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Μελέτη της Συνδεσιμότητας Τυχαίων Γεωμετρικών Γράφων με τη Βοήθεια της Θεωρίας Διήθησης

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Αφιερωμένο στο σκύλο μου τον Τάκη
και στο γάτο μου το Βαγγελάκι.

Εισαγωγή (εις την ελληνικήν γλώσσαν)

Τα ασύρματα δίκτυα είναι εγγενώς ασύμμετρα. Το φαινόμενο κατά το οποίο ο κόμβος j ακούει τον κόμβο i αλλά αυτός δε μπορεί να ακούσει τον κόμβο j είναι συχνό και οφείλεται στη χωρική συσχέτιση των παρεμβολών.

Παρ 'όλα αυτά, στα σημερινά δίκτυα συνηθίζεται να μη χρησιμοποιούνται οι σύνδεσμοι μονής κατεύθυνσης. Ο λόγος είναι κυρίως η δυσκολία επιβεβαίωσης μιας σωστής μετάδοσης, μιας και οι σύνηθεις μηχανισμοί γνωστοποίησης σωστής μετάδοσης (ARQ) δε μπορούν να χρησιμοποιηθούν. Στην παρούσα εργασία ενδιαφερόμαστε να εκτιμήσουμε το όφελος από τη χρησιμοποίηση κατευθυντικών συνδέσμων. Προτείνουμε έναν συνεργατικό σχήμα επιβεβαίωσης που παρέχει αξιόπιστη κατευθυντική επικοινωνία, και επικεντρωνόμαστε σε δυναμικά μοντέλα όπου η τοπολογία είναι κατευθυντική και χρονικά μεταβαλλόμενη.

Σε μια προσπάθεια να επεκτείνουν τα όρια της επικοινωνίας, διάφορες εργασίες τα τελευταία χρόνια προσπάθησαν να χρησημοποιήσουν κατευθυντικούς συνδέσμους. Παρά το γεγονός ότι οι προσπάθειες αυτές υπογραμμίζουν τη σημασία των κατευθυντικών συνδέσμων, δεν προσπαθούν ποσοτικοποιήσουν το όφελος που αυτές μπορεί να έχουν. Εμείς διαφοροποιούμαστε από προηγούμενες εργασίες μελετώντας ένα ALOHA μοντέλο όπου οι παρεμβολές μεταξύ κόμβων δημιουργούν ένα εξαιρετικά δυναμικό κατευθυνόμενο γράφο.

Η εργασία μας είναι σύμφωνη με άλλες δουλειές όπου εξετάζονται συνεργατικά σχήματα επιβεβαίωσης. Όμως, εκτός από το ότι σχηματίζουμε ένα εύχρηστο μαθηματικό μοντέλο το οποίο χρησιμοποιούμε για να εξαγάγουμε ποσοτικά αποτελέσματα, η εργασία μας διαφέρει ως προς το γεγονός ότι δεν απαιτείται από τους κόμβους συνεργασία για την αναμετάδοση δεδομένων, αλλά μόνο για μυνήματα επιβεβαίωσης σωστής μετάδοσης (ACK). Αυτή είναι μια σημαντική διαφορά που έχει ως αποτέλεσμα το σχήμα μας να απαιτεί πολύ λιγότερη μνήμη και ενέργεια από τους ασύρματους κόμβους. Ενώ επίσης είναι πολύ μικρή και η επιβάρυνση της ρυθμαπόδοσης αφού τα εν λόγω πακέτα ελέγχου έχουν μικρό μέγεθος.

Η συνεισφορά μας έγκειται στα εξής:

- Συμπληρώνουμε και επεκτείνουμε παλιότερα αποτελέσματα άλλων εργασιών με την εύρεση φραγμάτων για την πιθανότητα αποτυχημένης μετάδοσης σε δικτύο μορφής πλέγματος (lattice), και χρησιμοποιώντας αυτήν την πιθανότητα εξάγουμε αναλυτικές εκφράσεις για περισσότερο πολύπλοκες ποσότητες.

- Προτείνουμε έναν απλό μηχανισμό συνεργατικής επιβεβαίωσης (ACK) που δείχνει το όφελος των κατευθυντικών συνδέσμων.

- Ορίζουμε προβλήματα βελτιστοποίησης που αφορούν τη ρυθμαπόδοση και την καθυστέρηση πακέτων, μέσω της επίλυσης των οποίων εξάγουμε ποσοτικά αποτελέσματα που δείχνουν το προαναφερθέν όφελος.

- Δείχνουμε επίσης ότι οι κατευθυντικοί σύνδεσμοι είναι πολύ αποδοτικοί σε περιβάλλον οπου υπάρχουν διαλείψεις από πολυόδευση (fading).

Το κύριο σώμα της εργασίας ακολουθεί (ως 'παράρτημα').

Contents

1 Introduction

Wireless networks are inherently asymmetric. The phenomenon where node j listens to node i but i cannot listen to node j is frequent due to spatial correlation of interference and/or multipath fading, see for example [RM02, San07, MD02]. Mobile wireless networks are often represented by directed graphs varying over time. At each time epoch, a directional link from node i to j is formed if and only if j can decode a transmitted signal from i despite interference and fading.

Nevertheless, the current communication approach discards unidirectional links, that is, only the undirected subgraph of the original network is used. The reason behind this is mainly to avoid handling the complexity of announcing a correct transmission. As reported in [KK05, BLRS03, RCM02], topology control algorithms can be employed to form this undirected graph discarding any unidirectional links. In these references, it is stressed that unidirectional links can be problematic at layer-2 since straightforward ARQ approaches cannot be used. For example, a simple neighborhood discovery with hello messages will automatically discard the unidirectional links. In this work we are interested to gauge the benefit of utilizing unidirectional. We put forward a cooperative ACK scheme which provides reliable directed communications by the use of multihop ARQ.

Unidirectional links are notorious for disrupting layer-3 as well. In [MD02, Pra99] it is stressed that directional links in layer-3 can increase the cost of routing and cause malfunction of the routing protocol. We focus on dynamical models where the underlying topology is directional but time-varying as well. Thus, links from i to j and vice versa are realized now and then but not necessarily at the same time. Therefore, layer-3 considerations are largely separated from our work.

Recent work in percolation theory applied to wireless networks [DFM+05] has shown that in a large network where the locations of the nodes are distributed according to a Poisson point process and the transmissions interfere with each other, there exist critical thresholds for the density of nodes and the interference multiplicative constant that separate the subcritical from the supercritical region. For values in the supercritical region, the network forms a giant connected component where information can be sent arbitrarily far away. In [KY07b], this work is extended to directed graphs. In particular, it is shown that the network graph as well as the undirected subgraph (where the unidirectional links are discarded) have the same percolation thresholds. This fact implies that unidirectional links are not capable of extending the connectivity of a wireless network in the network-wide sense (they can however improve the connectivity locally). In our work we are interested in investigating the benefit of unidirectional links in throughput and packet delay in the network-wide sense.

In an effort to push the limits of communications, several works have lately considered the use of unidirectional links [Kut07, CHZ+09, WTB+08, ZGL08]. Although these efforts underline the importance of unidirectional links, they do not try to quantify the benefit that these might have. We deviate from prior work by considering a spatial ALOHA model where the nodes interact by means of per slot interference yielding a highly dynamical directed graph. During our analysis we utilize the work of [WAJ09] on interference analysis and we extend it in certain ways. By analyzing the ALOHA behavior, we have in mind WLAN networks which operate in a very similar way and we argue that the qualitative flavor of our results has direct implications in real-world WLAN networks.

Our work is in line with [AZKVA08, DLNsS05, CFG07, WCW+08] where cooperative ARQ schemes are investigated. Apart from the fact that we apply our scheme in a mathematically tractable model which we use to extract quantitative results, our work differs in the fact that we do not require from nodes to cooperate by means of retransmitting data but only ACKs. This is an important difference since our mechanism requires much less effort in terms of memory, energy dissipation and throughput overhead due to the small size of these control packets.

Our contribution lies in the following:

1. We supplement and extend results from [WAJ09] by finding bounds on the outage probability for the square lattice network, and using this probability we derive analytic expressions for more complex quantities.

2. We propose a simple mechanism of cooperative ACK that promises to leverage the benefit of unidirectional links.

3. We define optimization problems concerning per-user throughput and packet delay; by solving them we provide quantitative results of the benefit of unidirectional links in conjunction with the proposed mechanism.

4. We show that unidirectional links may have small impact in an interference limited environment, but they appear to be very efficient in alleviating multipath fading.

2 System Model

Schemes of Communication In this work we examine and compare two communication schemes in a multihop wireless network; specifically, a scheme (denoted b) that uses bidirectional links only, and a scheme (denoted u) that also uses unidirectional links. The underlying network can either be canonical (e.g. lattice) or random (e.g. point Poisson). Specifically, we consider networks where devices reside on each of the nodes of the square lattice, or on each of the nodes of a point Poisson process. What is more, all nodes transmit with the same power.

