

The simulation of low Mach flows: from the AUSM-IT flux scheme to ATCBC boundary conditions

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The simulation of low Mach flows: from the AUSM-IT flux scheme to ATCBC boundary conditions

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Inria Project-Team Cagire

(www.inria.fr/en/teams/cagire)

Lab of Mathematics and their Applications

Pau University – France

Universidad Nacional de Córdoba - Argentina

30 March 2017

Presentation Layout

1. The industrial context

2. A low Mach flux for low order FV method

3. Prescribing unsteady subsonic inlet boundary conditions

Different wall cooling approaches

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 Combustion chamber cooling through multiperforated surfaces « effusion cooling » → Canonical Air Fuel Crossflow (hot gases) E Flame Hot gases Effusion cooling Secondary flow (cold gases) Grazing waves \searrow Normal waves

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Acoustic sources

- Combustion noise (up to 80dB)
- Thermoacoustic instabilities (up to 140dB)
- **Mechanical vibrations**
- <u>.</u>
...

Acoustic modes:

- Longitudinal
- Radial
- **Azimuthal**

Wave types:

- **Steady (all modes)**
- Propagating (azimuthal modes essentially)

Illustration of a turbulent reacting flow with turbulence and deterministic motion obtained on the ORACLES test facility (Nguyen et al., 2009)

Exposure time: $1/50 s$ Exposure time: $1/2000 s$

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Low Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations

Low Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations

Using a single space scale and two time scales t_{conv} and $t_{ac} = t_{conv} / \hat{M}$,

any field variable, for insta the major players with

governing equations
 $_{conv}$ and $t_{ac} = t_{conv} / \hat{M}$,

is expanded as:
 $_{nc}$) + $\hat{M}^2 p^{(2)}(x, t_{conv}, t_{ac})$ + O

(chain rule):

-

ae simpler

abba – 30 March 2017 **c**: a better insight about the major players with
tic expansion of the governing equations
scale and two time scales t_{conv} and $t_{ac} = t_{conv} / \hat{M}$,
r instance for the pressure, is expanded as:
 \sum_{conv}, t_{ac}) + \hat{M} $p^{(1$ Let the major players with an
governing equations
 t_{conv} and $t_{ac} = t_{conv} / \hat{M}$,
is expanded as:
 $t_{ac} + \hat{M}^2 p^{(2)}(x, t_{conv}, t_{ac}) + O(\hat{M}^3)$
(chain rule):
 $-\frac{c}{\pi}$
the simpler
and $\sinh(2017)$ Low Mach flows: a better insight about the major players with an
asymptotic expansion of the governing equations
Using a single space scale and two time scales t_{conv} and $t_{ac} = t_{conv}/\hat{M}$,
any field variable, for instance $+O({\hat{M}}^3)$ Low Mach flows: a better insight about the major players with an
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Using a single space scale and two time scales t_{conv} and $t_{ac} = t_{conv} / \hat{M}$,
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 $_{nv}$, t_{ac}) + $O(M^3)$ *x* Mach flows: a better insight about the ma
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field variable, for instance for the pressure, is expande
field variable, for *t*_{ac}) + $O(\hat{M}^3)$
8
8 *x* in sight about the major play
 x and *t* ac = *t_{conv}* (*x* t ac = *t*_{conv} /
 i or the pressure, is expanded as:
 $\hat{M} p^{(1)}(x, t_{conv}, t_{ac}) + \hat{M}^2 p^{(2)}(x, t_{conv})$
 x and \hat{M} yields (chain rule):
 $\frac{c}{dt} = \frac{\partial$ by the set of the property of the product expansion of the product of the set of the set of the set of the set of th *tow* Mach flows: a better insight about the major players
asymptotic expansion of the governing equation
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asymptotic example space scale
ingle space scale
variable, for insta
 $y = p^{(0)}(x, t_{conv}, t_d)$
derivative at cons
 $\frac{\partial}{\partial t_{conv}} \frac{\partial t_{conv}}{\partial t} + \frac{\partial}{\partial t_d}$
w, whenever relev
 $t_{conv} \equiv t$ and $t_{ac} \equiv$
Pascal Bru Low Mach flows: a better insight about the major players with an
asymptotic expansion of the governing equations
Using a single space scale and two time scales t_{conv} and $t_{ac} = t_{conv}/\hat{M}$,
my field variable, for instance Low Mach flows: a better insight about the major players with an
asymptotic expansion of the governing equations
Using a single space scale and two time scales t_{com} and $t_{ac} = t_{com}/\hat{M}$,
any field variable, for instance

$$
\left. \frac{\partial}{\partial t} \right|_{\mathbf{x}, \hat{M}} = \frac{\partial}{\partial t_{conv}} \frac{\partial t_{conv}}{\partial t} + \frac{\partial}{\partial t_{ac}} \frac{\partial t_{ac}}{\partial t} = \frac{\partial}{\partial t} + \frac{1}{\hat{M}} \frac{\partial}{\partial t_{ac}}
$$

From now, whenever relevant, we shall retain the simpler $\tau_{conv} \equiv t$ and $t_{ac} \equiv \tau$

If one proceeds similarly as before, the zeroth, first and second order equations yields:

Continuity
 $\frac{\partial \rho^{(0)}}{\partial \tau} = 0 \Rightarrow \rho^{(0)} = \rho^{(0)}(x, t)$ **Continuity** Low Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations

Continuity
\n
$$
\frac{\partial \rho^{(0)}}{\partial \tau} = 0 \Rightarrow \rho^{(0)} = \rho^{(0)}(x, t)
$$
\n
$$
\frac{\partial \rho^{(1)}}{\partial \tau} + \frac{\partial \rho^{(0)}}{\partial t} + \nabla \cdot (\rho u)^{(0)} = 0
$$
\n
$$
\frac{\partial \rho^{(2)}}{\partial \tau} + \frac{\partial \rho^{(1)}}{\partial t} + \nabla \cdot (\rho u)^{(1)} = 0
$$
\nMomentum

Momentum

$$
\frac{\partial \mathcal{P}}{\partial \tau} + \frac{\partial \mathcal{P}}{\partial t} + \nabla \mathcal{P}(\rho \boldsymbol{u})^{(1)} = 0
$$

Monentum

$$
\nabla p^{(0)} = 0 \Rightarrow p^{(0)} = p^{(0)}(t)
$$

$$
\frac{\partial (\rho \boldsymbol{u})^{(0)}}{\partial \tau} = -\nabla p^{(1)} \Rightarrow \frac{\partial \boldsymbol{u}^{(0)}}{\partial \tau} = -\frac{1}{\rho^{(0)}} \nabla p^{(1)}
$$

$$
\frac{\partial (\rho \boldsymbol{u})^{(1)}}{\partial \tau} + \frac{\partial (\rho \boldsymbol{u})^{(0)}}{\partial t} + \nabla \mathcal{P}(\rho \boldsymbol{u})^{(0)} = -\nabla p^{(2)} + \frac{1}{Re} \nabla \mathcal{P}(\boldsymbol{x})^{(0)}
$$

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Low Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations

Energy equation

asymptotic expansion of the governing equations
\nEnergy equation
\n
$$
\frac{\partial(\rho E)^{(0)}}{\partial \tau} = 0
$$
\n
$$
\frac{\partial(\rho E)^{(1)}}{\partial \tau} + \frac{\partial(\rho E)^{(0)}}{\partial t} + \nabla.(\rho u H)^{(0)} = \frac{\gamma}{\gamma - 1} \frac{1}{\Pr Re} \nabla.(\lambda \nabla T)^{(0)} + (\rho q)^{(0)}
$$
\n
$$
\frac{\partial(\rho E)^{(2)}}{\partial \tau} + \frac{\partial(\rho E)^{(1)}}{\partial t} + \nabla.(\rho u H)^{(1)} = \frac{\gamma}{\gamma - 1} \frac{1}{\Pr Re} \nabla.(\lambda \nabla T)^{(1)} + (\rho q)^{(1)}
$$

