Countervailing Power?
Collusion in Markets with Decentralized Trade[*]

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Abstract

We consider the collective incentives of buyers and sellers to form cartels in markets where trade is realized through decentralized pairwise bargaining. Cartels are coalitions of buyers or sellers that limit market participation and compensate inactive members for abstaining from trade. In a stable market outcome, cartels set Nash equilibrium quantities and cartel memberships are immune to defections. We prove that the set of stable market outcomes is non-empty and we provide its full characterization. Stable market outcomes are of two types: (i) at least one cartel actively restrains trade and the levels of market participation are balanced, or (ii) only one cartel, eventually the cartel that forms on the long side of the market, is active and it reduces trade slightly below the opponent's.

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1 Introduction

Collective incentives to restrict trade have long been acknowledged as a prevalent phenomenon in markets. The inherent instability of cooperative agreements attempting to exploit such incentives has also been extensively discussed in the literature. A common feature of the models that address these issues consists in assuming that collusive practices arise only on one side of the market. For instance, particular emphasis has been given to the formation of cartels by oligopolistic …rms that face price-taking consumers.[1] More recently, developments in auction theory have addressed the issue of collusion among numerous buyers facing a single seller.[2]

When traders on both sides of a market behave strategically, both sides can in principle form cartels with the purpose of enhancing their collective market power with respect to the opponent's. Is it then possible that cartels emerge and persist on the two sides of a market? Is it possible for collusion to be a desirable phenomenon in this context?

These questions were raised long ago by Galbraith (1952), who claimed positive answers, in his theory of countervailing power. Galbraith asserted that "in the competitive model, the power of the …rm as a seller is checked or circumscribed by the competitor who o¤ers, or threatens to o¤er, a better bargain. The role of the buyer on the other side of such market is essentially a passive one. However, (...) the active restraint is provided not by the competitor but from the other side of the market by strong buyers".[3] Thus, the existence of market power on one side of the market would create an incentive for the other side to organize another position of power neutralizing the former. Countervailing power was seen behind the emergence of labor unions: "One …nds the strongest labor unions in the United States where markets are served by strong corporations. And it is not an accident that the large automobile, steel, electrical, farm machinery companies all bargain with powerful unions. It is the strength of the corporations in these industries that made it necessary for workers to develop the protection of countervailing power".[4] The retail market o¤ered another example of the operation of countervailing power. The great development of department-store chains, food chains, mail order houses was interpreted as the countervailing response of retailers, on consumers' behalf, to sellers' previously established positions of power.

Galbraith's claims were not sustained with a rigorous model. And the empirical evidence is somewhat controversial.[5] Yet, the preceding descriptions are suggestive and, despite their formal shortcomings, Galbraith's arguments had great impact on the development of economic policies in the second half of

[1] See d'Aspremont et al. (1983) and Donsimoni, Economides and Polemarchakis (1986) for a characterization of stable cartels in the context of oligopolistic markets.

[2] See McAfee and McMillan (1992).

[3] Galbraith (1952), p. 113.

[4] Galbraith (1952), p. 114-115.

[5] See Scherer and Ross (1990), ch. 14, and references therein.

the 20th century. Fifty years later, very little research has formally addressed the problem,[6] and it is still unclear whether the theory of countervailing power can stand a game-theoretic scrutiny. The present work aims at contributing to such analysis.

In this paper, we examine the problem of bilateral cartel formation in the context of decentralized exchange economies à la Rubinstein and Wolinsky (1985), with a continuum of homogeneous buyers and a continuum of homogeneous sellers. Buyers and sellers are randomly matched in pairs and bargain over the price to exchange one unit of an indivisible good. In markets that remain stationary at all rounds of trade (as in Rubinstein and Wolinsky [1985] where, at each round, new traders enter exactly in the same measure as satis…ed traders exit), the advantage of the short side of the market is not su¢cient to create collective incentives on either side to exclude some traders from the market.[7] However, such incentives do exists, and can be strong, in a market that does not remain stationary as it clears over several rounds of trade. Consequently, our analysis is carried out within environments where the relative measure of buyers to sellers changes across the di¤erent rounds of trade.

The market operates for …nitely many rounds, with no entry of new traders after the …rst round.[8] At each round, buyers and sellers search for a trading partner and, if they …nd a match, they bargain over the price at which to transact. If they reach an agreement, they trade and exit the market, otherwise they search again in the following round. Equilibrium prices at the di¤erent rounds of trade depend on the relative measures of buyers and sellers that are active in the market at those rounds. Agents in the short side of the market are able to apportion a bigger fraction of the surplus generated by trade.

Cartels are coalitions aiming to increase the collective bene…ts of their members, all of whom trade in the same side of the market. Actual cartels have a major impact both on the search and on the bargaining patterns of traders. Cartels may turn a market with decentralized trade into a market with centralized trade, substantially altering the process of price formation. In our model, however, cartels are endowed with much weaker prerogatives. We will assume that the only instrument that cartels have at their disposal is the restriction in the market participation of their members. Thus, each cartel chooses how many members, if any, to withdraw from the market and it redistributes its total payo¤ in order to compensate inactive members for their abstention. Given the indivisibility of the good traded, this is equivalent to cartels setting their own supply or demand. Even when cartels are active, prices are set through bilateral bargaining à la Rubinstein and Wolinsky (1985). These assumptions make our analysis tractable and our results independent of ad hoc assumptions concerning the process of price formation.

Moreover, we suppose that only one cartel can form on each side of the market and that a cartel might

[6] Bloch and Ghosal (2000), discussed below, represents the notable exception.

[7] See Bloch and Ghosal (2000) for a proof of this claim.

[8] The preceding description is best interpreted as the stage game of a repeated game, in which buyers and sellers repeatedly want to buy and sell, respectively, one unit of a good that perishes after two rounds of trade.

not have control over the whole population on its side. Outsiders of the cartel always participate in the market, whereby cartels actually determine the total quantities that will be supplied or demanded in the market. Given the potential measures of buyers and sellers and the levels of cartel memberships, we suppose that the two cartels play a non-cooperative game where the quantities supplied and demanded are set simultaneously. Non-members generally benefit from the formation of a cartel: they trade the indivisible good at the same price as cartel members, but they do not have to compensate inactive cartel members. This free-riding problem greatly limits the extent to which cartels can effectively reduce trade while expecting to maintain their memberships. Consequently, not all outcomes attained as equilibria of the quantity-setting game are equally relevant. A natural criterion for selecting among the equilibria of the quantity-setting game consists in requiring that they be supported by stable levels of cartel memberships.

Stable market outcomes are profiles of cartel memberships and an associated equilibrium of the quantity-setting game at which memberships are stable, in that cartel sizes do not trigger defections. We prove that the set of stable market outcomes is non-empty, and we provide its full characterization. Stable market outcomes can be of two different types.

The first type is such that at least one cartel actively restrains trade and such that the level of participation in the market is balanced on both sides, regardless of the potential sizes of supply and demand. Market outcomes might be inefficient when both cartels are active, because not all gains from trade are apportioned. Thus, using Galbraith's terminology, both sides exercise countervailing power. But when only one cartel (the one that forms in the long side of the market) is active, only one side of the market exercises countervailing power, restraining its participation up to the point at which supply and demand coincide. Consequently, this kind of stable market outcomes results in an efficient allocation and the effect of countervailing power is limited to a redistribution of the total surplus.

The second type of stable market outcomes is such that only one cartel (more likely the one that forms on the long side of the market) is active, which reduces its participation in the market so as to slightly undercut the opponent's. In this situation, only one side of the market exercises countervailing power and the total surplus is redistributed in favor of this side. The market outcome is not efficient, but the reduction in the quantity traded with respect to its potential total volume is not very big.

Our paper owes much to Bloch and Ghosal (2000) that precedes us in addressing the issue of cartel formation in the context of an exchange economy with bilateral trade and bargaining. Bloch and Ghosal (2000) considers the formation of cartels of buyers or sellers in markets with an equal and finite number of buyers and sellers. They show that cartels might be active on both sides of the market, but active cartels never withdraw more than one trader. Although there are many apparent differences between our work and that of Bloch and Ghosal, our results are closely related and, we believe, complementary to theirs. The peculiarity of our model can be ascribed to our assumption that cartels set continuous quantities. The continuum assumption, although debatable from a descriptive point of view (since bilateral collusion seems more likely in markets with small numbers of traders on each side), is crucial to attain a tractable analysis, and permits to analyze two-sided cartel activity in markets that are ex ante unbalanced, a case that is not addressed by Bloch and Ghosal (2000).