A "common" slot is partitioned into two parts (as depicted in Figure 1); first, the larger one where a data packet can be transmitted, and second, a smaller one where an ACK can be transmitted. While for scheme b the "common" slot is sufficient, for scheme u a "mini"-slot, that follows the "common" slot, is also required. During this "mini"-slot ACKs are flooded into the network, thus in contrast to b, in u an ACK may reach the transmitter that waits for it, in a later slot and possibly in a multihop fashion. A situation depicted in Figure 2.

Otherwise the behavior of the two schemes is the same; specifically, when a node gets a transmission opportunity for a data packet (i.e. for the first part of the "common" slot) it retransmits the packet it transmitted on its previous data transmission opportunity, or if this packet has already been acknowledged it moves on to a new packet.

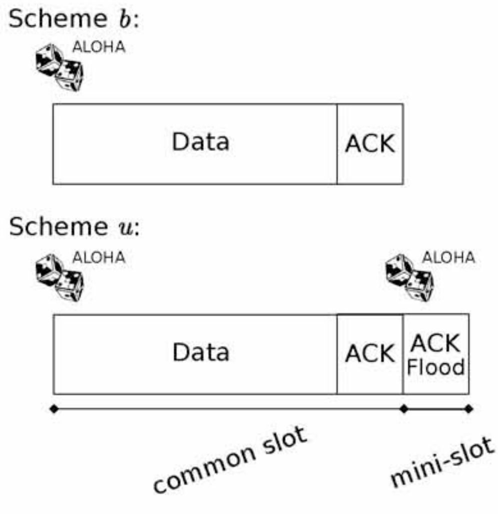

Figure 1: Frame structure. Scheme b uses the "common" slot alone, while scheme u requires a "mini"-slot too.

In both schemes, in order for a device to transmit (a data packet) during the first part of the "common" slot, it must receive a transmission oppor-

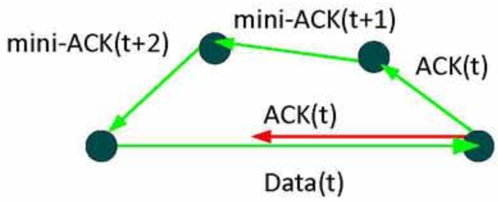

Figure 2: The forward transmission (data) is successful, the reverse (ACK) fails, and the ACK manages to reach the transmitter through the flooding.

tunity according to an ALOHA mechanism (with probability p_{data}); while in order for a device to transmit (an ACK) during the second part of the "common" slot, it must be one of the successful receivers of a data packet during the first part of the slot.

Now, in u, in order for a device to transmit during the "mini"-slot, it must receive a transmission opportunity according to an ALOHA mechanism (with probability p_{ACK}). During a "mini"-slot, many ACKs may be forwarded at the same time. Note, however, that there can be at most one ACK for each link, since, as already mentioned, the source of a link will not move on to a new packet if it has not acknowledged the previous one first. Given that there are at most n links in any given neighborhood, a hashing technique can be used for encoding the ACK identifiers such that a "mini"-slot length of $O(\log n)$ bits is sufficient to fit the required information. What is more, an ACK is ignored if a lot of time has passed since its transmission (e.g. after $5/p$ slots). When measuring the throughput of scheme u we will assume that the "mini"-slot has zero length, keeping in mind that the actual throughput is a bit smaller due to the overhead for announcing the identifiers.

What is more, since we are dealing with multihop flows, we need a policy that selects the path from source to destination, i.e. a routing protocol.

A key characteristic that lead us to expect better performance for scheme u, is the fact that the amount of time between two transmission opportunities for a certain transmitter is large (with mean inversely proportional to p_{data}). During this time period, it is hopeful that a multihop path appears, across which the ACK can be transported back to the transmitter using the flooding of the ACKs during the "mini"-slots.

In what follows we make frequent use of the symbol p_{data} and in order to simplify the appearance of the expressions we use the symbol p to signify the same thing. Moreover, when we refer to a lattice network we mean the square lattice network.

Assumptions and Remarks We make the following assumptions.

- We focus on a symmetric scenario where flows are initiated from all nodes towards all nodes, and the nodes are saturated (thus they always have packets to transmit). [1]
- No class forwarding is used, i.e., on each node, packets from all flows are waiting in the same queue.
- No packet scheduling among links takes place, since this is being taken care of by ALOHA.
- Packets are served in a FIFO manner. Each arbitrary packet, at each node, receives a queueing delay (waiting to be inserted in the MAC queue, i.e. waiting to be the next to go) and a MAC delay (waiting to successfully transmitted, which means that it also has to be successfully ACKed).
- Under natural homogeneous traffic and stability assumptions, we are to add a delay in the same manner at each node, thus we can assume that the queueing delay is zero without affecting the main conclusions of the comparison study. [2]
- Communications are full duplex, i.e. when a node transmits it may also receive. [3]

The set of our assumptions can be well understood from an alternative viewpoint. Consider a network where only one packet exists; the packet we are interested in. However, all nodes transmit (we need this to measure

[1] This way we guarantee that there is always traffic crossing any given node despite the routes selected. The number of packets waiting in a given queue is random but identically distributed over all queues in the network.

[2] Under a certain scheme (b or u) and for a certain assignment to the parameters of the system, the queueing delay for each node is, on average, the same. However, the delay under b and the one under u are not the same. Precisely speaking, for a certain assignment to the parameters of the system, the queueing delay under u is smaller than this under b; this is because the average number of retransmissions for an arbitrary packet under u is smaller (or equal) than this under b. Ultimately, this means that the improvement in delay that we measure under this assumption is a lower bound of the actual improvement. A small remedy is to partly consider the queueing delay and this is what we actually do (section 6.1).

[3] The analysis for half duplex communications is almost identical, however, it results in a more tedious exposition without giving more insight into the system. The only difference is that we need to augment the function of the outage probability q_d (defined in section 4) so as to consider that a transmission in order to be successful needs to be directed towards a receiver which is not transmitting at the current slot.

a proper interference level), according to the ALOHA mechanism; but all nodes except the one that holds the packet under consideration transmit in futility. Consequently, our packet is always the first to be transmitted on each situated node. Finally, we compute delay and throughput using this assumption. The explanation above is utilized to underline the practical importance of this otherwise impractical model.

3 Interference

In this section we derive some results concerning the interference experienced by nodes on the lattice network.

Let I be the interference to the node at the origin of the two dimensional point process Φ, where in the case of the square lattice network Φ is $\mathbb{L} \equiv \mathbb{Z}^2$. Let Φ^* be Φ without the origin. We have that

$$I = \sum_{x \in \Phi^*} l(||x||) 1_{\{x \text{ is transmitting}\}}.$$

Note that I is not actually the interference (as it also contains the signal of the transmitter), but the total signal received at any node, excluding the signal sent by itself.

3.1 Mean

The technique used in the following can be thought as taking larger and larger rings (more generally partitions) of nodes, and bounding them from below and above by using the distance of the closest and the furthest node in the ring respectively. We try two kinds of partitions.

The first try is similar to one in [HG09]. From it we get something like this:

$$2^{-\alpha/2} 8p\zeta(\alpha - 1) \leq \mathrm{E}[I] \leq 8p\zeta(\alpha - 1).$$

For this result the rings used are the sequentially larger squares of nodes on the lattice. For more details check out the appendix A. The quite more exact (and very accurate in general, especially for $a > 3$) formula from [HG09] is

$$\mathrm{E}[I] \approx p[4(1 + 2^{-\alpha/2}) + 8(5/4)^{-\alpha/2}(\zeta(\alpha - 1) - 1)]. \tag{1}$$

Another kind of rings are the (geometric) circular rings of width 1. Let $M_r \equiv |\{x \in \mathbb{L}^* : r \leq ||x|| < r + 1\}|$ denote the number of nodes in the ring whose inner circle is of radius r, and $E_r = \pi(r+1)^2 - \pi r^2 = \pi(2r+1)$ the area of the ring. It holds that $E_{r-\sqrt{2}} \leq M_r \leq E_{r+\sqrt{2}}$ (a result attributed to Hardy [Wei]). More specifically,

$$\begin{aligned} \mathrm{E}[I] \leq & p \sum_{k=1}^{\infty} M_k k^{-\alpha} \leq p \sum_{k=1}^{\infty} E_k k^{-\alpha} = p \sum_{k=1}^{\infty} \pi(2k+1) k^{-\alpha} \\ = & p\pi(2 \sum_{k=1}^{\infty} k^{1-\alpha} + \sum_{k=1}^{\infty} k^{-\alpha}) = p\pi(2\zeta(\alpha - 1) + \zeta(\alpha)). \end{aligned}$$

While a simple approximation is

$$\mathrm{E}[I] \approx p \sum_{k=1}^{\infty} M_{k-1} k^{-\alpha} \approx p \sum_{k=1}^{\infty} E_{k-1} k^{-\alpha} = p\pi(2\zeta(\alpha-1) - \zeta(\alpha)).$$

Alternatively, we can consider a heuristic approximate function like the following (which performs better than the above function)

$$\mathrm{E}[I] \approx p(2\pi\zeta(\alpha-1) - \zeta(\alpha)). \tag{2}$$

Both the above eq. 1 and 2 are a bit complicated to use because of $\zeta(s)$ which is an infinite sum. The following formula uses an integral instead of a sum, and the quite general idea that lead to eq. 2; this we will find quite handy in subsequent parts of the analysis of our system. Specifically, the approximation is

$$\mathrm{E}[I] \approx p\pi \int_{1}^{\infty} (2x-1) x^{-\alpha} dx + c = p\pi\Big(\frac{2}{a-2} - \frac{1}{a-1}\Big).$$

Here too we can consider a heuristic approximate function, like the following

$$\mathrm{E}[I] \approx p\Big(\frac{2\pi}{a-2} - \frac{1}{a-1}\Big) + c. \tag{3}$$

A good value for c is 3.2 (seems unhealthily close to π, but it is just a coincidence). This value actually is the difference between the values given by the formulas in eq. 2 and 3 for some small α.