State equations

\n
$$
\rho^{(0)}(t, x)T^{(0)}(t, x) = p^{(0)}(t)
$$
\n
$$
p^{(0)}(t) = (\gamma - 1)(\rho E)^{(0)}(t)
$$

Low Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations

The first order energy equation can be also expressed as:

$$
\frac{\partial p^{(1)}}{\partial \tau} + \gamma p^{(0)} \nabla . (u)^{(0)} = \gamma \frac{1}{\text{Pr Re}} \nabla . (\lambda \nabla T)^{(0)} + (\gamma - 1) (\rho q)^{(0)} - \frac{dp^{(0)}}{dt}
$$

Mach flows: a better insight about the major players v
asymptotic expansion of the governing equations
e first order energy equation can be also expressed as:
 $\frac{0}{\tau} + \gamma p^{(0)} \nabla \cdot (u)^{(0)} = \gamma \frac{1}{\text{Pr Re}} \nabla \cdot (2\nabla T)^{(0)} + (\gamma$ W Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations
The first order energy equation can be also expressed as:
 $\frac{\partial p^{(0)}}{\partial \tau} + \gamma p^{(0)} \nabla.(u)^{(0)} = \gamma \frac{1}{\text{PrRe}} \nabla.($ the divergence of the first order momentum equation yields: *p* Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations
 p first order energy equation can be also expressed as:
 $p^{(0)}$
 $p^{(0)} + \gamma p^{(0)} \nabla u^{(0)} = \gamma \frac{1}{\text{Pr Re}} \n$ Differentiating the above equation with respect to τ and substracting $\gamma p^{(0)}$ times ch flows: a better insight about the major players with an
asymptotic expansion of the governing equations
st order energy equation can be also expressed as:
 $\gamma p^{(0)} \nabla.(u)^{(0)} = \gamma \frac{1}{\text{PrRe}} \nabla.(\lambda \nabla T)^{(0)} + (\gamma - 1)(\rho q)^{(0)} -$ W Mach flows: a better insight about the major players with an
asymptotic expansion of the governing equations
The first order energy equation can be also expressed as:
 $\frac{\partial p^{(0)}}{\partial \tau} + \gamma p^{(0)} \nabla.(u)^{(0)} = \gamma \frac{1}{\text{PrRe}} \nabla.($ *u* Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations
 he first order energy equation can be also expressed as:
 $\frac{\partial p^{(0)}}{\partial \tau} + \gamma p^{(0)} \nabla u^{(0)} = \gamma \frac{1}{\text{PrRe}} \$ flows: a better insight about the major players with
mptotic expansion of the governing equations
rder energy equation can be also expressed as:
 ${}^{(0)}\nabla.(u)^{(0)} = \gamma \frac{1}{Pr Re} \nabla.(2\nabla T)^{(0)} + (\gamma - 1)(\rho q)^{(0)} - \frac{d\rho^{(0)}}{dt}$
thing W Mach flows: a better insight about the major players w
asymptotic expansion of the governing equations
The first order energy equation can be also expressed as:
 $\frac{\partial p^{(0)}}{\partial \tau} + \gamma p^{(0)} \nabla.(u)^{(0)} = \gamma \frac{1}{\text{PrRe}} \nabla.(\lambda \nabla$ Mach flows: a better insight about the major players with an
asymptotic expansion of the governing equations
 $\frac{dP^{(0)}}{dt} + \gamma p^{(0)} \nabla.(a)^{(0)} = \gamma \frac{1}{Pr Re} \nabla.(A \nabla T)^{(0)} + (\gamma - 1)(\rho q)^{(0)} - \frac{dp^{(0)}}{dt}$

fferentiating the above e W Mach flows: a better insight about the major players with an
asymptotic expansion of the governing equations
The first order energy equation can be also expressed as:
 $\frac{\partial p^{(0)}}{\partial \tau} + \gamma p^{(0)} \nabla.(u)^{(0)} = \gamma \frac{1}{\text{PrRe}} \nabla.($ *v* Mach flows: a better insight about the major players with an asymptotic expansion of the governing equations

the first order energy equation can be also expressed as:
 $\frac{p^{(0)}}{\partial \tau} + \gamma p^{(0)} \nabla \cdot (\mu)^{(0)} = \gamma \frac{1}{\text{PrRe}}$

$$
W
$$
 Mach flows: a better insight about the major players wit
asymptotic expansion of the governing equations
The first order energy equation can be also expressed as:

$$
\frac{\partial p^{(1)}}{\partial \tau} + \gamma p^{(0)} \nabla \cdot (\boldsymbol{u})^{(0)} = \gamma \frac{1}{\text{Pr Re}} \nabla \cdot (\lambda \nabla T)^{(0)} + (\gamma - 1)(\rho q)^{(0)} - \frac{dp^{(0)}}{dt}
$$

Differentiating the above equation with respect to τ and subtracting $\gamma p^{(0)}$
the divergence of the first order momentum equation yields:

$$
\frac{\partial^2 p^{(1)}}{\partial \tau^2} - \nabla \cdot (c^{(0)^2} \nabla p^{(1)})^{(0)} = (\gamma - 1) \frac{\partial (\rho q)^{(0)}}{\partial \tau} \text{ with } c^{(0)^2} = \gamma \frac{p^{(0)}(t)}{\rho^{(0)}(\boldsymbol{x}, t)}
$$

This is a wave equation and its source is the change **over acoustic time** of
the **leading order of the heat release rate.**
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This is a wave equation and its source is the change **over acoustic time** of the **leading order of the heat release rate.**

Effusion cooling: The related canonical flow is the jet in cross flow (JICF)

- " Examples of experiments:
	- Frick and Roshko (1994), Kelso et al. (1996), Smith and Mungal (1998), Gustafsson (2001), M'Closkey et al. (2001)**,** Most (2007), Michel (2010), Ali $(2010), \ldots$

Freely adapted from Frick and Roshko (1994)

From a real combustor to a baseline lab configuration: the test bench MAVERIC

Short term objective:

– Providing an experimental database with forcing and relevant conditions: non circular hole geometry and non conformal jet exit section with an acoustic forcing of the crossflow. This experimental database is meant to be first for RANS/DNS/LES assessment.

Long term objective :

– improving the understanding of the effect of the presence of the forcing may have on the unsteadiness of adiabatic cooling efficiency. Identifying optimal hole shapes (if any) compatible with the existing manufacturing process.

\mathbf{M} test benching to the configuration From a real combustor to a baseline lab configuration: the test bench MAVERIC

With the acoustic forcing system

→ Setting-up of automatic displacement system for optic measurement tools.

Recording system

 Laser Doppler Velocimetry (LDV) & Phase-Locked Particle image Velocimetry (PIV) measurements

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From a real combustor to a baseline lab configuration: the test bench MAVERIC

- " Pressure drop between lower and upper channel [10 140] Pa
- Main stream Reynolds number [2000 35000]
- Jet Reynolds number [1200 9000]

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Acoustic forcing: Experimental evidence of the flow response.

SPECTRA OF THE UPSTREAM PARIETAL PRESSURE DIFFERENCE

640 response. . Acoustic forcing: Experimental evidence of the flow

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Acoustic forcing: Experimental evidence of the flow response.

