Traffic control via moving bottleneck of coordinated vehicles

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Abstract: The possibility of properly controlling a moving bottleneck to improve the traffic flow is here considered. The traffic is represented by means of a macroscopic model able to take into account the interactions with the bottleneck. This latter interacts with the surrounding flow modifying the traffic density and the flow speed profiles. An optimal control problem is stated by using the speed of the moving bottleneck as control variable. Specifically in this paper the MPC (Model Predictive Control) approach is used in order to get a fuel consumption reduction when the traffic is congested due to the presence of a fixed bottleneck on the highway. In addition we have demonstrated that no increase of the travel time is caused by the control application. The concept illustrated in this paper suggests a future innovative traffic control approach. Indeed the prospective of exploiting special vehicles with manipulable speed to control the traffic flow is particularly attractive given the expected increasing penetration rate of autonomous vehicles in traffic networks in future years.

Keywords: Traffic control, moving bottleneck, MPC, fuel consumption

1. INTRODUCTION

Nowadays an always increasing attention is being payed to the study of traffic systems since the increasing number of vehicles traveling on roads frequently cause traffic jams and hard congestion on networks. These phenomena seriously affects people's quality of life, causing safety problems, pollution issues and affecting productivity. For this reason, an efficient traffic management could have a great socio-economical impact.
Different services from the domain of intelligent transport systems (ITS), such as traffic control systems, are currently studied and applied. But while in the past the typical traffic control solutions were mainly ramp-metering and variable speed limits, (Alessandri et al. (1998), Papageorgiou and Kotsialos (2002), Hegyi et al. (2005)), now, with the advent of intelligent vehicles and with their increasing level of autonomy, new control approaches can be envisaged.
In this paper, a strategy to control the vehicular traffic by controlling the speed of a moving bottleneck has been investigated. This is done relying on an existing model taken from Lebacque et al. (2014), Chalons et al. (2014) that adapts the classical LWR model, Lighthill and Whitham (1955), Richards (1956), to take into account interactions with a slow vehicle moving inside the cars flow. The basic idea is to exploit the controlled vehicle that interacts with the surrounding flow to modify in a proper way the traffic density in the overall section of the highway taken into consideration. Hence, we consider as control variable the speed of the moving bottleneck. In order to get benefits in terms of some prescribed cost functions, as the fuel consumption, we design an MPC based control law. The use of the MPC control methodology is quite common in traffic systems (see, among others, Baskar et al. (2008), Bianchi et al. (2013), Ferrara et al. (2015), Ferrara et al. (2016), Brandi et al. (2017)), yet it is quite new its application to the speed control of a moving bottleneck. Note that optimization based control solution are presented in Roncoli et al. (2015b), Roncoli et al. (2015a) and Iordanidou et al. (2015).
In this paper we consider the moving bottleneck as a single vehicle having a prespecified occupancy. Yet, the moving bottleneck dynamics can be detailed observing that it can be generated through a vehicles formation control, as discussed for instance in Varaiya (1993) and Ferrara and Vecchio (2008).
The present paper is organized as follows. In Section 2, a brief account of the macroscopic model used to describe the behavior of the traffic and the description of the simulation framework are given. In Section 3 the cost functionals that we have used to measure the traffic flow performance are introduced. Section 4 reports the sensitivity analysis of the cost functional depending on the bus speed and Section 5 shows the implemented control and the obtained results.

2. DESCRIPTION OF THE MODEL

The adopted model is the classical LWR, Lighthill and Whitham (1955), Richards (1956), a macroscopic scalar model based on the equation of conservation of cars.
Macroscopic traffic models, also called hydrodynamic models, describe the traffic flow like it was a fluid and

