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Is Bundling Anticompetitive?*

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Abstract

I analyze the implications of bundling on price competition in a market with complementary products. Using a model of imperfect competition with product differentiation, I identify the incentives to bundle for two types of demand functions and study how they change with the size of the bundle. With an inelastic demand, bundling creates an advantage over uncoordinated rivals who cannot improve by bundling. I show that this no longer holds with an elastic demand. The incentives to bundle are stronger and the market outcome is symmetric bundling, the most competitive one. Profits are lowest and consumer surplus is maximized.

JEL: L11, L13

Keywords: bundling, complementary goods, product differentiation

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1 Introduction

Bundling is a vertical practice and consists of selling together several products in the form of a package, regardless of whether consumers want to buy components from another seller. Examples of bundles are the desktop application packages Microsoft Office or Sun's Star Office.[1]

Firms often tie for efficiency reasons. However, there are also strategic reasons like market foreclosure or extension of monopoly power that raise concerns about the anticompetitive effects of the practice.[2] Recent economic theory also views bundling as a way of acquiring an advantageous market position when competing against uncoordinated component sellers, explaining in this manner the success of application bundles like MS Office.

The increasingly frequent use of bundling raises questions about its impact on competition and points out toward the necessity of economic analysis able to provide sound grounds for antitrust enforcement policies. In this paper I show that the potential of bundling to create a dominant market position crucially depends on the parameter conditions: A best response to rival bundling may be to bundle or not depending on the elasticity of demand.

I focus on the implications of bundling on complementary goods markets. In a model of

[1] A quick glance at the IT industries reveals that bundling is a common practice. Apple's i-Mac is an all-in-one computer, display and operating system (Mac OS X), the later being itself a bundle of traditional OS functions and DVD player, Media Player (QuickTime) and MS Internet Explorer. Real Networks offers various bundles of media capture, creation, presentation and delivery. In fact, Real One Player is a bundle of audio and video player, jukebox and media browser, like its main competitor, Windows Media Player. The examples do not limit to IT industries. Bundling of cars with radios or air conditioners was central to antitrust cases like *Town Sound and Custom Tops, Inc. v. Chrysler Motors Corp. (1992)* or *Heatransfer Corp. v. Volkswagenwerk AG* (1977).

[2] More recently, tying was subject to harsh treatment in the antitrust courts, in cases like, *Eastman Kodak v. Image Technical Service Inc. (1992),* and *E.C. v. Microsoft Corp. (2000)* where bundling of Windows OS with Windows Media Player was central to the case. (The decision in this last case was appealed in June 2004.) Also, bundling of Windows OS with Internet Explorer was part of the contrversial case *U.S. v. Microsoft Corp. (1998).*

imperfect competition with spatial product differentiation, I consider product bundles formed of two and, respectively, three components. Each component is produced by differentiated firms that differ across markets. The firms choose whether or not to bundle. Using two types of demand function, an elastic and an inelastic one, I identify how price elasticity affects the incentives to bundle.

Bundling increases price competition by decreasing product differentiation on the market. However, it has also a positive effect due to the internalization of the positive externality that a price decrease of one component has on the demand for the complementary ones. I show that the positive effect of bundling increases with the size of the system and, in the three component systems case, with elastic demand, is strong enough to offset the negative effects of an increase in price competition.

My model relates to the work of Nalebuff (2000), who analyses the effects of bundling on price competition in an inelastic demand setting. Once there are four or more items, the bundle seller does better than when he sells components individually and, moreover, bundle against component competition is a stable market outcome. This suggests that bundling can be an effective anticompetitive tool giving a firm that sells a bundle of complementary products a substantial advantage over rivals who sell the component products separately.

The present analysis shows that the *elastic demand setting* no longer supports these results. Here, *whenever there are incentives to bundle, the market outcome is bundle against bundle competition and bundling cannot be viewed as an anticompetitive tool.* With elastic demand, the changes in prices affect, not only the market sharing, but also the level of the demand at each location. In a zero elasticity setting, the demand for a product depends only on the price difference between the rival alternatives on the market. Then, the effects of a price decrease are partially offset by the response of the competitor. In a positive elasticity setting, the demand depends both on the price difference and the own price. Even though a price decrease generates a market sharing effect altered by the reaction of the rival, it has a direct positive market size effect unaltered by the competitors, because at each location more consumers buy.

I perform the related welfare analysis and show that the incentives to bundle are socially excessive. However, bundle against bundle competition (the market outcome with an elastic demand) generates higher consumer surplus than bundle against component competition (the market outcome with inelastic demand).

The present work closely relates to the work of Matutes and Régibeau (1992). They were the first to extend the monopoly bundling framework by considering a duopoly that produces in two complementary markets. Their main focus is the compatibility decision and the incentives to bundle. While my model is sharing certain features with their article, some of the most important differences are the consideration of duopolies varying across markets (resulting in a price coordination problem) and of three component systems, besides from the two component ones (allowing to analyze how the number of tied items affects the incentives to bundle and the market outcome).

Section 2 presents the model and section 3 examines the market equilibria in all considered cases. In section 4, I perform the welfare analysis. Finally, Section 5 discusses the results and section 6 draws final conclusions. Proofs are presented in the Appendix.

2 The Model

I consider bundles of two ($n = 2$) and, respectively, three ($n = 3$) components. In each component market there are two rival brands produced by firms A_i and, respectively, B_i ($i = \{1, ..., n\}$).[3] The A and B components are imperfect substitutes, while i and j ($i \neq j$)

[3] Duopolists serving different component markets are different firms.

are complementary components. Consumers get their valuation of the product only if all components are purchased. The product differentiation in each component market is modeled a la Hotelling. For each component I use a coordinate axis, where A brand is located at 0 and the B one is located at 1. The locations of all the systems form the vertices of a square (when $n = 2$) or cube (when $n = 3$) of volume 1. The transportation cost per unit of length is assumed to be 1. The production cost is assumed to be zero.[4]

Consumer locations are uniformly distributed on the square (cube). For a specific component a consumer location belongs to the interval $[0, 1]$. The coordinate gives the linear transportation cost incurred by the consumers at this specific location. Let x be the axis for the first component, y for the second one and z for the third component (this comes into play only for $n = 3$). Considering $n = 2$, a consumer located at (x, y), incurs a transportation cost of x if he buys the first component from the A firm, or, respectively, $1 - x$ if he buys the first component from the B firm, and a transportation cost of y if he buys the second component from the A firm or, respectively, $1 - y$ if he buys it from the B firm. When $n = 3$ a consumer located at (x, y, z) incurs, in addition, a cost of z, if buying the third component from A or $1 - z$, if buying it from B.[5]

Each consumer chooses one system that minimizes his perceived price (θ) equal to the price of the system plus the transportation cost. He buys only if this sum does not exceed his valuation of the product.

At each consumer location, there is a linear demand $D(\theta) = b - a\theta$, depending on the perceived price at this location.[6] I parametrize the slope to analyze how incentives to bundle

[4] With positive constant marginal cost the results would still hold as mark-ups over the cost.

[5] For example, the transportation cost is $x+1-y$ for bundle (A_1, B_2) and $x+1-y+z$ for bundle (A_1, B_2, A_3).

[6] "The way to justify a downward sloping demand at a given location is to envision a large number of consumers with different tastes for the system at this location." (J. Tirole, 1988, footnote 64, p. 335.) When $a = 0$, all consumers at a given location have the same valuation of the product.

change with price elasticity.[7] To study the effects of bundling on price competition, I restrict attention to covered market equilibria, where firms directly interact on the market. For this, at each location in the unit square (cube) there must be consumers who buy.[8] Consumers' valuation of the product (when $a = 0$) and the maximal valuation (when $a = 1$) should be high enough. When $a = 1$, not necessarily all consumers can buy, due to their heterogeneous valuations. For the simulation, the value $b = 10$ guarantees that the market is covered in equilibrium.[9]

For both inelastic and elastic demand: a) I identify the incentives to bundle and, b) I determine how the incentives to bundle change with the size of the bundle.