VISUALIZATION TO GET A FLAVOUR OF THE JICF (1-HOLE PLATE WITH FORCING @146 HZ)

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Acoustic forcing: Experimental evidence of the flow response.

Examples of extraction of the jets' coherent motion:

KIAI database - AF-CV-12R-60-146-24 , central jet of rows 3 and 5: phase average time variation incorporating four phase locked averages (0°, 90 ° , 180° and 270°, (statistics over 600 images per phase angle).

per phase angle).

Low Mach flow simulations : AUSM-IT, a discrete flux scheme for a low order FV colocated method

The continuous system of PDE's: Euler equations

$$
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0
$$
\n
$$
\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u}) = -\nabla p
$$
\n
$$
\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} H) = 0
$$
\n
$$
E = e + \frac{1}{2} ||\boldsymbol{u}||^2 = c_v T + \frac{1}{2} ||\boldsymbol{u}||^2 ; H = E + \frac{p}{\rho}
$$
\n
$$
p = (\gamma - 1)\rho e
$$

Together with proper initial and boundary conditions

Context and simplifying assumptions

Cell centered finite volume method with variables' co-Cell centered finite volume method with variables' co-
location on the domain $D = \bigcup_{i=1}^{i=N_{cell}} V_i$, coordinate system Ox $=\bigcup_{i=1}^{i}$ and normal basis $B = [e_1].$ Dual mesh cell

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Expression of the fluxes at the interfaces: momentum equation

Consider a first order Euler implicit time discretization.

 $_{+1} - (n+1)L$

$$
(\rho u)_i^{n+1} \simeq (\rho u)_i^n - \frac{\Delta t}{(x_{i+1/2} - x_{i-1/2})} \Big[(\rho u u)_{i+1/2}^{n+1} - (\rho u u)_{i-1/2}^{n+1} + (p)_{i+1/2}^{n+1} - (p)_{i-1/2}^{n+1} \Big]
$$

xpression of the fluxes at the interfaces: momentum equation

ler a first order Euler implicit time discretization.

the discrete system at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $\int_0^1 = (\rho u)_i^n - \frac{\Delta t}{(\lambda_{i+1/2} - \lambda_{i-1/2})} \Big[(\rho u u$ terfaces: momentum equation

discretization.
 $n+1)\Delta t$ reads as:
 $\begin{bmatrix}\n\frac{1}{2} - (\rho u u)_{i-1/2}^{n+1} + (p)_{i+1/2}^{n+1} - (p)_{i-1/2}^{n+1}\n\end{bmatrix}$

ing velocities and the face

To handle this for the

adly speaking two different The fluxes at the in

Euler implicit time

stem at time $t_{n+1} =$
 $\frac{\Delta t}{1/2 - x_{i-1/2}} \left[\left(\rho u u \right) \right]_i^r$

for the transpor

to be derived.

signal Bruel – Universidad Nacional d Expression of the fluxes at the interfaces: momentum equ

Consider a first order Euler implicit time discretization.

Thus, the discrete system at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $(\rho u)_i^{n+1} \approx (\rho u)_i^n - \frac{\Delta t}{(x_{i+1/2} - x_{i-1$ Expression of the fluxes at the interfaces: momentum equation

ider a first order Euler implicit time discretization.
 s, the discrete system at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $t_{n+1}^{n+1} = (\rho u)_{i}^{n} - \frac{\Delta t}{(x_{i+1/2} - x$ Expression of the fluxes at the interfaces: momentum equation

ider a first order Euler implicit time discretization.
 i, the discrete system at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $t_{n+1}^{n+1} \approx (\rho u)_{i}^{n} - \frac{\Delta t}{(x_{i+1/2}$ the fluxes at
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ystem at time
 $\frac{\Delta t}{\frac{1}{t+1/2} - x_{i-1/2}}$ [

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it time discretization.
 $t_{n+1} = (n+1)\Delta t$ reads as:
 $\rho u u_{i+1/2}^{n+1} - (\rho u u_{i-1/2}^{n+1} + \rho u)$
 sporting velocities
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is broadly speaking to

Nacional de Córdoba – 30 March Expression of the fluxes at the interfaces: momentum equation

msider a first order Euler implicit time discretization.

hus, the discrete system at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $u\Big|_{t}^{n+1} = (\rho u)_{t}^{n} - \frac{\Delta t}{(x_{i+1/2$ Expression of the fluxes at the interfaces: momentum equation
 Zonsider a first order Euler implicit time discretization.
 Thus, the discrete system at time $t_{n+1} = (n+1)\Delta t$ **reads as:**
 $\rho u_{i}^{n+1} = (\rho u)_{i}^{n} - \frac{\Delta t}{(\chi$ Expression of the fluxes at the interfaces: momentum equation

der a first order Euler implicit time discretization.

, the discrete system at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $\begin{aligned}\n\mu_1^2 &= (\rho u)_i^n - \frac{\Delta t}{(x_{i+1/2} - x_{i-1$ terfaces: momentum equation
 $(n+1)\Delta t$ reads as:
 $(n+1)\Delta t$ reads as:
 $\begin{bmatrix}\n\frac{1}{1+1/2} - (\rho uu)_{i-1/2}^{n+1} + (\rho)_{i+1/2}^{n+1} - (\rho)_{i-1/2}^{n+1} \n\end{bmatrix}$

ting velocities and the face

To handle this for the

badly speaking two he fluxes at the interfaces: momentum equation

Euler implicit time discretization.

stem at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $\frac{\Delta t}{1+1/2 - x_{i-1/2}} \left[\left(\rho u u \right)_{i+1/2}^{n+1} - \left(\rho u u \right)_{i+1/2}^{n+1} + \left(p \right)_{i+1/2}^{n+1} - \left(p \$ interfaces: momentum equation
 $=(n+1)\Delta t$ reads as:
 $\sum_{i=1/2}^{n+1} -(\rho uu)_{i-i/2}^{n+1} + (p)_{i+i/2}^{n+1} - (p)_{i-i/2}^{n+1}$

orting velocities and the face

d. To handle this for the

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and the Córdob of the fluxes at the interfaces: momentum equation

order Euler implicit time discretization.

te system at time $t_{n+1} = (n+1)\Delta t$ reads as:
 $-\frac{\Delta t}{(x_{i+1/2} - x_{i-1/2})} \Big[(\rho u u)_{i+i/2}^{n+1} - (\rho u u)_{i-i/2}^{n+1} + (p)_{i+i/2}^{n+1} - (p$ So, expressions for the transporting velocities and the face pressure have to be derived. To handle this for the transporting velocity, there is broadly speaking two different alternatives:

Expression of the fluxes at the interfaces

- 1. Either you assume that the transporting velocity is given a **dynamical meaning** and so consider that it satisfies an evolution equation obtained by discretizing the continuous momentum equation on the dual mesh \rightarrow this is the starting point of the **momentum interpolation method**,
- 2. Or you solve a **Riemann problem** derived from the charateristics system written at the interface between two adjacent control volumes (**Godunov type schemes**) .

