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CONFORMAL AND RELATED CHANGES OF METRIC ON THE PRODUCT OF TWO ALMOST CONTACT METRIC MANIFOLDS

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Abstract

This paper studies conformal and related changes of the product metric on the product of two almost contact metric manifolds. It is shown that if one factor is Sasakian, the other is not, but that locally the second factor is of the type studied by Kenmotsu. The results are more general and given in terms of trans-Sasakian, α-Sasakian and β-Kenmotsu structures.

1. Introduction

The product of an almost contact manifold M and the real line $\mathbf{R}$ carries a natural almost complex structure. When this structure is integrable the almost contact structure is said to be *normal*. In the contact case, a normal contact metric manifold is called a *Sasakian manifold*. Moreover the product of two almost contact manifolds also carries a natural almost complex structure whose integrability is equivalent to the normality of both almost contact structures [8]. However if one takes M to be an almost contact metric manifold and supposes that the product metric G on $M\times\mathbf{R}$ is Kaehlerian, then the structure on M is cosymplectic [4] and not Sasakian. On the other hand the second author pointed out in [9] that if the conformally related metric $e^{2t}G$, t being the coordinate on $\mathbf{R}$, is Kaehlerian, then M is Sasakian and conversely. In [2] Capursi showed that for the product of two almost contact metric manifolds, the product metric is Kaehlerian if and only if both factors are cosymplectic. This raises the still open question: What kind of change of the product metric will make both factors Sasakian? Here we study conformal and related changes of the product metric and show that if one factor is Sasakian the other is not, but that locally the second factor is of the type studied by Kenmotsu [6]. This structure will be described in section 3 and in section 4 we shall consider trans-Sasakian structures [9], which will be used for our main results in section 5.

2. Almost contact manifolds

An *almost contact manifold* is an odd-dimensional C^∞ manifold whose structural group can be reduced to $U(n) \times 1$. This is equivalent to the existence of a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η satisfying $\phi^2 = -I + \eta \otimes \xi$ and $\eta(\xi) = 1$. From these conditions one can deduce that $\phi\xi = 0$ and $\eta \circ \phi = 0$. A Riemannian metric g is *compatible* with these structure tensors if

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y)$$

and we refer to an *almost contact metric structure* (ϕ, ξ, η, g). Note also that $\eta(X) = g(X, \xi)$. For a general reference to the ideas of this section see [1].

Let M be an almost contact manifold and define an almost complex structure J on $M \times \mathbf{R}$ by

$$J(X, f\frac{d}{dt}) = (\phi X - f\xi, \eta(X)\frac{d}{dt}). \tag{2.1}$$

An almost contact structure is said to be *normal* if J is integrable. If in addition to the conditions for an almost contact metric structure, we have $d\eta(X,Y) = g(X, \phi Y)$, the structure is a *contact metric structure*, in particular if $dimM = 2n+1$, $\eta \bigwedge (d\eta)^n \neq 0$. Then a *Sasakian manifold* is a normal contact metric manifold. It is well known that the Sasakian condition may be expressed as an almost contact metric structure satisfying

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X,$$

again see e.g. [1].

More generally one has the notion of an *α-Sasakian structure* [5] which may be defined by the requirement

$$(\nabla_X \phi)Y = \alpha(g(X,Y)\xi - \eta(Y)X)$$

where α is a non-zero constant. Setting $Y = \xi$ in this formula, one readily obtains

$$\nabla_X \xi = -\alpha\phi X$$

3. Kenmotsu manifolds

In [10] Tanno classified connected almost contact metric manifolds whose automorphism groups have the maximum dimension. For such a manifold M, the sectional curvature of plane sections containing ξ is a constant, say c. If $c > 0, M$ is a homogeneous Sasakian manifold of constant ϕ-sectional curvature. If $c = 0, M$ is the product of a line or circle with a Kaehler manifold of constant holomorphic curvature. If $c < 0, M$ is a warped product space $\mathbf{R} \times_f \mathbf{C}^n$. In [6] Kenmotsu abstracted the differential geometric properties of the third case. In particular the almost contact metric structure in this case satisfies

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$$

and an almost contact metric manifold satisfying this condition is called a *Kenmotsu manifold* [5,6]. Kenmotsu proved in particular the following result.