For the first purpose (a), I compare the equilibrium outcomes of the three possible competition modes when pure bundling is available: component versus component competition (CvsC), bundle versus component competition (BvsC), and bundle versus bundle competition (BvsB).

To see how the number of components affects the incentives to bundle (b), I compare the results for two- and three-component systems.

I assume that, whenever firms bundle, the tied system is incompatible with the competing components, and that pure bundling is the only available strategy: A bundler does not sell separate components. Section 5 discusses the robustness of the results when these requirements are relaxed.

[7] When $a = 0$, the demand is inelastic and when $a = 1$, the demand is elastic.

[8] That is, at the most remote locations where a system is preferred (i.e., on the indifference line (plane)) there should be consumers who buy.

[9] In the inelastic case a change of b only changes the size of the market. In the elastic demand case, the same qualitative results can be obtained for any value of b, whenever the market is still fully covered at equilibrium.

3 The incentives to bundle

In this section I analyze the incentives to bundle for two types of demand function and two sizes of the bundle. There are incentives to bundle if the profit of the bundler in BvsC is higher than the aggregate profits of the potential bundlers in CvsC. If, in addition, aggregate profits of the component selling competitors of the bundler are higher than their profits in BvsB, then BvsC is a stable market outcome.

In CvsC, each consumer buys n complementary components, and he can mix and match to form his own system. Let p_i and q_i be the prices charged by the producers of the first component, A_i and, respectively, B_i.

In the presence of bundling, there are two possible cases. In BvsB, all A and, respectively, B firms coordinate their pricing decision and sell their components only as a bundle. In BvsC, only the A firms sell a bundle, and B firms continue to sell separate components. Consumers choose one of the two bundles (systems). Let p be the price of the coordinated A firm, q be the price of the B firms (in BvsB it is the price of the bundle, and in BvsC it is the sum of component prices, $\Sigma_i q_i$) and let Δ be the price difference between the two systems $q - p$.

In BvsC, the bundler internalizes the positive externality that one component seller has on the seller of a complementary good, while its competitors, selling separate components, neglect this effect. Then, $p < q = \Sigma q_i$ and $\Delta > 0$.[10]

At equilibrium should hold that $|q_i - p_i| \leq 1$ and $|\Delta| \leq n$, otherwise all consumers prefer to buy the lower price system.[11]

Each consumer buys the system with the smallest total cost, and the locations of the con-

[10] This is the result of the Cournot's dual problem and it still holds in a differentiated duopoly setting (Singh & Vives, 1984.)

[11] There can be no equilibrium with a monopolist on the market. The competitor would make zero profits, while undercutting the monopolist, can make positive profits.

sumers that buy the same system form an adjacent polygon (polyhedron) within the unit square (cube). When $a = 0$, at each such location the demand is equal to 10. When $a = 1$, the demand at each location depends on the perceived price.

Subsection 3.1 is devoted to the case of two-component systems and subsection 3.2 deals with three-component systems. In the Appendix, the results are grouped by competition mode.

3.1 Two-component system case

In the **absence of bundling** there are two A firms (A_1, A_2) and two B firms (B_1, B_2), each selling the corresponding individual component. There are four systems available on the market: $(A_1, A_2), (A_1, B_2), (B_1, A_2), (B_1, B_2)$. The perceived prices of a consumer located at (x, y) are: $p_1 + p_2 + x + y$ for system (A_1, A_2), $q_1 + q_2 + 1 - x + 1 - y$ for system (B_1, B_2), $q_1 + p_2 + 1 - x + y$ for system (B_1, A_2) and $p_1 + q_2 + x + 1 - y$ for system (A_1, B_2).

The vertical line $x = x_0 \equiv \frac{1+q_1-p_1}{2}$ separates the locations of consumers who buy different first component, and the horizontal line $y = y_0 \equiv \frac{1+q_2-p_2}{2}$ separates the locations of consumers who buy different second component.

For example, (see Figure 1A) locations of consumers who buy system (A_1, A_2) lie inside square $MNOP$. When $a = 1$, demand at a location is highest at point O (transportation cost is minimal), and lowest at point M (transportation cost is maximal).[12] Demand for system (A_1, A_2) is given by the volume depicted in Figure 1B.

From the computation of the demands and the profit maximization problem, follow the equilibria.

Proposition 1 *With two-component systems, in the absence of bundling, the equilibrium prices and the corresponding profits are:*

[12] In a symmetric equilibrium, the demand at points N and P, and along the line that connects them is equal.

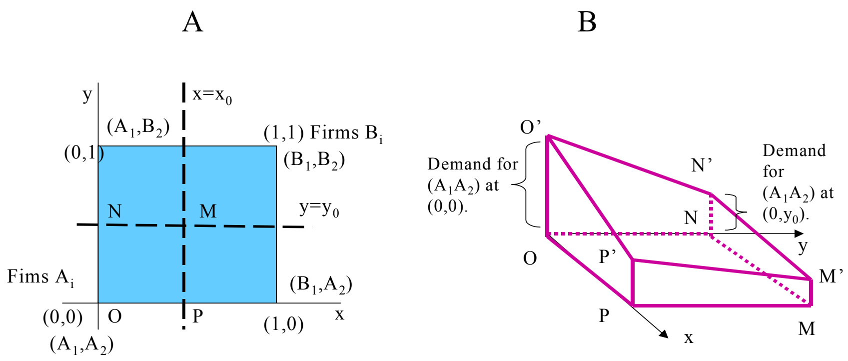

Figure 1: **A. Determination of locations of consumers who prefer same system. B. Computation of the demand for system (A_1,A_2) in the elastic demand case.**

a) when $a = 0$*,* $p_i = q_i = 1$ *and* $\pi_{Ai} = \pi_{Bi} = 5$*;*

b) when $a = 1$*,* $p_i = q_i = 0.91101$ *and* $\pi_{Ai} = \pi_{Bi} = 3.4974$

In the **presence of bundling**, locations where consumers are indifferent between the two systems are given by the intersection of the unit square with line $x + y = \frac{\Delta+2}{2}$. This line divides the unit square in two areas each formed by consumers who prefer the same system (see Figure 2A). Locations in the unit square that lie below the indifference line are served by firm A and the locations in the unit square that lie above are served by the B firms.

To have a covered market, when $a = 1$, the zero demand line should lie further from the seller's location than the indifference line.[13]

When both A **and** B **firms bundle**, there is a symmetric equilibrium. The indifference line coincides with $x + y = 1$, as $\Delta = 0$. For instance, in Figure 2A, locations of consumers who buy bundle A lie within triangle ORQ and, at equilibrium, locations are split equally between

[13] Then at all locations of consumers who prefer a certain bundle, there are buyers who purchase. Zero demand line of a system is given by $x + y = 10 - p_s$, where p_s is the system price.

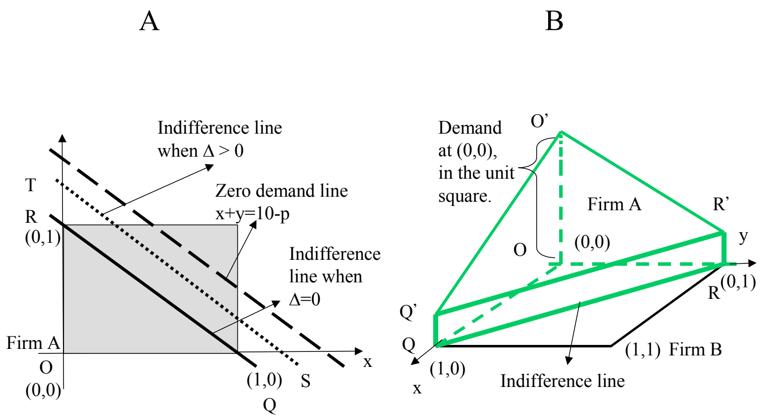

Figure 2: **A. Determination of locations of consumers who prefer same system in the presence of bundling. B. Demand for bundle A, with elastic demand, under symmetric bundling.**

firms. When $a = 1$, demand is highest at point O (transportation cost is minimal) and lowest at points R and Q (and along the indifference line that connects them where transportation cost is constant and maximal). Figure 2B depicts the volume that gives total demand for bundle A in this setting.