Momentum interpolation method

For the flux of momentum, let's re-write it formally as :

 $\left(\rho uu\right)^{n+1}_{i=1/2}=\left(\rho u\right)^{n+1}_{i=1/2}u^{n+1}_{i=1/2}$ and conversely $\left(\rho uu\right)^{n+1}_{i=1/2}=\left(\rho u\right)^{n+1}_{i=1/2}u^{n+1}_{i=1/2}$ $(\,\rho u\,)^{n+1}_{i+1/2}$ is the sou Momentum interpolation method

Ilux of momentum, let's re-write it formally as :
 $\frac{1}{1/2} = (\rho u)^{n+1}_{i-1/2} u^{n+1}_{i-1/2}$ and conversely $(\rho uu)^{n+1}_{i+1/2} = (\rho u)^{n+1}_{i+1/2} u^{n+1}_{i+1/2}$

onsider that $(\rho u)^{n+1}_{i+1/2}$ is the Momentum interpolation method

flux of momentum, let's re-write it formally as :
 $\sum_{i=1/2}^{n+1} a_{i-1/2}^{n+1} a_{i-1/2}^{n+1}$ and conversely $(\rho uu)_{i+1/2}^{n+1} = (\rho u)_{i+1/2}^{n+1} u_{i+1/2}^{n+1}$

onsider that $(\rho u)_{i+1/2}^{n+1}$ 1 : the set $1/2$ is tric sou *n n n n n n i i i i i i n i uu u u uu u u* **Example 10** Momentum, let's re-write it formally as :
 $\rho uu \big|_{i=1/2}^{n+1} = (\rho u)_{i=1/2}^{n+1} u_{i-1/2}^{n+1}$ and conversely $(\rho uu)_{i+1/2}^{n+1} = (\rho u)_{i+1/2}^{n+1} u_{i+1/2}^{n+1}$

then, consider that $(\rho u)_{i\pm 1/2}^{n+1}$ is the s $\left[\rho u\right]_{_{i+1/2}}$ is the sol Momentum interpolation method

flux of momentum, let's re-write it formally as :
 $\sum_{-1/2}^{n+1} = (\rho u)_{i-1/2}^{n+1} u_{i-1/2}^{n+1}$ and conversely $(\rho uu)_{i+1/2}^{n+1} = (\rho u)_{i+1/2}^{n+1} u_{i+1/2}^{n+1}$

consider that $(\rho u)_{i+1/2}^{n$ Momentum interpolation method
 \mathbf{r}^{H} . Flux of momentum, let's re-write it formally as :
 \mathbf{r}^{H} .
 \mathbf $+1$. \cdot . \cdot . \cdot . Momentum interpolation method

v of momentum, let's re-write it formally as :
 $= (\rho u)_{i-l/2}^{n+1} u_{i-l/2}^{n+1}$ and conversely $(\rho uu)_{i+l/2}^{n+1} = (\rho u)_{i+l/2}^{n+1} u_{i+l/2}^{n+1}$

ider that $(\rho u)_{i \pm 1/2}^{n+1}$ is the sought **tra** Then, consider that $(\rho u)_{i+1/2}^{n+1}$ is the sought **transported** quantity 1 is the tro $1/2$ IS LIIE LEAL 1 can hot $1/2$ Call De Li $n+1$ is the t $i \pm 1/2$ ¹⁵ $u_{i+1/2}^{n+1}$ can be th $u_{i\pm1/2}^{n+1}$ is the **transporting** velocit
ation on the primal cell, $\ u_{i\pm1/2}^{n+1}$ c $_{\pm1/2}$ is liie lle and that $u_{i\pm 1/2}^{n+1}$ is the **transporting** velocity. On the ground of the sole
discretization on the primal cell, $u_{i\pm 1/2}^{n+1}$ can be thought of as an interpolated quantity based on the cell based values . entum interpolation method

um, let's re-write it formally as :
 $\sum_{i=1/2}^{t+1}$ and conversely $(\rho uu)_{i+1/2}^{n+1}$
 $\sum_{i=1/2}^{n+1}$ is the sought **transported**
 usporting velocity. On the ground

mal cell, $u_{i\pm 1/2}$ entum interpolation method

m, let's re-write it formally as

⁺¹

¹/_{i+1/2} and conversely $(\rho uu)_{i+1/2}^{n+1}$
 $)_{i\pm1/2}^{n+1}$ is the sought **transported

sporting** velocity. On the grou

mal cell, $u_{i\pm1/2}^{n+1}$ c *i i i u Interp u u* We shall denote it by .

We shall denote it by
$$
u_{i\pm 1/2}^{*,n+1} = Interp\left(u_i^{n+1}, u_{i\pm 1}^{n+1}\right)
$$
.

Momentum interpolation method

Then we make the following approximations:

1) The transporting velocity is treated explicitly in

time so
$$
u_{i\pm 1/2}^{*,n+1} \approx u_{i\pm 1/2}^{*,n}
$$

2) An upwind first order expression in space is retained for the transported quantity.

So one gets the following expressions (positive velocities):

momentum interpolation method

\nThen we make the following approximations:

\n1) The transporting velocity is treated explicitly in time so
$$
u_{i\pm1/2}^{*,n+1} \approx u_{i\pm1/2}^{*,n}
$$

\n2) An upward first order expression in space is retained for the transported quantity.

\nSo one gets the following expressions (positive velocities):

\n
$$
(\rho uu)_{i-1/2}^{n+1} \approx (\rho u)_{i-1}^{n+1} u_{i-1/2}^{*,n} = B_i
$$

\n
$$
(\rho uu)_{i+1/2}^{n+1} \approx (\rho u)_{i}^{n+1} u_{i+1/2}^{*,n} = (\rho u)_{i}^{n+1} A_i = B_{i+1}
$$

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Momentum interpolation method

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ds now as:
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 $u)_i^{n+1} A_i + (p)_{i+1/2}^{n+1} - (p)_{i-1/2}^{n+1} + \frac{\$ **Momentum interpolation method**
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 Momentum interpolation method

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ssh and the A_i *is* stand for the transporting velocities therein, so v
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ll reads now **Example 11 Example 11 Example 12 Example 12** *x* **z** *Momentum interpolation method***

x** *z z <i>z <i>z z z z z z z z z z z z z <i>z z z <i>z***** *<i>z <i>z* *<i>z z <i>z <i>z* *<i>z* Momentum interpolation method

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and the A_i 's stand for the transporting velocities therein, so with

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nsporting velocities therein, so with

bmentum equation on a primal mesh
 $B_i + (p)_{i+i/2}^{n+1} - (p)_{i-i/2}^{n+1}$
 $\frac{\Delta x}{\Delta t} \Big[(\rho u)_{i}^{n+1} - (\rho u)_{i}^{n} \Big]$ or
 $\rho u_{$ Momentum interpolation method

e B_i 's stand for the momentum flux at the interfaces of the primal

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and for the transporting velocities therein, so with

discretized momentum equation on a primal mesh
 $\left[(\rho u)^{n+1}_{i} A_{i} - B_{i} + (p)^{n+1}_{i+1/$ Momentum interpolation method

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 I_i 's stand for the transporting velocities therein, so with

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 B_i 's stand for the momentum flux at the interfaces of the primal

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the B_i 's stand for the momentum flux at the interfaces of the primal

sh and the A_i 's stand for the transporting velocities therein, so with

see notations, the discretized momentum equa these notations, the discretized momentum equation on a primal mesh
cell reads now as:

$$
(\rho u)_i^{n+1} = (\rho u)_i^n - \frac{\Delta t}{\Delta x} \Big[(\rho u)_i^{n+1} A_i - B_i + (\rho u)_{i+1/2}^{n+1} - (\rho u)_{i-1/2}^{n+1} \Big]
$$

or equivalently:

$$
B_{i} = (\rho u)_{i}^{n+1} A_{i} + (p)_{i+1/2}^{n+1} - (p)_{i-1/2}^{n+1} + \frac{\Delta x}{\Delta t} \Big[(\rho u)_{i}^{n+1} - (\rho u)_{i}^{n} \Big] \text{ or}
$$

$$
B_{i} = B_{i+1} + (p)_{i+1/2}^{n+1} - (p)_{i-1/2}^{n+1} + \frac{\Delta x}{\Delta t} \Big[(\rho u)_{i}^{n+1} - (\rho u)_{i}^{n} \Big]
$$

