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Skar, Asmus; Poulsen, Peter Noe; Olesen, John Forbes

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Asmus Skar[1], Peter Noe Poulsen[2], John Forbes Olesen[2]

[1]Section for Geotechnics and Geology, Department of Civil Engineering, Nordvej, Building 119, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark

[2]Section for Structural Engineering, Department of Civil Engineering, Brovej, Building 118, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark

e-mail: asska@ byg.dtu.dk, pnp@byg.dtu.dk, jfo@byg.dtu.dk

Asmus Skar, MSc, PhD, Postdoc, DTU. Areas of interest: Pavement mechanics, computational modelling, FEM, constitutive modelling, fracture mechanics, forward and inverse pavement analysis.

Peter Noe Poulsen, MSc, PhD, Associate Professor, DTU. Areas of interest: non-linear FEM, FELA, fracture mechanics of concrete and reinforced concrete structures, constitutive modelling.

John Forbes Olesen, MSc, PhD, Associate Professor, DTU. Areas of interest: Computational structural modelling, FEM, FELA, non-linear fracture mechanics of concrete and reinforced concrete structures.

1. Introduction

Concrete structures, e.g. concrete bridges, rigid and semi-rigid pavements, are subjected to cyclic loading from moving vehicles. This type of loading results in initiation of bending cracks in the quasi-brittle cemented material. Subsequently, these cracks propagate, which might lead to failure of the structure. The cyclic behaviour of concrete materials has mainly been studied subjected to fatigue loading in direct tensile, flexural or indirect tensile loading, see e.g. Cornelissen [1]. These types of experiments have typically been used to establish *Wöhler* type of fatigue relationships, or so-called *S-N* curves, and provide some information about the number of cycles to failure and the damage development. However, these tests do not distinguish between crack initiation and crack propagation or elastic and inelastic work.

In order to create a simple and robust modelling framework for engineering application and inverse analysis of experimental results, this paper presents a multi-scale cyclic cohesive model. The model consists of three levels, i.e. (i) fiber, (ii) hinge and (iii) beam level, respectively.

At the lowest level a fiber of concrete material including a crack is considered and a stress-mean strain relationship is established. The cyclic behaviour of the crack is incorporated into the fiber response considering the loading process below the monotonic softening curve, i.e. fatigue crack growth after crack initiation. Reduction of the crack bridging stress during cyclic loading is determined applying an energy based approach, relating damage of the cohesive potential to the accumulated work during the fatigue loading process.

The fiber stress-mean strain relationship is incorporated in a hinge as proposed by Skar et al. [2]. Hinge models have been effectively applied for studying crack growth phenomena in both plain concrete beams [3]-[4], as well as reinforced and fiber-reinforced concrete beams [5]-[7]. For the hinge, which is a finite part of the beam, a relationship between the generalized sectional forces and strains is established.

For structural analysis, the hinge model is then applied as a constitutive model in a non-linear beam element as first proposed by Olesen and Poulsen [8]. Although the underlying description of the hinge is based on the formation of discrete cracks the constitutive behaviour of the hinge is smeared (smooth). This particular feature is practical and effective as it requires no a-priori knowledge of the crack pattern. Skar et al. [9] showed that this model is able to predict the stress distribution and stiffness during crack development, resulting in a precise prediction of the crack-opening.

The presented model accounts for the material behaviour in all the cracked phases, linking the development of the fracture process zone and damage of the existing fracture process zone to the monotonic material characteristics in a rational manner. The model represent an extension and generalization compared to previous models developed by the Authors, see [10], including pre-peak initiation of cracks and more complex stress-crack opening behaviour.

2. Fiber level

The uni-axial tensile behaviour of the concrete material is modelled according to the fictitious crack model by Hillerborg et al. [11]. The linear elastic pre-crack state is described by the elastic modulus, E_c. The uni-axial tensile strength is denoted by f_t and the corresponding strain by ε_{ct}. However, as micro cracks form at a lower level than the tensile strength, the proposed model is generalized to account for the damage process during pre-peak crack initiation. This behaviour can be described by exponential or multilinear stress-crack opening relationships, e.g.

$$\sigma(w) = b_i + a_i w \qquad\qquad 0 \leq w \leq w_c \qquad\qquad (1)$$

where w is the crack opening, w_c is the final zero-stress displacement, a_i is the slope and b_i the intersection of the tangent line segment and the ordinate for a given point on softening curve as shown in Figure 1.