Theorem. (Kenmotsu [6]) *Let M be a Kenmotsu manifold. Then for any point $p \epsilon M$, there is a neighborhood U of p which is a warped product $(-\varepsilon, \varepsilon) \times_f V$ where $f(t) = ce^t$ on the interval $(-\varepsilon, \varepsilon)$ and V is a Kaehler manifold.*

Again one has the more general notion of a *β-Kenmotsu structure* [5] which may be defined by

$$(\nabla_X \phi)Y = \beta(g(\phi X, Y)\xi - \eta(Y)\phi X)$$

where β is a non-zero constant. From the condition one may readily deduce that

$$\nabla_X \xi = \beta(X - \eta(X)\xi).$$

4. Trans-Sasakian manifolds

In the classification of Gray and Hervella [3] of almost Hermitian manifolds there appears a class, W_4, of Hermitian manifolds which are closely related to locally conformally Kaehler manifolds. An almost contact metric structure (ϕ, ξ, η, g) on M is *trans-Sasakian* [9] if $(M \times \mathbf{R}, J, G)$ belongs to the class W_4, where J is the almost complex structure on $M \times \mathbf{R}$ defined by (2.1) and G is the product metric on $M \times \mathbf{R}$. This may be expressed by the condition

$$(\nabla_X \phi)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X)$$

for functions α and β on M, and we shall say that the trans-Sasakian structure is of *type* (α, β); in particular, it is normal and it generalizes both α-Sasakian and β-Kenmotsu structures. From the formula one easily obtains

$$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi),$$

$$(\nabla_X \eta)(Y) = -\alpha g(\phi X, Y) + \beta(g(X, Y) - \eta(X)\eta(Y)),$$

$$(\nabla_X \Phi)(Y, Z) = \alpha(g(X, Z)\eta(Y) - g(X, Y)\eta(Z)) - \beta(g(X, \phi Z)\eta(Y) - g(X, \phi Y)\eta(Z))$$

where Φ is the fundamental 2-form of the structure, given by $\Phi(X, Y) = g(X, \phi Y)$. Hence

$$(\nabla_X \Phi)(X, \xi) = -\alpha, \ (\nabla_X \eta)(X) = \beta$$

for X orthogonal to ξ, and $g(X, X) = 1$. Then

$$\delta\Phi(\xi) = 2n\alpha, \ \delta\eta = -2n\beta$$

where δ is the codifferential of g and $\dim M = 2n + 1$. Moreover $d\eta = \alpha\Phi$. If α is a non-zero constant, Φ is closed and one has (cf. [1], p.53)

$$\begin{aligned} g((\nabla_X \phi)Y, Z) &= d\eta(\phi Y, X)\eta(Z) - d\eta(\phi Z, X)\eta(Y) \\ &= \alpha g(X, Y)\eta(Z) - \alpha g(X, Z)\eta(Y) \end{aligned}$$

Then

$$(\nabla_X \phi)Y = \alpha(g(X,Y)\xi - \eta(Y)X)$$

Thus $\beta = 0$ and therefore a trans-Sasakian structure of type (α, β) with α a non-zero constant is α-Sasakian.

Example. Let (x, y, z) be cartesian coordinates on $\mathbf{R}^3$ and put

$$\xi = \frac{\partial}{\partial z}, \; \eta = dz - y\,dx$$

$$\phi = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & -y & 0 \end{pmatrix}, \; g = \begin{pmatrix} e^z + y^2 & 0 & -y \\ 0 & e^z & 0 \\ -y & 0 & 1 \end{pmatrix}$$

Then $\delta\Phi(\xi) = -\frac{1}{e^z}, \delta\eta = -1$ and (ϕ, ξ, η, g) is a trans-Sasakian structure on $\mathbf{R}^3$ of type $(-\frac{1}{2e^z}, \frac{1}{2})$.

The relation between trans-Sasakian, α-Sasakian and β-Kenmotsu structures was recently discussed by Marrero [7].