Momentum interpolation method: primal mesh view

Momentum interpolation method: dual mesh equation

The same type of discretized momentum equation is postulated on the dual mesh, namely:

is postulated on the dual mesh, namely:
\n
$$
B_{i+1/2} = (\rho u)_{i+1/2}^{n+1} A_{i+1/2} + (p)_{i+1}^{n+1} - (p)_{i}^{n+1} + \frac{\Delta x}{\Delta t} \Big[(\rho u)_{i+1/2}^{n+1} - (\rho u)_{i+1/2}^{n} \Big]
$$

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Momentum interpolation method: relating primal and dual cells quantities

 $_{1/2}$ and $B_{i+1/2}$ Where $A_{i+1/2}$ and $B_{i+1/2}$ are supposed to be such that (Rhie and Chow, 1993): $i_{i+1/2}$ and B_i $A_{i+1/2}$ and B $_{+1/2}$ and $B_{i+1/2}$ are

Where
$$
A_{i+1/2}
$$
 and $B_{i+1/2}$ are supposed to be such that (Rhie and Chow, 1993):
\n
$$
A_{i+1/2} = 2 \frac{A_i A_{i+1}}{A_i + A_{i+1}}
$$
\n
$$
\frac{B_{i+1/2}}{A_{i+1/2}} = \frac{1}{2} \left(\frac{B_i}{A_i} + \frac{B_{i+1}}{A_{i+1}} \right)
$$
\nwhich leads to:
\n
$$
(\rho u)_{i+1/2}^{n+1} = \frac{1}{2} \left[\frac{(\rho u)_{i-1}^{n+1} u_{i-1/2}^{n+1}}{u_{i+1/2}^{n+1}} + \frac{(\rho u)_{i}^{n+1} u_{i+1/2}^{n+1}}{u_{i+3/2}^{n+1}} \right] - \frac{1}{A_{i+1/2}} \left[(\rho u)_{i+1}^{n+1} - (\rho u)_{i}^{n+1} \right]
$$
\n
$$
- \frac{1}{A_{i+1/2}} \frac{\Delta t}{\Delta x} \left[(\rho u)_{i+1/2}^{n+1} - (\rho u)_{i+1/2}^{n} \right]
$$

Momentum interpolation method: the final expression of the primal cell face velocity

$$
(u)_{i+1/2}^{n+1} = (\rho u)_{i+1/2}^{n+1} / \rho_{i+1/2}^{n+1}
$$
 where $\rho_{i+1/2}^{n+1} = \frac{1}{2} [\rho_i^{n+1} + \rho_{i+1}^{n+1}]$

and the ρ_i^{n+1} are calculated through the continuity equation discretized as the momentum on the primal mesh. ρ_i are calculated t

Momentum interpolation method: the final expressi

primal cell face velocity
 $\int_{1/2}^{1} = (\rho u)_{i+1/2}^{n+1} / \rho_{i+1/2}^{n+1}$ where $\rho_{i+1/2}^{n+1} = \frac{1}{2} [\rho_i^{n+1} + \rho_{i+1}^{n+1}]$

the ρ_i^{n+1} are calculated through the con Momentum interpolation method: the final expressi

primal cell face velocity
 $\int_{1/2}^{1} = (\rho u)^{n+1}_{1+1/2} / \rho^{n+1}_{i+1/2}$ where $\rho^{n+1}_{i+1/2} = \frac{1}{2} [\rho^{n+1}_{i} + \rho^{n+1}_{i+1}]$

the ρ^{n+1}_{i} are calculated through the conti We are done with the interface velocity on the primal mesh but not with the Momentum interpolation method: the final express

primal cell face velocity
 $n+1$
 $n+1$ Momentum interpolation method: the final expression of the

primal cell face velocity
 $u\big|_{i=i/2}^{n+1} = (\rho u)_{i+i/2}^{n+1} \cdot \rho_{i+1/2}^{n+1}$ where $\rho_{i+1/2}^{n+1} = \frac{1}{2} [\rho_i^{n-1} + \rho_{i+1}^{n+1}]$

and the ρ_i^{n+1} are calculat Momentum interpolation method: the final expression of the

primal cell face velocity
 $\mu_{1/2}^{(1)} = (\rho u)_{i+\nu/2}^{n+1} / \rho_{i+\nu/2}^{n+1}$ where $\rho_{i+\nu/2}^{n+1} = \frac{1}{2} [\rho_i^{n+1} + \rho_{i+1}^{n+1}]$

the ρ_i^{n+1} are calculated throu Momentum interpolation method: the final expression of the

primal cell face velocity
 $\mu_{1/2}^{(+)} = (\rho u)_{i+1/2}^{m+1} / \rho_{i+1/2}^{m+1}$ where $\rho_{i+1/2}^{m+1} = \frac{1}{2} [\rho_i^{n+1} + \rho_{i+1}^{m+1}]$

the ρ_i^{m+1} are calculated throu omentum interpolation method: the final expression of the

primal cell face velocity
 $= (\rho u)_{i+i/2}^{n+1} / \rho_{i-i/2}^{n+1}$ where $\rho_{i+i/2}^{n+1} = \frac{1}{2} [\rho_i^{n-1} + \rho_{i+1}^{n+1}]$
 ρ_i^{n+1} are calculated through the continuity e Momentum interpolation method: the final expression of the

primal cell face velocity
 $(u)_{i_1i_2}^{n+1} = (\rho u)_{i_1i_2}^{n+1} \neq \rho_{i+12}^{n+1}$

and the ρ_i^{n+1} are calculated through the continuity equation discretized

as pressure interface ! To calculate the latter, an AUSM⁺ expression the primal cell face velocity
 $u v_{n+1/2}^{n+1} / \rho_{n+1/2}^{n+1}$ where $\rho_{n+1/2}^{n+1} = \frac{1}{2} [\rho_{n}^{n+1} + \rho_{n+1}^{n+1}]$
 $u v_{n+1/2}^{n+1} / \rho_{n+1/2}^{n+1}$ where $\rho_{n+1/2}^{n+1} = \frac{1}{2} [\rho_{n}^{n+1} + \rho_{n+1}^{n+1}]$
 u are calculat Momentum interpolation method: the final expression of the

primal cell face velocity
 $\mu_{11/2}^{+1} = (\rho u)_{i_11/2}^{n+1} / \rho_{i_11/2}^{n+1}$ where $\rho_{i_11/2}^{n+1} = \frac{1}{2} [\rho_i^{n+1} + \rho_{i_11}^{n+1}]$

the ρ_i^{n+1} are calculated Momentum interpolation method: the final expression of the

primal cell face velocity
 $f_{i+i/2}^{n+1} = (\rho u)_{i+i/2}^{n+1} / \rho_{i+1/2}^{n+1}$ where $\rho_{i+1/2}^{n+1} = \frac{1}{2} [\rho_i^{n+1} + \rho_{i+1}^{n+1}]$

and the ρ_i^{n+1} are calculated t omentum interpolation method: the final expression of the

primal cell face velocity
 $=(\rho u)^{n+1}_{i+1/2} / \rho^{n+1}_{i+1/2}$ where $\rho^{n+1}_{i+1/2} = \frac{1}{2} [\rho^{n+1}_{i} + \rho^{n+1}_{i+1}]$
 ρ^{n+1}_{i} are calculated through the continuity od: the final expression of the

ce velocity
 $\frac{1}{2} \left[\rho_i^{n+1} + \rho_{i+1}^{n+1} \right]$

ontinuity equation discretized

on the primal mesh but not with the

er, an AUSM⁺ expression

with:
 $\frac{2}{1} - 1$

with $|M| < 1$

de Cór 1 method: the final expression of the

cell face velocity
 $\rho_{i+1/2}^{n+1} = \frac{1}{2} \left[\rho_i^{n+1} + \rho_{i+1}^{n+1} \right]$

gh the continuity equation discretized

mesh.