A plot of the quantities corresponding to eq. 1, 2, 3, and the real value of $\mathrm{E}[I]$ (truth is, for small α it's not so real 'cause the sum takes forever to converge) are depicted in Figure 3 and 4.

3.2 Variance

Variance can be found in a way analogous to the one used to find the mean.

$$\begin{aligned}
\mathrm{Var}(I) &= \mathrm{Var}\Big(\sum_{x\in\mathbb{L}^*} l(||x||) 1_{\{x \text{ is transmitting}\}}\Big) \\
&= \sum_{x\in\mathbb{L}^*} l(||x||)\, \mathrm{Var}(1_{\{x \text{ is transmitting}\}}) \\
&= \sum_{x\in\mathbb{L}^*} l(||x||) p(1-p) \\
&= (1-p)\, \mathrm{E}[I]
\end{aligned}$$

Note that the case is quite different for the Poisson network, 'cause the positions of the nodes are random and Campbell's theorem must be used.

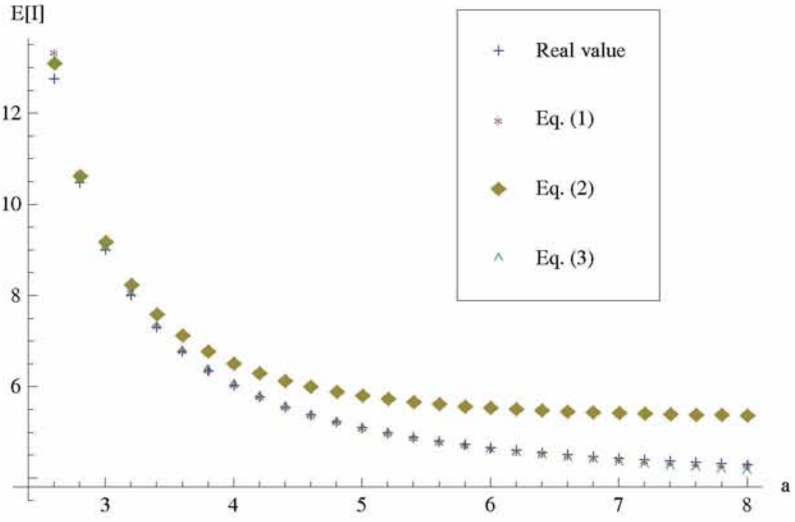

Figure 3: E[I] on the square lattice.

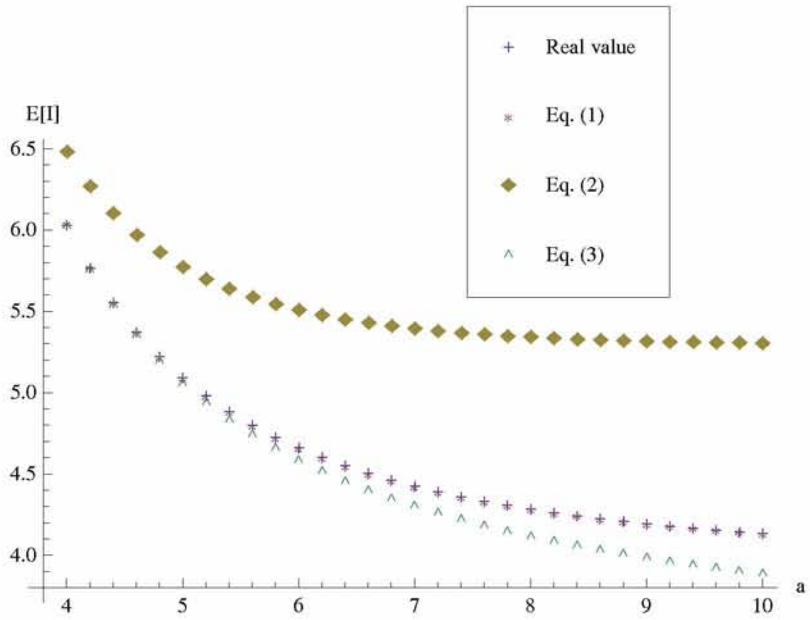

Figure 4: E[I] on the square lattice (detail).

4 Outage Probability

In this section we derive results for the outage probability on the lattice network. We also give analogous results, found in [WAJ09], for the case of a point Poisson network.

4.1 Lattice Network

Let q_d denote the outage probability corresponding to the transmission from a node $t \in \mathbb{L}^*$ to a receiver at the origin, where d is the distance from t to the origin, i.e. the length of the link. As outage probability is defined the probability that the receiver fails to receive the message. Then

$$q_d \equiv \mathbb{P}(\mathrm{SIR} < \beta) = \mathbb{P}\Big(\frac{l(d)}{I - l(d)} < \beta\Big) = \mathbb{P}\Big(Y_d > \frac{1}{\beta}\Big),$$

where

$$Y_d \equiv \frac{I - l(d)}{l(d)} = \frac{\sum_{x \in \mathbb{L}^*} l(||x||) 1_{\{x \text{ is transmitting}\}} - l(d)}{l(d)}. \tag{4}$$

Here we follow closely [WAJ09] and break interference to two parts. The trick is to consider as nodes that are near, only the nodes that are near enough to prohibit any reception, even if only one of them is open.

Let $\mathcal{N}_d$ be the set of the nodes with the property that each of them, if open, contributes that much interference that the node at the origin cannot receive from node t. It can be computed by

$$\mathcal{N}_d = \Big\{x \in \mathbb{L}^* : \frac{l(d)}{l(||x||)} < \beta, x \neq t\Big\}.$$

Note that $\mathcal{N}_d$ does not contain neither the node at the origin (as $\mathbb{L}^*$ is used, instead of $\mathbb{L}$), nor t. If $l(d) = d^{-\alpha}$, then

$$|\mathcal{N}_d| = \Big|\Big\{x \in \mathbb{L}^* : ||x|| < R, x \neq t\Big\}\Big| \approx \pi R^2 - 2,$$

where $R = d\sqrt[\alpha]{\beta}$ is the radius of $\mathcal{N}_d$. [4]

Let us break Y_d as follows.

$$Y_d^n \equiv \frac{\sum_{x \in \mathcal{N}_d} l(||x||) 1_{\{x \text{ is transmitting}\}} - l(d)}{l(d)}$$

and

$$Y_d^f \equiv \frac{\sum_{x \in \mathbb{L}^* - \mathcal{N}_d} l(||x||) 1_{\{x \text{ is transmitting}\}}}{l(d)}.$$

[4] The approximation above follows from the solution to Gauss's Circle Problem. Refer to [Wei] for more details and exact solutions to the latter.

It should be obvious that $Y_d = Y_d^n + Y_d^f$. [5]

Furthermore, we can trivially derive from eq. 4 that

$$Y_d^f = Y_d - Y_d^n = \frac{I - l(d)}{l(d)} - Y_d^n, \tag{5}$$

which would be handy if it comes to be easier to handle I and Y_d^n, than Y_d^f.

4.1.1 Lower Bound

We have that

$$q_d = \mathbb{P}\Big(Y_d^n + Y_d^f > \frac{1}{\beta}\Big) > \mathbb{P}\Big(Y_d^n > \frac{1}{\beta}\Big) \equiv q_d^l,$$

and we can find q_d^l easily as pie. Precisely speaking,

$$q_d^l = 1 - \mathbb{P}(\text{none of the } m \in \mathcal{N}_d \text{ is transmitting}) = 1 - (1-p)^{|\mathcal{N}_d|}. \tag{6}$$

4.1.2 Upper Bound

Following [WAJ09], we can express q_d as follows

$$q_d = q_d^l + (1 - q_d^l)\mathbb{P}\Big(Y_d^f > \frac{1}{\beta}\Big). \tag{7}$$

Now we're after an upper bound for $\mathbb{P}\Big(Y_d^f > \frac{1}{\beta}\Big)$. The following we're gonna need.

$$\begin{aligned}
\mathrm{E}[Y_d^f] &= \mathrm{E}\Big[\frac{\sum_{x\in\mathbb{L}^*-\mathcal{N}_d} l(||x||)1_{\{x \text{ is transmitting}\}}}{l(d)}\Big] \\
&= \frac{1}{l(d)} \sum_{x\in\mathbb{L}^*:\frac{l(d)}{l(||x||)}>\beta} l(||x||)p \\
&\overset{e.g.}{=} p\frac{1}{d^{-\alpha}} \sum_{x\in\mathbb{L}^*:\frac{d^{-\alpha}}{||x||^{-\alpha}}>\beta} ||x||^{-\alpha} \\
&= pd^{\alpha} \sum_{x\in\mathbb{L}^*:||x||>R} ||x||^{-\alpha},
\end{aligned} \tag{8}$$

where $R = d\sqrt[\alpha]{\beta}$ is the radius of $\mathcal{N}_d$.