The rest of the paper is organized as follows. The basic model of decentralized trade is presented in Section 2. In Section 3, the non-cooperative game played by the cartels is described, and the notions of cartel stability are introduced. The equilibria of the quantity-setting game played by the cartels are characterized in Section 4. Stable market outcomes are described in Section 5.

2 Decentralized trade

Consider a market with a continuum of identical sellers and a continuum of identical buyers. Each seller owns one unit of a homogeneous indivisible good and his valuation of the good is normalized at zero. Each buyer owns one unit of a perfectly divisible commodity and his valuation for the indivisible good is normalized at one. All agents are perfectly patient.[9]

The market operates for two trading rounds $t = 1; 2$.[10] It is assumed that a measure b of buyers and a measure s of sellers enter the market in the first round, and that the market is potentially unbalanced. No new agents enter after the first round. In each round, buyers and sellers are randomly matched in pairs and each pair bargain over the surplus generated by the indivisible good.

The matching mechanism is characterized by non-negligible search frictions. At each round t; the traders on the short side of the market do not find a trading partner with certainty, but only with probability $\gamma \in (0; 1)$. The traders on the long side of the market are matched with an even smaller probability $\mu_t = \gamma \frac{\min\{b_t; s_t\}}{\max\{b_t; s_t\}}$; being b_t and s_t the measures of active buyers and sellers at round t:

When a buyer and a seller get matched, they bargain a price to trade the indivisible object. At either round of trade, the bargaining game consists in an ultimatum game. Namely, a fair lottery selects one of the parties to propose a partition of the surplus; the other party responds by accepting the offer or by rejecting it. Upon acceptance, the agents trade and leave the market. Upon rejection in the first period, the match breaks and the agents return to the market, searching for other partners in the second round of trade. A rejection in the second round implies that the game ends without trade for the given match. The payoffs for trading at price $p \in [0; 1]$ at either round are p to the seller and $1 - p$ to the buyer. The utility associated with no trade is zero.

At the unique subgame perfect equilibrium of the bargaining game, the proposer offers the responder a share equal to the latter's expected value of returning to the pool of unmatched agents in the following

[9] Introducing pure time preferences that are common for all agents would not alter the qualitative features of the results.

[10] This assumption is made for tractability. The results can be generalized to the case in which the market operates for more trading rounds:

round, and the responder accepts. In other words, the responder is given his outside option at that period and the proposer gets the residual surplus. Consequently, there exists a unique market equilibrium such that, at each round, all pairs of traders immediately agree on the same price.[11] Since the populations of buyers and sellers typically change from one round to the next, the bargaining pairs face endogenous and time-varying outside options.

Suppose that sellers are initially the sort side of the market, i.e. $s \leq b$. At period t, outside options can be defined recursively as

$$x^L_{B;t} = \quad = \theta_{t+1}\left(\frac{1}{2}\left(1 - x^S_{S;t+1}\right) + \frac{1}{2}x^L_{B;t+1}\right) + (1-\theta_{t+1})\, x^L_{B;t+1}$$

for buyers, and

$$x^S_{S;t} = \gamma\left(\frac{1}{2}\left(1 - x^L_{B;t+1}\right) + \frac{1}{2}x^S_{S;t+1}\right) + (1-\gamma)\, x^S_{S;t+1}$$

for sellers, where the superscripts L and S stand for long and short side of the market, respectively.

Starting from the last period and substituting backwards, one can compute the agent's present discounted value of participating in a two-rounds market, which is

$$\pi^L_B = x^L_{B;0} = \frac{\theta_1(2-\gamma)+(2-\theta_1)\theta_2}{4} \tag{1}$$

for buyers, and

$$\pi^S_S = x^S_{S;0} = \frac{\gamma(4-\gamma-\theta_2)}{4} \tag{2}$$

for sellers. Note that both π^L_B and π^S_S depend on the initial measures of buyers b and sellers s, through the matching probabilities $\theta_1 = \gamma\frac{s}{b}$ and $\theta_2 = \gamma\frac{(1-\gamma)s}{b-\gamma s}$. Symmetric expected values can be computed if $b < s$.

Thus, in general, the individual expected gains to a buyer, when the initial measure of buyers is b and the initial measure of sellers is s, can be written as

$$\pi_B(b;s) = \begin{cases} \pi^S_B(b;s) & \text{if } b \leq s \\ \pi^L_B(b;s) & \text{if } b \geq s \end{cases};$$

where

$$\pi^S_B(b;s) \equiv \gamma\frac{(4-\gamma)(s-\gamma b)-(1-\gamma)\gamma b}{4(s-\gamma b)} \quad \text{and} \quad \pi^L_B(b;s) \equiv \gamma\frac{s}{b}\frac{(3-2\gamma)(b-\gamma s)+(1-\gamma)b}{4(b-\gamma s)};$$

and where $\pi^S_B(b;s) = \pi^L_B(b;s)$ for $b = s$.

[11] The assumption that traders use a (random proposer) ultimatum game is made for expositional clarity. Other bargaining games yield the same market equilibrium. See Ponsati (2002) for a proof that under infinite horizon alternating offers barganing, ultimatum strategies can still prevail and yield a unique market equilibrium as well.

As for the group expected gains of traders on one side of the market, there might exist collective incentives to exclude some agents from trade. This occurs because buyers' (sellers') collective utility generally increases when the measure of buyers (sellers) trading on the market is reduced, given the measure of active sellers (buyers).

In a market where the initial measures of traders is $(b;s)$; the group expected payoff to the buyers that enter the market and face a measure s of sellers is simply $\pi_B(b;s)b$. When the buyers are the long side of the market, that is when $b \geq s$; their joint utility is

$$\pi_B^L(b;s)b = \gamma s \frac{(3-2\gamma)(b-\gamma s)+(1-\gamma)b}{4(b-\gamma s)} ;$$

whose derivative with respect to b takes value

$$\frac{\partial}{\partial b}\pi_B^L(b;s)b = -\frac{(1-\gamma)\gamma^2 s^2}{4(b-\gamma s)^2} < 0 .$$

It immediately follows that a decrease in the measure of active traders is always collectively beneficial for the long side of the market. Conversely, when $b \leq s$; the collective utility of the agents in the short side of the market is

$$\pi_B^S(b;s)b = b\gamma \frac{(4-\gamma)(s-\gamma b)-(1-\gamma)\gamma b}{4(s-\gamma b)} .$$

It is easy to check that $\pi_B^S(b;s)b$ is a strictly concave function in all its domain and that it reaches a maximum at

$$\hat{b}(s) \equiv s\frac{(5-2\gamma)-\sqrt{(1-\gamma)(5-2\gamma)}}{\gamma(5-2\gamma)} .$$

Note that the above maximum lies outside the relevant range when $\hat{b}(s) \geq s$; that is when

$$\gamma \leq \frac{7-\sqrt{17}}{4} = 0.71922 \equiv \underline{\gamma}. \qquad (3)$$

For further reference, we will say that search frictions are high when condition (3) is met, otherwise we will say that the search frictions are low.

Therefore, collective incentives to restrict market participation are present, and they might exist even for the short side of the market. It seems then natural to analyze whether coalitions that attempt to restrict trade are sustainable or not, and what impact they have on market performance. We address these issues in the following sections.

3 Cartel Games

We assume that one and only one cartel operates on each side of the market. The buyers' cartel controls $\mu_B b$ buyers, and $\mu_S s$ sellers belong to the sellers' cartel, where $0 < \mu_B; \mu_S \leq 1$. There are $(1-\mu_B)b$ free buyers and $(1-\mu_S)s$ free sellers, who operate as independent traders and remain active in the market as long as they have trade to carry out.