velocity on the primal mesh but not with the

e the latter, an

\n Momentum interpolation method: the final expres problem is given by:\n
$$
\text{Poisson}^{\text{m}}(u)_{i+1/2}^{n+1} = \left(\rho u\right)_{i+1/2}^{n+1} / \rho_{i+1/2}^{n+1}
$$
\n

\n\n We have calculated through the continuity equation as the momentum on the primal mesh.\n We are done with the interface velocity on the primal mesh pressure interface! To calculate the latter, an AUSM⁺expr is used (Liou, 1996),:\n $\left(p\right)_{i+1/2}^{n} = f_p^+(M_i)(p)_i^n + f_p^-(M_{i+1})(p)_{i+1}^n$ \n

\n\n with $|M| < 1$ \n

\n\n The second Bruel–Universal Nacional de Córdoba – 30 March 2017.\n

Momentum interpolation method: example of results for a 2D pulse simulations (second order discretization, from Moguen et

Initial conditions:

 $(x-0.5)^{2} + (y-0.5)$ Initial conditions:
 $\rho(x, y, t = 0) = \rho_0 + \delta \rho$ $\rho(x, y, t = 0) = \rho_0 +$
 $u(x, y, t = 0) = u_0$ $u(x, y, t = 0) = u_0$
 $v(x, y, t = 0) = v_0$ $p(x, y, t = 0) = v_0$
 $p(x, y, t = 0) = p_0 + \delta p$ 2 $($ $)$ $($ $)$ $($ $)$ 2 2 ² with c^2 and $\delta \rho = \delta p / c_0^2$ with $c_0^2 = \gamma p_0 / \rho_0$ 5 and $\omega p = o p / c_0$ with $c_0 = \gamma p_0 / p_0$
 $M_0 \approx 10^{-5}$, mesh size 500 x 500, $\binom{p}{0.5}^2 + \left(y - 0.5\right)$ $200 \exp \left[-\frac{(x-0.5)^2 + (y-0.5)^2}{0.05^2}\right]$ 0.05 domain size 1m x1m Acoustic C FL number : 20 $(x-0.5)^{2} + (y)$ $p(x, y, t = 0) = p_0 + o p$
 $\delta p = 200 \exp \left[-\frac{(x - 0.5)^2 + (y - 0.5)^2}{2.027} \right]$ δ = 0) = v_0
= 0) = p_0 + δp $= 200 \exp \left[-\frac{(x-0.5)^2 + (y-0.5)^2}{0.05^2}\right]$ $\begin{bmatrix} -\frac{(n-5.6)(1/5-5.6)}{0.05^2} \end{bmatrix}$

 $\varrho_0 = 1.2046 \text{ kg m}^{-3}$, $u_0 = v_0 = 0.3088610^{-2} \text{ m s}^{-1}$, $p_0 = 101300 \text{ Pa}$

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Evolution of the momentum interpolation method for low Mach number Riemann problem (from Moguen et al., 2015a)

Fig. 1. Low Mach number acoustic Riemann problem with a two-shock solution (d. Table 1), second-order discretization [7]. Rhie-Chow momentum interpolation solution (6), cf. Eq. (5), vs. exact solution (solid line) at time $t = 0.01$ s.

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Dispersion Effect !

Evolution of the momentum interpolation method for low Mach number Riemann problem (from Moguen et al., JCP, 2015a)

Density $(kg/m³)$ Velocity (m/s) 25.25 0.6 25.2 0.5 25.15 25.1 0.4 25.05 0.3 25 0.2 24.95 0.2 0.4 0.6 0.8 0.2 0.4 0.6 0.8 0 1 $\bf{0}$ 1 Pressure (Pa) Energy (J) 14200 1406 1405 14150 1404 14100 1403 1402 14050 1401 14000 1400 0.2 0.4 0.6 0.8 0.2 0.4 0.6 0.8 0 1 $\bf{0}$ 1

Centering the velocity in the face velocity expression

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Using the momentum interpolation as a guide to improve a Godunov type scheme (from Moguen et al., JCP, 2015b) for low Mach number

AUSM⁺-up mass flux (Liou, 2006)

$$
(\varrho v)_{i+1/2} = (\varrho cM)_{i+1/2} - \frac{K_p \max\{1 - \sigma \overline{M}^2, 0\}}{c_{i+1/2} f_c(M_0)} (p_{i+1} - p_i),
$$

Mimicking the form obtained when using the momentum interpolation method

(Moguen et al., 2015a) AUSM-IT (IT : Inertia Term) mass flux

$$
(\varrho v)^{n+1}_{i+1/2} = (\varrho c M)^{n+1}_{i+1/2} - \frac{K \max\{1-\sigma \overline{M}^2,0\}}{c_{i+1/2}^{n+1} f_c(M_0)} \bigg\{ p^n_{i+1} - p^n_i + \frac{\Delta x}{\Delta t} [(\varrho v)^{n+1}_{i+1/2} - (\varrho v)^n_{i+1/2}] \bigg\}
$$

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Beneficial effect: the correct low Mach asymptotic behavior at the discrete level is recovered.

■ Setting

$$
M = v_{ref} / \sqrt{p_{ref}/\rho_{ref}}
$$

\n
$$
\rho = \rho_{ref}(\rho^{(0)} + M\rho^{(1)} + M^2\rho^{(2)} + ...)
$$

\n
$$
v = v_{ref}(v^{(0)} + Mv^{(1)} + M^2v^{(2)} + ...)
$$

\n
$$
p = p_{ref}(p^{(0)} + Mp^{(1)} + M^2p^{(2)} + ...)
$$

 \blacksquare ξ = Mx (variable for the large acoustic length scale) $\rho^{(l)} = \rho^{(l)}(x, \xi, t), v^{(l)} = v^{(l)}(x, \xi, t), p^{(l)} = p^{(l)}(x, \xi, t), l = 0, 1, 2$ ■ Substitution of the expansions in the Euler equations and averaging on the small convective length scale leads to the acoustic wave equation :

$$
\partial_t v^{(0)} + \partial_{\xi} p^{(1)} / \varrho^{(0)} = 0
$$

$$
\partial_t p^{(1)} + \gamma p^{(0)} \partial_{\xi} \widetilde{v^{(0)}} = 0
$$

The discrete (at first order) acoustic energy level is conserved with AUSM-IT

Substituting $v^{(0)}$ and $p^{(1)}$ into the low Mach number form of the AUSM-IT scheme:

$$
v_{i+1/2} = \frac{v_i + v_{i+1}}{2} - \frac{1}{2(\varrho c)_{i+1/2} M_r^2} (p_{i+1} - p_i) - \frac{St_r \Delta x}{2c_{i+1/2} M_r} \partial_t v_{i+1/2}
$$

$$
p_{i+1/2} = \frac{p_i + p_{i+1}}{2}
$$

and setting $\Delta \xi = M_r \Delta x$ (Δx at the convective length scale), the equivalent equation for $p^{(1)}$ (2nd order error) is :

$$
\text{St}_{\text{r}}\,\partial_t p^{(1)} + \gamma p^{(0)}\nabla_{\xi} \cdot \widetilde{v^{(0)}} = \frac{c^{(0)}\Delta x}{2M_{\text{r}}} \left(\nabla_{\xi} \cdot \nabla_{\xi} p^{(1)} + \varrho^{(0)} \text{St}_{\text{r}}\,\partial_t \nabla_{\xi} \cdot \widetilde{v^{(0)}}\right)
$$