Using the volumes I compute the demands for the two bundles. From the corresponding first order conditions of the profit maximization problem I obtain the equilibrium prices.

Proposition 2 *With two-component bundles, when both A and B firms bundle, the equilibrium prices and the corresponding profits are:*

a) when $a = 0$, $p = q = 1$ *and* $\pi_A = \pi_B = 5$;

b) when $a = 1$, $p = q = 0.9265$ *and* $\pi_A = \pi_B = 3.8946$.

Asymmetric bundling is the last competition mode to consider. Figure 2A depicts the market sharing in this case, where the indifferent consumers locations lie on segment ST.

Using the market sharing between the two available systems, demands faced by firm A and by the B firms can be computed.[14] While A firms coordinate their pricing decision, each B firm chooses separately the price of the component it sells.[15] The profit maximization problem gives the equilibrium in this case.

Proposition 3 *With two-component bundles, under asymmetric bundling, the equilibrium prices and the corresponding profits are:*

a) when $a = 0$, $p = 1.4533$, $q_i = 0.86332$ $(\Sigma q_i = 1.7266)$ *and* $\pi_A = 9.117$, $\pi_{Bi} = 3.2173$ $(\Sigma_i \pi_{Bi} = 6.4346)$;

b) when $a = 1$, $p = 1.3045$, $q_i = 0.7971$ $(\Sigma q_i = 1.5943)$ *and* $\pi_A = 6.5736$, $\pi_{Bi} = 2.2849$ $(\Sigma_i \pi_{Bi} = 4.5697)$.

With two-component bundles, independent of the elasticity of demand, the highest profits are obtained when all firms sell separate components and consumers are allowed to mix and match. There are no incentives to bundle in this case. Moreover, assuming that A firms bundled, the B firms do not have incentives to bundle as their profits in BvsC are higher than their profits in BvsB.

BvsB is the strongest competition attainable in this setting and makes the price of the bundle fall to about half the price of the system in CvsC. It also makes the profits of the bundlers be almost half of their aggregate profits in the absence of bundling.

[14] When $a = 0$, the demand for the bundle is the volume of the triangular prism with base SOT (see Fig. 2B*)* and height 10 minus the volumes of two triangular prisms with same height whose bases lie beyond the unit square. When $a = 1$, the geometric figure is similar to the one in Fig. 2B, requiring the above-mentioned adjustments.

[15] Each B firm maximizes its profits only with respect to own price. The resulting price of the B system will be the sum of the individual prices of the two components.

3.2 Three-component system case

In this part I consider bundles formed of three goods. **In the absence of bundling,** there are three A firms (A_1, A_2, A_3) and three B firms (B_1, B_2, B_3), each selling one component. Thus, there are eight systems available on the market. The perceived price of a consumer located at (x, y, z) is, for example, $p_1 + p_2 + q_3 + x + y + 1 - z$ for buying (A_1, A_2, B_3).

The planes $x = x_0 \equiv \frac{1+q_1-p_1}{2}$, $y = y_0 \equiv \frac{1+q_2-p_2}{2}$, and $z = z_0 \equiv \frac{1+q_3-p_3}{2}$ separate the locations of consumers who prefer different first, second and, respectively, third component.

The planes $x = x_0$, $y = y_0$ and $z = z_0$ separate the locations of consumers who buy different components. The locations of consumers that buy the same system form an interior parallelepiped with three faces adjacent to the unit cube. Intuitively, the geometric representation is an extension of Figure 1 to three dimensions. When $a = 1$, total demand for a system is given by the volume of a four dimensional figure.[16] After deriving the functional forms of the demands, next result follows.

Proposition 4 *With three-component systems, in the absence of bundling, the equilibrium prices and the corresponding profits are:*

a) when $a = 0$, $p_i = q_i = 1$ *and* $\pi_{Ai} = \pi_{Bi} = 5$;

b) when $a = 1$, $p_i = q_i = 0.89735$ *and* $\pi_{Ai} = \pi_{Bi} = 2.9424$.

In the **presence of bundling**, with an elastic demand at each location, the covered market condition requires the indifference plane to lie closer to the location of the system than the zero demand plane.[17] The set of locations where consumers are indifferent between the two bundles

[16] Sectioning horizontally these adjacent parallelepipeds I obtain a rectangle. At each location in this rectangle there is an elastic demand. The demand corresponding to such section is given by the volume of the flat top pyramid similar to $MNOPM'N'P'O'$ in Figure 1B. Integrating over all sections, I compute the demands for the systems.

[17] The zero demand plane of a system is given by $x + y + z = 10 - p_s$, where p_s is the price charged for the

is given by the intersection of the unit cube with the plane $x + y + z = \frac{\Delta+3}{2}$. The locations in the unit cube that lie below the indifference plane are served by A firm and the locations above are served by the B firm(s).

When both A and B firms bundle there is a symmetric equilibrium. The equation of the indifference plane is $x + y + z = \frac{3}{2}$. When $a = 1$, the demand faced by any of the bundlers is given by the volume of a four-dimensional figure.[18]

Proposition 5 *With three-component bundles, when both A and B firms bundle, the equilibrium prices and the corresponding profits are:*

a) when $a = 0$, $p = q = 1.333$ *and* $\pi_A = \pi_B = 6.6665$;

b) when $a = 1$, $p = q = 1.1903$ *and* $\pi_A = \pi_B = 4.5922$.

Under **BvsC,** $\Delta = \Sigma_i q_i - p > 0$ and $a = 1$, the demand is given by the volume of a four dimensional figure to be computed using the steps described in BvsB case. The profit maximization problem gives the equilibrium in this case, where each independent seller (B_i) maximizes with respect to its own price, without considering the impact of his pricing strategy on the other complementary component sellers, B_j $(i \neq j)$.

Proposition 6 *With three-component bundles, under asymmetric bundling, the equilibrium prices and the corresponding profits are:*

a) when $a = 0$, $p = 2.094$, $q_i = 0.88493$ $(\Sigma_i q_i = 2.6548)$ *and* $\pi_A = 14.72$, $\pi_{Bi} = 2.6287$ $(\Sigma_i \pi_{Bi} = 7.8861)$;

b) when $a = 1$, $p = 1.7671$, $q_i = 0.7898$ $(\Sigma_i q_i = 2.3694)$ *and* $\pi_A = 8.8302$, $\pi_{Bi} = 1.5066$ $(\Sigma_i \pi_{Bi} = 4.5198)$.

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system.

[18] Using the sectioning method presented above and integrating over all sections, the demands for the two bundles can be computed. In this case, the demand corresponding to an arbitrary section is the volume of a flat top pyramid resembling $OQRO'R'Q'$ in Figure 2B.

With an *inelastic demand* and three-component bundles, still there are no incentives to bundle: While aggregate profits of potential bundlers, in CvsC are equal to 15, in BvsC, the bundler makes a profit of 14.72. Moreover, if A firms bundle, the B firms do not have incentives to bundle because their profits are higher in BvsC than in BvsB. Symmetric bundling is the strongest attainable competition mode, profits of the bundlers fall by more than half of their aggregate profits in the absence of bundling.

Nalebuff (2000) shows that for bundles of more than 4 components there are incentives to bundle, and the component seller competitors of a bundler do not have incentives to bundle. This makes bundling be an efficient tool to depress rivals' profits, while increasing the profits of the bundler, when large bundles are involved. The competitors, moreover, are worse off if they decide to bundle their components as well.

