Technical University of Denmark

DTU

Offset Risk Minimization for Open-loop Optimal Control of Oil Reservoirs

Capolei, Andrea; Christiansen, Lasse Hjuler; Jørgensen, John Bagterp

DTU Library
Technical Information Center of Denmark

Offset Risk Minimization for Open-loop Optimal Control of Oil Reservoirs [⋆]

A. Capolei[∗] L.H. Christiansen[∗] J.B. Jørgensen[∗]

[∗] *Department of Applied Mathematics and Computer Science & Center for Energy Resources Engineering, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark (e-mails: {acap, lhch, jbj }@dtu.dk)*

Abstract: Simulation studies of oil field water flooding have demonstrated a significant potential of optimal control technology to improve industrial practices. However, real-life applications are challenged by unknown geological factors that make reservoir models highly uncertain. To minimize the associated financial risks, the oil literature has used ensemble-based methods to manipulate the net present value (NPV) distribution by optimizing sample estimated risk measures. In general, such methods successfully reduce overall risk. However, as this paper demonstrates, ensemble-based control strategies may result in individual profit outcomes that perform worse than real-life dominating strategies. This poses significant financial risks to oil companies whose main concern is to avoid unacceptable low profits. To remedy this, this paper proposes *offset risk mimimization*. Unlike existing methodology, the offset method uses the NPV *offset* distribution to minimize risk *relative* to a competing reference strategy. Open-loop simulations of a 3D two-phase synthetic reservoir demonstrate the potential of offset risk minimization to significantly improve the worst case profit offset relative to real-life best practices. The results suggest that it may be more relevant to consider the NPV offset distribution than the NPV distribution when minimizing risk in production optimization.

Keywords: Optimal control, Model-based control, Production control, Risk, Stochastic modelling.

1. INTRODUCTION

Industrial strategies of oil field water flooding rely on reactive control to shut in producer wells as they become unprofitable. To enhance production, the oil literature has proposed optimal control technology, including nonlinear model predictive control (NMPC). The use of NMPC is referred to as closed-loop reservoir management (CLRM) (Jansen et al., 2009). The goal of CLRM is to determine the optimal operating profile that maximizes a key performance indicator (KPI) over the reservoir life-cycle, e.g, the cumulative oil recovery or a financial measure such as the net present value (NPV). CLRM consists of 1) an optimizer that uses the reservoir model to determine the optimal operating profile by solving a constrained open-loop optimization problem and 2) a state estimator for history matching to update the reservoir model as new data becomes available. This paper focuses on the optimizer, i.e. feedback and state-estimation is not considered. In the oil literature, this open-loop optimal control problem is referred to as life-cycle production optimization. The problem corresponds to computing the a priori optimal operating profile before the oil recovery process has begun and feedback becomes available. While simulation studies have demonstrated a significant potential of production optimization to increase overall profit, real-life applications are challenged by a wide range of uncertainties

tied to reservoir simulation. To address the challenges of uncertainty, the oil literature has considered *ensemble-based* methods. Such methods represent the uncertainty by approximating the continuous NPV distribution by a finite number of possible outcomes, i.e., by an *ensemble of realizations*. To minimize risk, the ensemble members are combined to form a sample estimated risk measure that is optimized over the reservoir life-cycle. Popular ensemble-based methods include robust optimization (RO) (Van Essen et al. (2009)), mean-variance optimization (MVO) (Bailey et al. (2005), Capolei et al. (2015b)) and conditional value-at-risk optimization (CVaRO) (Capolei et al. (2015a), Siraj et al. (2015), Codas et al. (2016)). Such methods have proven to reduce overall risk relative to real-life dominating strategies of reactive control. However, ensemble-based control strategies may still result in individual profit outcomes that perform worse than reactive control. For reservoir asset managers whose primary concern is profit loss, this poses a significant risk of unacceptable low profit realizations. Therefore, despite overall lower risk, oil companies may be inclined to discard ensemble-based methodology. To meet this challenge, this paper proposes *offset risk minimization*. The offset approach seeks to determine the control strategy that minimizes the risk of performing worse than a competing reference strategy. To this end, the method maximizes the worst-case outcome of the *NPV offset distribution*. As opposed to methods of the oil literature, the offset approach mitigates the risk of low profit realizations *relative* to the

competing reference strategy. In this way, the risk of profit loss relative to industrial standards is minimized. Using an ensemble of 100 realizations of a 3D synthetic reservoir, open-loop simulations demonstrate the potential of offset risk minimization to significantly increase the offset worst-case scenario relative to reactive control. Compared to the conventional use of the NPV distribution, the results suggest that the NPV offset distribution may be more relevant for risk mitigation in life-cycle production optimization. The paper is organized as follows. In section 2, life-cycle production optimization under uncertainty is formulated as a risk minimization problem. Section 3 introduces off-set risk minimization. Numerical results are presented in Section 4 and conclusions are made in Section 5.