The present study aims at creating a simple modelling framework for engineering design purposes, and not models that exactly describes the concrete unloading-reloading hysteresis loops found from experiments, see e.g. Hordijk [12]. For simplicity unloading towards the origin is assumed as a starting point for the model development, as shown in Figure 2.
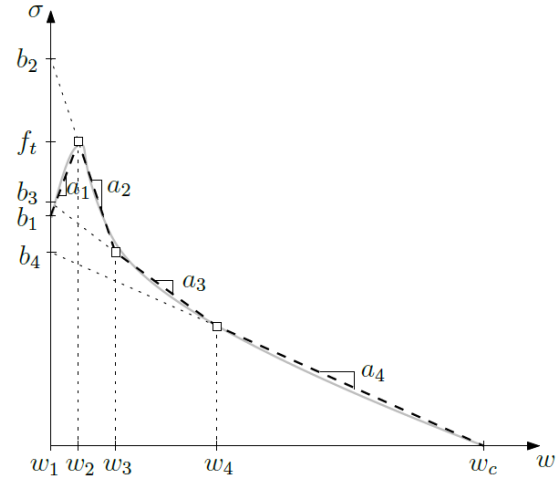
Figure 1 . Definition of parameters for softening law selected: multi-linear idealisation (black dashed) of exponential softening curve (grey solid) using four linear line segments.

(a)   (b)
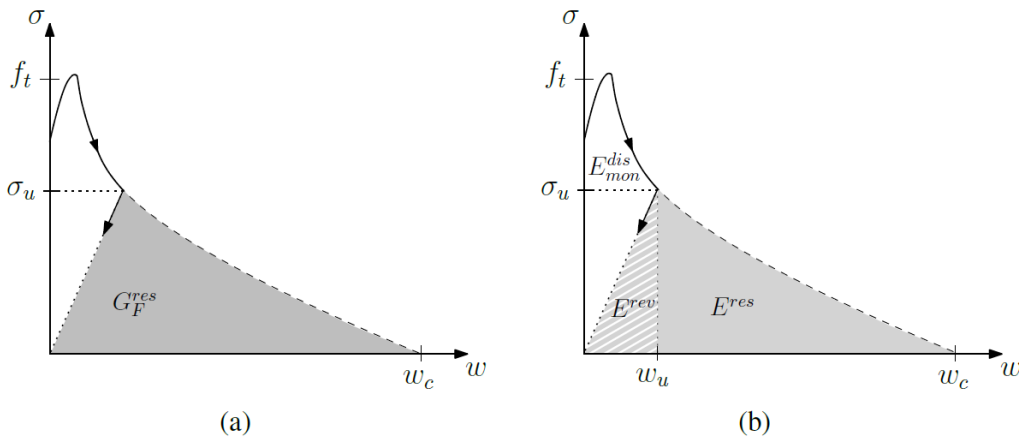
Figure 2. Energy considerations used for description of cyclic behaviour of concrete material in uni-axial tension at the start of fatigue analysis: (a) Residual fracture energy and (b) Dissipated, reversible and residual energy.

The cohesive potential, i.e. the residual fracture energy, G_F^{res}, at the point where a fiber of concrete material enters the fatigue phase is given as, see Figure 2 (a),

$$G_F^{res} = G_F - E_{mon}^{dis} = E^{rev} + E^{res} \qquad\qquad (2)$$

where E_{mon}^{dis} is the dissipated energy from monotonic crack-opening process, E^{rev} is the reversible elastic fracture energy upon the first load cycle and E^{res} is the residual fracture energy upon the first load cycle.