With the equivalent equation for $\widetilde{v^{(0)}}$ (2nd order error), the right-hand side is zero

■ Consequence : The acoustic wave equations are retrieved (2nd order error)

The discrete (at first order) acoustic energy level is conserved with AUSM-IT

Suppose that $\rho^{(0)}$ et $c^{(0)}$ are constant in time and space Acoustic energy (T: unit torus, assuming periodic boundary conditions) :

$$
E_a = \int_{\mathbb{T}} \left[\frac{1}{2} \widetilde{\varrho^{(0)}} \|\widetilde{v^{(0)}}\|^2 + \frac{1}{2} \frac{(p^{(1)})^2}{\widetilde{\varrho^{(0)}} (c^{(0)})^2} \right]
$$

$$
d_t E_a = \widetilde{\varrho^{(0)}} \int_{\mathbb{T}} \widetilde{v^{(0)}} \cdot \partial_t \widetilde{v^{(0)}} + \frac{1}{\widetilde{\varrho^{(0)}} (c^{(0)})^2} \int_{\mathbb{T}} p^{(1)} \partial_t p^{(1)}
$$

With the equivalent equations (2nd order error) :

$$
d_t E_a = 0
$$

Without an inertia term, acoustic energy is dissipated with the decrease rate :

$$
\mathrm{d}_{t}E_{a} = -\frac{\Delta x}{2 \operatorname{St}_{\mathrm{r}}\widetilde{\varrho^{(0)}}c^{(0)}\mathrm{M}_{\mathrm{r}}}\int_{\mathrm{T}}||\nabla_{\xi}p^{(1)}||^{2} \leq 0
$$

Behavior of AUSM+-up for propagating wave (inlet velocity forcing): evidence of a quite strong dissipation

Behavior with AUSM-IT for the same propagating wave

Low Mach turbulent flow unsteady simulation: ATCBC, a proposal for the boundary condition prescription.

Brief recap: simulations of turbulent flows

Option A - Modeling, discretizing, solving (most widely used)

Step A-1 *Turbulence modeling*

 Reynolds Averaged Navier-Stokes (RANS) - Unconditional NS ensemble averaging (temporal filtering over all time scales of fluctuations if ergodicity property is fulfilled). The resulting solution is a **steady** averaged solution.

 Unsteady Reynolds Averaged Navier Stokes (URANS) - Ensemble conditional averaging at a continuously varied given phase angle of a coherent mono-harmonic motion, artificially introduced (forcing) or naturally present in the flow. The unsteadiness is that of conditionnally **averaged** fields (Phase locking). This is directly related to the triple decomposition proposed by Hussain and Reynolds (JFM, 1971).

 Large-eddy simulations (LES) - NS space (most of the time) filtering with a compact filter. The resulting 3D **instantaneous** (filtered) flow fields realizations are **unsteady**.

Brief recap: simulations of turbulent flows

Option **A** (continued):

Step A-2 *Discretization*

Finite differences, finite volumes, finite elements, high performance computing strategy (**LES**), method of solution (explicit, implicit), mesh generation (structured vs unstructured) management issues (related to HPC) .

o **Step A-3** *Simulation*

Run, collect, post-process and analyze the results.

Brief recap: simulations of turbulent flows

 Option **B** - Discretizing, solving i.e. **DNS** (3D, used for simple geometries and not too high a turbulent Reynolds number):

o Step **B-1** *Discretization*

Finite differences, finite volumes, finite elements, spectral methods, high performance computing strategy (unavoidable), method of solution (explicit, implicit), mesh generation (structured vs unstructured) and management issues (related to HPC).

o Step **B-2** *Simulation*

Run, collect (a real issue !!), post-process and analyze the results.

This is the purpose of the AeroSol Library developed by the Cagire team (in partnership with Cardamon Inria team)

Prescribing boundary conditions for low Mach flow simulations

To determine the **number of physical boundary conditions** to be imposed, the guide is the **wave pattern system analysis** at the considered boundary. The sign of the eigenvalues of the Euler flux jacobian matrix and their multiplicity order is the guide (Poinsot and Lele, 1992)

Prescribing boundary conditions for low Mach flow simulations

The combinations are numerous, see Poinsot and Lele (JCP, 1992) for more details about well posedness vs implementation feedback

Number of physical boundary conditions @outlet for inert NS, (Practically 4 imposed) 1 Euler like + 3 viscous A possible set: Static pressure imposed @infinity and three viscous conditions (zero normal derivative of τ_{xy} , τ_{xz} and of the normal heat flux) f physical
conditions
r inert NS,
4 imposed)
+ 3 viscous
ple set:
re imposed
three viscous
zero normal
 τ_{xy}, τ_{xz} and of
heat flux) At the inlet, how to generate $u'(x,t)$, $v'(x)$,

Approach: choice of the triple decomposition proposed by Hussain and Reynolds (1970) *u'(x,t), v'(x,t), w'(x,t)?*
triple decomposition proposed by
 $u'_{s}(t)$ (assuming that $\overline{u'_{p}u'_{s}} = 0$)
and $u'_{s}(t)$ be generated?

$$
u'(t) = u'_{p}(t) + u'_{s}(t) \quad \text{(assuming that } \overline{u'_{p}u'_{s}} = 0)
$$

derate $u'(x, t)$

of the triple
 (1970)
 $\int_{p}^{b} (t) dt + u'_{s}(t) dt$
 $\int_{a}^{b} f(t) dt$

and
 $\int_{a}^{b} f(t) dt$ now to generate $u'(x,t)$, $v'(x,t)$, $w'(x,t)$?

: choice of the triple decomposition proposed by

Reynolds (1970)
 $u'(t) = u_p(t) + u_s'(t)$ (assuming that $\overline{u_p u_s} = 0$)

: how can $\overline{u_p}(t)$ and $\overline{u_s}(t)$ be generated ?

leterm ate $u'(x,t)$, $v'(t)$

the triple decord

(0)

(1) $\left(u''(t) - u''(t) \right)$

(1) $\left(u''(t) - u''(t) \right)$

(2) $\left(u''(t) - u''(t) \right)$

(2) $\left(u''(t) - u''(t) \right)$

(3) $\left(u''(t) - u''(t) \right)$

(3) $\left(u''(t) - u''(t) \right)$

(3) $\left(u''(t) - u''(t) \right)$

(the inlet, how to generate $u'(x,t)$, $v'(x,t)$, $w'(x,t)$?
 Approach: choice of the triple decomposition proposed by

ussain and Reynolds (1970)
 $u'(t) = u\frac{1}{\rho}(t) + u_s(t)$ (assuming that $\frac{u}{u_p u_s} = 0$)

Question: how can $u\$ inlet, how to generate $u'(x,t)$, $v'(x,t)$, $w'(x,t)$?
 proach: choice of the triple decomposition proposed by
 $u'(t) = u_p(t) + u_s'(t)$ (assuming that $\overline{u_p u_s} = 0$)

nestion: how can $u_p'(t)$ and $u_s'(t)$ be generated?