2. LIFE-CYCLE PRODUCTION OPTIMIZATION UNDER UNCERTAINTY

Oil recovery by water flooding uses injection wells to dynamically inject water into the reservoir to displace hydrocarbons towards a set of production wells. The well injection strategy is referred to as the operating profile, u. The goal of life-cycle production optimization is to determine the operating profile that maximizes profit, ψ, over the reservoir life by solving the optimal control problem (Brouwer and Jansen, 2004; Sarma et al., 2005; Nævdal et al., 2006; Foss and Jensen, 2011; Völcker et al., 2011; Capolei et al., 2013):

$$\max_{u \in \mathcal{U}} \psi(u; \theta). \tag{1}$$

Here $\mathcal{U}$ expresses linear decision constraints and $\theta \subset \mathbb{R}^m$ represents geological, petrophysical and economical model parameters. In this paper, profit is given by the cumulative NPV, i.e.,

$$\psi(u, \theta) = \sum_{k=0}^{N-1} \frac{\Delta t_k}{(1+d)^{\frac{t_{k+1}}{\tau}}} \left[\overbrace{\sum_{j \in \mathcal{P}} r_o \, q_{o,j}\big(u_k, x_{k+1}(u, \theta)\big)}^{\text{value of produced oil}} \right.$$
$$\left. - \overbrace{\sum_{j \in \mathcal{P}} r_{wP} \, q_{w,j}\big(u_k, x_{k+1}(u, \theta)\big)}^{\text{cost of separating produced water}} \right.$$
$$\left. - \overbrace{\sum_{j \in \mathcal{I}} r_{wI} \, q_j\big(u_k, x_{k+1}(u, \theta)\big)}^{\text{cost of injecting water}} \right]. \tag{2}$$

Here r_o, r_{wp} and r_{wi} denote the oil price, the water separation cost, and the water injection cost, respectively; $q_{w,i}$ and $q_{o,i}$ are the volumetric water and oil flow rates at producer i; q_l is the volumetric well injection rate at injector l; d is the discount factor, N is the number of control steps and $\Delta_{t_k} = t_{k+1} - t_k$ denotes the length of the time step. Well flow rates are computed using the Peaceman well model (Peaceman, 1983). For each time-step, t_k, the state-space variables, $x_k = x(t_k)$, denote reservoir pressures and fluid saturations whereas $u_k = u(t_k)$ represents a zero-order-hold parametrization of the well controls. The states x_k are computed by a two-phase immiscible flow model based on mass conservation and Darcy's law for porous media. Relative permeabilities are described by the Corey model. See e.g. Aziz and Settari (1979); Chen et al. (2006); Chen (2007); Völcker et al. (2009).

2.1 Risk mitigation by ensemble-based methods

The inaccessible geographical location of oil fields severely limits the amount of available geological data. Consequently, reservoir model parameters such as permeability, porosity and initial states are often highly uncertainty. The control strategy that solves (1) therefore imposes significant risks of profit loss and becomes unreliable for practical purposes. To reduce the financial risks of model discrepancies with real-life reservoirs, the oil literature has proposed ensemble-based production optimization. Ensemble-based methods represent geological uncertainty by a discrete set of equiprobable model realizations

$$\theta_{n_d} = \{\theta^1, \theta^2, ..., \theta^{n_d}\} = \{\theta^i\}_{i=1}^{n_d}. \tag{3}$$

The ensemble (3) is used to approximate the continuous NPV probability distribution by the related finite set of profit outcomes

$$\psi_{n_d} = \{\psi^i\}_{i=1}^{n_d}, \psi^i = \psi(u; \theta^i), \ 1 \le i \le n_d. \tag{4}$$

To minimize risk, the idea is to manipulate the discrete NPV profit distribution (4) by formulating an appropriate optimal control problem. To this end, it is customary to use a risk measure $\mathcal{R} : \psi_{n_d} \to \mathbb{R}$ to replace the overall profit distribution and quantify risk in terms of the scalar objective, $\mathcal{R}(\psi)$:

$$\min_{u \in \mathcal{U}} \mathcal{R}(\psi(u; \theta_{n_d})). \tag{5}$$

Figure 1 illustrates the key features of ensemble-based production optimization.