But, when *demand is elastic*, for bundles of three-components, the profits of a bundler competing against components exceed aggregate profits that complementary component sellers obtain in the absence of bundling. In effect, the incentives to bundle are stronger in this setting. In addition, when there are incentives to bundle, BvsC is not a stable outcome, because component sellers have incentives to bundle as well. The market outcome is BvsB, the most competitive one. Hence, once an elastic demand is considered, bundling cannot be viewed as an efficient anticompetitive tool. It also suggests that, with elastic demand, for larger sizes of bundles, there are incentives to bundle, and the market outcome is characterized by strong competition, unlike the inelastic demand case.

CvsC, where consumers are allowed to mix and match components, counts with highest product differentiation and, therefore, with weakest price competition. Bundling decreases product differentiation and leads to stronger price competition, reducing profits. But, it also helps to internalize positive price externality that complementary goods sellers have on each other, augmenting profits. Incentives to bundle result from the trade off between these two opposed effects. The positive impact on profits gets stronger as bundle size increases because a decrease in the price of a component favors a larger number of complementary components.

Similarly, symmetric bundling affects positively previously uncoordinated competitors of a bundler, making them internalize the positive price externality they have on each other. Also, with inelastic demand, under BvsB, the higher incentives to undercut prices[19] hurt both firms and make the competitors of a bundler worse off than under BvsC.

With *inelastic demand*, whenever there are incentives to bundle, the market outcome of the bundling game (BvsC) favors the bundler and is detrimental to his competitors. In the presence of bundling, the demands depend only on the price difference (Δ) and not on the own price. A decrease in price affects demand only to the extend to which it acts upon the price difference (Δ). Then, under symmetric bundling, part of the internalized price externality is offset by the price cut of competitors.

With *elastic demand*, incentives to bundle are stronger and the best response to bundling is to bundle. An important difference is that aggregate demand depends on both the price difference and own price. As before, the decrease in price due to bundling affects the demand through the price difference (Δ), *but it also has a direct effect.* This makes the incentives to bundle be stronger with an elastic demand (they already exist for bundles of three components, unlike the inelastic setting). In addition, the incentives to undercut prices created by symmetric bundling, are attenuated by the increase in the demand at each location due to lower prices. This makes the competitors of a bundler better off when selling a bundle. As a result, the the bundling game resembles a Prisoners' Dilemma.

[19]Starting from BvsC, BvsB generates a price decrease relatively to the price of the system previously sold by independent sellers. This decrease, further decreases Δ, makes the initial bundler loose market share and determines him to undercut prices. In his turn the last bundler has incentives to undercut and so on.

Although, for computational reasons, I performed the analysis only for bundles of two and three components, I believe that same result holds for arbitrarily large bundles.

4 Welfare analysis

To complete the analysis, I assess the impact of bundling on the social welfare, computed as the sum of consumer surplus and profits.

With *inelastic demand*, given that the prices are just a transfer from consumers to firms and that market is fully covered, total surplus depends on the total transportation cost incurred by the consumers. Social welfare is maximized when the total transportation cost is minimized, case that turns out to be CvsC, independently of bundle size. Moreover, total surplus in CvsC is equal to the socially optimal level. BvsB results in the highest level of consumers surplus and, although, firms obtain their lowest profits, total welfare exceeds the one created by BvsC. With bundles of two and three components, there are no incentives to bundle and equilibrium welfare is socially optimal. Once larger bundles are considered, the market outcome is BvsC, and total surplus is lowest.

With *elastic demand,* the price directly affects the levels of social welfare and even surplus maximizing competition mode is below the welfare optimal levels. When demand is elastic, CvsC creates highest total surplus. BvsB, the market outcome when larger bundles are considered, results in a higher welfare than BvsC. BvsB depresses profits, but consumers' gain offsets producers' loss. Hence, when there are incentives to bundle and demand at each location is elastic, the market outcome creates the highest welfare attainable in a bundling setting.[20]

Tables 1 and 2 present consumer surplus, profits and total welfare, for inelastic and, respectively, elastic demand, and Table 3 presents optimal welfare levels.[21] The computation can be

[20] Still, this level of welfare is below the one created under CvsC, and below optimal level.

[21] I also present optimal welfare in the presence of bundling (in BvsB), though is always below the levels in

found in the Appendix.

Table 1: Welfare levels with inelastic demand

No. of components	$n = 2$			$n = 3$		
Competition	CvC	BvB	BvC	CvC	BvB	BvC
Cons. Surplus	75	83.33	77.61	62.5	75.72	66.10
Total Profits	20	10	15.52	30	13.33	22.60
Total Surplus	95	93.33	93.16	92.5	89.06	88.70

Table 2: Welfare levels with elastic demand

No. of components	$n = 2$			$n = 3$		
Competition	CvC	BvB	BvC	CvC	BvB	BvC
Cons. Surplus	29.49	35.36	31.26	21.53	29.8	23.89
Total Profits	13.99	7.78	11.14	17.65	9.18	13.35
Total Surplus	43.48	43.14	42.41	39.18	38.98	37.24

Table 3: Welfare optimal levels

Demand	Inelastic				Elastic			
No. of comp.	$n = 2$		$n = 3$		$n = 2$		$n = 3$	
Competition	CvC	BvB	CvC	BvB	CvC	BvB	CvC	BvB
Welfare	95	93.33	92.5	89.06	45.14	43.58	42.81	39.70

5 Results and discussion

Using the equilibrium profits corresponding to each of the cases studied in Sections 3 and 4, I construct the normal form representation of the bundling game. This may be related to a one shot game or to a sequential move one. The players of the hypothetical game are the A firms

CvsC (to which I refer in the text whenever welfare optimal levels are mentioned).

Table 1: Normal forms with inelastic demand

		$n=2$				$n=3$	
		A				A	
		BU	DB			BU	DB
B	BU	$(5,5)$	$(9.11,6.43)$	B	BU	$(6.66,6.66)$	$(14.72,7.88)$
	DB	$(6.43,9.11)$	$(10,10)$		DB	$(7.88,14.72)$	$(15,15)$

Table 2: Normal forms with elastic demand

		$n=2$				$n=3$	
		A				A	
		BU	DB			BU	DB
B	BU	$(3.89,3.89)$	$(6.57,4.56)$	B	BU	$(4.59,4.59)$	$(8.83,4.51)$
	DB	$(4.56,6.57)$	$(6.99,6.99)$		DB	$(4.51,8.83)$	$(8.82,8.82)$

and the B firms, they can choose between two possible actions *Bundle* (BU) and *Don't bundle* (DB) and the resulting payoffs are their profits. Table 4 and 5 present the payoff matrices.

With two and three component bundles, in the inelastic demand setting, the outcome of the game is CvsC.[22] With elastic demand and two component bundles, there are no incentives to bundle. But, with three component bundles, market outcome is BvsB. This last case resembles a Prisoner's Dilemma and shows that, when demand is elastic and there are incentives to bundle, the market outcome is the most competitive one. These results assess that once an elastic demand is considered, bundling can no longer be regarded as an anticompetitive device used to create a favorable market position, as it leads to the strongest possible competition.

This work focuses on extending the inelastic demand setting to an elastic one. However, for this purpose uses several other restrictions that maybe important for the results. Although a

[22] Nalebuff (2000) shows that for larger bundles the outcome is BvsC.

more flexible setting would make the analysis more general, the complexity of the underlying computation required some simplifications of the model.

For tractability, I assume that the bundle is incompatible with rival components in BvsC, and that pure bundling is the only strategy available to a bundler. If these assumptions are relaxed, the profits of all firms are higher in BvsC. Then, incentives to bundle increase when competing against separate components, and decrease when competing against a bundle. Whether the results are robust to these extensions depends on the magnitude of the gain in profit and on how it changes with bundle size. The overall effect is rather ambiguous.[23]

Though I deal with effects of bundling in a *post entry* set-up, present analysis can provide intuition on the potential of the practice as a barrier to entry. Monopolists selling complementary components do better coordinating their price decision and, under the threat of entry, competition with a bundle decreases entrant's profits. Then, bundling has an anticompetitive effect, making the incumbent look tougher and increasing the range of fixed costs where entry is deterred. With elastic demand and larger bundles, firms can credibly commit to bundling, in order to discourage entry. With inelastic demand, commitment to bundling is not credible when the incumbent is facing coordinated entry, but bundling offers an important first mover advantage in front of uncoordinated entry.