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In incremental unloading-reloading schemes, shown as the grey curve in Figure 3 (a), the gradient at the onset of unloading would decline with decreasing stress level due to the incremental deterioration of the fracture energy. Similarly, the gradient at the onset of reloading, would be equal to the final unloading stiffness, and then decline with increasing stress level. In the model presented here, a more simplified format is selected, assuming a constant unloading and reloading stiffness, i.e. unloading towards origin and reloading towards a fixed point on the monotonic curve, respectively. The fixed point on the monotonic curve is controlled by the residual fracture energy, G_F^{res}, after a cyclic loading process. The residual fracture energy is updated at the onset of reloading. This is exemplified for the reloading process A-B in Figure 3 (a), comparing the simple constant stiffness scheme proposed (black line) and incremental stiffness scheme (gray line) where the deterioration is exaggerated.
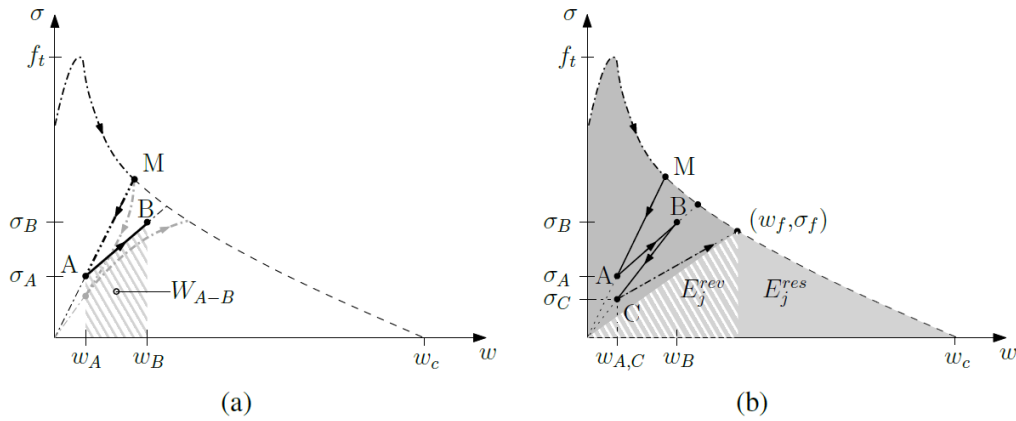
Figure 3. a) Proposed simplified constant stiffness scheme (black line) compared to a more realistic incremental stiffness scheme (gray line) showing the work of the cyclic reloading process A-B (shaded gray). (b) Sketch of arbitrary load case M-A-B-C: Unloading with constant damage (towards origin) and reloading with damage of loading process below the monotonic curve (towards (w_f, σ_f)).

In order to determine the cyclic damage an arbitrary load case is considered, see Figure 3 (b). After unloading from point M on the monotonic curve, the fiber enters cyclic loading. The residual fracture energy for a point j is given by the residual fracture energy at the onset of cyclic loading, here denoted point i, and the accumulated work. A simple linear relationship is proposed

$$G_{F,j}^{res} = G_{F,i}^{res} - k_{fat} \sum W \qquad (3)$$

where k_{fat} is the fatigue damage parameter. The fatigue damage parameter is a key calibration parameter of the model and is used to scale the fracture energy dissipated during fatigue loading as a fraction of accumulated work. $\sum W$ is the accumulated work, i.e. for the loading process M-A-B, shown in Figure 3 (a),

$$\sum W = W_{M-A} + W_{A-B} \tag{4}$$

The work for a load increment, e.g. W_{A-B} is found from trapezoidal calculation.

Considering a fiber in point C, the work from the loading process M-A-B-C is added to the accumulated work upon reloading from C. Reloading from point C takes place towards the fixed point on the monotonic failure envelope (w_f, σ_f) found from a simple bisection interpolation scheme, i.e. solving the equation

$$f(w_f) = G_{F,j}^{res} - \left(E_j^{rev}(w_f) + E_j^{res}(w_f) \right) = 0 \tag{5}$$

The stiffness during cyclic loading depends on whether the cohesive surface opens or closes, i.e.

$$\dot{\sigma} = \begin{cases} K_s^- \dot{w} & \dot{w} < 0 \\ K_s^+ \dot{w} & \dot{w} > 0 \end{cases} \text{, where } \begin{array}{l} K_s^- = \frac{\sigma_u - \sigma_u^k}{w_u} \\ K_s^+ = \frac{\sigma_f - \sigma_r}{w_f - w_r} \end{array} \tag{6}$$

where K_s^- and K_s^+ are the unloading and reloading stiffnesses, respectively. σ_u and σ_r are the stresses at the onset of unloading and reloading, respectively, and w_u and w_r are the crack-openings at the onset of unloading and reloading, respectively. σ_u^k is a fixed negative intersecting point on the ordinate for more precise prediction of the concrete cyclic response. For secant unloading σ_u^k is zero. The stress during unloading and reloading is then given as

$$\sigma = \begin{cases} K_s^- w & \dot{w} < 0 \\ K_s^+(w - w_r) & \dot{w} > 0 \end{cases} \tag{7}$$

The fatigue crack growth process in concrete may be divided into a crack initiation phase and a crack development phase. In order to test the proposed methodology, the latter phase is considered, avoiding influence of pre-peak fatigue damage on the results. This means that the maximum stress σ_{max} at some point has reached the tensile strength f_t, where a crack in the concrete material has initiated.

