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VIBRATION CONTROL IN PERIODIC STRUCTURES BY PIEZOELECTRIC RL SHUNTS

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Key words: Waves, Periodic structure, Damping, Piezoelectric shunt, Modal analysis.

Summary. Within the framework of periodic structures, the calibration of RL shunted piezoelectric inclusions is investigated with respect to maximum damping of a particular wave form. A finite element setting is assumed, with local shunted inclusions inside the unit cell. The effect of the shunts is represented for a targeted wave form, characterized by its short-circuited eigenvalue problem and two correction coefficients, representing the influence from residual modes, not addressed by the supplemental damping. Calibration formulae are finally derived for the shunt inductance L and resistance R. The presentation contains dispersion diagrams and vibration amplitude curves for the optimally calibrated RL shunt system in a 1-D periodic structure with local piezoelectric inclusions.

1 PERIODIC STRUCTURE

The equilibrium equation for a structure may be determined by vanishing virtual work in a finite element formulation with element displacements contained in the element displacement vector $\mathbf{u}_e$. The element virtual work can then be expressed as

$$\delta W_e = \delta \mathbf{u}_e^T \Big([-\omega^2 \mathbf{m}_e + \mathbf{k}_e] \mathbf{u}_e + \mathbf{b}_e f_e - \mathbf{f}_e \Big) \tag{1}$$

with element mass matrix $\mathbf{m}_e$, stiffness matrix $\mathbf{k}_e$, participation vector $\mathbf{b}_e$, piezoelectric force f_e and external load $\mathbf{f}_e$. Wave propagation for a periodic structure is commonly analyzed by representing the spatial dependence of the wave via the normalized wavenumber $\gamma \ell_e$ in the factorized form

$$\mathbf{u}_e = \mathbf{E}(\gamma \ell_e) \mathbf{A}_e \mathbf{u} \tag{2}$$

Here $\mathbf{A}_e$ extracts the element displacements from the global displacement vector $\mathbf{u}$, while the wavenumber dependent exponential matrix $\mathbf{E}(\gamma \ell_e)$ represents the spatial advancement of the wave through the element.

The total work is obtained by summation over the elements in the periodic structure

$$\delta W = \sum_e^{N_e} \delta W_e = \delta \mathbf{u}^T \Big((-\omega^2 \mathbf{M} + \mathbf{K}) \mathbf{u} - \mathbf{W} \mathbf{f} \Big) = 0 \tag{3}$$

introducing the global mass and stiffness matrices

$$\mathbf{M} = \sum_{e}^{N_e} (\mathbf{EA}_e)^T \mathbf{m}_e (\mathbf{EA}_e) \quad , \quad \mathbf{K} = \sum_{e}^{N_e} (\mathbf{EA}_e)^T \mathbf{k}_e (\mathbf{EA}_e) \tag{4}$$

while the piezoelectric forces are represented by the term

$$\mathbf{Wf} = \sum_{p}^{N_p} (\mathbf{EA}_p)^T \mathbf{b}_p f_p = \sum_{p}^{N_p} \mathbf{w}_p f_p, \quad \mathbf{w}_p = (\mathbf{EA}_p)^T \mathbf{b}_p \tag{5}$$

with summation over the reduced number of elements representing a piezoelectric inclusion. The equilibrium equation from (3) can then be written in homogeneous form[1]

$$(-\omega^2 \mathbf{M} + \mathbf{K})\mathbf{u} + \mathbf{Wf} = \mathbf{0} \tag{6}$$

The construction of the unit cell is shown in Fig. 1 for a unidirection rod-type structure, where the gray areas represent piezoelectric inclusions and (c) illustrates the periodicity.