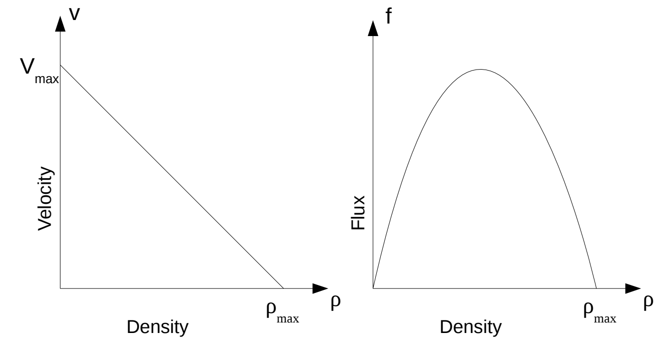


Fig. 1. Quadratic fundamental diagram

dynamical variables are locally aggregated quantities. The macroscopic quantities describing the model are the density of the traffic $\rho = \rho(t, x)$, defined as the number of vehicles per unit length of the road,the average speed $v = v(t, x)$ at which people drive and the flux that is given by $f(\rho, v) = \rho \cdot v$.
The equation describing the model is the scalar conservation of cars

$$\frac{\partial \rho}{\partial t} + \frac{\partial f(\rho)}{\partial x} = 0 \quad (1)$$

The corresponding law that express the the flux as a function of the density is called fundamental diagram. Several fundamental diagrams have been proposed in order to try to describe the real world traffic in the most accurate way. In this paper we use the model proposed by Greenshield in 1934 that assumes a linear dependence of the speed on the traffic the density

$$v(\rho) = v_{max}\left(1 - \frac{\rho}{\rho_{max}}\right) \quad (2)$$

As a result, the fundamental diagram is a quadratic function (Figure 1).

$$f(\rho) = \rho v_{max}\left(1 - \frac{\rho}{\rho_{max}}\right) \quad (3)$$

The presence of a slow moving vehicle, let us call it a bus, and the way of interaction with the surrounding traffic has been study.
In Monache and Goatin (2014) a strongly coupled PDE-ODE system is exploited to model the influence of a slow and large vehicle.
The model consists of a PDE (Partial Differential Equation) that describes the general trend of the traffic density accordingly to the LWR model and an ODE (Ordinary Differential Equation) which takes into account the trajectory of the bus. The fully coupled PDE-ODE model is

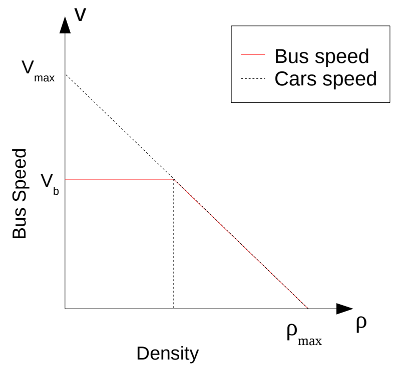


Fig. 2. Speed of the bus and of the car (Fig.1 in Chalons et al. (2014))

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial f(\rho)}{\partial x} = 0 \\ \rho(0, x) = \rho_0(x), \\ f(\rho(t, y(t))) - \dot{y}(t)\rho(t, y(t)) \leq \dfrac{\alpha \rho_{max}}{4 v_{max}}(v_{max} - \dot{y}(t))^2 \\ \dot{y}(t) = \omega(\rho(t, y(t)+)) \\ y(0) = y_0 \end{cases} \quad (4)$$

for $x \in \mathbb{R}$ and $t > 0$. Above, $\rho = \rho(t, x) \in [0, \rho_{max}]$ is the traffic density, ρ_{max} is its maximum attainable value and ρ_0 its initial value. The flux function $f : [0, \rho_{max}] \to \mathbb{R}$ is supposed to be strictly concave and such that $f(0) = f(\rho_{max}) = 0$ and it is given by $f(\rho) = \rho \cdot v(\rho)$.
The mean traffic speed $v(\rho)$ is given by Eq. (2) and v_{max} is its maximum allowed value. The time-dependent variable $y = y(t)$ indicates the bus position and ω the bus speed law.
The bus has its own maximal speed V_b that is smaller than the maximal speed of the traffic flow ($V_b \leq v_{max}$). Therefore the bus moves at its maximal speed when the traffic is not congested and it adapts its velocity accordingly to the surrounding traffic when it is too congested (Fig. 2). The bus cannot overtake the cars. The trend of the bus velocity then is given by:

$$\omega(\rho) = min(V_b, v(\rho)) \quad (5)$$

The bus is considered as a moving capacity restriction and this is expressed by the third equation of (4) that is a local constraint on the flux, where $\alpha \in]0, 1[$ is the reduction rate of the road capacity due to the presence of the bus. A numerical scheme to compute approximate solutions of (4) is designed in Chalons et al. (2014).
In this paper we have chosen a particular framework in order to simulate situations that are close to the real world. Specifically in the following we consider a section of highway of length $L = 50$ km with three lanes, uniform road condition with no on- or off- ramps or other inhomogeneities (Fig. 3).
In order to use parameters able to reflect the real world traffic flow, according to the choices made in Ramadan and Seibold (2017), the maximum flux speed has been chosen equal to $v_{max} = 140\ km/h$ and the maximum density equal to $\rho_{max} = 400\ vehicles/km$. This is justified by considering 5 m of average vehicle length and its 50% of safety distance.
As initial data we consider a constant initial density ρ_0

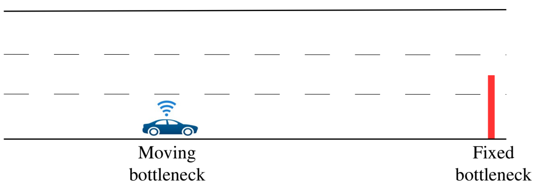


Fig. 3. Simulation framework

equal to the 30% of the maximum density ρ_{max}.
T_f is the simulation time horizon and it has been chosen equal to one hour.
The boundary condition are given by two flux functions, $f_{in}(t)$ and $f_{out}(t)$:

$$f_{in} = \begin{cases} f_{max} & if\ t < 0.5 \cdot Tf \\ 0 & if\ t > 0.5 \cdot Tf \end{cases} \tag{6}$$

$$f_{out} = 0.5 \cdot f_{max} \qquad \forall t \in [0, T_f] \tag{7}$$

3. TRAFFIC PERFORMANCE INDEXES

In order to evaluate the performance of any kind of control on traffic flow, some suitable cost functionals have to be defined. They have to be minimized or maximized along the solution of the problem consisting of the LWR model with the given initial and boundary condition and constraints. For both people traveling in the traffic and road managers, the travel time is a key quantity that has to be analyzed.
If a microscopic model is used, the definition of the travel time appears to be obvious since it is simply derived in terms of trajectories duration. On the other hands, using macroscopic model, we have to define the way of measuring the Travel Time. We have considered the Average Travel Time (ATT) that is calculated on a spatio-temporal region $[x_1, x_2[\times [t_1, t_2[$

$$ATT = \int_{x_1}^{x_2} \int_{t_1}^{t_2} \frac{1}{v(\rho(x,t))} dx dt \tag{8}$$

A further cost functional that has been taken into account is the length of the traffic jam that forms in the proximity of a chosen fixed bottleneck i.e. a car accident or an obstacle along the highway.
At every time t, the n cells that present a density ρ_{out} corresponding to the outflow F_{out} (Fig. 4) at the fixed bottleneck, so the cells that are interested by the jam at time t, are counted.
An average of the jam length on the time is calculated. By calling dl the length of a single cell, $n(t)$ the number of cells in which the congestion is present at time t and T_f the simulation time, the considered cost functional is

$$\sum_{t=1}^{T_f} \frac{n(t) \cdot dl \cdot dt}{T_f} \tag{9}$$

The road transport is one of the main source of air pollution, therefore it has become of paramount importance to find a way that leads to a reduction of its contribution. For this reason we consider the fuel consumption as a cost functional.