Proposition 4.1. (Marrero [7]) *Let M be a 3-dimensional Sasakian manifold with structure tensors (ϕ, ξ, η, g), $f > 0$ a non-constant function on M and $\bar{g} = fg + (1-f)\eta \otimes \eta$. Then $(\phi, \xi, \eta, \bar{g})$ is a trans-Sasakian structure of type $(\frac{1}{f}, \frac{1}{2}\xi(\ell n\, f))$.*

Proposition 4.2. (Marrero [7]) *A trans-Sasakian manifold of dimension ≥ 5 is either α-Sasakian, β-Kenmotsu or cosymplectic.*

5. A Study of $M_1 \times M_2$

Let M_1 and M_2 be almost contact metric manifolds with structure tensors $(\phi_i, \xi_i, \eta_i, g_i)$, $i = 1, 2$. Define an almost complex structure J on $M_1 \times M_2$ by

$$J(X_1, X_2) = (\phi_1 X_1 - e^{-2\mu}\eta_2(X_2)\xi_1, \phi_2 X_2 + e^{2\mu}\eta_1(X_1)\xi_2)$$

where μ is a function on $M_1 \times M_2$. That $J^2 = -I$ is easily checked. Let $\tilde{g}$ be the Riemannian metric on $M_1 \times M_2$ defined by

$$\tilde{g}((X_1, X_2), (Y_1, Y_2)) = e^{2\rho} g_1(X_1, Y_1) + e^{2\tau} g_2(X_2, Y_2)$$

where ρ and τ are functions on $M_1 \times M_2$. Then $\tilde{g}$ is Hermitian with respect to J, i.e.

$$\tilde{g}(J(X_1, X_2), J(Y_1, Y_2)) = \tilde{g}((X_1, X_2), (Y_1, Y_2))$$

if and only if

$$\mu = \frac{1}{2}(\rho - \tau).$$

Let ∇^1, ∇^2 and $\tilde{\nabla}$ denote the Riemannian connections of g_1, g_2 and $\tilde{g}$ respectively. Now taking X_1 and Y_1 as vector fields tangent to M_1 and independent of M_2 and similarly for X_2 and Y_2 we give the connection $\tilde{\nabla}$ explicitly:

$$\tilde{\nabla}_{(X_1,0)}(Y_1,0) =$$
$$(\nabla^1_{X_1}Y_1 + (X_1\rho)Y_1 + (Y_1\rho)X_1 - g_1(X_1,Y_1)\mathrm{grad}^1\rho, -e^{2(\rho-\tau)}g_1(X_1,Y_1)\mathrm{grad}^2\rho)$$
$$\tilde{\nabla}_{(0,X_2)}(0,Y_2) =$$
$$(-e^{2(\tau-\rho)}g_2(X_2,Y_2)\mathrm{grad}^1\tau, \nabla^2_{X_2}Y_2 + (X_2\tau)Y_2 + (Y_2\tau)X_2 - g_2(X_2,Y_2)\mathrm{grad}^2\tau)$$
$$\tilde{\nabla}_{(X_1,0)}(0,Y_2) = ((Y_2\rho)X_1, (X_1\tau)Y_2)$$
$$\tilde{\nabla}_{(0,X_2)}(Y_1,0) = ((X_2\rho)Y_1, (Y_1\tau)X_2)$$

Now taking $\mu = \frac{1}{2}(\rho - \tau)$, we compute the covariant derivative of J.

$$\begin{aligned}(\tilde{\nabla}_{(X_1,0)}J)(Y_1,0) = ((&\nabla^1_{X_1}\phi_1)Y_1 + (\phi_1Y_1\rho)X_1\\ &- (Y_1\rho)\phi_1X_1 - g_1(X_1,\phi_1Y_1)\mathrm{grad}^1\rho\\ &+ g_1(X_1,Y_1)\phi_1\mathrm{grad}^1\rho + e^{\rho-\tau}(\xi_2\rho)\eta_1(Y_1)X_1\\ &- e^{\rho-\tau}(\xi_2\rho)g_1(X_1,Y_1)\xi_1,\\ &\ e^{\rho-\tau}(\nabla^1_{X_1}\eta_1)(Y_1)\xi_2\\ &- e^{\rho-\tau}(Y_1\rho)\eta_1(X_1)\xi_2 + e^{\rho-\tau}(\xi_1\rho)g_1(X_1,Y_1)\xi_2\\ &- e^{2(\rho-\tau)}g_1(X_1,\phi_1Y_1)\mathrm{grad}^2\rho + e^{2(\rho-\tau)}g_1(X_1,Y_1)\phi_2\mathrm{grad}^2\rho)\end{aligned} \tag{5.1}$$