6 Conclusions

The present paper uses a model of imperfect competition with product differentiation in complementary goods markets in order to determine the effects of bundling on competition It considers

[23]However, examples of incompatible bundles are physically integrated TV sets and DVD players, or monitors and computers. Pure bundling may be found in the car industry, due to the practice of adding new facilities to the basic product, or in IT industry where many applications are not available outside the bundle.

two types of demands and two bundle sizes. The incentives to bundle increase with the size of the system, and they are stronger when demand is elastic than when it is inelastic.

For three component bundles, with elastic demand, there are already incentives to bundle against rivals selling separate components, unlike the inelastic demand case where there are no such incentives, although this size of the bundle is close to their existence. Whenever there are incentives to bundle in the elastic demand case, the stable market outcome is bundle against bundle competition, leading to lowest prices. This is contrary to inelastic demand case where bundling can be used to depress rivals profits, and the stable market outcome is bundle versus components competition.

This paper suggest that, unless entry decisions are at stake, bundling cannot be considered a way to achieve an advantageous position, once elastic demands are allowed. Not only is potential anticompetitiveness of bundling particularly sensitive to demand elasticity, but, under certain conditions, the practice may foster price competition. The results contribute to determine the boundaries of anticompetitive product bundling and may be useful for competition policy in information technology industries, or wherever there is evidence of this practice.

7 Appendix

7.1 Tables of results by competition mode

Component versus component

Demand	Inelastic		Elastic	
No. of comp.	$n = 2$	$n = 3$	$n = 2$	$n = 3$
$p_i = q_i =$	1	1	0.911	0.897
$\Sigma_i p_i = \Sigma_i q_i =$	2	3	1.822	2.692
$D_{Ai} = D_{Bi} =$	5	5	3.839	3.279
$\pi_{Ai} = \pi_{Bi} =$	5	5	3.497	2.942
$\Sigma_i \pi_{Ai} = \Sigma_i \pi_{Bi} =$	10	15	6.994	8.827

Bundle versus component

Demand	Inelastic		Elastic	
No. of comp.	$n = 2$	$n = 3$	$n = 2$	$n = 3$
$p =$	1.453	2.094	1.304	1.767
$q_i =$	0.863	0.884	0.797	0.789
$\Sigma_i q_i =$	1.726	2.654	1.594	2.369
$D_A =$	6.273	7.029	5.039	4.997
$D_{Bi} =$	3.726	2.970	2.864	1.907
$\pi_A =$	9.117	14.72	6.573	8.830
$\pi_{Bi} =$	3.217	2.628	2.283	1.506
$\Sigma_i \pi_{Bi} =$	6.434	7.886	4.567	4.519

Bundle versus bundle

Demand	Inelastic		Elastic	
No. of comp.	$n=2$	$n=3$	$n=2$	$n=3$
$p=q=$	1	1.333	0.926	1.190
$D_A=D_B=$	5	5	4.203	3.858
$\pi_A=\pi_B=$	5	6.666	3.894	4.592

7.2 Proofs of Propositions

Proof of Proposition 1. The demand for A_1 will be given by:

$$\int_0^{x_0}\left[\begin{array}{c}\int_0^{y_0} 10-a\left(p_1+p_2+x+y\right)dy+\\ \int_{y_0}^1 10-a\left(p_1+q_2+x+1-y\right)dy\end{array}\right]dx=$$
$$-\frac{-1-q_1+p_1}{8}\left(-2a-aq_1-3ap_1-2ap_2-2ap_2q_2+ap_2^2-2aq_2+aq_2^2+40\right).$$

The demand for B_1 will be given by:

$$\int_{x_0}^{1}\left[\begin{array}{c}\int_0^{y_0} 10-a\left(p_2+q_1+1-x+y\right)dy+\\ \int_{y_0}^1 10-a\left(q_1+q_2+1-x+1-y\right)dy\end{array}\right]dx=$$
$$\frac{1-q_1+p_1}{8}\left(-2a-3aq_1-ap_1-2aq_2-2ap_2+aq_2^2-2ap_2q_2+40+ap_2^2\right).$$

The demand for A_2 will be given by:

$$\int_0^{y_0}\left[\begin{array}{c}\int_0^{x_0} 10-a\left(p_1+p_2+x+y\right)dx+\\ \int_{x_0}^1 10-a\left(p_2+q_1+1-x+y\right)dx\end{array}\right]dy=$$
$$-\frac{1+q_2-p_2}{8}\left(2a+aq_2+3ap_2+2ap_1+2ap_1q_1-ap_1^2+2aq_1-aq_1^2-40\right).$$

The demand for B_2 will be given by:

$$\int_{y_0}^{1}\left[\begin{array}{c}\int_{x_0}^{1} 10-a\left(q_1+q_2+1-x+1-y\right)dx+\\ \int_0^{x_0} 10-a\left(p_1+q_2+x+1-y\right)dx\end{array}\right]dy=$$
$$\frac{-1+q_2-p_2}{8}\left(2a+3aq_2+ap_2+2aq_1+2ap_1-40-aq_1^2+2ap_1q_1-ap_1^2\right).$$

The system of equations formed by the FOCs and the symmetric equilibrium conditions lead to the candidate equilibrium prices. ■

Proof of Proposition 2. I compute the demand functions for more general values of Δ. They are also used to determine the equilibrium in BvsC. For their derivation I use Figure 2.

$D_A = -\frac{1}{24}(\Delta+2)^2(a\Delta - 30 + 3ap + 2a) -$

$2\max\left\{0, -\frac{1}{24}\Delta^2(a\Delta + 3a - 30 + 3ap)\right\} =$

$$= \begin{cases} -\frac{1}{24}(\Delta+2)^2(a\Delta - 30 + 3ap + 2a) & \text{if} \quad \Delta < 0 \\ \left[\begin{array}{l} -\frac{1}{24}(\Delta+2)^2(a\Delta - 30 + 3ap + 2a) \\ -\, 2\left(-\frac{1}{24}\Delta^2(a\Delta + 3a - 30 + 3ap)\right) \end{array} \right] & \text{if} \quad \Delta \geq 0 \end{cases}$$

$D_B = \frac{1}{24}(\Delta-2)^2(a\Delta + 30 - 3aq - 2a) -$

$2\max\left\{0, \frac{1}{24}\Delta^2(a\Delta - 3a + 30 - 3aq)\right\} =$

$$= \begin{cases} \frac{1}{24}(\Delta-2)^2(a\Delta + 30 - 3aq - 2a) & \text{if} \quad \Delta \geq 0 \\ \left[\begin{array}{l} \frac{1}{24}(\Delta-2)^2(a\Delta + 30 - 3aq - 2a) \\ -2\left(\frac{1}{24}\Delta^2(a\Delta - 3a + 30 - 3aq)\right) \end{array} \right] & \text{if} \quad \Delta < 0 \end{cases}$$

At $p = q : D_A = -\frac{4}{24}(-30 + 3ap + 2a)$ and $D_B = -\frac{4}{24}(-30 + 3aq + 2a)$.

The side derivatives of D_A and D_B are equal at $\Delta = 0$. $\left(D'_A = D'_B = \frac{1}{2}ap - 5.\right)$ The functions are continuous and differentiable.

At a symmetric equilibrium, by the profit maximization, the FOC becomes:

$p\left(\frac{1}{2}ap - 5\right) - \frac{4}{24}(-30 + 3ap + 2a) = 0 \;\Leftrightarrow\; \frac{1}{2}ap^2 - 5p + 5 - \frac{1}{2}ap - \frac{1}{3}a = 0.$

Substituting for a I obtain the candidate equilibria. ■

Proof of Proposition 3. From the Cournot's dual problem it follows that $\Delta > 0$.