Cartels play a quantity-setting game where they simultaneously choose their participation level in the market. Given the bilateral nature of trade and the indivisibility of the traded good, each cartel restricts the quantity actually traded by withdrawing some measure of its members from the market. In particular, the buyers' cartel sets the measure of its active members, i.e. it sets its demand $q_B \in [0;\mu_B b]$; and the sellers' cartel sets its supply $q_S \in [0;\mu_S s]$: Since free traders always participate in trade, the total market demand is given by $\beta = q_B + (1-\mu_B)b$ and the total market supply is equal to $\sigma = q_S + (1-\mu_S)s$: Therefore it is as if the buyers' cartel sets market demand $\beta \in [(1-\mu_B)b; b]$ and the sellers' cartel chooses market supply $\sigma \in [(1-\mu_S)s; s]$: The aggregate cartel payoffs are then redistributed equally among cartel members in order to compensate the inactive agents for their abstention.[12]

For each profile of cartel memberships $\mu = (\mu_B;\mu_S)$, the payoffs of the quantity-setting game G^μ; where both cartels act simultaneously, can be expressed as functions of market demand and supply as

$$B(\beta;\sigma) = \begin{cases} (\beta-(1-\mu_B)b)\,\pi_B^S(\beta;\sigma) & \text{if } (1-\mu_B)b \le \beta \le \sigma \\ (\beta-(1-\mu_B)b)\,\pi_B^L(\beta;\sigma) & \text{if } \sigma \le \beta \le b \end{cases}$$

and

$$S(\beta;\sigma) = \begin{cases} (\sigma-(1-\mu_S)s)\,\pi_S^S(\beta;\sigma) & \text{if } (1-\mu_S)s \le \sigma \le \beta \\ (\sigma-(1-\mu_S)s)\,\pi_S^L(\beta;\sigma) & \text{if } \beta \le \sigma \le s \end{cases};$$

respectively, for the buyers' and the sellers' cartels.

Let $\beta(\sigma)$ and $\sigma(\beta)$ denote the best responses of the buyers' and sellers' cartels, respectively, in the game G^μ: If necessary, we will use the notation $\beta^{\mu_B}(\sigma)$ and $\sigma^{\mu_S}(\beta)$ to stress that, for a given quantity traded by the opponent, the best response of the buyers' (sellers') cartel depends on the cartel's own size but not on the other cartel's membership level. Similarly, the payoff functions will be denoted by $B^{\mu_B}(\beta;\sigma)$ and $S^{\mu_S}(\beta;\sigma)$.

A market outcome is a profile $(\mu_B;\mu_S;\beta;\sigma)$ such that $(\beta;\sigma)$ is a Nash equilibrium of G^μ; that is, $\sigma = \sigma(\beta)$ and $\beta = \beta(\sigma)$ for the given cartel sizes.

A market outcome is a reasonable prediction for the operation of the market only when cartels can be expected to maintain their membership levels. Cartels might not preserve their sizes, either because some of their members wish to defect and become free traders, or because some free traders wish to join the cartel. In the present model, since cartels generate positive externalities that benefit outsiders on the same side of the market, the incentives of insiders to defect from the cartel may strongly undermine the stability of a market outcome. Conversely, the incentives of outsiders to join the cartel will not

[12] It is assumed that each cartel can enforce the exclusion of traders, i.e. its members cannot sneak in the market when they have been ordered to stay out and they cannot organize parallel trade of the excluded quantities.

be a concern. With a continuum of non-atomic agents, the defection of a single agent has a negligible impact on the market outcome, therefore the notion of stability must not rely directly on immunity from unilateral deviations.

Our concept of cartel stability first postulates the absence of incentives to deviate by coalitions of small but strictly positive measure. Given a market outcome $(\mu_B; \mu_S; \beta; \sigma)$; we will say that an ε-coalition (that is a measure $\varepsilon > 0$ of agents) of buyers in the cartel benefits from defecting the cartel if and only if

$$\pi_B(\beta_{\mu_B-\varepsilon}(\sigma);\sigma) > \frac{\pi_B(\beta;\sigma)(\beta-(1-\mu_B)b)}{\mu_B b}:$$

Similarly, an ε-coalition of buyers that are outsiders to the cartel benefits from joining the cartel if and only if

$$\frac{\pi_B(\beta_{\mu_B+\varepsilon}(\sigma);\sigma)(\beta_{\mu_B+\varepsilon}(\sigma)-(1-\mu_B-\varepsilon)b)}{(\mu_B+\varepsilon)b} > \pi_B(\beta;\sigma):$$

The conditions under which an ε-coalition of sellers benefits from a deviation can be expressed analogously. Observe that, in assessing whether an ε-coalitional deviation is profitable or not at the market outcome $(\mu_B; \mu_S; \beta; \sigma)$, the quantity supplied or demanded on the other side of the market is taken as given. Moreover, it is assumed that a cartel can perfectly observe if it is affected by a deviation, and the change in the cartel's size will in turn alter the cartel's optimal response to the quantity set by the counterpart.

A cartel is ε-stable at the profile $(\mu_B; \mu_S; \beta; \sigma)$ if there exists a positive but arbitrarily small measure ε of agents such that no ε-coalition of its members (non-members) benefits from defecting (joining) the cartel. A profile $(\mu_B; \mu_S; \beta; \sigma)$ is an ε-stable market outcome if and only if: (i) it is a market outcome, (ii) both cartels are ε-stable:

We will say that the profile $(\mu_B; \mu_S; \beta; \sigma)$ is a stable market outcome if it is the limit of ε-stable market outcomes when $\varepsilon \rightarrow 0$:

The remainder of the paper is devoted to proving that the sets of ε-stable and stable market outcomes are non-empty, and to their characterization. This is done in two steps. In the first step, market outcomes for generic cartel memberships $(\mu_B; \mu_S)$ are characterized. In the second step, the constraints of stability at the market outcomes are explored, providing the desired characterization and the proof of existence.

4 Market Outcomes

At this stage, we will explore the properties and the existence of market outcomes, that is of Nash equilibria of the quantity-setting game G^{μ} with given measures of cartel memberships.

The following assertions about the properties of the buyers' cartel payoffs, $B(\beta;\sigma)$, are useful to gain some intuition about the results. The proofs are straightforward and will be omitted for the sake of brevity. Similar claims hold for $S(\beta;\sigma)$:

Claim 1 The payoff function $B(\beta;\sigma)$ is continuous at all $(\beta;\sigma)$, since $\pi^L(x;x) = \pi^S(x;x)$ for all $(x;x)$.

Claim 2 When $\sigma > 0$, the payoff function $B(\beta;\sigma)$ is strictly increasing at $\beta = (1-\mu_B)b$. Therefore, when supply is positive, there are always some members of the buyers' cartel who actively trade.

Claim 3 When $\beta \leq \sigma$, the payoff function $B(\beta;\sigma)$ is strictly concave. Therefore, if $B(\beta;\sigma) \geq B\left(\beta';\sigma\right)$ for all $\beta' \leq \sigma$, then β is such that

$$\beta = \min\left\{\widehat{\beta}(\sigma);\sigma\right\};$$

where $\widehat{\beta}(\sigma)$ solves

$$\frac{\partial}{\partial\beta}\left[(\beta-(1-\mu_B)b)\,\pi_B^S(\beta;\sigma)\right] = 0\,. \tag{4}$$

Claim 4 When $\beta \geq \sigma$, any critical point of the payoff function $B(\beta;\sigma)$ is a minimum. Therefore, if $B(\beta;\sigma) \geq B\left(\beta';\sigma\right)$ for all $\beta' \geq \sigma$, either $\beta = b$ or $\beta = \sigma$.

Taking into account the above claims, one can conclude that $\beta(\sigma)$, the best reply of the buyers' cartel to any level of market supply, is one of the following three: (i) to be inactive, that is not to withdraw any members and set $\beta(\sigma) = b$; (ii) to match the opponents' quantity, that is to set demand exactly equal to supply $\beta(\sigma) = \sigma$, or (iii) to undercut the quantity traded on the sellers' side and to set demand below supply at a level satisfying condition (4), i.e. $\beta(\sigma) = \widehat{\beta}(\sigma) < \sigma$. When does each one of the above choices prevail?[13]

Suppose first that undercutting is not a payoff-maximizing solution and that the cartel's decision is based on the comparison between its payoffs when being inactive and when matching the sellers' offer, which are

$$B(b;\sigma) = b\mu_B\pi_B^L(b;\sigma) \quad \text{and} \quad B(\sigma;\sigma) = (\sigma-(1-\mu_B)b)\,\pi_B^L(\sigma;\sigma)\;;$$

respectively. Denote by $\sigma_{I\sim M}$ the supply level which satisfies that $B(b;\sigma) = B(\sigma;\sigma)$, whereby the buyers' cartel is exactly indifferent between being inactive and matching total supply at $\sigma = \sigma_{I\sim M}$. Then $B(b;\sigma) > B(\sigma;\sigma)$ if and only if

$$\sigma < \frac{2(2-\gamma)(1-\mu_B)b}{\gamma(1+(1-\mu_B)(3-2\gamma))} \equiv \sigma_{I\sim M}\;; \tag{5}$$

where $\sigma_{I\sim M}$ is a decreasing function of the buyers's cartel membership μ_B. The above condition is trivially satisfied when $\sigma_{I\sim M} > s$ or equivalently when

$$0 < \mu_B < \frac{2(2-\gamma)\left(1-\gamma\frac{s}{b}\right)}{2(2-\gamma)-\gamma\frac{s}{b}(3-2\gamma)} \equiv \mu_B^{I\sim M} < 1.$$

[13] In the sequel, the indeces I, M and U will refer to, respectively, a cartel being inactive, matching the quantity traded on the other side of the market, or undercutting it.

