

# Magneto-Mechanical Model of Passive Magnetic Axial Bearings versus the Eccentricity Error, Part I: Physical Mathematical Model

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**Abstract** — In this paper we illustrate a particular analytical numerical model of passive magnetic bearings with axial magnetization. The approach is based on the magnetic charges method. This method avoids the utilization of the finite element analysis. In relation to the system geometry, we find explicit formulations for computing magnetic fields by simple numerical integrations. A detailed magnetostatic model is developed and the nonlinearity of the magnetization vector  $\mathbf{M}$  of the ring magnets can be considered by a very simple modification of the equations illustrated. The equations can be immediately implemented in a mathematical software and only few minutes are sufficient to obtain the results.

**Index Terms** — Levitation, magnetic bearings, magnetostatic field, natural frequencies, stiffnesses.

## I. INTRODUCTION

The magnetic levitation allows the suspension of one object above another without the two coming into contact. There are several studies and applications of this phenomenon [1], [2] and one of the best known is represented by passive magnetic bearings [3-6]. Generally these bearings can be of two types depending on the direction of polarization of the rings: axial or radial. In both cases, the forces that keep the rings separate are repulsive. Therefore, the rings of these bearings are arranged with the same poles facing each other. The value of these repulsive forces depends on the air gap between the facing surfaces. The air gap changes as a function of the applied forces. Consequently, it is possible to define a bearing stiffness which varies depending on the magnitude of the load applied and/or by the mutual position of the rings. Since the rotating rings of the magnetic bearings are always keyed to a shaft on which other elements are also fixed, an elastic system characterized by a certain stiffness and mass is defined. Therefore, we can evaluate the natural vibration frequencies of this mechanical system. These frequencies depend on the stiffness and mass suspended by the magnetic levitation. Since the stiffness changes with the

mutual position of each pair of facing rings, the stiffness and the natural frequency of the system vary versus the applied load. Thus, in general, with regard to each stationary working condition of the magnetic bearings, a natural frequency of the system is fixed. In this respect, we have developed a model based on magnetic charges to evaluate stiffnesses and natural frequencies of a magnetic levitation system with a passive axial magnetic bearing. We note that the same procedure can be easily extended to calculate the above mentioned stiffness and frequencies also for passive radial magnetic bearing.

## II. CONFIGURATION OF THE SYSTEM

Figure 1 shows the case study. The polarized ring A is fixed. The moving ring of the bearing is denoted by B. The two rings have the facing surfaces polarized with the same pole. The polarized ring B can rotate around its own axis with a certain angular velocity  $\omega$  and is positioned at a distance  $t$  from ring A. Therefore  $t$  is the air gap of the bearing. This air gap can also be considered as a translation degree of freedom of the system. The axes of the two rings are parallel but, in general not coaxial. An eccentricity  $e$  is defined:  $e$  represents a coaxiality error. The ring B supports a mass  $m$  whose value is equal to the sum of all the masses rigidly integral with the same ring B. The vertical force  $\mathbf{F}$  is the axial force applied to the bearing. The dashed segments  $a$  and  $b$  represent the two circumferences that pass through the section centers of gravity of the polarized rings. The sections of these rings have been considered to be identical for both rings A and B. The shape of the sections is rectangular. Figure 2 illustrates the magnetization vector  $\mathbf{M}$  of A and B. The  $\mathbf{M}$  direction is defined by different values of the angle  $\alpha$ . Three cases have been considered:  $\alpha = 90, 60, 30$  degrees. The discrete variability of the angle  $\alpha$  has only been considered for illustrating a general procedure to obtain the equations of the field and the forces when the magnetization  $\mathbf{M}$  depends on the same  $\alpha$  and possibly on the radius. For simplicity, such procedure is illustrated by fixing the module of  $\mathbf{M}$  to a constant value. Moreover, its direction does not change when the planes  $\pi_1$  and  $\pi_2$ ,

to which  $\mathbf{M}$  belongs, radially move towards or away from the respective axes of the magnets A and B (see Fig. 2). If  $\mathbf{M}$  depends on  $\alpha$  and the radius  $r$  (the distance from the axes of the magnets), in all the integrals indicated in the following Section III,  $\mathbf{M}$  and the other quantities versus  $\alpha$  and  $r$  must remain under the integral sign (in this case the magnetization model is not linear). Moreover, also the volume charge density  $\rho_M(\mathbf{P})$  defined in the generic point P (end of the vector  $\mathbf{P}$ ) of the permanent magnets has to be considered. In the case study ( $\mathbf{M}$  and  $\alpha$  are constants), since the magnetization model is linear, the inclined magnetization can be decomposed in an axial and a circular component independent of  $\alpha$  and  $r$ . The circular component defines a flux inside the magnet and does not generate any external magnetic field outside the same magnet. Therefore, the circular components of  $\mathbf{M}$  of the two polarized rings cannot interact since they produce no field and force outside the magnets. This consideration will be also illustrated by the numerical examples.