2.2 Specific risk measures and ensemble-based methods

Risk measures quantify the stochastic profit, ψ, by a numerical value, $\mathcal{R}(\psi)$, which serves as a surrogate for the overall profit distribution. The quantification of risk allows for fast and efficient decision-making. In particular, risk assessment of two scenarios, ψ' and ψ'', reduces to comparing the values $\mathcal{R}(\psi')$ and $\mathcal{R}(\psi'')$. However, the quality of the risk assessment heavily depends on the properties of the risk measure in question. The following briefly discusses the risk measures and related ensemble-based method used in this paper. Capolei et al. (2015a) provide a detailed overview of risk quantification in production optimization.

Robust optimization (RO) (Van Essen et al., 2009) refers to the ensemble-based method that maximizes the life-cycle sample estimated *expected return*, i.e.,

$$\mathcal{R}_{RO} := -\frac{1}{n_d} \sum_{i=1}^{n_d} \psi^i. \tag{6}$$

As a drawback, the expected profit is a risk neutral measure (Capolei et al., 2015a). As such, RO does not directly account for important risk indicators such as the lowest profit outcome.

Worst-case optimization (WCO) (Alhuthali et al., 2010) focuses solely on maximizing the lowest profit outcome, i.e.,

$$\mathcal{R}_{WCO} := -\min_{\theta^i} \psi(u; \theta^i) = -\tilde{\psi}. \tag{7}$$

Here $\tilde{\psi}$ denotes the lowest profit realization associated with the ensemble, i.e., $\tilde{\psi} \le \psi^i$, $1 \le i \le n_d$. The restriction to a single profit outcome implies that the measure is blind
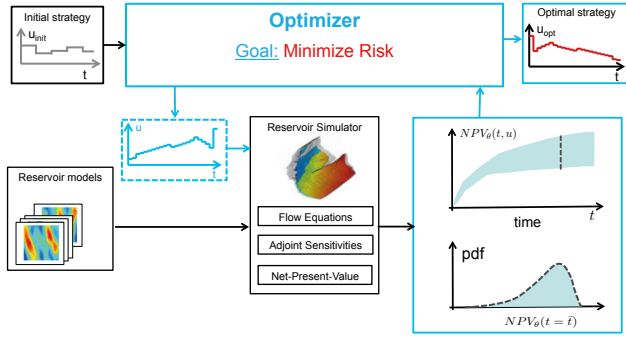
Fig. 1. Outline of ensemble-based production optimiza-
tion, that consists of two key parts: 1) a reservoir
simulator that, given an ensemble of reservoir models
and a control input, computes the profit probability
distribution (black arrows and boxes) and 2) an opti-
mizer that uses the profit distribution to compute the
control strategy that minimizes risk as measured by
$\mathcal{R}$ (blue arrows and boxes).

to features of the NPV probability distribution. Conse-
quently, the risk quantification may be too conservative
and expected return may be compromised.

Conditional value-at-risk optimization (CVaRO) maxi-
mizes the sample estimated average of the $\alpha \cdot 100\%$ lowest
outcomes, $\{\tilde{\psi}^i\}_{i=1}^{n_\alpha}$, i.e.

$$\mathcal{R}_{\text{CVaR},\alpha} := -\frac{1}{n_\alpha} \sum_{i=1}^{n_\alpha} \tilde{\psi}^i, \ \alpha \in (0,1). \quad (8)$$

Here $\tilde{\psi}^i$ denotes the ith lowest profit outcome associated
with the ensemble. By design, CVaRO_α accounts for the
entire α-tail of the profit distribution. When α increases,
CVaRO_α includes more of the profit distribution. As such,
emphasis is gradually moved towards promoting expected
return. In particular, for $n_\alpha := n_d$, CVaRO_α reduces to
RO (6). On the other hand, when α decreases, CVaRO_α
includes only low profit realizations and in the extreme
case of $n_\alpha := 1$, CVaRO_α reduces to WCO (7).
As a drawback, $\text{CVaR}_\alpha\big(\psi(u,\theta)\big)$ is non-differentiable with
respect to the controls, u, for any $\alpha \neq 1$ (Christiansen
et al., 2016). The non-differentiability may interfere with
the optimization procedure (5). This potentially leads to
suboptimal solutions. As a way to overcome this issue,
Rockafellar and Uryasev (2002) and Rockafellar and Roy-
set (2010) show that the minimization problem (5) is
equivalent to the following smooth problem

$$\min_{c \in \mathbb{R}, u \in \mathcal{U}, y \in \mathbb{R}^{n_d}} -c + \frac{1}{\alpha \cdot n_d} \sum_{i=1}^{n_d} y_i, \quad (9a)$$

$$s.t. \qquad y_i \geq c - \psi(u, \theta^i), \quad i = 1, \ldots, n_d, \quad (9b)$$

$$y_i \geq 0, \qquad i = 1, \ldots, n_d, \quad (9c)$$

This paper uses the formulation (9) to minimize CVaR_α,
whenever $n_\alpha \neq 1$.