The demand for bundle A will be given by:

$D_A = -\frac{1}{24}(q_1 + q_2 - p + 2)^2(a(q_1 + q_2 - p) - 30 + 3ap + 2a)$

$+\frac{2}{24}(q_1 + q_2 - p)^2(a(q_1 + q_2 - p) + 3a - 30 + 3ap)\,.$

The demand for the second firms' system is given by:

$D_B = \frac{1}{24}(q_1 + q_2 - p - 2)^2(a(q_1 + q_2 - p) + 30 - 3a(q_1 + q_2) - 2a)\,.$

The two B firms are maximizing separately, with respect to their own price.

The system of FOC of the profit maximization problem gives the candidates for equilibrium prices. ■

Proof of Proposition 4. The total demand of firm A_1will be:

$$D_{A1} = -\frac{-1-q_1+p_1}{8}\left(\begin{array}{c} -3ap_1 - 2aq_2 + 40 - 2ap_3q_3 - 2ap_2q_2 + aq_2^2 + \\ ap_2^2 - 2aq_3 + aq_3^2 - aq_1 + ap_3^2 - 2ap_2 - 2ap_3 - 3a \end{array}\right).$$

The total demand of firm A_2 will be:

$$D_{A2} = \frac{-q_2+p_2-1}{8}\left(\begin{array}{c} 3ap_2 + 2ap_1q_1 + 2aq_3 - aq_3^2 + 2ap_3 - ap_1^2 + 2ap_1 \\ +aq_2 + 2ap_3q_3 + 2aq_1 - ap_3^2 - 40 - aq_1^2 + 3a \end{array}\right).$$

The total demand of firm A_3 will be:

$$D_{A3} = -\frac{-q_3+p_3-1}{8}\left(\begin{array}{c} -3ap_3 + ap_1^2 - 2ap_2q_2 + 40 - 2aq_2 - 2ap_2 - 3a - \\ 2ap_1q_1 + aq_2^2 + aq_1^2 - 2aq_1 - 2ap_1 - aq_3 + ap_2^2 \end{array}\right).$$

The demand for firm B_1 will be:

$$D_{B1} = \frac{1-q_1+p_1}{8}\left(\begin{array}{c} -ap_1 - 2aq_2 + 40 - 2ap_3q_3 - 2ap_2q_2 - 3a - 3aq_1 \\ +ap_2^2 - 2aq_3 + aq_2^2 - 2ap_3 + ap_3^2 - 2ap_2 + aq_3^2 \end{array}\right).$$

The demand for firm B_2 will be given by:

$$D_{B2} = -\frac{-q_2+p_2+1}{8}\left(\begin{array}{c} ap_2 + 3a - ap_3^2 - 40 - aq_1^2 + 2ap_3 - ap_1^2 + 3aq_2 + \\ 2aq_3 - aq_3^2 + 2aq_1 + 2ap_1 + 2p_3aq_3 + 2p_1aq_1 \end{array}\right).$$

The demand for firm B_3 will be given by:

$$D_{B3} = \frac{1-q_3+p_3}{8}\left(\begin{array}{c} -ap_3 + ap_1^2 + 40 - 3a - 3aq_3 + aq_2^2 + ap_2^2 - 2aq_2 \\ -2aq_1 - 2q_1ap_1 - 2p_2aq_2 + aq_1^2 - 2ap_2 - 2ap_1 \end{array}\right).$$

The solution to the system of FOC of the profit maximization problem gives the candidates to equilibrium prices. ■

Proof of Proposition 5. I present the demands for more general values of Δ, so they serve also to determine the equilibrium in BvsC.

$$D_A(p,q) = \begin{cases} -\frac{1}{384}(\Delta+3)^3(3a\Delta+8ap+9a-80) & \text{if} \quad \Delta \leq -1 \\ \begin{bmatrix} -\frac{1}{384}(\Delta+3)^3(3a\Delta+8ap+9a-80) \\ +\frac{3}{384}(\Delta+1)^3(3a\Delta+11a+8ap-80) \end{bmatrix} & \text{if} \quad -1 \leq \Delta \leq 1 \\ \begin{bmatrix} -\frac{1}{384}(\Delta+3)^3(3a\Delta+8ap+9a-80) \\ +\frac{3}{384}(\Delta+1)^3(3a\Delta+11a+8ap-80) \\ -\frac{3}{384}(\Delta-1)^3(3a\Delta-80+13a+8ap) \end{bmatrix} & \text{if} \quad \Delta \geq 1 \end{cases}$$

$$D_B(p,q) = \begin{cases} -\frac{1}{384}(\Delta-3)^3(3a\Delta+80-9a-8aq) & \text{if} \quad -\Delta \leq -1 \\ \begin{bmatrix} -\frac{1}{384}(\Delta-3)^3(3a\Delta+80-9a-8aq) \\ +\frac{3}{384}(\Delta-1)^3(3a\Delta+80-11a-8aq) \end{bmatrix} & \text{if} \quad -1 \leq -\Delta \leq 1 \\ \begin{bmatrix} -\frac{1}{384}(\Delta-3)^3(3a\Delta+80-9a-8aq) \\ +\frac{3}{384}(\Delta-1)^3(3a\Delta+80-11a-8aq) \\ -\frac{3}{384}(\Delta+1)^3(3a\Delta-13a-8aq+80) \end{bmatrix} & \text{if} \quad -\Delta \geq 1 \end{cases}$$

I look for a symmetric equilibrium. At $\Delta = 0$ the demand is continuous and differentiable:

$D_A(\Delta = 0) = -\frac{1}{2}ap - \frac{35}{64}a + 5$

$D'_A(\Delta = 0) = D'_B(\Delta = 0) = \frac{1}{16}a - \frac{15}{4} + \frac{3}{8}ap$

FOC becomes: $\frac{\partial \pi_A}{\partial p} = 5 - \frac{35}{64}a - \frac{7}{16}ap - \frac{15}{4}p + \frac{3}{8}p^2 a = 0$

Substituting for a, gives the equilibrium price candidates. ■

Proof of Proposition 6. I use throughout the maximization problem the fact that $1 \geq \Delta > 0$. I show that there is no other equilibrium in the following two lemmas.

When $0 < \Delta < 1$ (that is, $-1 < \Delta < 1$), the demands of the two firms become:

$D_A = -\frac{1}{384}(q_1+q_2+q_3-p+3)^3(3a(q_1+q_2+q_3-p)+8ap+9a-80)$

$+\frac{3}{384}(q_1+q_2+q_3-p+1)^3(3a(q_1+q_2+q_3-p)+11a+8ap-80)$,

$$D_B = -\frac{1}{384}(q_1+q_2+q_3-p-3)^3 \begin{pmatrix} 3a(q_1+q_2+q_3-p) \\ +80-9a-8a(q_1+q_2+q_3) \end{pmatrix}$$

$$+\tfrac{3}{384}\left(q_1+q_2+q_3-p-1\right)^3\left(\begin{array}{c} 3a\left(q_1+q_2+q_3-p\right)+80 \\ \\ -11a-8a\left(q_1+q_2+q_3\right) \end{array}\right).$$

The system of FOC of the profit maximization problem gives the candidates for equilibrium prices. ■

Lemma 1 *For $n=3$, in BvsC, there is no equilibrium when $\Delta=1$.*

Proof. I prove for the case of $a=1$. The proof for $a=0$, follows the same steps.