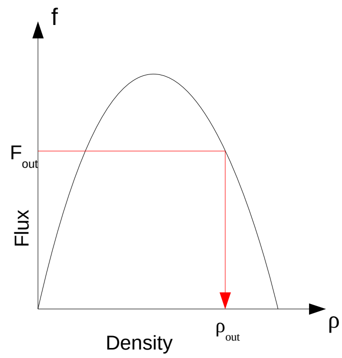


Fig. 4. Density of the jam due to the imposition on the outgoing flux

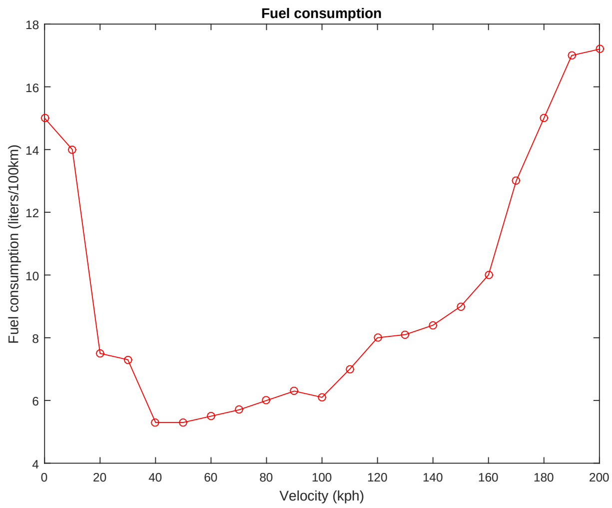


Fig. 5. Approximated average trend of fuel consumption versus cruise speed for steady-speed driving

In order to achieve this purpose, systems able to precisely estimate vehicles emissions are needed. Nowadays, different kind of vehicles emission models exist exploiting both microscopic and macroscopic approaches (see, for instance, Ferrara et al. (2017)).
Belonging to macroscopic models there is the class of average speed models that uses as input the average speed driven on a certain portion of road. The model outputs are local emission factors, describing fuel consumption or emissions in kg (or liters) per meter, volume or mass of consumed fuel or emitted pollutant per kilometer and per vehicle, on average Treiber and A.Kesting (2013). Basically, these types of models derive the average emissions of a certain pollutant depending only on the average speed during a trip.
In this paper we have used the fuel consumption model described in Ramadan and Seibold (2017). Fuel consumption curves of four different vehicles have been taken into account in Berry (2010), providing fuel consumption efficiency data.
Looking at the fuel characteristics reported in Berry (2010), reproduced in approximated way in Fig. 5 where only an average trend is shown, it is possible to note that vehicles consume more fuel per distance at very low and very high speed.
In Ramadan and Seibold (2017) the fuel consumption

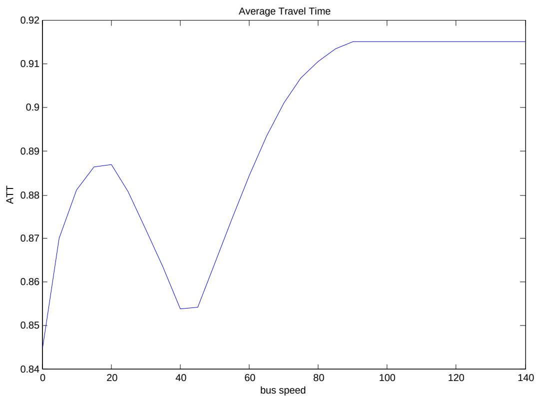


Fig. 6. Average Travel Times trend when the velocity of the bus varies between 0 km/h and 140 km/h

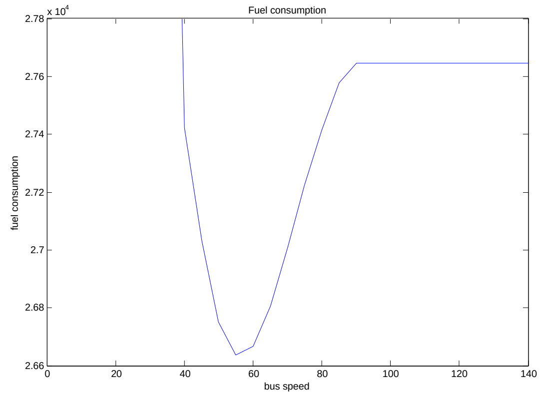


Fig. 7. Fuel consumption trend when the velocity of the bus varies between 0 km/h and 140 km/h