The proposed methodology is tested and compared to fatigue tests of cracked plain concrete cylinders in uni-axial tension reported in Plizzari et al. [13]. Cylinders with a length of 210 mm, diameter of 80 mm and a notch depth of 4 mm were considered. The crack mouth opening displacement was measured over a length of 35 mm. In these tests only the cracked phase was considered, see example in Figure 4.
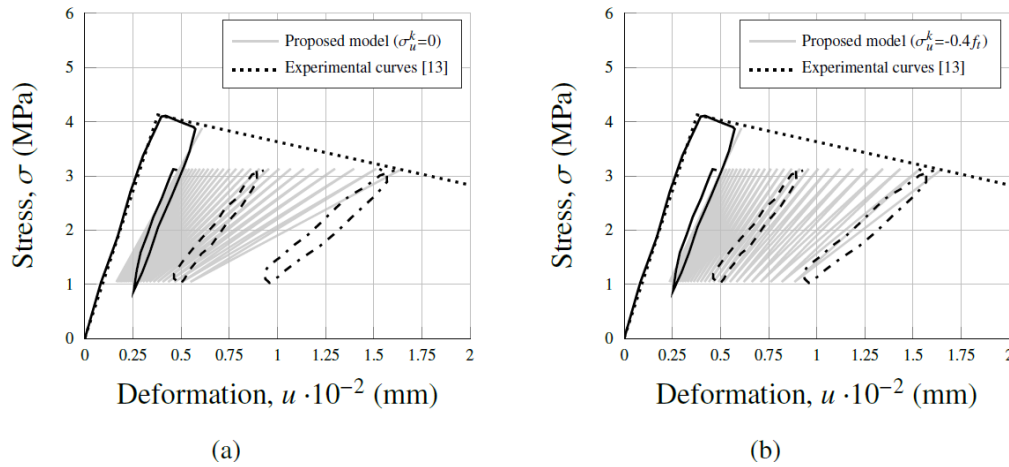
Figure 4. The proposed cyclic cohesive model (gray) compared to fatigue cracking hysteresis loops at the start (black solid), intermediate (black dashed) and end of fatigue analysis (black dashed dotted) reported in [13]: (a) unloading towards or origin (σ_u^k=0) and (b) unloading towards σ_u^k=-0.4f_t for load cycles N: [1-280-480...5080]. Specimen geometry (L/d): 210 x 80 x 50 mm^3 notch depth, a_0=4 mm and s=35 mm. Mechanical properties f_c=42.1 MPa, f_t=4.25 MPa, G_F=151 N/m, w_c=0.341 mm (bilinear softening).

From Figure 4 (a) it is observed that the proposed secant unloading scheme does not comply perfectly with experimental curves due to the secant unloading scheme selected. However, it was proposed in [2] to define a fixed negative intersecting point σ_u^k of -0.4f_t on the ordinate, for which results are shown in Figure 4 (b). In this model the parameter σ_u^k is fitted and the response simplified, i.e. this model does not take into account the stiffening of concrete in the tension-compression transition. With these modifications the model format proposed is suitable for more precise prediction of the concrete cyclic response.

In the three experiments evaluated, the fatigue damage parameter k_{fat} is fitted to 3.4·10^{-3}, 2.2·10^{-3} and 0.3·10^{-3} for failure after 344, 645 and 5081 load cycles, respectively. The fitted values are based on the assumption that a specimen fails in fatigue when the reloading curve re-joins the monotonic envelope after undergoing stress controlled fatigue loading. The scatter in these values is mainly related to the variation of concrete tensile strength measured to app. 2.4, 3.7 and 4.3 MPa in the three experiments, respectively.

The study in [13] is one of very few studies in the literature which report data suitable for evaluation of cyclic concrete material behaviour at constitutive level. Thus, the experimental basis for evaluation of the fatigue damage parameter k_{fat} is limited at present. In order to further evaluate the proposed method, or fatigue crack growth phenomena in general, more extensive fracture testing is required.