$\Delta=q-p=q_1+q_2+q_3-p=1\Rightarrow q=p+1$

$\pi_A=p\left(-\tfrac{1}{384}(4)^3(3-71+8p)+\tfrac{3}{384}(2)^3(3+8p-69)\right)=\tfrac{173}{24}p-\tfrac{5}{6}p^2$

Consider a small deviation of firm A to a higher price, $p'=p+\varepsilon$, for $\varepsilon>0$, very small. Then, $-1\leq\Delta<1$, and demand faced by firm A is:

$D_A=-\tfrac{1}{384}\left(q_1+q_2+q_3-p+3\right)^3\left(3\left(q_1+q_2+q_3-p\right)-71+8p\right)$

$+\tfrac{3}{384}\left(q_1+q_2+q_3-p+1\right)^3\left(3\left(q_1+q_2+q_3-p\right)+8p-69\right).$

Notice that $\Delta'=q-p'=1-\varepsilon$. Then,

$$\pi'_A=(p+\varepsilon)\left(\begin{array}{c} -\tfrac{1}{384}(1-\varepsilon+3)^3\left(3(1-\varepsilon)-71+8(p+\varepsilon)\right) \\ \\ +\tfrac{3}{384}(1-\varepsilon+1)^3\left(3(1-\varepsilon)+8(p+\varepsilon)-69\right) \end{array}\right)=$$

$\tfrac{173}{24}p-\tfrac{5}{6}p^2+\varepsilon\left(\tfrac{173}{24}-\tfrac{11}{3}p+\tfrac{1}{4}p^2\right)+\varepsilon^2\left(-\tfrac{17}{6}-\tfrac{9}{16}p+\tfrac{1}{8}p^2\right)+$

$\varepsilon^3\left(-\tfrac{13}{16}+\tfrac{13}{24}p-\tfrac{1}{24}p^2\right)+\varepsilon^4\left(\tfrac{5}{12}-\tfrac{13}{192}p\right)-\tfrac{5}{192}\varepsilon^5.$

In order for such deviation to be profitable should hold that $\pi'_A-\pi_A>0\Leftrightarrow$

$\left(\tfrac{173}{24}-\tfrac{11}{3}p+\tfrac{1}{4}p^2\right)+\varepsilon\left(-\tfrac{17}{6}-\tfrac{9}{16}p+\tfrac{1}{8}p^2\right)+$

$\varepsilon^2\left(-\tfrac{13}{16}+\tfrac{13}{24}p-\tfrac{1}{24}p^2\right)+\varepsilon^3\left(\tfrac{5}{12}-\tfrac{13}{192}p\right)-\tfrac{5}{192}\varepsilon^4>0.$

Or, when $\varepsilon\to 0$, in the limit the above expression becomes:

$\tfrac{173}{24}-\tfrac{11}{3}p+\tfrac{1}{4}p^2=\tfrac{1}{4}\left(p-\tfrac{22}{3}+\tfrac{\sqrt{898}}{6}\right)\left(p-\tfrac{22}{3}-\tfrac{\sqrt{898}}{6}\right)>0.$

Then, there are incentives to deviate whenever $p<\tfrac{22}{3}-\tfrac{\sqrt{898}}{6}=2.3389$. (1)

Consider a small deviation of firm A to a smaller price, $p''=p-\varepsilon$, for $\varepsilon>0$, very small. Then,

$\Delta > 1$, and demand faced by firm A is:

$D_A = -\frac{1}{384}(\Delta+3)^3(3\Delta-71+8p)+\frac{3}{384}(\Delta+1)^3(3\Delta+8p-69)-\frac{3}{384}(\Delta-1)^3(3\Delta-67+8p)$.

Notice that $\Delta'' = 1+\varepsilon$. Then,

$$\pi''_A = (p-\varepsilon)\left(\begin{array}{c} -\frac{1}{384}(1+\varepsilon+3)^3(3(1+\varepsilon)-71+8(p-\varepsilon)) \\ +\frac{3}{384}(1+\varepsilon+1)^3(3(1+\varepsilon)+8(p-\varepsilon)-69) \\ -\frac{3}{384}(1+\varepsilon-1)^3(3(1+\varepsilon)-67+8(p-\varepsilon)) \end{array}\right) =$$

$\frac{173}{24}p - \frac{5}{6}p^2 + \varepsilon\left(-\frac{173}{24}+\frac{11}{3}p-\frac{1}{4}p^2\right)+\varepsilon^2\left(-\frac{17}{6}-\frac{9}{16}p+\frac{1}{8}p^2\right)+$

$\varepsilon^3\left(\frac{13}{16}-\frac{1}{24}p-\frac{1}{48}p^2\right)+\varepsilon^4\left(-\frac{1}{12}+\frac{13}{384}p\right)-\frac{5}{384}\varepsilon^5.$

In order for the deviation to be profitable, should be that $\pi''_A - \pi_A > 0$

$\pi''_A - \pi_A = \left(-\frac{173}{24}+\frac{11}{3}p-\frac{1}{4}p^2\right)+\varepsilon\left(-\frac{17}{6}-\frac{9}{16}p+\frac{1}{8}p^2\right)+$

$\varepsilon^2\left(\frac{13}{16}-\frac{1}{24}p-\frac{1}{48}p^2\right)+\varepsilon^3\left(-\frac{1}{12}+\frac{13}{384}p\right)-\frac{5}{384}\varepsilon^5 > 0.$

When $\varepsilon \to 0$, in the limit, the above expression becomes:

$-\frac{173}{24}+\frac{11}{3}p-\frac{1}{4}p^2 > 0 \Leftrightarrow \frac{1}{4}\left(\frac{22}{3}-\frac{\sqrt{898}}{6}-p\right)\left(p-\frac{22}{3}-\frac{\sqrt{898}}{6}\right) > 0.$

So, there are incentives to deviate whenever $p \in (2.3389, 10]$. (2)

By (1) and (2), I am left with checking the incentives to deviate at $p = 2.3389$, where $q = 3.3389$ and $q_i = 1.113$, $i = 1,2,3$. Then, at $\Delta = 1$:

$\pi_{B1} = \left[-\frac{1}{384}(-2)^3[3+71-8(q_1+q_2+q_3)]\right]q_1 = \frac{37}{24}q_1 - \frac{1}{6}q_1^2 - \frac{1}{6}q_1q_2 - \frac{1}{6}q_1q_3.$

Consider a deviation of firm B_1 from q_1 to $q_1' = 1.113-\varepsilon$. As $p = 2.3389$ and $\Delta = 1-\varepsilon$:

$$\pi'_{B1} = (q_1-\varepsilon)\left[\begin{array}{c} -\frac{1}{384}(1-\varepsilon-3)^3(3(1-\varepsilon)+71-8(q_1+q_2+q_3-\varepsilon)) \\ +\frac{3}{384}(1-\varepsilon-1)^3(3(1-\varepsilon)+69-8(q_1+q_2+q_3-\varepsilon)) \end{array}\right] =$$

$\left(\frac{37}{24}q_1-\frac{1}{6}q_1^2-\frac{1}{6}q_1q_2-\frac{1}{6}q_1q_3\right)+$

$\varepsilon\left(-\frac{37}{24}+\frac{31}{12}q_1+\frac{1}{6}q_2+\frac{1}{6}q_3-\frac{1}{4}q_1^2-\frac{1}{4}q_1q_3-\frac{1}{4}q_1q_2\right)+$

$\varepsilon^2\left(-\frac{29}{12}+\frac{25}{16}q_1+\frac{1}{4}q_2+\frac{1}{4}q_3-\frac{1}{8}q_1^2-\frac{1}{8}q_1q_3-\frac{1}{8}q_1q_2\right)+$

$\varepsilon^3\left(-\frac{21}{16}-\frac{1}{6}q_1+\frac{1}{8}q_2+\frac{1}{8}q_3+\frac{1}{24}q_1^2+\frac{1}{24}q_1q_3+\frac{1}{24}q_1q_2\right)+$

$\varepsilon^4\left(+\frac{7}{24}-\frac{13}{192}q_1-\frac{1}{24}q_2-\frac{1}{24}q_3\right)+\frac{5}{192}\varepsilon^5.$

For firm B_1 to deviate, should hold that:

$\pi'_{B1} - \pi_{B1} > 0 \Leftrightarrow$

$\left(-\frac{37}{24} + \frac{31}{12}q_1 + \frac{1}{6}q_2 + \frac{1}{6}q_3 - \frac{1}{4}q_1^2 - \frac{1}{4}q_1q_3 - \frac{1}{4}q_1q_2\right) +$