(*t*) is d the $u'(x,t)$, $v'(x,t)$,

he triple decompo

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(*p*) (t) and $u'_s(t)$ be

so it is quite eas *e* inlet, how to generate $u'(x,t)$, $v'(x,t)$, $w'(x,t)$?
 pproach: choice of the triple decomposition proposed

sain and Reynolds (1970)
 $u'(t) = u_p(t) + u_s'(t)$ (assuming that $\overline{u_p u_s} = 0$

equestion: how can $u_p'(t)$ and $u_s'($ *u*['](*x*,*t*), *v*['](*x*,*t*), *w*['](

triple decompositic
 t) $\frac{d}{dt}$ *t*) (assuming
 t) and $\frac{d}{dt}$ _{*s*} (*t*) be gen

o it is quite easy, b the inlet, how to generate $u'(x,t)$, $v'(x,t)$, $w'(x,t)$?
 Approach: choice of the triple decomposition proposed by

ussain and Reynolds (1970)
 $u'(t) = u'_{p}(t) + u'_{s}(t)$ (assuming that $\overline{u'_{p}u'_{s}} = 0$)

Question: how can \overline{u} \overline{u} \overline{u} \overline{u} \overline{d} $\overline{$ (*x, t*), *w*'(*x, t*)?

composition proposed by

(assuming that $\overline{u_p u_s} = 0$)

(*t*) be generated?

te easy, but $u_s(t)$ is not.

At the inlet, how to generate $u_s(x,t)$?

Precursor simulation: perform a separate simulation and store the results in a plane that will be used to feed the current simulation (computationaly expansive, simple geometries only) $u'_{s}(x,t)$?
ion and store the result
utationaly expansive, si

Recycling on the fly: select a plane at some downstream section in the current simulation and regularly reinject the (scaled) values at the inlet (Simple geometries, absence of strong pressure gradients, pb with recycling frequency for aeroacoustics).

Synthetic turbulence generation (STG): white noise, stochastic differential equations, digital filtering, synthetic eddy method (the artificial nature of turbulence leads to long adaptation length, but can be improved).

Volumic forcing: add a force in the momentum equation (efficient when combined with STG)

Vortex generating devices: simple but provide quite long adaptation length.

(See Shur et al., Flow Turbulence and Combustion 93:63-92 (2014) and the references therein for a more complete overview)

Example of choice of STG: the synthetic eddy method

Eddy method (SEM): direct injection at the inlet plane of analytically defined structures that reproduce to some extent the coherent structures of the turbulent flow.

References

SEM Basic form: (Jarrin *et al.*, *Int. J. Heat Fluid Fl.*, 2006), (Jarrin, *PhD of the University of Manchester*, 2008), (Jarrin *et al.*, *Int. J. Heat Fluid Fl.*, 2009) **SEM for wall-bounded flows:** (Pamiès *et al., Phys. Fluids, 2009)* **Divergence free SEM** *(*Poletto *et al., Flow, Turbulence and Combustion,* 2013*)*

Experiment – simulation: example of connection to develop unsteady inlet BC for DNS with the SEM (inlet of MAVERIC rig)

Measurements: provide some of the targeted values for the SEM

⁵¹ Pascal Bruel – Universidad Nacional de Córdoba – 30 March 2017

Incoprorating a STG approach into a compressible Euler solver: the ATCBC method at a subsonic inlet (Moguen et al., 2014)

Consider a one-dimensional flow governed by the Euler equation for simplicity, thus the temporal rates of change of the wave amplitudes are given by:

Encoprorating a STG approach into a compressible Euler solver: the ATCBC method at a subsonic inlet (Moguen et al., 2014)

\nConsider a one-dimensional flow governed by the Euler equation for simplicity, thus the temporal rates of change of the wave amplitudes are given by:

\n
$$
L_1 = (\nu - c)(\frac{1}{\rho c}\partial_x p - \partial_x \nu) \qquad \text{upstream travelling acoustic wave}
$$
\n
$$
L_2 = \nu(\partial_x \rho - \frac{1}{c^2}\partial_x p) \qquad \text{entropy wave}
$$
\n
$$
L_3 = (\nu + c)(\frac{1}{\rho c}\partial_x p + \partial_x \nu) \qquad \text{downstream travelling acoustic wave}
$$
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These *Li*'s satisfy the following LODI system at the inlet:

$$
\partial_t \rho + \frac{\rho}{2c} (L_1 + L_3) + L_2 = 0 \quad (1a)
$$

$$
\partial_t v + \frac{1}{2} (L_3 - L_1) = 0 \quad (1b)
$$

$$
\partial_t p + \frac{\rho c}{2} (L_1 + L_3) = 0 \quad (1c)
$$

Subsonic inlet (1D): Two Li's have to be imposed.

Main hypothesis (**H1**) **behind the ATCBC method to avoid reflection**: at the inlet, the gap (if any) between the targeted velocity v † coming from the STG process and the current one v is solely attributed to the upstream travelling acoustic wave.

Methodology to check the proposal coherence at low Mach: two-scale asymptotic analysis of the LODI system.

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Incoprorating a STG approach into a compressible solver: the ATCBC method at a subsonic inlet

From (1b), **H1** is enforced by assuming that:

$$
\partial_t (\nu - \nu^{\dagger}) + \frac{1}{2} (0 - L_1) = 0
$$

which is achieved by imposing that: $-2 \partial v^{\dagger} = L$,

compressible solver: the ATCBC method
orced by assuming that:
 $(0 - L_1) = 0$
 $-2 \partial_t v^{\dagger} = L_3$
i is derived by 1) assuming a frozen
asymptotically expanding L_2 which
0). The remaining boundary condition is derived by 1) assuming a frozen turbulence hypothesis (H2) and 2) asymptotically expanding L_2 which finally yields $L₂ = 0$ (at order -1 and 0).

Example of different tests when setting-up the ATCBC method

Harmonic inlet signal

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Recent Cagire references related to low/all Mach number flow simulation issues

- 1. Dellacherie, S., Jung, J., Omnes, P., Raviart, P.-A. (2016) *Construction of modified Godunov type schemes accurate at any Mach number for the compressible Euler system*, Mathematical Models and Methods in Applied Sciences, Vol. 26, No. 13, pp. 2525–2615.
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- 3. Delmas, S. (2015) « *Simulation numérique directe d'un jet en écoulement transverse à bas nombre de Mach en vue de l'amélioration du refroidissement par effusion des chambres de combustion aéronautiques* », PhD thesis, Pau and Pays de l'Adour University, France.
- 4. Moguen, Y., Bruel, P., Dick, E. (2015a) "*Solving low Mach number Riemann problems by a momentum interpolation method*", Journal of Computational Physics, Vol. 298, pp. 741-746.
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- 7. Moguen, Y**.,** Bruel, P., Dick, E. (2013) "*Semi-implicit characteristic-based boundary treatment for acoustics in low Mach number flows*", Journal of Computational Physics, Vol. 255, pp. 339- 361.
- 8. Moguen, Y., Dick, E., Vierendeels, J., Bruel, P. (2013) "*Pressure-velocity coupling for unsteady low Mach number flow simulations: an improvement of the AUSM+-up scheme",* Journal of Computational and Applied Mathematics, Vol. 246, pp. 136-143.
- 9. Moguen, Y., Kousksou, T., Bruel, P., Vierendeels, J. and Dick, E. (2012) "*Pressure-velocity coupling allowing acoustic calculation in low Mach number flow*", Journal of Computational Physics, Vol. 231, pp. 5522-5541.