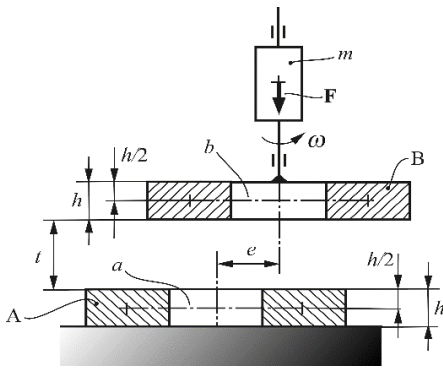


Fig. 1. Axially polarized rings with eccentricity  $e$ .

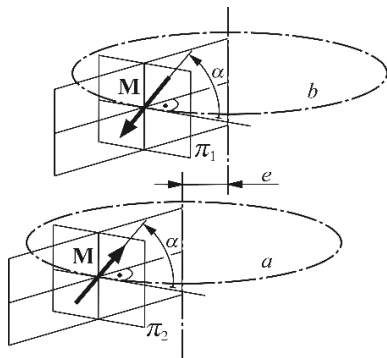


Fig. 2. Direction of the magnetization vectors  $\mathbf{M}$  in the two polarized rings.

### III. EVALUATION OF THE MAGNETIC FIELD

The calculation of the levitation forces has been performed by using the magnetostatic model and the

magnetic charge method [7-9]. The surface charge density  $\sigma_M(\mathbf{P})$  and the volume charge density  $\rho_M(\mathbf{P})$ :

$$\sigma_M(\mathbf{P}) \equiv \mathbf{M}(\mathbf{P}) \cdot \hat{\mathbf{n}}, \quad (1)$$

$$\rho_M(\mathbf{P}) \equiv -\nabla \cdot \mathbf{M}(\mathbf{P}), \quad (2)$$

were considered. This method can be considered a valid alternative to the finite element method that is often utilized [10], [11]. As a matter of fact, the time computation and the accuracy of the results can improve, even though an analytical formulation is necessary.

#### A. Surface charge density $\sigma_M(\mathbf{P})$ for the polarized rings A and B

In Fig. 3 an infinitesimal element of the magnet A is illustrated. The point P represents the center of the element. The element shows six infinitesimal faces denoted by  $dS_1, dS_2, \dots$ , and  $dS_6$ . The correspondent normal versors are  $\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \dots$ , and  $\hat{\mathbf{n}}_6$ . The expressions of the versors can be suitably expressed versus the angle  $\theta$ . The magnetization vector  $\mathbf{M}(M_x, M_y, M_z)$  is applied to the point P of the infinitesimal magnet illustrated in Fig. 3. The moduli with the signs  $M_x, M_y$ , and  $M_z$  of the components of  $\mathbf{M}$  can be expressed versus the angles  $\theta$  and  $\alpha$  (see Fig. 4). By using Eq. (11), we obtain the six surface charge densities  $\sigma_{MAi}$  relative to the surfaces  $dS_i$  ( $i=1, 2, \dots, 6$ ) of the infinitesimal magnet A:

$$\sigma_{MA1} = M \sin \alpha, \quad (3)$$

$$\sigma_{MA2} = -M \sin \alpha, \quad (4)$$

$$\sigma_{MA3} = -M \cos \alpha, \quad (5)$$

$$\sigma_{MA4} = M \cos \alpha. \quad (6)$$

For the surfaces  $dS_5$  and  $dS_6$ ,  $\sigma_{MA5}$  and  $\sigma_{MA6}$  are equal to zero ( $\mathbf{M}$  is always perpendicular to the normal straight line of the surfaces  $dS_5$  and  $dS_6$ ). The surface charge densities  $\sigma_{MBi}$  of the polarized ring B are obtained by changing the sign of the  $\sigma_{MAi}$ .