$\varepsilon\left(-\frac{29}{12} + \frac{25}{16}q_1 + \frac{1}{4}q_2 + \frac{1}{4}q_3 - \frac{1}{8}q_1^2 - \frac{1}{8}q_1q_3 - \frac{1}{8}q_1q_2\right) +$

$\varepsilon^2\left(-\frac{21}{16} - \frac{1}{6}q_1 + \frac{1}{8}q_2 + \frac{1}{8}q_3 + \frac{1}{24}q_1^2 + \frac{1}{24}q_1q_3 + \frac{1}{24}q_1q_2\right) +$

$\varepsilon^3\left(+\frac{7}{24} - \frac{13}{192}q_1 - \frac{1}{24}q_2 - \frac{1}{24}q_3\right) + \frac{5}{192}\varepsilon^4 > 0.$

Or, when $\varepsilon \to 0$, in the limit:

$-\frac{37}{24} + \frac{31}{12}q_1 + \frac{1}{6}q_2 + \frac{1}{6}q_3 - \frac{1}{4}q_1^2 - \frac{1}{4}q_1q_3 - \frac{1}{4}q_1q_2 > 0.$

Thus, $p = 2.3389$ and $q_1 = q_2 = q_3 = 1.113$ cannot be an equilibrium, because any of the B_i firms has incentives to deviate to a price higher than 1.113. (3)

When $q_2 = q_3 = 1.113$

$-\frac{37}{24} + \frac{31}{12}q_1 + \frac{1}{6}q_2 + \frac{1}{6}q_3 - \frac{1}{4}q_1^2 - \frac{1}{4}q_1q_3 - \frac{1}{4}q_1q_2 > 0$

$\Leftrightarrow 2.026\,8q_1 - 0.25q_1^2 - 1.170\,7 > 0 \Leftrightarrow 7.481\,3 > q_1 > 0.625\,94.$

So, there are incentives to deviate whenever $q_i \in (0.625\,94, 7.481\,3]$. (3)

$(1), (2)$ and (3) complete the proof that there can be no equilibrium when $\Delta = 1$. ■

Lemma 2 *For $n = 3$, in BvsC, there is no equilibrium when $\Delta > 1$.*

Proof. The demand functions are:

$$D_A = -\frac{1}{384}(q_1 + q_2 + q_3 - p + 3)^3 (3a(q_1 + q_2 + q_3 - p) + 8ap + 9a - 80)$$
$$+\frac{3}{384}(q_1 + q_2 + q_3 - p + 1)^3 (3a(q_1 + q_2 + q_3 - p) + 11a + 8ap - 80)$$
$$-\frac{1}{384}(q_1 + q_2 + q_3 - p - 1)^3 (3a(q_1 + q_2 + q_3 - p) - 80 + 13a + 8ap),$$

$$D_B = -\frac{1}{384}(q_1 + q_2 + q_3 - p - 3)^3 \begin{pmatrix} 3a(q_1 + q_2 + q_3 - p) + 80 - 9a \\ -8a(q_1 + q_2 + q_3) \end{pmatrix}.$$

Using the demand and deriving the candidates for equilibrium prices from the system of FOC of the profit maximization problem, can be shown that there is no equilibrium consistent with the covered market conditions and with $\Delta > 1$. ■

Remark 1 *Using the same steps as the ones in Lemma 1, it can be shown that there is no equilibrium with* $\Delta = -1$.

7.3 Welfare analysis: Consumer surplus

I present the derivation of the consumer surplus results for an elastic demand. Similarly, results in Table 1 can be derived.

Two components and elastic demand. Table 2 for $n = 2$.

1. CvsC: Consumer surplus corresponding to one system at a symmetric equilibrium is:

$CS(p) = \frac{1}{2}\int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}}(10 - p_i - x - y)^2\,dxdy = \dfrac{1}{2}\left(\frac{2167}{96} - \frac{19}{4}p_i + \frac{1}{4}p_i^2\right).$

At equilibrium, total consumer surplus is 29.497.

2. BvsB: Consumer surplus corresponding to one firm is:

$CS = \frac{1}{2}\int_0^1\int_0^{1-x}(10 - p - x - y)^2\,dydx = \frac{523}{24} - \frac{14}{3}p + \frac{1}{4}p^2.$

At equilibrium, total consumer surplus is 35.364.

3. BvsC: Firm A bundles. At equilibrium, the indifference line is $x + y = \frac{q_1+q_2-p+2}{2} = 1.1449$.

The consumer surplus generated by the bundler is:

$CS = \frac{1}{2}\int_0^1\int_0^{1.1449-y}(10 - p - x - y)^2\,dxdy -$

$\frac{1}{2}\int_1^{1.1449}\int_0^{1.1449-x}(10 - p - x - y)^2\,dydx =$

$27.149 - 5.8665p + .3172p^2.$

B firms sell separate components. The consumer surplus generated by the component sellers is:

$CS = \frac{1}{2}\int_0^{0.8551}\int_0^{0.8551-y}(10 - p - x - y)^2\,dxdy =$

$32.525 - 6.8953p + .3656p^2.$

At equilibrium, total consumer surplus is 31.267. ■

Three components and elastic demand. Table 1 for $n=3$.

1. CvsC: Consumer surplus corresponding to an arbitrary system is:

$CS(p)=\frac{1}{2}\int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}}(10-p-x-y-z)^2\,dxdydz=\frac{685}{64}-\frac{37}{16}p+\frac{1}{8}p^2$

Total consumer surplus is 21.534.

2. BvsB: At equilibrium, $x+y+z=1.5$.

The consumer surplus generated by one firm is:

$CS(p)=\frac{1}{2}\int_0^{1}\int_0^{1.5-z}\int_0^{1.5-z-x}(10-p-x-y-z)^2\,dydxdz-$

$\int_1^{1.5}\int_0^{1.5-z}\int_0^{1.5-z-x}(10-p-x-y-z)^2\,dydxdz.$

Total consumer surplus is 29.8.

3. BvsC: Firm A bundles. The indifference line is given by $x+y+z=1.8012$.

The consumer surplus related to the bundle is:

$CS_A=\frac{1}{2}\int_0^{1}\int_0^{1.8012-z}\int_0^{1.8012-z-x}(10-p-x-y-z)^2\,dydxdz-$

$\int_1^{1.8012}\int_0^{1.8012-z}\int_0^{1.8012-z-x}(10-p-x-y-z)^2\,dydxdz.$

B firms sell separate components. The consumer surplus corresponding to B system is:

$CS_B=\frac{1}{2}\int_0^{1}\int_0^{1.1988-z}\int_0^{1.1988-z-x}(10-p-x-y-z)^2\,dydxdz-$

$\int_1^{1.1988}\int_0^{1.1988-z}\int_0^{1.1988-z-x}(10-p-x-y-z)^2\,dydxdz.$

At equilibrium, total consumer surplus is 23.893. ■

7.4 Compatibility of the components

I present in this appendix the equilibrium that would emerge in bundle versus component competition, were the products compatible. Some of the consumers buy both the bundle and a component and, free disposing a component of the bundle, create a new system closer to their ideal one.

Proposition 7 *When products are compatible:*

a) With inelastic demand and $n = 2$, , *at equilibrium* $p = 1.3895$ *and* $q_1 = q_2 = .80805$. *Corresponding profits are* $\pi_A = 8.5606$ *and* $\pi_{Bi} = 3.2644$.

b) With elastic demand and $n = 2$, *at equilibrium,* $p = 1.2648$ *and* $q_1 = q_2 = .75287$. *Corresponding profits are* $\pi_A = 6.3512$ *and* $\pi_{Bi} = 2.3913$.

c) With inelastic demand and $n = 3$, *at equilibrium,* $p = 1.8512$ *and* $q_1 = q_2 = q_3 = .75824$. *Corresponding profits are* $\pi_{Bi} = 2.7964$ *and* $\pi_A = 12.459$.

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