[5] The $-l(d)$ term at the numerator of Y_d went to Y_d^n; i.e. we assume that $\mathcal{N}_d$ contains the receiver. From the definition of $\mathcal{N}_d$ we see that this is the case iff $\frac{l(d)}{l(d)} < \beta \Leftrightarrow \beta > 1$.

The derivation in eq. 8 is exact; however, it's not yet practical, so here follows an upper bound

$$\begin{aligned}\mathrm{E}[Y_d^f] \leq& p\pi d^\alpha (2\sum_{k=R}^{\infty} k^{1-\alpha} + \sum_{k=R}^{\infty} k^{-\alpha})\\ \leq& p\pi d^\alpha (2\zeta(\alpha-1, \lfloor R \rfloor) + \zeta(\alpha, \lfloor R \rfloor)),\end{aligned}$$

which is rough. [6]

It would be interesting to find $\mathrm{E}[Y_d^f]$ using eq. 5, but then we'd need to handle Y_d^n. That's not an easy task either.

Following subsection 3.2, we also get this:

$$\mathrm{Var}(Y_d^f) = (1-p)\,\mathrm{E}[Y_d^f].$$

Upper Bound through Chebyshev's Inequality Using Chebyshev's inequality we get

$$\begin{aligned}\mathbb{P}\Big(Y_d^f > \frac{1}{\beta}\Big) \leq& \mathbb{P}\Big(|Y_d^f - \mathrm{E}[Y_d^f]| > \frac{1}{\beta} - \mathrm{E}[Y_d^f]\Big)\\ \leq& \frac{\mathrm{Var}(Y_d^f)}{\Big(\frac{1}{\beta} - \mathrm{E}[Y_d^f]\Big)^2} = (1-p)\frac{\mathrm{E}[Y_d^f]}{\Big(\frac{1}{\beta} - \mathrm{E}[Y_d^f]\Big)^2}.\end{aligned} \tag{9}$$

From eq. 7 and 9 follows that

$$\begin{aligned}q_d^{u,Chebyshev} =& q_d^l + (1-q_d^l)\mathbb{P}\Big(Y_d^f > \frac{1}{\beta}\Big)\\ =& 1-(1-p)^{|\mathcal{N}_d|} + (1-p)^{|\mathcal{N}_d|}\mathbb{P}\Big(Y_d^f > \frac{1}{\beta}\Big)\\ =& 1-(1-p)^{|\mathcal{N}_d|}\Big(1-\mathbb{P}\Big(Y_d^f > \frac{1}{\beta}\Big)\Big)\\ \leq& 1-(1-p)^{\pi R^2-2}\Big(1-(1-p)\frac{\mathrm{E}[Y_d^f]}{\Big(\frac{1}{\beta} - \mathrm{E}[Y_d^f]\Big)^2}\Big).\end{aligned}$$

We get a good bound if $(1-p)\dfrac{\mathrm{E}[Y_d^f]}{(1/\beta - \mathrm{E}[Y_d^f])^2}$, which bounds $\mathbb{P}\Big(Y_d^f > \dfrac{1}{\beta}\Big)$, is close to zero. Thus, we get a good bound when we have a small probability of a failed outage caused by distant nodes.

[6] Note that $\zeta(s,q) = \sum_{l=0}^{\infty} 1/(q+l)^s$ is the Hurwitz zeta function.

Upper Bound through Chernoff Bound We find an upper bound for $\mathbb{P}\Big(Y_d^f > \dfrac{1}{\beta}\Big)$ for the case of $d = 1$.

For every $t > 0$, we have that

$$\mathbb{P}\Big(Y_1^f > \frac{1}{\beta}\Big) = \mathbb{P}\Big(e^{tY_1^f} > e^{t/\beta}\Big) \le \frac{\mathrm{E}[e^{tY_1^f}]}{e^{t/\beta}},$$

where the last inequality follows from Markov's inequality.

Moreover

$$\mathrm{E}[e^{tY_1^f}] = \mathrm{E}[e^{\sum_{k\in\mathbb{L}^*-\mathcal{N}_d} 1_{\{k \text{ is on}\}} t||x_k||^{-\alpha}}] = \prod_{k\in\mathbb{L}^*-\mathcal{N}_d} \mathrm{E}[e^{1_{\{k \text{ is on}\}} t||x_k||^{-\alpha}}],$$

where

$$\begin{aligned}\mathrm{E}[e^{1_{\{k \text{ is on}\}} t||x_k||^{-\alpha}}] =& 1 - p + pe^{t||x_k||^{-\alpha}} = 1 + p(e^{t||x_k||^{-\alpha}} - 1)\\ \le& e^{p(e^{t||x_k||^{-\alpha}}-1)}.\end{aligned}$$

Thus, we have

$$\begin{aligned}\mathrm{E}[e^{tY_1^f}] \le& \prod_{k\in\mathbb{L}^*-\mathcal{N}_d} e^{p(e^{t||x_k||^{-\alpha}}-1)} = \Big(\prod_{i=0}^{\infty}\prod_{j=1}^{\infty} e^{p(e^{t\sqrt{i^2+j^2}^{-\alpha}}-1)}\Big)^4\\ =& e^{4p\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}(e^{t\sqrt{i^2+j^2}^{-\alpha}}-1)},\end{aligned}$$

for every $t > 0$.

For $a = 3$ and $b = 1$ we get the upper bound depicted in Figure 5. Note that Chebyshev's bound is very bad. Simulations show that for $d = 1$, $a = 3$ and $b = 1$ throughput and delay are optimized concurrently for p close to 0.1, and the corresponding q_d is around 0.4.

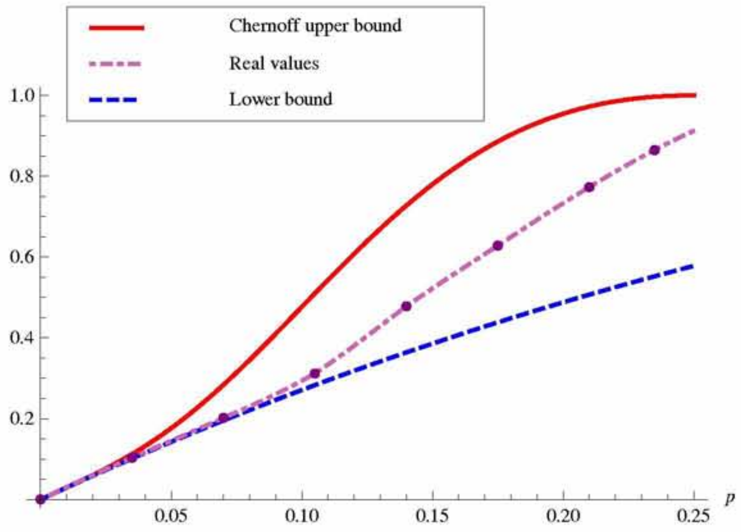

Figure 5: The real values of q_d (obtained numerically), and the upper (Chernoff) and lower bounds.

4.2 Poisson Network

Lower Bound Assume a point Poisson process of density λ The analogous lower bound (analogous to the one for the lattice network) is

$$q^l(\lambda) = 1 - \mathbb{P}[|\mathcal{N}_d| = 0] = 1 - e^{-\lambda p \pi R^2},$$

where $R = d\sqrt[\alpha]{\beta}$ is the radius of $\mathcal{N}_d$.

Upper Bounds From eq. 8 we have

$$\begin{aligned}\mathrm{E}[Y_d^f] = pd^\alpha \int_R^\infty \lambda(2\pi x)x^{-\alpha}dx = 2\pi\lambda pd^\alpha \int_R^\infty x^{1-\alpha}dx = 2\pi\lambda pd^\alpha \frac{R^{2-\alpha}}{\alpha-2} \\ = 2\pi\lambda pd^\alpha \frac{d\sqrt[\alpha]{\beta}^{2-\alpha}}{\alpha-2} = 2\pi\lambda pd^2 \frac{\beta^{(2-\alpha)/\alpha}}{\alpha-2}.\end{aligned}$$

Using Capbell's theorem we can see that

$$\mathrm{Var}(Y_d^f) = \pi d^2 \lambda p \frac{\beta^{\frac{2}{\alpha}-2}}{\alpha-1}.$$

Now we can calculate the Chebyshev bound; we have

$$q_d^{u,Chebyshev}(\lambda) \leq \pi d^2 \beta^{2/\alpha}\lambda p + \frac{\mathrm{Var}(Y_d^f)}{(1/\beta - \mathrm{E}[Y_d^f])^2}.$$

For the Chernoff bound, which is very complicated, refer to [WAJ09].