#### B. Volume charge density $\rho_M(\mathbf{P})$ for the polarized rings A and B

By observing Figs. 3 and 4 we obtain:

$$M_x = -M \cos \alpha \frac{p_y}{\sqrt{p_x^2 + p_y^2}}, \quad (7)$$

and

$$M_y = M \cos \alpha \frac{p_x}{\sqrt{p_x^2 + p_y^2}}, \quad (8)$$

where  $p_x, p_y$ , and  $p_z$  are the components of the vector  $\mathbf{P}$  that identifies the point P. By using Eq. (2) and by deriving

Eqs. (7) and (8) with respect to  $p_x$ , and  $p_y$ , respectively, we obtain:

$$\frac{\partial M_x}{\partial p_x} = M \cos \alpha \frac{p_x p_y}{(p_x^2 + p_y^2)^{3/2}}, \quad (9)$$

and

$$\frac{\partial M_y}{\partial p_y} = -M \cos \alpha \frac{p_x p_y}{(p_x^2 + p_y^2)^{3/2}}. \quad (10)$$

The partial derivative  $\partial M_z / \partial p_z$  is equal to zero. By substituting Eqs. (9), (10), and  $\partial M_z / \partial p_z = 0$  in Eq. (2), we note that volume charge density  $\rho_{MA}(\mathbf{P})$  is always equal to zero. For the magnet B we obtain the same result, i.e.,  $\rho_{MB}(\mathbf{P}) = 0$ , whatever the value of  $\alpha$  is.

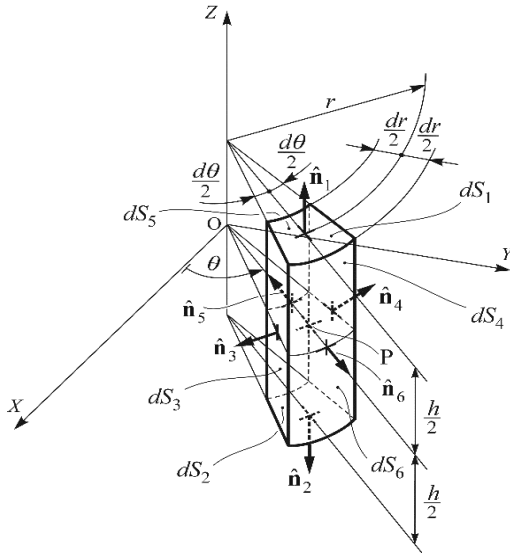


Fig. 3. Infinitesimal element of the polarized ring A with versors outgoing from the surfaces.

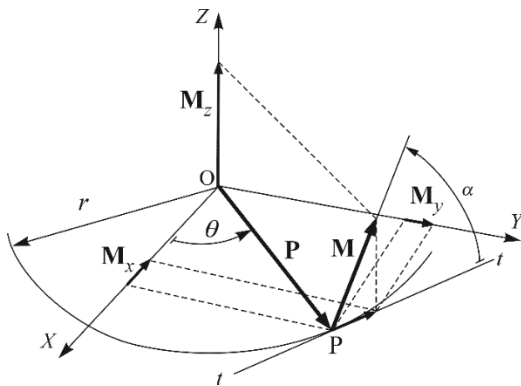


Fig. 4. Magnetization vectors components  $\mathbf{M}_x$ ,  $\mathbf{M}_y$ , and  $\mathbf{M}_z$ , in the generic point P of the magnet A (see Fig. 3).  $t-t$  is the tangent to the circumference of radius  $r$  in P ( $\mathbf{P}$  is always perpendicular to  $t-t$ ).

### C. Surfaces $dS_1, dS_2, \dots$ , and $dS_4$

In order to evaluate the magnetic induction generated by the magnet A and the forces/moments applied to the magnet B, since  $\sigma_{MA5}$ ,  $\sigma_{MB5}$ ,  $\sigma_{MA6}$ , and  $\sigma_{MB6}$  are equal to zero, we evaluate the only expressions of the surfaces  $dS_1, dS_2, \dots$ , and  $dS_4$ . By observing Fig. 5, we can define the expressions of the infinitesimal surfaces  $dS_i$  with  $i=1, 2, \dots, 4$  versus  $d\theta$ ,  $dr$  and  $h$ . Denoting by  $\mathbf{P}_i(p_{xi}, p_{yi}, p_{zi})$  the vectors that identify the centers  $P_i$  of the above-mentioned surfaces  $dS_i$ , we obtain the expression of the components  $p_{xi}$ ,  $p_{yi}$ , and  $p_{zi}$  in function of  $\theta$ ,  $r$  and  $h$ .

### D. Evaluation of the magnetic induction $\mathbf{B}(\mathbf{P}')$

In order to evaluate forces and moments applied to the magnet B, four contributions  $\mathbf{B}_1(\mathbf{P}')$ ,  $\mathbf{B}_2(\mathbf{P}')$ ,  $\dots$ ,  $\mathbf{B}_4(\mathbf{P}')$  of the magnetic induction have to be considered.  $\mathbf{P}'$  is the vector that identifies the point where the magnetic induction will be computed is given by [4]:

$$\mathbf{B}_i(\mathbf{P}') = \frac{\mu_0}{4\pi} \int_{S_i} \frac{\sigma_{MA_i}(\mathbf{P})(\mathbf{P}' - \mathbf{P})}{|\mathbf{P}' - \mathbf{P}|^3} dS_i, \quad (11)$$