$$\begin{aligned}(\tilde{\nabla}_{(0,X_2)}J)(0,Y_2) = (&-e^{\tau-\rho}(\nabla^2_{X_2}\eta_2)(Y_2)\xi_1 + e^{\tau-\rho}(Y_2\tau)\eta_2(X_2)\xi_1\\ &- e^{\tau-\rho}(\xi_2\tau)g_2(X_2,Y_2)\xi_1 - e^{2(\tau-\rho)}g_2(X_2,\phi_2Y_2)\mathrm{grad}^1\tau\\ &+ e^{2(\tau-\rho)}g_2(X_2,Y_2)\phi_2\mathrm{grad}^1\tau,\\ &\ (\nabla^2_{X_2}\phi_2)Y_2 + (\phi_2Y_2\tau)X_2\\ &- (Y_2\tau)\phi_2X_2 - g_2(X_2,\phi_2Y_2)\mathrm{grad}^2\tau + g_2(X_2,Y_2)\phi_2\mathrm{grad}^2\tau\\ &- e^{\tau-\rho}(\xi_1\tau)\eta_2(Y_2)X_2 + e^{\tau-\rho}(\xi_1\tau)g_2(X_2,Y_2)\xi_2)\end{aligned} \tag{5.2}$$

$$\begin{aligned}(\tilde{\nabla}_{(X_1,0)}J)(0,Y_2) = (&-e^{\tau-\rho}\eta_2(Y_2)\,\nabla^1_{X_1}\,\xi - e^{\tau-\rho}(\xi_1\rho)\eta_2(Y_2)X_1\\ &+ e^{\tau-\rho}\eta_1(X_1)\eta_2(Y_2)\mathrm{grad}^1\rho + (\phi_2Y_2\rho)X_1 - (Y_2\rho)\phi_1X_1,\\ &\ e^{\rho-\tau}\eta_1(X_1)\eta_2(Y_2)\mathrm{grad}^2\rho - e^{\rho-\tau}(Y_2\rho)\eta_1(X_1)\xi_2)\end{aligned} \tag{5.3}$$

$$\begin{aligned}(\tilde{\nabla}_{(0,X_2)}J)(Y_1,0) = (&-e^{\tau-\rho}\eta_1(Y_1)\eta_2(X_2)\mathrm{grad}^1\tau\\ &+ e^{\tau-\rho}(Y_1\tau)\eta_2(X_2)\xi_1,\\ &\ e^{\rho-\tau}\eta_1(Y_1)\,\nabla^2_{X_2}\,\xi_2 + e^{\rho-\tau}(\xi_2\tau)\eta_1(Y_1)X_2\\ &- e^{\rho-\tau}\eta_1(Y_1)\eta_2(X_2)\mathrm{grad}^2\tau + (\phi_1Y_1\tau)X_2 - (Y_1\tau)\phi_2X_2)\end{aligned} \tag{5.4}$$

We now suppose that $(M_1 \times M_2, J, \tilde{g})$ is Kaehlerian and study the question of M_1 being trans-Sasakian. If M_1 is trans-Sasakian of type (α, β), the first component of (5.1) becomes

$$\begin{aligned}\alpha(g_1(X_1,Y_1)\xi_1 - \eta_1(Y_1)X_1) + \beta(g_1(\phi_1 X_1, Y_1)\xi_1 - \eta_1(Y_1)\phi_1 X_1)\\ + (\phi_1 Y_1\rho)X_1 - (Y_1\rho)\phi_1 X_1 - g_1(X_1,\phi_1 Y_1)\mathrm{grad}^1\rho\\ + g_1(X_1,Y_1)\phi_1\mathrm{grad}^1\rho + e^{\rho-\tau}(\xi_2\rho)\eta_1(Y_1)X_1\\ - e^{\rho-\tau}(\xi_2\rho)g_1(X_1,Y_1)\xi_1 = 0\end{aligned}$$