3. Hinge level

For analysis of bending in beam structures, the proposed uni-axial cyclic cohesive model in Section 2 is incorporated into a hinge model. The basic assumption of the hinge model is the fact that the presence of a crack influences the overall stress and strain field of a structure only locally. The discontinuity created by the crack is expected to vanish outside a certain width. The width s between two such sections embracing one crack defines a hinge.

In the present study the hinge formulation proposed by Skar et al. [2] is applied. This hinge consists of fibers of concrete material, shown in Figure 5.
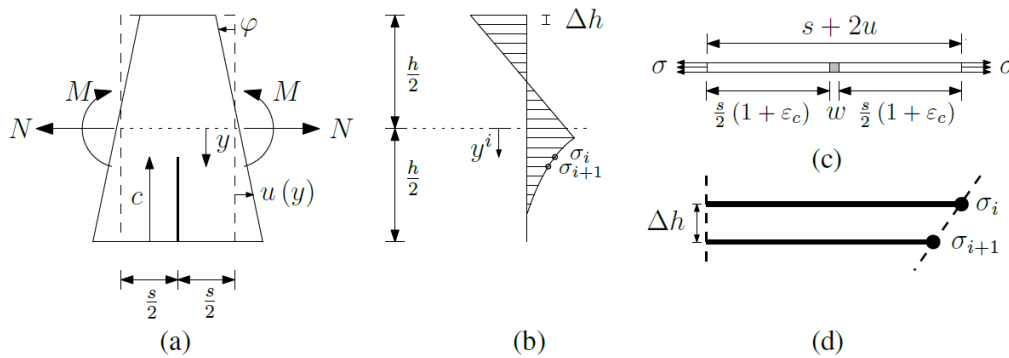

Figure 5. Fiber hinge model: (a) Beam segment with constant sectional forces and deformation of cracked beam segment. (b) Hinge stress distribution after initiation of cracking showing the individual fibers (*n*=24, whereof 4 stress free). (c) Material fiber in uni-axial tension: loaded state beyond peak-load showing crack deformations. (d) Geometrical definition of one hinge strip (interpolation of stresses between two fibers). From Skar et al. [2].

The tensile behaviour of the hinge may be established by considering a fiber of material in uni-axial tension as shown in Figure 5 (c). The elongation of the fiber located at y can be expressed in terms of the mean normal strain, i.e. $\bar{\varepsilon}(y) = \bar{\varepsilon}_0 + \bar{\kappa}y$. Where $\bar{\varepsilon}_0$ is the mean normal strain at the beam axis, and $\bar{\kappa}$ the mean curvature of the hinge. In the cracked state, the crack opening and the corresponding stress in the fiber are given as

$$w_i = s \frac{E_c\bar{\varepsilon}(y)-b_i}{E_c+a_is} \quad , \quad \sigma_i = E_c \frac{b_i+a_is\bar{\varepsilon}(y)}{E_c+a_is} \quad \quad 0 \le w_i \le w_c$$

$$w_i = s\bar{\varepsilon}(y) \quad , \quad \sigma_i = 0 \quad \quad w_c \le w_i$$

(8)

The hinge is divided into, $n + 1$, number of fibers with the strip height Δh between fibers, shown in Figure 5 (b). The sectional forces with respect to $y = 0$ are then a sum of the contributions from all, n, strips and may be calculated from

$$N(\bar{\varepsilon}_0,\bar{\kappa}) = t \int_{-h/2}^{h/2} \sigma_c \, dy = \sum_{i=1}^n N_i$$

$$M(\bar{\varepsilon}_0,\bar{\kappa}) = t \int_{-h/2}^{h/2} \sigma_c y \, dy = \sum_{i=1}^n M_i$$

(9)

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where N_i and M_i are the normal force contribution and moment contribution from each strip, respectively. It was found in [2] that sufficient accuracy of the hinge response can be obtained with app. 10-30 fibers.

The functionality of the proposed cyclic hinge model is tested on a single hinge subjected to 25 cycles (N=25) with hinge mean curvature between 1 and 4 mm^{-1} as shown in Figure 6.