with  $i=1, 2, \dots, 4$ .  $\mu_0$  is the free space permeability. The volume contribution to  $\mathbf{B}_i(\mathbf{P}')$  is always equal to zero because  $\rho_{AM}(\mathbf{P}) = 0$ . By substituting Eqs. (3)-(6) and the expressions of  $p_{xi}$ ,  $p_{yi}$ , and  $p_{zi}$  versus  $\theta$ ,  $r$  and  $h$  in Eq. (11), we achieve the components  $B_{xi}(\mathbf{P}')$ ,  $B_{yi}(\mathbf{P}')$ , and  $B_{zi}(\mathbf{P}')$  of  $\mathbf{B}_i(\mathbf{P}')$ . For example, the components  $B_{xi}(\mathbf{P}')$  is the following:

$$B_{xi}(\mathbf{P}') = \frac{\mu_0 M \sin \alpha}{4\pi} \int_0^{2\pi} \int_{r_i}^{r_e} \frac{(p'_x - r \cos \theta) r}{\left[ (p'_x - r \cos \theta)^2 + (p'_y - r \sin \theta)^2 + (p'_z - h/2)^2 \right]^{3/2}} d\theta dr. \quad (12)$$

The other components have a similar formulation. The components  $B_x(\mathbf{P}')$ ,  $B_y(\mathbf{P}')$  and  $B_z(\mathbf{P}')$  of the resultant magnetic induction  $\mathbf{B}(\mathbf{P}')$  in the generic point  $\mathbf{P}'$  of the B magnet surfaces are obtained by adding the correspondent components  $B_{xi}(\mathbf{P}')$ ,  $B_{yi}(\mathbf{P}')$ , and  $B_{zi}(\mathbf{P}')$  with  $i=1, 2, \dots, 4$ . Since the sign of  $B_{xi}(\mathbf{P}')$ ,  $B_{yi}(\mathbf{P}')$ , and  $B_{zi}(\mathbf{P}')$  is opposite to the sign of  $B_{xi+1}(\mathbf{P}')$ ,  $B_{yi+1}(\mathbf{P}')$ , and  $B_{zi+1}(\mathbf{P}')$ , respectively, when  $i$  is equal to 3 and 4 and the corresponding moduli are equal to each other, we have:

$$B_x(\mathbf{P}') = B_{x1}(\mathbf{P}') + B_{x2}(\mathbf{P}'), \quad (13) \quad \text{and analogous expressions of } B_y(\mathbf{P}') \text{ and } B_z(\mathbf{P}').$$