Hence the buyers' cartel always prefers to remain inactive rather than to match total supply if its membership level is low enough, i.e. when $\mu_B < \mu_B^{I\to M}$.

Let us now address the case in which undercutting could be the payoff-maximizing decision. Observe that a solution to (4) satisfying $\beta < \sigma$ cannot exist when γ is too small, namely under high search frictions. Indeed, condition (4) yields as the unique solution

$$\hat{b}(\sigma) \equiv \frac{\sigma(5-2\gamma) - \sqrt{\sigma(1-\gamma)(5-2\gamma)(\sigma-\gamma(1-\mu_B)b)}}{\gamma(5-2\gamma)} \qquad (6)$$

and under high search frictions, i.e. $\gamma \leq \underline{\gamma}$, it is always the case that $\hat{b}(\sigma) > \sigma$.[14] For low frictions, i.e. $\gamma > \underline{\gamma}$, the cartel's decision depends on the comparison between its payoffs when undercutting and when matching the sellers' offer, which are

$$B(\hat{b}(\sigma);\sigma) = \left(\hat{b}(\sigma) - (1-\mu_B)b\right)\pi_B^S(\hat{b}(\sigma);\sigma) \quad \text{and} \quad B(\sigma;\sigma) = (\sigma-(1-\mu_B)b)\pi_B^S(\sigma;\sigma),$$

respectively. The inequality $B(\sigma;\sigma) < B(\hat{b}(\sigma);\sigma)$, together with $\frac{\partial}{\partial\sigma}\hat{b}(\sigma) > 0$ and $\hat{b}(\sigma) < \sigma$, is satisfied if and only if

$$\sigma > \frac{(1-\mu_B)\gamma b}{(1-(5-2\gamma)(1-\gamma))} \equiv \sigma_{M\to U}. \qquad (7)$$

The function $\sigma_{M\to U}$ is decreasing in μ_B and thus condition (7) never holds if $\sigma_{M\to U} \geq s$, that is if

$$0 < \mu_B \leq \frac{\gamma - (1-(5-2\gamma)(1-\gamma))\frac{s}{b}}{\gamma} \equiv \mu_B^{M\to U} < 1.$$

Moreover, $\sigma_{I\to M} \leq \sigma_{M\to U}$ holds if and only if

$$0 < \mu_B \leq \frac{2(2-\gamma)\left(\gamma^2-(1-(5-2\gamma)(1-\gamma))\right)}{\gamma^2(3-2\gamma)} \equiv \bar{\mu},$$

a condition which is always satisfied if $\bar{\mu} \geq 1$, that is when search frictions are not too low, that is when

$$\underline{\gamma} < \gamma \leq 0.78203 \equiv \bar{\gamma},$$

in which case both $\mu_B^{I\to M} \leq \mu_B^{M\to U}$ and $\bar{\mu} > \mu_B^{M\to U}$ hold. When $\gamma > \bar{\gamma}$ search frictions will be called very low.

When $\bar{\mu} < \mu_B \leq 1$ (implying that market frictions are very low), the payoff-maximizing decision of the buyers' cartel cannot be to match σ, and it is solely based on the comparison between its payoffs from being inactive and from undercutting the quantity offered. Let $\sigma_{I\to U}$ be defined as the solution to $B(b;\sigma) = B(\hat{b}(\sigma);\sigma)$, where $\sigma_{M\to U} < \sigma_{I\to U} < \sigma_{I\to M}$ always holds for $\bar{\mu} < \mu_B \leq 1$.[15] Moreover, define $\mu_B^{I\to U}$ as the solution to $\sigma_{I\to U} = s$, with $\sigma_{I\to U} < s$ if and only if $\mu_B > \mu_B^{I\to U}$. Then reducing demand

[14] This involves some tedious, but otherwise straightforward algebra. See page 7 for the definition of high search frictions.

[15] We omit the analytical expression for $\sigma_{I\to U}$, as it is uninformatively complicated, being $\sigma_{I\to U}$ one of the roots of a fourth-degree polinomial equation.

to $\hat{\beta}(\sigma)$ is the cartel's optimal choice when sellers' supply is such that $\sigma \geq \sigma_{I \to U}$, whereas remaining inactive is the cartel's payoff-maximizing solution for $\sigma < \sigma_{I \to U}$.

The optimal decisions of the sellers' cartel are characterized analogously, with $\beta_{I \to M}; \beta_{M \to U}; \beta_{I \to U}$ and $\mu_S^{I \to M}; \mu_S^{M \to U}; \mu_S^{I \to U}$ defined symmetrically.

This completes the proof of Lemma 1 that follows.

For the remainder of the paper, and without loss of generality, we maintain the assumption that sellers are the short side of the market and we normalize the measure of the long side, that is $s \leq b = 1$.

Lemma 1 (a) When search frictions are high, i.e. $\gamma \leq \underline{\gamma}$, the best reply function of the buyers' cartel is

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \min\{\sigma_{I \to M}; s\} \\ \sigma & \text{if } \sigma \geq \min\{\sigma_{I \to M}; s\} \end{cases} \tag{a}$$

and similarly the best reply of the sellers' cartel is

$$\sigma(\beta) = \begin{cases} s & \text{if } \beta < \min\{\beta_{I \to M}; 1\} \\ \min\{\beta; s\} & \text{if } \beta \geq \min\{\beta_{I \to M}; 1\} \end{cases}.$$

(b) When search frictions are low, i.e. $\gamma > \underline{\gamma}$; and $\mu_B \leq \bar{\mu}$; the best reply function of the buyers' cartel is

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \min\{\sigma_{I \to M}; s\} \\ \sigma & \text{if } \min\{\sigma_{I \to M}; s\} \leq \sigma < \min\{\sigma_{M \to U}; s\} \\ \hat{\beta}(\sigma) & \text{if } \sigma \geq \min\{\sigma_{M \to U}; s\} \end{cases} \tag{b}$$

and similarly (for $\mu_S \leq \bar{\mu}$ and $\gamma > \underline{\gamma}$); the sellers' cartel best reply function is

$$\sigma(\beta) = \begin{cases} s & \text{if } \beta < \min\{\beta_{I \to M}; 1\} \\ \min\{\beta; s\} & \text{if } \min\{\beta_{I \to M}; 1\} \leq \beta < \min\{\beta_{M \to U}; 1\} \\ \min\left\{\hat{\sigma}(\beta) = \frac{\beta(5-2\gamma) - \sqrt{\beta(1-\gamma)(5-2\gamma)(\beta-\gamma(1-\mu_S)s)}}{\gamma(5-2\gamma)}; s\right\} & \text{if } \beta \geq \min\{\beta_{M \to U}; 1\} \end{cases}$$

(c) When market frictions are very low, i.e. $\gamma > \bar{\gamma}$; and $\bar{\mu} < \mu_B \leq 1$; the best reply function of the buyers' cartel consists in

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \min\{\sigma_{I \to U}; s\} \\ \hat{\beta}(\sigma) & \text{if } \sigma \geq \min\{\sigma_{I \to U}; s\} \end{cases} \tag{c}$$

and similarly (when $\bar{\mu} < \mu_S \leq 1$) the sellers' cartel best reply function is

$$\sigma(\beta) = \begin{cases} s & \text{if } \beta < \min\{\beta_{I \to U}; 1\} \\ \min\{\hat{\sigma}(\beta); s\} & \text{if } \beta \geq \min\{\beta_{I \to U}; 1\} \end{cases}.$$

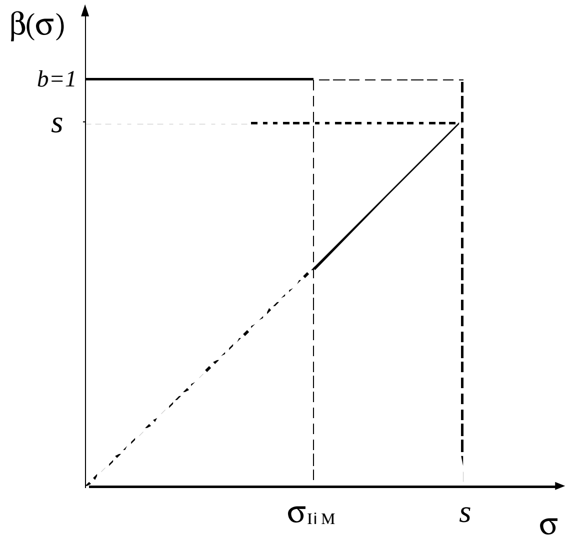

Figure 1: (a) The reaction function of a moderate buyers' cartel.