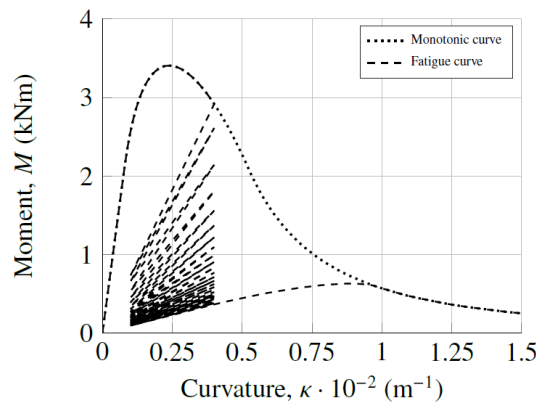**Figure 6. Cyclic behaviour of hinge under constant curvature control: Fatigue damage parameter k_{fat}=0.25. Hinge dimensions (h/t): 0.20 x 0.10 m². Hinge parameters: n=50 and s=H/2. Material properties: E_c=30 GPa, f_t=3.5 MPa, G_F=150 N/m and w_c=0.1 mm (linear softening).**

The evolving stress-mean strain response for the upper quarter hinge fiber, at the position $y = -h/4$, and the bottom hinge fiber, at the position $y = h/2$, during the cyclic loading process is shown in Figure 7 (a) and (b).
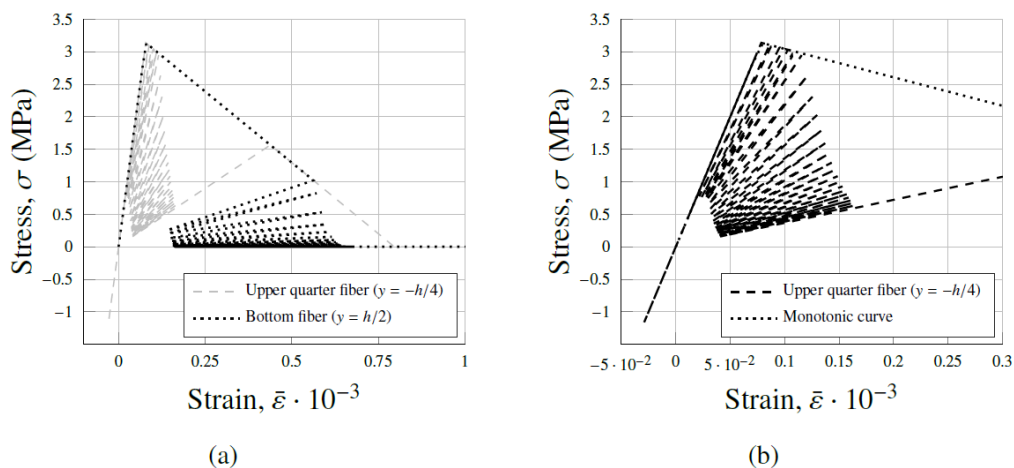

(a)

(b)

Figure 7. Behaviour of hinge fibers during cyclic loading: (a) Bottom fiber (black) and upper quarter fiber (grey). (b) Close-up of the upper hinge fiber at position y=-$h/4$ going through different phases during cyclic loading of hinge.

From Figure 7 (a) it is observed that the bottom fiber enters the fatigue phase after the first monotonic load step. As fibers on the lower part of the hinge deteriorate, new fibers in the upper part of the hinge are activated, see Figure 7 (b). The upper hinge fiber is first in

compression, then in linear elastic tension, before entering a short stage of low-cyclic monotonic damage, i.e. development of the fracture process zone. Finally, the fiber enters the fatigue phase. This behaviour highlights the importance of a consistent format for numerical simulations of concrete material subjected to cyclic loading, accounting for all the different cracked phases in a unified manner.

4. Beam level

Under constant moment, e.g. between the loaded points in Figure 8 (a), the beam sections at the midpoint between cracks will, due to the periodicity of the cracks, remain plane during deformation of the beam. This justifies the application of the hinge model. For the beam parts outside the loaded points, the moment distribution is no longer constant. Such phenomena can be handled with appropriate numerical tools, i.e., the finite element (FE) method, as exemplified for a single beam element in Figure 8 (b).
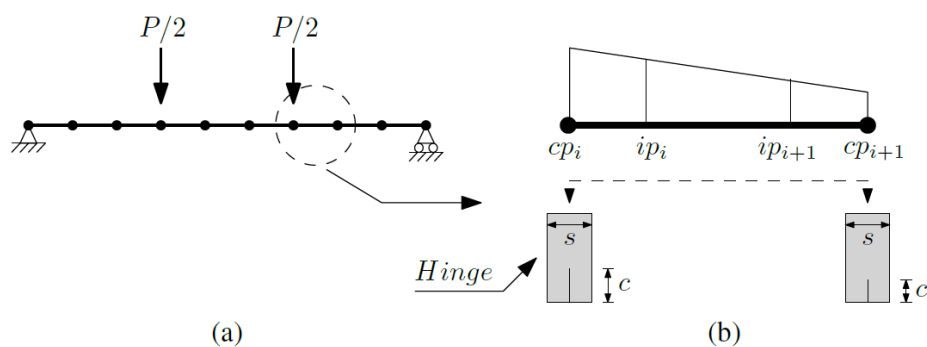