3. OFFSET RISK MITIGATION

Simulations studies have demonstrated the potential of
ensemble-based methodology to reduce overall risk of
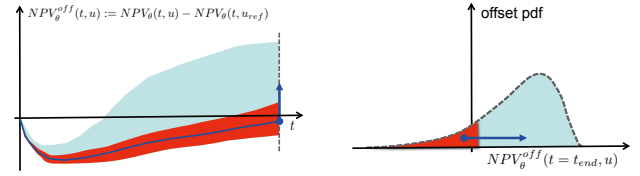profit loss relative to real-life dominating practices such

Fig. 2. NPV offset risk mitigation. NPV offset uncertainty
band versus time (left). NPV offset probability dis-
tribution function over the reservoir lifetime (right).
Risk is reduced by maximizing the lifecycle average
value (blue circles) of the $\alpha\%$ lowest offset profits (red
areas).

as reactive control. However, the conventional ensemble-
based methods and the associated risk measures take
no precautions to avoid profit loss *relative* to a com-
peting strategy. Consequently, despite overall lower risk,
ensemble-based control strategies may lead to individual
profit outcomes that perform significantly worse than a
given competing industrial control strategy, u_{ref}. Such
unacceptable low profit outcomes impose risks of profit
loss. Overall, the risk may be small compared to the
gains that stand to be made. However, to oil companies,
risk of profit loss outweighs potential profit gains. Conse-
quently, ensemble-based methods may be considered too
risky relative to conventional reactive control. To meet this
challenge, this paper proposes *offset risk minimization* as
a mean to reduce risk of profit loss *relative* to industrial
standards.

3.1 The profit offset distribution

Unlike ensemble-based methods that rely on the NPV
profit distribution, the offset approach uses the profit *offset*
distribution $\psi_{off_{n_d}} = \{\psi_{off}^i\}_{i=1}^{n_d}$, where:

$$\psi_{off}^i(u; \theta^i) = \psi(u; \theta^i) - \psi(u_{ref}; \theta^i), \ 1 \leq i \leq n_d. \quad (10)$$

Here u_{ref} denotes a competing reference strategy. For
a given control strategy, u, the profit offset distribution
provides a complete picture of the risk profile relative
to the industrial reference case. The offset distribution
therefore provides management with a tool for assessing
new methodology relative to existing practices. In this
regard, two distributions are of special interest: the tail
profit offset distribution,

$$\{\psi_{off}(u; \theta^i) | \psi_{off} < 0\}, \quad (11)$$

and the upper tail profit offset distribution,

$$\{\psi_{off}(u; \theta^i) | \psi_{off} \geq 0\}. \quad (12)$$

These distributions represent, respectively, the distribu-
tion of the profit loss and the profit gain with respect to
the reference profit.

3.2 Offset risk minimization

Offset risk minimization seeks to determine the operating
profile, u, that minimizes the risk of performing worse
than a competing reference strategy. As opposed to the
conventional approach of minimizing risk of the profit
distribution (5), the offset approach minimizes risk of the
profit *offset* distribution

$$\min_{u \in \mathcal{U}} \mathcal{R}(\psi_{off}(u; \theta_{n_d})). \quad (13)$$

In this way, the risk of profit loss relative to industrial standards is minimized. Fig. 2 illustrates the idea of offset risk minimization.

3.3 Worst case offset risk minimization

In oil reservoir management, new methodology is typically not judged based on the chance of increased expected return, but rather on the risk of performing worse than existing practices. In particular, reservoir asset managers primarily focus on risks of low profit realizations. Therefore, this paper uses the offset approach with the worst-case risk measure (7) to maximize the worst profit outcome of the offset distribution:

$$\max_{u \in \mathcal{U}} \inf_{\theta} \left(\psi_{off}(u, \theta) \right) = \max_{u \in \mathcal{U}} \min_{i=1,\ldots,n_d} \left(\psi_{off}(u, \theta^i) \right). \tag{14}$$

The optimization problem (14) is non-smooth. However, for $n_\alpha = 1$, the numerical solution of (14) is equivalent to the solution of the smooth constrained optimization problem

$$\min_{u \in \mathcal{U}} \left[-\inf_{i=1,\ldots,n_d} \left(\psi_{off}(u, \theta^i) \right) \right] = \min_{u \in \mathcal{U}} \left[\text{CVaR}_\alpha \left(\psi_{off}(u, \theta) \right) \right], \tag{15}$$

Consequently, the maximization of the worst case offset profit can be regarded as an offset profit CVaR minimization problem that can be solved by (9).