We will say that a cartel is moderate when, irrespectively of its size, the only relevant options that it faces are either to be inactive or to match the opponent's quantity. A cartel is moderate if and only if search frictions are high, whereby either both the buyers' and the sellers' cartels are moderate or none is. Observe that a moderate cartel will always be inactive if the fraction of its members is such that $\mu_i < \mu_i^{I»M}$; with $i = B; S$: A cartel will be called radical if its best reply might consist in either being inactive or undercutting the opponent's quantity (but not matching). A cartel is radical if and only if its size is high enough, i.e. $\bar{\mu} < \mu_i \leq 1$; with $i = B; S$ (a condition implying that market frictions are very low). A radical cartel will always prefer to be inactive if the proportion of its membership is such that $\bar{\mu} < \mu_i \leq \mu_i^{I»U}$: Finally, we will say that a cartel is flexible if it can potentially respond to the opponent's quantity by staying inactive, matching, or undercutting. A cartel is always flexible if search frictions are not too low and it is flexible if and only if search frictions are low and $\mu_i \leq \bar{\mu}$. Nevertheless, a flexible cartel will always remain inactive if $\mu_i < \mu_i^{I»M}$; and it will always prefer to match rather than undercut the opponent's volume of trade if $\mu_i^{I»M} \leq \mu_i \leq \mu_i^{M»U}$. In these cases, the flexible cartel actually behaves as a moderate one.

Example 1 below displays the buyers' cartel reaction functions under the different scenarios contemplated in Lemma 1.

Example 1 (a) Suppose that the buyers' cartel is moderate (equivalently search frictions are high, i.e. $\gamma \leq \underline{\gamma}$; and $\sigma_{I»M} < s$): Then its reaction function is as the one displayed in Figure 1. Note that any cartel membership $\mu_B < \mu_B^{I»M}$ would yield $\sigma_{I»M} > s$ and thus matching σ would no longer be a relevant

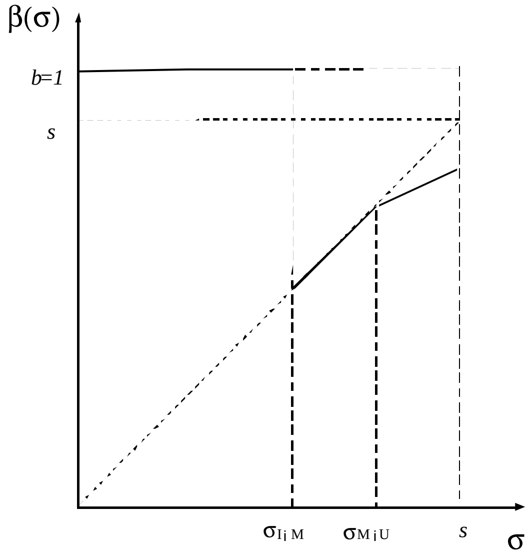

Figure 2: (b) The reaction function of a flexible buyers' cartel.

option for the buyers' cartel and $\beta(\sigma) = 1$ for all σ.

(b) Consider now a flexible buyers' cartel and suppose that $\sigma_{I\sim M} < \sigma_{M\sim U} < s$, in which case the buyers' cartel reaction function is represented as in Figure 2. In the event that $\sigma_{I\sim M} < s < \sigma_{M\sim U}$, or else that $\mu_B^{I\sim M} < \mu_B < \mu_B^{M\sim U}$, undercutting would not be payoff-maximizing and the buyers' cartel best reply function would look like the moderate cartel's, displayed in Figure 1. Furthermore, if $\mu_B \leq \mu_B^{I\sim M}$ and $\sigma_{I\sim M} \geq s$, then the buyers' best response would be $\beta(\sigma) = 1$ for all σ.

(c) Finally consider a radical buyers' cartel. Its reaction function can be represented as in Figure 3 as long as $\mu_B > \mu_B^{I\sim U}$ holds, otherwise $\beta(\sigma) = 1$ for all σ.

The best response functions characterized in Lemma 1 generate two distinct types of market outcomes.

On the one hand, the best responses may overlap for a non-empty interval along the diagonal. In this case, all market outcomes yield a perfect match in the measures of active buyers and sellers, and at least one cartel actively restrains trade. This symmetric trade scenario is attained when search frictions are high and $\sigma_{I\sim M} \leq s$ holds (no requirement is needed on $\beta_{I\sim M}$). When search frictions are low, an outcome with symmetric participation can prevail only if both cartels are flexible (i.e. if both $\sigma_{I\sim M} \leq \sigma_{M\sim U}$ and $\beta_{I\sim M} \leq \beta_{M\sim U}$ hold, a constraint binding only under not too low frictions), and if and only if the best response functions overlap along the diagonal, that is

$$\underline{q} \equiv \min\{\max\{\beta_{I\sim M};\sigma_{I\sim M}\};s\} \leq \min\{\beta_{M\sim U};\sigma_{M\sim U};s\} \equiv \overline{q}. \tag{8}$$

On the other hand, when the best response functions do not overlap along the diagonal, they may intersect at the boundaries. This yields a unique market outcome with asymmetric participation where

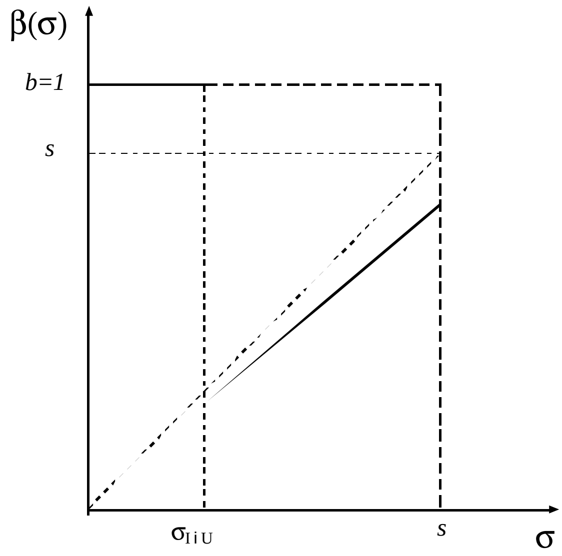

Figure 3: (c) The reaction function of a radical buyers' cartel.

one cartel is inactive; the other cartel may restrain its market participation so that it slightly undercuts the opponent's, or it may also remain inactive.[16]

The magnitude of absolute cartel memberships, μ_B and $\mu_S s$, determines whether the Nash equilibrium of the game G^μ exists, and, if so, whether it is a symmetric market outcome or an asymmetric one. In particular, if absolute membership levels are close to each other and sufficiently high, then condition (8) holds and symmetric market outcomes, with at least one cartel restraining its market participation, are attained. Conversely, if cartel memberships are sufficiently different, a unique Nash equilibrium prevails in which the quantities traded are asymmetric, and where only the larger cartel restrains trade. Finally, for intermediate cases, the game G^μ might not have a Nash equilibrium. Of course, when cartel sizes $(\mu_B; \mu_S s)$ do not support a market outcome, then no profile $(\mu_B; \mu_S; \beta; \sigma)$ can be an ε-stable market outcome (neither a stable market outcome).

The proposition below characterizes the Nash equilibria of the game G^μ, and gives the necessary and sufficient conditions for their existence. The proof follows straightforwardly from inspection of the best response functions.[17]

Proposition 1 (a) Let search frictions be high (i.e. $\gamma \leq \underline{\gamma}$; implying that both cartels are moderate). If being inactive and matching are both payoff-maximizing options for the buyers' cartel, i.e. $\mu_B \geq \mu_B^{I\lambda M}$,

[16] For the sake of completeness, let us point out that, when the best response functions intersect at the boundaries, there also exists a unique market outcome with symmetric participation such that the sellers' cartel is inactive and the buyers' cartel matches s.

[17] See Example 2 below.