2 PIEZOELECTRIC SHUNT FORCE

In (6) the vector $\mathbf{f}$ may contain electromechanical forces f_p from several piezoelectric elements within the unit cell. An electromechanical force is defined as

$$f_p = \theta V \qquad , \qquad p = 1, 2, \dots N_p. \tag{7}$$

where V is the voltage across the element nodes, while θ represents the electromechanical coupling coefficient. The piezoelectric coupling is governed by the electrical balance equation,

$$Q = -\theta \mathbf{b}_p^T \mathbf{u}_p + CV \tag{8}$$

where Q is the charge in the element, while C is the capacitance of the piezoelectric element associated with blocked strain conditions.
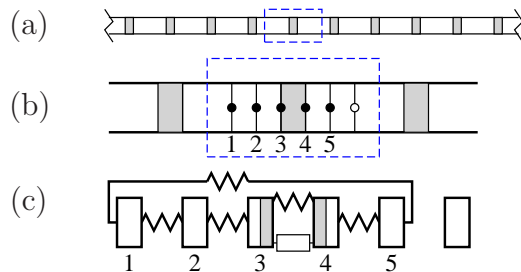

Figure 1: (a) Periodic bar with (b) unit cell and (c) representative spring-mass system.

The shunt properties are expressed by the impedance relation $V = -i\omega Z(\omega)Q$, whereby

$$\Big(1 + i\omega C Z(\omega)\Big)\mathbf{f} = \theta^2 C^{-1} i\omega C Z(\omega)\mathbf{v} \tag{9}$$

in which the piezoelectric forces are collected in the vector $\mathbf{f} = [f_1, f_2, \ldots, f_{N_p}]^T$, while

$$\mathbf{v} = \mathbf{W}^T \mathbf{u} \quad , \quad \mathbf{W} = \big[\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_{N_p}\big] \tag{10}$$

represents the displacement vector for the piezoelectric inclusions.

3 MODAL EQUATION

The free vibration properties (ω_j and $\mathbf{u}_j$) for the unit cell structure with piezoelectric inclusions, periodic boundary conditions and short-circuited electrodes is governed by

$$\big(\mathbf{K} - \omega_j^2 \mathbf{M}\big)\mathbf{u}_j = \mathbf{0} \tag{11}$$

The piezoelectric displacement vector $\mathbf{v}$ is approximated by the contribution $\mathbf{v}_r q_r$ from the targeted mode $j = r$ and supplemental flexibility from the non-resonant modes ($j \neq r$). This augmented modal representation can be written as[2,3]

$$\mathbf{v} = \mathbf{v}_r q_r - \Big(\frac{1}{k_r \kappa_r'} - \frac{1}{\omega^2 m_r \mu_r'}\Big)\mathbf{f} \tag{12}$$

where $\mathbf{v}_r = \mathbf{W}^T \mathbf{u}_r$, while κ_r' and μ_r' represent apparent flexibility and inertance ratios associated with the influence of residual modes. Elimination of $\mathbf{v}$ in (9) gives

$$\Big[1 + \frac{\theta^2}{Ck_r}\frac{1}{\kappa_r'} + \frac{1}{i\omega C Z(\omega)} - \frac{\theta^2}{Ck_r}\frac{\omega_r^2}{\omega^2}\frac{1}{\mu_r'}\Big]\mathbf{v}_r^T \mathbf{f} = \frac{\theta^2}{Ck_r}k_r \nu_r^2 q_r \tag{13}$$

in which the modal participation factor is $\nu_r = \sqrt{\mathbf{v}_r^T \mathbf{v}_r}$. In (13) the flexibility correction modifies the apparent electromechanical coupling, while the inertance correction in the last term represents an artificial inductance that alters the shunt frequency. The corresponding structural equation of motion follows from a modal representation of (6),

$$(-\omega^2 m_r + k_r)q_r + \mathbf{v}_r^T \mathbf{f} = 0 \tag{14}$$

The two coupled equations (13) and (14) comprise an eigenvalue problem, governing the wave propagation in the unit cell. The residual mode correction coefficients κ_r' and μ_r' can be determined by introducing a pure inductive shunt $Z(\omega) = i\omega L$ for each of the n_p inclusions. Hereby, the structural frequency ω_r splits into $n_p + 1$ frequencies around ω_r, of which ω_A and ω_B are associated with structural vibration modes, while the remaining modes are highly damped. The two frequencies determine intermediate coefficients