4. NUMERICAL RESULTS

The following case study demonstrates the potential of offset risk minimization to reduce the risks of low profit realizations relative to academic and industrial best practices. Firstly, the offset approach is used to maximize the worst-case offset profit relative to reactive control over the reservoir life-cycle. Secondly, the offset approach is compared to RO, WCO and CVaRO to illustrate the main benefits of offset risk mitigation relative to conventional ensemble-based methods.

4.1 Reservoir model description

The numerical simulations use the standard version of the Egg model (Jansen et al., 2014). This model has been used in a number of publications as a benchmark to test optimal control methodologies (Van Essen et al., 2009). The Egg model is a synthetic reservoir model consisting of $60 \times 60 \times 7 = 25.200$ grid cells of which 18.553 cells are active. The reservoir is produced for 3.600 days under water flooding conditions. It contains eight water injectors and four producers, which are completed in all seven layers. The bhps of the producer wells are kept fixed at 395 bar and the water injection rates are subject to control with a sample time of 90 days. The water injection rates are bound to be in the interval $[0, 79.5]\,\text{m}^3/\text{day}$. Fig. 3 shows the well setup. Model uncertainty is represented by an ensemble of 100 permeability realizations. Table 1 provides petrophysical and economical simulation parameters. Reservoir fluid flow is simulated using a two phase (oil and water) immiscible flow model with zero capillary pressure and incompressible fluids and rocks.
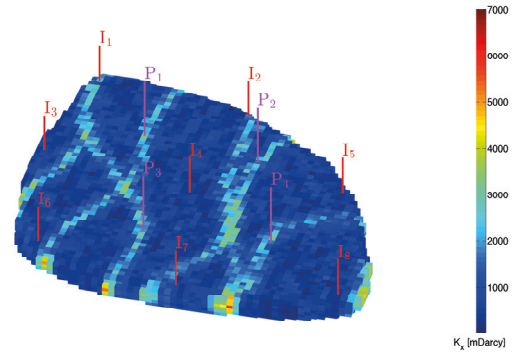

Fig. 3. Permeability field of the ensemble and well setup for the case study.

Table 1. Petro-physical and economical model parameters

	Description	Value	Unit
h	Grid-block height	4	m
$\Delta x, \Delta y$	Grid-block length/width	8	m
ϕ	Porosity	0.2	-
c_o	Oil compressibility	$1.0 \cdot 10^{-10}$	Pa^{-1}
c_r	Rock compressibility	0	Pa^{-1}
c_w	Water compressibility	$1.0 \cdot 10^{-10}$	Pa^{-1}
μ_o	Oil dynamic viscosity	$5 \cdot 10^{-3}$	Pa $\cdot$ s
μ_w	Water dynamic viscosity	$1.0 \cdot 10^{-3}$	Pa $\cdot$ s
k_{ro}^0	End-point relative permeability, oil	0.8	-
k_{rw}^0	End-point relative permeability, water	0.75	-
n_o	Corey exponent, oil	4.0	-
n_w	Corey exponent, water	3.0	-
S_{or}	Residual oil saturation	0.1	-
S_{ow}	Connate water saturation	0.2	-
p_c	Capillary pressure	0	Pa
P_{init}	Initial reservoir pressure (top layer)	$40 \cdot 10^6$	Pa
$S_{w,0}$	Initial water saturation	0.1	-
p_{bhp}	Production well bottom hole pressures	$39.5 \cdot 10^6$	Pa
$q_{wi,min}$	Minimum water injection rate for well	0	m^3/day
$q_{wi,max}$	Maximum water injection rate for well	79.5	m^3/day
r_{well}	Well-bore radius	0.1	m
T	Simulation time	3600	day
N	Number of control steps	40	-
r_o	Oil price	126	USD/m^3
r_{wP}	Water separation cost	19	USD/m^3
r_{wI}	Water injection cost	6	USD/m^3
d	Discount factor	0	-