Setting $X_1 = Y_1$ and orthogonal to ξ_1, the ξ_1-component yields

$$\xi_2\rho = \alpha e^{\tau-\rho} \tag{5.5}$$

Setting $Y_1 = \xi_1$ and taking X_1 orthogonal to ξ_1 we then have

$$\xi_1\rho = -\beta \tag{5.6}$$

Setting $X_1 = \xi_1$ and $Y_2 = \xi_2$ in the first component of (5.3) we obtain

$$\mathrm{grad}^1\rho = (\xi_1\rho)\xi_1 \tag{5.7}$$

Conversely if $\mathrm{grad}^1\rho = -\beta\xi_1$ and $\xi_2\rho = \alpha e^{\tau-\rho}$, where α and β are functions on M_1, it is easy to see that $(\nabla^1_{X_1}\phi_1)Y_1 = \alpha(g_1(X_1,Y_1)\xi_1 - \eta_1(Y_1)X_1) + \beta(g_1(\phi_1 X_1, Y_1)\xi_1 - \eta_1(Y_1)\phi_1 X_1)$. Note also from the second component of (5.3) we have immediately that $\mathrm{grad}^2\rho = (\xi_2\rho)\xi_2$. Thus we have the following proposition.

Proposition 5.1. *Suppose that $(M_1 \times M_2, J, \tilde{g})$ is Kaehlerian. Then M_1 is trans-Sasakian of type (α, β) if and only if $grad^1\rho = -\beta\xi_1$ and $\xi_2\rho = \alpha e^{\tau-\rho}$ in which case $grad^2\rho = \alpha e^{\tau-\rho}\xi_2$.*

If $\beta = 0$, it follows from (5.6) and (5.7) that ρ is independent of M_1 and we have the following corollary.

Corollary. *Suppose that $(M_1 \times M_2, J, \tilde{g})$ is Kaehlerian and α is a non-zero constant. Then M_1 is α-Sasakian if and only if $grad^1\rho = 0$ and $\xi_2\rho = \alpha e^{\tau-\rho}$.*

Remark. Suppose that M_1 is α-Sasakian and let X_1 be a local coordinate field on M_1. Then $0 = X_1\xi_2\rho = \alpha e^{\tau-\rho}(X_1\tau)$ and hence τ is also independent of M_1. Thus we have that if $(M_1 \times M_2, J, \tilde{g})$ is Kaehlerian and M_1 is α-Sasakian, M_2 cannot be α-Sasakian for any constant; for then ρ and τ would also be independent of M_2 and hence constant on $M_1 \times M_2$. This would then give $\alpha = 0$ on M_1 by (5.5), a contradiction.

Similarly to Proposition 5.1 we have the following.

Proposition 5.2. *Suppose that $(M_1 \times M_2, J, \tilde{g})$ is Kaehlerian; then M_2 is trans-Sasakian of type (α, β) if and only if $\xi_1 \tau = -\alpha e^{\rho-\tau}$ and $grad^2 \tau = -\beta \xi_2$.*

Corollary. *Suppose that $(M_1 \times M_2, J, \tilde{g})$ is Kaehlerian; then M_2 is β-Kenmotsu if and only if $\xi_1 \tau = 0$ and $grad^2 \tau = -\beta \xi_2$.*

Now, let us consider again the almost contact metric manifolds M_1 and M_2 and the almost Hermitian manifold $(M_1 \times M_2, J, \tilde{g})$. Suppose that

$$\mathrm{grad}^1 \rho = -\beta_1 \xi_1, \ \mathrm{grad}^2 \rho = \alpha_1 e^{\tau-\rho} \xi_2$$

$$\mathrm{grad}^1 \tau = -\alpha_2 e^{\rho-\tau} \xi_1, \ \mathrm{grad}^2 \tau = -\beta_2 \xi_2$$

where α_1, β_1 are functions on M_1 and α_2, β_2 are functions on M_2. If M_1 and M_2 are trans-Sasakian then it is seen directly that all the components of (5.1)-(5.4) vanish, giving the following result.

