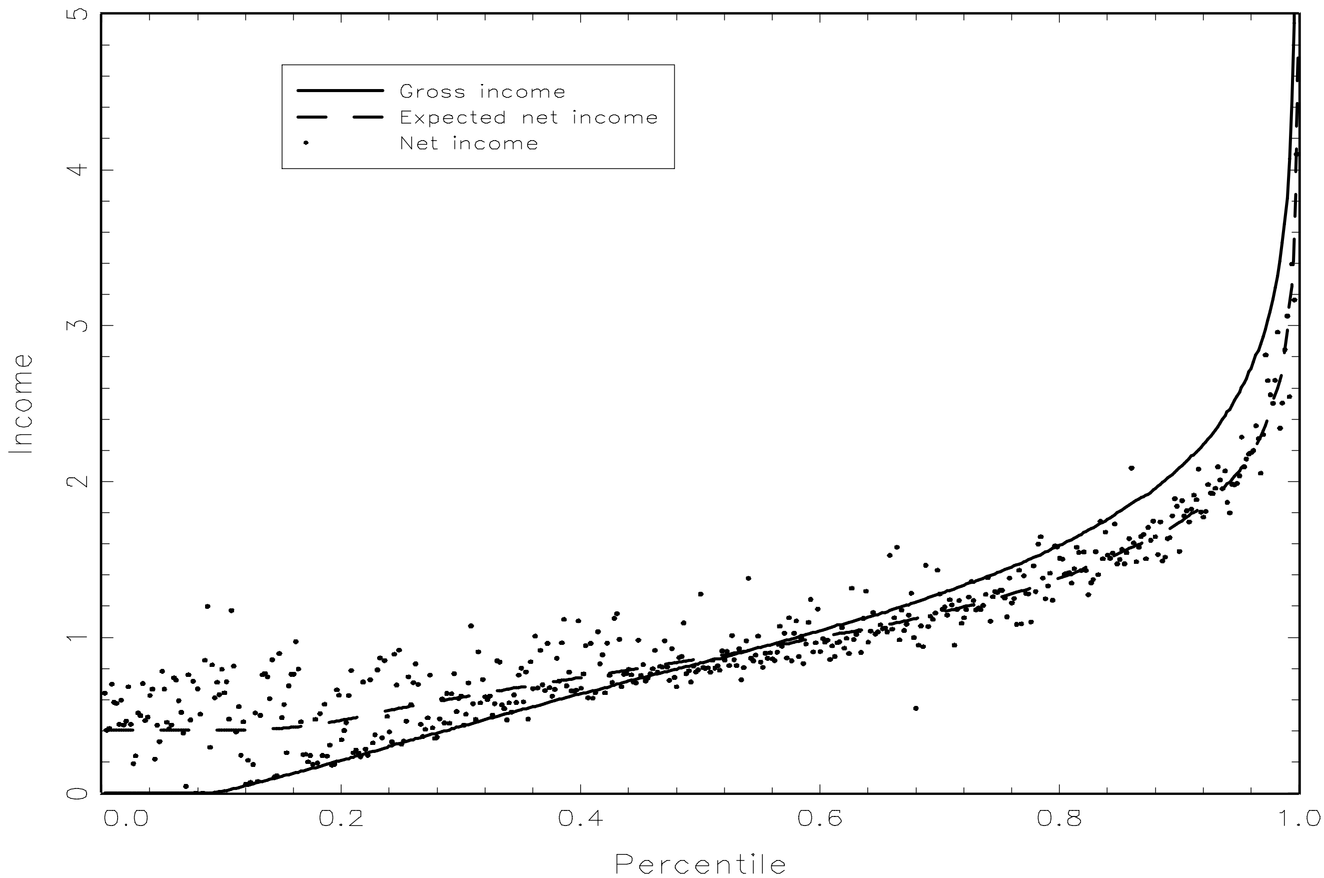

Figure 2: Index of classical horizontal inequity H by choice of parameter, 1994

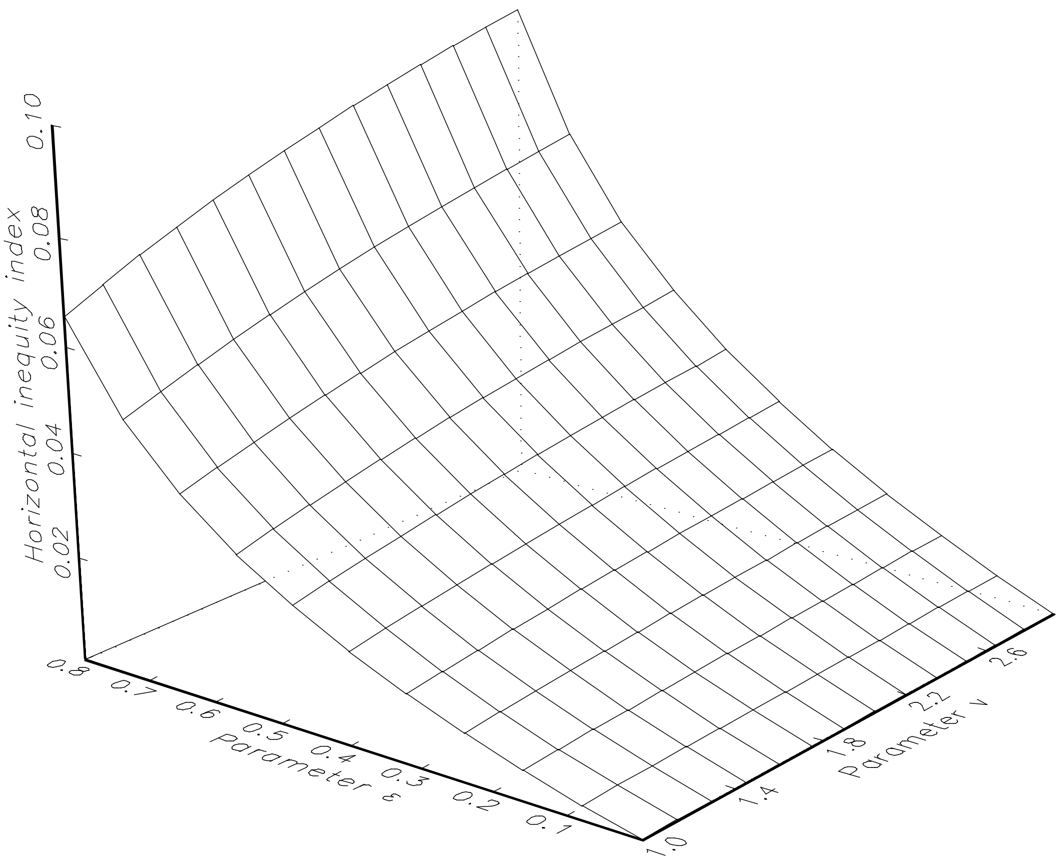