5 Probability of an Unsuccessful Attempt

In this section we derive results for the probability of an unsuccessful attempt (to be defined) on the lattice network.

Let ϕ_d^m denote the probability of an unsuccessful attempt to relay a packet, across a link of length d, and get it acknowledged; the superscript $m \in \{b, u\}$ denotes the scheme used. According to our model, if at the moment when the transmitter gets a transmission opportunity, it has not yet received an ACK for its last transmitted packet, then the last attempt is considered to be unsuccessful.

5.1 Scheme b

Let $Y_d(t)$ and $Y_d(r)$ be the Y_d value (as defined in section 4) at the transmitter and at the receiver respectively. Now, let ξ_t and ξ_r be the events $\{Y_d(t) > \frac{1}{\beta}\}$ and $\{Y_d(r) > \frac{1}{\beta}\}$ respectively. Note that we consider these two events in different moments in time. Specifically, we care for ξ_r when t sends a data packet, and for ξ_t when r transmits the corresponding ACK.

We have that

$$\begin{aligned} \phi_d^b &= \mathbb{P}\big[\xi_r \cup \xi_t\big] = \mathbb{P}\big[\xi_r\big] + \mathbb{P}\big[\overline{\xi_r} \cap \xi_t\big] \\ &= \mathbb{P}\big[\xi_r\big] + \mathbb{P}\big[\overline{\xi_r}\big]\mathbb{P}\big[\xi_t|\overline{\xi_r}\big] = q_d + (1 - q_d)\rho, \end{aligned} \tag{10}$$

where $\rho = \mathbb{P}\big[\xi_t|\overline{\xi_r}\big]$ which corresponds to the event depicted in Figure 6.

Figure 6: The forward transmission (data) is successful while the reverse (ACK) fails.

There is at most one successful receiver in $N_d(t)$ (namely r, the receiver designated to t), thus when r transmits the ACK for the first time, there can be no "near" interferers to t. Therefore

$$\rho = \mathbb{P}\big[\xi_t^n \cup \xi_t^f|\overline{\xi_r}\big] = \mathbb{P}\big[\xi_t^f|\overline{\xi_r}\big].$$

5.1.1 Lower Bound

We do not yet have a lower bound. A lower bound for ϕ_d^b would be very important as it would allow us to derive a lower bound for the performance gain of the system; specifically, for the values of T_g and D_g, defined in section 6.3.

5.1.2 Upper Bound

We get an upper bound by considering the (of zero probability) case where all transmissions are successful (i.e. all receivers received a packet successfully). In order to give rise to this realization of the receivers, we choose nodes (the transmitters) with probability p each, and then, for each one, randomly (uniformly) set one of its four nearest neighbors open.

In the following where "a priori" is used, it corresponds to the lack of knowledge about the realization of the transmitters. The realization of the receivers (a priori) is a random ALOHA realization, with probability less than p ('cause two transmitters may choose the same receiver). That is because (a priori) every node has the same probability to be a receiver.

Therefore, in the case where all transmissions are successful, a random transmitter experiences a less intense interference setting, but of the exact same nature with the receivers. Specifically in both cases a random ALOHA realization, with no more information, determines the interference seen by the nodes.

Thus, we have

$$\rho = \mathbb{P}\big[\xi_t^f | \overline{\xi_r}\big] \leq \mathbb{P}\big[\xi_r^f\big] = \mathbb{P}\Big(Y_d^f > \frac{1}{\beta}\Big).$$

5.2 Scheme u

Now, let V_k be the event that conditioned on the fact that the transmission was successful and the ACK failed to reach t at the first slot (i.e. the slot where the data packet transmission took place), k slots later the ACK has not arrived yet. We have that

$$\begin{aligned}
\phi_d^u =& \mathbb{P}\big[\xi_r\big] + \mathbb{P}\big[\overline{\xi_r} \cap \xi_t\big] \sum_{k=1}^{\infty} (1-p)^{k-1} p \mathbb{P}\big[V_k\big] \\
\Big[=& q_d + (\phi_d^b - q_d) p \sum_{k=1}^{\infty} (1-p)^{k-1} \mathbb{P}\big[V_k\big] \Big] \\
=& q_d + (1-q_d)\rho p \sum_{k=1}^{\infty} (1-p)^{k-1} \mathbb{P}\big[V_k\big] \\
=& q_d + (1-q_d)\rho\gamma, \qquad (11)
\end{aligned}$$

where $\gamma = p\sum_{k=1}^{\infty}(1-p)^{k-1}\mathbb{P}\big[V_k\big]$ is the probability of an unsuccessful attempt conditioned on the fact that the transmission was successful but the transmitter failed to receive an ACK during the first slot. A comparison between the expressions of ϕ_d^b and ϕ_d^u makes evident the relation between them. Specifically, $\rho\gamma$ plays the same role in ϕ_d^u as ρ plays in ϕ_d^b. [The second part of the derivation follows from eq. 10.]

6 Metrics

Let $\overline{D^m}$ denote the expected delay of delivery per flow over unit distance. Precisely speaking, $\overline{D^m}$ is the expected value of the time it takes for an arbitrary packet to traverse one of the links, of the multihop route of a certain flow, for the first time (i.e. it is irrelevant whether the ACK sent by the receiver is received by the transmitter, and subsequently whether a retransmission occurs), divided by the length of the distance between the transmitter and the receiver.

Moreover, let $\overline{P^m}$ be the expected number of transmissions per packet delivery over unit distance. Similarly to $\overline{D^m}$, $\overline{P^m}$ is also the expected value of a quantity corresponding to a link, divided by the length of the link. However, in this case the quantity of interest is the number of transmissions required, in order for the packet to traverse the link and for the respective ACK (either during the "common" slot or through the flood) to reach the transmitter.

Now if we interpret $1/\overline{P^m}$ as the probability of a transmission to be successful, we can express the throughput (as packets over distance) in a box of n nodes as $np/\overline{P^m}$. Thus per-user throughput is proportional to $\overline{T^m} \equiv p/\overline{P^m}$, and from here on we will refer to $\overline{T^m}$ as the throughput.

6.1 Delay

In order to assist the analysis of the delay we introduce two FIFO queues for each node as follows. First, a queue denoted Q_n holds the (new) data packets that have not yet been transmitted. [7] Second, a queue denoted Q_r holds the packets that are to be retransmitted, i.e. packets that have already been transmitted but have not been ACKed yet. Note that an immediate result of our retransmission strategy is that the number of packets $|Q_r|$ in Q_r is at most one, as a node will not try to transmit a packet unless all previous packets have already been ACKed. [8]

When a node gets a transmission opportunity, if there is a packet in Q_r it transmits (specifically, retransmits) this packet, while if there is no packet in Q_r it transmitts the first packet in Q_n. If the packet in Q_n is transmitted but not successfully ACKed (according to the scheme used) then it is placed

[7] Note that a packet having been transmitted does not mean that it has also been successfully ACKed, thus a packet that has not been transmitted is a packet that has never been transmitted.

[8] These queues do not correspond to the network and MAC layer queues. Specifically, packets that are in Q_r are sure to be in the MAC layer queue, and all packets that are in Q_n are in the network layer queue, except for the first one which, when Q_r is empty, is in the MAC layer queue.

in Q_r.

We partly ignore the queueing delay, in the manner implied in paragraph Assumptions and Remarks of section 2. Specifically, we start measuring the delay for a packet starting from the moment when the packet is the first packet in Q_n, which means that if there is a packet in Q_r waiting for an ACK, it may delay the transmission of this first packet in Q_n as the packet in Q_r has priority.

The distribution of the delay $D^m(d)$ of a packet that has just taken the first position in Q_n, conditioned on the fact that $|Q_r| = 0$, follows. This is the time it takes for a packet sent by the transmitter to reach the receiver for the first time; the superscript $m \in \{b, u\}$ in $D^m(d)$ denotes the scheme used. We have that

$$D^m(d) = \sum_{k=1}^{n+1} G_k(p), \text{ with probability } (q_d)^n(1 - q_d),$$

where $G_k(p)$ are i.i.d. geometric random variables. [9]

As noted above, when a node gets a transmission opportunity, it may have one packet (at most) in its Q_r queue. For the mean per-link delay, we have the following.

$$\begin{aligned}
\mathrm{E}[D^m(d)] &= (1 + \mathbb{P}[|Q_r| \neq 0]) \sum_{n=0}^{\infty} (q_d)^n (1 - q_d) \sum_{k=1}^{n+1} \frac{1}{p} \\
&= (1 + \mathbb{P}[|Q_r| \neq 0]) \frac{(1 - q_d)}{p} \sum_{n=0}^{\infty} (q_d)^n (n + 1) \\
&= (1 + \mathbb{P}[|Q_r| \neq 0]) \frac{1}{p(1 - q_d)}.
\end{aligned}$$

The mean per-link delay over distance is

$$D^m \equiv \frac{\mathrm{E}[D^m(d)]}{d}.$$

Now we model Q_r using the Markov chain depicted in Figure 7. Notice that transitions are made during ALOHA slots only. As the routing is static, a node transmits along hops of length d alone, thus

$$\mathbb{P}[|Q_r| \neq 0] = \mathbb{P}[|Q_r| = 1] = \phi_d^m.$$

[9] Notice that in both schemes, a node does not know if the ACK it sent after receiving a packet was received by the transmitter of the packet, thus it relays the packet anyway. Furthermore, as a result of a certain flow always using the same path, if the node has already received a certain packet, in subsequent receptions of this packet (as a result of retransmissions) the node discards the packet and sends an ACK back to the transmitter.