efficiency has been multiplied by the vehicles speed to obtain the fuel consumption rate (Liters/hours), then the curves have been averaged and approximated by using a sixth order polynomial K(v) (see Fig. 5 in Ramadan and Seibold (2017)) that express the the fuel consumption as function of the speed:

$$K(v) = 5.7 \cdot 10^{-12} \cdot v^6 - 3.6 \cdot 10^{-9} \cdot v^5 + 7.6 \cdot 10^{-7} \cdot v^4 - \\ - 6.1 \cdot 10^{-5} \cdot v^3 + 1.9 \cdot 10^{-3} \cdot v^2 + 1.6 \cdot 10^{-2} \cdot v + 0.99 \quad (10)$$

Where $K(v)$ is expressed in $[Liters/hr]$ and v in $[km/hr]$.

By recalling the feature of the LWR model for which the velocity depends only on the density, it is possible to derive the trend of fuel consumption as a function of the traffic density. We have chosen to use the quadratic fundamental diagram with linear velocity function $v = v(\rho)$, so we re-parametrized (10) in terms of the density ρ by using the velocity function (2) and we obtain a function $fc(\rho) = K(v(\rho))$ that represents the fuel consumption rate of one vehicles as a function of the traffic density at the vehicle's position. The total fuel consumption rate $TFC(\rho)$ therefore will be obtain by multiplying $fc(\rho)$ by ρ.

4. SENSITIVITY ANALYSIS

By exploiting the model described in Section 2, a sensitivity analysis has been done relying on the cost functionals introduced in Section 3. We vary the speed V_b between 0 km/h and 140 km/h with a 2 km/h variation, for each value of V_b a simulation is run and the cost functionals are evaluated. In the following simulations the framework described in Section 2 with the occupancy ratio α fixed at 0.6 and the bus initial position at 2 km is considered.
The trend of the ATT is shown in Fig. 6. For very low bus speed, the ATT is bigger because such a speed for the bus, seen as a obstacles from the vehicles point of view, avoids the reaching of the end of the road for the cars. Anyway, a speed for a vehicle lower than 40 km/h for an highway is not realistic. It shows a point of minimum for the speed of the bus close to 40 km/h and it is constant at its maximum value beyond 90 km/h, that being a speed comparable to the one of the traffic free flow, makes the bus do not affect the traffic. In Fig. 7 the trend of the TFC depending on the bus speed is shown. According to the fuel consumption cost

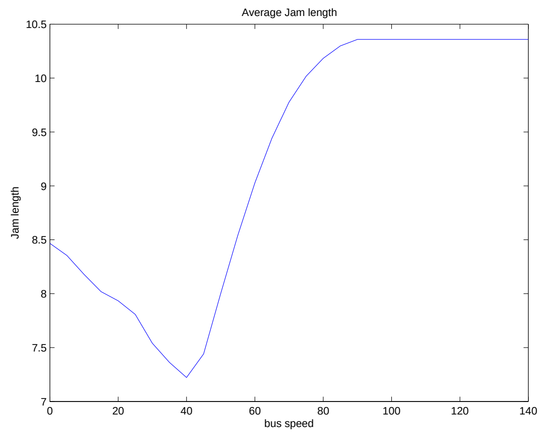


Fig. 8. Jam length trend when the velocity of the bus varies between 0 km/h and 140 km/h

functional we are using, that implies more consumption at very low and very high speed, the trend shows high value of the functional for low and high speed of the bus. A minimum is present at a speed of around 60 km/h.

In Fig. 8 the trend of the jam length at the fixed bottleneck is reported. The functional is constant at its maximum value when the bus speed is beyond the 90 km/h and it is no more able to affect the traffic flow in a considerably way. There is a point of minimum at 40 km/h.