4.2 Numerical optimization method

The optimization problem (13) is solved using a gradient based optimization algorithm provided by `MATLABs` optimization toolbox (MATLAB, 2014). Given an iterate of the optimizer, $\psi(u, \theta^i)$ is computed by solving the flow equations using MRST (Lie et al., 2012). The gradient, $\nabla_u \psi$ is computed by the adjoint method (Jørgensen, 2007; Völcker et al., 2011; Capolei et al., 2012a,b; Jansen, 2011; Sarma et al., 2005; Suwartadi et al., 2012). An optimal solution is reported if the KKT conditions are satisfied to within a relative and absolute tolerance of 10^{-6}. The current best but non-optimal iterate is returned in cases for which the optimization algorithm uses more than 400 iterations, the relative change in the cost function is less than 10^{-6}, or the relative change in the step size is less than 10^{-10}. These stopping criteria are independent, i.e. when one of the criteria is satisfied, the optimizer stops. Furthermore, the cost function is normalized to improve convergence. The normalization consists of dividing by 10^6 such that the objective function is appropriately scaled.

Table 2. Key performance indicators for the NPV offset distribution.

Control strategy	$\mathrm{E}_\theta(\psi_{off})$ 10^6 USD	$\inf(\psi_{off})$ 10^6 USD	$\beta := \mathrm{Prob}[\psi_{off} < 0]$	$\mathrm{E}_\theta[\psi_{off}|\psi_{off} < 0]$ 10^6 USD	$\mathrm{E}_\theta[\psi_{off}|\psi_{off} \geq 0]$ 10^6 USD
w.c. opt.	1.06	-1.11	9%	-0.41	1.20
c.s. 20%	1.24	-0.92	8%	-0.41	1.39
RO	1.44	-1.48	15%	-0.53	1.79
offset w.c. opt.	0.99	-0.35	8%	-0.22	1.10

4.3 Worst-case offset risk minimization

Fig. 4 compares the profit offset realizations associated with 1) the worst case offset optimization strategy (offset w.c. opt), 2) the worst case optimization strategy (w.c. opt), 3) the CVaR$_{20\%}$ optimization strategy (c.s. 20%), and 4) the robust optimization strategy (RO). All strategies produce realizations that perform worse than reactive control. However, as indicated by the 5th percentile, the worst case offset optimization strategy manages to significantly reduce both the number of negative offset realizations and the the amount of potential profit loss compared to the ensemble-based strategies. In this way, the worst case offset optimization solution represents the strategy that reduces risk of profit loss relative to reactive control to the largest extend. Fig. 5 confirms these observations. In particular, for low risk levels, $\alpha < 0.2$, all control strategies risk to perform worse than reactive control. Nevertheless, the offset worst case optimization strategy offers the lowest risk. As a minor drawback, the low risk of profit loss comes at the price of overall lowest expected return.

Table 2 quantifies the above observations by comparing key performance indicators for the profit offset worst case optimization strategy and the ensemble-based methods. The first column compares expected returns, $\mathrm{E}_\theta(\psi_{off})$ and the second column compares the worst case offset profit outcomes, $\inf(\psi_{off})$. The results confirm that the worst case optimization strategy offers the lowest potential profit loss at the cost of the lowest expected return. The fourth to the sixth column report the probability of negative offset profits, $\beta = \mathrm{Prob}[\psi_{off} < 0]$, the average offset profit of the negative offsets profits, $\mathrm{E}_\theta[\psi_{off}|\psi_{off} < 0]$, and the average offset profit of the positive offset profits, $\mathrm{E}_\theta[\psi_{off}|\psi_{off} \geq 0]$. The results show that the offset worst case optimization strategy has a mere 8% chance of yielding a negative offset profit of -0.22 mio USD, but a 92% chance of yielding positive offset profits with an average value of 1.10 mio USD. This implies that the offset worst case optimization strategy provides 1) the lowest risk of profit loss and 2) at the same time, has a high probability (92%) of outperforming reactive control. The price to be paid is that the offset worst case optimization provides the lowest average positive offset profit. This implies that the offset worst case optimization stands to improve reactive control by the smallest amount on average.