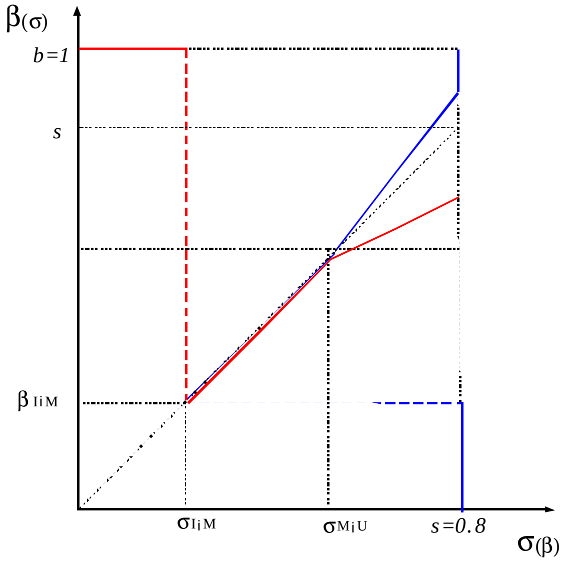

Figure 4: (a) Multiplicity of equilibria.

then any strategy pair $(\beta;\sigma) = (q;q)$; with $q \in [\underline{q}; s]$; is a NE. Otherwise, the strategy pair $(\beta;\sigma) = (1;s)$ is the unique NE.

(b) Let search frictions be low, i.e. $\gamma > \underline{\gamma}$. Then, a strategy pair $(\beta;\sigma) = (q;q)$; with $q \in [\underline{q};\overline{q}]$; is a Nash equilibrium if and only if both cartels are flexible, i.e. $\mu_B;\mu_S \leq \overline{\mu}$ and (8) is satisfied. Otherwise, the unique Nash equilibrium is

$$(\beta;\sigma) = \begin{cases} \left(\hat{\beta}(s);s\right) & \text{if } \max\{\sigma_{M\to U};\sigma_{I\to U}\} \leq s \text{ and } \hat{\beta}(s) < \min\{\beta_{I\to M};\beta_{I\to U}\} \\ (1;\hat{\sigma}(1)) & \text{if } \max\{\beta_{M\to U};\beta_{I\to U}\} \leq s \text{ and } \hat{\sigma}(1) < \min\{\sigma_{I\to M};\sigma_{I\to U};s\} \\ (1;s) & \text{if } s < \min\{\sigma_{I\to M};\sigma_{I\to U}\} \text{ and } \hat{\sigma}(1) \geq s. \end{cases} \quad (9)$$

No NE exists when the conditions in (9) are not met.

Example 2 considers the range of market outcomes for flexible cartels.

Example 2 Let $s = \frac{4}{5}$ be the ex ante measure of sellers, let search frictions be equal to $\gamma = \frac{3}{4}$ and let the proportion of buyers in the cartel be $\mu_B = \frac{20}{21}$.

(a) If the proportion of sellers in the cartel is $\mu_S = \frac{31}{33}$ then the game $G^{\frac{20}{21};\frac{31}{33}}$ is such that all pairs $(\beta;\sigma) = (q;q)$ with $q \in \left[\frac{4}{27};\frac{2}{7}\right]$ represent equilibrium outcomes. This result is shown in Figure 4.

(b) Assume that $\mu_S = \frac{23}{50}$, then the unique equilibrium of game $G^{\frac{22}{21};\frac{23}{50}}$ is given by the pair $\left(\hat{\beta}(s);s\right) = (0.78;0.8)$ and is shown in Figure 5.

(c) Finally, letting $\mu_S = \frac{5}{6}$, it is straightforward to check that the game $G^{\frac{23}{21};\frac{5}{6}}$ has no Nash equilibrium, as shown in Figure 6.

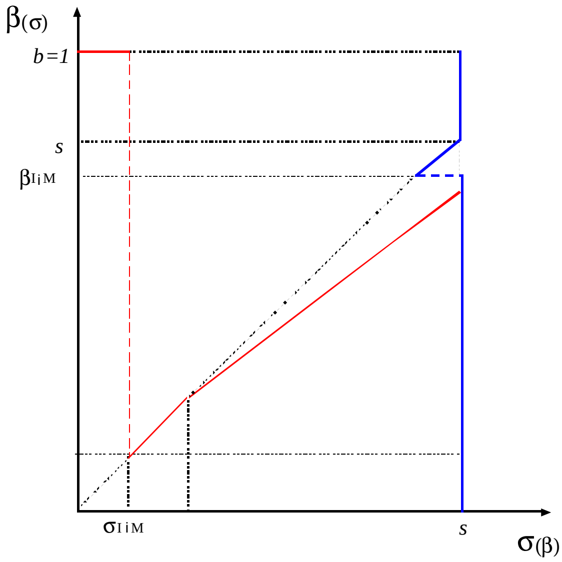

Figure 5: (b) Uniqueness of the equilibrium.

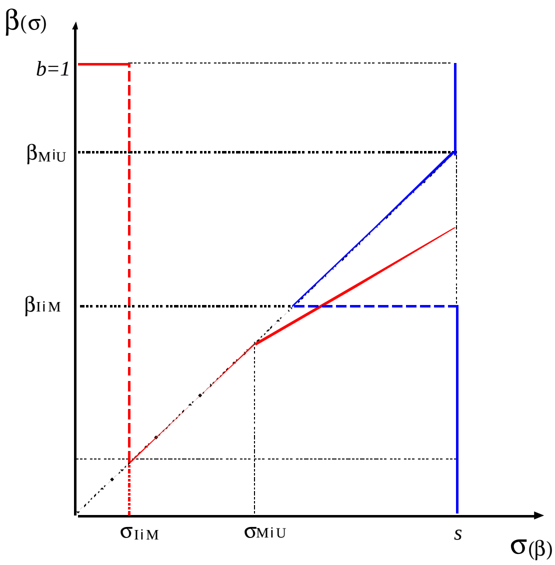

Figure 6: (c) Non-existence of an equilibrium.

5 Stability

Depending on the magnitudes of absolute cartel memberships μ_B and $\mu_S s$, a broad set of Nash equilibria might exist. But not all market outcomes are equally relevant, because some of them cannot be supported by stable levels of cartel memberships.

Indeed, given a profile $(\mu_B; \mu_S)$, at most a Nash Equilibrium of the cartel game is supported by stable membership levels. We will show that, for a wide set of parameter configurations, there exist ε-stable (and stable) market outcomes with at least one active cartel. At these market outcomes, the quantities traded must be symmetric, regardless of the potential measures of traders on each side of the market. If frictions are low, ε-stable (and stable) market outcomes, where only one cartel is active, exist as well. At these outcomes, the active cartel is the one with greater absolute membership and it reduces its supply or demand so as to slightly undercut the counterpart's (unconstrained) market participation.

For further reference, observe that condition $\hat{q}(1) < s$ is satisfied if and only if

$$\mu_S > \frac{(4-\gamma) - (3(3-\gamma) - (5-2\gamma)s^{\gamma})s^{\gamma}}{(1-\gamma)s^{\gamma}} = \underline{\mu}_S :$$

Our next result establishes necessary and sufficient conditions for ε-stability.

Proposition 2 (i) A market outcome with symmetric market participation and at least the buyers' cartel being active, i.e. a profile $(\mu_B; \mu_S; q; q)$ with $q \leq s$, is ε-stable if and only if either (a) trade is set at the level $\beta = \sigma = \beta_{I \sim M} = \sigma_{I \sim M} \leq s$ and neither cartel is radical being $\mu_B^{I \sim M} \leq \mu_B \leq \overline{\mu}$ and $\mu_S^{I \sim M} \leq \mu_S \leq \overline{\mu}$; or (b) trade is set at $(s; s)$ and $\mu_B^{I \sim M} = \mu_B$ and $\mu_S < \min\left\{\mu_S^{I \sim M}; \mu_S^{I \sim U}\right\}$.
(ii) A market outcome with asymmetric participation and the buyers' cartel only being active, i.e. a profile $\left(\mu_B; \mu_S; \hat{q}(s); s\right)$; is ε-stable if and only if cartels are not moderate, the sellers' cartel prefers to be inactive, i.e. $\mu_S < \min\left\{\mu_S^{I \sim M}; \mu_S^{I \sim U}\right\}$; and either $\mu_B = \mu_B^{I \sim U}$, or $\mu_B = \mu_B^{M \sim U} + \varepsilon$ with $s - \varepsilon \leq \hat{q}(s) < s$.
(iii) A market outcome with asymmetric participation and the sellers' cartel only being active, i.e. a profile $(\mu_B; \mu_S; 1; \hat{q}(1))$; is ε-stable if and only if cartels are not moderate, buyers always prefer to be inactive, i.e. $\mu_B < \min\left\{\mu_B^{I \sim M}; \mu_B^{I \sim U}\right\}$; and either $\mu_S = \mu_S^{M \sim U} + \varepsilon > \underline{\mu}_S$ (in this case $s - \varepsilon < \hat{q}(1) < s$ for $s \in (s_{\varepsilon}; 1]$; with s_{ε} solving $\mu_S^{M \sim U} + \varepsilon = \underline{\mu}_S$) or $\mu_S = \mu_S^{I \sim U} > \underline{\mu}_S$:

Proof. See Appendix A. ■

Observe that an ε-stable outcome with only the sellers' cartel being active might exist when the market is balanced, i.e. if $s = b = 1$; or only if the total measure of sellers s is infinitesimally smaller than the total measure of buyers b. In this case the sellers' cartel is active but only marginally so.