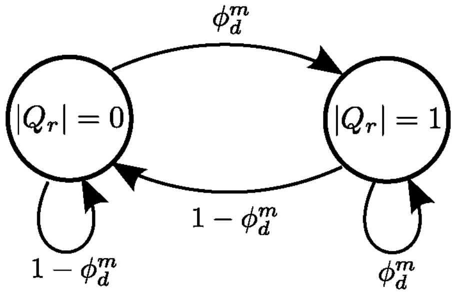

Figure 7: Markov chain corresponding to the behavior of Q_r during ALOHA slots.

For long paths, we have

$$\overline{D^m} = \mathrm{E}_{\mathrm{x}\in\Phi}[D^m] = \mathrm{E}_{\mathrm{x}\in\Phi}\left[\frac{\mathrm{E}[D_l^m(d)]}{d}\right] = \mathrm{E}_{\mathrm{x}\in\Phi}\left[\frac{1+\phi_d^m}{p(1-q_d)d}\right],$$

where Φ is the set of the nodes of the network[10], and in the case of the lattice network it holds that

$$\overline{D^m} = \frac{1+\phi_d^m}{p(1-q_d)d}.$$

6.1.1 Alternative Modeling of the Delay

We may consider only the MAC layer delay, i.e. the time from entering the MAC queue until the packet is received for the first time by the next node. Then the delay would be

$$\overline{D^m} = \mathrm{E}_{\mathrm{x}\in\Phi}\left[\frac{1}{p(1-q_d)d}\right],$$

which implies that, since q_d is the same for both u and b, the MAC delay is identical for the two methods. This approach makes more clear that the benefit of our scheme comes from the queueing delay, which we partially characterize with the above approach. In order to follow this alternative approach, the saturation assumptions should be relaxed in order to allow for the study of the queueing delay. In the rest of the text we assume the first approach for modeling the delay.

[10]Note that ϕ_d^m, q_d, and d are random variables in the case where the locations of the nodes are random. What is more, d depends on the routing protocol, and for a certain routing protocol d is determined by the relative locations of all nodes in the network. Moreover, for an arbitrary link and the corresponding receiving node, ϕ_d^m and q_d are also determined by the relative locations of the nodes in the network, as the latter affect the interference the receiving node experiences. Nevertheless, notice that in the case of the lattice network each node has the same relative position, thus they all have the same ϕ_d^m, q_d, and d.

6.2 Throughput

For the per-link number of transmissions per packet, we have the following.

$$P^m(d) = G(1-\phi_d^m),$$

where $G(p)$ is a geometric random variable, and

$$\mathrm{E}[P^m(d)] = \frac{1}{1-\phi_d^m}.$$

For long paths, we have that

$$\overline{P^m} = \mathrm{E}_{\mathrm{x}\in\Phi}\left[\frac{\mathrm{E}[P^m(d)]}{d}\right] = \mathrm{E}_{\mathrm{x}\in\Phi}\left[\frac{1}{(1-\phi_d^m)d}\right],$$

where Φ is the set of the nodes of the network, and in the case of the lattice network it holds that

$$\overline{P^m} = \frac{1}{(1-\phi_d^m)d}.$$

And as we already mentioned

$$\overline{T^m} = p/\overline{P^m}.$$

6.3 Performance Gain

For the case of the lattice network, we define the delay gain as

$$D_g \equiv \frac{\overline{D^u}-\overline{D^b}}{\overline{D^b}} = \frac{\phi_d^u-\phi_d^b}{1+\phi_d^b} = -\frac{1-\gamma}{1+\dfrac{1+q_d}{(1-q_d)\rho}}, \tag{12}$$

and the throughput gain as

$$T_g \equiv \frac{\overline{T^u}-\overline{T^b}}{\overline{T^b}} = \frac{\phi_d^b-\phi_d^u}{1-\phi_d^b} = \frac{\rho}{1-\rho}(1-\gamma). \tag{13}$$

6.4 Remarks on Delay and Throughput

Scheme u realizes at least the performance of scheme b, both in terms of throughput and delay. This is apparent from the fact that T_g is always positive and D_g always negative (as can be seen by eq. 12 and 13), which is a natural as $\phi_d^u \geq \phi_d^b$ (as can be seen by eq. 10 and 11). Simulations indicate that a certain assignment of values to the parameters of the system (i.e. the routing protocol and p), corresponds to the optimization of both delay and throughput for both schemes. Precisely speaking, for all the assignments in a small neighborhood around the said assignment, the metrics take values very close to the optimal ones. For example, see Figure 10.

For the case of the lattice network, we justify why the said assignment optimizes both delay and throughput under a certain scheme as follows. We have that

$$\overline{D^b} = \frac{1+\phi_d^b}{p(1-q_d)d} = \frac{1}{pd}\Big(\frac{2}{1-q_d} - (1-\rho)\Big).$$

Let us assume that $(1-\rho)$ is small compared to $2/(1-q_d)$. Then it holds that $\overline{D^b}$ is roughly proportional to $1/(p(1-q_d)d)$, which is a quantity to which $\overline{T^b}$ is clearly inversely proportional. Thus $\overline{D^b}$ is inversely proportional to $\overline{T^b}$, and as we want to minimize $\overline{D^b}$ and maximize $\overline{T^b}$, we can deduce that the two metrics are going to get their optimal values for the same set of parameters. In our simulations, it is the case that $(1-\rho)$ is small enough compared to $2/(1-q)$, in order for the above phenomenon to occur. An analogous statement holds for the case of $\overline{D^u}$ and $\overline{T^u}$.

7 Formulation of the Optimization Problems

In this section, we formulate some optimization problems that can help compare the performances of the two schemes with respect to throughput and/or delay. Note that both $\overline{D^m}$ and $\overline{T^m}$ are functions of the routing protocol and the ALOHA probabilities (p_{data} and p_{ACK}) which are the parameters over which we optimize. The optimal choices for these parameters may be different for each scheme.

One way to formulate the problem is to define an objective function which has the form $w\overline{T^m} - (1-w)\overline{D^m}$ with $w \in [0,1]$, where of special interest are the cases for $w = 1$ (maximize $\overline{T^m}$) and $w = 0$ (minimize $\overline{D^m}$), corresponding to the cases of delay tolerant and sensitive systems, respectively.

As already mentioned (section 6.4), simulations indicate that a certain assignment of values to the parameters of the system (i.e. the routing protocol and the ALOHA probabilities) for fixed α and β, corresponds to the optimization of both delay and throughput for both schemes (that is, it optimizes $\overline{D^b}$, $\overline{T^b}$, $\overline{D^u}$ and $\overline{T^u}$ all at once). This ultimately suggests that the comparison of the two schemes boils down to optimizing any of the metrics (throughput or delay) for any of the schemes. This way we can solve any problem from the family described above.

7.1 Using the Lattice Network

On the lattice network the available hop lengths for each device are fixed and as such, we expect an optimal routing path towards a distant destination to be formed by links of equal length d. Consider a flow with source and destination on the same line of a square grid at distance $x \gg 1$ where 1 is the distance between two neighboring nodes on the grid. Without loss of generality let the source be at the origin and the destination at $\{x, 0\}$. We can define the cost of traveling from a node at $\{m, 0\}$ to destination as $J(m)$. Evidently $J(x) = 0$ and $J(0)$ is the total cost for this flow. From the theory of dynamic programming, if we take the limit of $x \to \infty$, the optimal policy (routing policy in our case) is a stationary policy, i.e. making the same decision at each routing hop. This implies that the rule for selecting the best hop length at each point is always the same (indeed, independently of how many steps we have made we are always faced with the same optimization problem again and again). The difference in case of finite horizon is that when approaching the destination, the horizon biases the optimal routing and thus changes the optimal hop distance making it a bit smaller. More info can be found in [Ber95], and especially theorem 7.2.1 p408. For the rest of the text, as regards lattice networks, we will consider routing protocols that make the same selection (i.e. neighbors at the same distance) again and again even if this means that the destination is overtaken by some remaining distance.

7.2 Using the Poisson Network

The analysis of routing on a Poisson network is notoriously difficult (see for example [BBM09]), thus we rely on a greedy heuristic routing protocol. The proposed protocol is easily implementable (as it is based on metrics that we can actually compute), and achieves satisfactory performance.