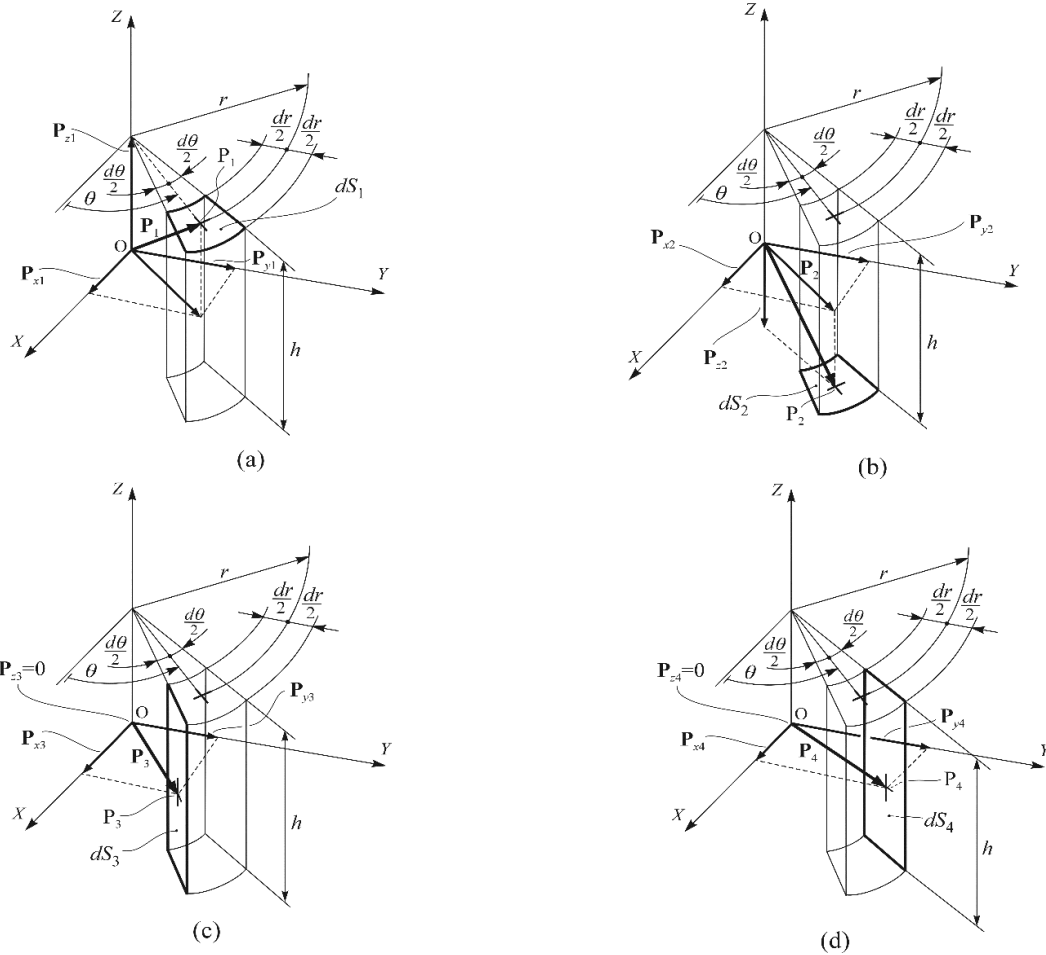


Fig. 5. Evaluation of: (a)  $dS_1$ , (b)  $dS_2$ , (c)  $dS_3$ , and (d)  $dS_4$  with the relative  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$  centres (see also Fig. 3) in the magnet A.

#### IV. EVALUATION OF FORCES AND MOMENTS APPLIED TO THE POLARIZED RING B

With reference to Fig. 6, the infinitesimal resultant force  $d\mathbf{F}$  applied from the magnet A to a generic infinitesimal element of the magnet B is obtained by adding four force  $d\mathbf{F}_i$  ( $i=1, 2, \dots, 4$ ):

$$d\mathbf{F} = d\mathbf{F}_1 + d\mathbf{F}_2 + d\mathbf{F}_3 + d\mathbf{F}_4. \quad (14)$$

Each of them is applied to the correspondent surfaces  $dS_i$  that define the infinitesimal element of the polarized ring B (see Fig. 6). We observe that these surfaces have the same expressions of the correspondent surfaces defined for the magnet A. Since the surfaces charges densities  $\sigma_{MB5}$  and  $\sigma_{MB6}$  are equal to zero, the surfaces  $dS_5$  and  $dS_6$  relative to the ring B do not give any contribution to  $d\mathbf{F}$ . By denoting  $\mathbf{P}'_i(p'_{xi}, p'_{yi}, p'_{zi})$  the vectors that identify the centers  $P'_i$  of the above-mentioned surfaces  $dS_i$  ( $i=1, 2, \dots, 4$ ), we can define the expressions of  $p'_{xi}$ ,

$p'_{yi}$ , and  $p'_{zi}$  versus  $p_{xi}$ ,  $p_{yi}$ ,  $p_{zi}$ ,  $e$ , and  $t$  (see Fig. 6). The forces  $d\mathbf{F}_i$  are applied to the points  $P'_i$ . The evaluation of  $d\mathbf{F}_i$  is performed by the following relation:

$$d\mathbf{F}_i = \sigma_{Bi}(\mathbf{P}'_i)\mathbf{B}(\mathbf{P}'_i) dS_i, \quad (15)$$

where  $i=1, 2, \dots, 4$ . By using Eq. (13) and the analogous expressions of  $B_y(\mathbf{P}')$  and  $B_z(\mathbf{P}')$ , integrating Eq. (15), we compute the moduli with the signs  $F_{xi}$ ,  $F_{yi}$ , and  $F_{zi}$  of the  $\mathbf{F}_i$  components. For example, the components  $F_{x1}$  and  $F_{x3}$  are the following:

$$F_{x1} = -M \sin \alpha \int_0^{2\pi} \int_{r_i}^{r_e} B_x(\mathbf{P}'_1) r d\theta dr, \quad (16)$$

$$F_{x3} = M \sin \alpha \int_{r_i}^{r_e} B_x(\mathbf{P}'_3) h dr. \quad (17)$$

The other components have an analogous formulation. Therefore, by adding the four forces  $\mathbf{F}_i$  ( $F_{xi}$ ,  $F_{yi}$ ,  $F_{zi}$ ) we obtain the resultant force applied to the ring B.