5. CONCLUSION

This paper has introduced and investigated offset risk minimization for life-cycle production optimization under geological uncertainty. Using 100 realizations of a 3D synthetic reservoir, open-loop simulations have demonstrated the potential of offset risk minimization to reduce the risk of low profit outcomes relative to the industrial standards of reactive control. To illustrate benefits over conventional risk mitigation methods, the offset approach was compared to a representative selection of ensemble-based strategies.
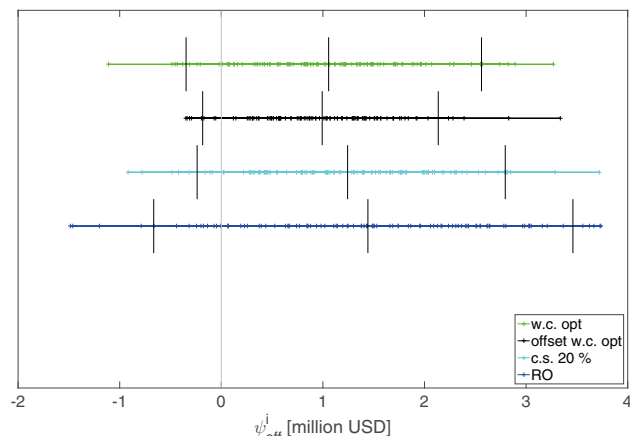

Fig. 4. Strip charts of the NPV offset distributions. The black vertical lines indicate the 5th percentile, the mean, and the 95th percentile of the profit offset distribution.
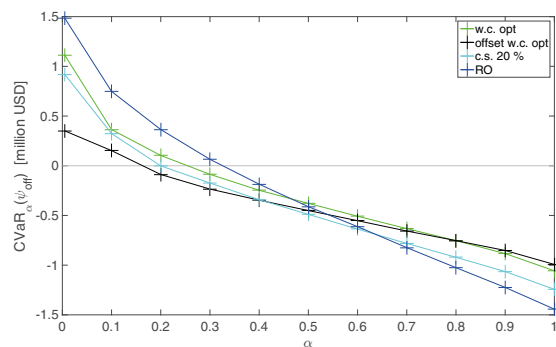

Fig. 5. Plot of $\mathrm{CVaR}_\alpha(\psi_{off})$ as a function of the risk level, α.

Based on the numerical results, the following conclusion can be made:

- Among the strategies considered in this paper, the offset worst case optimization strategy offers the lowest risk of profit loss relative to reactive control.
- Compared to the ensemble-based strategies, the worst case offset optimization strategy manages to significantly reduce both the number of negative offset realizations and the amount of potential profit loss.
- The low risk of profit loss comes at the price of overall lowest expected return.
- Due to oil companies main concern of avoiding unacceptable low profits, the results suggest that it may be more relevant to consider the NPV offset distribution than the NPV distribution when minimizing risk in production optimization.

As a minor drawback, the offset worst case optimization strategy could not ensure zero probability of yielding lower profit realizations than the reactive strategy. This is most

likely because reactive control relies on feedback. Future work seeks to explore the benefits of combining the offset risk minimization procedure with feedback using a receding horizon implementation of combined data assimilation and optimization.

REFERENCES

Alhuthali, A.H., Datta-Gupta, A., Yuen, B., and Fontanilla, J.P. (2010). Optimizing smart well controls under geologic uncertainty. *Journal of Petroleum Science and Engineering*, 73, 107–121.

Aziz, K. and Settari, A. (1979). *Petroleum reservoir simulation*. Applied Science Publishers.

Bailey, W.J., Couët, B., and Wilkinson, D. (2005). Framework for field optimization to maximize asset value. *SPE Reservoir Evaluation & Engineering*, 8(1), 7–21.

Brouwer, D.R. and Jansen, J.D. (2004). Dynamic optimization of waterflooding with smart wells using optimal control theory. *SPE Journal*, 9(4), 391–402. SPE-78278-PA.

Capolei, A., Foss, B., and Jørgensen, J.B. (2015a). Profit and risk measures in oil production optimization. In *Proceedings of the 2nd IFAC Workshop on Automatic Control in Offshore Oil and Gas Production*, volume 48, 214–220. Florianópolis, Brazil.

Capolei, A., Stenby, E.H., and Jørgensen, J.B. (2012a). High order adjoint derivatives using esdirk methods for oil reservoir production optimization. In *ECMOR XIII, 13th European Conference on the Mathematics of Oil Recovery*.

Capolei, A., Suwartadi, E., Foss, B., and Jørgensen, J.B. (2013). Waterflooding optimization in uncertain geological scenarios. *Computational Geosciences*, 17(6), 991–1013.

Capolei, A., Suwartadi, E., Foss, B., and Jørgensen, J.B. (2015b). A mean-variance objective for robust production optimization in uncertain geological scenarios. *Journal of Petroleum Science and Engineering*, 125, 23–37.

Capolei, A., Völcker, C., Frydendall, J., and Jørgensen, J.B. (2012b). Oil reservoir production optimization using single shooting and ESDIRK methods. In *Proceedings of the 2012 IFAC Workshop on Automatic Control in Offshore Oil and Gas Production*, 286–291. Trondheim, Norway.