$$\kappa_* = \Big[1 - \Big(\frac{\omega_A}{\omega_r}\Big)^2\Big]\Big[\Big(\frac{\omega_B}{\omega_r}\Big)^2 - 1\Big] \quad , \quad \mu_* = \Big[1 - \Big(\frac{\omega_r}{\omega_B}\Big)^2\Big]\Big[\Big(\frac{\omega_r}{\omega_A}\Big)^2 - 1\Big] \tag{15}$$

which are subsequently used to determine the actual correction coefficients

$$\frac{1}{\kappa_r'} = \frac{1}{\kappa_*} - \frac{k_r}{\theta^2 C^{-1}} \quad , \quad \frac{1}{\mu_r'} = \frac{1}{\mu_*} - \frac{k_r}{\theta^2 \omega_r^2 L} \tag{16}$$

4 CALIBRATION

The desired resonant vibration damping is obtained by shunts with a resistor R and an inductor L placed in parallel in the shunt, whereby the impedance is given as

$$\frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{i\omega L} \tag{17}$$

Upon substitution, the flexibility relation (13) can be expressed in short form as

$$\left[\frac{1}{\kappa_r} + \frac{\omega_r}{i\omega\beta_r} - \frac{\omega_r^2}{\omega^2\mu_r}\right]\mathbf{v}_r^T\mathbf{f} = k_r\nu_r^2 q_r \tag{18}$$

in which the resulting stiffness, damper and inertance ratios are defined as

$$\frac{1}{\kappa_r} = \frac{k_r}{\theta^2 C^{-1}} + \frac{1}{\kappa_r'} \quad , \quad \frac{1}{\beta_r} = \frac{k_r}{\theta^2\omega_r R} \quad , \quad \frac{1}{\mu_r} = \frac{k_r}{\theta^2\omega_r^2 L} + \frac{1}{\mu_r'} \tag{19}$$

with correction coefficients κ_r' and μ_r' determined in (16).

Effective vibration control of the targeted mode $j = r$ is obtained by initially requiring that the damping should be identical in the two non-redundant wave forms. This is for the parallel RL shunt secured by the simple relation

$$\mu_r = \kappa_r \quad \Rightarrow \quad \frac{1}{\omega_r^2 CL} = 1 + \frac{\theta^2}{k_r C}\left(\frac{1}{\kappa_r'} - \frac{1}{\mu_r'}\right) \tag{20}$$

which determines the shunt inductance L. The maximum damping is then obtained at the bifurcation point in a complex root locus diagram, associated with

$$\beta_r = \sqrt{\tfrac{1}{4}\kappa_r} \quad \Rightarrow \quad \frac{1}{\omega_r CR} = \sqrt{4\frac{\theta^2}{k_r C}\left(1 + \frac{\theta^2}{k_r C}\frac{1}{\kappa_r'}\right)} \tag{21}$$

explicitly calibrating the shunt resistance R. Alternatively, the factor $\frac{1}{4}$ can be increased to $\frac{1}{2}$, whereby optimal vibration amplitude mitigation is instead obtained[2,3].

REFERENCES

[1] Høgsberg J (2017) Vibration damping in periodic 1d-structures by R-shunted piezo-electric elements. *Proceedings of SMART 2017*, 5-8 June, 2017, Madrid.

[2] Krenk S and Høgsberg J (2016) Tuned resonant mass or inerter-based absorbers: unified calibration with quasi-dynamic flexibility and inertia correction. *Proceedings of the Royal Society A - Mathematical, Physical and Engineering Sciences* 472: 20150718 (23pp). DOI: 10.1098/rspa.2015.0718.

[3] Høgsberg J and Krenk S (2017) Calibration of piezoelectric RL shunts with explicit residual mode correction. *Journal of Sound and Vibration* 386: 65-81.