The routing protocol works as follows. If t is the transmitter, x_d the destination of the flow, and y the next hop, then the latter is determined by

$$\begin{aligned}\underset{y}{\operatorname{argmax}}\{(1-q_d)c,\ & \text{where } d=||t-y||,\\ & c=||t-x_d||-||y-x_d||\}.\end{aligned}$$

Hence, the node that maximizes the product of forward success probability $(1-q_d)$ and the decrease of the distance to the destination c, is the one chosen as the next hop. Note that the aforementioned protocol is a greedy one as it chooses as next hops, only nodes that are closer to the destination than the current transmitter.

In Figure 8 are depicted the links (which are a result of the aforementioned routing protocol) for a network of 90 nodes. Note that the finite space where the network resides forms a torus and the arrows on the figure may not correspond to the smallest distance between two given nodes.

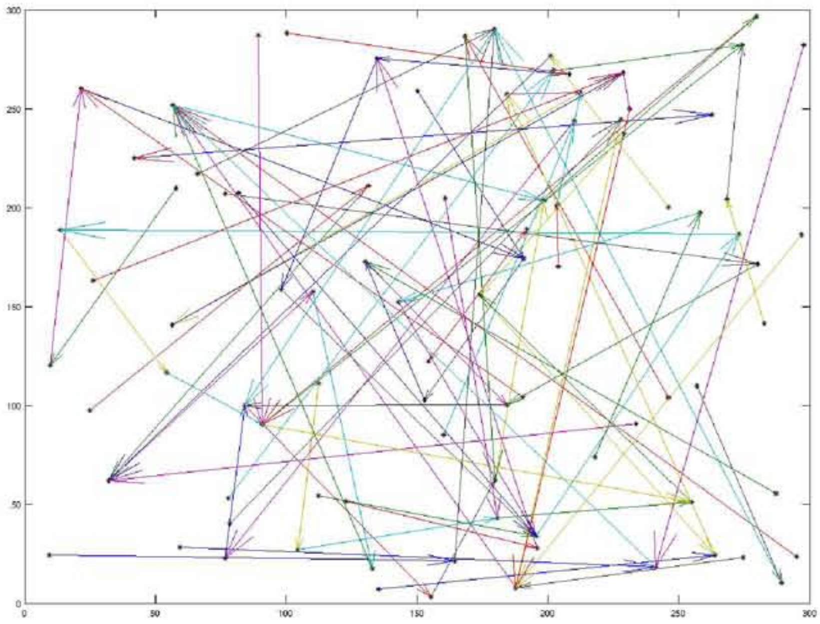

Figure 8: The links of a Poisson network chosen according to the routing protocol introduced in section 7.2.

8 Simulation Results

We run simulations for both lattice and Poisson networks. For the lattice network, we consider a routing protocol that uses hops of equal length; simulations indicate that for both schemes the optimal choice is $d = 1$. On the other hand, for the Poisson network we use the routing protocol that we described in section 7.2. We run simulations for the suboptimal case for scheme u where $p_{\text{data}} = p_{\text{ACK}} = p$.

All the values for T_g and D_g shown below, correspond to a comparison between the optimal values of throughput and delay for schemes b and u. In the tables that follow p^* denotes the optimal p; the optimal d is always 1, as already mentioned, and is not shown in the tables. Lastly, T_g, D_g, ρ, γ, q_d are given in percentages.

8.1 Using the Attenuation Function $l(d) = d^{-a}$

In this case, as can be seen in Table 1, the throughput gain (T_g) and the delay one (D_g), for both network types, are at most about 7.5% and −2.5%, respectively. The reason behind this performance is that ρ (i.e. the probability of broken reverse path conditioned on the fact that the forward one is working) is small; specifically around 10%. Hence, the improvement in performance may not be satisfactory, but this is only because in this setup ρ is small, which means that unidirectional links are rare. This is particularly clear if one considers eq. 12 and 13.

The results are even poorer for the Poisson network, and we conclude that in this setting the use of scheme u is not worthwhile.

Table 1: Lattice network and $l(d) = d^{-a}$

α	p^*	T_g	D_g	ρ	γ	q_d
2.3	1/20	4.5	−2.5	10	60	22
3	1/8	6.5	−2.5	11	49	40
4.5	1/4	7.5	−1.5	10	35	60

8.2 Using the Attenuation Function $l(d) = Zd^{-a}$ (Fading)

We assume a flat fading channel where the envelope is Rayleigh distributed, and thus the power is exponentially distributed. We define Z to be an exponentially distributed random variable with variance $\text{Var}(Z)$. In this case

Table 2: Lattice network and $l(d) = Zd^{-a}$

α	$\mathrm{Var}(Z)$	p^*	T_g	D_g	ρ	γ	q_d
2.3	10^{-2}	1/16	83	−9	65	55	41
3	10^{-1}	1/8	81	−9	63	52	45
3	10^{-2}	1/8	81	−9	63	55	41
3	10^{-3}	1/8	83	−9	65	55	41
4.5	10^{-2}	1/4	116	−9	65	38	60

Table 3: Poisson network and $l(d) = Zd^{-a}$

α	$\mathrm{Var}(Z)$	p^*	T_g	D_g	ρ	γ	q_d
2.3	10^{-2}	1/26	148	−3	79	63	85
3	10^{-1}	1/34	142	−3	81	58	85
3	10^{-2}	1/32	135	−3	81	60	87
4.5	10^{-2}	1/56	132	−3	77	57	83

ρ increases substantially which in turn results in an improvement in T_g and D_g. The following results, as shown in Tables 2 (lattice) and 3 (Poisson), correspond to simulations with fading of variance $\mathrm{Var}(Z)$ from 10^{-3} to 10^{-1}, for α from 2.3 to 4.5, and $\beta = 1$.

For the case of the lattice network, T_g is around 80% (or more for α near 4.5) and D_g is around −9% (a decrease in delay, thus an improvement). The corresponding ρ, γ, and q_d are around 65%, 55%, and 40%, respectively (except for α near 4.5, where results are even more favorable for scheme u).

While for the case of the Poisson network, T_g is around 140% and D_g around −3%. The corresponding ρ, γ, and q_d are around 80%, 58%, and 85%, respectively.

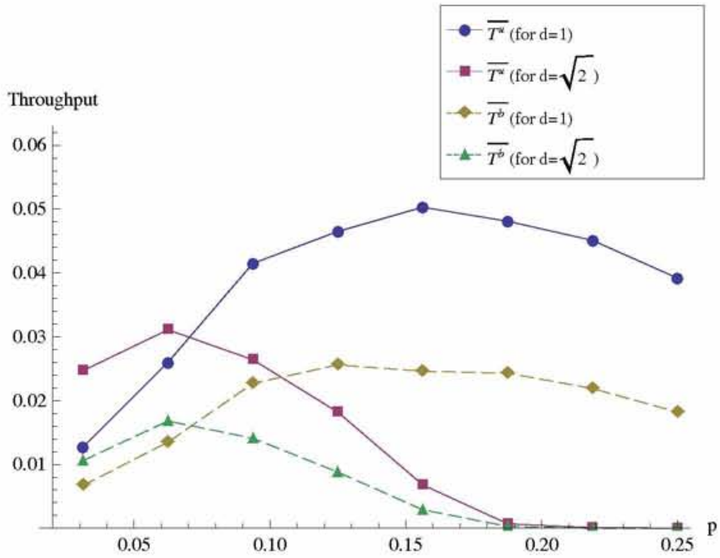

Figure 9: $\overline{T^u}$ and $\overline{T^b}$ for $d \in \{1, \sqrt{2}\}$, on a lattice network with $\alpha = 3$, $\beta = 1$, and fading of variance 10^{-2}. Both schemes achieve their maximum throughput for $d = 1$ and the corresponding optimal p^* is around $1/8$.

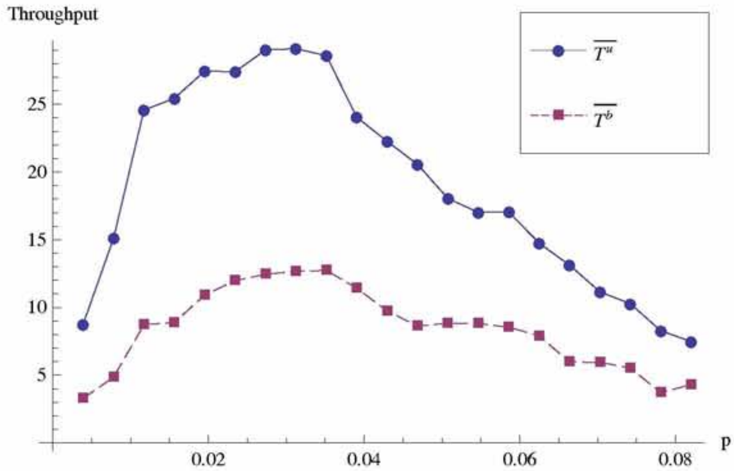

Figure 10: $\overline{T^u}$ and $\overline{T^b}$ on a Poisson network with $\alpha = 3$, $\beta = 1$, and fading of variance 10^{-2}; using a fixed routing protocol (described in section 7.2). Both throughputs are optimized around $p^* = 1/32$.