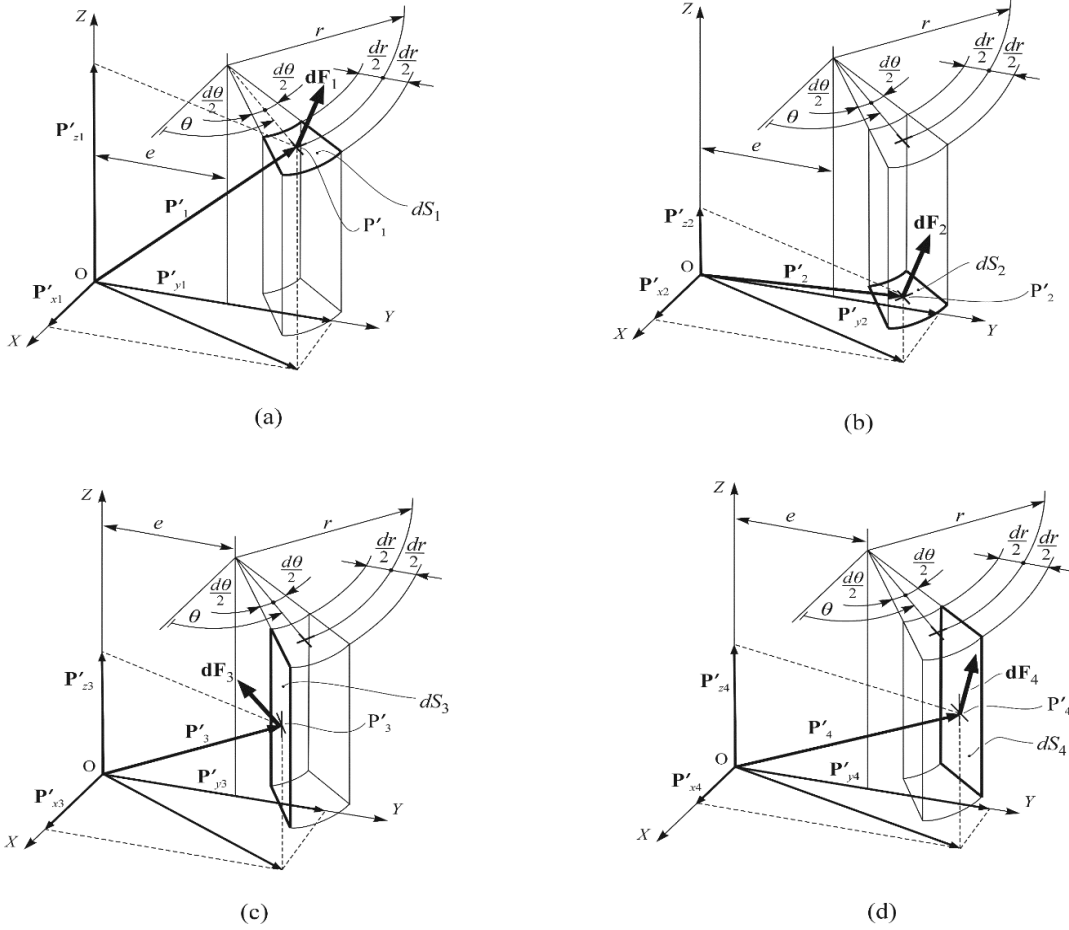


Fig. 6. Infinitesimal forces: (a)  $d\mathbf{F}_1$ , (b)  $d\mathbf{F}_2$ , (c)  $d\mathbf{F}_3$ , and (d)  $d\mathbf{F}_4$  applied to the corresponding centres  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$  of the surfaces  $dS_1$ ,  $dS_2$ ,  $dS_3$ , and  $dS_4$  of the infinitesimal element of the magnet B.

## V. EVALUATION OF THE TORQUE APPLIED TO THE POLARIZED RING B

In order to check the correctness of the physical mathematical model, it is suitable to verify the law of energy conservation. This check can be performed by computing the moment component  $\tau_z$  along the axis Z applied from the ring A to the ring B.  $\tau_z$  must always be equal to zero, whatever the values of  $\alpha$  and  $e$  are. If this condition is not met, the law of energy conservation is not verified and the model is wrong (the ring B spontaneously rotates). The computation of  $\tau_z$  is performed by integrating the following relation:

$$d\tau_z = d\tau_{z1} + d\tau_{z2} + d\tau_{z3} + d\tau_{z4}, \quad (18)$$

where

$$d\tau_{zi} = d\mathbf{F}_i \times \mathbf{P}'_i, \quad (19)$$

and  $i=1, 2, \dots, 4$ .  $d\tau_{zi}$  represents the moment around the axis Z generated from the force  $d\mathbf{F}_i$  applied to the

corresponding surface  $dS_i$  of the infinitesimal element of the ring B. Therefore, by Eqs. (18) and (19) we obtain the following modulus with sign of  $\tau_z$ :

$$\begin{aligned} \tau_z = M \sin \alpha \int_0^{2\pi} \int_{r_i}^{r_e} r [B_x(r \cos \theta, r \sin \theta + e, \frac{h}{2} + h + t)(r + e) \sin \theta - \\ B_y(r \cos \theta, r \sin \theta + e, \frac{h}{2} + h + t) r \cos \theta] - \\ r [B_x(r \cos \theta, r \sin \theta + e, \frac{h}{2} + t)(r + e) \sin \theta - \\ B_y(r \cos \theta, r \sin \theta + e, \frac{h}{2} + t) r \cos \theta] d\theta dr. \end{aligned} \quad (20)$$