Figure 8. Sketch of hinge model implemented in simply supported beam under four point loading: (a) overview of beam structure, (b) underlying discrete formulation of cracks at constitutive points, cp, and smeared constitutive behaviour obtained from interpolation between constitutive points, ip.

The proposed hinge model is implemented in a user-built finite element code, hereafter referred to as the FEM hinge. Detailed description of the implementation of the model can be found in Skar et al. [10] and Skar [14].

The functionality of the proposed numerical hinge model for simulation of the cyclic fracture behaviour of a three point beam in the fatigue phase is demonstrated by comparing crack length versus number of load cycles with the experimental results reported in Toumi and Bascoul [15], as shown in Figure 9.
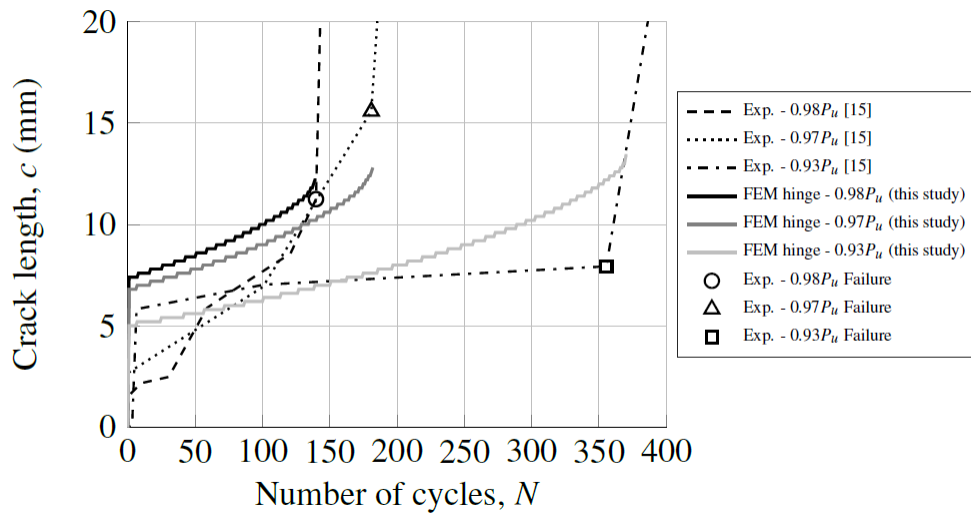
Figure 9. FEM hinge versus experimental and model curves reported in [15]: Crack length versus number of load cycles. Where the crack length c is taken as the progressive depth to the crack tip.

It is observed from Figure 9 that the FEM hinge model is able to capture the fatigue crack growth development during cyclic loading after an initial phase of app. 50 load cycles. Moreover, some characteristic features of the model are shown: (i) the initial crack length increases with increasing load (P_u), (ii) the crack growth rate increases with increasing load (P_u) and the fatigue life increases for decreasing load (P_u). It is also observed that the numerical crack growth curves resemble the experimental curves, and that the combination of all three models is able to give a close prediction of the number of load repetitions to failure.

5. Conclusions

A simple energy based approach for damage evolution during cyclic loading was proposed, requiring only a single model parameter additional to the monotonic parameters. The selected format is general and consistent, and ensures that damage during cyclic loading in fatigue is restricted to the monotonic failure envelope, i.e. the damage state after arbitrary loading is associated with a monotonic loading process that leads to the same damage state. The proposed model shows satisfactory results when compared to experiments with cracked plain concrete cylinders in uni-axial tension. The finite element hinge adequately describes the fatigue crack growth of plain concrete beams under three point loading. Main characteristic features, such as initial crack length and fatigue crack growth rate, can be simulated with the finite element hinge model. The results obtained are encouraging and show that the methodology is well suited for practical use, e.g. inverse analysis of experiments and forward analysis of fatigue crack growth in concrete beam structures.

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