5. THE DESIGNED CONTROL

In this Section we design a speed control for the bus with the aim of influencing the entire traffic system into which the bus is moving. Basically the aim is to find a control law for the velocity of the bus that is able to modify the density profile on the portion of highway in order to achieve benefits in terms of a chosen cost functional.
Considering a generic system, assuming to be at the initial time instant t_0, the aim is to compute an “optimal control” $u_0(t)$, $t \in [t_0, T]$ minimizing a chosen performance index, also called cost function. To this aim we chose to use the MPC (Model Predictive Control) approach, since it can deal with nonlinear systems, multi-criteria optimization, and constraints, Liu et al. (2017).
The MPC works by choosing the optimal value of the control variable on the base of the evaluation of the cost functional over a prediction of the evolution of the system.

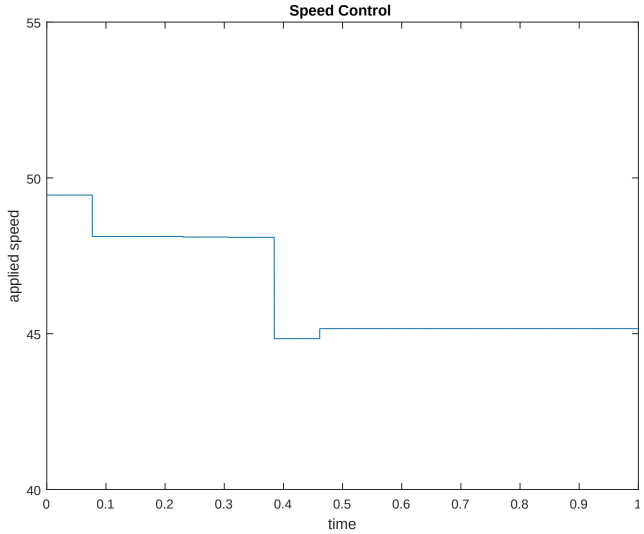
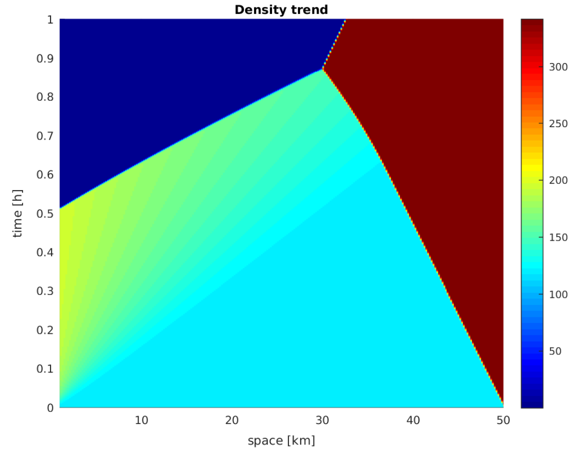


Fig. 9. Applied maximum bus velocity

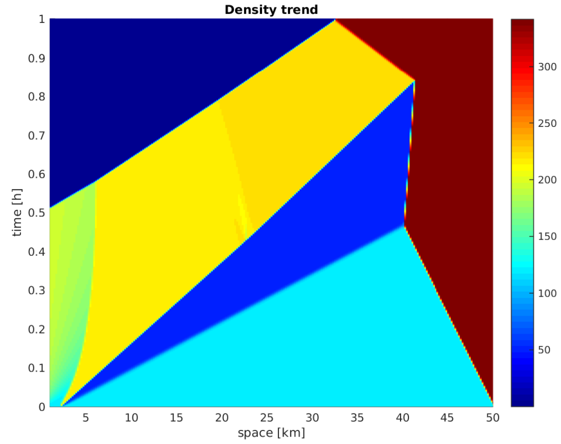
Unlike for linear systems where the prediction of the state along the considered horizon depends linearly on the future control moves and explicit formula exist, this problem can only be solved numerically, by means of iterative optimization algorithms requiring, at each iteration, a simulation run of the system. Since our model is strongly non linear, this latter approach must be followed and we will use the LWR model for the prediction. Let us recall the trend of the speed of the bus considered in the model. The bus moves at its maximum speed while the traffic is not congested, otherwise it adapts its velocity accordingly to the surrounding traffic. The control acts on the maximum value of the bus speed, modifying it according to the prediction.