9 Conclusion and Future Work

A simple cooperative scheme is introduced for acknowledging a successful reception using diversity along multiple paths. We examine an ALOHA protocol that is based on ARQ operation for error detection on both a lattice and a Poisson network. We derive analytical expressions to get an insight into the behavior of the system. We find that the said scheme does not offer a substantial improvement in the absence of fading, as in this case unidirectional links are sparse. Then we take into account Rayleigh fading on the network, which results in the appearance of many unidirectional links. In this latter case we find through simulations that the throughput is increased almost 100%. Our results indicate that WLANs in the presence of interference and fading may benefit greatly by introducing a smart cooperative mechanism for handling asymmetry in the network graph.

In future work, one may inspect other topology models (e.g. by using different underlying networks than Poisson and lattice networks, or by using a different than ALOHA mechanism for the MAC layer), or the case for wifi networks. Specifically, one may find ρ and γ for other network topologies and identify the gain of unidirectional links. Moreover, the analysis of the system using fading may be undertaken ([HG09, WAJ09]). What is more, a lower bound for ϕ_d^b may be found which would assist in deriving analytic lower bounds for T_g and D_g. Additionally, we may relax the assumption of saturated sources and consider ALOHA stability jointly with the optimization of our problem; then it will be possible to determine the queueing delay improvement that our scheme provides due to higher efficiency. Lastly, the system may be implemented and tested on real-world conditions.

Appendices

A Details for Section 3.1

$$
\begin{aligned}
\mathrm{E}[I] = &\sum_{k \in \mathbb{L}^*} l(||x_k||)p \\
=& p\big[4l(1) + 4l(\sqrt{1^2+1^2}) + \\
& 4l(2) + 8l(\sqrt{1^2+2^2}) + 4l(\sqrt{2^2+2^2}) + \\
& 4l(3) + 8l(\sqrt{1^2+3^2}) + 8l(\sqrt{2^2+3^2}) + 4l(\sqrt{3^2+3^2}) + \\
& \dots \\
=& p\sum_{k=1}^{\infty}\big[8\sum_{m=1}^{k-1} l(\sqrt{m^2+k^2}) + 4\big(l(k) + l(\sqrt{2}k)\big)\big] \\
\overset{e.g.}{=}& p\sum_{k=1}^{\infty}\big[8\sum_{m=1}^{k-1} (\sqrt{m^2+k^2})^{-\alpha} + 4\big(k^{-\alpha} + (\sqrt{2}k)^{-\alpha}\big)\big] \\
=& p\sum_{k=1}^{\infty}\big[8\sum_{m=1}^{k-1} (\sqrt{m^2+k^2})^{-\alpha} + 4(1 + (\sqrt{2})^{-\alpha})k^{-\alpha}\big] \\
\leq& p\sum_{k=1}^{\infty}\big[8\sum_{m=1}^{k-1} (\sqrt{m^2+k^2})^{-\alpha} + 4(1 + 1)k^{-\alpha}\big] \\
=& 8p\sum_{k=1}^{\infty}\sum_{m=0}^{k-1} (\sqrt{m^2+k^2})^{-\alpha}
\end{aligned}
$$

At the inequality we added

$$
\begin{aligned}
p\sum_{k=1}^{\infty}\big[4(1-(2^{-\alpha/2}))k^{-\alpha}\big] =& 4(1-(2^{-\alpha/2}))p\sum_{k=1}^{\infty} k^{-\alpha} \\
\leq& 4(1-(2^{-\alpha/2}))p, \text{for } a > 2.
\end{aligned}
$$

We have that

$$
\begin{aligned}
8p\sum_{k=1}^{\infty}\sum_{m=0}^{k-1}(\sqrt{k^2+k^2})^{-\alpha} \leq \mathrm{E}[I] \leq 8p\sum_{k=1}^{\infty}\sum_{m=0}^{k-1}(\sqrt{0^2+k^2})^{-\alpha} \\
8p\sum_{k=1}^{\infty} k(\sqrt{k^2+k^2})^{-\alpha} \leq \mathrm{E}[I] \leq 8p\sum_{k=1}^{\infty} k(\sqrt{0^2+k^2})^{-\alpha} \\
8p\sum_{k=1}^{\infty} 2^{-\alpha/2}k^{1-\alpha} \leq \mathrm{E}[I] \leq 8p\sum_{k=1}^{\infty} k^{1-\alpha} \\
2^{-\alpha/2}8p\zeta(\alpha-1) \leq \mathrm{E}[I] \leq 8p\zeta(\alpha-1).
\end{aligned}
$$

The above can be thought as taking larger and larger rings (more generally partitions) of nodes and bounding them from below and above by using the distance of the closest and the furthest node in the ring respectively. The rings used above are the sequentially larger squares of nodes on the lattice.

B Initial Idea

The initial idea for the current work follows.

Let the node locations lie on a uniform square grid of given density.

- model #1: In a slotted system each node transmits at a given slot with probability p (similar to ALOHA). The channel is one and all other transmissions are considered as interference to any given receiver.
- model #2: The system is still slotted, but the nodes operate on the same channel using for example CDMA. Instead, the nodes aim to save power by turning off the transmitter at each slot with probability p.

Let $SIR_{ij} = l_{ij}/\sum_k (l_{kj} 1_{\{k \text{ is on}\}})$ and l_{ij} is the path which is calculated based on grid geometry. Also, power is fixed. A directional link is set from i to j iff $SIR_{ij} \geq b$.

Both models are the same in mathematical terms but are able to describe different and interesting setups. We know that percolation exists in this model in a mean sense. I.e. if we think of a random graph generated at each slot, the supergraph of these graphs has an infinite component a.s. (see Olivier's thesis for a non-rigorous proof). There is a number of interesting questions to be answered however:

- 1) In such a setting, do we gain something from directional communications? To see this we can ask for example what is the distribution of a link appearing unidirectional or bi-directional. Are they similar? Although Yeh has provided a generic answer, this question is not the same and in not the same framework.
- 2) What are the properties that relate to delay-tolerant behavior of the system. If I am interested to send a packet x away, what is the proper tactics for sending this packet? How long should i wait for it.
- 3) In terms of saving energy: how bad does the random energy saving is doing in comparison to a pre-scheduled system?

More questions can be set in the future.

Differences from prior work: The latency paper is doing similar stuff but without interference. We have fixed positions so more things can be calculated. (I know that you would like to have an underlying process.... but). Maybe we dont care too much about percolation in this. We would like to verify that Yeh's work is in the right direction, i.e. that indeed directional communications do not offer anything even though the property of links being asymmetric would imply the opposite.

Intuition: there are a lot to say about, delay tolerant networks, energy saving and connectivity with directional communications.

Technical note to start with: In `http://www.ece.drexel.edu/weber/publications.html` you can find a paper that deals with separating interference in near and far parts. A premature version was published in the ITW 2009 Volos. The far interference generated outside a radius which is variance dependent, can be replaced with a mean term which in our case would be something simple (a functional on the grid geometry). The inner part should probably be calculated in a more explicit way. Letsee it together and discuss it if you like it.

C Trying to Find the Distribution of Interference (Abandoned)

Let d_k denote the distance between the nodes in the k-th ring and the origin, while $M_k(n)$ is defined above. [This is a sketch – it can work using bounds instead of the approximation.]

$$\begin{aligned} I =& \sum_{m\in\mathbb{L}^*} l(||x_m||)1_{\{m \text{ is on}\}} \\ \approx& \sum_{k=1}^{\infty} l(d_k)B(M_k(n),p) \\ \overset{e.g.}{=}& \sum_{k=1}^{\infty}(ck)^{-\alpha}B(8k,p) \\ =& c^{-\alpha}\sum_{k=1}^{\infty}k^{-\alpha}B(8k,p) \end{aligned}$$

Now this is a pain in the ass; it seems that there are no results about weighted sums of Bernoulli random variables. However, it may be approximated. More specifically, let X_k or $X_{k,m}$ be Bernoullis and check out the following (there are no results here – just some possibly TODO stuff – also notice that the sums below are finite):

$$\sum_{k=1}^{N} X_k k \tag{14}$$

$$\sum_{k=1}^{N} X_k k^{-\alpha} \tag{15}$$

$$\sum_{k=1}^{N} k^{-\alpha} (\sum_{m=1}^{k} X_{k,m}) \tag{16}$$

Using the script weighted_bernoullis.m I tried to visualize the distribution for some of the above random variables. The behavior of the r.v. corresponding to 14 is not too complicated and maybe could be approximated (also check out this: `http://mathforum.org/kb/message.jspa?messageID=7062739`). Moreover, the behavior of the r.v. corresponding to 15 seems to have some self-similar properties. Peak points are evident at the values of the larger weights. Also, α makes much difference; this is also the case for 16, which gets quite more complicated. We wanted to check if in some way the Central Limit Theorem holds. We know that the clt does not hold in its simple form, and also neither the Lyapunov nor the Lindeberg conditions (for generalized clt...).

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