In relation to the law of energy conservation the value of  $\tau_z$  computed by Eq. (20) must be equal to zero, whatever the angle  $\alpha$  of the magnetization  $\mathbf{M}$  is (see Fig. 2). Eq. (20) has been numerically evaluated and in Part II we briefly discuss this aspect. The values of  $\tau_z$  versus  $\alpha$  and  $e$  obtained are very small and confirm the

previous statement.

## VI. AXIAL/RADIAL STIFFNESSES AND NATURAL FREQUENCIES

### A. Stiffnesses

In general, the computation of the stiffness  $K$  is based on the following relation:

$$K = \frac{\partial F(p)}{\partial p}, \quad (21)$$

where  $F(p)$  is the force versus the parameter  $p$  that defines the degree of freedom (DOF) along which the stiffness is computed. In the present study we evaluate the axial stiffness  $K_t$  along the axis  $Z$  versus the air gap  $t$ :

$$K_t = \frac{\partial F_z(t)}{\partial t}, \quad (22)$$

and the radial stiffness  $K_e$  along the axis  $Y$  where the eccentricity  $e$  is defined:

$$K_e = \frac{\partial F_y(e)}{\partial e}. \quad (23)$$

The evaluation of  $K_e$  can be interesting also when we study an axial magnetic bearing. As a matter of fact,  $K_e$  has to be considered together with the other radial stiffnesses of the two radial bearings keyed on the shaft. The dynamic behaviour of the system also depends on  $K_e$ .

### B. Natural frequencies

The natural frequencies of a system depend on its mass and stiffness. From the modelization point of view, the number of these frequencies is equal to the number of degrees of freedom of the model. In relation to the device schematized in Fig. 1, we can consider various models. The choice of the model is strictly connected to the dynamic behaviour of the real system that we want study. If a rigid body schematization of the real system is acceptable and the radial bearings of the vertical shaft have a very high radial stiffness, we can modelize the structure by one degree of freedom (DOF) model (the DOF along the axis  $Z$ ). If the flexural stiffness of the shaft is not high and there are radial excitation forces, it is necessary to introduce new DOFs. Moreover, also if the radial stiffness of the radial bearing is not high, other radial DOFs associated with these bearing have to be considered. We observe that the system can become very complex. The vibrational behavior will depend on nonlinear magnetic stiffnesses and also small chaotic precessional motions can rise. In a demanding practical application, this kind of motions can be due to the alignment errors of the shaft (concentricity, circularity, perpendicularity, plumb, straightness, see Fig. 7 [12]). In the present study we can limit ourselves to two simple cases. The first one considers a model with a DOF only along the axis  $Z$ . In the second case the model has a DOF

only along the axis  $Y$ . The two models are illustrated in Figs. 8 (a) and 8 (b), respectively. The model of Fig. 8 (a) can be used to study the dynamical behaviour of a device where all the stiffnesses are much higher than the stiffness  $K_t$  defined by Eq. (22). In Fig. 8 (a)  $m_{tot}$  represents the total suspended mass. Conversely, Fig. 8 (b) shows a model to study a system with a shaft that can only horizontally translate. By this schematization we again assume that the stiffness of all parts of the device are very high with respect to the radial stiffness  $K_e$  furnished by Eq. (23). In this case the translation DOF could be due to the radial clearances of the radial bearings. These clearances would allow a small horizontal translation of the rigid shaft. Therefore, the shaft horizontally translates during its rotation. Small rotations around the centres of the bearing could also occur. Nevertheless, if the flexural stiffness of the shaft is high, in general the influence of the corresponding rotational DOFs on the vibration behavior is negligible. With reference to this hypothesis and overall for simplicity, we can consider the simplified model illustrated in Fig. 8 (b). The system would normally be studied by using complex modelizations based on rotor dynamics (see, for example, [13]). The four masses indicated in Fig. 8 (b) represent the point masses to modelize, for example, the rotating mass of a hydrounit for electric generation (see Fig. 9 [12]). If we assume to substitute the oleodynamic thrust bearing (see particular C in Fig. 9) with a passive magnetic axial bearing (see Fig. 1), we can suitably fix the values of  $m_1, m_2, \dots$ , and  $m_4$  versus the masses of the various rotating parts of the hydrounit [mass of the thrust bearing, shafts, rotor, turbine (not illustrated), etc.]. Therefore, the mass of the polarized ring B indicated in Fig. 1 contributes to defining the mass  $m_2$  shown in Fig. 8 (b). With reference to the two models illustrated in Fig. 8 we evaluate the corresponding natural angular frequencies  $\omega_{em_h}$ ,  $\omega_{em_{tot}}$ ,  $\omega_{tm_h}$ , and  $\omega_{tm_{tot}}$  of the system by the following relations:

$$\omega_{em_h} = \sqrt{\frac{K_e(e)}{m_1 + m_2 + m_3 + m_4}}, \quad (24)$$

$$\omega_{em_{tot}} = \sqrt{\frac{K_e(e)}{m_{tot}}}, \quad (25)$$

$$\omega_{tm_h} = \sqrt{\frac{K_t(t)}{m_1 + m_2 + m_3 + m_4}}, \quad (26)$$

$$\omega_{tm_{tot}} = \sqrt{\frac{K_t(t)}{m_{tot}}}. \quad (27)$$

As soon as  $m_{tot}, m_1, m_2, \dots$ , and  $m_4$  have been fixed and the stiffnesses  $K_t$  and  $K_e$  are known [see Eqs. (22) and (23)], we can compute the natural angular frequencies

versus the air gap  $t$  and the eccentricity  $e$  [when we evaluate  $K(t)$  we fix a certain value of  $e$  and vice versa].

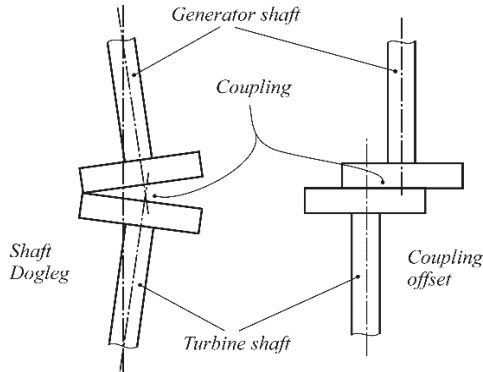


Fig. 7. Alignment errors of a turbine and generator shafts of a hydrounit [12].

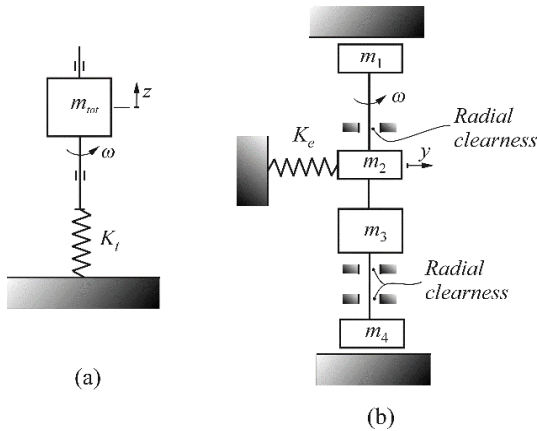


Fig. 8. Simplified physical model of the system with one (a) vertical and (b) horizontal DOF.

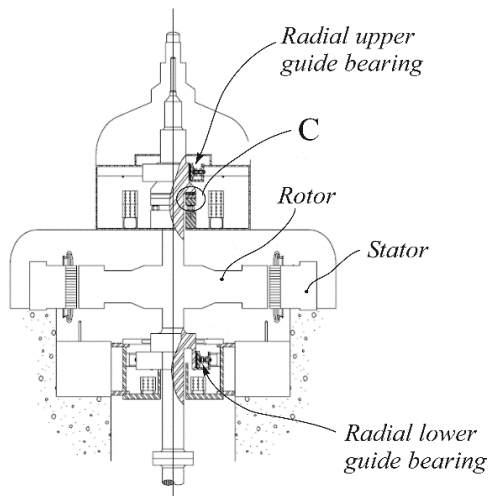


Fig. 9. A typical vertical disposition rotor/stator with thrust bearing C of a hydrounit for electric generation [12].

**VII. CONCLUSION**

A detailed formulation for evaluating forces, moments, stiffnesses and natural frequencies of a thrust magnetic bearing has been presented. Equations for checking the correctness of the analysis based on the magnetic charges method was considered. A mechanical model referred to a vertical disposition of a hydrounit for electric generation for performing the numerical calculations illustrated in Part II has been developed.

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#### **Supplementary materials**

All the details of the analytical formulations can be requested to the author at [muscia@units.it](mailto:muscia@units.it).



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