At the time step $k = 0$ the simulation starts and, on the basis of the initial data, a prediction of the future evolution of the situation is accomplished by simulating the system over a chosen time interval, the prediction horizon ΔT=15 minutes. The chosen cost functional, the TFC, is evaluated on the prediction as a function of the control variable, the bus speed. Hence this last function is minimized with respect to the speed to obtain constant optimal value. This speed is then applied to the controlled vehicle, it is kept constant for a time interval $\Delta\tau$= 5 minutes and the state is updated. At the end of this period $\Delta\tau$, another prediction is done by considering the present value of the density and of the inflow as initial and boundary data for the model and again a constant value of the velocity is obtained by minimizing the cost functional over the prediction horizon with respect to the bus speed. The result of the minimization is applied to the controlled vehicle for $\Delta\tau$, at the end of which we start again a prediction, and so on. At the end we come up with a piecewise constant bus speed control law that is reported in Fig.9. The minimization is done by considering as constraints a limitation on the bus speed, between 30 km/h and 80 km/h.

Let us now make a comparison between this last case with the active control and the simple case in which the maximum bus speed is constant and equal to 80 km/h. The trend of the vehicles density on the portion of the road during the simulation is reported in Fig. 10(a) in the case of maximum speed constant to 80 km/h and in Fig. 10(b) for the controlled case in which the bus maximum speed is imposed by the MPC.



(a) Uncontrolled case



(b) Controlled case

Fig. 10. Density trend as function of time and space in the uncontrolled and controlled case

The bus interacts with the traffic flow slowing it down in both cases but from Figures 10(a), 10(b) the shape of the density appears to be really different in the two cases. The presence of the bus speed control causes a stronger braking to the traffic flow that it is represented by parts of the figure at low density, depicted in yellow. This last appears to be considerably bigger in the controlled case. Moreover, the portion of the graphic at highest density, the red one representing the jam that forms at the fixed bottleneck at the end of the road, is more extended in the uncontrolled case. The cost functional corresponding to the two case of study have been evaluated and a comparison is reported in table 5. We get some improvements in the cost functional computed in the controlled case. Starting from the TFC, the one that is minimized to get the control sequence, it was $2.7413 \cdot 10^4$ liters in the first case and now it is $2.6852 \cdot 10^4$ and therefore 561 liters of fuel have been saved. This corresponds to a percentage reduction of the 2.05% and leads to an economical return for the drivers but also to a reduction of the pollution.

6. CONCLUSION

In this paper we have explored the idea of using some special vehicles regarded as moving bottlenecks in order to control the traffic flow, in a context of a macroscopic traffic modeling.
The control we have implemented aims at reducing the fuel consumption of vehicles and hence their emissions, but by exploiting the same control framework, other objectives (e.g. travel time minimization) could be attained. Simulation evidence shows that the control via moving bottleneck has not only benefits on the reduction of fuel consumption, the cost functional minimized by the control, but it also reduces the travel time.
The proposed control via moving bottlenecks would be of low cost implementation in a scenario in which the penetration rate of autonomous vehicles becomes not negligible. Creating formations, such as platoons, of autonomous vehicles, one could generate different moving bottlenecks the velocity of which could be controlled relying on local conditions of traffic. In addition, even the the length of the moving bottleneck, i.e. the platoon of vehicles, adjustable in time, could be used as a second control variable so as to have a stronger impact on the traffic flow.

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Table 1. Comparison between cost functionals in the controlled and uncontrolled case.

	ATT [h]	jam [km]	TFC [liters]	TFC reduction %
Uncontrolled	0.9107	10.18	$2.7413 \cdot 10^4$	0
Controlled	0.8579	7.66	$2.6852 \cdot 10^4$	2.05

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