Figure 3: Index of reranking R by choice of parameter, 1994

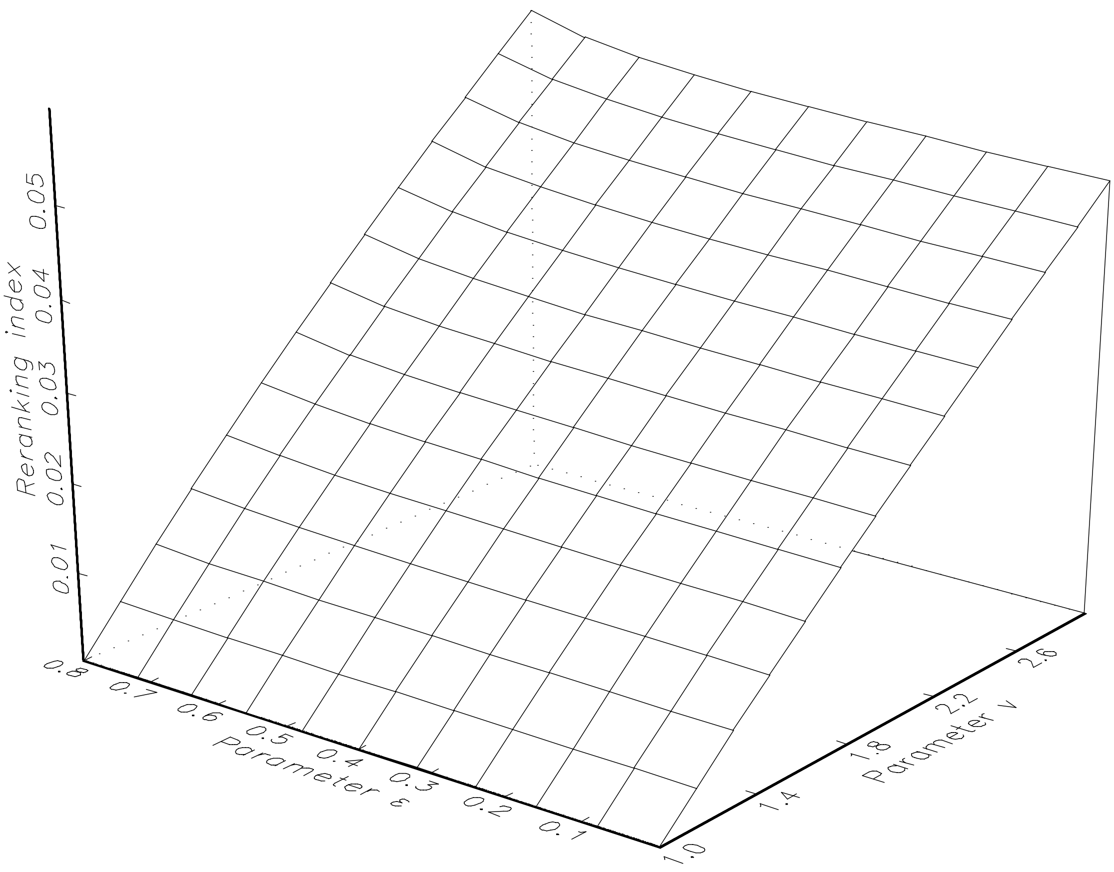

Figure 4: Classical horizontal inequity by percentile, 1981 to 1994 ($\epsilon = 0.4, v = 1.5$)