Proposition 5.3. *If one of the following three conditions is satisfied, the other two are equivalent:*

(a) $(M_1 \times M_2, J, \tilde{g})$ is Kaehlerian

(b) The structures on M_1 and M_2 are trans-Sasakian (of types (α_1, β_1) and (α_2, β_2) respectively)

(c) $grad^1 \rho = -\beta_1 \xi, grad^2 \rho = \alpha_1 e^{\tau-\rho} \xi_2, grad^1 \tau = -\alpha_2 e^{\rho-\tau} \xi_1, grad^2 \tau = -\beta_2 \xi_2$.

We now turn to our main result.

Theorem. *Let M_1 and M_2 be almost contact metric manifolds and U a coordinate neighborhood on M_2 such that $\xi_2 = \frac{\partial}{\partial t}$. Consider the change of metric $\tilde{g} = e^{2\rho} g_1 + e^{2\tau} g_2$ on $M_1 \times U$ given by*

$$\rho = \ell n(k - \frac{\alpha}{\beta} e^{-\beta t}), \tau = -\beta t$$

where $\alpha \neq 0, \beta \neq 0$ and k are constants such that ρ is defined on U. Then $(M_1 \times U, J, \tilde{g})$ is Kaehlerian if and only if the structure on M_1 is α-Sasakian and the structure on U is a β-Kenmotsu.

Proof: First note that $\xi_2 \rho = \alpha e^{\tau-\rho}$ and $\xi_2 \tau = -\beta$. Now suppose that the structure on $M_1 \times U$ is Kaehlerian. Then from the first component of (5.1) we see that M_1 is α-Sasakian. Now in the first component of (5.3) choose Y_2 orthogonal to ξ_2; then $(\phi_2 Y_2 \rho) X_1 - (Y_2 \rho) \phi_1 X_1 = 0$ from which we have $Y_2 \rho = 0$. Therefore

$$0 = \frac{\alpha e^{-\beta t}}{e^{\rho}} (Y_2 t)$$

giving $Y_2 t = 0$ and hence $Y_2 \tau = 0$. Thus

$$\mathrm{grad}^2 \tau = -\beta \xi_2.$$

Now using the second component of (5.2) we have

$$(\nabla^2_{X_2}\phi_2)Y_2 = \beta(g_2(\phi_2 X_2, Y_2)\xi_2 - \eta_2(Y_2)\phi_2 X_2),$$

i.e. the structure on U is β-Kenmotsu.

Conversely since the structure on U is β-Kenmotsu, $\nabla^2_{X_2}\xi_2 = \beta(X_2 - \eta_2(X_2)\xi_2)$ from which $d\eta_2 = 0$. Thus the subbundle $\eta_2 = 0$ is integrable. Therefore $Y_2 t = 0$ for any vector field Y_2 orthogonal to ξ_2 and hence

$$\mathrm{grad}^2\tau = -\beta\xi_2$$

and

$$\mathrm{grad}^2\rho = \alpha e^{\tau-\rho}\xi_2.$$

Moreover $\mathrm{grad}^1\rho = \mathrm{grad}^1\tau = 0$. Now, by using Proposition 5.3 we see that $(M_1 \times U, J, \tilde{g})$ is Kaehlerian. ■

Remarks. 1. The conformal change $\rho = \tau = t_2$ gives M_1 Sasakian and U (-1)–Kenmotsu which means that $(\phi_2, -\xi_2, -\eta_2, g_2)$ is a Kenmotsu structure. The choice $\rho = \ell n(k - e^{-t_2}), \tau = -t_2$ gives M_1 Sasakian and U Kenmotsu directly.

2. The fact that the theorem is local in regard to the second manifold M_2 is not unnatural. Even for $M_1 \times \mathbf{R}$, the 1-dimensional case for M_2, note that the Hopf manifold $S^{2n+1} \times S^1$ is locally conformally Kaehler but not globally conformally Kaehler.

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