Also note that among ε-stable market outcomes with asymmetric participation, only those where the fraction of cartel members is $\mu_i = \mu_i^{I \sim U}$; with $i = B; S$, survive in the limit as $\varepsilon \to 0$.

A characterization of the set of stable market outcomes is presented in the next proposition, whose proof is immediate and therefore omitted.

Proposition 3 (i) The set of stable market outcomes with symmetric participation and at least the buyers' cartel being active coincides with the ε-stable ones.

(ii) A market outcome with asymmetric participation and the buyers' cartel only being active is stable if and only if $\mu_B = \mu_B^{\text{l»U}}$ and $\mu_S < \min\left\{\mu_S^{\text{l»M}}; \mu_S^{\text{l»U}}\right\}$:

(iii) A market outcome with asymmetric participation and the sellers' cartel only being active is stable if and only if $\mu_S = \mu_S^{\text{l»U}} > \underline{\mu}_S$ and $\mu_B < \min\left\{\mu_B^{\text{l»M}}; \mu_B^{\text{l»U}}\right\}$:

Remark 1 Note that $\beta = \beta_{\text{l»M}} = \sigma_{\text{l»M}} = \sigma$ is equivalent to

$$0 < \mu_B = 1 - \frac{(1-\mu_S)s}{1+(1-\mu_S)(3-2\gamma)(1-s)} < 1 ; \qquad (10)$$

so that, to every pair of cartel memberships satisfying (10), there corresponds a stable market outcome with symmetric trade. Moreover, there always exist pairs of cartel memberships meeting the stability requirements for a market outcome with asymmetric trade.

The level of trade at stable market outcomes with symmetric participation might be inefficient. Moreover, condition (10) is compatible with market outcomes in which the cartels control more and more traders and force very limited market participation. In the limit, the market is driven to the collapse, given that the profile $(1; 1; 0; 0)$, is a stable market outcome for any level of market frictions.

Corollary 1 The set of stable market outcomes with symmetric trade includes inefficient profiles such that $0 \le \beta_{\text{l»M}} = \sigma_{\text{l»M}} < s$: The degree of inefficiency is unbounded.

Nonetheless, Proposition 3 does not rule out market outcomes at which quantities are set equal to the measure of the short side, $\beta = \sigma = s$; thus the following holds.

Corollary 2 Stable market outcomes include efficient profiles where $\mu_B = \mu_B^{\text{l»M}}$ and either $\mu_S = \mu_S^{\text{l»M}}$ (which exists if and only if $s = b = 1$) or $\mu_S < \min\left\{\mu_S^{\text{l»M}}; \mu_S^{\text{l»U}}\right\}$: It is an outcome with symmetric participation where the sellers' cartel is not active and the buyers' is active only to assure that demand equals supply.

At stable market outcomes with symmetric participation and both cartel being active, the cartel on the short side always withdraws from the market fewer traders than the cartel on the long side does. Does the cartel on the long side necessarily control more traders than its opponent? Does it need to encompass a relatively higher proportion of traders? These questions are answered in the following corollary.

Corollary 3 At stable market outcomes with symmetric trade and both cartel being active: (i) the buyers' cartel controls more traders than the sellers' cartel, i.e. $\mu_B > s\mu_S$; (ii) the buyers' cartel always controls a larger fraction of the total population on its side than the sellers' cartel, i.e. $\mu_B > \mu_S$:

Proof. Taking into account condition (10), the inequality $\mu_B > s\mu_S$ holds if and only if

$$s\mu_S < 1 - \frac{(1-\mu_S)s}{1+(1-\mu_S)(3-2\gamma)(1-s)}; \tag{11}$$

and it is straightforward to check that (11) holds for all μ_S and s. Inequality $\mu_S < \mu_B$ also holds for all $s < 1$. ■

At stable outcomes with asymmetric trade, the only active cartel slightly undercuts the opponent's participation level: Even though the active cartel actually withdraws a non-negligible measure of its members, efficiency is only marginally affected, since the level of realized trade decreases only by a small amount. The distribution of surplus between buyers and sellers is again substantially altered in favor of the side of the market where the cartel is active.

We may now summarize our results. There are stable market outcomes where both sides exercise countervailing power. In this case, countervailing power might be the source of (potentially severe) market inefficiency. However, in markets that are not balanced ex ante (maybe because the short side has effective means to prevent entry without compensation to excluded potential traders) there exist stable outcomes at which the exercise of countervailing power by the long side affects the distribution of surplus without damaging efficiency. We find that these market outcomes (partially) vindicate Galbraith's claims that countervailing power plays a desirable role in some markets.

A Proof of Proposition 2

It is clear that there are no situations where a positive measure ε of independent buyers or sellers have incentive to deviate and to join the cartel on their side of the market. Indeed, outsiders obtain a higher payoff than insiders provided that the cartel is active. Hence, the only relevant deviations are instances in which a positive measure ε of cartel members wish to defect from the cartel.

(a) Consider first market outcomes with symmetric trade.

- These outcomes can arise when reaction functions overlap along the diagonal, i.e. for cartel memberships such that $\mu_B, \mu_S \le \bar{\mu}$. In particular, assume that the fractions of cartel members are such that

  $$\max\{\beta_{I\cap M}, \sigma_{I\cap M}\} = \sigma_{I\cap M} \le \min\{\beta_{M\cap U}, s\} = \min\{\beta_{M\cap U}, \sigma_{M\cap U}, s\}$$

  whereby equilibrium strategies are such that $(\beta, \sigma) = (q, q)$, with

  $$\sigma^{\mu_B}_{I\cap M} = \frac{2(2-\gamma)(1-\mu_B)}{\gamma(1+(1-\mu_B)(3-2\gamma))} \le q \le \min\{\beta_{M\cap U}, s\}.$$

  Suppose further that, at a given market outcome, a strictly positive measure ε of cartel members leave the buyers' cartel. When such a defection occurs, the best reply function of the buyers' cartel shifts towards the right, due to a decrease in cartel membership from μ_B to $\mu_B - \varepsilon$. If the equilibrium measure of active traders q is such that $\sigma_{I\sim M}^{\mu_B-\varepsilon} \le q \le \min\{\beta_{M\sim U}; s\}$; then we claim that leaving the cartel is beneficial. Indeed, after the defection, the buyers' cartel continues to set its measure of active traders equal to $\beta^{\mu_B-\varepsilon}(q) = q$, which yields per capita payoffs $\pi_B(q;q)$ to outsiders (and to defecting cartel members). Prior to the defection, the individual payoff to cartel members is $\frac{B^{\mu_B}(q;q)}{\mu_B} < \pi_B^L(q;q)$: However, if q is such that $\sigma_{I\sim M}^{\mu_B} \le q < \sigma_{I\sim M}^{\mu_B-\varepsilon}$; then the buyers' cartel breaks down completely as a consequence of the defection, and it plays $\beta^{\mu_B-\varepsilon}(q) = 1$: In this situation, defecting buyers would each receive a payoff equal to $\pi_B^L(1;q)$; and a defection would not be profitable if $\frac{B^{\mu_B}(q;q)}{\mu_B} \ge \pi_B^L(1;q)$; which is the case. Conversely, it remains profitable for members of the sellers' cartel to leave their cartel, and consequently a defection of ε members does not induce the sellers' cartel to modify the chosen measure q of active sellers, being $\beta_{I\sim M} \le \sigma_{I\sim M}$. The same reasoning applies when $\max\{\beta_{I\sim M}; \sigma_{I\sim M}\} = \beta_{I\sim M} \le \min\{\sigma_{M\sim U}; s\}$: Thus, in general, when $\mu_B; \mu_S \le \overline{\mu}$; both cartels are stable if and only if

$$\beta = \beta_{I\sim M} = \frac{2(2-\gamma)(1-\mu_S)s}{\gamma(1+(1-\mu_S)(3-2\gamma))} = \frac{2(2-\gamma)(1-\mu_B)}{\gamma(1+(1-\mu_B)(3-2\gamma))} = \sigma_{I\sim M} = \sigma ;$$

which implies

$$\mu_B = 1 - \frac{(1-\mu_S)s}{1+(1-\mu_S)(3-2\gamma)(1-s)} :$$

A stable market outcome with symmetric trade thus exists when $\mu_B^{I\sim M} \le \mu_B \le \overline{\mu}$ and $\mu_S^{I\sim M} \le \mu_S \le \overline{\mu}$.