Chen, Z. (2007). *Reservoir Simulation. Mathematical Techniques in Oil Recovery*. SIAM, Philadelphia, USA.

Chen, Z., Huan, G., and Ma, Y. (2006). *Computational Methods for Multiphase Flows in Porous Media*. SIAM.

Christiansen, L.H., Capolei, A., and Jørgensen, J.B. (2016). Time-explicit methods for joint economical and geological risk mitigation in production optimization. *Journal of Petroleum Science and Engineering*, 146, 158–169.

Codas, A., Hanssen, K.G., Foss, B., Capolei, A., and Jørgensen, J.B. (2016). Multiple shooting applied to robust reservoir control optimization including output constraints on coherent risk measures. *Computational Geosciences*. Submitted.

Foss, B. and Jensen, J.P. (2011). Performance analysis for closed-loop reservoir management. *SPE Journal*, 16(1), 183–190. SPE-138891-PA.

Jansen, J.D. (2011). Adjoint-based optimization of multiphase flow through porous media - A review. *Computers & Fluids*, 46, 40–51.

Jansen, J.D., Douma, S.D., Brouwer, D.R., Van den Hof, P.M.J., Bosgra, O.H., and Heemink, A.W. (2009). Closed-loop reservoir management. In *2009 SPE Reservoir Simulation Symposium*, 856–873. The Woodlands, Texas, USA. SPE 119098-MS.

Jansen, J.D., Fonseca, R.M., Kahrobaei, S., Siraj, M.M., Essen, G.M.V., and Van den Hof, P.M.J. (2014). The egg model - a geological ensemble for reservoir simulation. *Geoscience Data Journal*, 1(2), 192–195.

Jørgensen, J.B. (2007). Adjoint sensitivity results for predictive control, state- and parameter-estimation with nonlinear models. In *Proceedings of the European Control Conference 2007*, 3649–3656. Kos, Greece.

Lie, K.A., Krogstad, S., Ligaarden, I.S., Natvig, J.R., Nilsen, H.M., and Skaflestad, B. (2012). Open source matlab implementation of consistent discretisations on complex grids. *Computational Geosciences*, 16(2), 297–322.

MATLAB (2014). *version 8.3.0.532 (R2014a)*. The MathWorks Inc., Natick, Massachusetts.

Nævdal, G., Brouwer, D.R., and Jansen, J.D. (2006). Waterflooding using closed-loop control. *Computational Geosciences*, 10, 37–60.

Peaceman, D.W. (1983). Interpretation of well-block pressures in numerical reservoir simulation with nonsquare grid blocks and anisotropic permeability. *SPE Journal*, 23, 531–543.

Rockafellar, R.T. and Royset, J.O. (2010). On buffered failure probability in design and optimization of structures. *Reliability Engineering and System Safety*, 95, 499–510.

Rockafellar, R. and Uryasev, S. (2002). Conditional value-at-risk for general loss distributions. *J. Bank. Financ.*, 26(7), 1443–1471.

Sarma, P., Aziz, K., and Durlofsky, L.J. (2005). Implementation of adjoint solution for optimal control of smart wells. In *Proceedings of the SPE Reservoir Simulation Symposium*. The Woodlands, Texas. SPE-92864.

Siraj, M.M., Van den Hof, P.M.J., and Jansen, J.D. (2015). Risk management in oil reservoir water-flooding under economic uncertainty. In *54th IEEE Conference on Decision and Control*, 7542–7547. Osaka, Japan.

Suwartadi, E., Krogstad, S., and Foss, B. (2012). Nonlinear output constraints handling for production optimization of oil reservoirs. *Computational Geosciences*, 16, 499–517.

Van Essen, G.M., Zandvliet, M.J., Van den Hof, P.M.J., Bosgra, O.H., and Jansen, J.D. (2009). Robust waterflooding optimazation of multiple geological scenarios. *SPE Journal*, 14(1), 202–210. SPE-102913-PA.

Völcker, C., Jørgensen, J.B., and Stenby, E.H. (2011). Oil reservoir production optimization using optimal control. In *50th IEEE Conference on Decision and Control and European Control Conference*, 7937–7943. Orlando, Florida.

Völcker, C., Jørgensen, J.B., Thomsen, P.G., and Stenby, E.H. (2009). Simulation of subsurface two-phase flow in an oil reservoir. In *Proceedings of the European Control Conference 2009*, 1221–1226. Budapest, Hungary.