- Consider now the case in which a market outcome with symmetric trade arises when the reaction functions intersect at the boundaries. It is straightforward to check that deviations are not profitable if and only if the sellers' cartel is inactive and the buyers' cartel just matches s; that is for $\sigma_{I\sim M} = s$ and $\mu_S < \min\left\{\mu_S^{I\sim M}; \mu_S^{I\sim U}\right\}$.

(b) Secondly, consider market outcomes with asymmetric trade and suppose that the strategy pair $\left(\widehat{\beta}(s); s\right)$ is played.

- Suppose, for the time being, that $\max\{\sigma_{M\sim U}; \sigma_{I\sim U}\} = \sigma_{M\sim U}$ and consider a potential defection from the buyers' cartel. After the deviation, the measure of buyers in the cartel becomes $\mu_B - \varepsilon$ and the cartel's best reply function shifts slightly towards the right. Such a defection has three possible consequences, depending on the magnitude of $\mu_B - \varepsilon$. (i) The buyers' cartel continues to respond to the total quantity s offered by setting $\beta(s) = \widehat{\beta}^{\mu_B-\varepsilon}(s)$, and the equilibrium of the quantity-setting game is $\left(\widehat{\beta}^{\mu_B-\varepsilon}(s); s\right)$: This occurs when $\sigma_{M\sim R}^{\mu_B-\varepsilon} < s$; or equivalently when $\mu_B - \varepsilon > \mu_B^{M\sim U}$; and when $\widehat{\beta}^{\mu_B-\varepsilon}(s) < \min\{\beta_{I\sim M}; \beta_{I\sim U}\} \le s$: We claim that, when no cartel is active on the supply side, it is profitable for a measure $\varepsilon > 0$ of members of the buyers' cartel to defect. Observe that the per capita utility of outsiders after the defection is equal to

$$\pi_B^S\left(\hat{\beta}^{\mu_B-\varepsilon}(s);s\right) = \frac{\gamma(5-2\gamma)(s-\gamma(1-\mu_B+\varepsilon))-\sqrt{(s(1-\gamma)(5-2\gamma)(s-\gamma(1-\mu_B+\varepsilon)))}}{4(s-\gamma(1-\mu_B+\varepsilon))}; \qquad (12)$$

whereas the per capita utility that cartel members receive prior to the defection is

$$\frac{\pi_B\left(\hat{\beta}^{\mu_B}(s);s\right)}{\mu_B} = \frac{(s-(1-\mu_B)\gamma)(5-2\gamma)+s(1-\gamma)-2\sqrt{s(1-\gamma)(5-2\gamma)(s-(1-\mu_B)\gamma)}}{4\mu_B}. \qquad (13)$$

Furthermore note that $\pi_B^S\left(\hat{\beta}^{\mu_B}(s);s\right) > \frac{\pi_B\left(\hat{\beta}^{\mu_B}(s);s\right)}{\mu_B}$ always holds and that $\pi_B^S\left(\hat{\beta}^{\mu_B-\varepsilon}(s);s\right)$ is decreasing in ε. Thus, for ε small enough, expression (12) is strictly greater than (13). (ii) After the defection, the buyers' cartel demands $\hat{\beta}^{\mu_B-\varepsilon}(s)$ but no equilibrium of the quantity-setting game exists. This is the case when $\min\{\beta_{I\cup M};\beta_{I\cup U}\} < s$ and $\min\{\beta_{I\cup M};\beta_{I\cup U}\} \leq \hat{\beta}^{\mu_B-\varepsilon}(s) < s$. This situation could then be discarded. (iii) The buyers' cartel sets $\beta(s) = s$ and the equilibrium of the game is (s,s). Then, it must be that $\sigma_{M\cup U}^{\mu_B-\varepsilon} \geq s$, or equivalently that $\mu_B - \varepsilon \leq \mu_B^{M\cup U}$, and that $\min\{\beta_{I\cup M};\beta_{I\cup U}\} > s$. When this defection occurs, cartel members have individual payoff (13) before the defection, which is greater than $\pi_B(s,s)$, the outsiders' individual payoff. Therefore, if cartel memberships are such that $\mu_B = \mu_B^{M\cup U} + \varepsilon$ and $\mu_S < \min\left\{\mu_S^{I\cup M};\mu_B^{I\cup U}\right\}$, the profile $\left(\mu_B;\mu_S;\hat{\beta}(s);s\right)$ is ε-stable.

• When instead $\max\{\sigma_{M\cup U};\sigma_{I\cup U}\} = \sigma_{I\cup U}$ and a defection from the buyers' cartel occurs, the following cases have to be considered. (i) The buyers' cartel continues to respond setting $\beta(s) = \hat{\beta}^{\mu_B-\varepsilon}(s)$, and the outcome of the quantity-setting game is $\left(\hat{\beta}^{\mu_B-\varepsilon}(s);s\right)$. If $\sigma_{I\cup U}^{\mu_B-\varepsilon} < s$, which implies $\hat{\beta}^{\mu_B-\varepsilon}(s) < \min\{\beta_{I\cup M};\beta_{I\cup U};s\}$, then the deviation is profitable. (ii) The buyers' cartel plays $\hat{\beta}^{\mu_B-\varepsilon}(s)$, but no Nash equilibrium exists. (iii) When $\sigma_{I\cup U}^{\mu_B-\varepsilon} \geq s$, or equivalently $\mu_B - \varepsilon \leq \mu_B^{I\cup U}$, the defection from the cartel triggers the response $\beta^{\mu_B-\varepsilon}(s) = 1$, in which case the equilibrium outcome is $(1,s)$ and the deviating members are not better off. Then the profile $\left(\mu_B;\mu_S;\hat{\beta}(s);s\right)$ is an ε-stable market outcome for $\mu_B = \mu_B^{I\cup U}$ and $\mu_S > \max\left\{\mu_S^{I\cup M};\mu_S^{I\cup U}\right\}$.

(c) Finally, consider market outcomes of the form $(\mu_B;\mu_S;1;\hat{\sigma}(1))$. In order for such a profile to be an ε-stable market outcome, it must be the case that the strategy pair $(1;\hat{\sigma}(1))$ be a Nash equilibrium of the quantity-setting game G^{μ}. Recall that $\hat{\sigma}(1) < s$ if and only if

$$\mu_S > \frac{(4-\gamma)-(3(3-\gamma)-(5-2\gamma)s\gamma)s\gamma}{(1-\gamma)s\gamma} = \underline{\mu}_S.$$

Therefore, the profile $\left[\mu_B;\mu_S^{M\cup U}+\varepsilon;1;\hat{\sigma}(1)\right]$, where $\mu_B \leq \mu_B^{I\cup M}$, is ε-stable only if the condition $\underline{\mu}_S < \mu_S^{M\cup U} + \varepsilon < 1$ is also satisfied, which is the case if and only if $s > s^*$, where s^* solves

$\mu_S^{M,\sigma U} + \varepsilon = \underline{\mu}_S$ and is such that

$$s_\varepsilon \equiv \frac{(4-\gamma)+\gamma^2(5-2\gamma)+\varepsilon\gamma(1-\gamma)-\sqrt{(1-\gamma)\left(\gamma^2(1-\gamma)\varepsilon^2+2\gamma((5-2\gamma)\gamma^2+(4-\gamma))\varepsilon+(1-\gamma)(4+3\gamma-2\gamma^2)^2\right)}}{2(5-2\gamma)\gamma^2};$$

with $s_\varepsilon < 1$ being true for all $\varepsilon > 0$: In addition. the profile $\left(\mu_B; \mu_S^{I,\sigma U}; 1; \pi(1)\right)$ represents an ε-stable market outcome if and only if $\mu_S = \mu_S^{I,\sigma U} > \underline{\mu}_S$ and $\mu_B < \min\left\{\mu_B^{I,\sigma M}; \mu_B^{I,\sigma U}\right\}$:

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